DIRECTOR OF INDUSTRIES
Bengal

A DICTIONARY

OF

APPLIED PHYSICS

MACMILLAN AND CO., LIMITED
LONDON • BOMBAY • CALCUTTA • MADRAS
MELBOURNE

THE MACMILLAN COMPANY
NEW YORK • BOSTON • CHICAGO
DALLAS • SAN FRANCISCO

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

A DICTIONARY

OF

APPLIED PHYSICS

EDITED BY

SIR RICHARD GLAZEBROOK

K.C.B., D.Sc., F.R.S.

DIRECTOR OF INDUSTRIES
Bengal.

IN FIVE VOLUMES

VOL. III

METEOROLOGY, METROLOGY, AND
MEASURING APPARATUS

MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON
1923

COPYRIGHT

PRINTED IN GREAT BRITAIN

LIST OF CONTRIBUTORS

METEOROLOGY AND METROLOGY

ATTWELL, SYDNEY W., M.B.E., M.I.Mech.E. . . . . . . S. W. A.

Surveying Tapes and Wires.

BOYS, CHARLES VERNON, F.R.S. . . . . C. V. B.

Earth, Measurement of Mean Density of.

Quartz Fibres, Preparation of.

Radio-micrometers.

BRODETSKY, SELIG, M.A., Ph.D. . . . . . S. B.

Nomography.

BRUNT, DAVID, CAPTAIN, M.A., B.Sc. . D. B.

Atmosphere, Physics of the.

COBLENTZ, WM. W. (Bureau of Standards, U.S.A.), Ph.D. . W. W. C.

Radiation, Measurement of Solar.

CONSTABLE, E. G. . . . . E. G. C.

Watches and Chronometers.

CORLESS, RICHARD, O.B.E., M.A.

Meteorological Instruments.

DARWIN, Sir HORACE, K.B.E., M.A., F.R.S., and MASON, CECIL C., O.B.E., M.A. . . . . H. D., C. C. M.

Instruments, Design of Scientific.

DE GRAAF HUNTER, JAMES, M.A., Sc.D., F.Inst.P. . DE G. H.

Gravity Survey.

Trigonometrical Heights and Refraction.

DINES, W. H., F.R.S. . W. H. D.

Air, Investigation of the Upper.

Radiation.

GOULD, F. A., M.A. . F. A. G.

Balances.

Barometers and Manometers.

GRIFFITHS, EDGAR A. Meters. Radiant Heat and its Spectrum Distribution.	E. A. G.
HINKS, ARTHUR ROBERT, C.B.E., M.A., F.R.S. Latitude and Longitude. Maps and Map Projection.	A. R. H.
HORSBURGH, ELLICE M., M.A., D.Sc., A.M.Inst.C.E. Calculating Machines.	E. M. H.
JOHNSON, W. H., B.Sc. Comparators. Line Standards of Length.	W. H. J.
JOLLY, H. L. P., M.A. Observations, Combination of.	H. L. P. J.
KNOTT, CARGILL G. (The late), D.Sc., LL.D., F.R.S., F.R.S.E. Earthquakes and Earthquake Waves.	C. G. K.
LEVY, HYMAN, M.A., D.Sc., F.R.S.E. Integration, Mechanical Methods of.	HY. L.
MATTHEWS, DONALD J., F.I.C. Oceanography.	D. J. M.
MITCHELL, A. CRICHTON, D.Sc., and J. J. SHAW Seismometry.	C. M., J. J. S.
OWEN, GEORGE A. Weighing Machines.	G. A. O.
ROLT, FREDERICK HENRY, M.I.E.E., B.Sc., A.C.G.I., A.M.I.Mech.E. Gauges.	F. H. R.
SAMPSON, RALPH ALLEN, D.Sc., F.R.S. Clocks and Time-keeping.	R. A. S.
SEARS, JOHN EDWARD (Jr), C.B.E., M.A., Deputy Warden of the Standards Metrology.	J. E. S.
SELBY, F. J., C.B.E., M.A. Tides and Tide Prediction.	F. J. S.
SHAW, Sir NAPIER, LL.D., D.Sc., F.R.S. Atmosphere, Thermodynamics of the.	
SIMMONDS, C. (The late) (the Government Laboratory). Revised by CONNAH, JAMES, B.Sc., late Deputy Government Chemist Alcoholometry. Saccharometry.	C. S.

SKINNER, SYDNEY, M.A., F.Inst.P.	S. S.
Humidity.	
STOTT, VERNEY, B.A., F.Inst.P.	V. S.
Hydrometers.	
Volume, Measurement of.	
TURNER, ALMA, A.M.I.Mech.E.	A. T.
Draughting Devices.	
TURNER, H. H., D.Sc., F.R.S.	H. H. T.
Micrometers.	
WHIPPLE, F. J. W.	F. J. W. W.
Meteorological Optics.	
WILSON, C. T. R., M.A., F.R.S.	C. T. R. W.

DIRECTOR OF INDUSTRIES
Bengal

DIRECTOR OF INDUSTRIES
Bengal

DICTIONARY OF APPLIED PHYSICS

ABACUS—AIR, THE INVESTIGATION OF THE UPPER

—— A ——

Abacus. See "Calculating Machines," § (1) (i.).

Absorption, Atmospheric:
- Effect of water vapour on. See "Radiation," § (2) (ii.).
- Of Solar Radiation. See *ibid.* § (1) (i.) and § (3) (ii.).

Absorptivity of Dry Air. See "Radiation," § (2) (ii.).

Acme Standard Thread. See "Gauges," § (47).

Adding Machines. See "Calculating Machines," § (7).

Adiabatic or Isentropic Processes in the Atmosphere. See "Atmosphere, Thermodynamics of the," §§ (6), (7), and Sections IV., V., VI., and *Figs.* 15, 16, 17.

Adiabatic Lapse of Temperature. See "Atmosphere, Physics of," §§ (3), (5), (6).

Adiabatic Temperature Gradient of the Sea. See "Oceanography, Physical," § (49).

Air:
- Evidence for descent of, when cooled. See "Atmosphere, Thermodynamics of the," § (24).
- Flow of, over isobars. See *ibid.* § (16).
- Relation of horizontal and vertical flow of. See *ibid.* § (16).
- See also "Convection," "Eviction," etc.

Air, Conductivity of. See "Atmospheric Electricity," § (11).
- Near a thunderstorm. See *ibid.* § (4).

Air, Density of, in grm. per litre, tabulated. See "Balances," Table IV.

Air, Dry:
- Adiabatic lines for. See "Atmosphere, Thermodynamics of the," *Fig.* 15.
- Characteristic equation of. See *ibid.* § (18) and Table V.
- Density of. See *ibid.* § (2).
- Entropy-temperature diagram for. See *ibid.* §§ (19), (22).
- Isentropic equation of. See *ibid.* § (19).
- Physical constants of. See *ibid.* § (2) and Table I.
- Specific heat of. See *ibid.* § (2).
- Thermodynamic properties of. See *ibid.* IV., *Fig.* 16.

AIR, THE INVESTIGATION OF THE UPPER

I. Methods of Investigation—Instruments used—The Interpretation of the Records

§ (1) **Methods.**—About twenty-five years ago the late Laurance Rotch of Blue Hill Observatory, near Boston, U.S.A., started a systematic inquiry into the conditions prevailing in the free atmosphere. For more than a hundred years previously occasional attempts had been made to ascertain the electrical conditions, the temperature, and the strength of the wind, by Franklin, Watson, Glaisher, Archibald and others, but the inception of the present methods is certainly due to Rotch.

The matters to which attention has chiefly been directed are the chemical composition of the atmosphere, its temperature and humidity, the velocity and direction of the air current, and latterly, since this is of great importance for the navigation and safe landing of aeroplanes, the height and density of clouds.

Observations have been obtained in various ways, chiefly by the use of kites, captive balloons, kite balloons (a combination of a kite and a balloon), registering-balloons (or balloon sondes), pilot balloons, and latterly aeroplanes.

Before describing the instruments used to record the temperature and humidity it will be well to notice the means of raising them.

Laurance Rotch commenced his investigation with the ordinary flat kite with a tail, held by the best and strongest string obtainable, but he soon discarded these for the box form of kite, which had been recently invented by Hargreaves in Australia, held by steel wire in place of the cord, and no doubt he owed his subsequent success to the change. He also employed a steam engine for winding it.

For many years kites continued to be the only practical means of raising self-recording instruments into the air, and although they are now being superseded by aeroplanes they are still used for the purpose. The early pioneers in kite flying for scientific purposes were subject to many worries and difficulties, and were liable, most unwillingly, to inflict inconvenience and considerable risk upon their neighbours, risks from which they were free themselves, with the exception of injury from lightning. The chief obstacle to raising a kite beyond two or three thousand feet is the horizontal force produced by wind pressure upon the retaining string or wire, the mere weight of the string is of quite trifling importance in comparison. Hence the necessity to use some material that will have great strength combined with small bulk Steel music wire best meets these conditions and has been universally employed for kite flying. It can be obtained in lengths of six or more miles in one piece, and with a diameter of only one thirty-second of an inch will stand a tensile force of 250 lbs. But it will be readily seen that a mile or two of such wire lying across roads subject to ordinary traffic may do serious harm. Amongst other accidents that have occurred may be mentioned the killing of a horse by an electric current brought down from the conductors of an overhead tramway line. This occurred in Germany. In France the wire lying across a railway became entangled between the eccentrics on the driving axle of a locomotive, and such a quantity of wire became wound round the axle that it rendered the engine useless and the train was held up.

§ (2) KITES.—The kites used in the investigation of the upper air require certain characteristics some of which are mutually antagonistic, so that, as in many other matters, a compromise has to be arranged. The danger of dropping the wire across a road arises in three ways. The wind may become too light to allow the kite to fly; it may become so strong that the kite becomes unstable and dives to the ground; the wire may break near the winch, either from some injury or a kink having been formed in it, or from inability to stand the tensile force put upon it by the kites. To fly in a light wind a kite must be of light weight, and if so, it is not strong enough to stand the stress put upon it by a strong wind, but becomes deformed and hence unstable. For the first kilometre or so of height the strength of the wind is known from the surface wind, but the uppermost of a train of kites may reach 5 or 6 km., and the wind at such heights cannot be inferred with any certainty from the surface wind.

Some form of the box or Hargreave kite (*Fig.* 1) has been used almost universally, but the details have been considerably modified by the staff at each separate aeronautical station, and it is noteworthy that each station has shown a decided tendency to keep to its own particular kite. Neither is the reason far to seek.

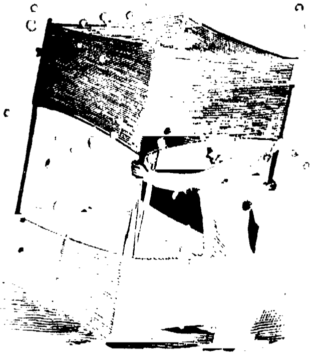


FIG. 1.—Kite used in England.

The peculiarities of a kite, especially the objectionable ones, become well known to those who use it, and they learn by experience how to deal with them, whereas with a strange kite, even though on the whole it might be a better one, the accident of dropping or breaking the wire would quite likely occur before the staff were used to the kite. It is a case of the proverb—Better the devil you do know than the devil you do not know. It is not possible to get a comparison of the goodness of the kites from the results obtained at the different stations, partly because the suitability of the weather to kite flying is widely different in different countries, but chiefly because the average height attained is closely dependent upon the risk of accident that is habitually taken. It is easy to add kites on the line and attain a great height, the difficulty is to get back the kites and wire uninjured. Although the wire may break, the record from the instrument carried is generally recovered intact.

The Hargreave or box kite consists of two separate cells. It is like a box with the two ends gone, and a piece cut of the middle, excepting for the main sticks of the framework, which keep the ends apart. In the Continental and American form the cells are rectangular with the depth about half or one third the width. In the English form the section is approximately a rhomboid with the short diameter equal to either side. In the Cody kite that was used at Aldershot for man-lifting in the years before the war, the cells were rectangular, and an addition of large wings on the sides of the upper cell was made, so that the kite looked much like a large bird.

The illustration shows the form of kite that was used in England. For full information about what can be done with kites the publications giving details of the work at Lindenburg, Germany, should be read. At this station, by means of kites and captive balloons, for a long series of years ascents have been made daily without a break. The loss of material in the form of kites and wire has been large, but excellent series of records have been obtained.

§ (3) Captive Balloons.—Since a kite cannot be raised on calm days, its place has often been taken by a captive balloon. In quite calm weather a captive balloon may reach a very good height, as it will rise almost vertically, but any appreciable wind will exert a horizontal pressure on it in excess of its own lift, thus greatly reducing its height. On the return of the balloon it is usual to withdraw the gas into a gasometer. But the same gas cannot be used indefinitely. Hydrogen diffuses outwards through the envelope, and also it appears that the gases of the atmosphere diffuse inwards, for after a comparatively short time the gas in the gasometer is found to have increased in specific gravity, so that it will not give the necessary lifting power to the balloon, and fresh gas must be supplied.

§ (4) Kite Balloons.—The kite balloon seems to have been first used in Germany. It has not been extensively employed in England to obtain observations of temperature and humidity, but has been developed for military purposes and has given valuable results. It is a combination of a kite and a balloon, so that it has the power of rising in a calm in virtue of the buoyancy of the light gas it contains, and unlike the spherical balloon it will continue flying at a good angle in a strong wind on precisely the same principle as that on which a kite depends. In shape it is somewhat like a sausage, being of stream-line form,[1] so as to offer the minimum of resistance when head to the wind. There are fins behind and the shape is maintained by the air pressure inside. The dynamic lift and the stability in wind depend on the mode of attachment of the bridle as in the ordinary kite. In strong winds the lift due to the gas is insignificant compared to that due to the wind.

§ (5) Registering Balloons.—Registering balloons, or balloon sondes, are small balloons made usually of thin sheet-rubber. At average pressures and temperatures one cubic foot of commercial hydrogen will exert a lift of about one ounce, that being the difference between the weight of a cubic foot of air and a cubic foot of commercial hydrogen. The rubber balloons are securely tied up round the neck at starting, and assuming that the gas inside is at the same temperature as the air outside, and that the pressure inside is not increased to any appreciable extent by the tension of the rubber, the ratio of the density of the hydrogen to that of the air in which the balloon floats remains unchanged with height, and hence the same quantity of hydrogen gives the same lift. The limit to the height reached for balloons of the same kind is therefore set by the diameter at which the balloon bursts. In this connection the following figures are of interest. The diameter of a balloon is doubled by the expansion of the gas inside when it has reached a height of 16·4 km., trebled at 24·2, and quadrupled at 30 km. These figures are for average conditions of temperature and pressure in the south-east of England, and assume that the rubber allows no leak of hydrogen during the ascent and exerts no pressure on the gas inside. Since the weight of a balloon varies as the area of the fabric used, that is as the square of the diameter, and the lift as the cube of the diameter, it is obvious that much greater heights will be obtained by increasing the size of the balloons. In practice, however, the quality of the rubber and the workmanship are of such importance that they almost swallow up for a considerable range the question of size. Like a chain the strength of a balloon is that of its weakest part, so that one defective half inch on one of the many seams is fatal.

It is not absolutely necessary to use rubber, but it has great advantages. It gives a nearly uniform rate of ascent from the ground up to the highest point reached, and this ensures an efficient ventilation of the thermograph. On the other hand, a paper or gutta-percha balloon reaches a height where it is more or less in equilibrium and floats at that height. If this occurs in the day-time the thermograph is more influenced by solar radiation than by the temperature of the air, and temperatures of 50° C. too high may be recorded. Moreover, a considerable time may elapse before the balloon falls, in which it may drift a long distance down the wind, a result that in England is likely to lead to its loss in the Channel or North Sea.

The balloons that have been mostly used in England have weighed from 9 to 13 ounces, the load they have to carry is 3 ounces, and the free lift given has been made, as the result of experience, equal to the weight of the balloon. Thus a total lift of about 25 ounces, secured by 25 cubic feet of hydrogen, is used. This requires a diameter of 43·5 inches, which is about double the unstretched diameter of the balloon. To reach a height of 16·4 km., which was about the average height for England before the war, the rubber before giving way must be stretched to four times its unstretched length in each direction, that is, reduced to $\frac{1}{16}$ of its thickness. It is not surprising that good rubber and good workmanship are required to make a satisfactory balloon. The conditions on the Continent are much the same, excepting that the meteorograph to be carried up is much heavier. This requires a larger balloon and the diameter at starting is about 2 metres.

---

[1] See "Aeronautics," Vol. V.

The meteorograph in England is hung below the balloon by a thread of 40 metres length, so that it may be free from the wake of hot air that will be left by a balloon rising in full sunshine, and it carries a label offering a reward of 5s. to the finder if he will return it uninjured. On the Continent about 90 per cent are returned, in England many are lost in the sea, and only from 60 to 70 per cent are returned.

§ (6) Pilot Balloons.—A knowledge of the velocity and direction of the air currents in the higher strata of the atmosphere in all conditions of weather is necessary for the safe navigation of aeroplanes, but the only simple and reliable method is by means of pilot balloons, and this method unfortunately fails just when it is most wanted, namely, at times when low clouds are prevalent.

Pilot balloons were in use before the war, and they have been employed most extensively during the war, so that a very large mass of information has accumulated and is awaiting some one to sift and classify it. There is a close connection between the direction and strength of the upper winds and the position of the isobars on the corresponding weather chart, but the finer details are still confused and undiscovered.

A pilot balloon is a small india-rubber balloon of about 18″ or 2′ diameter at starting, and weighing about 30 grammes; it is filled with hydrogen until somewhat more than double its natural diameter, and securely tied up. When no leaks develop in the rubber it rises with a uniform, or very nearly uniform, velocity, and is kept in view in the field of one or sometimes two theodolites. It is obvious that with one theodolite the only observation made is the direction of the line of sight; there is no linear dimension to form a scale as a basis for determining the velocity. This is supplied by assuming a uniform ascensional velocity. With two theodolites the base line or distance between the theodolites forms the scale.

(i.) *One-Theodolite Method.*—From the results of many pilot-balloon ascents made with two theodolites and from many direct experiments in closed buildings such as the Albert Hall, where the time of ascent from the floor to the ceiling can be measured by a stopwatch, a formula has been deduced giving the ascensional velocity in terms of the weight and the free lift. There are some curious anomalies in the motion of a sphere through a fluid; near to certain critical sizes and speeds an increase of the load may even lead to an increase in the ascensional velocity, and hence the formula is only valid for balloons of about the usual size.

The formula is

$$V = qL^{\frac{1}{2}}/(W+L)^{\frac{1}{3}},$$

where q is a constant equal to 84 for V in metres per minute or 275 in feet per minute,
L is the net free lift in grammes,
W is the dead weight of the balloon and its attachments in grammes.

The free lift and the weight are easily measured and then the ascensional velocity is known, multiplying by the time from the start the height is known, and then from the observation of the angular altitude and azimuth made by means of the theodolite the precise position is known, and thus the mean velocity and mean direction of motion between any two observations is readily calculated. The observations are in general made at one-minute intervals.

The assumption made that the ascensional velocity is uniform may be justified for still air or for air with no vertical motion, but it is not true for bright sunny days, since on such days convection currents are generally present. It has been found that inland the balloon rises more quickly in the first kilometre of its rise than in the subsequent kilometres, and the following explanation has been given. Every rising current of air must be compensated by an equal descending current elsewhere, and the flow of air near the earth's surface will tend to be from the foot of the descending current towards the ascending current. A balloon therefore when near the earth's surface will tend to be carried by the winds to places where there is an ascending current, and this tendency will lead to a systematic increase of the ascensional velocity. This explanation is confirmed by the fact that over the sea the increased rate of rising in the first kilometre is not found.

(ii.) *Two Theodolites.*—Using two theodolites, the exact course of the balloon can theoretically be determined; there is indeed one superfluous quantity for solving the equations giving the three co-ordinates of position. For if the azimuths of the balloon from the two ends of the base line are determined the foot of the vertical through the balloon is found by solving a triangle with one side and all the angles known, and then either observation of angular altitude gives the height of the balloon. A similar result follows, though not so simply if two altitudes and one azimuth are observed.

So long as the distance of the balloon is not too large compared with the length of the base line, and especially when the track of the balloon is approximately at right angles to it, quite exact determinations can be made, but this latter condition cannot always be secured in practice. Captain Cave, the pioneer in work with pilot balloons in England, states in his book (*The Structure of the Atmosphere in Clear Weather*, Cambridge University Press) that for considerable heights he considers the one-theodolite method the more accurate. No doubt in the first kilometre or two convection currents do cause errors, but these errors are casual and have a very trifling effect upon the means. Also the one-theodolite method requires less time to work up the observations, a point that may in some circumstances be of considerable importance.

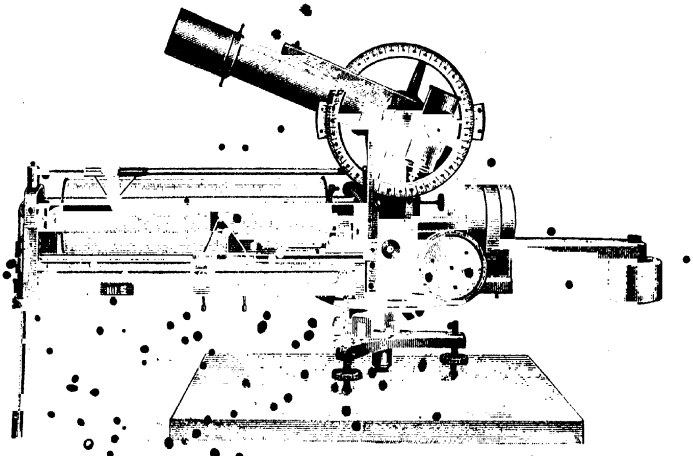


Fig. 2.—Self-recording Theodolite.

(iii.) *The Tail Method.*—The plan has been adopted in some cases of hanging a visible object by a thread of known length and observing the angle subtended by the thread with a micrometer eyepiece in the theodolite. The plan is useful, though it obviously fails when the balloon is near the zenith, but the difficulty is that the hanging object swings about like a pendulum, and hence the observed subtended angle is systematically too small and also difficult to measure owing to its rapid variation.

(iv.) *Self-recording Theodolite.*—To reduce the number of observers and to obtain continuous records instead of observations at one minute intervals a self-recording instrument was designed by Mr. J. S. Dines. It is self-recording in the sense that the balloon is kept at the centre of the cross wires by the observer, while the azimuth and altitude angles are recorded by two pens using differently coloured inks on a long clock drum. The instrument is shown in the illustration (*Fig.* 2), and is quite simple in principle. The pen carriages run on guides parallel to the axis of the drum, which is horizontal, and they are moved by light chains which are coiled in a groove attached to the divided circles of the theodolite. The chains are guided by jockey pulleys where required and are kept taut by a hanging weight at the far end. Thus a change of azimuth, say, uncoils a certain amount of one chain and the free carriage moves an equal distance parallel to the axis of the clock drum. The azimuth and altitude at any given time can be read to about 10 minutes of arc.

§ (7) Aeroplanes.—The use of aeroplanes for obtaining observations up to the height they can reach is self-evident, and inasmuch as they are likely to be used in increasing numbers, so that observations may be a sort of by-product, it is probable that they will more or less supersede kites and captive balloons. Self-recording instruments may be carried by an aeroplane, or direct observation of the temperature shown by a thermometer can be made by the observer. The essential point is that the thermometer, whether self-recording or otherwise, shall be so carried that it may be uninfluenced by radiation or the hot gases from the engine.

In the Annual Supplement of the *Geophysical Journal* for 1918, issued by the Meteorological Office monthly, mean temperatures of the upper air over Martlesham Heath and South Farnborough are given. The values have been obtained by aeroplanes, and the number of ascents combined on which they are based at 4·5 km. exceeds two hundred.

§ (8) Meteorographs.—The instruments used for determining the temperature and humidity of the upper air are of various kinds, but may be divided into two classes, those which use a clock and write on a moving surface, and those which simply draw a graph showing the temperature and the relative humidity in terms of the extension of an aneroid box under the decreasing pressure. The instruments used with kites also mostly give a record of the wind velocity.

The difficulties met with in designing a suitable instrument, more particularly for use with registering balloons, have been considerable. It must be light, though an extra half-pound in the case of kites is not of serious import; it must be capable of bearing rough usage and of recording at very low temperatures.

The instruments used on the Continent and in America, both for kites and for registering balloons, are of standard type, but made as light as possible. The difficulty of finding an ink that will flow at very low temperatures is met by using a needle point writing on paper or metal foil lightly covered by soot from a lamp. The record is set as soon as it is obtained by immersion in a thin shellac varnish. The clocks are especially made to withstand the action of extreme cold on the springs. The three pens, pressure temperature and humidity, write on the same clock drum, but inasmuch as low temperature and low pressure must inevitably occur together, considerable overlapping of these two scales is allowed, so as to save length in the clock drum. Some five inches in length is given to each scale, and this has to cover a range of 1000 mb. pressure and at least 100° C. of temperature. It follows that the temperature can be obtained from the chart without difficulty to within one degree, but on the pressure scale 4 mb. go to one-fiftieth of an inch, which is about the limit to which the chart can be read at all accurately. With the heights that can be reached with a kite or aeroplane the range of pressure and temperature is much less, so that a more open scale can be obtained.

In England different types of instruments are in use both for kites and balloons. In the kite meteorograph the pens write upon a circular disc of stiff paper turning on a pin through its centre, instead of upon a sheet of paper wrapped round a drum. The disc is driven by a small cheap clock, such as used to be obtainable for a few shillings, the small milled wheel provided to set the hands being used to produce a friction drive. The pressure and humidity pens write on one side and the temperature and wind-force pens write on the other side of the centre. The whole instrument occupies a shallow flat box about 3 × 12 × 14 inches, which can be conveniently secured in the centre of the kite.

The balloon meteorograph (Fig. 3) is of altogether different construction and has no clock. It gives a graph scratched by a sharp metal point on a silver-plated surface, from which under a low-power microscope the temperature can be obtained. A single aneroid box is used of about 2¾ in. in diameter, made of thin German silver; the box is not exhausted of air, but is sealed up with the opposite faces held close together. Since the metal is thin the extension of the box depends largely on the elasticity of the enclosed air rather than on the elasticity of the metal, and thus an open scale free from much hysteresis is obtained with the additional advantage of a light box. The box is placed between two levers connected at one end by a spring, and as the box opens under the decreasing atmospheric pressure the other ends of the levers separate. One lever carries a small metal plate the size of a postage stamp, on which the record is made, the other carries the arrangement for the thermograph and hygrograph.

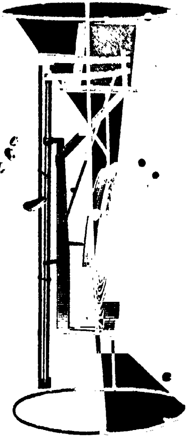


FIG. 3.
Balloon Meteorograph.

The thermograph depends on the expansion of a thin strip of German silver some four inches long compared with that of an invar rod. The expansion, or rather contraction, of the German silver strip is multiplied up some ten times by a light lever of thin steel, the far end of which is sharpened and turned down to form the scratching point. As the aneroid box opens the point is drawn over the small plate scratching, if the temperature is constant, an arc of a circle. A variation of the temperature causes motion of the point at right angles to the arc. No pivots are used in the construction of the instrument, their place being taken by springs. The object of the plating is to secure a surface quite free from scratches and not readily corrodible, the latter requirement being necessary because the instrument may have to lie out exposed to the weather for a considerable time before it is found. Traces have been deciphered after a year's exposure.

§ (9) INTERPRETATION OF THE RECORDS.—The records of the recording instruments and of the observed angles of the theodolites in the case of pilot balloons have to be expressed finally gives values of the temperature, humidity, wind velocity and wind direction that prevailed at the various levels at the instant the recording instrument or the pilot balloon passed through them. The recording instruments give a double record, one for the ascent and one for the descent, but since as a rule the differences are small, it suffices to publish the mean of the two. The records of the wind are obtained directly in terms of the height, but the records of temperature and humidity are obtained primarily in terms of pressure, or to be more precise, in terms of aneroid box extension, which unfortunately is not strictly proportional to change of pressure, and owing to hysteresis lags somewhat behind the change of pressure that causes it.

The following is briefly the method of working up the record of a balloon meteorograph used in England:

The instrument is calibrated a short time before each ascent is made, and the calibration marks are made on the same metal plate which is to be used for the ascent. These marks consist of three lines of equal temperature at about 10° C., -20° C., and -50° C., with cross lines on each temperature mark at 900, 800, 700—to 100 millibars of pressure. This provides 27 standard points of reference through or between which the actual trace will be made. On recovering the instrument after an ascent the first process is to transfer these standard marks and the actual trace to squared paper. This is done under a low-power microscope with a micrometer eyepiece and a moving stage. The co-ordinates are temperature obtained from the micrometer and aneroid box extension obtained from the motion of the stage, which by means of a micrometer screw can be read to ·01 mm. Thereby interpolation the temperature corresponding to any pressure can be read off from the diagram that has been drawn on the squared paper.

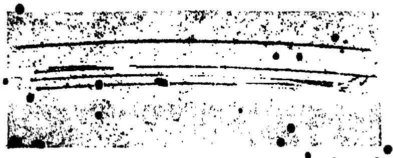


FIG. 4.—Copy of Actual Record from a Balloon Meteorograph. Enlarged five times.

The next process is to obtain the heights from the pressure. This is given by the formula $h = 6{\cdot}0674\,T \log_{10} (p_0/p)$, where h is the height in kilometres, p_0 the surface pressure, p the pressure at the height h, and T the harmonic mean of the absolute temperatures. For heights over which the range of temperature is not large, the arithmetical mean of the temperatures may be substituted for the harmonic mean without serious error.

The computation may be made by means of tables, but a graphical process using semi-logarithmic paper is the most convenient and is accurate to about 1 millibar.

§ (10) ACCURACY OF THE OBSERVATIONS.—There are two independent lines of evidence, both of which lead to the conclusion that, when it is finally published, the probable error in the temperature does not exceed about 1° C. They are, firstly, the good agreement between records that are obtained from the same place at short intervals of time or from places near together at the same time; and, secondly, from the high correlation coefficients which exist between the pressures and temperatures. Such high correlation coefficients could not be found if the error in the measurement of either variable were appreciable when compared with its standard deviation. With regard to height, it is calculated from the pressure. In the first few kilometres of height a difference of one millibar of pressure corresponds to a difference of about 10 metres, but at 20 km. a difference of 1 mb. corresponds to about 130 metres, and consequently the recorded values of great heights are more or less uncertain.

Full details of the instruments and methods used in Great Britain have been published, mostly by the Meteorological Office. A list is given below.[1]

II. THE TEMPERATURE CONDITIONS IN THE UPPER AIR

§ (11) TEMPERATURE AND HEIGHT.—Observations on the temperature and humidity of the upper air have been made by means of registering balloons in many localities, including a few in the Antarctic by Dr. Simpson, and the following is a brief summary of the results; but it is only in Europe that the observations are at all numerous.

From sea-level upwards to about three kilometres the fall of temperature with height, which has latterly come in England to be called the "lapse rate," is more or less irregular, but has on the whole a value of rather over 5° C. per kilometre, or 3° F. per 1000 feet. In the first kilometre it depends greatly upon local conditions, upon whether the day be clear or cloudy, upon whether the time be early morning or afternoon, and, on the coast, upon whether the wind is from the shore or off the sea. In spring and summer on sunny days the lapse rate in the afternoon for the first kilometre will generally be as large as 9° or 10° C. But, on the other hand, between sunset and sunrise on clear nights the temperature at one kilometre height will probably be higher than that near the ground. These discrepancies are due to the fact that the diurnal change of temperature, due to the sun's heat by day and absence at night, is very shallow and does not, as a rule, extend to any appreciable extent much over one kilometre. Thus the conditions which govern the temperature close to the ground are quite different to those which prevail higher up.

---

[1] Meteorological Office—
M.O. 202.
M.O. No. 210, *Geophysical Memoirs*, No. 6.
M.O. 223, Section 11, *The Computer's Handbook*.
Do., Subsection 1.
Do., Subsection 1 (*continued*).
M.O. 232 j, *Professional Notes*, No. 10.
"Second Report on Wind Structure," by J. S. Dines, *Technical Report of the Advisory Committee for Aeronautics*, 1910–11.
Q.J.R. Met. Soc., vol. xxxix, No. 166, "Rate of Ascent of Pilot Balloons."

Inversions, as they are called—that is to say, cases in which the temperature increases with height—are common in the lower strata; they are generally found in anticyclonic regions and are perhaps permanent in places where the surface temperature is very low, as in Canada, Siberia, or the Antarctic in winter.

From three kilometres and upwards to a height which depends on the latitude, but is for Europe about 10·5 km., a lapse rate of from 6° to 7° C. per kilometre is found and inversions of temperature are rare. All this part of the atmosphere in which a lapse rate is found is called the troposphere. Above it up to the greatest height to which sufficient observations have been made—that is, to about 20 km. (12½ miles)—the temperature does not change much with height, although it may vary through 20° C. or more with time. This part of the atmosphere is called the stratosphere. The boundary between the two is in general sharply defined, and often there is an inversion at the boundary.

The thickness of the troposphere is subject to large variation both in time and space. In the equatorial regions it is about 16 km. There are not enough observations to allow a very precise value to be given. For latitude 50° over Europe the value is 10·5 km., for Scotland and Petrograd a little under 10 km. So far as is known the value depends on the latitude and not at all on the longitude. This refers to average values. The changes with time are of two kinds in temperate latitudes. There is an increase in summer and a decrease in winter, the range being nearly proportional to the range in the surface temperature. Thus in Canada the annual range is some 4 km.; in England, though well marked, it is less than 2 km. The other change occurs with changing types of weather. The troposphere is thick over anticyclonic areas, thin over cyclonic areas. Thus in England, over a deep cyclonic depression the value may be as low as 7·5 km., and over an anticyclone as high as 12·5 km. The variations due to pressure are much larger than those due to change of season.

The changes of temperature are as follows: In the troposphere there is a seasonal change of temperature, so that the lapse rate is the same with the exception mentioned above both summer and winter. The changes with barometric variation are such that the troposphere is cold over a cyclonic area and warm over an anticyclonic. The seasonal change in the stratosphere over Europe is in the same sense but greatly reduced in magnitude; over Canada (Toronto) the stratosphere seems to be coldest in the summer, warmest in the winter. With barometric changes the conditions in the troposphere and stratosphere are inverted; the stratosphere is warm over the cyclone, cold over the anticyclone. With change of latitude the troposphere is naturally warmer as we go towards the equator, but the reverse is the case with the stratosphere. In low latitudes at 17 km. the temperature is as low as 193° *a*, - 80° C., the lowest natural temperature ever recorded on the earth; over Scotland and Petrograd it is 223, - 50° C., and the rise of temperature with increasing latitude is plainly apparent from the observations from places so near together as London, Manchester, and Scotland.

The changes of temperature between the troposphere and stratosphere are conveniently summed up in the statement that the mean temperature of the air column taken with regard to height from the ground to 20 km. is very nearly constant both with regard to change of latitude and change of barometric conditions; when the bottom is cold the top is warm, and conversely. Since the pressure at 20 km., calculated from the surface pressure, depends chiefly on the mean temperature of this air column, it follows that whereas at the height of the homogeneous atmosphere, viz. 9 to 10 km., there are large differences of pressure from place to place and time to time, at 20 kilometres there is a very great uniformity.

Except near the earth's surface there is very close correlation between the pressure and the temperature of the air, but with the same exception it is strange to find little or no correlation between the direction of the air motion and the temperature. However, there is not space to discuss the question here.

The following table gives the mean temperatures in absolute measure, the mean pressure in millibars and the mean densities in grammes per cubic metre. It is taken from *The Characteristics of the Free Atmosphere*, M.O. 220 C, which can be obtained from the Meteorological Office and which contains further information on the subject.

Height in Kilometres.	England (South-east).			Europe.			Canada (Toronto).			Equator.		
	T.	P.	D.	T.	P.	D.	T.	P.	D.	T.	P.	D.
20	219	55	87	219	55	87	214	54	88	193	53	96
19	219	64	102	219	64	102	215	63	102	193	63	113
18	219	75	119	219	75	119	214	74	121	193	75	135
17	219	88	139	219	88	139	211	87	144	193	90	162
16	219	102	162	219	102	162	211	102	169	195	107	191
15	219	120	191	219	120	191	211	120	198	198	128	225
14	219	140	223	219	140	223	212	142	233	203	152	261
13	219	164	261	219	164	261	214	167	268	211	178	294
12	219	192	30[illegible]	218	19[illegible]	307	216	195	314	219	209	331
11	220	224	355	219	225	358	219	228	365	227	244	374
10	22[illegible]	261	409	222	262	411	223	266	415	235	283	419
9	228	303	463	227	305	467	229	309	470	243	327	469
8	234	352	524	233	353	528	236	358	528	251	376	522
7	241	407	589	241	408	590	243	413	592	258	430	581
6	248	469	658	248	470	661	251	475	662	265	491	645
5	255	538	735	255	538	735	258	543	733	272	558	714
4	262	615	819	261	614	819	264	618	815	279	632	789
3	268	699	909	267	699	913	270	703	905	285	713	871
2	273	795	1014	272	794	1017	275	798	1011	290	803	968
1	278	900	1128	277	899	1128	278	903	1134	295	903	1067
0	282	1014	1253	281	1014	1258	282	1017	1258	300	1012	1174

W. H. D.

Air, Moist:

Characteristic equation of. See "Atmosphere, Thermodynamics of the," § (20) and Table V.

Entropy-temperature diagram for. See *ibid.* § (22) and *Figs.* 16, 17.

Indicator diagram for. See *ibid.* § ([illegible]3) and *Fig.* 17.

Isentropic equation of. See *ibid.* § (21).

Physical constants of. See *ibid.* § (2) and Table I.

Thermodynamical properties of. See *ibid.* V.

Aircraft, meteorological instruments for use in. See "Meteorological Instruments," VIII.

Air-earth Current. See "Atmospheric Electricity," § (10).

Air-meter: an instrument designed to measure the flow of air. See "Meteorological Instruments," § ([illegible]8).

Airy's Table: a numerical relation suggested by Sir George Airy in 1867 as the basis of conversion from pressures to heights. See "Barometers and Manometers," § (16) (iii.).

Albedo: a word used to denote the amount of reflected light. The light of the sun reflected by the moon is known as the albedo of the moon.

Albedo of cloud. See "Meteorological Optics," § (16) (iv.).

Alcohol. Specific gravity of alcohol and water mixtures. See "Alcoholometry," § (4).

ALCOHOLOMETRY

§ (1) Introductory.—In the days of the alchemists various rough-and-ready methods were employed for testing the strength of spirits. Thus Lully directs his readers to moisten a piece of cloth with *aqua vitae* and apply a lighted taper: if the cloth ignited, the spirit was to be regarded as *aqua vitae rectificata*, or strong spirit. Another method was to pour oil into the spirit; if the latter were strong, it floated on the surface of the oil; if weak, it rested beneath the oil. Basil Valentine, in the fifteenth century, described a mode of testing alcohol by deflagration. He judged the strength of *aqua vitae* by igniting a certain volume of it; if the whole burned away it was pure spirit; if more than half burned off, the spirit was strong; if less than half, it was weak, and needed further rectification.

After the introduction of gunpowder the "proof" test came into use. It was performed by moistening a little powder with the alcohol and applying a light. Rapid combustion implied a "high proof" spirit; failure to burn, or burning only with difficulty, indicated a weak alcohol; whilst if the mixture burned steadily but slowly, the spirit was regarded as "good, rightfull, and of vertue." The formation of bubbles or "beads" when the spirit was shaken in a glass vessel, and the length of time during which the bubbles persisted, gave to a practised observer a rough idea of the strength of a spirit (*Preuve d'Hollande*). This test is occasionally used, in default of better means, even at the present day.

In the year 1666, some friction arose between importers of French brandy and the customs officials of this country as to the rate of duty chargeable on the liquor. There were two rates, 4d. per gallon and 8d., for liquors of different qualities, and the revenue officials, guided by the sense of taste in levying the duties, decided that French brandy ought to pay the higher amount. This decision was contested by the importers, but was eventually ratified and made statutory by an Act passed in 1670. Presently, however, fraudulent merchants attempted by various devices to disguise the real strength of their brandies, so that other tests besides that of taste had to be resorted to, and recourse was had to those mentioned above. But the tests were crude, and the results often capricious; and with the increasing importance of alcohol taxation the need for better methods of evaluation became more and more apparent. Towards the close of the century, therefore, a good deal of attention was devoted to the question of obtaining an areometer or hydrometer suitable for testing spirits, since the use of this instrument appeared to offer an easy and accurate method of appraising alcoholic strengths. Boyle, who seems to have been the first to apply the principle of the hydrometer to the testing of distilled liquors, had described his instrument in 1675.[1] After various improvements, the hydrometer came into use for revenue purposes in the early part of the eighteenth century, as described below.

§ (2) Spirit Balances.—At first the hydrometer appears to have been employed in a qualitative way, as an adjunct to the oil and gunpowder tests for distinguishing between different liquors, rather than for the actual estimation of alcoholic strength. For this latter purpose various kinds of "spirit balance" were introduced. Desaguliers[2] remarks that "the hydrometer has only been used to find whether any one liquor is specifically heavier than another, but not to tell how much, which cannot be done without a great deal of trouble even with a nice instrument. The hydrostatical balance has supplied the place of the hydrometer, and shows the different specific gravities of fluids by a very great exactness."

Special forms of the balance were devised for testing spirits. Ramsden's "balance hydrométrique" (1792) was one of the first of these: other forms were Hooke's, Millar's, Murray's, and a "statical hydrometer" introduced by Adie. Some of these instruments, as Hooke's and Adie's, determined spirit strengths from specific gravity on the principle of weighing a "sinker" in the spirit. Thus Hooke's "poise" was a large pear shaped glass bulb, equal in volume to about one-third of a gallon; this, hung from one arm of a balance, was placed in the spirit, and counterpoised; the weight required indicated the strength of the liquor. Others were, essentially, a simple form of balance in which a vessel was suspended from one arm of a pivoted beam, and counterbalanced by a sliding weight on the other arm. Spirit strengths were found by comparing the weight of spirituous liquor which the vessel would hold with the weight of its water-content. In some instruments the beam itself was graduated to show various alcoholic strengths directly. In Murray's balance the weight was fixed, and the vessel containing the spirit was movable along the beam to points which showed "proof" and strengths "under" or "over" proof, as the case might be, when equilibrium was established. These earlier forms of spirit-balance do not appear to have been in general use for commercial spirit-assaying. They were employed, rather, in special cases, and for purposes of checking, where more accurate results than those given by the hydrometer were required. Thus in 1790 we find Blagden (Report to Royal Society) advising that a balance should be supplied to each important centre where alcohol duties were collected, in order to settle any disputes which might arise between the trading community and the revenue officials. Later, the well-known "Mohr" hydrostatic balance came into use for ascertaining the specific gravities of liquids, and this instrument, or the "Westphal" modification of it, is frequently employed for alcohol determinations at the present day.

Meanwhile, however, various investigations had been in progress, with the object of determining the true specific gravity of alcohol and of its aqueous solutions on the one hand, and of perfecting the hydrometer on the other. Attention must now be given to the results of these inquiries.

§ (3) Contraction.—When alcohol and water are mixed, a rise in the temperature of the liquid occurs, and on cooling the mixture to the original temperature there is found to be a contraction of the total volume. The specific gravity of the mixture, therefore, is greater than the arithmetical mean of the specific gravities of the two constituents. The amount of contraction depends upon the relative proportions of the two liquids. Calculated as a percentage on the sum of the initial volumes, the maximum contraction is given by a mixture containing one molecule of alcohol and three molecules of water, the amount of the contraction being then 3·64 per cent at 15·56° C. Or, expressed in another manner, the maximum contraction is obtained by mixing 52 volumes of alcohol with 48 volumes of water, and the volume of the resulting mixture (at 20° C.) is 96·3 instead of 100.

---

[1] *Phil. Trans.* x. 329.

[2] *Ibid.*, 1730, xxxvi. 277.

When 100 c.c. of alcohol, at 15·56° C., are mixed with increasing quantities of water, the maximum *amount* of contraction, 9·08 c.c., is reached when the mixture corresponds with one molecule of alcohol to eight molecules of water. This is to be distinguished from the maximum *percentage* contraction mentioned above.

The results of a series of experiments are shown in the subjoined table (J. Holmes):

TABLE I

CONTRACTION OF ALCOHOL-WATER MIXTURES. Temp. 15·56° C.

Mixture. One Mol. Alcohol to	Actual Contraction: Initial Volume of Alcohol 100 c.c.	Percentage Contraction, calculated on Sum of Initial Volumes.
½ mol. water	1·21 c.c.	0·99
1 „	3·79 „	2·89
3 „	7·03 „	3·64
6 „	8·87 „	3·09
8 „	9·08 „	2·61
12 „	8·78 „	1·86
20 „	7·88 „	1·09

On account of the irregularities in the amount of contraction shown by mixtures of alcohol and water, the specific gravities of such mixtures cannot be calculated accurately from a knowledge of the proportions of water and alcohol present. In constructing specific gravity tables for aqueous alcohol, therefore, it is necessary to determine the specific gravities by actual experiments on a series of mixtures having a known composition, and differing only so much as will allow of intermediate values being interpolated with a sufficient degree of accuracy.

(4) SPECIFIC GRAVITY OF ALCOHOL, AND OF ALCOHOL-WATER MIXTURES.—The founder of alcoholometry was George Gilpin, clerk to the Royal Society in the latter part of the eighteenth century. Dissatisfaction having arisen over the processes in use for assessing the amount of duty payable on spirits, the Government of the day applied to the Society for assistance in the matter. Dr. Chas. Blagden, the secretary, helped to arrange experiments, and in 1790 presented a "Report on the best method of proportioning the Excise on Spirituous Liquors."[1] It was recognised that "no method can be accurate except one based upon specific gravities"; and the experiments consequently took the form of determining the specific gravities of a long series of mixtures containing known weights of "alcohol" and water. The experiments, commenced by a Swiss chemist, Dr. Dollfuss, were carried out by Gilpin, who in 1794 presented the results in a set of "Tables for Reducing the Quantities by Weight, in any mixture of pure Spirit and Water, to those by Measure; and for Determining the Proportion, by Measure, of each of the two Substances in such Mixtures."

The "alcohol" used by Gilpin had the specific gravity 0·82514 at 15·56°/15·56° C. It would contain, therefore, only 88·97 per cent by weight of absolute alcohol. Determinations were made at each temperature-interval of 5° F. between 30° F. and 100° F.; so that the experiments provided data for calculating the thermal expansion of aqueous alcohol within this range of temperature, and also the amount of contraction of the mixtures, as well as their specific gravities.

Starting with a weight of alcohol denoted by 100 parts, this quantity was at first kept constant, and a series of determinations made after adding to it 5, 10, 15 . . . 100 parts by weight of water; afterwards the water was kept constant at 100 parts, and the alcohol decreased by differences of 5 parts from 95 to 0. Gilpin's actual results need not be given here: for present-day use they require to be corrected, since the "alcohol" on which they are based was not absolute alcohol. With this and other corrections, they are embodied in the revised table subjoined (Table II.).

Lowitz, of St. Petersburg, shortly afterwards obtained alcohol much stronger than Gilpin's. He treated spirit of wine with potassium carbonate, poured the partly dehydrated alcohol off, and distilled it until not more than two-thirds of the spirit had passed over. The product[2] had the specific gravity 0·791 at 16° R. (=20° C.), or 0·79484 when calculated to 15·56°/15·56° C. Lowitz's alcohol would thus contain 99·6 per cent of absolute alcohol.

Tralles of Berlin (1811) utilised Gilpin's tables, together with results obtained by himself and others due to Lowitz, in compiling a series of alcohol tables for the Prussian Government. He, also, had succeeded in obtaining a much stronger alcohol than Gilpin's. It had the specific gravity 0·7939 at 15·56°/4° C., or 0·7946 at 15·56°/15·56° C., which corresponds with 99·68 per cent of anhydrous alcohol by weight. Tralles recalculated Gilpin's results on the basis of this stronger alcohol, and constructed a table showing percentage of alcohol by volume at 15·56° C., referred to water at 4° C. as unity, as well as several other tables for use at different temperatures in connection with his hydrometer.[3] These tables and instrument were used officially in Germany during the greater part of the nineteenth century. Brix (1847) recalculated Tralles's main table, taking water at 15·56° C. as unity.

---

[1] *Phil. Trans.*, 1790, lxxx. 321.

[2] *Crell's Annalen*, 1796, i. 195.

[3] *Gilbert's Annalen*, 1811, xxxviii. 349.

In 1824, Gay-Lussac published a treatise upon the centesimal alcoholometer, together with tables calculated for the temperature, 15°/15° C.: these have formed the basis of French alcoholometry. The alcohol taken as basis had the specific gravity 0·7947 at the temperature mentioned, in accordance with results obtained by Rudberg.

Fownes[1] in 1847 described experiments "on the values in absolute alcohol of spirits of different specific gravities." He found the specific gravity of absolute alcohol to be 0·7938 at 15·56°/15·56° C.

About the same time another important determination was carried out by Drinkwater.[2] He dehydrated strong spirit first with freshly-ignited potassium carbonate, and then with quicklime; and obtained a nearly absolute alcohol with the specific gravity 0·79381 at 15·56°/15·56° C. This result is practically identical with that found by Fownes, but both are slightly higher than the value obtained later by Mendeléeff, and now generally accepted as the true value.

Mendeléeff's classical research upon alcohol was carried out after careful consideration of previous investigations, and of all the possible sources of error. The results obtained for the specific gravity, referred to water at 4° C. as unity, were:

At 0° C.	0·80625
„ [illegible]° C.	0·80207
„ 10° C.	0·79788
„ 15° C.	0·79367
„ 20° C.	0·78945
„ 25° C.	0·78522
„ 30° C.	0·78096

These values correspond with 0·79359 at 15·56°/15·56° C. (in air); and this is accepted as probably the most accurate value, although it may be mentioned that a somewhat lower result (0·7935) has been obtained by Squibb.[3]

For calculating the specific gravity, d_t, at temperatures other than 0° C., Mendeléeff gives the following equation:[4]

$$d_t = 0{\cdot}80625 - 0{\cdot}0008340t - 0{\cdot}00000029t^2.$$

An account of the research is given by V. Richter.[5]

As regards the specific gravity of aqueous solutions of alcohol, Gay-Lussac's tables have been, and still are, employed for fiscal purposes in France and several other continental countries, but they have undergone more than one revision—the last, and chief one, in 1884. Tralles's tables were used in Germany down to the year 1888, when they were superseded by others, calculated mainly from Mendeléeff's results by the Normal Eichungs Kommission. In the United Kingdom, Sikes's tables (*v. infra*) have been employed since the year 1816. Following, however, upon certain provisions in the Finance Act of 1907, a revision and extension of Sikes's original tables was undertaken by Sir Edward Thorpe and his staff at the Government Laboratory, London, at the request of the Customs and Excise authorities, and the revised tables were issued in the year 1912. For the purpose of this revision the work of Blagden and Gilpin, Drinkwater, Mendeléeff, and the Kaiserliche Normal Eichungs Kommission was utilised, after full consideration, as embracing the most trustworthy data. It may be remarked that Mendeléeff was so well satisfied with Gilpin's and Drinkwater's work, that he incorporated among his own results many of those obtained by these two investigators for spirituous mixtures of low strength. The accompanying table gives, in an abbreviated form, the specific gravities of aqueous solutions of alcohol according to Thorpe's revision (Table II.).

TABLE II

SPECIFIC GRAVITY AT 15·56°/15·56° C. OR 60°/60° F.

Specific Gravity in Air at 15·56°/15·56° C.	Percentage of Proof Spirit.	Percentage of Alcohol.	
		By Weight.	By Volume at 15·56° C.
0·79359	175·35	100·00	100·00
0·794	175·21	99·87	99·92
0·796	174·52	99·22	99·52
0·798	173·80	98·57	99·12
0·800	173·07	97·91	98·70
0·802	172·33	97·25	98·28
0·804	171·56	96·57	97·84
0·806	170·77	95·89	97·39
0·808	169·96	95·20	96·93
0·810	169·13	94·50	96·45
0·812	168·28	93·80	95·97
0·814	167·41	93·08	95·47
0·816	166·51	92·36	94·97
0·818	165·60	91·63	94·45
0·820	164·67	90·90	93·92
0·822	163·72	90·16	93·38
0·824	162·75	89·41	92·83
0·826	161·76	88·65	92·26
0·828	160·75	87·88	91·69

---

[1] *Phil. Trans.*, 1847, cxxxvii. 249.

[2] *Mem. Chem. Soc.*, 1848, iii. 447.

[3] *Ephemeris of Materia Medica*, 1884, p. 541.

[4] *Untersuchungen über die Verbindungen des Alkohols mit Wasser*, St. Petersburg, 1865.

[5] *Ann. d. Physik*, 1869, cxxxvii. 103, 230.

TABLE II (*continued*)

Specific Gravity in Air at 15·56°/15·56 °C.	Percentage of Proof Spirit.	Percentage of Alcohol.	
		By Weight.	By Volume at 15·56° C.
0·830	159·73	87·11	91·11
0·832	158·69	86·34	90·52
0·834	157·63	85·56	89·91
0·836	156·56	84·78	89·30
0·838	155·47	83·99	88·68
0·840	154·37	83·20	88·06
0·842	153·25	82·40	87·42
0·844	152·12	81·60	86·77
0·846	150·97	80·79	86·12
0·848	149·80	79·98	85·46
0·850	148·62	79·17	84·78
0·852	147·43	78·35	84·11
0·854	146·23	77·53	83·42
0·856	145·01	76·71	82·73
0·858	143·78	75·88	82·03
0·860	142·54	75·05	81·32
0·862	141·28	74·22	80·61
0·864	140·02	73·39	79·89
0·866	138·74	72·55	79·16
0·868	137·46	71·72	78·43
0·870	136·16	70·88	77·69
0·872	134·84	70·04	76·94
0·874	133·53	69·19	76·19
0·876	132·19	68·35	75·45
0·878	130·86	67·51	74·68
0·880	129·50	66·66	73·91
0·882	128·14	65·81	73·13
0·884	126·77	64·96	72·34
0·886	125·37	64·10	71·55
0·888	123·97	63·24	70·75
0·890	122·56	62·38	69·95
0·892	121·14	61·52	69·14
0·894	119·70	60·66	68·33
0·896	118·26	59·80	67·50
0·898	116·81	58·93	66·67
0·900	115·33	58·06	65·83
0·902	113·84	57·18	64·98
0·904	112·35	56·31	64·13
0·906	110·82	55·42	63·26
0·908	109·29	54·54	62·39
0·910	107·74	53·65	61·51
0·912	106·20	52·77	60·63
0·914	104·63	51·88	59·74
0·916	103·05	50·98	58·83
0·918	101·43	50·08	57·92
0·920	99·80	49·17	56·99
0·922	98·16	48·25	56·05
0·924	96·49	47·33	55·10
0·926	94·80	46·40	54·14
0·928	93·09	45·47	53·16
0·930	91·36	44·53	52·18
0·932	89·61	43·59	51·18
0·934	87·81	42·62	50·15
0·936	85·97	41·64	49·10
0·938	84·10	40·65	48·04
0·940	82·19	39·65	46·95
0·942	80·22	38·64	45·85
0·944	78·26	37·60	44·71
0·946	76·21	36·54	43·54
0·948	74·12	35·46	42·35
0·950	71·98	34·37	41·13
0·952	69·76	33·25	39·87
0·954	67·48	32·09	38·57
0·956	65·09	30·90	37·20
0·958	62·60	29·66	35·79
0·960	60·03	28·39	34·33
0·962	57·33	27·06	32·79
0·964	54·51	25·68	31·18
0·966	51·53	24·23	29·48
0·968	48·38	22·71	27·69
0·970	45·14	21·14	25·83
0·972	41·77	19·53	23·91
0·974	38·35	17·90	21·96
0·976	34·87	16·25	19·98
0·978	31·42	14·51	18·00
0·980	27·99	12·99	16·04
0·982	24·66	11·42	14·13
0·984	21·44	9·91	12·29
0·986	18·34	8·46	10·51
0·988	15·38	7·08	8·80
0·990	12·53	5·76	7·18
0·992	9·82	4·51	5·63
0·994	7·24	3·31	4·14
0·996	4·73	2·17	2·71
0·998	2·33	1·07	1·34
1·000	0·00	0·00	0·00

TEMPERATURE CORRECTIONS.—If for any reason the specific gravity of the alcohol is not taken at the standard temperature, it is necessary to include a correction to compensate for the deviation. The correction is greater at higher strengths than at lower, as will be seen from the table given below, which is used in the following manner:

The difference between the standard temperature and the actual temperature of the observation is multiplied by the appropriate factor, taken from the table. If the actual temperature is higher than the standard, the product is added to the observed specific gravity; if it is lower, the product is subtracted. The unit throughout is water at the *standard* temperature, 15·56° C. (60° F.).

TABLE III

TEMPERATURE CORRECTIONS

Specific Gravity.	Correction for	
	1° F.	1° C.
0·794	0·00046	0·00083
0·864	45	81
0·889	44	79
0·902	43	77
0·912	42	76
0·921	41	74
0·928	40	72
0·935	39	70
0·940	38	68
0·943	37	67
0·946	36	65
0·949	35	63
0·951	34	61
0·953	33	59
0·955	32	58
0·957	31	56
0·959	30	54
0·961	29	52
0·962	28	50
0·963	27	49
0·965	26	47
0·966	25	45
0·967	24	43
0·968	23	41
0·969	22	40
0·970	21	38
0·971	20	36
0·973	19	34
0·974	18	32
0·975	17	31
0·976	16	29
0·977	15	27
0·978	14	25
0·980	13	23
0·981	12	22
0·983	11	20
0·985	10	18
0·987	0·00009	16
0·990	8	14
0·995	7	13
1·000		

Osborne, M'Kelvy, and Bearce have given the following data,[1] which allow of the specific gravity of aqueous alcohol mixtures being calculated for any temperature between 10° and 40° C.

The density of alcohol at 25° was found to be 0·78506 gram per c.c.

The specific gravity of various mixtures of alcohol and water was determined at 10°, at 40°, and at each interval of 5° between these temperatures. From the results, the values of the coefficients in the following equation were calculated:

$$D_t = D_{25} + \alpha(t-25) + \beta(t-25)^2 + \gamma(t-25)^3.$$

Here D_t is the specific gravity at any temperature t between 10° and 40°, and D_{25} is the specific gravity at 25°. The values of the coefficients α, β, and γ are given in the following table.

Alcohol per cent by Weight.	Sp. Gr. at 25°. Grams per c.c.	$\alpha \times 10^7$.	$\beta \times 10^8$.	$\gamma \times 10^{10}$.
0·000*	0·997077	−2565	−484	+319
4·907	0·988317	−2684	−502	+311
9·984	0·980461	−3119	−484	+258
19·122	0·967648	−4526	−393	+180
22·918	0·962133	−5224	−351	+100
30·086	0·950529	−6431	−226	+ 47
39·988	0·931507	−7488	−145	− 4
49·961	0·909937	−8033	−128	− 24
59·976	0·887051	−8358	−121	− 24
70·012	0·863380	−8581	−117	− 9
80·036	0·839031	−8714	−108	− 69
90·037	0·813516	−8746	− 93	− 51
99·913	0·785337	−8593	− 57	− 63

* Water.

With the aid of this table and the following one giving an extended range of alcohol percentages, the specific gravity of any mixture of ethyl alcohol and water, at any temperature between 10° and 40°, can be calculated from the foregoing equation.

Alcohol per cent by Weight.	Sp. Gr. at 25°. Grams per c.c.	Alcohol per cent by Weight.	Sp. Gr. at 25°. Grams per c.c.
0	0·997077	55	0·898502
2	0·993359	60	0·886990
5	0·988166	65	0·875269
6	0·986563	70	0·863359
10	0·980434	75	0·851336
15	0·973345	80	0·839114
20	0·966592	85	0·826596
25	0·958946	90	0·813622
30	0·950672	95	0·799912
35	0·941459	98	0·791170
40	0·931483	99	0·788135
45	0·920850	100	0·785058
50	0·909852		

Table IV. on the following page shows the specific gravity, referred to water at 4° C. as unity, of aqueous alcohol at seven different temperatures.

§ (5) HYDROMETERS ("AREOMETERS").—The general principle of the hydrometer are dealt with elsewhere in this work (see article "Hydrometers"). Here we need only consider the forms which have been more particularly associated with the development of alcoholometry. The first of these,[2] "Boyle's Bubble," was composed of two glass bulbs, surmounted by a glass stem. The lower and smaller bulb

---

[1] *Bull. U.S. Bureau of Standards*: Scientific Paper No. 197.

[2] *Phil. Trans.*, 1675, x. 329.

TABLE IV

DENSITY (IN GRAMS PER C.C.) OF ALCOHOL-WATER MIXTURES

Per cent Alcohol by Weight.	Temperature ° C.						
	10°.	15°.	20°.	25°.	30°.	35°.	40°.
0	0·99973	0·99913	0·99823	0·99708	0·99568	0·99406	0·99225
5	·99098	·99032	·99938	·98817	·98670	·98501	·98311
10	·98393	·98704	·98187	·98043	·97875	·97685	·97475
15	·97800	·97669	·97514	·97334	·97133	·96911	·96670
20	·97252	·97068	·96864	·96639	·96395	·96134	·95856
25	·96665	·96424	·96168	·95895	·95607	·95306	·94991
30	·95977	·95686	·95382	·95067	·94741	·94403	·94055
35	·95162	·94832	·94494	·94146	·93790	·93425	·93051
40	·94238	·93882	·93518	·93148	·92770	·92385	·91992
45	·93226	·92852	·92472	·92085	·91692	·91291	·90884
50	·92162	·91776	·91384	·90985	·90580	·90168	·89750
55	·91055	·90659	·90258	·89850	·89437	·89016	·88589
60	·89927	·89523	·89113	·88699	·88278	·87851	·87417
65	·88774	·88364	·87948	·87527	·87100	·86667	·86227
70	·87602	·87187	·86766	·86340	·85908	·85470	·85025
75	·86408	·85988	·85564	·85134	·84698	·84257	·83809
80	·85197	·84772	·84344	·83911	·83473	·83029	·82578
85	·83951	·83525	·83095	·82660	·82220	·81774	·81322
90	·82654	·82227	·81797	·81362	·80922	·80478	·80028
95	·81278	·80852	·80424	·79991	·79555	·79114	·78670
100	·79784	·79360	·78934	·78506	·78075	·77641	·77203

contained mercury, the larger contained water; these liquids served as ballast to maintain the instrument erect when in use. Placed in water, it sank only so far as to cover the bulbs, leaving the whole of the stem exposed. Placed in strong alcohol, it sank till only the top of the stem remained uncovered. In mixtures of alcohol and water it sank to intermediate positions depending upon the proportions of the two ingredients. The larger the proportion of alcohol, the deeper the "bubble" sank.

The possibility of adapting this instrument for use in assaying spirits was quickly perceived, and various improvements were suggested. *Moncony* (1679) described a glass areometer terminating below in a small loop carrying a silver hook, by means of which a graduated series of silver rings could be attached (1 grain, ½ grain, and so on), thus affording a means of making comparative measurements and extending the range of the instrument. Later,[1] the stem of the areometer was roughly graduated by means of "small bits of glass, of different colours, stuck on the outside"; or it was marked off into degrees by lines. The form probably used by revenue officers in the period 1730–1760 had a graduated stem, tapering to a point.

Clarke's hydrometer was brought under the notice of the Royal Society by Desaguliers in 1730, and was in use for revenue work some thirty years later, though it did not receive statutory sanction until 1787. It was made of copper, and had an attachment at the base to which different weights could be screwed; these allowed the total mass of the instrument to be varied, and gave it a greater range in use. One especial feature may be noted: the hydrometer was provided with special "weather weights," for use when the weather was "hot," "warm," "cold," and so on, thus introducing a rough correction for variations of specific gravity due to changes in temperature. There were three marks on the stem, one showing "proof" strength, the others one-tenth under proof and one-tenth over proof respectively (*i.e.* 10 per cent under and over proof). The idea of making a correction for temperature had already been utilised in *Martin's* hydrometer, an instrument devised for finding the specific gravity of spirits, and furnished with a "scale of lines," on the principle of the slide rule, for converting the hydrometer readings into spirit strengths. Clarke's instrument was provisionally sanctioned as the official hydrometer in 1787 (27 Geo. III. c. 31). This sanction was continued from time to time until 1801, when the provision was made permanent (41 Geo. III. c. 97).

Dicas's hydrometer must also be briefly mentioned, since, although it does not appear to have been much used in this country, it was adopted in 1790 as the legal instrument for alcohol-testing in the United States, and employed there for some sixty years. It was made of gilded brass or copper, and the weights were not submerged as in Clarke's instrument, but placed on the top of the stem. The readings, like those of Martin's hydrometer, were translated into spirit strengths by means of a slide rule. Dicas's instrument was eventually superseded in the United States by *Greiner's* "thermo-alcoholometer," an instrument graduated to show percentages of Tralles's alcohol, and carrying a Fahrenheit thermometer in its lower part. This in turn was replaced by a hydrometer constructed by *Tagliabue*, showing percentages of (U.S.) proof spirit: this is the form still employed in the United States.

---

[1] *Phil. Trans.*, 1730, xxxvi. 277.

Both in Scotland and in Ireland Clarke's hydrometer was used soon after its adoption in England; but later two other instruments were officially recognised in Ireland—first, *Hyatt's*, and then, in 1802, *Speer's* hydrometer. In the latter instrument, more particularly, the claim was made for an improvement in allowing for changes of temperature. Clarke's method, in fact, had become too cumbersome, with its unwieldy notation and its multiplicity of weights to meet different requirements—there were 54 weights supplied with the official instrument, and in 1816 Sikes's hydrometer superseded it, as well as the others. This instrument is the one still employed for fiscal purposes in the United Kingdom, and on account of its importance it must be described at some little length. Before doing this, however, mention may be made of *Lovi's beads*, devised by Dr. Wilson of Glasgow, and patented by Mrs. I. Lovi, a glassworker, and J. R. Irving in 1805. These "beads" are small hollow glass balls of different sizes and weights, and marked in a graduated series to show either specific gravities or spirit strengths. On placing these balls in a spirituous liquid the one which just floats steadily in any position shows the specific gravity of the liquid, or its proof strength, according to the marking of the "bead."

§ (6) SIKES'S HYDROMETER.—In the year 1802, improved hydrometers and methods of assaying spirits were inquired for by the Treasury, and a scientific committee was appointed to adjudicate upon the various proposals submitted. The instrument and tables tendered by Bartholomew Sikes, a former Secretary of Excise, were selected. Sikes was well acquainted with Blagden and Gilpin's published experimental results, and availed himself of them when necessary. His fiscal alcoholic strengths appear, indeed, to have been calculated mainly from Gilpin's data.

Sikes's hydrometer[1] consists of a gilded brass bulb, 1·5 inches in diameter, to the bottom of which is affixed a short, tapering rod ending in a pear-shaped counterpoise, whilst on the top is a thin stem of rectangular section, 3·5 inches long, marked off into 10 equidistant spaces or "degrees." Each degree is subdivided into fifths. The degrees are marked from 0 to 10, beginning at the top of the stem, and are of arbitrary value—that is, they do not by themselves express the strength of the spirit or its specific gravity, but are correlated with the tables supplied with the instrument. The readings on the stem are called the "Indication," and corresponding with each indication-number the tables show the strength of the spirit tested, in terms of "proof," "over proof," or "under proof." With its 10 divisions, each having 5 subdivisions, the instrument gives 50 indication-numbers, namely 0, 0·2, 0·4, 0·6, and so on up to 10·0. At the temperature 60° F., these indications correspond with strengths of spirit from 67·0 over proof down to 58·2 over proof. For lower strengths than these the instrument is too light: it will not sink in the weaker, and therefore heavier, alcohol. To meet this difficulty, weights are employed, which rest on the counterpoise when in use, and are therefore immersed in the liquid tested. Nine such weights are provided, numbered 10, 20, 30, and so on, up to 90. Their volumes and masses are so related to those of the hydrometer that they furnish a continuous series of indication-numbers, aggregating 500 for the whole range, and allowing of the instrument being used for the determination of alcoholic strengths ranging from 70 over proof down to *nil*.

In addition to the nine weights, a brass cap is supplied which fits on to the top of the stem of the hydrometer. The weight of this cap is exactly one-twelfth of that shown by the instrument and the weight No. 60 taken together. If this cap is placed on the instrument, together with the weight 60, it will sink the hydrometer in distilled water at 51° F. down to a certain mark on the stem at the division 0·8—that is, the "indication" shown is 60·8. This mark is called the "proof mark." If the cap is removed, and the instrument with weight 60 placed in *proof spirit* at 51° F., the indication will be found as before—viz. 60·8. As the same volume of liquid is displaced in the two experiments, but the weight supported in the second case is only $\frac{12}{13}$ths of that in the first, it follows that the density of the proof spirit at 51° F. is $\frac{12}{13}$ths of that of water at the same temperature.

The ordinary Sikes's hydrometer cannot be used with very strong spirits—*e.g.* those of strength upwards of 70 o.p. (= 96·95 per cent of alcohol by volume). It has therefore been supplemented by a smaller instrument of similar design, known as the "light hydrometer" or the "A" instrument. This extends the range up to 73·5 o.p. at 60° F., corresponding with 98·94 per cent of alcohol by volume, and up to 74·0 o.p. at 30° F.

In using Sikes's hydrometer, after the temperature of the liquid has been observed the instrument is to be immersed until the whole divided part of the stem is wet; the pressure required for this will serve as a guide in selecting the proper weight. After attaching this weight, the hydrometer is again immersed to the division O, and allowed to rise slowly to the resting-point. The eye is then brought to the level of the surface of the liquid, and the part of the stem cut by that surface, *as seen from below*, is noted. The reading of the stem thus given is added to the number of the weight to obtain the indication.

---

[1] See also "Hydrometers" for a consideration of sources of error.

As regards reading hydrometers in general, see the articles on "Saccharometry" and "Hydrometers": ordinary hydrometers are immersed up to the resting-point only.

§ (7) PROOF SPIRIT.—In this country, and also in the United States and in Holland, fiscal charges on alcohol, as well as many commercial transactions, are not based directly upon the percentage of actual alcohol contained in spirituous liquors, but upon the proportion of aqueous alcohol of a certain strength, termed "proof" spirit. The strength adopted as "proof," however, is not the same in the three countries. Confining our attention for the moment to the United Kingdom, proof spirit is, legally, spirit of the strength denoted as proof by Sikes's hydrometer (Spirits Act, 1880, sec. 134). Another legal definition makes proof spirit "that which at the temperature of 51° by Fahrenheit's thermometer weighs exactly twelve-thirteenth parts of an equal measure of distilled water" (56 Geo. III. c. 140). The temperature of the water is not expressly stated in this definition, but it was given as 51° F. in tables issued by Sikes for use with his instrument, and is the one adopted.

Drinkwater, in 1848, published the results of a very careful investigation upon the composition of proof spirit, prepared, according to the above definition, from the "absolute" alcohol which he had obtained as already described. Calculating the expansion between 51° and 60° F. from Gilpin's data, he concluded that proof spirit consisted of 49·24 per cent of absolute alcohol by weight, and 50·76 per cent of water; that its specific gravity at 60°/60° F. was 0·91984; and that the strength of the absolute alcohol was 75·25 degrees over proof. These values have been slightly modified by Thorpe's recent revision, and the figures now accepted for proof spirit are:

Spec. gravity, 0·91976 at 15·56°/15·56° C.; percentage of alcohol by weight, 49·28; or by volume at 15·56° C., 57·10. Strength of absolute alcohol, 75·35 degrees over proof.

Subjoined is a table showing the percentage of proof spirit corresponding with each integral indication-degree of Sikes's hydrometer at 60° F. The complete tables show each fifth of a degree, and cover a range of temperature from 30° F. to 100° F.

TABLE V

PROOF SPIRIT STRENGTH CORRESPONDING WITH THE INDICATIONS OF SIKES'S HYDROMETER. Temp. 60° F.

Indication.	Strength Over Proof.	Indication.	Strength Over Proof.
Light Hydrometer		Ordinary Hydrometer	
A 0	73·5	45	19·7
1	72·9	46	18·3
2	72·2	47	17·0
3	71·6	48	15·6
4	71·0	49	14·3
5	70·3	50	12·9
6	69·6	51	11·5
7	68·9	52	10·1
8	68·2	53	8·7
9	67·5	54	7·3
Ordinary Hydrometer		55	5·8
		56	4·4
0	66·7	57	2·9
1	66·0	58	1·4
2	65·2		*Under Proof*
3	64·4	59	0·2
4	63·6	60	1·7
5	62·8	61	3·3
6	61·9	62	4·8
7	61·1	63	6·4
8	60·2	64	8·1
9	59·3	65	9·7
10	58·4	66	11·4
11	57·6	67	13·1
12	56·7	68	14·9
13	55·7	69	16·7
14	54·8	70	18·6
15	53·8	71	20·5
16	52·9	72	22·4
17	51·9	73	24·4
18	50·9	74	26·4
19	49·9	75	28·5
20	48·9	76	30·7
21	47·9	77	32·9
22	46·8	78	35·3
23	45·8	79	37·7
24	44·7	80	40·3
25	43·6	81	42·9
26	42·5	82	45·7
27	41·4	83	48·6
28	40·3	84	51·7
29	39·1	85	54·8
30	38·0	86	58·2
31	36·9	87	61·5
32	35·7	88	65·0
33	34·6	89	68·4
34	33·4	90	71·9
35	32·2	91	75·2
36	31·0	92	78·4
37	29·8	93	81·4
38	28·5	94	84·4
39	27·3	95	87·3
40	26·0	96	90·0
41	24·8	97	92·6
42	23·6	98	95·1
43	22·3	99	97·6
44	21·0	100	100·0

"Over" and "Under" Proof.—If an over proof strength is added to 100, the sum represents the number of volumes of spirit at proof strength which 100 volumes of spirit at that particular over-proof strength would make. Thus 100 vols. of spirit at 25° over proof would make 125 vols. of proof spirit.

If an *under* proof strength is *subtracted* from 100, the remainder shows the volumes of proof spirit which are contained in 100 vols. at that particular under-proof strength. Thus 100 vols. of spirit at 25 degrees under proof contain 75 vols. of proof spirit.

The sum and the remainder show, in fact, the percentages of alcohol, calculated as proof spirit, in the stronger and the weaker spirits, respectively. The following examples show how to calculate the equivalent proof quantity of any given volume of over-proof or under-proof spirit: (1) Given 120 gallons of alcohol, at strength 6·5 o.p. The equivalent proof gallons are (100+6·5) per cent of 120=127·8 proof gallons. (2) Given 150 gallons at 12·5 u.p. The equivalent proof gallons are (100−12·5) per cent of 150 or 131·25 proof gallons.

"Proof spirit" terminology is confusing and cumbersome, but there is no loss of scientific precision in thus adopting a diluted alcohol as the unit of measurement, instead of absolute alcohol. By means of its specific gravity the unit of diluted alcohol can be defined to any degree of accuracy required. "There appears to be no reason, either philosophical or practical, why the alcohol should be considered as absolute. A definite mixture of alcohol and water is as invariable in its value as absolute alcohol can be. . . . A diluted alcohol is, therefore, that which is recommended by us as the only excisable substance." (Report by Committee of Royal Society to the Treasury, 1833.)

The legal authority for the use of Sikes's hydrometer for fiscal purposes in the United Kingdom is contained in the Spirits Act, 1880, s. 134: "All spirits shall be deemed to be of the strength denoted by Sikes's hydrometer . . . in accordance with the table lodged with the Commissioners [of Inland Revenue]. Revised tables were published in 1912; the legalisation of these is contained in Sec. 19 of the Finance (No. 2) Act, 1915. It may, however, be noted that the revenue authorities may, by regulations, now authorise the use of *any* means approved by them for ascertaining the strength or weight of spirits (Finance Act, 1907).

Sikes's tables are so constructed as to show, for a given spirit, the same strength at whatever temperature (within the limits of the tables) the strength is taken. A spirit, for instance, which shows 62·0 degrees over proof at 60° F. will show the same strength at 55° F. or at 65° F., though its "indication" will be different. The tables, however, do not take account of changes in volume due to alterations of temperature. This is certainly a defect, but the errors arising therefrom, which will be sometimes in favour of the revenue and sometimes against it, are not regarded as of much practical importance.

§ (8) USE OF SIKES'S HYDROMETER IN FINDING THE VOLUME OF SPIRITS FROM THEIR WEIGHT.—Experiments have been made to determine the specific gravities which correspond with the indication-numbers of Sikes's hydrometer. The results are embodied in a statutory table, which is authorised for use in evaluating spirits, and is employed in ascertaining the quantity of spirits in casks by the method of weighing (Schedule to Spirits Act, 1880). A revision of the original table was published in the year 1916, and an abbreviation of it is reproduced here.

TABLE VI

WEIGHT OF SPIRITS PER GALLON BY SIKES'S HYDROMETER

Ind.	Wt. per Gallon, lb.	Ind.	Wt. per Gallon, lb.	Ind.	Wt. per Gallon, lb.
A Hydrometer		Ordinary Hydrometer		Ordinary Hydrometer	
0	7·991	25	8·585	63	9·276
1	8·007	26	8·602	64	9·295
2	8·024	27	8·620	65	9·314
3	8·040	28	8·638	66	9·333
4	8·057	29	8·656	67	9·352
5	8·073	30	8·674	68	9·371
6	8·090	31	8·690	69	9·390
7	8·107	32	8·708	70	9·410
8	8·123	33	8·726	71	9·430
9	8·140	34	8·744	72	9·449
10	8·157	35	8·762	73	9·468
Ordinary Hydrometer		36	8·780	74	9·487
		37	8·798	75	9·506
0	8·157	38	8·816	76	9·526
1	8·174	39	8·834	77	9·545
2	8·190	40	8·852	78	9·565
3	8·207	41	8·869	79	9·584
4	8·224	42	8·887	80	9·604
5	8·241	43	8·905	81	9·624
6	8·258	44	8·924	82	9·643
7	8·275	45	8·942	83	9·662
8	8·293	46	8·960	84	9·682
9	8·310	47	8·979	85	9·702
10	8·326	48	8·997	86	9·721
11	8·342	49	9·016	87	9·741
12	8·359	50	9·035	88	9·761
13	8·376	51	9·052	89	9·782
14	8·394	52	9·071	90	9·801
15	8·411	53	9·089	91	9·821
16	8·428	54	9·108	92	9·840
17	8·446	55	9·126	93	9·860
18	8·463	56	9·145	94	9·880
19	8·481	57	9·164	95	9·900
20	8·498	58	9·183	96	9·920
21	8·514	59	9·202	97	9·940
22	8·532	60	9·220	98	9·961
23	8·549	61	9·239	99	9·981
24	8·567	62	9·257	100	10·001

If the decimal point be moved one place to the left, the numbers showing weights per gallon will represent specific gravities, for 1 gallon of distilled water weighs 10 lbs.

The method of using this table is as follows. Suppose that a cask has been weighed first empty and then when filled with spirit, and the weight of the latter thus found to be 600 lbs. Its indication is, say 7·0. Then from the table, the weight of the spirit per gallon is 8·275 lbs. The volume of the spirit is therefore 600÷8·275=72·5 gallons. These bulk gallons, of which the strength is known from the indication and the temperature, are then converted into the equivalent *proof* gallons in the manner already shown. Thus if the temperature is 60° F., the indication being 7·0, the strength is found from the table (*ante*) to be 61·1 over proof. The equivalent number of proof gallons is therefore 161·1 per cent of 72·5=116·8.

In practice, the actual division (weight of spirit in lb.÷weight per gallon) is obviated by the use of tables (Loftus's tables) which have been worked out for the purpose.

§ (9) ALCOHOLOMETRY IN FOREIGN COUNTRIES.—In France, the assessment of spirit duties is made with the centesimal alcoholometer and tables of Gay-Lussac, which date from the year 1824. The range of the alcoholometer (hydrometer) extends from "water" to "absolute alcohol," and is divided into 100 degrees, each degree representing 1 per cent of alcohol by volume at the temperature 15° C. Three separate instruments are used to cover this range. One extends from 0° to 35°; the next from 35° to 70°; and the third from 70° to 100°. If the spirit tested is at the temperature 15° C., the reading of the instrument shows the percentage of alcohol by volume directly: a reading of 40, for instance, indicates that the spirit contains 40 per cent of alcohol by volume.

At temperatures higher or lower than 15° the readings are termed "apparent degrees," and Gay-Lussac's chief table (*Table de la force réelle des liquides spiritueux*) gives the true percentage of alcohol corresponding with these "apparent" or "observed" readings. It shows also the corresponding correction for the change in volume which the spirituous liquid has undergone with the variation of temperature from the standard. The true quantity of alcohol can thus be calculated.

As already mentioned, the alcohol used as the basis of Gay-Lussac's original tables had the specific gravity 0·7947 at 15° C. referred to water at the same temperature as unity. In 1884, however, it was decreed that the graduation of alcoholometers should be based upon a new "table of the densities of mixtures of water and absolute alcohol" drawn up by the National Bureau of Weights and Measures, in which the specific gravity of the absolute alcohol is given as 79·433 at 15° *in vacuo*, water at the same temperature being taken as 100. The effect is to show slightly lower values than those of the original instruments, with a maximum difference of 0·4 per cent.

With slight adaptations, Gay-Lussac's instrument and tables are also used for fiscal purposes in Belgium, Norway, and Sweden. In Spain, both Gay-Lussac's and an earlier French hydrometer (Cartier's) are employed.

In Germany, Tralles's alcoholometer and tables were used during the greater part of the nineteenth century, and the instrument is still employed officially in Italy, and commercially in Russia. It is a glass hydrometer showing directly the percentage of alcohol by volume at 15·6° C. The alcohol taken as basis had the specific gravity 0·7946 at 15·6°/15·6° C. For use at other temperatures, tables were supplied.

Tralles's system has been superseded in Germany by the adoption of an alcoholometer graduated to show percentages of alcohol *by weight* at 15° C. The tables adopted are based upon the results of Mendeléeff's investigations. The official alcoholometers are made of glass, and contain a thermometer in the lower part, so that the one instrument shows both the temperature of the spirit and its alcoholic strength. Although the latter is taken by weight, for the purpose of charging duty the results are converted into *volumes* of absolute alcohol (at 15·6°) by means of tables which show the number of litres of absolute alcohol corresponding with any given number of kilograms of the spirit tested.

In what was formerly the Austrian Empire, Meisner's areometer was used for the assay of spirits. It is an instrument very similar to that of Tralles, but indicating percentages of alcohol both by weight and by volume. The alcohol on which the tables are based had the sp. gr. 0·795 at 12°/12° R. (15°/15° C.).

In Holland the official hydrometer is devised upon a different plan from any of the foregoing. The stem is graduated in terms of the volume of the instrument below the zero mark, each degree being one-hundredth part of this volume. The graduation is thus not arbitrary as with Sikes's instrument, nor does it show percentages of alcohol directly, like Gay-Lussac's or Tralles's alcoholometers. Tables are supplied which convert the indications of the hydrometer into percentage of alcohol by volume at 15° C.; but the standard adopted for fiscal charges is a "proof" spirit, which at 15° contains 50 per cent by volume of absolute alcohol.

A metal hydrometer essentially similar to that of Sikes is used officially in Russia. It is graduated, however, in the reverse manner to Sikes's instrument, water being represented by zero on the Russian hydrometer and strong spirit by 100. The zero mark, therefore, is at the bottom of the stem. Tralles's alcohol is taken as the standard, and the tables used with the instrument show percentage of this alcohol by volume at 12⅘° R. (15·6° C.).

In the United States, the Customs duties upon spirits were formerly levied in terms of *alcohol* percentages. Objections, however, were made to this practice, on the ground that it did not conform to trade usage, which was to buy and sell in terms of *proof spirit*. After inquiry, therefore, by a Committee appointed in 1866 to examine into the whole question of testing spirit strengths, it was decided that "the duties on all spirits shall be levied according to their equivalent in proof spirit," and this system has continued in use to the present time. Tralles's alcohol was taken as the standard, and the United States' proof spirit contains one-half of its volume of this alcohol at 15·6° C. Gilpin's results were largely used in compiling the tables for use with the hydrometers, of which there is a series covering a range of graduations from 0° to 200°. At the standard temperature, 60° F. (15·6° C.), distilled water is represented by 0° on the hydrometer scale, proof spirit by 100°, and Tralles's alcohol by 200°.

In Switzerland, Beck's hydrometer, a modified form of Baumé's instrument, is employed for alcohol testing. Its zero point corresponds with the specific gravity of water at 12·5° C., and the indication 44° with that of Tralles's alcohol.

Table VII. shows for the principal foreign countries the alcoholic strengths corresponding with various percentages of British proof spirit.

§ (10) Alcohol Conversion Equations.—The quantitative relations between specific gravity, proof spirit, and percentage of alcohol by volume and by weight can be summarised in the following equations:

Let S denote the sp. gr. of a specimen of alcohol; P the percentage of proof spirit by volume; V the percentage of alcohol by volume; W the percentage by weight, and G the grams per 100 c.c.

Then $P = 1\cdot7535V = 2\cdot2095WS$;

$V = 0\cdot5703P = 1\cdot2601WS$;

$W = P/S \times 2\cdot2095 = 0\cdot7936V/S$;

and $G = 0\cdot7928V = 0\cdot4521P$.

TABLE VII

FOREIGN ALCOHOLIC STRENGTHS CORRESPONDING WITH BRITISH PROOF VALUES

Great Britain. Proof Spirit. Per cent.	France, Belgium. Alcohol by Volume at 15°.	Italy, Russia, Austria (Tralles). Alcohol by Volume at 15·6°.	United States. Proof Spirit.	Germany. Alcohol by Weight.
5	2·8	2·9	5·7	2·3
10	5·6	5·7	11·4	4·6
15	8·5	8·6	17·2	6·9
20	11·3	11·4	22·8	9·2
25	14·2	14·3	28·6	11·6
30	17·1	17·2	34·4	13·9
35	19·9	20·1	40·2	16·4
40	22·7	22·9	45·8	18·7
45	25·5	25·6	51·5	21·0
50	28·4	28·6	57·3	23·5
55	31·3	31·5	63·0	25·9
60	34·2	34·4	68·8	28·4
65	37·1	37·3	74·7	29·6
70	39·9	40·1	80·1	33·4
75	42·7	42·9	85·8	35·9
80	45·6	45·8	91·4	38·5
85	48·3	48·5	97·0	41·1
90	51·2	51·4	102·8	43·9
95	54·0	54·2	108·5	46·5
100	56·9	57·1	114·2	49·3
105	59·8	60·0	120·0	52·1
110	62·7	62·9	125·7	55·0
115	65·6	65·7	131·3	57·9
120	68·5	68·6	137·0	60·8
125	71·3	71·4	142·8	63·9
130	74·1	74·2	148·4	67·0
135	77·0	77·1	154·2	70·2
140	79·8	79·9	159·9	73·4
145	82·7	82·6	165·6	76·7
150	85·5	85·6	171·3	80·1
155	88·4	88·5	177·1	83·7
160	91·2	91·3	182·7	87·3
165	94·1	94·2	188·3	91·1
170	97·0	97·2	194·3	95·3

C. S.

Alcoholometry. See "Hydrometers," § (19).

Altimeter Scales, Toussaint's Exponential Formula as the Basis of. See "Barometers and Manometers," § (17).

Altimeters, Errors of. See "Atmosphere, Physics of," § (4).

Altimetry, Barometric: the determination of height from observations of pressure (and temperature). See *ibid.* § (4).

American Units of Volume. See "Volume, Measurements of," § (4).

Anemobiagraph: an instrument designed to record the velocity or the force and sometimes also the direction of the wind. See "Meteorological Instruments," § (20) (ii.).

Anemometers: instruments for measuring the velocity or the force of the wind.
Exposure of. See "Meteorological Instruments," §§ (20) (ii.), (23).

Anemometers, Types of:
I. Dines pressure tube:
Calibration of. See "Meteorological Instruments," § (20) (ii.).
Comparison with records of cup-anemometer. See *ibid.* § (20) (v.).
Description of. See *ibid.* § (20) (ii.).
Exposure. See *ibid.*
Recorder. See *ibid.*

II. Fan. See "Meteorological Instruments," § (18).

III. Pressure plate:
Osler pressure plate. See "Meteorological Instruments," § (19) (ii.).
Swinging plate. See *ibid.* § (19) (i.).

IV. Robinson cup:
Comparison with records of pressure-tube anemometer. See "Meteorological Instruments," § (20) (iv.).
Conversion factors for different types of. See *ibid.* § (17) (ii.), (iii.), (iv.), (v.).
Description of. See *ibid.* § (17) (i.).
Recording form of. See *ibid.* § (17) (vi.).
Pressure tube. See "Anemometers," "Dines Pressure Tube."

Aneroid, English Altimeter: basis of graduation of, tabulated. See "Barometers and Manometers," § (16) (iii.) (*a*) Table VII.

Aneroid as Altimeter: effect of changes in the internal structure of the metal of the vacuum-box. See *ibid.* § (18) (iii.).
Errors due to "creep" or hysteresis. See *ibid.* § (18) (iv.).

Aneroid Mechanism, Adaptation of, to a uniform scale of heights. See *ibid.* § (18) (i.).

Angle (of Screw Thread), Definition of. See "Metrology," vii. § (23) (i.).

Angle, Measures of. The symbol π is used to denote the ratio of the circumference of a circle to its diameter.

$$\pi = 3{\cdot}14159265,$$
$$\log \pi = 0{\cdot}49715,$$
$$\frac{1}{\pi} = 0{\cdot}318309886,$$
$$\log\left(\frac{1}{\pi}\right) = \bar{1}{\cdot}50285.$$

(i.) *The Radian.*—The unit of measurement for angles is the radian, which is equal to the angle subtended at the centre of a circle by an arc of length equal to the radius.

$$\begin{aligned} \pi \text{ radians} &= 180^\circ, \\ 1 \text{ radian} &= 57{\cdot}29578^\circ \\ &= 57^\circ\ 17'\ 44{\cdot}81'', \\ 1^\circ &= 0{\cdot}017453 \text{ radians.} \end{aligned}$$

(ii.) *The Point.*—Wind direction is often measured in points where

$$1 \text{ point} = \tfrac{1}{32}\,(360^\circ) = 11\tfrac{1}{4}^\circ.$$

See Vol. I., "Measurement, Units of."

Angström's Compensating Pyrheliometer. See "Meteorological Instruments," § (28).

Anthelion: a patch of light appearing at the point of the sky opposite to and at the same altitude as the sun. See "Meteorological Optics," § (22) (i.).

Anticyclone. An anticyclone is a region where the pressure is high relative to the surrounding pressures. There are two belts of anticyclones forming almost complete girdles around the earth in latitudes 30° N. and 30° S., but in higher latitudes isolated anticyclones occur. In the latter the central regions are distinguished by closed isobars of a roughly circular or oval form.

Near the centre of an anticyclone the winds are light and irregular, with frequent calms, but further out the winds arrange themselves clockwise around the isobars, the surface winds crossing the isobars at a small angle and blowing out of the high pressure region. The anticyclone is conventionally regarded as a region of settled weather, but in practice almost any kind of weather, short of gales, may occur in its area. In summer the weather is frequently fine, but much cloud may occur, with rain in the outer portions. In a winter anticyclone we may find either a sky overcast with stratus (anticyclonic gloom) or warm sunny days followed by frosty nights, with thick fog in the morning.

Anticyclones are usually greater in extent than cyclones and they move much more slowly. When once set up, an anticyclone may remain almost stationary for days, or in some cases for several weeks. The general direction of motion is towards the east or north-east. It has been noted that very cold anticyclones usually move faster than warm anticyclones.

Decumulation of air in. See "Atmosphere, Thermodynamics of the," §§ (15) and (16).
Distribution of pressure, temperature, and density, and height of tropopause in. See *ibid.* § (5), Table III.
Distribution of realised entropy in. See *ibid.* § (6), *Fig.* 10.

Effect of temperature on vertical extent of. See "Atmosphere, Thermodynamics of the," § (7).

General characteristics of. See "Atmosphere, Physics of," § (18).

Pressure gradient in. See *ibid.* § (16).

Types of. See *ibid.* § (16).

Vertical flow in. See "Atmosphere, Thermodynamics of the," § (16).

Wind in. See "Atmosphere, Physics of," §§ (9), (16).

ANTONIUS DE DOMINUS. Demonstration of rainbow. See "Meteorological Optics," § (14).

AQUEOUS VAPOUR, ABSORPTION OF RADIATION BY. See "Radiation," § (2) (ii.).

ARCS:

Circumzenithal. See "Meteorological Optics," § (21) (ii.).

Kern's. See *ibid.* § (21) (iii.).

Of Lowitz. See *ibid.* § (20) (iv.).

Paranthelic. See *ibid.* § (22) (iv.).

Parry's upper. See *ibid.* § (20) (vi.).

Tangent (upper and lower). See *ibid.* §§ (20) (v.), (21) (iv.).

ARIES, FIRST POINT OF: basis of sidereal time at any place. See "Clocks and Time-keeping," § (1).

ARITHMOMETERS. See "Calculating Machines," § (4).

The Thomas. See *ibid.* § (2) (i.).

ASSMANN PSYCHROMETER:

Measurement of temperature by. See "Meteorological Instruments," § (6).

Description and use of. See "Humidity," § (9).

ATMOSPHERE:

Absorption of solar radiation by. See "Radiation," §§ (1) (i.), (3) (ii.).

As a heat-engine. See "Atmosphere, Thermodynamics of the," §§ (1), (24) *et seq.*

Circulation of:

Effect of rotation of the earth. See "Atmosphere, Physics of," § (8).

General:

At sea-level. See "Atmosphere, Thermodynamics of the," ii.

At 8 km. See *ibid.* § (9).

Kinetic energy of. See *ibid.* § (9).

Local. See *ibid.* II.

Dynamics of. See "Atmosphere, Physics of," § (8).

Equilibrium of:

Adiabatic or convective. See *ibid.* §§ (2), (6) (i.).

Isothermal or conductive. See *ibid.* § (6) (ii.).

Radiative. See *ibid.* § (6) (iii.).

Friction of. See *ibid.* § (14).

Kinetic energy of. See "Atmosphere, Thermodynamics of the," § (9).

Origin and maintenance of the electrical field of. See "Atmospheric Electricity," § (15).

Oscillations of. See "Atmosphere, Physics of," § (17).

Reflection of solar radiation by. See "Radiation," §§ (3) (ii.), (4) (i.).

Resilience of the. See "Atmosphere, Thermodynamics of the," § (14).

Revolving fluid in. See "Atmosphere, Physics of," § (15). See also "Density," "Eddy-motion," "Pressure," "Temperature."

Stability of. See "Atmosphere, Thermodynamics of the," § (7).

Thermodynamical processes in. See *ibid.* VI., §§ (22), etc. See also "Pressure," "Radiation," "Convection," etc.

Transmission of solar radiation by. See "Radiation," §§ (3) (ii.), (4) (i.).

Water-vapour in. See "Humidity," § (16).

Wave-motion in. See "Atmosphere, Physics of," § (17).

ATMOSPHERE, PHYSICS OF THE

I. STATICAL ASPECTS

§ (1) PRESSURE.—If we consider an atmosphere at rest horizontally and vertically, the vertical pressure on unit surface must be equal to the weight of the superincumbent atmosphere. Since the pressure on unit area of a surface centred at a particular point in a fluid is independent of the orientation of that unit area of surface, it follows that in speaking of the pressure at a point in a fluid we need not specify any particular direction. The pressure of the atmosphere is measured by means of a barometer. The standard type of mercury barometer consists essentially of a tube from which all air has been excluded, inverted over a well of mercury.[1] The pressure of the air maintains the mercury in the tube at a higher level than the free surface, and the difference in level of the top of the mercury in the tube and the well affords a direct measure of the atmospheric pressure. What is directly measured in this instrument is the length of a column of mercury, but it must be remembered that the pressure is the weight of the column of mercury per unit area of cross-section. Its dimensions are those of weight/area, MLT^{-2}/L^2 or $ML^{-1}T^{-2}$. Pressure may be measured in terms of any convenient unit of length, such as the inch or millimetre, or the scale may be so graduated as to give the pressure in terms of a statical unit of pressure, such as the millibar (see "Bar" and "Millibar.") The millibar is a pressure of 1000 dynes per square centimetre.

[1] See "Barometers and Manometers."

§ (2) Variation of Pressure with Height.—From the definition of the pressure at any point as the weight of a vertical column of unit cross-section with its base at the point considered, we can immediately derive the differential equation for the variation of pressure with height. Let p, ρ, T be the pressure, density, and absolute temperature in degrees Centigrade at a height z, $p+dp$ the pressure at height $z+dz$. Then $-dp$ is the weight of a disc of air of unit area, and of thickness dz.

$$dp = -\rho g dz,$$

$$\frac{dp}{dz} = -\rho g. \quad . \quad . \quad . \quad (1)$$

The equation cannot be integrated until we express ρ and g as functions of z. The value of g is to some extent variable both with latitude and height z above the ground, but these variations are small in comparison with the magnitude of g itself, and will be neglected in the subsequent discussion. In equation (1) substitute for ρ from the gas equation $p=\mathrm{R}\rho\mathrm{T}$.

$$\frac{dp}{dz} = -\frac{pg}{\mathrm{RT}} \text{ or } \frac{1}{p}\frac{dp}{dz} = -\frac{g}{\mathrm{RT}} \quad . \quad (2)$$

If T can be expressed as a function of height z, equation (2) can be integrated. The relation between T and z will depend on the mechanism which controls the transference of heat from one layer of the atmosphere to another. If conduction were the controlling factor, the atmosphere would be in isothermal equilibrium, T being constant at all heights. Equation (2) then yields on integration

$$\log_e p = -\frac{gz}{\mathrm{RT}} + \text{const}.$$

If p_0 be the pressure at $z=0$, the last equation becomes

$$z = \frac{\mathrm{RT}}{g}(\log_e p_0 - \log_e p)$$

$$z = \frac{\mathrm{RT}}{g \log_{10} e}(\log_{10} p_0 - \log_{10} p). \quad . \quad (3)$$

The height in feet is given by

$$z = 221 \cdot 1\mathrm{T}(\log p_0 - \log p),$$

and in metres by

$$z = 67 \cdot 4\mathrm{T}(\log p_0 - \log p).$$

Actually the atmosphere is not in isothermal equilibrium, and a closer approximation is obtained by assuming a linear relation between T and z, *i.e.* by assuming that the temperature T decreases uniformly with increasing height. Let T_0 be the temperature at the ground, and at height z let $\mathrm{T}=\mathrm{T}_0-\beta z$. Equation (2) yields on substitution for T

$$\frac{1}{p}\frac{dp}{dz} = -\frac{g}{\mathrm{R}}\frac{1}{\mathrm{T}_0-\beta z},$$

which can be immediately integrated

$$\log p = +\frac{g}{\mathrm{R}\beta}\log(\mathrm{T}_0-\beta z) + \text{const.},$$

$$\mathrm{T}_0-\beta z = \mathrm{A}p^{\frac{\mathrm{R}\beta}{g}},$$

or if p_0 is the pressure corresponding to $z=0$,

$$\frac{\mathrm{T}_0-\beta z}{\mathrm{T}_0} = \left(\frac{p}{p_0}\right)^{\frac{\mathrm{R}\beta}{g}}. \quad . \quad . \quad . \quad (4)$$

Thus

$$\frac{\beta z}{\mathrm{T}_0} = 1 - \left(\frac{p}{p_0}\right)^{\frac{\mathrm{R}\beta}{g}} = 1 - \left(\frac{p}{p_0}\right)^{0 \cdot 293\beta},$$

if β is measured in degrees C per 100 metres. If we assume[1] as the value for β 6°·5 C. per kilometre, we find for the height in metres the expression

$$z = 153 \cdot 85\mathrm{T}_0\left\{1 - \left(\frac{p}{p_0}\right)^{\cdot 190}\right\}.$$

§ (3) Stability of Equilibrium.—If the atmosphere is in statical equilibrium, the pressure at all points at the same level must be equal, since molecular motion always works in the direction of equalising any horizontal differences of pressure. The variation of pressure in the vertical is given by equation (1), § (2)

$$\frac{dp}{dz} = -g\rho.$$

The fact that the resultant force on a floating body is equal to the weight of fluid displaced requires that the weight of any portion of air in equilibrium must be equal to the weight of an equal volume of the surrounding air. Hence the density of the air must be uniform at any given level. If the atmosphere is homogeneous, it then follows from the gas equation $p=\mathrm{R}\rho\mathrm{T}$ that the temperature is also uniform at any given level.

To determine whether the state of equilibrium in any case is stable, unstable, or neutral, we consider what happens to a small volume-element of the air displaced vertically from its original position. If it is displaced upward it will return to its original level, or be further displaced from its original level according as it is surrounded in its displaced position by air lighter or heavier than itself. The problem of an isolated mass of air moving vertically will be considered in a wider aspect before being applied to the problem in question.

Let p, ρ, T be the pressure, density, and temperature at height z. Let p, ρ', T′, be the pressure, density, and temperature of a small mass of air moving vertically. The acceleration of the moving mass is $-g+(\rho/\rho')g$, or $-g(1-\mathrm{T}'/\mathrm{T})$. In moving upwards the small mass takes at all levels the pressure of the surrounding air. It will be supposed to move adiabatically, *i.e.* without any interchange of heat with the surrounding air. In moving from z to $z+dz$ its pressure is changed by

---

[1] "Air, Investigation of the Upper" (Lapse rate), § (11).

$$dp = -g\rho dz = -\frac{gp}{RT}dz. \quad . \quad . \quad . \quad (1)$$

Let the volume of unit mass of the moving air change from v to $v+dv$ in the same interval. The amount of heat gained is

$$dQ = C_v dT' + Apdv,$$

where C_v is the specific heat at constant volume, and A the reciprocal of the mechanical equivalent of heat.

But $pv = RT'$ and $pdv = RdT' - vdp$,

$$\therefore\ dQ = (C_v + AR)dT' - Avdp.$$

If the pressure had remained constant, we should have had

$$dp = 0 \text{ and } dQ = C_p dT'.$$

Hence $\qquad C_p = C_v + AR$

and $\qquad dQ = C_p dT' - Avdp.$

Since the motion is adiabatic, $dQ = 0$, and

$$dT' = -\frac{A}{C_p}vdp = -\frac{A}{C_p}\frac{dp}{\rho'} = -\frac{gA\rho}{C_p\rho'}dz$$

$$= -\frac{gA}{C_p}\frac{T'}{T}dz. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This equation expresses the rate of change of temperature of the moving air in terms of the ratio of the temperature of the moving air to that of the surrounding air at the same level.

If now we consider a small mass of air originally at level z to be displaced adiabatically to $z+dz$, its change of temperature will be

$$dT' = -\frac{gA}{C_p}dz \quad . \quad . \quad . \quad . \quad (3)$$

from equation (2), since originally $T' = T$. The displaced air will return to its original position if

$$-\frac{dT}{dz} < -\frac{dT'}{dz};$$

it will be displaced further if

$$-\frac{dT}{dz} > -\frac{dT'}{dz};$$

and it will remain in its displaced position if

$$-\frac{dT}{dz} = -\frac{dT'}{dz}.$$

The equilibrium then is stable, unstable or neutral according as $-dT/dz$ is less than, greater than or equal to gA/C_p. The limiting value of the rate of change of temperature with height which gives neutral (or adiabatic, or convective) equilibrium is $Ag/C_p = g/R\ (C_p - C_v)/C_p = 0{\cdot}00986$, corresponding almost exactly to a fall of temperature of 1° for each 100 metres.

If θ denote the potential temperature, defined as the temperature which will be taken up by the element of air concerned if brought adiabatically to some standard pressure, it is evident that in the case of adiabatic equilibrium the potential temperature is the same at all heights. In general the equilibrium of the atmosphere will be stable, neutral, or unstable according as $d\theta/dz$ is $>$, $=$, or $<$ 0.

§ (4) BAROMETRIC ALTIMETRY.—Equations (2) and (3) of § (2) can be applied to compute the height z at which a particular pressure p is observed, provided the mean temperature T is known. If, for example, p_0 be the pressure at the ground ($z=0$) and the atmosphere be assumed isothermal, at temperature T, equation (3) can be directly applied to compute z.

The ordinary altimeter of commerce is an aneroid barometer with a pressure scale graduated in inches, and movable height scale engraved on the outer edge of the dial. The height scale is graduated by the use of equation (3) of § (2), the temperature usually being assumed uniform at 50° F. or 10° C. Allowance for variations in ground pressure is made by rotating the height scale until its zero is opposite the new ground pressure indicated by the index hand.[1]

Two classes of error are inherent in this type of altimeter. In the first place, if the mean temperature from the ground to the height of the observations differs from the standard temperature 10° C., an error is immediately introduced. Putting in the values of the constants involved, equation (3) may be written

$$z = C_p T(\log p_0 - \log p),$$

where $C_p = 67{\cdot}4$ gives the height in metres, and $C_p = 221{\cdot}1$ gives the height in feet. An increase of 1° in T from the standard value 283° abs. increases the height corresponding to pressure p by 1/283, or with sufficient accuracy by 1/300, of the value read off the altimeter scale. The rule for correcting for temperature can thus be stated in a simple form suitable for mental computation. For every 1° in/decrease of temperature from 283° abs. or 10° C. add/subtract $\frac{1}{300}$th of the height read off from the altimeter scale. If the temperatures are on the Fahrenheit scale the corresponding corrections are $\frac{1}{540}$th of observed height for each degree above or below 50° F.

Actually the temperature of the atmosphere is never uniform at all heights. When observations of temperature in the upper air are available, it is usual to take the mean temperature over the range considered and to correct for the deviation of this mean temperature from the standard by the method outlined above.

---

[1] See "Barometers and Manometers," § (16), etc.

The second type of error is introduced by the adjustment of the height scale to changing ground pressures. Let the altimeter height scale be graduated by the use of equation (3) § (2) for a ground pressure p_0. If the height scale be adjusted to give zero height at a ground pressure p_0', then the height interval from p_0' to p will be the same as the height interval from p_0 to $p+p_0-p_0'$, on the unadjusted scale. The apparent height z' is therefore given by

$$z'=\frac{\mathrm{RT}}{g\log e}\{\log p_0-\log(p+p_0-p_0')\}.$$

The true height is

$$z=\frac{\mathrm{RT}}{g\log e}(\log p_0'-\log p).$$

Error in assumed height

$$=z'-z=\frac{\mathrm{RT}}{g\log e}\log\frac{p_0}{p_0'}\frac{p}{p+p_0-p_0'}.$$

This error is zero at the ground ($p=p_0'$), but increases steadily with height, and may amount to over 1000 feet at 15,000 feet, if the ground pressure differs by as much as 1½ inches from the normal.

§ (5) OBSERVED TEMPERATURE GRADIENTS.—The mean distribution of temperature with height[1] in the atmosphere is best explained by reference to the attached diagram (*Fig.* 1), which shows the mean distributions for summer and winter up to a height of 18 km. for the four stations Munich, Strassburg, Trappes, and Uccle.[2] The most striking feature of the diagram is the sudden change in the slope of the curves at a height of about 11 km. Above this limit the temperature remains sensibly constant or even increases slightly, in striking contrast with the steady decrease of temperature from the ground up to the 11 km. limit. The upper region of constant temperature is called the stratosphere, the lower region of decreasing temperature being known as the troposphere. The region of sudden cessation of the lapse of temperature is called the tropopause.

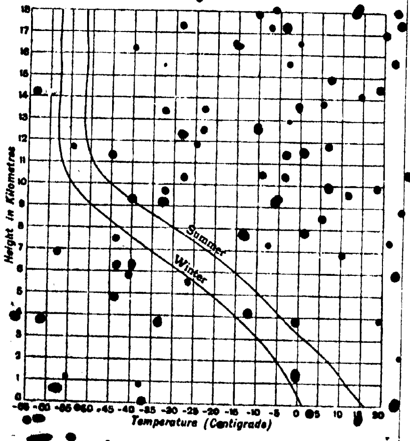


FIG. 1.

The suddenness of the change of lapse of temperature is somewhat masked in the diagram by the process of taking mean temperatures, since the height of the tropopause depends upon season, pressure conditions, and latitude. It is higher in summer than in winter, higher above high pressure regions than above low pressure regions, and higher above the Equator than above polar regions.

Even within the troposphere the temperature lapse shows several striking features. It will be noted that the curves for winter and summer are roughly parallel, showing that the temperature distribution throughout the atmosphere is determined by the same factors, conduction, convection, and radiation, in winter and in summer. The diagram shows that up to 2 or 3 kilometres the rate of fall of temperature is less in winter than in summer. This is due in large part to the frequent inversions of temperature near the ground in winter (see "Inversion").

Between 4 and 8 km. the lapse rate approaches more nearly the "adiabatic" rate (see § (3)). These levels are beyond the reach of surface inversions, and are scarcely affected by turbulent motions produced at the ground. Beyond 8 km. the mean lapse rate decreases steadily, until at about 11 km. it becomes zero.

The curves in *Fig.* 1 represent the mean of a large number of observations. The temperature-height curve for a particular occasion, while following the main features of the curves shown in *Fig.* 1, may show considerable differences in detail. Inversions are frequent at the tops of layers of cloud of certain types, and in winter they occur frequently near the ground. Also outside the regions showing inversions the temperature lapse may show considerable variations from the mean value.

§ (6) THEORETICAL DISCUSSION OF THE TEMPERATURE DISTRIBUTION. (i.) *Isothermal or Conductive Equilibrium.*—By the medium of molecular motion heat is transferred by conduction through the air from the regions of higher to regions of lower temperature, without any mass transfer of air.

Air is, however, an extremely poor conductor of heat, and Exner[3] states that if the diurnal variation of temperature at the ground were transferred upwards only by conduction its amplitude at 1 metre would amount to only ¼ of the amplitude at the ground. Beyond the first few metres the effect would be entirely negligible. We may therefore conclude that conduction is at most an unimportant factor in the vertical transference of temperature. Its effects would be entirely masked by the more rapid transference of heat due to convection, evaporation, and condensation.

---

[1] See "Air, Investigation of the Upper," § (11).

[2] W. J. Humphreys, *Bulletin Mount Weather Observatory*, ii. 183.

[3] F. M. Exner, *Dynamische Meteorologie*, p. 57.

(ii.) *Adiabatic or Convective Equilibrium.*—The first effect of insolation of the earth's atmosphere is to heat those layers of the atmosphere in contact with the earth's surface. If the air near the ground is sufficiently heated it attains a higher potential temperature than the air above it, and the slightest disturbance will then cause it to rise. During the day such ascending currents are frequent in the atmosphere. They move upward without appreciable loss of heat by radiation or conduction, and so act as carriers of heat from lower to higher layers. This process of transfer of heat by mass transfer is known as convection of heat, and the ascending currents are called convection currents.[1] The rate of change of temperature with height within a convection current is given by equation (2), § (3).

The effect of convection currents in the atmosphere, continued over a sufficiently long time, is to produce an increasingly close approximation towards adiabatic or convective equilibrium, in which an element of air moving vertically without loss or gain of heat takes at all heights the temperature of the surrounding air at the same level. It has been shown that when this condition is satisfied the lapse rate of temperature is approximately 1° C. in 100 metres. When the atmosphere is in adiabatic equilibrium any element of air displaced vertically will remain in its new position since there is no force tending to restore it to its original position, or to displace it further. It follows that adiabatic equilibrium is, from its definition, neutral equilibrium.

It has been shown (§ (3) equation (2)) that for adiabatic equilibrium $-dT/dz = Ag/C_p$. By integration this yields $T = T_0 - (Ag/C_p)z$. To express the pressure p as a function of z, substitute for T in the statical equation

$$\frac{1}{p}\frac{dp}{dz} = -\frac{g}{RT} = -\frac{g}{R}\frac{1}{T_0 - (Ag/C_p)z}.$$

On integration this yields

$$\log p = \text{const.} + \frac{C_p}{AR}\log\left(T_0 - \frac{Ag}{C_p}z\right).$$

If p_0 be the pressure at $z = 0$, this gives

$$\log\frac{p}{p_0} = \frac{C_p}{AR}\log\frac{T_0 - (Ag/C_p)z}{T_0},$$

$$\left(\frac{p}{p_0}\right)^{\frac{AR}{C_p}} = \frac{T_0 - (Ag/C_p)z}{T_0} = \frac{T}{T_0} = \frac{p}{p_0}\frac{\rho_0}{\rho}. \qquad (1)$$

For dry air, $AR/C_p = 0{\cdot}2884$, and the equation connecting pressure, temperature, and density becomes

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{0\cdot2884} = \frac{T_0 - (Ag/C_p)z}{T_0}. \qquad (2)$$

Along with this we have the adiabatic equation $p = k\rho^\gamma$ which, putting in the value for γ, may be written

$$\frac{p}{\rho^{1\cdot41}} = \text{constant.} \qquad (3)$$

If in equation (2) p_0 be taken as the standard pressure, T_0 becomes the potential temperature θ. Hence

$$\theta = T\left(\frac{p_0}{p}\right)^{0\cdot2884}.$$

Substituting $T = p/R\rho$, we find

$$\theta = \frac{p_0^{0\cdot2884}}{R}\frac{p^{0\cdot7116}}{\rho}$$

or
$$p = H(\rho\theta)^{1\cdot41}, \qquad (4)$$

where H is a constant.

Equation (4) gives the relation between pressure, density, and potential temperature.

(iii.) *Radiative Equilibrium.*—The accepted explanation of the existence of the stratosphere or isothermal layer occurred almost simultaneously to E. Gold[2] and W. J. Humphreys.[3] The basis of their theory is as follows: Within the stratosphere vertical convection cannot occur to any marked extent, and the temperature distribution must be determined almost entirely by radiation and absorption. If the radiation and absorption are equal the temperature of any portion of the atmosphere remains constant.[4]

The chief difficulty which is met in any direct application of mathematical analysis to this problem lies in our imperfect knowledge of the constants of radiation and absorption which are involved. The discussion here given is a variant of Emden's proof,[5] due to F. M. Exner.[6] The high temperature radiation of the sun is essentially short-waved, while the radiation of the earth and of the earth's atmosphere is essentially long-waved, the maximum of solar radiation being at wave-length $0{\cdot}7\mu$, and the maximum of terrestrial radiation at 10μ. The short- and long-waved radiations are assumed to have coefficients of absorption k_1 and k_2 respectively, and the atmospheric absorption is assumed to be entirely due to water vapour. This assumption is in fair agreement with observations.

---

[1] "Heat, Convection of," Vol. I.

[2] *Proc. Roy. Soc.*, 1909, lxxxii. A, 43.

[3] *Astrophysical Journal*, 1909, xxix. 14.

[4] This principle was previously applied by Schwarzschild in a discussion of the equilibrium of the solar atmosphere, *Nachrichten K. Gesell. zu Gottingen*, 1906, p. 45.

[5] *Sitzber. k. b. Akad. Wiss. München*, 1913, heft I.

[6] F. M. Exner, *Dynamische Meteorologie*, p. 59 *et seq.*

Consider the absorption and radiation in a thin layer of unit area containing a mass dm of water vapour (*Fig.* 2). The bounding surfaces will be assumed plane, the curvature of the earth being neglected.

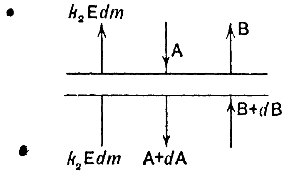


FIG. 2.

Let A_1 = downward moving solar radiation,
A_2 = downward moving terrestrial or atmospheric radiation,
B = upward moving terrestrial or atmospheric radiation,
E = radiation from unit surface of a black body at the temperature of the layer considered $=\sigma T^4$, where σ is Stefan's constant.

In *Fig.* 2, $A = A_1 + A_2$.

The unit of mass of water vapour, denoted by m, is so chosen that $m=0$ at the upper limit of the atmosphere, and $m=1$ at the earth's surface. The symbol m denotes the mass of water contained in the whole atmosphere above a particular level.

By Kirchoff's law, the emission from the upper and lower surfaces of the layer considered is $k_2 Edm$.

Since the temperature of the atmosphere and earth as a whole is to be regarded as constant, the total amount of radiation moving inward across a sphere concentric with the earth is equal to the amount moving outward across the same sphere. Hence

$$A_1 + A_2 = B \quad . \quad . \quad . \quad . \quad (1)$$

and

$$dA_1 + dA_2 = dB. \quad . \quad . \quad . \quad (2)$$

The change in downward moving radiation is due partly to the partial absorption of A_1 and A_2 in the layer, and partly to the addition of the downward radiation from the lower surface of the layer.

$$dA_1 + dA_2 = -k_1A_1dm - k_2A_2dm + k_2Edm, \quad (3)$$

$$dB = k_2Bdm - k_2Edm. \quad . \quad . \quad (4)$$

In the first of these equations the long-waved radiation may be separated from the short-waved radiation, so that

$$dA_1 = -k_1A_1dm \text{ or } A_1 = Ie^{-k_1m}, \quad . \quad (5)$$

I being the value of A_1 at the outer limit of the atmosphere.

Adding equations (3) and (4), we find, using equations (2) and (5),

$$\begin{aligned} 2(dA_1 + dA_2) = 2dB &= dA_1 + dA_2 + dB \\ &= (-k_1A_1 - k_2A_2 + k_2B)dm \\ &= -(k_1 - k_2)A_1dm \\ &= -(k_1 - k_2)Ie^{-k_1m}dm. \end{aligned}$$

Integrating the last equation, we find

$$A_1 + A_2 = B = \tfrac{1}{2}I\frac{k_1 - k_2}{k_1}e^{-k_1m} + \text{constant}.$$

When $m=0$, $\quad A_1 + A_2 = B = I$,

$$\therefore A_1 + A_2 = B = \tfrac{1}{2}I\left[1 + \frac{k_2}{k_1} - \left(\frac{k_2}{k_1} - 1\right)e^{-k_1m}\right]. \, (6)$$

Substituting for B in equation (4), we find

$$E = \sigma T^4 = \tfrac{1}{2}I\left[1 + \frac{k_2}{k_1} - \left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)e^{-k_1m}\right]. \quad (7)$$

According to Abbot and Fowle, solar radiation is diminished by $\frac{1}{10}$th, and terrestrial radiation by $\frac{9}{10}$ths in passing through the whole atmosphere. Hence $e^{-k_1} = 0{\cdot}9$. This gives $e^{-k_2} = 0{\cdot}1$, and $k_1 = 0{\cdot}105$, $k_2 = 2{\cdot}30$. Since k_1m is always less than $0{\cdot}105$ we may for all practical purposes substitute $e^{-k_1m} = 1 - k_1m$, so that the temperature is given by

$$\sigma T^4 = \tfrac{1}{2}I\left\{1 + k_1m + \frac{k_1}{k_2}(1 - k_1m)\right\}. \quad . \quad (8)$$

At the outer limit of the atmosphere $m=0$ and $T = 216°$ abs., $= -57°$ C. At ground level $m=1$, and $T = 288°$ abs., $= 15°$ C., in close agreement with the observed mean value of 14° C.

The temperatures at intervening points cannot be directly deduced until we can state a relation between height h and the mass of water vapour m above this level. Hann[1] has given the empirical formula $e = e_0 10^{-\frac{h}{6000}}$ for vapour pressure at height h. It follows that $m = 10^{-\frac{h}{6000}}$. By a straightforward application of this equation and of the T equation Exner computes a table giving the temperature at different heights. The values are shown diagrammatically in *Fig.* 3. The temperature at 10 km. is given as $-55°$ C., in exact agreement with the observed values. Comparing the diagrams of *Figs.* 1 and 3, we see that radiation alone, if convection were absent, would give much lower temperatures at heights of 2 to 8 km. than the observed temperatures. The temperature lapse at levels of about 2 to 5 km. would be so great that instability would arise through the superposition of dense cold air on lighter air.

---

[1] *Lehrbuch der Meteorologie*, 1906, p. 170.

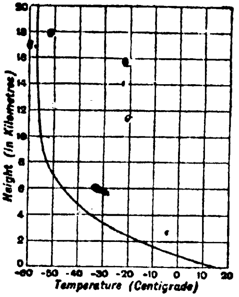


FIG. 3.

There is thus an essential antagonism between radiative and convective equilibrium.

Emden suggests (*loc. cit.* p. 115) that the instability produced by radiation from moderate levels of the atmosphere during the early part of the night accounts for the occurrence of maximum frequency of thunderstorms over the sea during the second half of the night.

§ (7) THE VARIATION OF DENSITY WITH HEIGHT.—The law of variation of density with height has already been derived in the two cases of isothermal and convective equilibrium. It is found that the atmosphere does not in fact approximate to either of these conditions. Observation shows that up to about 25,000 feet the mean density can be very accurately represented by the empirical formula

$$\rho=\rho_0 \, . \, 10^{-0\cdot144 \, . \, 10^{-4}h}, \qquad (1)$$

where ρ_0 is the density at the ground, and h the height in feet.[1] Above 25,000 feet the accuracy of the above formula decreases at first slowly, then rapidly, and above 35,000 feet it ceases to be even an approximate formula for the actual variation. Beyond 35,000 feet it is found that the density is very accurately represented by

$$\log\frac{\rho_0}{\rho}=0\cdot4535+0\cdot21 \, . \, 10^{-4}(h-35,000). \qquad (2)$$

Such formulae as are given above are of particular use in ballistic work. For if a standard pressure and temperature be assumed at the ground (usually 30 inches and 60° F.), the corresponding distribution of temperature can be deduced from equation (1) or (2) combined with the pressure-height relation $dp/dz=-g\rho$. Equation (1) corresponds approximately to a temperature lapse of 2° F. for every 1000 feet of elevation, or 1° C. for 275 metres of elevation.

II. DYNAMICAL ASPECTS

§ (8) EQUATIONS OF MOTION.—Consider a particle at a point O in latitude ϕ. Take rectangular axes Ox, Oy, Oz through this point, rotating with the earth, Oz being the vertical to the earth's surface at O, and Ox and Oy being drawn towards west and south respectively. Let the component velocities be u, v, w. Let the forces acting on unit mass of the particle have components X, Y, Z respectively along the three axes. Then if ω be the angular velocity of the earth, the equations of motion are[2]

$$\frac{du}{dt}-2\omega w\cos\phi-2\omega v\sin\phi=\mathrm{X},$$

$$\frac{dv}{dt}+2\omega u\sin\phi=\mathrm{Y},$$

$$\frac{dw}{dt}+2\omega u\cos\phi=-g+\mathrm{Z}.$$

If there is no vertical motion, $w=0$, and the first and second equations may be written

$$\frac{du}{dt}-2\omega v\sin\phi=\mathrm{X},$$

$$\frac{dv}{dt}+2\omega u\sin\phi=\mathrm{Y}.$$

The effect of the rotation of the earth is represented by the second term on the left-hand side of each of these equations. It is equivalent to accelerations, $2\omega v\sin\phi$ along Ox, and $-2\omega u\sin\phi$ along Oy, whose resultant is an acceleration $2\omega\mathrm{V}\sin\phi$, V being the resultant velocity of the particle. The resultant acceleration is at right angles to the direction of motion, and in the northern hemisphere is directed towards the right of an observer looking forward in the direction of motion.

The result derived above is of fundamental importance in meteorology. It may be stated in the following form: The effect of the rotation of the earth upon air moving with velocity V is equivalent to a force of $2\omega\mathrm{V}\rho\sin\phi$ per unit volume acting at right angles to V, and directed to the right of an observer looking forward in the direction of motion. If this deviating force be included in [illegible] statement of forces acting upon each element of air, the rotation of the axes is immediately accounted for, and the subsequent discussion may be regarded as referred to fixed axes.

§ (9) STEADY MOTION IN FRICTIONLESS FLUID. GRADIENT WIND.—The results of § (8) can be applied to discuss the motion of air over the surface of the earth. For air near the ground it is necessary to consider the effects of friction with the earth's surface and the turbulence produced thereby. We shall first consider the motion of air when these forces are neglected. The results so obtained will be applicable to the winds at heights sufficiently great to be removed from the action of turbulence at the ground.

---

[1] *The Application of Meteorology to Gunnery* (Wedderburn), p. 4.

[2] See Routh's *Rigid Dynamics*, ii. 28.

Steady motion under the conditions prescribed above will be a motion under balanced forces. These forces are (*a*) the gradient of pressure of magnitude P, say, directed towards low pressure, and (*b*) the centrifugal force, of magnitude $\rho(v^2/r)$, where r is the radius of curvature of the path. Consider first the case of air moving along a great circle; its path lies in a plane through the centre of the earth. In this case r is infinitely large, so that the centrifugal force vanishes and the gradient of pressure only need be considered. Let V be the velocity of the wind. Then the deviating force due to the rotation of the earth is $2\omega V\rho \sin\phi$ at right angles to V. This must be balanced by the gradient of pressure P. Hence $P = 2\omega V\rho \sin\phi$ and its direction is at right angles to V, and towards the left. Steady motion under the action of the pressure gradient P is therefore at right angles to the gradient, or along the isobar, with low pressure to the left, and the velocity of the motion is $P/2\omega\rho \sin\phi$. The wind which balances the gradient of pressure is called the "geostrophic" wind.

When the path of the air is not a great circle, but a small circle of angular radius r, it is necessary to consider the centrifugal force involved in the motion. If R be the radius of the earth, the radius of the small circle is R sin r, and the centrifugal force is $V^2/R \sin r$, and the effective component of the centrifugal force, which is tangential to the earth's surface, is of amount $V^2 \cos r/(R \sin r)$ or $V^2/(R \tan r)$.

If the angle r is very small, we may take R tan r in place of R sin r to be the linear radius of the circle. It is in no sense necessary to the argument that the motion should be completely around the small circle. It suffices that the small circle considered should be the circle of curvature at the point of the path considered.

For steady motion the gradient of pressure must balance the algebraic sum of the deviating force due to the earth's rotation, and the centrifugal force. The first of these is of magnitude $2\omega V\rho \sin\phi$ and perpendicular to the direction of V, and directed towards the low pressure; while the second is of magnitude $\rho V^2/R \tan r$, also acting at right angles to wind, and directed towards the centre of the curvature. Two cases arise, according as the centrifugal force opposes or reinforces the gradient of pressure, or according as the low pressure is on the concave or convex side of the isobar; in other words, according as the pressure distribution is cyclonic or anticyclonic. The equations which determine the wind under the two sets of conditions are:

Anticyclonic—

$$\frac{P}{\rho} = 2\omega V \sin\phi - \frac{V^2}{R}\cot r, \quad . \quad . \quad . \quad (1)$$

Cyclonic—

$$\frac{P}{\rho} = 2\omega V \sin\phi + \frac{V^2}{R}\cot r. \quad . \quad . \quad . \quad (2)$$

The wind derived from the solution of the appropriate one of these equations is called the *gradient wind*. It differs from the geostrophic wind in that it takes the curvature of the path into account. In medium and high latitudes the first term on the right-hand side of these equations is usually much more important than the second term, and a good approximation to the wind at a height sufficiently great to be out of reach of the effect of turbulence at the ground is got by taking only the first term into consideration, *i.e.* the geostrophic wind is assumed.

In low latitudes, on account of the factor sin ϕ involved, the geostrophic term becomes unimportant, and the second term only need be retained in the computation of the wind. The wind so derived is called the *cyclostrophic wind*. It may be noted in passing that when the geostrophic component is neglected the cyclostrophic wind is only real for cyclonic curvatures, so that small closed anticyclonic isobars are not possible in equatorial regions.

It is usually considered that the geostrophic wind is a close approximation to the wind at 1500 – 3000 feet. This is based on a detailed comparison by E. Gold (*Barometric Gradient and Wind Force M.O.* 190) of the geostrophic wind computed from M.S.L. isobars with the observed wind at 500 metres. Gold found remarkably close agreement, but on the average the velocities observed at 500 metres were slightly lower than the computed wind.

§ (10) Wind in the Troposphere: Effect of Horizontal Temperature Gradients.—It may be accepted as a basic principle that in any layer out of reach of the effects of surface turbulence the wind will approximate to the geostrophic wind computed from *the gradient of pressure for that particular layer*. But the distribution of pressure at any level depends not only on the distribution of pressure at mean sea-level, but also on the distribution of temperature in the intervening layers.

Let p, ρ, T denote the pressure, density, and temperature at a point x, y, z, the axis of z being vertical, and the axes of x and y any convenient rectangular axis in the horizontal plane. Then in general p, ρ, T are functions of all three coordinates x, y, z. Let u, v be the components of the geostrophic wind parallel to the axes of x and y. Then

$$\left.\begin{aligned} 2\omega\rho u\sin\phi &= -\frac{dp}{dy} \\ 2\omega\rho v\sin\phi &= \frac{dp}{dx} \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

Substituting $\rho = \frac{p}{\mathrm{RT}}$,

$$\left.\begin{aligned} \frac{2\omega\sin\phi}{\mathrm{R}}\frac{u}{\mathrm{T}} &= -\frac{1}{p}\frac{dp}{dy} \\ \frac{2\omega\sin\phi}{\mathrm{R}}\frac{v}{\mathrm{T}} &= \frac{1}{p}\frac{dp}{dx} \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

From the fundamental statical equation

$$\frac{dp}{dz} = -g\rho$$

it follows that

$$\frac{1}{p}\frac{dp}{dz} = -\frac{g}{\mathrm{RT}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Differentiating this equation with respect to x, and substituting from (2)

$$\frac{g}{\mathrm{R}}\frac{1}{\mathrm{T}^2}\frac{d\mathrm{T}}{dx} = \frac{d}{dx}\left(\frac{1}{p}\frac{dp}{dz}\right) = \frac{d^2}{dxdz}\left(\log p\right) = \frac{d}{dz}\left(\frac{1}{p}\frac{dp}{dx}\right)$$

$$= \frac{2\omega\sin\phi}{\mathrm{R}}\frac{d}{dz}\left(\frac{v}{\mathrm{T}}\right),$$

or

$$\left.\begin{aligned} \frac{d}{dz}\left(\frac{v}{\mathrm{T}}\right) &= \frac{g}{2\omega\sin\phi}\frac{1}{\mathrm{T}^2}\frac{d\mathrm{T}}{dx} \\ \text{Similarly}\quad \frac{d}{dz}\left(\frac{u}{\mathrm{T}}\right) &= -\frac{g}{2\omega\sin\phi}\frac{1}{\mathrm{T}^2}\frac{d\mathrm{T}}{dy} \end{aligned}\right\} \quad . \quad . \quad . \quad (4)$$

When there is no horizontal gradient of temperature, or $\frac{d\mathrm{T}}{dx} = \frac{d\mathrm{T}}{dy} = 0$, equations (4) show that the geostrophic wind at any point is proportional to the absolute temperature at that point.

Equations (4) may also be written

$$\frac{du}{dz} = \frac{u}{\mathrm{T}}\frac{d\mathrm{T}}{dz} - \frac{g}{2\omega\sin\phi}\frac{1}{\mathrm{T}}\frac{d\mathrm{T}}{dy},$$

$$\frac{dv}{dz} = \frac{v}{\mathrm{T}}\frac{d\mathrm{T}}{dz} + \frac{g}{2\omega\sin\phi}\frac{1}{\mathrm{T}}\frac{d\mathrm{T}}{dx}.$$

On substituting for u and v from equations (1), and putting $g = -(1/\rho)\,(dp/dz)$, we may write these equations in the form

$$\left.\begin{aligned} -\frac{du}{dz} &= \frac{1}{2\rho\mathrm{T}\omega\sin\phi}\left\{\frac{dp}{dy}\frac{d\mathrm{T}}{dz} - \frac{dp}{dz}\frac{d\mathrm{T}}{dy}\right\} \\ \frac{dv}{dz} &= \frac{1}{2\rho\mathrm{T}\omega\sin\phi}\left\{\frac{dp}{dx}\frac{d\mathrm{T}}{dz} - \frac{dp}{dz}\frac{d\mathrm{T}}{dx}\right\} \end{aligned}\right\} \quad . \quad (5)$$

The condition for constant geostrophic velocity at all heights is

$$\frac{dp}{dx} : \frac{dp}{dy} : \frac{dp}{dz} = \frac{d\mathrm{T}}{dx} : \frac{d\mathrm{T}}{dy} : \frac{d\mathrm{T}}{dz}.$$

This is the condition that the tangent planes to the isobaric and isothermal surfaces should coincide at all points, or that the isobaric surfaces should also be isothermal surfaces.

Integrating equation (4) we find

$$\left.\begin{aligned} \frac{u}{\mathrm{T}} &= \frac{u_0}{\mathrm{T}_0} - \frac{g}{2\omega\sin\phi}\int_{z_0}^{z}\frac{1}{\mathrm{T}^2}\frac{d\mathrm{T}}{dy}dz \\ \frac{v}{\mathrm{T}} &= \frac{v_0}{\mathrm{T}_0} + \frac{g}{2\omega\sin\phi}\int_{z_0}^{z}\frac{1}{\mathrm{T}^2}\frac{d\mathrm{T}}{dx}dz \end{aligned}\right\}, \quad . \quad . \quad (6)$$

where u_0, v_0 are the components of the geostrophic wind at height z_0, and T_0 the corresponding absolute temperature.

Thus the geostrophic wind at height z is made up of

(*a*) A component equal to the geostrophic wind at level z_0 reduced in the ratio T/T_0.

(*b*) A "thermal wind" whose components are

$$-\frac{g\mathrm{T}}{2\omega\sin\phi}\int_{z_0}^{z}\frac{1}{\mathrm{T}^2}\frac{d\mathrm{T}}{dy}dz \text{ and } \frac{g\mathrm{T}}{2\omega\sin\phi}\int_{z_0}^{z}\frac{1}{\mathrm{T}^2}\frac{d\mathrm{T}}{dx}dz.$$

The magnitude of the thermal wind will increase steadily with height within regions where the horizontal gradient of temperature maintains its general direction unchanged. The signs of the components of the "thermal" wind show that it circulates around low temperature in the same sense as the ordinary geostrophic wind circulates around low pressure, *i.e.* with low values to the left. If the temperature decreases towards north the thermal wind is west to east. When the gradients of temperature and pressure are parallel the wind increases with height. When these two gradients are opposed to one another, the wind will decrease with height, and may be reversed at moderate heights if the temperature gradient is relatively large.

The results derived above are not easy to apply to the computation of the geostrophic wind at different levels, on account of the lack of observations of the distribution of temperature in the vertical. They may be directly applied, however, to the large-scale variations of temperature of the globe. In the troposphere the mean latitude variation of temperature is a steady decrease from equator to pole. The effect of this is to superpose upon the winds at low levels a component from west to east increasing with height. This is in agreement with observed facts, accounting in a simple manner for the general increase with height of westerly winds, and the general decrease with height of easterly winds.

For descriptions of detailed investigations of the variation of wind with height, reference may be made to the work of Cave, Dines, Dobson, and Gold.[1] A useful summary is given in the second paper by G. M. B. Dobson referred to above. It is there shown that the wind shows a general tendency to increase with height within the troposphere, while at the base of the stratosphere the velocity falls off rapidly, indicating a reversal of the horizontal temperature gradient within the stratosphere.

If the wind distribution at different heights be known from observation, and the vertical temperature distribution be known or assumed to have mean values, the horizontal temperature gradients can be computed by the use of equations (4) and (6). An example is given below, where the horizontal temperature distribution is calculated from the results of a pilot balloon ascent followed by Capt. Cave on April 29, 1908, at Ditcham Park. Equations (4) may be written in the form

$$d\left(\frac{u}{T}\right) = -\frac{g}{2\omega\sin\phi}\frac{1}{T^2}\frac{dT}{dy}dz.$$

In the example $dz = 1$ km. = 1000 metres, and mean values of T are taken from the *Computer's Handbook*, Section II. p. 55, Table IV. The component velocities to E and N, u and v, are shown in the third and seventh columns.

Ht. in Km.	T.	u.	$100\frac{u}{T}$.	Difference.	$\frac{dT}{dy}10^5$.	v.	$100\frac{v}{T}$.	Difference.	$\frac{dT}{dx}10^5$.	10^5q.	ψ.
6	246	17·7	7·19	..	..	−10·3	−4·15	..	..	..	..
	250	..	..	2·03	1·45	..	..	−1·17	0·84	1·67	30°
5	252	13·0	5·16	..	..	−7·5	−2·98	..	..	..	..
	255	..	..	1·92	1·44	..	..	−2·40	1·80	2·30	51°
4	259	8·4	3·24	..	..	−1·5	−0·58	..	..	..	..
	262	..	..	0·82	0·65	..	..	−0·81	0·64	0·91	44°
3	265	6·4	2·42	..	..	+0·6	+0·23	..	..	..	..
	267	..	..	−0·36	−0·30	..	..	−0·77	0·63	0·70	116°
2	270	7·5	2·78	..	..	+2·7	+1·00	..	..	..	..
	273	..	..	1·22	1·05	..	..	+0·09	−0·08	1·05	356°
1	276	4·3	1·56	..	..	+2·5	+0·91	..	..	..	..

The last column gives the direction of the resultant temperature gradient (increasing temperature) and the last column but one gives its magnitude in degrees Centigrade per 100 km. The calculation assumes that from 1 km. upward the effects of turbulence are negligible, and that all changes in wind are due to the horizontal distribution of temperature. This example gives an idea of the magnitude [illegible] velocity effects produced by temperature gradients—*e.g.* from 4 k. to 6 k. the average temperature gradient is about 2° C. per 100 k. from N.E. and it produces a change in u of +9·3, and a change in v of −8·8, or gives an added wind of about 12 metres per second from S.E. in 2 k. of height.

§ (11) WIND IN THE STRATOSPHERE.—Dobson's analysis of the upper wind observations contained in the publications of the International Commission on Scientific Aeronautics, showed that in the upper region of the troposphere the wind increased steadily, reaching a maximum at the base of the stratosphere, after which there was a pronounced decrease in velocity. These conclusions confirmed those previously stated by Cave (*loc. cit.* p. 61). The change in the nature of the wind variation on entering the stratosphere can only be accounted for by a reversal of the temperature gradient in the lower layers of the stratosphere. With increasing height the pressure gradient must also decrease rapidly.

Observations reaching heights well within the stratosphere are of necessity rare; the discussion of even the small number of observations available has brought to light a number of remarkable facts. It appears that above the column of cold air which represents the low pressure in the troposphere, there is a column of warm air in the stratosphere, while the reverse holds for regions of high pressure. At great heights in the stratosphere there is a tendency for equalisation of temperature in the horizontal as well as in the vertical direction.

§ (12) VARIATION OF WIND WITH HEIGHT IN THE LOWER STRATA.—Hitherto only winds at heights removed from the effects of surface friction have been considered. We shall now discuss briefly the nature of the variations of wind in the strata immediately above the ground. With increasing height it is found that the wind veers steadily, while the velocity increases rapidly in the first 60 or 70 metres, afterwards more slowly up to about 500 metres, at which height the wind usually approximates to the geostrophic wind. The changes beyond 500 metres usually depend largely upon the temperature distribution, and have already been discussed in the preceding sections.

---

[1] C. J. P. Cave, *The Structure of the Atmosphere in Clear Weather*, Cambridge University Press. J. S. Dines, *Advisory Committee for Aeronautics—Reports and Memoranda*, various reports on wind structure. G. M. B. Dobson, *Q. J. Roy. Met. Soc.* 1914, xl. 123, and April 1920. E. Gold, *Barometric Gradient and Wind Force M.O.*, No. 190—*The International Kite and Balloon Ascents M.O.*; *Geophysical Memoirs*, No. 5. Sir N. Shaw, *Manual of Meteorology*, part iv.

Numerous formulae have been put forward as approximate statements of the mean variation of wind in the first 500 metres. Sir N. Shaw and Capt. C. J. P. Cave,[1] in examining the results of observations at Ditcham Park, found that the variation of wind with height could be expressed with fair accuracy by the formula $V_H = (H + a)/aV_0$, where V_H denotes the velocity at height H above the anemometer, V_0 is the velocity recorded by the anemometer, and a is a constant depending on the exposure of the anemometer. A similar formula was found to hold for Pyrton Hill and Brighton, but not so satisfactorily for observations at Glossop Moor in Derbyshire, a high-level station.

Hellman's[2] observations over flat meadow land at Nauen at heights of 2 m., 16 m., and 32 m. above the ground conformed to an empirical formula $v = kH^{\frac{1}{5}}$. In a later paper[3] Hellman gave instead a formula $V = a \log (H + c) + b$, where a, b, and c are constants. This is a more general formula than that of Chapman (M.O. London, Prof. Notes No. 6, 1919), $V = a \log H + b$. Chapman showed that his formula fits with fair accuracy a number of observations made by Dines, Dobson, and Cave. It was found, however, that Chapman's formula gave a very poor fit for the variation of wind with height at Butler's Cross near West Lavington, on Salisbury Plain.

None of these formulae, or of the many others which have been put forward as statements of the variation of the wind with height in the lower layers, afford any indication of the physical causes which underlie the variations. The whole question was put on a basis of physical reasoning for the first time by G. I. Taylor's papers on Eddy Motion. Taylor explained the variation of wind with height as a direct product of turbulence in the atmosphere produced by friction at the ground.

§ (13) TURBULENT MOTION IN THE ATMOSPHERE.—When a fluid moves over an uneven surface the motion is turbulent, as when a stream flows over an uneven bed. The small eddies formed near a projecting rock show most distinctly, but disturbances from uniform motion also occur down near the bed of the stream, and can be made visible by dropping fine sand into the stream.

Similar eddies form in any current of air moving over the surface of the earth. Normally the pattern of the eddies is not visible, but it is made visible to the eye in the trail of smoke from a factory chimney, a garden fire, or a moving steamer. It may be noted in passing that eddies do not form in the trail of smoke from a steamer moving in still air. Under such conditions the trail of smoke forms a uniform flat ribbon sometimes stretching for some miles.

The effect of eddies passing a particular point is to produce instantaneous changes in velocity and direction of the wind, and the effect on an anemometer record is to give a ribbon of varying width for both velocity and direction records, instead of the straight lines which would record constant winds. Usually a broad velocity trace is associated with a broad direction trace. Taylor[4] showed by simple calculation that the mean deviations from the average wind, along and perpendicular to the average direction, were equal. In the same report Taylor also showed from observations of a tethered balloon that the mean deviation of the vertical component of wind was equal in magnitude to the mean deviation of the cross-wind component. If we call the difference between the instantaneous wind and the average wind the eddy wind, it may therefore be taken as fairly well established that the components of the eddy winds in three perpendicular directions have the same mean value, or, in other words, that there is equipartition of energy of the eddy winds in all three dimensions. This is strictly analogous to the equipartition of molecular energy in three dimensions. In Taylor's discussion of eddy motion the eddies form the mechanism which transmits heat, humidity, or horizontal momentum upward or downward in the atmosphere. It is not possible to apply mathematical reasoning to the history of an isolated eddy, but it may be applied to discuss the effect of a large number of eddies, just as the kinetic theory deals with mean motions of molecules.

The power of the atmosphere to transmit heat, etc., vertically is represented by a constant K, the eddy diffusivity, which is roughly equal to $\frac{1}{2}\bar{w}d$, where $\bar{w}$ is the mean vertical component of velocity, and d the mean diameter of the eddies.[5] If θ be the potential temperature at height z, the rate of transmission of heat across unit area of horizontal surface at this height is $K\rho\sigma(d\theta/dz)$. The rate of gain of heat by a disc of unit area and thickness dz is

$$K\rho\sigma\frac{d^2\theta}{dz^2}dz = \rho\sigma\frac{d\theta}{dt}dz, \text{ or } K\frac{d^2\theta}{dz^2} = \frac{d\theta}{dt} \quad (1)$$

This equation is of the same form as the equation for conduction of heat in a solid of conductivity K, except that in the solid θ would become the actual temperature.

Taylor[6] first applied this theory of vertical diffusion of heat in a discussion of observations which he made on SS. *Scotia* off the Great Banks of Newfoundland. Numerous cases were observed of fog associated with inversions of temperature at the surface. Taylor traced backward the history of the air currents involved. In a typical case of an inversion of temperature up to 400 metres, above which the temperature lapse approached the value of the dry adiabatic rate up to 750 metres, the wind current after coming from the interior of Labrador had for three days passed over water which was cooler than itself. The cooling of the air near the sea surface was transmitted upward by eddy motion, and the application of a formula due to Taylor (*loc. cit.*) connecting the height z to which eddy motion would reach in time t, $z^2 = 4Kt/\rho\sigma$, gave a direct means of computing K. The mean value of K derived from such observations was $3 \cdot 10^3$.

---

[1] *Advisory Committee for Aeronautics—Reports and Memoranda*, 1909, No. 9.

[2] *Met. Zeitschrift*, 1915.

[3] *Preuss. Akad. Wiss.*, Berlin, 10, 1917.

[4] *Add. Com. Aer. R. and M.*, No. 345.

[5] G. I. Taylor, "Eddy Motion in the Atmosphere," *Phil. Trans.* A, 1914, ccxv. 1.

[6] *Report of SS. Scotia to Board of Trade*, 1913.

Observations of diurnal variation of temperature at heights of 123, 197, and 302 metres above the base of the Eiffel Tower afforded a means of deducing the value of K over Paris. If in the last equation we put $b^2 = \pi/TK$ a solution of the equation is given by $\theta = Ae^{-bz}\sin(2\pi(t/T) - bz)$. The solution may be made to represent the diurnal variation, which is almost accurately a sine curve, by making $T = 24$ hours. The range of variation is $2Ae^{-bz}$, and the ratio of the total range at two known heights z_1 and z_2 is $e^{-b(z_1 - z_2)}$, from which the value of b and hence that of K may be immediately deduced. Taylor[1] gives a table of values of K deduced by this method for different ranges of heights for each month of the year. The values of K so deduced were greatest in summer, and showed an increase with height in summer, and a decrease with height in winter. The mean value of K so deduced was 10^5. It should be noted that the value of K varied in the same sense as the difference in temperature between the top and base of the Tower. This accords with what might have been anticipated *a priori*, since the frequent inversions of temperature in winter tend to check the upward motion of eddies, while the nearer approach to adiabatic lapse in summer favours the spontaneous formation and upward motion of eddies.

§ (14) **VARIATION OF WIND WITH HEIGHT IN THE SURFACE LAYERS.**—In the discussion of eddy motion in the atmosphere Taylor showed that if u and v be the components of velocity parallel to horizontal axes x and y, the rates of gain of momentum parallel to these axes, in unit volume at height z, are $K\rho(d^2u/dz^2)$, $K\rho(d^2v/dz^2)$, K being the constant used in discussing temperature variations. If the motion be steady the resultant gain or loss of momentum must balance the other forces acting on the element of volume considered. The eddy diffusion of momentum can be regarded as an internal frictional force, and is in fact the "virtual friction" of Ekman.[2]

Now consider the conditions for steady motion of an element of air at O moving in the direction OP (*Fig.* 4). Let Ox be tangential to the mean sea-level isobar,

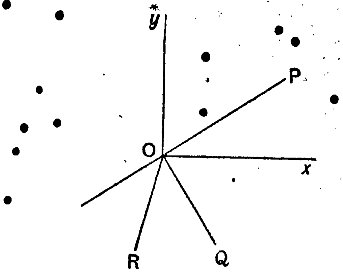


FIG. 4.

Oy perpendicular to it, in the direction of decreasing pressure. Let O be at height z above the ground. The element at O is acted upon by (1) the gradient of pressure along Oy, of magnitude $2\omega G\rho\sin\phi$, where G denotes the geostrophic wind velocity; (2) the deviating force due to the earth's rotation, acting along OQ, whose components are $2\omega\rho v\sin\phi$, and $-2\omega\rho u\sin\phi$; and (3) the virtual friction along OR, whose components are $K\rho(d^2u/dz^2)$ and $K\rho(d^2v/dz^2)$.

Resolving along Ox and Oy we find as conditions for steady motion:

$$\left.\begin{aligned} -K\rho\frac{d^2u}{dz^2} &= 2\omega v\rho\sin\phi \\ -K\rho\frac{d^2v}{dz^2} &= -2\omega u\rho\sin\phi + 2\omega G\rho\sin\phi \end{aligned}\right\} . \quad (1)$$

Multiplying the second equation by i, and adding to the first, and writing $V = u + iv$ we find:

$$\frac{d^2V}{dz^2} = 2i\frac{\omega\sin\phi}{K}(V - G) = (1+i)^2B^2(V - G), \quad (2)$$

where B^2 is written for $\omega\sin\phi/K$.

If we assume that B and G do not vary with height the solution of this equation is

$$V - G = C_1e^{(1+i)Bz} + C_2e^{-(1+i)Bz}. \quad (3)$$

Since we cannot admit of the velocity varying indefinitely with height, we must have $C_1 = 0$. If α denote the angle between the wind at the ground ($z = 0$) and the tangent to the isobar we find[3] on introducing the condition that the slipping at the ground is in the direction of strain that the value of C_2 is $\sqrt{2}G\sin\alpha e^{i\{\alpha + (3\pi/4)\}}$.

$$u - G + iv = V - G = \sqrt{2}G\sin\alpha e^{-Bz}e^{i\left(\alpha + \frac{3\pi}{4} - Bz\right)}, \quad (4)$$

or

$$\left.\begin{aligned} u &= G - \sqrt{2}G\sin\alpha e^{-Bz}\cos\left(\alpha - \frac{\pi}{4} - Bz\right) \\ v &= \sqrt{2}G\sin\alpha e^{-Bz}\sin\left(\alpha - \frac{\pi}{4} - Bz\right) \end{aligned}\right\}, \quad (5)$$

which are equivalent to Taylor's equations for u and v.

---

[1] *Proc. Roy. Soc.* 94 A, p. 141.

[2] *Arkiv f. Mat. Ast. och Fysik*, 1905, Bd. 2, H. 1-2.

[3] See Brunt, "Internal Friction in the Atmosphere," *Q.J. Roy. Met. Soc.*, April 1920, xlvi.

The exponential form admits of a simple geometrical interpretation. If in *Fig.* 5 OG

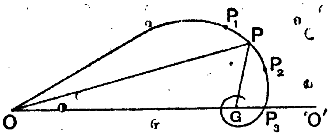


FIG. 5.

represents the gradient wind G in magnitude and direction, and OP represents the wind at height z in magnitude and direction, the vectorial difference PG is represented by the right-hand side of equation (4) so that $PG = \sqrt{2}G\ e^{-Bz} \sin \alpha$, and, therefore, decreases exponentially with increasing height. Also the angle $PGO' = \alpha + (3\pi/4) - Bz$, so that PG rotates uniformly with increasing z. These laws of variation of PG are equivalent to stating that P sweeps out an equiangular spiral[1] to which the vector representing the surface wind is a tangent. The angle between PG and the tangent at P is $\pi/4$.

This spiral affords the simplest method of summarising the distribution of wind with height which is represented by the equation (5). At the ground ($z=0$) the velocity is $G(\cos \alpha - \sin \alpha)$. With increasing elevation the velocity increases, at first rapidly, then more slowly, veering steadily the while. At P_1 it attains the velocity of the geostrophic wind. At P_2 it reaches a maximum, and at P_3 it first attains the direction of the geostrophic wind. The height of P_3 (H_3, say) is given by

$$\alpha + \frac{3\pi}{4} - BH_3 = 0 \text{ or } H_3 = \frac{\alpha + (3\pi/4)}{B}. \quad (6)$$

To find the height of P_1 (say H_1) join P_1G and bisect it at M, and join OM (*Fig.* 6). Then

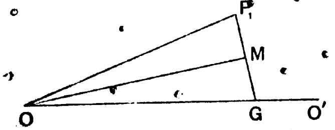


FIG. 6.

using relations previously deduced, we have

$$P_1G = 2MG = 2OG \cos OGM = 2OG \cos PGO,$$

or

$$\sqrt{2}G \sin \alpha e^{-BH_2} = 2G \cos\left(\alpha - \frac{\pi}{4} - BH_2\right)$$

$$\sin \alpha \,.\, e^{-BH_2} = \sqrt{2} \cos\left(\alpha - \frac{\pi}{4} - BH_2\right). \quad (7)$$

If α be known, and also H_1 and H_3, the heights at which the wind attains the direction and the magnitude respectively of the geostrophic wind, equations (6) and (7) afford two independent determinations of B. Observations made by Dobson at Upavon afford estimates of H_1 and H_3, the mean values being about 300 metres and 800 metres respectively. The values of K so deduced were $6{\cdot}2.10^4$, $5{\cdot}0.10^4$, and $2{\cdot}8.10^4$ for strong, moderate, and light winds respectively. These values are somewhat higher than the mean value observed over the sea, 3.10^3, but are nearer to the mean value of 10^5 obtained for eddy diffusivity over Paris from a discussion of diurnal variations of temperature.

Akerblom,[2] from a comparison of the velocity and direction of the wind at the base and at the top of the Eiffel Tower, using equations equivalent to our equations (1), deduced values of K, $6{\cdot}5.10^4$ during winter, and $9{\cdot}23.10^4$ during summer. In a similar way Hesselberg and Sverdrup (*loc. cit.*) deduced the value $K = 5.10^4$ from an analysis of pilot balloon ascents made at Lindenberg.

The "virtual friction" or resultant loss of momentum has been represented by $K\rho(d^2V/dz^2)$. From equations (2) and (4)

$$\begin{aligned} K\rho\frac{d^2V}{dz^2} &= 2i\rho\omega \sin \phi(V - G) \\ &= 2\sqrt{2}i\rho\omega \sin \phi G \sin \alpha e^{-Bz + i\left(\alpha + \frac{3\pi}{4} - Bz\right)} \\ &= 2\sqrt{2}\rho\omega \sin \phi G \sin \alpha e^{-Bz} e^{i(\alpha + \pi - Bz)}. \end{aligned}$$

The virtual friction acts, therefore, at right angles to PG, and at the ground ($z=0$) it therefore acts at an angle of 45° with the surface wind. But if, in *Fig.* 4, R represents the virtual friction, R is the resultant of the pressure gradient and the deviating force due to the earth's rotation. The pressure gradient can be computed from a chart, and the deviating force can be computed from the observed wind, and so R can be found. Akerblom (*loc. cit.*) computed the magnitude and direction of R at the top and at the base of the Eiffel Tower. The magnitude of R is proportional to e^{-Bz}, and, therefore, the ratio of R for base and top of the tower $= e^{-B(z_1 - z_0)}$, and since $z_1 - z_0 = 286$ metres, B can be computed. Also the angle between R at the top and base of the tower $= B(z_1 - z_0)$, and from this the values of B can again be computed. The values of K deduced from the change in magnitude and from the change in direction of R were consistent, yielding $K = 6{\cdot}8.10^4$ for winter, and $K = 9{\cdot}2.10^4$ for summer.

The values of K referred to above are collected in a table for purposes of comparison.

	C.G.S. units.
At sea over Great Banks from temperature distribution	3.10^3
Over Salisbury Plain from wind distribution	5.10^4

---

[1] This is discussed fully and compared with observations by Hesselberg and Sverdrup, *Beit. Phys. Atm.*, 1916, vii. 156.

[2] Upsala, *Soc. Scient. Acta*, 1908, ser. iv. vol. ii. No. 2.

	C.G.S. units.
At Eiffel Tower. Taylor's determination from temperature distribution . .	10.10^4
At Eiffel Tower. Akerblom's determination from wind distribution . .	$7{\cdot}6.10^4$
At Eiffel Tower. Brunt's determination from internal friction	$7{\cdot}4.10^4$
Lindenberg, Hesselberg, and Sverdrup from wind distribution . . .	5.10^4

As might be expected the eddy conductivity is less over the sea than over grassland, and greater over Paris than over either. The closeness of the mean values obtained from the Eiffel Tower observations of temperature and of wind distribution indicate that Taylor's hypothesis that the same agency transmits both temperature and momentum is substantially correct, or that the same constant K is involved in the equations for transmission of heat and momentum.

Taylor's theory thus gives a dynamical reason for the difference between the structure of the lower strata of the atmosphere over sea and land, over town and plain. Since the coefficient K varies with the lapse rate of temperature and with velocity, the wind structure will vary with the time of day and with the season.

It is customary to speak in a rather loose manner of the "regions beyond the reach of turbulence," and it is of some importance to define the height at which the effect of turbulence becomes negligible. This may be defined as the height at which PG becomes negligible in comparison with OG (*Fig.* 5). The wind direction first reaches the geostrophic direction at a height $\frac{3}{4}\pi + \alpha/B$. If we assume $K = 10^{-5}$, $B = 2{\cdot}4.10^{-5}$, and with $\alpha = 22\frac{1}{2}°$, this gives for the height H_1 1145 metres. At this height $e^{-Bz} = {\cdot}064$, and $P_3G = \sqrt{2} \sin \alpha \; {\cdot}064G$ or $P_3G = {\cdot}033G$. At 1500 metres $e^{-Bz} = {\cdot}027$, and $\sqrt{2}e^{-Bz} \sin \alpha = {\cdot}0139$. These figures would indicate that the geostrophic wind should be taken as a measure of the wind about 1000 to 1500 metres. At 500 metres $e^{-Bz} = {\cdot}305$ and $\sqrt{2} \sin \alpha e^{-Bz} = {\cdot}156$, and $\angle PGO' = 90°$, $OP = 1{\cdot}01G$, and $\angle POG = 9°$. Thus at 500 metres the wind should be slightly above the geostrophic wind and should be at an angle of 9° to the isobars, blowing into the region of low pressure. The effect of turbulence will die out at a lower height if B is increased or K decreased. If K is halved B is increased 41 per cent, and the height at which turbulence is negligible is 70 per cent of the original height.

§ (15) THE PERSISTENCE OF MOVING CYCLONES.—The cyclones observed on the synoptic chart usually have a motion of translation to east or east-north-east in the northern hemisphere. It is necessary to consider, therefore, the conditions under which a moving cyclone can persist. The question has recently been discussed by Sir Napier Shaw in two papers. In the first[1] he assumed that the velocity of the wind in the cyclone was everywhere constant. The general result of this discussion was that revolving fluid should be looked for in the secondaries which form in the outer regions of large depressions rather than in the central portions of the depressions themselves. In a later paper[2] a cyclone was assimilated to a spinning horizontal cartwheel endowed with a motion of translation in its own plane. This simple conception helps to explain a number of the salient features of the cyclone, and will be considered in some detail.

In *Fig.* 7 let O be the centre of the rotating disc, moving along OA with velocity U, and let the angular velocity of the disc be ζ. There will be a point O′ on the line through O perpendicular to OA, such that $OO'.\zeta = U$. The point O′ will be the instantaneous centre of the disc, and it is a matter of simple geometry to prove that the instantaneous motion of any other point P in the disc is $\zeta O'P$, perpendicular to OP, equivalent to an instantaneous rotation with velocity ζ about the point O′. The point O′ will thus be the centre of the winds in the cyclone, or the kinematic centre, while O is the centre of the revolving fluid or the "tornado" centre. It will be seen later that the isobars form a system of concentric circles, whose centre is on O′O produced.

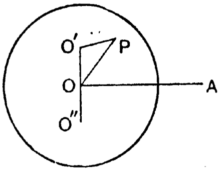


FIG. 7.

It is necessary to show that such a system of motions as is contemplated in the last paragraph is dynamically possible. The problem will here be treated as a two-dimensional problem starting from the Eulerian equations referred to uniformly rotating axes, following the lines of a discussion by Lord Rayleigh.[3] Surface friction and turbulence are neglected, and the atmosphere is treated as incompressible and inviscid. The results of the discussion are therefore only applicable at heights removed from the effects of surface turbulence.

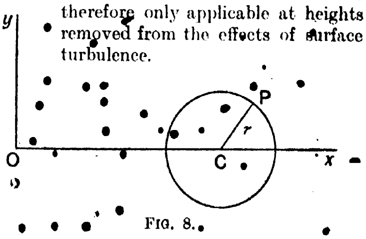


FIG. 8.

In *Fig.* 8 let Ox, Oy be rectangular axes fixed relative to the earth, and rotating with it. They therefore have a uniform rotation of $\omega \sin \phi$ about O. The motion to be investigated is that of a ring of particles of radius r, whose centre C moves with uniform velocity U along Ox, while the ring rotates with uniform angular velocity ζ relative to the moving axes. Let O be the initial position of C at time $t=0$. Then $OC = Ut$. The component velocities of P along Ox, Oy are

---

[1] *Proc. Roy. Soc.* A, 1917, xciv. 34.

[2] *M.O. Geophysical Memoirs*, No. 12.

[3] *Phil. Mag.*, 1919, xxxviii. 420.

$$u = U - \zeta y, \qquad v = \zeta(x - Ut). \quad . \quad . \quad (1)$$

The Eulerian equations give [1]

$$\left.\begin{aligned} \frac{1}{\rho}\frac{dp}{dx} &= \omega^2 \sin^2 \phi x + 2\omega v \sin \phi - \frac{Du}{Dt} \\ \frac{1}{\rho}\frac{dp}{dy} &= \omega^2 \sin^2 \phi y - 2\omega u \sin \phi - \frac{Dv}{Dt} \end{aligned}\right\} . \quad (2)$$

The last terms in these two equations are the component accelerations relative to C, and are therefore equal to $-\zeta^2(x - Ut)$ and $-\zeta^2 y$ respectively.

In equations (2) substitute for u, v, Du/Dt, Dv/Dt. Then

$$\left.\begin{aligned} \frac{1}{\rho}\frac{dp}{dx} &= \omega^2 \sin^2 \phi x + 2\omega\zeta(x - Ut) \sin \phi + \zeta^2(x - ut) \\ \frac{1}{\rho}\frac{dp}{dy} &= \omega^2 \sin^2 \phi y - 2\omega(U - \zeta y) \sin \phi + \zeta^2 y \end{aligned}\right\} (3)$$

But $\qquad r^2 = (x - UT)^2 + y^2$

and $\qquad rdr = (x - Ut)dx + ydy.$

From equations (3)

$$\frac{dp}{\rho} = \frac{1}{\rho}\frac{dp}{dx}dx + \frac{1}{\rho}\frac{dp}{dy}dy, \quad . \quad . \quad . \quad (4)$$

$$\frac{dp}{\rho} = \omega^2 \sin^2 \phi(xdx + ydy) + 2\omega \sin \phi U dy$$
$$+ (2\omega \sin \phi\zeta + \zeta^2)rdr. \quad (5)$$

If this equation can be integrated the motion contemplated is dynamically possible. So far it has only been assumed that ζ is constant for the ring of particles of radius r. The result shown in equation (4) allows of ζ varying with r. In the special case considered by Shaw ζ is constant. Equation (5) then yields on integration

$$\frac{p}{\rho} = \tfrac{1}{2}\omega^2 \sin^2 \phi(x^2 + y^2) + 2\omega \sin \phi U y$$
$$+ \tfrac{1}{2}(2\omega \sin \phi\zeta + \zeta^2)\{(x - Ut)^2 + y^2\} + \text{const.} \quad (6)$$

When the cyclone is at rest $U=0$, and

$$\frac{p}{\rho} = \tfrac{1}{2}(\omega \sin \phi + \zeta)^2(x^2 + y^2) + \text{const.} \qquad (7)$$

The terms in $\omega^2 \sin^2 \phi$ may be neglected by comparison with the terms in $\omega \sin \phi . \zeta$ and ζ^2, and equation (6) may be written

$$\frac{p}{\rho} = \text{constant} + (\omega\zeta \sin \phi + \tfrac{1}{2}\zeta^2)$$
$$\left\{(x - Ut)^2 + \left(y - \frac{\omega U \sin \phi}{\omega\zeta \sin \phi + \frac{1}{2}\zeta^2}\right)^2\right\}. \quad (8)$$

The system of motions contemplated in *Fig.* 10 is thus dynamically possible. Equation (8) shows that the isobars form a system of concentric circles, whose centre is at a point distant $\omega U \sin \phi/\omega\zeta \sin \phi + \frac{1}{2}\zeta^2$ below the centre of the revolving disc. This point is called the dynamic centre of the moving cyclone.

The effect of uniform translation in a straight line upon a system of circular isobars is to displace the centre of isobars to the right of the line of motion of the original centre, and to displace the centre of winds in the opposite direction. It is worthy of note that on a synoptic chart it should be possible to detect immediately the centre of winds and the centre of isobars, but not the tornado centre, or centre of rotating fluid, since this point is not distinguished by any special feature on the chart.

Equation (8) was derived from equation (6) by neglecting the terms in $\omega^2 \sin^2 \phi$. The pressure field of the moving system is thus equivalent to the superposition of an uncompensated field of pressure $2\omega\rho Uy \sin \phi$ on the original circular field of the cyclone at rest. The superposed field has a gradient $2\omega\rho U \sin \phi$ corresponding to a geostrophic velocity U along the axis of x. The dynamical system represented by a set of circular isobars moving with uniform velocity may therefore be regarded as equivalent to fluid rotating about an axis to the left of the line of motion of the dynamic centre, and set in motion by an uncompensated linear field of pressure.

There remains the consideration of the relation of the cyclone to its environment. The assumption of constant vorticity ζ throughout the cyclone involves a discontinuity at the boundary, unless the region of constant vorticity is surrounded by a region of decreasing vorticity and decreasing velocity in which the conditions merge slowly into those of the environment. Shaw [2] states: ". . . the distribution of velocity in the ordinary cyclones of our maps suggests the 'simple vortex' with velocity A/r for the outer margin of a cyclone." This is essentially the same conception as that of Oberbeck.[3] Oberbeck showed that if in a stationary cyclone the velocity were proportional to the distance from the centre in the inner region, and inversely proportional to the distance from the centre in the outer region, there would be no essential discontinuity at the boundary. But there is a striking distinction between the inner and outer regions, in that the inner region has one definite kinematic centre and one definite dynamic centre, while each ring of the outer region has its own kinematic and dynamic centres, whose positions are functions of the radius r of the ring.

Some investigations of Lord Rayleigh on revolving fluid in the atmosphere[4] would indicate that the effect of convection along the core of the revolving fluid would be to produce convergence towards the axis of rotation. The direct consequence of the convergence would be an increase of the pressure gradient in the cyclone, and a superposition upon the normal cyclone (with constant angular velocity) of a simple vortex, so that the tangential velocity v would be given by $v = \zeta r + Ar^{-1}$. There is no adequate means of deciding what happens to the ascended air at the top of the core, though the anvil-shaped thundercloud may provide an analogy on a small scale. It would thus appear that the region of the tornado centre is the obvious region in which to look for convection in the normal cyclone. If local convection should occur outside the central region of the revolving fluid, it would presumably cause a local circulation which in the case of prolonged convection would give rise to a secondary depression.

---

[1] See Lamb's *Hydrodynamics*, 1916, § 207.

[2] *Manual of Meteorology*, part iv. p. 150.

[3] Sprung, *Lehrbuch der Meteorologie*, p. 145.

[4] *Proc. Roy. Soc.* A, 1917, xciii. 148.

It is not possible to say at what height the column of revolving fluid begins. It may possibly come into existence as the result of prolonged convection on a large scale. The rotation leads to the superposition of a field of circular isobars upon the original field of pressure, and the revolving fluid is displaced along the direction of the original isobars. This may possibly account for the existence of "revolving fluid" in secondaries formed within larger depressions, and in large cyclones embedded in fields of straight or slightly curved isobars.

§ (16) THE ANTICYCLONE.—The equation connecting the wind velocity with gradient of pressure is

$$\frac{P}{\rho} = 2\omega V \sin\phi - \frac{V^2}{R}\cot r.$$

If the equation be solved as a quadratic for V,

$$V = R\omega \sin\phi \tan r \pm R\tan r\sqrt{\omega^2 \sin^2\phi - \frac{P\cot r}{R\rho}}.$$

The roots become imaginary if $P\cot r/R\rho$ exceed $\omega^2 \sin^2\phi$. Thus in an anticyclone the pressure gradient cannot exceed a small limiting value, whereas in the cyclone there is no such limit set to the gradient of pressure. This simple relation was adduced by Gold[1] to explain the well-known fact that only light airs can exist in the central regions of anticyclones.

Hanzlik[2] distinguished two types of anticyclones, cold and warm. The cold anticyclone is shallow and moves rapidly. The rapid motion appears to be necessary to the maintenance of low temperature. If the cold anticyclone slows down, the pressure in the centre rises steadily, and as the anticyclone is becoming stationary it begins to become warm. The warm anticyclone reaches to greater heights, and its motion is indefinite. It may come over Central Europe as a fully developed warm anticyclone, or it may originate from a cold anticyclone as suggested above.

Hanzlik's investigations were based on observations made at mountain stations, ranging from the Brocken (1143 metres) to Sonnblick (3106 metres). He found that the axis of the "anticyclonic whirl" was usually inclined towards the north-west. The front is colder, cloudier, and more humid than the rear in both cold and warm anticyclones.

It will be noted that the velocity of translation is of fundamental importance in Hanzlik's classification. The cold anticyclone remains cold only because it moves rapidly. Hanzlik explained the difference between the anticyclones of N. America and those of Europe on this basis, most of the former being rapidly-moving and cold.

The problem of the translation of an anticyclone has not been discussed in the manner applied to cyclones in § (15). Since the direction of rotation is opposite to that of the cyclone, the pressure equation for an anticyclone rotating as a solid disc should be derivable from the cyclonic equation by changing the sign of ζ. The resulting equation is

$$\frac{P}{\rho} = \text{const.} - (\omega \sin\phi\,\zeta - \tfrac{1}{2}\zeta^2)\left\{\left(y + \frac{\omega U \sin\phi}{\omega \sin\phi\,\zeta - \frac{1}{2}\zeta^2}\right)^2 + (x - Ut)^2\right\}.$$

This will represent an anticyclonic distribution of pressure if

$$\zeta < 2\omega \sin\phi.$$

No estimates of the possible value of ζ in an anticyclone are available, but it seems improbable that the anticyclone is to be explained by the last equation.

§ (17) ATMOSPHERIC OSCILLATIONS. (1.) *Free Periods of the Atmosphere.*—The free periods of the elastic oscillations of the earth's atmosphere were first discussed by Rayleigh.[3] A preliminary investigation showed that the vertical oscillations were of very short period and could be neglected. Assuming the atmosphere to form an isothermal spherical shell, and neglecting the effects of rotation and friction, Rayleigh[4] showed that the free periods of the horizontal oscillations are given by $\tau_n = 2\pi r/a\sqrt{n(n+1)}$, where n is an integer, r the radius of the earth, and a the velocity of sound. Taking the observed value for a, which is equivalent to assuming the oscillations to be adiabatic, Rayleigh found for the first 3 periods 23·8 hours, 13·7 hours, and 9·7 hours. It appears probable that oscillations of such long period would be isothermal rather than adiabatic, so that for a the Newtonian value should be assumed. With this substitution the periods are increased to 28·1, 16·2, and 11·5 hours.

---

[1] *Barometer Gradient and Wind Force, M.O.*, No. 190.

[2] *Denkschriften k. Akad. Wiss. Wien*, 1909, lxxxiv. 163-256.

[3] *Collected Papers*, iii. 335.

[4] *Vide Theory of Sound*, § 333.

Margules[1] reconsidered the whole question, and discussed the effect of rotation and friction upon the periods and amplitudes. He showed that the effect of rotation would be to decrease the periods. Rotation reduces the second period of 16·2 hours to 12·3 hours for oscillations along the meridians, and to 11·94 hours for oscillations along the parallels of latitude. The effect of friction is to increase the periods slightly, the effect being proportionately greater for the longer periods. The effect of the large coefficient of friction 0·0001 upon a westward-moving wave of period 13·9 hours is to increase the period to 14·6 hours, so that it may be assumed that the period of 11·94 hours will still be in the neighbourhood of 12 hours when friction is taken into account. This westward-moving double wave has two maxima and two minima upon each parallel of latitude. The observed semi-diurnal pressure wave is probably produced by resonance between this free oscillation and the pressure variation produced by the semi-diurnal temperature oscillation.[2] It is worthy of note that Margules found no free period near 24 hours.

(ii.) *Waves in Surfaces of Discontinuity.*—Helmholtz[3] demonstrated that at the boundary of two media of unequal densities moving with unequal velocities, a sharply defined surface of discontinuity would be formed. Such conditions resemble those which hold when a wind blows over the surface of water, and a series of waves will form in the surface of discontinuity moving forward in the direction of the more rapidly moving medium.

Such a surface of discontinuity will always tend to form in the earth's atmosphere at the bounding surface between the East to West circulation of the polar regions, and the West to East circulation of mid-latitudes. Bjerknes ascribes the formation of cyclones to the waves which form in this surface. Helmholtz showed from considerations of stability that the surface of discontinuity should be inclined to the earth's surface at a smaller angle than the elevation of the celestial pole, except in the special case when the discontinuity is simply a discontinuity of velocity. In the latter case the surface becomes cylindrical, with its axis parallel to the axis of the earth.

Helmholtz also explained the wave-like form of some clouds as due to waves in surfaces of discontinuity in the atmosphere. Such waves will form in the surface of separation of two media of different densities moving with different velocities. So long as the amplitude of oscillation remains small by comparison with the wave-length, the waves retain the approximate form of a sine curve. With increasing amplitude they depart more and more from this form, and eventually break.

The dynamical theory of such waves is extremely complex, and only the main result can be quoted here. Helmholtz[4] and Wien[5] showed that if s_1 be the density of the upper medium, s_2 that of the lower medium, v_1 the horizontal velocity of the wave relative to the upper medium, v_2 the velocity relative to the lower medium, and λ the wave-length of the progressive wave,

$$s_1v_1^2+s_2v_2^2=\frac{g\lambda}{2\pi}(s_2-s_1).$$

Actual observation only yields v_1+v_2, but if the velocity of the wave be assumed to be the same in the two media, or

$$v_1=v_2=\tfrac{1}{2}v,$$

then $$(s_1+s_2)v^2=\frac{2g\lambda}{\pi}(s_2-s_1)$$

will give the wave-length.

Exner,[6] using an analytical method due to Schmidt, re-derived the main Helmholtz-Wien results. Taking the axis of z vertical, and x, y, axes in the horizontal plane, he showed that if there is no velocity parallel to the y axis, and the motion is irrotational, the period T is given by

$$T=\sqrt{\frac{2\pi\lambda}{g}\,\frac{s_2+s_1}{s_2-s_1}}.$$

Reference may be made to a paper by Lamb,[7] in which the theory of waves at a surface of discontinuity, and of waves in an atmosphere with any assumed constant temperature lapse, is developed at some length.

§ (18) SYNOPTIC CHARTS.—The synoptic chart consists of a map on which are inscribed opposite each observing station the barometric pressure, reduced to mean sea-level; the wind indicated in direction by an arrow, the strength on the Beaufort scale being denoted by the number of barbs on the arrow; the temperature; the weather denoted in Beaufort letters; and the tendency, or the change of pressure during the last three hours. The isobars, or lines of equal pressure, are then drawn. These lines resemble contour lines on a map, and they usually take well-defined shapes. The two main types of pressure distribution are the cyclone or depression, and the anticyclone: centres of low and high pressure respectively. Abercromby, in his treatise on *Weather*, distinguished the following seven types of pressure distribution:

---

[1] *Sitzber. Wiener Akad.* ci. part ii.*a*, 597; cii. part ii.*a*, 11 and 1369. Margules' papers are not easy reading, but a very clear account of this work is given by Trabert in *Meteorologische Zeitschrift*, 1903, pp. 481-501.

[2] *Vide* Simpson, "The Twelve-Hourly Barometer Oscillation," *Quarterly Jour. Roy. Met. Soc.* xliv. 1.

[3] *Über atmosphärische Bewegungen*, Ges. Abh. iii. 289.

[4] *Berlin Sitz.-Ber.*, 1889 and 1890.

[5] *Ibid.*, 1894 and 1895.

[6] Exner, *Dynamische Meteorologie*, p. 280.

[7] *Proc. Roy. Soc.* A, lxxxiv. 551.

(i.) *The Cyclone or Depression.*—Distinguished by closed circular or oval isobars enclosing a centre of low pressure, having a total diameter of anything from 100 to 1000 miles, or even more. This is the travelling storm of extra-tropical latitudes. It is a region of strong winds, and generally bad weather. The winds blow round the isobars counterclockwise, and slightly across the isobars into the region of low pressure. Depressions move across the map at rates varying up to thirty miles per hour, the general direction of drift being W.S.W. to E.N.E. Their movements are, however, somewhat irregular. Van Bebber classified the paths of cyclones and showed that in summer the paths converge towards the Arctic seas, but in winter a greater number move towards Central Siberia. (See "Cyclone.")

(ii.) *Anticyclones.*—Distinguished by oval or irregularly shaped isobars, enclosing regions of high pressure. They are usually larger in extent than cyclones. They move very slowly, and may remain practically stationary for days, as much as ten days or more in exceptional cases. They are distinguished by calms or light winds in the central region, and moderate winds in the outer regions, blowing clockwise round the isobars, and slightly out of the high pressure. They are regions of settled weather, but they often bring much fog and cloud in winter. (See "Anticyclone.")

(iii.) *Secondary Depression.*—Sometimes near the outer edge of a depression, most frequently on the southern side, is noted a bulge in the isobars, inside which may appear a secondary centre of low pressure. The secondary may have its own wind circulation, or may only produce a variation in the wind distribution of the main depression. Usually the winds are light or moderate, but the secondary depression usually brings fog in winter, heavy rain, and not infrequently thunderstorms in summer.

(iv.) *V-shaped Depression.*—A variant of the secondary, in which the isobars take the form of a V, usually pointing south. The passage of the trough of the V brings heavy squalls or driving rain, while the wind veers rapidly from a southerly direction to N.W. accompanied by rapid clearing, and a marked fall of temperature.

(v.) *Wedge of High Pressure.*—The region between two depressions. Its front is marked by rapid clearing, and a rapid rise of the barometer, followed by a rapid drop of the barometer, a change of wind from N.W. to S., and clouding over of the sky, indicating the approach of another depression.

(vi.) *Col.*—The region between two anticyclones. It is marked by light airs, and winds converging from the two anticyclones. It occasionally brings brilliantly fine weather, but is usually cloudy, with fogs in cold weather, and thunderstorms in summer.

(vii.) *Straight Isobars.*—It sometimes happens that the isobars are practically straight over a considerable range between a cyclone and an anticyclone. Over the British Isles this happens most frequently with low pressure to north and high pressure to south. It is marked by westerly winds and considerable variety of weather. Straight isobars running north and south, with high pressure to west and low pressure to east, usually bring squally wind, much precipitation, and snow in winter.

Further details of the distribution of weather in the different types of pressure distributions will be found in Abercromby, *Weather*; W. N. Shaw, *Forecasting Weather*; and *Meteorological Glossary*, article "Isobars." E. Gold, *Aids to Forecasting*, M.O. 220f., gives fifteen types of pressure distribution, and an outline of the weather associated with each.

§ (19) TYPES OF WEATHER.—Although cyclones move much more rapidly than anticyclones, it is the anticyclones which determine the type or the general outline of the weather. Four main types of weather can be distinguished according to the position of the dominant anticyclone in the neighbourhood.

(i.) *Southerly type*, with the dominant anticyclone to the east or south-east of the British Isles. Depressions which approach from the Atlantic pass off northwards. It is not usually a very persistent type. The eastern edge of the southerly current has clear skies, but rather cold weather; the western edge has cloudy weather and some rain.

(ii.) *Westerly type*, with an anticyclone extending from the region of the Azores towards S.W. Europe. This condition favours the passage of depressions from the Atlantic in an easterly direction, and produces unsettled weather over the British Isles for periods of as much as six weeks at a time.

(iii.) *Northerly type*, with an anticyclone extending from Greenland in a southerly direction to the west of the British Isles, bringing a cold northerly current along its eastern edge, and favouring the passage of depressions in a southerly or south-easterly direction. These depressions frequently pass over the North Sea, bringing wet weather over Great Britain.

(iv.) *Easterly type*, with an anticyclone over Scandinavia or to the north of the British Isles, and low pressure over France. This produces easterly or north-easterly winds, and the cold weather associated with the east winds of March. The southern edge of the easterly current brings rain, which is frequently very persistent, and snow in winter. Nearer the centre of the anticyclone the weather is bright and cold.

§ (20) WEATHER FORECASTING.—Since the weather associated with a particular type of pressure distribution is usually of a definitely known type, the forecaster's main problem is to forecast the changes in the pressure distribution, or the movements of cyclones and anticyclones. The chief aid to the solution of the problem is the distribution of barometric tendency on the synoptic chart. Regions of negative tendencies indicate regions over which depressions are advancing, or from which anticyclones are receding, while regions of positive tendencies indicate regions over which anticyclones are advancing, or from which depressions are receding. As some depressions move very rapidly the forecaster has to be on guard against surprise by an unforeseen depression.

The first sign of the advance of a depression from the Atlantic over the British Isles is a rapid drift of cirrus from the direction of the depression. The wind backs to east of south over the western coasts of Ireland, and the barometer begins to fall over the same region. The rate of fall of the barometer gives an indication of the rapidity of the advance of the depression. Once the forecaster has determined the changes which are taking place in the pressure distribution, and the rapidity of the change, he can assign the probable general course of the weather. In practice a considerable amount of experience is necessary before a forecaster can with any degree of certainty predict the rapidity with which the pressure changes move across the regions concerned. Difficulties also arise from the fact that depressions do not always follow straight paths, and do not advance with uniform velocity.

On the whole, depressions tend to move so as to keep both high pressure and high temperature on their right. If high pressures and low temperatures are associated together, the rate of motion of the depressions will be slow. It must be admitted, however, that the use of surface temperatures in forecasting does not lead to very definite results for the British Isles. Aitken has suggested that a depression will move in the direction of the strongest winds, and it may be noted in passing that this fits in with the theory of the moving cyclone given in § (15). Upper winds strengthening usually denote a circulation round an advancing depression. Some French meteorologists use as an aid in forecasting the ratio of wind to barometric tendency, high values of this ratio indicating the slow passing away of a depression, low positive values indicating rapid passage of a depression, and low negative values indicating a region threatened by an advancing depression. Such rules may be of value in special cases, but it is to be feared that when the barometric tendencies fail to give a true indication of coming changes, all other rules fail.

Dr. Nils Ekholm suggested that a chart should be drawn indicating the changes in pressure since the last time of observation, lines of equal change, or isallobars, being drawn, enclosing regions of rising and of falling pressures. These regions are very clearly defined on the isallobaric chart. They drift across the map in a more regular and persistent manner than cyclones and anticyclones, and for considerable periods they follow the same path.

Apart from the question of forecasting the pressure distribution there arise difficulties where the details of the coming weather are concerned. The element which can be forecasted with most confidence is wind. In winter it is extremely difficult to forecast snow. The physical conditions which determine whether precipitation will take the form of rain or snow have not yet been fully formulated. In general, it is not easy to forecast the amount of precipitation which a depression will bring. The problem will probably be solved by the multiplication of observations of temperature in the upper air. If the lapse rate of temperature in the lower strata is well below the adiabatic rate heavy precipitation should be improbable, while a lapse rate near the adiabatic should favour precipitation. A lapse rate greater than the adiabatic indicates instability, which may, other conditions favouring, lead to thunderstorms.

In forecasting cloud amounts use is made of the well-marked diurnal variation.[1] There is a well-marked tendency for cloud to [illegible] at sunset, particularly Cu., St.-Cu., and to a less extent stratus and alto-stratus. Use should be made of this diurnal variation only in conjunction with a synoptic chart, since the changes of pressure may modify the diurnal changes on any particular occasion.

§ (21) NORWEGIAN METHODS OF FORECASTING.—Norwegian meteorologists are at present engaged on the development of what promises to be a very fruitful line of attack upon the problem of forecasting. They regard the weather of the northern hemisphere as largely dependent upon the existence of a surface of discontinuity between polar and equatorial air. Helmholtz[1] showed that such a surface of discontinuity should tend to form, and Professor Bjerknes and his associates claim that it can be detected at the earth's surface as a line of discontinuity in atmospheric conditions. The polar air is cold, dry, and transparent, often moving from an easterly point, and the equatorial air warm, moist, with poor visibility, always blowing from a westerly point. The line of discontinuity which is called the "polar front" passes through the centres of cyclones, connecting the centre of one cyclone with those of the preceding and following cyclones. The part from the centre to the front margin of the cyclone is called the steering surface or "anaphalanx," and the part from the centre to the rear margin the squall surface or "kataphalanx." All the weather incidental to the passage of the depression is referred to the anaphalanx and kataphalanx.

---

[1] See Brunt, Prof. Notes, No. 1 (Met. Office), tables for Kew and Greenwich showing relation between wind direction and cloud amount.

In *Fig.* 9 OA and OB represent the anaphalanx and kataphalanx respectively. The equatorial current meets the polar current sideways at OA, and climbs up over it, giving a broad band of rain over the shaded region to northward of OA. The cold air bends round in a southerly direction to the west of O, and attacks the warm current in the flank, pushing under it and raising it, giving a narrow band of rain, shown shaded on the western side of OB. The heavy rain associated with the depression should be limited to the shaded regions. Light showers may occur elsewhere, but not heavy rain.

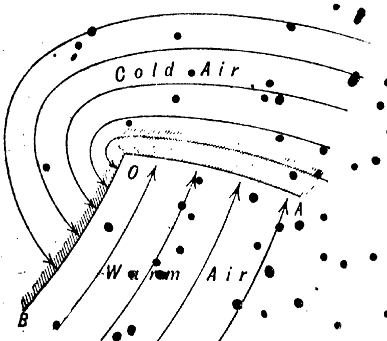


FIG. 9.

At present the investigations of the Norwegian meteorologists are incomplete, and no judgement is possible as to the final value of their ideas, so far as application to the weather of the British Isles is concerned. It is a practical problem to trace the polar front in the whole of its course round the globe, and a network of observers will be required to keep track of its changes from day to day.

It remains to be seen whether a cyclone is to be accounted for as an almost stationary wave in the polar front. This would appear to require that the axis of the cyclone should always be inclined in a northerly direction.

§ (22) THUNDERSTORMS. — Thunderstorms[2] are associated with the formation of heavy cumulus or cumulo-nimbus clouds. They frequently advance across country in a long narrow belt moving transversely to its length. The first sign of the approach of a thunderstorm is the formation of heavy clouds of cumulus type, with round heads, and very sharply defined edges. The wind freshens, blowing at first towards the advancing storm, while the barometer falls slowly. At the onset of the storm the wind changes its direction, and blows from the storm outward, and the barometer rises rapidly through from one to three millibars. Heavy rain falls, usually mixed with hail, accompanied by lightning and thunder. After some fluctuations during the passage of the storm the barometer becomes steady, usually at a slightly higher level than before the storm. The temperature is usually high before the onset of the storm, but begins to fall when the wind reverses its direction, and after some fluctuation during the passage of the storm becomes steady at a lower level than before the storm. The rain is heaviest when the storm is overhead, and lightning and thunder are then most intense, but there may be a continuance of rain for some hours after the storm proper has passed, with no accompanying thunder and lightning.

The necessary condition for the occurrence of a thunderstorm is the production of convection on a large scale, in air of a sufficiently high degree of humidity to produce towering cumulus clouds. Capt. C. K. M. Douglas,[3] in a discussion of upper air data relating to thunderstorms, points out that in severe thunderstorms the base of the thunder-clouds is usually between 5000 and 6000 feet, the tops attaining 20,000 feet in summer, and 15,000 feet in winter. He adds that it appears to be a necessary condition for the development of the storms that there should be at least one damp layer as low as 6000 feet. The higher the humidity the greater is the chance of a thunderstorm.

---

[1] Helmholtz, "Über atmosphärische Bewegung," *Sitzber. K. Pr. Akad. Wiss.*, 1888; see also Emden, *Gaskugeln*, chap. xviii. B; also a paper by Shaw, *Proc. Roy. Soc.* lxxiv. 20.

[2] See also "Atmospheric Electricity," § (2), etc.

[3] Meteorological Office, Professional Notes, No. 8.

Convection on the requisite scale may be produced by

(i.) Strong surface heating of air by insolation on clear days with light surface winds.

(ii.) The overrunning of a warm layer of air by a layer at a lower potential temperature.

(iii.) The underrunning of a surface layer of warm damp air by a colder layer which causes it to ascend.

These three causes will be considered in turn.

(i.) *Convection due to Surface Heating.*—This can only be effective when the pressure distribution is irregular and the surface winds are light. In still, sunny weather the air in contact with the ground may in places be heated to such an extent that it becomes potentially warmer than the air above it, with the result that it rises. But it is not possible to suppose that instability on a large scale can be directly produced in this manner. Any element of air which is lighter than the surrounding air should rise the moment it is disturbed by the slightest turbulence. There is no reason for supposing that it maintains its position until a large reservoir of instability is produced and develops violent motions on a large scale. It seems rather that convection is first set up on a small scale, a big bubble of the warm air ascending and being replaced by warm air drawn in from around the ascending column, with a compensating settling down of air from above. The ascending column will continue to rise so long as it is surrounded by air denser than itself. When the vertical motion ceases, the air from the ascending column spreads out horizontally. The convection will attain very great heights if the upper air is very cold. The process is therefore most likely to occur in early summer (May or June) before the upper air has attained its maximum summer temperature. If the ascending air is very damp it quickly reaches its dew point, and a cloud is formed by condensation. The convection column and the cloud are carried forward by the general current of air above, and in consequence the indraught of warm air is chiefly from the front, while the cold air from above settles down in the rear of the ascending column. The rise and descent of air, once established, will be self-perpetuating so long as a supply of warm surface air is available to renew the ascending branch. It should be noted also that instability can be produced when a layer of saturated air (or a cloud) is below a layer of dry air, and the two strata are lifted bodily. The lifting causes the upper layer to cool at the dry adiabatic rate, while the lower cools at the saturated adiabatic rate, *i.e.* the upper layer cools nearly twice as rapidly as the lower, so that a stage of instability is soon attained, leading to violent convection. Strata of unequal humidity may be caused to ascend by the upward push of convection currents from the ground, and in such a case the resulting convection strengthens the effect of the surface convection currents.

Thunderstorms due to convection currents produced by surface heating occur at those times of day when convection is greatest, in the middle of summer afternoons.

The essential condition for the occurrence of convection extending to great heights is that there should be a rapid decrease of temperature with height. In the case considered above, of thunderstorms on land, this condition is produced by heating of the lower layers by insolation. The necessary instability may also be produced by the cooling of the upper layers by radiation, and this can happen above the sea at night, since the diurnal variation of the surface layers of air over the sea is very slight, thus accounting for the frequent occurrence of thunderstorms at sea during the latter part of the night.

(ii.) *Overrunning of a Warm Layer by a Cold Current.*—When this occurs, the upper cold air is heavier than the lower warm air, and instability results. The upward movement is usually started by local irregularities of pressure, and once started it develops with great rapidity, producing violent storms. Over England thunderstorms of this class occur most frequently in conjunction with powerful south-westerly currents in the upper air, above warm southerly or more particularly south-easterly currents at the ground. The heating of the surface currents by insolation is not a necessary feature of such storms, and as might be expected they occur as frequently by night as by day. Summer thunderstorms which occur in cols, between two anticyclones, are frequently of this class.

(iii.) *Underrunning of a Warm Current by a Cold Current.*—In this case the underrunning cold current causes the warm current to ascend. This forced convection may only produce cloud, or it may be sufficient to produce thunderstorms. Much of the rain associated with the passage of the trough of a depression may be ascribed to this cause, a cold westerly or north-westerly current undercutting the warmer south-[illegible]ly current of the south-eastern quadrant of the depression. This appears to be the genesis of coastal thunderstorms frequently produced by the passage of troughs of depressions, in late summer and autumn. In slow-moving depressions the south-westerly current is also frequently very cold, and it may produce thunderstorms by underrunning the warm south-easterly current of the front of the depression. Thunderstorms of this class are not usually very violent, but occasionally violent storms occur near the centre of a depression. They may occur at night or by day, in summer or winter.

In the ascending current of air, produced in any one of the three ways considered above, considerable condensation must occur at relatively low levels. The drops of water are held up by the ascending current if the velocity of ascent is sufficiently great. Lenard[1] showed that an ascending current of 8 metres/sec. would prevent drops of any size from falling, while a stronger current would carry the drops upward. Lenard showed that drops have a limiting size, beyond which they become unstable and break into smaller drops. The upward current in the thundercloud is not steady, but blows in gusts and lulls, so that the drops of water may rise and fall repeatedly, coalescing and breaking up again through collision or instability. The drops which get to the edge of the ascending current or reach the head of the current and spread out horizontally with it fall to the ground, giving the heavy rain of the thunder-squall. Simpson[2] showed that the ascending current of the front of the storm is a sufficient mechanism to account for the large amount of electricity which is separated in the storm. By means of a series of experiments Simpson showed that each time a drop breaks up, the air takes a negative charge and the water drops a positive charge. The process of coalescence and redivision of the drops therefore produces an increasing charge of positive electricity on the drops, the charged drops combining with facility to form large drops. Simpson showed that the process was capable of producing charges quantitatively of the order of those which he measured at Simla. The negative charges which are presumably in the form of free ions will be carried upward with the full velocity of the rising current, and will soon outstrip the positively charged drops. These free ions must be rapidly absorbed by the cloud particles, and the charged drops rapidly combine to form large drops, giving a considerable rainfall. The negatively charged drops will be distributed through the cloud-cap over a considerable region outside the zone of the ascending currents, and will give a more uniform rain than the positively charged drops, which on account of their intimate connection with the ascending currents tend to give the heavier rain associated with the front of the storm.

The formation of hail in a thunderstorm is in itself a proof of the existence of strong vertical currents. If the cumulus cloud is sufficiently high the condensation in its upper region will be in the form of snow, and any drops of water attaining this level will be frozen immediately, and will gather a coating of snow or ice in any further wanderings. If later on an incipient hailstone gets into a lull in the upward current it falls down and may again get into the region of raindrops, where it gathers a coating of water. If it again gets into an upward current it is lifted into the region of snow where the coating of water is frozen and a fresh layer of snow added. The process may be repeated over and over again until the hailstone gets out of reach of the ascending current, or becomes too big to be held up. Its structure, consisting of alternate layers of ice and snow, testifies to its history, and at the same time affords a convincing proof of the existence of strong ascending currents in the thunder-cloud.

Photographs of lightning taken with a rotating camera show that it consists mainly of unidirectional discharges. The path of the discharge is built up piecemeal by progressive ionisation, beginning with a small branching spark, followed in a small fraction of a second by another which is somewhat longer, and the process continues until the main line of the discharge is built up. A photograph of a flash of lightning bears a close resemblance to a map showing a river and its tributaries. The accumulation of a large amount of electricity in a small space which is incidental to the passage of lightning, produces a strong repulsion of electrified particles, so that a compression wave of air of explosive violence is produced, followed by a wave of rarefaction, succeeded by waves of less intensity. W. Schmidt showed that only a very small part of this energy takes the form of audible waves, so that we hear only a very small part of thunder. It may be noted that lightning affects wireless apparatus, and a directional wireless set can be adapted to detection of thunderstorms. By the use of a number of directional stations it has been found possible to follow the course of thunderstorms for long distances.

The rapid drop of temperature noted at the onset of a thunderstorm is in large part to be ascribed to the evaporation of the falling rain. The descending current of air which settles down to fill the place of the ascending air is dynamically warmed, and is below saturation, so that it is capable of producing considerable evaporation of the falling raindrops, its own temperature being reduced thereby.

The rapid rise of barometer noted at the onset of a storm is probably due chiefly to the interference with the horizontal flow of air produced by friction between the thunderstorm gust and the ground. Other factors contributing to the increase of pressure would be the vertical wind pressure due to descending air, the lowering of temperature, and the decrease in absolute humidity. D. B.

---

[1] *Met. Zeitschrift*, 1904, xxi. 249-262.

[2] *Phil. Trans. Roy. Soc.*, 1909, ccix. A, 379-413; also *Phil. Mag.*, 1915, xxx. 1.

Atmosphere, Pressure of:

Methods of measuring. See "Atmosphere, Physics of," § (1).

Variation with height. See *ibid.* § (2).

Atmosphere, Standard: a unit of pressure employed by the physicist, defined as the pressure due to the weight of 760 mm. of mercury at 0° C. and at sea-level in latitude 45°. See "Barometers and Manometers," § (2) (ii.).

ATMOSPHERE, THERMODYNAMICS OF THE

I. Introduction

§ (1) The Atmosphere as a Heat-engine.—From the thermodynamical point of view the atmosphere may be regarded as a heat-engine, or an assembly of heat-engines, of vast dimensions, of which air, containing more or less moisture, is the working substance. The ordinary cycle of a heat-engine comprises the transfer of heat to the working substance at high temperature from the source, the reduction of temperature by expansion, the retransfer of heat from the working substance at low temperature, the elevation of temperature by compression, and the performance of work in the course of the completed cycle.

Bearing in mind the several stages of the process of the reversible cycle of an ideal engine, the place of the cylinder or chamber in which the working substance is enclosed, and that of the piston by which mechanical work is done, are both supplied by the environment of the working air. The environment, of itself, forms, effectively, an elastic piston: after the air has left the ground there is no direct communication of heat from outside except by radiation, and the transference of heat by conduction or molecular diffusion between the working substance and its environment is so slow that we may regard the changes in the mixture of dry air and moisture which constitutes the working substance as taking place under adiabatic conditions except in so far as the duration of the operations is long enough for the effects of radiation to become important. But the dry air regarded separately may appropriate the latent heat of any water that is condensed and discarded, so that the adiabatic conditions have to be interpreted cautiously.

The essential difficulty of the general problem is, in fact, to specify the conditions for the transfer of heat and performance of work in the atmosphere in such a form as to bring the quantities within the possibilities of computation.

We know of places where heat passes between the air and the surface of land or sea, and we know that the process of radiation into space disposes of a large quantity of heat from all parts of the atmosphere. Some of it may have been contributed locally by direct radiation from the sun, but the loss is only completely compensated when the heat deriving indirectly from the sun through the agency of warmed land or water is added. We are unable strictly to identify the air which gains heat in one locality with that which previously lost heat, or will lose it subsequently, in another. If we wish to deal numerically with the processes we must first come to some understanding as to what the cycle of operations is.

Regarding the atmosphere as a whole, the transfer of energy either as heat or work from a selected portion to its environment is only the redistribution of energy between the different parts of the working substance, and the problem reduces itself to considering the transfer across the boundaries; and in this aspect the heat taken in is made up of: (*a*) the balance of heat communicated from the ground, or the sea, to the air which lies upon them or moves over them; (*b*) the balance of gain and loss by radiation from sun to earth and from earth and air to space. The difference of these at any time is accounted for by the variation in the kinetic energy of the winds or the work done thereby: but in the long run the total thermal effect is immeasurably small, and the solar energy which passes through the atmosphere simply maintains the *status quo ante*. We cannot deal, therefore, with the relation between heat and work by regarding the whole atmosphere as a unit. Meteorological literature provides only a general equation for balance of radiation, and the distribution of temperature incidental thereto (§ (11)). The dynamical effects, which are patent to all, and are by common consent attributed to the heating produced by the sun combined with the cooling by radiation into space, are dependent upon the differential treatment of different parts of the atmosphere. To bring the process under computation we must suppose a portion of air to be isolated and trace the thermal changes which it experiences.

If, then, we regard the phenomena exhibited by separate masses of air we have no difficulty in finding evidence of all the separate stages of the thermal cycle of a heat-engine. We know that air is warmed by contact with warmed solid or liquid surfaces, and that it is cooled by contact with cooled surfaces. We may also suppose that it is warmed and cooled by its own absorption of solar radiation and by radiative emission of its own energy respectively, though we cannot assign any very satisfactory numerical values to the experience of any particular specimen. The dynamical effects which represent the work done by the working substance as the result of the thermodynamical operations are represented by the variation of the kinetic energy of the winds, which are capable of doing a vast amount of useful work on sails and windmills, and still more obviously display on occasions a good deal of destructive energy. There is a constant dissipation of the kinetic energy through the formation of transient eddies due to surface obstacles or to effective discontinuities arising from the differences of motion of adjacent or superjacent layers, ultimately lost by viscosity. But a considerable amount of the kinetic energy is conserved in the form of the main air currents of the globe, or in the form of cyclones and cyclonic depressions, which differ from the transient eddies in utilising favourable conditions to maintain their form, structure, and a large part of their velocity for days or even weeks.

In so far as the general circulation, or the local circulations, are permanent, we may, if we please, assume that they require a certain supply of energy for their maintenance and, failing any such, that they would decay rapidly; but presumably not nearly as much is required per unit of energy maintained as that which must be supplied to maintain a cliff-eddy—that is, the eddy which is formed at any time at the edge of a cliff in the face of a strong wind. It draws its energy persistently from the wind, and ceases immediately when the wind drops.

Another of the familiar experiences of the thermodynamic operations of the atmosphere is the formation of clouds and the fall of rain and from these we may conclude that convection is operative in the atmosphere on a very large scale, for rain is now regarded as produced almost exclusively by dynamical cooling due to the reduction of temperature on reduction of pressure. The reduction of pressure is for the most part attributed to the increase of elevation; rainfall can therefore be regarded as definite evidence of ascended air.

(2) Units of Measurement and Physical Constants for Dry Air and Mixtures of Air and Water-vapour.—We remark at once that the numerical values of the quantities with which we are concerned in any discussion of the practical aspects of thermodynamics are very large.

For meteorological work the most convenient scale of temperature is the scale of centigrade degrees, measured from 273° below the normal freezing-point of water. For all practical purposes this scale agrees with the scale of the hydrogen thermometer and with the absolute thermodynamic scale; it is, however, not strictly speaking "absolute," and therefore to avoid misunderstanding it is desirable to have a separate name and a separate notation for it. Following the practice of the *Computer's Handbook of the Meteorological Office*, we shall call it the "tercentesimal scale," and denote the temperature on that scale by the unit symbol a without any sign of degrees. The chief advantage of the name is that it is not used for any other purpose, and is therefore not likely to cause confusion. The algebraic symbol for temperature on this scale will be T.

In expressing pressures we shall follow the practice adopted by the Meteorological Office from the commencement of observational work upon the upper air in this country by using 1000 dynes per square centimetre as the working unit. In this article, as in the practice of the Meteorological Office, it is called the millibar.

A cube of air 10 metres in the side weighs about 1·25 metric tons. In summer on the average in this country it would contain 10 kilogrammes of water-vapour, and would supply water enough to cover the base of the cube with rain to the depth of 0·1 mm.

All the water except ·03 gramme per cubic metre could be extracted by reducing the temperature to 222a, by elevating it to about 11,000 metres, where the pressure would be 232 mb.; hence, confining attention to a limited area, a fall of rain of 1 millimetre would correspond with the desiccation of a column of air 100 metres high and no more. We have to consider whether such a process may be regarded as natural. One millimetre of rainfall over a square kilometre represents a million kilogrammes or a thousand metric tons. The dynamical equivalent of the thermal energy set free by the condensation of water to the extent of a millimetre of rainfall over a square kilometre is 6×10^{11} thermal units or $2{\cdot}5 \times 10^{12}$ joules, about a million horse-power-hours. And as the practical unit of area for the fall of rain may be regarded as a hundred kilometres square, the energy with which we have to deal in the ordinary way is of the order of ten thousand million horse-power-hours. It must be remembered that when rainfall is produced energy to the corresponding extent must be disposed of. It is not uncommon to find suggestions that air may be "supersaturated" before rainfall. There is no evidence in support of the view, but even if it were true the disposal of the energy is not avoided; it must have taken place in order to produce the supersaturated air.

The properties of dry air and of water-vapour and of mixtures of the two under various conditions of temperature and pressure are familiar subjects of physical investigation into the details of which we need not enter, but it will be desirable to give the numerical results which for meteorological purposes must be extended to temperatures below the ordinary limits of experiment, because the temperatures of the atmosphere range from about 180a to 310a, and the necessary extension of thermal lines which we actually use carries our tables to 350a.

The properties of air containing various amounts of moisture upon which we rely for numerical computations are set out in the following Table I.:

TABLE I

TABLE OF PHYSICAL CONSTANTS FOR DRY AIR AND MIXTURE OF AIR AND WATER-VAPOUR

Temperature Tercentesimal Scale.	Saturation Pressure.	Density of Water-vapour.	Temperature Tercentesimal Scale.	Saturation Pressure.	Density of Water-vapour.
a.	mb.	g/m³.	a.	mb.	g/m³.
350	419·14	259·5	264	2·8591	3·247
348	385·73	240·2	262	2·3970	1·983
346	354·58	222·1	260	2·0037	1·670
344	325·59	205·1	258	1·6699	1·403
342	298·62	189·2	256	1·3874	1·174
340	273·56	174·3	254	1·1490	·980
338	250·31	160·5	252	·9486	·816
336	228·77	147·5	250	·7805	·677
334	208·81	135·5	248	·6399	·559
332	190·36	124·2	246	·5230	·461
330	173·32	113·8	244	·4258	·378
328	157·59	104·1	242	·3454	·309
326	143·09	95·11	240	·2792	·252
324	129·76	86·77	238	·2248	·205
322	117·510	79·10	236	·1803	·166
320	106·262	71·96	234	·1440	·133
318	95·949	65·39	232	·1145	·107
316	86·512	59·34	230	·0907	·0855
314	77·883	53·75	228	·0716	·0680
312	70·008	48·62	226	·0561	·0538
310	62·831	43·93	224	·0439	·0425
308	56·298	39·61	222	·0341	·0333
306	50·363	35·67	220	·0264	·0260
304	44·976	32·06	218	·0204	·0203
302	40·098	28·77	216	·0156	·0157
300	35·686	25·78	214	·0119	·0120
298	31·704	23·06	212	·0091	·0093
296	28·114	20·58	210	·0068	·0070
294	24·885	18·34	208	·0051	·0053
292	21·984	16·32	206	·0037	·0039
290	19·384	14·48	204	·0028	·0030
288	17·057	12·83	202	·0021	·00225
286	14·979	11·35	200	·0015	·0016
284	13·127	10·02	198	·0011	·0012
282	11·479	8·821	196	·00079	·00087
280	10·017	7·751	194	·00057	·00064
278	8·721	6·798	192	·00040	·00045
276	7·575	5·947	190	·00029	·00033
274	6·565	5·193	188	·00020	·00023
273	6·106	4·847	186	·00014	·00016
272	5·6261	4·482	184	·000096	·00011
270	4·7696	3·829	182	·000065	·00007[illegible]
268	4·0327	3·260	180	·000044	·000053
266	3·4004	2·770			

Note.—The values of saturation pressure from 204a to 350a are taken from the *Smithsonian Meteorological Tables, 1918*, and converted from millimetres to millibars. 1 mm. = 1·333224 mb. From 180a to 202a the formula $\log e = \log e_0 + 9{\cdot}632\,(1 - {\cdot}00035t)(t/T)$ is used where e is the vapour pressure and $t = T - 273$. The values for the density are computed from the formula $\Delta = {\cdot}622e/(RT)$, where $R = 2{\cdot}870$.

We add some other constants for air and water and water-vapour which will be required in the course of the work:

Density of dry air at 1000 mb. and 290a = 1·201 g/m³.
Specific heat of dry air at constant pressure = ·2417.

Specific heat of dry air at constant volume = ·1715.

Ratio of the specific heats, $\gamma = 1{\cdot}40$.

The constant for computing potential temperature $(\gamma - 1)/\gamma = {\cdot}286$.

Specific heat of water-vapour at constant pressure, ·4652 at 373a.

Specific heat of water-vapour at constant volume, ·340.

Specific heat of water, 1.

Specific heat of ice at 260a, ·502.

Latent heat of water-vapour at temperature 273a, 597 cal.

Latent heat of water-vapour at temperature 373a, 539 cal.

Latent heat of water, 79·77 cal.

1 cal. = 4·18 joules.

The weight in grammes of water which saturates one kilogramme of dry air at temperatures between 180a and 350a is given in Table IV., and the constant R′ of the characteristic equation of air with different quantities of water-vapour in Table V. § (20).

II. The Facts which call for Explanation: the General Circulation and the Local Circulations

§ (3) Sketch of the Distribution of the Meteorological Elements—Temperature, Moisture, Rainfall, Pressure, and Winds.—We may regard the whole sequence of events in the atmosphere as part of a continuous thermodynamic process, to which the kinetic energy of the winds belongs, and of which the direct transference of heat to and from the atmosphere are essential steps, involving the consecutive changes in the distribution of temperature, water-vapour, pressure, and wind that provide the weather for every part of the world. It will be useful briefly to review the facts which have been elicited by observation, classified in such a manner as to represent the general subject from the thermodynamic point of view.

We do not propose to start "from rest." We have no information as to any primal condition of rest from which the whole sequence started, nor need we seek any. We may take instead as our standard of reference the "normal" distribution obtained by combining mean values of the elements derived from observations of standard type extending over a long series of years. It would, of course, be unjustifiable to base any rigorous course of reasoning on the supposition that the distribution of mean values exactly represents the most frequent or any other type of actual distribution. The distribution represented by averages may never have occurred at all, and may indeed be an impossible situation, but still it will be convenient to have it in mind, and regard the actual condition at any time as representing departures from mean values. It is better to do so than to assume an initial condition of rest which is certainly much further from actual truth and which would require the solution of the problem through an infinity of ages to arrive at the conditions as we find them to-day.

In order to keep a natural sequence in mind we may begin with the distribution of temperature over the surface (*Figs.* 1 to 4) and in the upper air (Table II.), and follow it next with the distribution of water-vapour (*Fig.* 5), because those elements represent the store of thermal energy in the atmosphere. To these we add a representation of the distribution of rainfall (*Fig.* 6) as an index of the activity of convection. We can then proceed to the distribution of pressure (*Fig.* 7) which is controlled by the distribution of temperature and so pass to the distribution of kinetic energy represented at any moment by the winds. We leave the maps, with the explanations which accompany them, to speak for themselves.

§ (4) General Circulation and Local Circulations.—The winds corresponding with the distribution of the elements representing the average conditions may be referred to as the general circulation. On the other hand, each cyclonic distribution of pressure has a distribution of related winds which are sufficiently similar to a selected type to justify classification according to types, and application of general thermodynamical principles to the types. The typical circulations which are thus indicated may be called local circulations. It is desirable to consider these separately.

Confining our attention, therefore, for the present to the general circulation, the sequence of cause and effect seems rational if we consider the general lines of the distribution of temperature due to the radiation received from the sun on the one hand, and the equal loss by radiation into space on the other, to be the first stage. These vary slowly with the seasons and may be subject to slight temporary variations between day and night, from day to day, or from year to year, but in their general features they are permanent. The distribution of land and water, which is also permanent, governs certain features of the distribution of temperature and also the distribution of water-vapour by which vast quantities of heat are transferred to the atmosphere in a latent form.

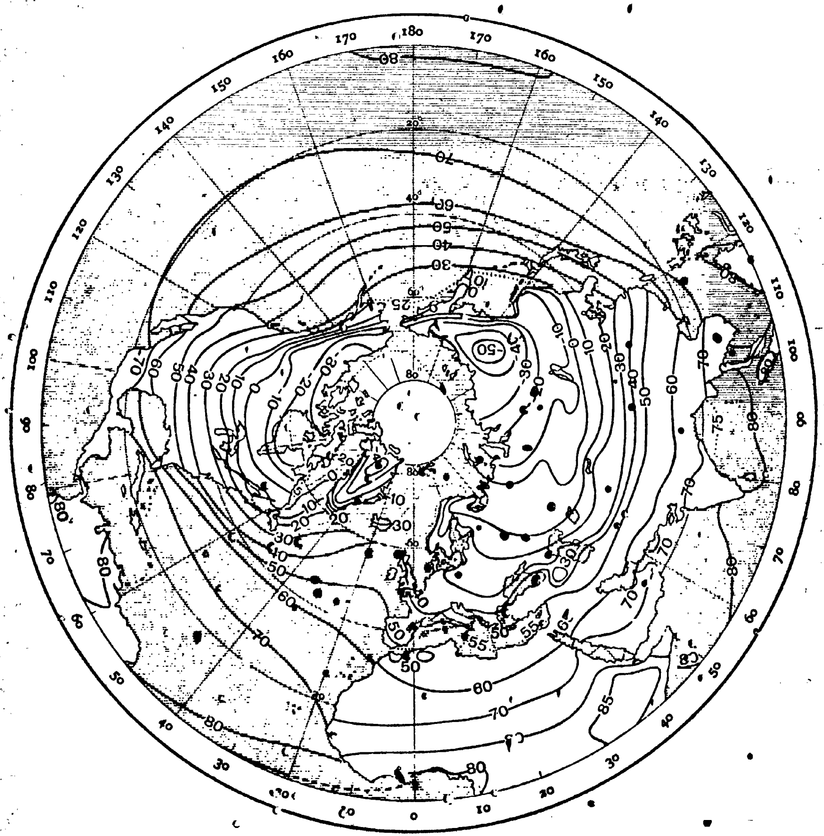
Based upon the distribution of these elements at the surface, we can recognise the main features of the distribution of the same elements in the upper air so far as our knowledge extends. For present purposes we may regard the level of 20 kilometres as the boundary. The figures for temperature at successive kilometres of height are set out in Table II.

FIG. 1.—MEAN TEMPERATURE OF THE AIR IN THE NORTHERN HEMISPHERE. ISOTHERMS FOR INTERVALS OF TEN DEGREES FAHRENHEIT.

Temperatures over the land have been reduced to sea-level.
Isotherms over land are plotted independently of those over sea.

The modified shading of the sea in the polar regions shows the probable range of ice between summer and winter.

NORTHERN HEMISPHERE



Equivalents below Freezing-point.

°F.	a.	°F.	a.
-50	227·4	0	255·2
-40	233·0	10	260·8
-30	238·6	20	266·3
-20	244·1	30	271·9
-10	249·7	32	273·0

JANUARY.

Equivalents above Freezing-point.

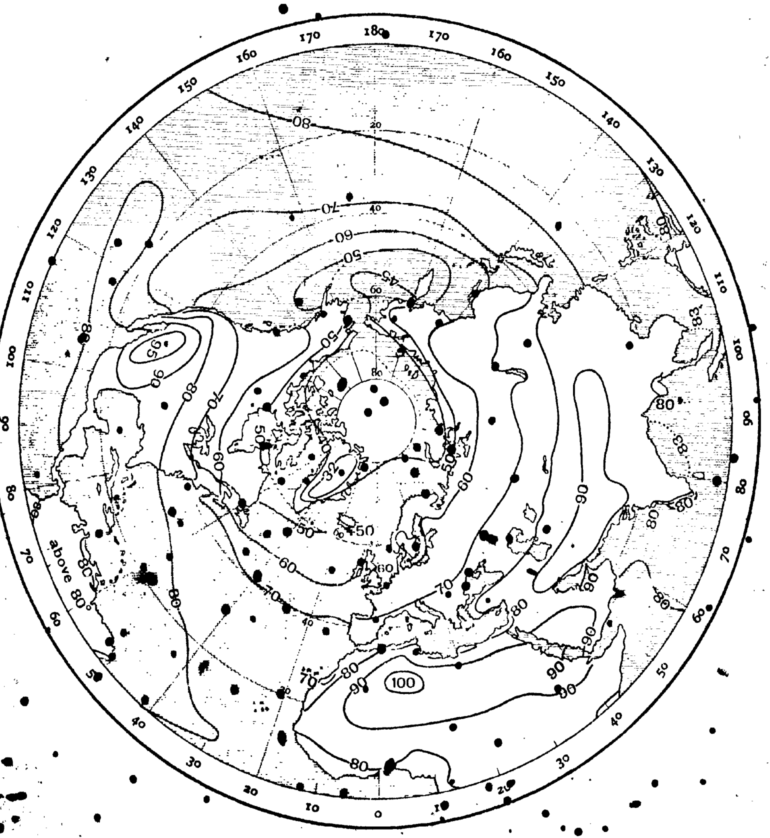
°F.	a.	°F	a.
32	273·0	70	294·1
40	277·4	80	299·7
50	283·0	90	305·2
60	288·6	100	310·8

FIG. 2.—MEAN TEMPERATURE OF THE AIR IN THE NORTHERN HEMISPHERE. ISOTHERMS FOR INTERVALS OF TEN DEGREES FAHRENHEIT.

Temperatures over the land have been reduced to sea-level.
Isotherms over land are plotted independently of those over sea.

The modified shading of the sea in the polar regions shows the probable range of ice between summer and winter.

NORTHERN HEMISPHERE



Equivalents below Freezing-point.

°F.	a.	°F.	a.
−50	227·4	0	255·2
−40	233·0	10	260·8
−30	238·6	20	266·3
−20	244·1	30	271·9
−10	249·7	32	273·0

JULY

Equivalents above Freezing-point.

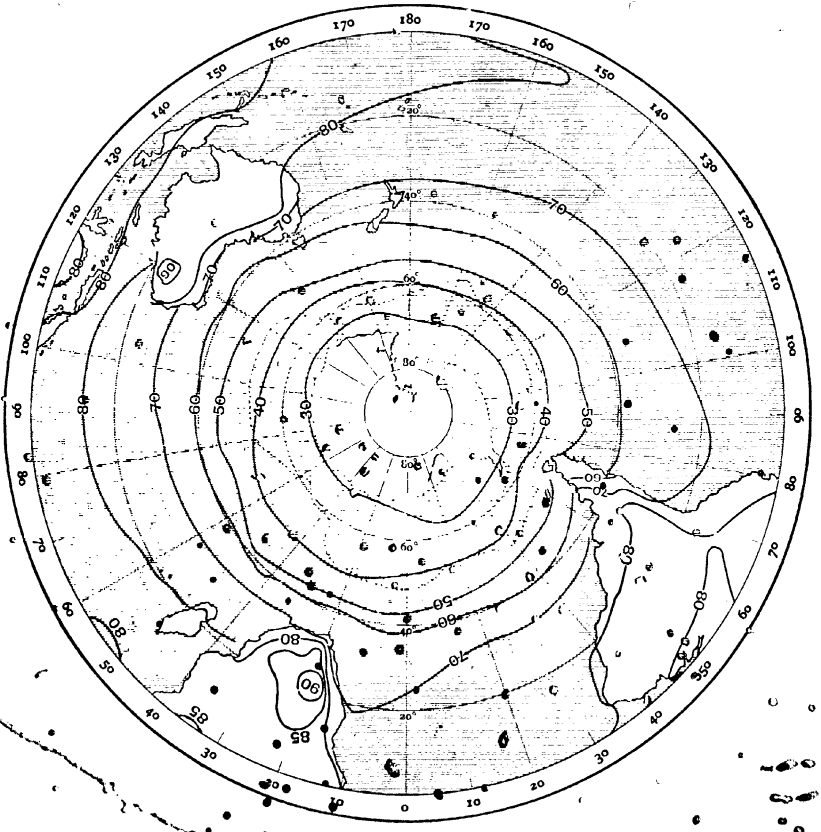
°F.	a.	°F.	a.
32	273·0	70	294·1
40	277·4	80	299·7
50	283·0	90	305·2
60	288·6	100	310·8

FIG. 3.—MEAN TEMPERATURE OF THE AIR IN THE SOUTHERN HEMISPHERE. ISOTHERMS FOR INTERVALS OF TEN DEGREES FAHRENHEIT.

Temperatures over the land have been reduced to sea-level.
Isotherms over land are plotted independently of those over sea.

The modified shading of the sea in the polar regions shows the probable range of ice between summer and winter.

SOUTHERN HEMISPHERE



Equivalents below Freezing-point.

°F.	a.	°F.	a.
-50	227·4	0	255·2
-40	233·0	10	260·8
-30	238·6	20	266·3
-20	244·1	30	271·9
-10	249·7	32	273·0

JANUARY.

Equivalents above Freezing-point.

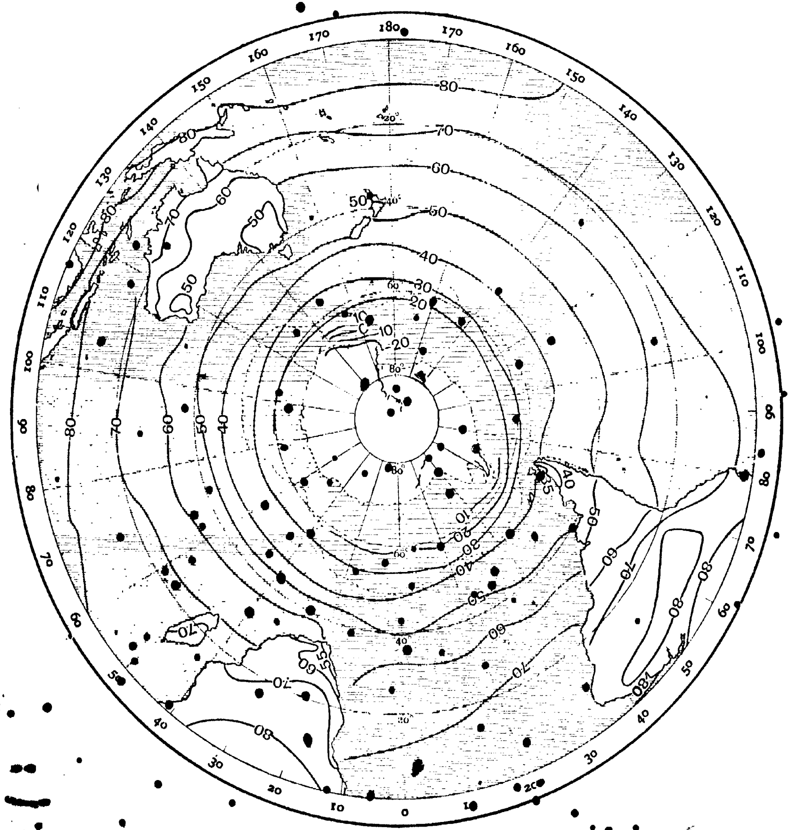
°F.	a.	°F.	a.
32	273·0	70	294·1
40	277·4	80	299·7
50	283·0	90	305·2
60	288·6	100	310·8

FIG. 4.—MEAN TEMPERATURE OF THE AIR IN THE SOUTHERN HEMISPHERE. ISOTHERMS FOR INTERVALS OF TEN DEGREES FAHRENHEIT.

Temperatures over the land have been reduced to sea-level.
Isotherms over land are plotted independently of those over sea.

The modified shading of the sea in the polar regions shows the probable range of ice between summer and winter.

SOUTHERN HEMISPHERE



Equivalents below Freezing-point.

°F.	a.	°F	a.
-50	227·4	0	255·2
-40	233·0	10	260·8
-30	238·6	20	266·3
-20	244·1	30	271·9
-10	249·7	32	273·0

JULY

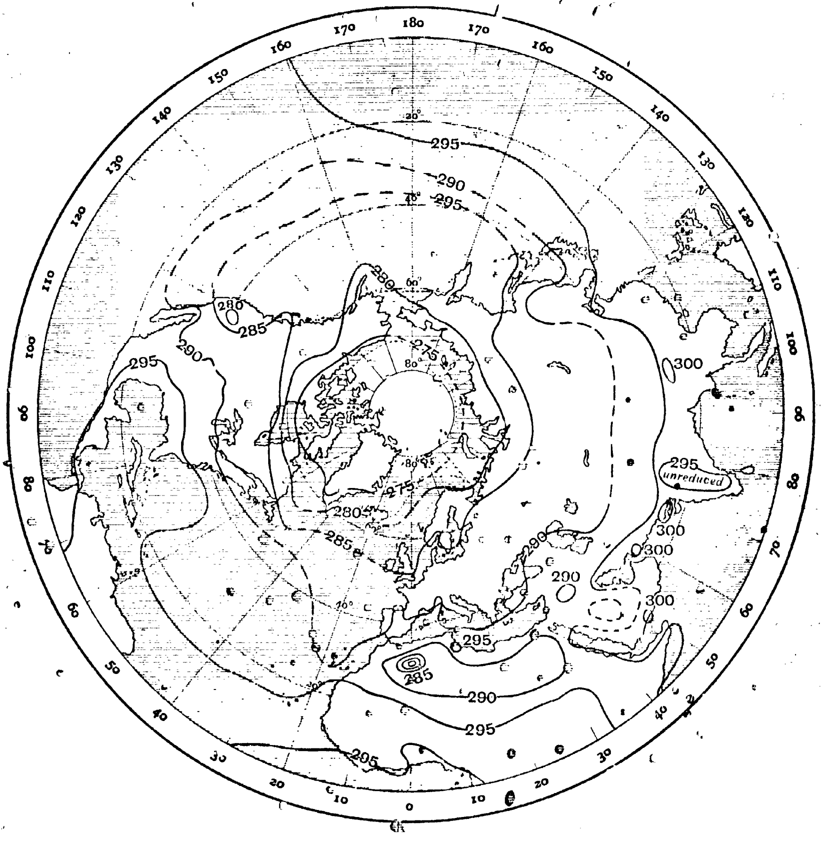
Equivalents above Freezing-point.

°F.	a.	°F.	a.
32	273·0	70	294·1
40	277·4	80	299·7
50	283·0	90	305·2
60	288·6	100	310·8

FIG. 5.—TEMPERATURE OF SATURATION OR DEW-POINT.

Vapour pressure has been reduced to sea-level by the formula $e_0 = e_h(1 + \cdot 0004h)$, where h is in metres.

NORTHERN HEMISPHERE



Pressure of Aqueous Vapour and Density at Saturation.

Dew-point. a.	Pressure. mb.	Density. g/m³.
275	7·1	5·6
280	10·0	7·8
285	14·1	10·7

JULY

Pressure of Aqueous Vapour and Density at Saturation.

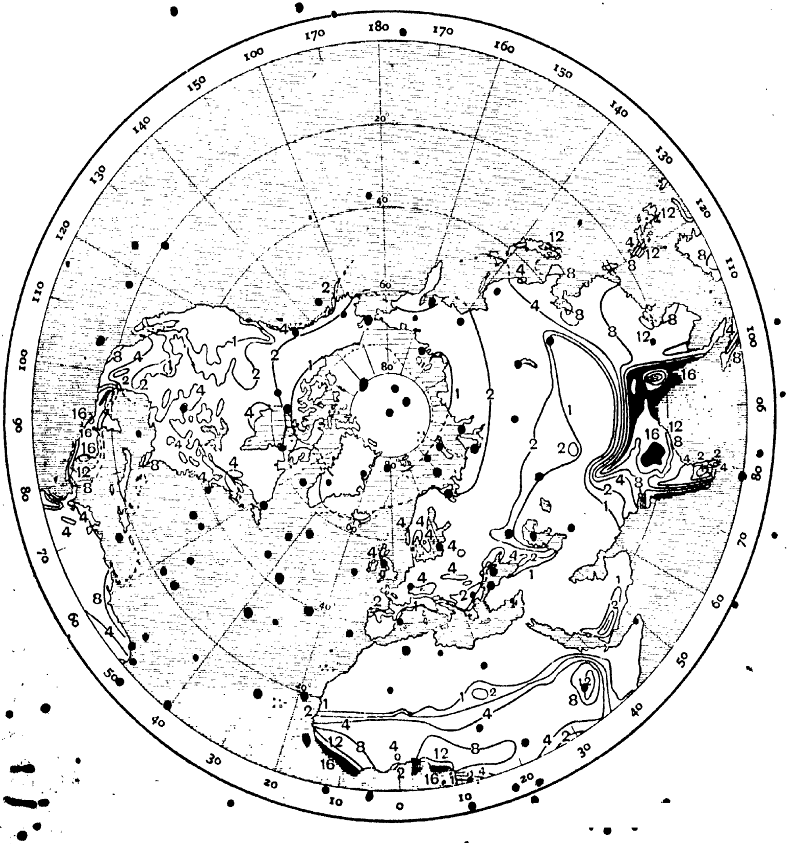
Dew-point. a.	Pressure. mb.	Density. g/m³.
290	19·4	14·5
295	26·5	19·5
300	35·7	25·8

FIG. 6.—MEAN RAINFALL IN THE NORTHERN HEMISPHERE IN INCHES.

Isohyets are drawn for 1, 2, 4, 8, 12, 16 inches.

Black blocked-in indicates rainfall above 16 inches.

NORTHERN HEMISPHERE



JULY

Equivalents.

in.	mm.
1	2·5
2	5·1
4	10·2

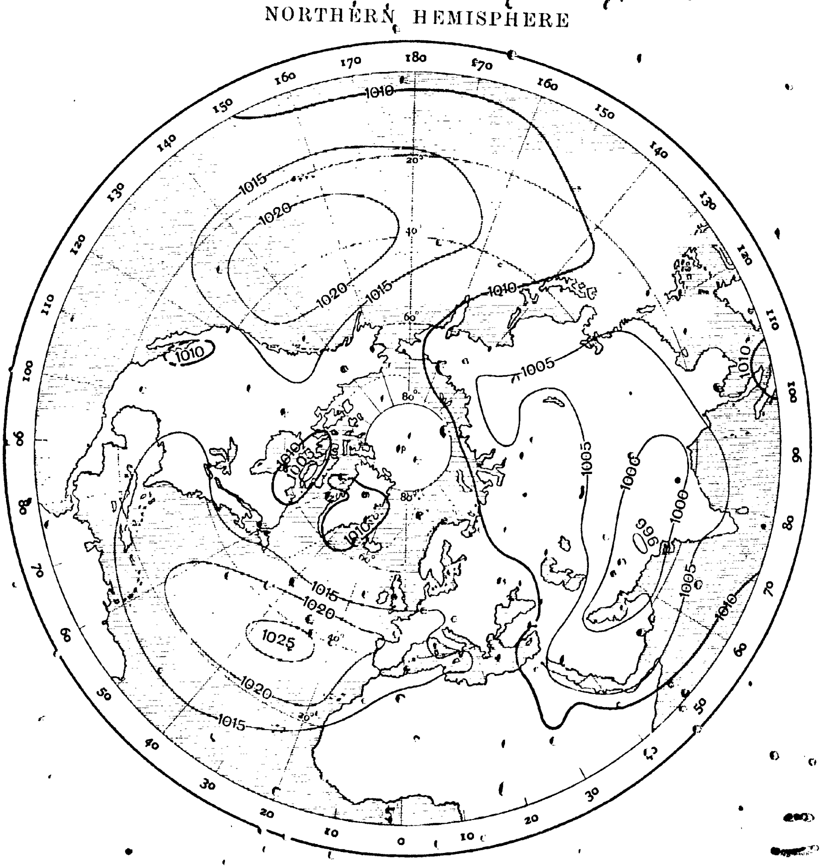
Equivalents.

in.	mm.
8	20·3
12	30·5
16	40·6

FIG. 7.—MEAN PRESSURE AT SEA-LEVEL IN THE NORTHERN HEMISPHERE.

Isobars are drawn for intervals of 5 millibars.

WINDS.—The distribution of pressure shown in this map serves as a guide to the GENERAL CIRCULATION of air in the strata near sea-level (see § (8)).



Equivalents.

mb.	in.	mm.
1030	30·42	772·6
1025	30·27	768·8
1020	30·12	765·1
1015	29·97	761·3

JULY

Equivalents.

mb.	in.	mm.
1010	29·83	757·6
1005	29·68	753·8
1000	29·53	750·1
996	29·41	747·1

TABLE II

TEMPERATURE IN THE UPPER AIR (IN DEGREES OF THE TERCENTESIMAL SCALE)

Black type signifies a region in which the lapse-rate of temperature is zero (approximately) or negative.

SUMMER. Lat. 45° to 90°.

Height km.	M'Murdo Sound.[1] Lat. 78° S.	Arctic Sea.[2] Lat. 77° N.	Kiruna.[3] Lat. 68° N. (7 Obs.).	Pavlovsk.[4] Lat. 59° 41′ N. (19 Obs.).	Ekaterinburg.[3] Lat. 57° N. (3 Obs.).	Kutchino.[4] Lat. 55° 45′ N. (10 Obs.).	England.[5] Lat. Approx. 52° N.	Europe.[6]	Pavia.[7] Lat. 45° 11′ N. (15 Obs.).
20	..	..	(August)	..	..	..	..	**224**	..
19	..	..	..	..	..	..	..	**223**	..
18	..	..	..	..	..	..	..	**223**	..
17	..	..	..	..	..	..	..	**222**	..
16	..	..	..	..	..	..	..	**221**	..
15	..	..	..	..	..	..	..	**221**	..
14	..	..	**225**	..	**225**	..	**222**	**221**	..
13	..	..	**222**	..	**225**	..	**222**	**220**	..
12	..	..	**222**	**225**	**225**	**223**	**222**	**220**	219·5
11	..	**221**	**224**	**225**	**224**	**223**	**222**	223	223
10	..	**218**	228	227	223	226	226	228	229
9	..	224	235	230	229	231	233	235	237
8	..	233	243	236	238	238	240	242	244
7	..	241	250	243·5	245·5	246	247	250	251
6	228	248	257	250·5	253·5	253	254	257	258
5	236	255	263·5	257	259	260	261	263	265
4	241	261·5	269	263·5	264	266	267	269	270
3	246	267	273	269	270	272	273	274	276
2	252	272	278	275	277·5	278	278	279	283
1	259	274·5	282	281	284·5	284	283	285	289
Surface or sea-level	265·5 ..	278 ..	284 *0·5 km.*	288 ..	294 *·268 km.*	291 ..	289 ..	288 ..	293 *Surface*

---

[1] British Antarctic Expedition, 1910-1913, *Meteorology*, 6 summer ascents, ii. 284—calculated from mean lapse-rate.

[2] *Beiträge zur Physik der freien Atmosphäre*, Band vi., "Aerologische Studien im arktischen Sommer," H. Hergesell, p. 249.

[3] Summaries by H. H. Hildebrandsson, *Geografiska Annaler*, 1920, Häft 2, 110. For original observations at Kiruna see *Sondages aériens à Kiruna*, 1907–9, by [illegible] H. Maurice. *Nova Acta Soc. Reg. Scient. Ups.*, 1917, Ser. IV. iii. No. 7.

[4] *Meteorologische Zeitschrift*, 1911, xxviii. No. 1, 1-16, by M. Rykatchew (junior). Table 5.

[5] *The Characteristics of the Free Atmosphere*, by W. H. Dines, M.O. 220c, p. 54. (Means for June, July, August.)

[6] Summaries from Humphreys, *Physics of the Air*, p. 54. From average lapse-rates of temperature at Trappes, Uccle, Strassburg, Munich.

[7] *Meteorologische Zeitschrift*, 1911, xxviii. No. 6, 261-265, by Dr. A Wagner.

TABLE II (*continued*)

TEMPERATURE IN THE UPPER AIR, (IN DEGREES OF THE TERCENTESIMAL SCALE)

Black type signifies a region in which the lapse-rate of temperature is zero (approximately) or negative.

SUMMER. Lat. 0° to 45°.

Height km.	Canada.[1] Lat. 43° N.	United States.[2] Lat. Approx. 40° N.	St. Louis.[3] Lat. 38° 38′ N.	North Atlantic. Lat.[4] 60°-30° N. (11 Obs.).	North Atlantic. Lat.[5] 20°-40° (10 Obs.).	North Atlantic. Lat.[4] 10°-20°. (10 Obs.).	Batavia.[6] Lat. 6° S.	Victoria Nyanza.[7] Lat. 0°.
20	..	**222**	..	..	..	..	**197**	**195**
19	..	**220**	..	..	..	..	**191**	**196**
18	**214**	**217·5**	..	..	..	..	**186**	**197·5**
17	**212**	**216**	..	..	..	..	187	199
16	**210·5**	**215**	..	..	..	..	191	203
15	**210**	**217**	222	204	..	204	197	207
14	**209**	219	217	215	209	210	205	211
13	214	223	214	216	216	215	213	217
12	218	226	217	220	219·5	221	221·5	223
11	223	232·5	222·5	227	227	229·5	230	230
10	230	239	230	235	235	238	239	238
9	237·5	246	239	243	242	249	246	245
8	244·5	252	247	252	252	256	253	252
7	251	259	255·5	259	259	262	260	259
6	259	264·5	263	266	266	268	266	265
5	265	270·5	270	263	272	272	271·5	270
4	272	277	275	277	277	278·5	277	276
3	277	283	280	282	282	285	282·5	282
2	282·5	290	284	287	288	289	288	289
1	288	295	292	290·5	292	293	293	..
Surface or sea-level	293 *·3 km.*	298 ..	298 *·167 km.*	299 ..	298 ..	298·5 ..	299 ..	296 *1·171 km.*

---

[1] *Upper Air Investigation in Canada*, Part I., by J. Patterson, Ottawa, 1915, p. 16. (Means for June, July, August.)

[2] *Monthly Weather Review*, Jan. 1918, xlvi. No. 1, 11-20. (The values are the means for the stations Fort Omaha, Indianapolis, Huron, Avalon.)

[3] *Annals of the Astronomical Observatory of Harvard College*, lxviii. Part I. 68.

[4] *Beiträge zur Physik der freien Atmosphäre*, 1912, Band iv. "Temperatur- und Druckgefälle in grossen Höhen," by A. Peppler, p. 22.

[5] Summaries by H. H. Hildebrandsson, *Geografiska Annaler*, 1920, Haft 2, 110.

[6] *Koninklijk Magnetisch en Meteorologisch Observatorium te Batavia*, 1916, by W. van Bemmelen, p. 27. The summer values are means for December, January, February.

[7] *Ergebnisse der Arbeiten des K. Preuss. Aeronaut. Obs. bei Lindenberg. Bericht über die aerologische Expedition nach Ostafrika im Jahre 1908*, by Arthur Berson. (Calculated from table of lapse-rates, p. 76. Observations from July to October.)

Note.—The observations from which the mean values over the North Atlantic were obtained are given in *Travaux scientifiques de l'Observatoire de Météorologie dynamique de Trappes*, 1909, iv. Paris.

TABLE II (*continued*)

TEMPERATURE IN THE UPPER AIR (IN DEGREES OF THE TERCENTESIMAL SCALE)

Black type signifies a region in which the lapse-rate of temperature is zero (approximately) or negative.

WINTER. Lat. 0° to 45°.

Height km.	Victoria Nyanza.[1] Lat. 0°.	Batavia.[2] Lat. 6° S.	St. Louis.[3] Lat. 38° 38′ N.	United States.[4] Lat. Approx. 40° N.	Canada.[5] Lat. 43° N.	Height km.
20	**195**	**203**	..	**219**	..	20
19	**196**	**200**	..	**218**	..	19
18	**197 5**	**194**	..	**218**	..	18
17	199	**189**	..	**218**	..	17
16	203	192	..	**218**	..	16
15	207	198	**211**	**218**	**215**	15
14	211	204	**211**	**219**	**216**	14
13	217	212	**214·5**	**220**	**217**	13
12	223	220	**215·5**	**221**	**217**	12
11	230	228	217	**222**	**216**	11
10	238	237	221	224	219	10
9	245	245	226	228	223	9
8	252	253	231	234	229	8
7	259	258	240	240	237	7
6	265	265	249	247	244·5	6
5	270	271	255	254	251	5
4	276	276	261·5	260	258	4
3	282	282	266	265	265	3
2	288	288	270	270	269	2
1	289	293	272	271·5	272	1
Surface or sea-level	396 *1·171 km.*	299 ..	274 *·167 km.*	27[illegible] ..	272 *·3 km.*	Surface or sea-level

---

[1] *Ergebnisse der Arbeiten des K. Preuss. Aeronaut. Obs. bei Lindenberg. Bericht über die aerologische Expedition nach Ostafrika im Jahre 1908*, by Arthur Berson. (Calculated from table of lapse-rates, p. 76. Observations from July to October.)

[2] *Koninklijk Magnetisch en Meteorologisch Observatorium te Batavia*, 1916, by W. van Bemmelen, p. 27. The winter values are means for June, July, August.

[3] *Annals of the Astronomical Observatory of Harvard College*, lxviii. Part I, 68.

[4] *Monthly Weather Review*, Jan. 1918, xlvi. No. 1, 11-20. (The values are the means for the stations Fort Omaha, Indianapolis, Huron, Avalon.)

[5] *Upper Air Investigation in Canada*, Part I., by J. Patterson, Ottawa, 1915, p. 16. (Means for December, January, February.)

TABLE II (*continued*)

TEMPERATURE IN THE UPPER AIR (IN DEGREES OF THE TERCENTESIMAL SCALE)

Black type signifies a region in which the lapse-rate of temperature is zero (approximately) or negative.

WINTER. Lat. 45° to 90°.								
Height km.	Pavia.[1] Lat. 45° 11′ N. (15 Obs.).	Europe.[2]	England.[3] Lat. Approx. 52° N.	Kutchino.[4] Lat. 55° 45′ N. (6 Obs.).	Ekaterinbourg.[5] Lat. 57° N. (5 Obs.).	Pavlovsk.[4] Lat. 59° 41′ N. (16 Obs.).	Kiruna.[5] Lat. 68° N. (23 Obs.).	M'Murdo Sound.[6] Lat. 78° S.
20	..	**215**	..	..	..	..	..	..
19	..	**215**	..	..	..	..	..	..
18	..	**215**	..	..	..	..	..	..
17	..	**215**	..	..	..	..	..	..
16	..	**216**	..	..	..	..	..	..
15	..	**216**	..	..	..	..	..	..
14	..	**216·5**	**216**	..	..	..	**214**	..
13	..	**217**	**216**	..	..	..	**218**	..
12	**206**	**216**	**217**	**214**	..	**217**	**216**	..
11	**207**	**216**	**217**	**213**	**210**	**217**	**214**	..
10	214	219	220	**214**	**212**	**218**	**214**	..
9	222	223	224	219	216	**219**	221·5	..
8	230	229	230	225·5	223	223	226	..
7	237	236	237	233	232	229	232·5	..
6	245	244	244	240	239	236	239	..
5	252	251	250	247	246	243	244	..
4	259	258	257	253	253	248	251	..
3	264·5	264	263	258	259	254·5	256	236
2	270	269	267	263	265·5	260	259	239
1	273	272	271	**265**	**268**	263	**264**	**243**
Surface or sea-level	274 *Surface*	275 ..	276 ..	**263** ..	**265** *·268 km.*	265·5 ..	**254** *0·5 km.*	**238** ..

---

[1] *Meteorologische Zeitschrift*, 1911, xxviii. No. 6, 264-265, by Dr. A. Wagner.

[2] Summaries from Humphreys, *Physics of the Air*, p. 54. From average lapse-rates of temperature at Trappes, Uccle, Strassburg, Munich.

[3] *The Characteristics of the Free Atmosphere*, by W. H. Dines, M.O. 220c, p. 54. (Means for December, January, February.)

[4] *Meteorologische Zeitschrift*, 1911, xxviii. No. 1, 1-16, by M. Rykatchew (junior). Table 5.

[5] Summaries by H. H. Hildebrandsson, *Geografiska Annaler*, 1920, Haft 2, 110. For original observations at Kiruna see *Sondages aériens à Kiruna*, 1907–9, by H. Maurice. *Nova Acta Soc. Reg. Scient. Ups.*, 1917, Ser. IV. iii. No. 7.

[6] British Antarctic Expedition, 1910–1913, *Meteorology*, 4 winter ascents, ii. 275—calculated from mean lapse-rate.

§ (5) THE GEOGRAPHICAL DISTRIBUTION OF TEMPERATURE.—The distribution of temperature at the surface may be illustrated by the normal isotherms for January and July on the maps of the Northern Hemisphere (*Figs.* 1, 2). The points to bear in mind are first the gradient of temperature from the equatorial to the polar regions and the striking difference between the range of temperature displayed in the two months, 135° F. in January as against 68° F. in July; secondly, the discontinuity of temperature at the coast lines which is manifested by the break between the parts of the isotherms for the same temperature over the land and over the sea. These features of the distribution of temperature at the surface find expression in the general circulation of the atmosphere in various ways which will be noted later.

Above the surface, in the upper air to the level of 20 kilometres the general lines of the distribution can be inferred from the figures quoted in Table II. for the average temperatures in summer and winter at various stations in different parts of the world. In the table the stations are grouped according to latitude irrespective of hemisphere.

A study of the figures shows that from the ground upward the temperature falls at the rate of 5a or 6a or 7a per kilometre until the fall is brought to a sudden stop. We may call the fall of temperature per kilometre the "lapse-rate" of temperature, avoiding the word gradient, which has often been used in this connection, because we require it for the change of temperature in the horizontal.

An exception to this rule is offered by the polar regions in winter, where the temperature is lowest at the surface and increases upward to the first kilometre at least, although beyond that level the general law of fall of temperature with height becomes operative. Corresponding exceptions are often exhibited locally in valleys at night, where the cold air accumulates in consequence of the loss of heat by radiation from the ground. The cold layer thus formed is very favourable for the formation of fog.

The lapse-rate at the surface under summer conditions is generally between 5a and 6a per kilometre. It may be subject to variation and even reversed locally in certain layers as greater heights are reached, but on the average the rate increases with height and becomes 6a, 7a, or 8a per kilometre before it suddenly ceases where a surface of demarcation is reached which we call the *tropopause*. Above that surface of demarcation the isothermal surfaces are normally vertical, or nearly so, and below it they are nearly horizontal.

Thus the atmosphere is divided into two distinct parts: the upper, called by Teisserenc de Bort the *stratosphere*, where the isothermal surfaces are vertical and there is no fall of temperature with height; the lower, called the *troposphere*, in which the isothermal surfaces are nearly horizontal and the maximum change of temperature is in the vertical. The figures which relate to the stratosphere in Table II. are in black type, as are also those for the polar regions, where also there is no lapse of temperature near the surface in winter, but on the contrary a rise with height.

The tropopause separates the lower portion, the troposphere, with its temperatures arranged in horizontal layers, from the upper, the stratosphere, with its temperatures arranged in vertical sheets.

The fall of temperature with height is a natural consequence of any mechanical process of mixing that would tend to realise the conditions of convective equilibrium. Hence we may conclude that the cessation of fall of temperature at the tropopause means that the automatic process of mixing which operates in the lower atmosphere does not run beyond that level and that the stratosphere is free from that kind of disturbance. The conditions which tend towards convective equilibrium in the atmosphere are those of turbulence, which may be set up by the relative motion of streams of air with regard to the ground, or with regard to each other, and may therefore be induced by convection. Such conditions are apparently inoperative in any general sense in the stratosphere, and consequently we may regard the stratosphere as normally free from the convection or relative motion which produces mixing in the vertical.

An important point to notice about the tropopause as indicated in Table II. by the upper line of separation between black type and ordinary type is that its height above the ground or above sea-level (we have not evidence enough at present to distinguish) is dependent upon latitude, being highest in the equatorial region and lowest in the polar region. The differences are very considerable, the lowest figures shown for the height of the tropopause being 8-9 kilometres at Pavlovsk in winter, and the highest 17-18 kilometres over Victoria Nyanza or Batavia in the northern summer. The intermediate regions with some exceptions, which may perhaps be connected with associated high or low pressures, show intermediate heights, and the gradual slope downward of the tropopause from the equator to the pole as a normal condition seems very probable.

We may, however, here note that any general law of that kind has to take account of the disturbance caused by the features of the variable distribution of pressure in cyclones and anticyclones. Mr. W. H. Dines has classified the observations of the upper air in England according to the surface pressure at the time of observation,[1] and from his results we derive the following figures (Table III.):

TABLE III

PRESSURE, TEMPERATURE, AND DENSITY IN REGIONS OF HIGH PRESSURE AND LOW PRESSURE

Height.	High Pressure.			Low Pressure.		
	Pressure.	Temperature.	Density.	Pressure.	Temperature.	Density.
m.	mb.	Tercentesimal Scale.	g/m³.	mb.	Tercentesimal Scale.	g/m³.
14	143	**215**	232	135	**224**	210
13	168	**215**	272	158	**225**	245
12	197	**217**	316	184	**225**	285
11	231	220	350	214	**224**	333
10	269	225	427	249	**225**	389
9	313	231	472	289	**226**	446
8	362	238	530	337	228	516
7	417	246	589	390	234	582
6	478	253	658	451	242	648
5	547	259	734	519	249	724
4	623	265	818	594	256	808
3	708	271	906	678	263	898
2	802	276	1010	772	270	997
1	908	279	1135	875	276	1105
0	1026	..	..	989	..	..

From these figures it appears that the tropopause is drawn down from 12 kilometres to 8 kilometres by the transition from high pressure of 1026 mb. to a low pressure of 989 mb., that in the troposphere air is warmer under high pressure than under low pressure, and that in the stratosphere the reverse is the case, high pressure is *cold* relatively to low pressure. We may perhaps infer that the sequence of pressure which we experience is dictated mainly by the stratosphere.

The figures of Table II. also show that in the equatorial regions there is a continuous range of temperature with height from the highest figure 300a to the lowest figures of the whole table. If we regard the distribution as represented by isothermal surfaces we can see that isothermal surfaces for each temperature cross the equator. Those that reach the tropopause and there turn up and become vertical are successively warmer as we proceed farther north. Hence we reach the important conclusion that the distribution of temperature with which we are familiar at the surface, namely highest temperature at the equator and lowest in the polar regions,
kills completely reversed within the range of 20
Themetres.
distributiomost important characteristics of the
ately from tn of water-vapour follow immedi-
because the northe distribution of temperature,
cannot exceed the mal quantity of water-vapour
saturction of the ai amount necessary for the
r at the temperatures
indicated in Table II. Since the pressure of saturation of water-vapour falls off approximately in geometrical progression as the temperature falls in arithmetical progression, the most striking feature of the distribution of water-vapour is the rapidity with which its density diminishes with height. A general impression of the distribution can be conveyed by a diagram such as *Fig.* 8 representing the density of water-vapour at different levels by the number of horizontal lines of dots per kilometre of height. The thickness of the cloud of dots produced in this way for a selected number of stations referred to in Table II. represents in an effective manner the relative importance of different layers of the atmosphere in respect of water-vapour. It is true that the air in question is not always nor even generally saturated with water-vapour, and perhaps an allowance of one quarter of the number of lines might be made on that account, but the reader is left to introduce that correction at his discretion.

§ (6) POTENTIAL TEMPERATURE OR REALISED ENTROPY IN THE ATMOSPHERE.—Another important feature of the figures included in Table II. is the information which they disclose as to the extent to which the atmosphere differs from the condition of convective equilibrium. It should be remembered that a fluid is in convective or *labile* equilibrium[2] when any element of it moving vertically without loss or gain of heat takes at all heights the temperature of the surrounding air at the same level.[3] Hence in an atmosphere in convective equilibrium any part of the air warmed, however slightly, goes to the top, and cooled, however slightly, to the bottom. But convective equilibrium as applied to the atmosphere is a term of ambiguous meaning, because the atmosphere always contains water-vapour, generally not very far from the point of saturation. Often it is actually saturated, or condensation may be in progress. The heat derived from the changes in the water-vapour modifies the equilibrium conditions. In the atmosphere convective equilibrium is different for upward motion and for downward motion. If saturated air moves upward and suffers reduction of pressure and consequent reduction of temperature, the lapse of temperature is affected by the setting free of the latent heat of

---

[1] *Phil. Trans. Roy. Soc.*,
Geophysical Memoirs, No. 13, Series A, ccxl. 253;
No. 220c. M.O. publication

[2] For a fuller discussion see § (13).

[3] See "Atmosphere, Physics of the," § (6), p. 25.

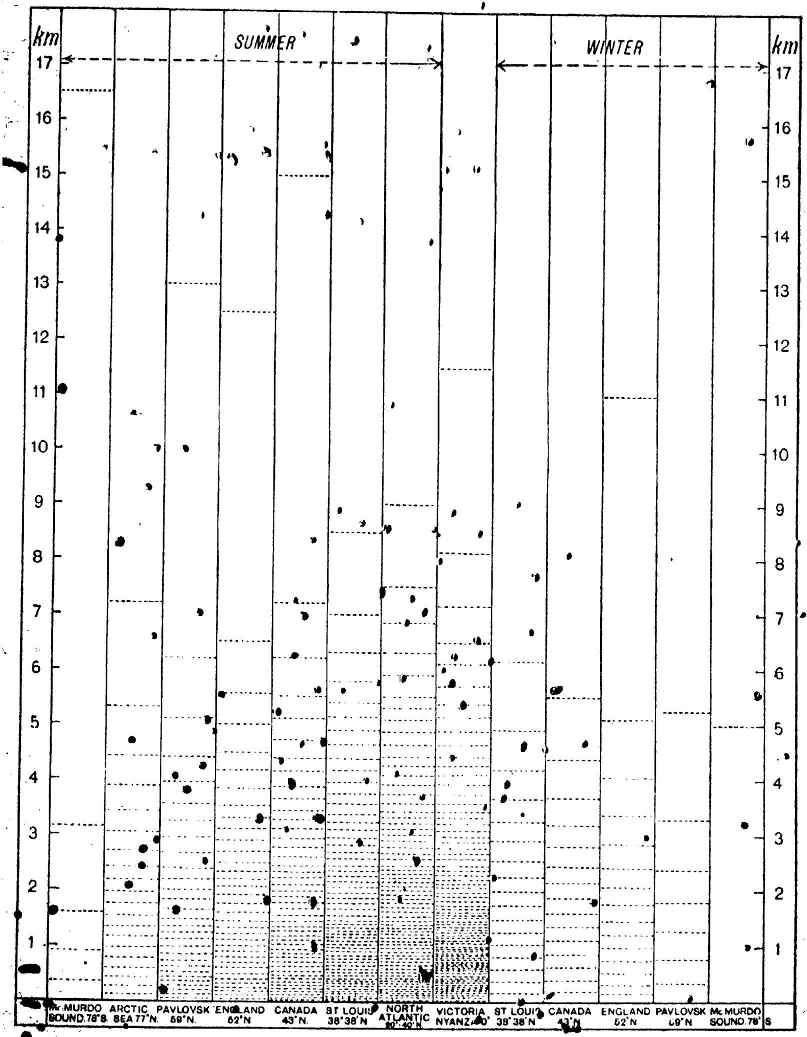


FIG. 8.—Density of Water-vapour for Saturated Air at different Levels at Selected Stations in different Latitudes in Summer and Winter.

The number of lines in any column represents the equivalent, in millimetres of rainfall, of the water-vapour in saturated air at the normal temperatures given in Table II. The extension to sea-level at land-stations is by extrapolation.

evaporation and an adiabatic change of special character occurs for which equations will be given subsequently. Convective equilibrium for upward motion implies change of temperature according to the adiabatic for saturated air (see § (22)). On the other hand, if the saturated air is moved downward, and increase of pressure occurs, then the presence of the water-vapour is of very little importance; the change of temperature produced by the change of pressure is in accordance with the adiabatic change for dry air with the coefficient ·286 for its ratio to the percentage change of pressure. We have to borrow in this section some of the results of thermodynamic theory which are the subject of later sections ((18) and (19)), but the idea of defining temperature in the atmosphere with reference to some standard condition of pressure is of such general importance that the borrowing may be excused. The table of figures shows that in the upper air of our latitude the average change of temperature with height is that of saturated air going upward, not that of air coming downward. Hence in summer, in this country saturated air can on the average go up automatically, but it cannot come down again, and the same is true of air at the equator, whereas in the polar regions we can scarcely see our way to conditions favourable for air to rise whether it be saturated or not. Thus the consideration of temperature with reference to the saturation-adiabatic for ascent and the dry adiabatic for descent is of great importance in considering the statical stability of the atmosphere. It has in consequence been usual to represent the condition of the atmosphere at any point of low pressure in the upper air by recording the temperature which the air would show if the pressure were increased to a standard pressure under adiabatic conditions. The temperature adjusted in that manner was called "potential temperature" by von Bezold.[1] It may obviously be closely connected with the entropy which the air possesses in virtue of its temperature alone without reckoning any further store of entropy available in the event of the condensation of its remaining water-vapour. Regarding, therefore, dry air as the working substance, according to the isentropic equation pv^γ = constant, we get by aid of the characteristic equation $pv = \mathrm{RT}$

$$\frac{p\mathrm{T}^\gamma}{p^\gamma} = \text{constant},$$

$$\text{i.e.}\qquad \frac{\mathrm{T}}{p^{(\gamma-1)/\gamma}} = \text{constant} = \frac{\theta}{p_0^{(\gamma-1)/\gamma}},$$

where θ is the potential temperature and p_0 is the standard pressure. Whence

$$\frac{d\theta}{\theta} = \frac{d\mathrm{T}}{\mathrm{T}} - \frac{\gamma-1}{\gamma}\frac{dp}{p}.$$

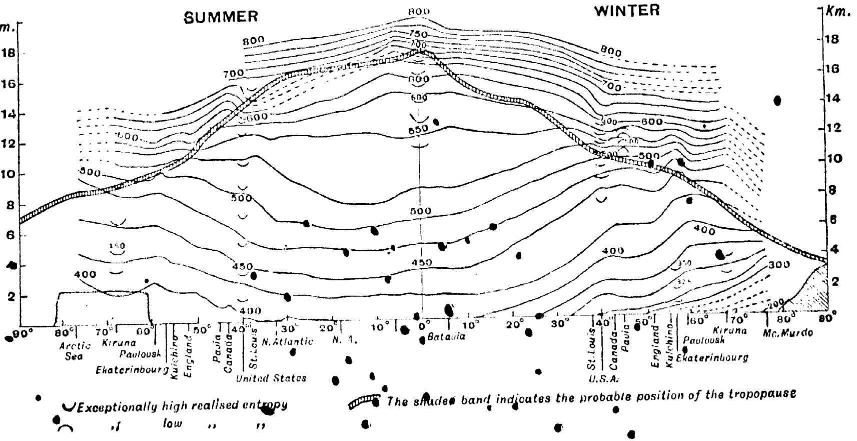


FIG. 9.—Distribution of Realised Entropy, in Joules per Tercentesimal Unit of Temperature, in the Upper Air over the Globe.

The realised entropy is related to the potential temperature by the equation $\phi = c_p \log_e \theta + \text{const.}$

The standard state from which the entropy is measured is when T = 200a and standard pressure is 1000 mb.

The diagram represents a section of the atmosphere from pole to pole; the shaded area near the north pole represents the section of the massif of Greenland, and that at the south pole the Antarctic Continent.

The indications of exceptionally high realised entropy over the equator are derived from the observations on Victoria Nyanza.

---

[1] *Sitzber. Berliner Akad.*, 1888, xlvi. 1189, "Zur Thermodynamik der Atmosphäre," also in von Bezold's *Gesammelte Abhandlung*, Vieweg und Sohn, Braunschweig, 1906, p. 128, tr. C. Abbé, *Mechanics of the Earth's Atmosphere*, p. 243.

The energy equation is

$$dQ = c_p dT - Avdp$$
$$= c_p dT - \frac{ART}{p}dp,$$

where A is the thermal equivalent of a unit of work. Hence if ϕ denote the entropy,

$$d\phi = \frac{dQ}{T} = c_p\frac{dT}{T} - \frac{ARdp}{p}$$
$$= c_p\frac{dT}{T} - (c_p - c_v)\frac{dp}{p}$$
$$= c_p\frac{dT}{T} - \frac{\gamma - 1}{\gamma} \cdot \frac{c_p dp}{p}$$
$$= c_p\frac{d\theta}{\theta},$$

therefore integrating,

$$\phi = c_p \log_e \theta + \text{const.}$$

or changes of entropy from an agreed standard are expressed by the natural logarithm of the potential temperature multiplied by the specific heat at the standard pressure.

We may call the entropy expressed in this manner the *realised entropy*, in contra-distinction from the total entropy of the air which includes the entropy of the water still existing in the form of vapour.

The normal condition of the atmosphere in respect of the distribution of realised entropy computed in this manner is represented by the diagram *Fig.* 9.

The relative conditions in high and low pressure are similarly shown in *Fig.* 10.

§ (7) STATICAL STABILITY OF THE ATMOSPHERE.—It will be understood that if the atmosphere were in convective equilibrium for dry air or for downward motion the realised entropy would be the same at all heights, for the changes are all adiabatic, hence the number of lines for any locality in this diagram is an indication of the stability of the air for downward motion or for upward motion on the one side of the limit of saturation.

It should be noted that in the stratosphere the lines are separated from one another by very small intervals, and we correctly infer therefrom that the stratosphere is characterised by great stability. In order to replace the air of any locality by air beneath it, that air would have to acquire the entropy suitable for the place it would have to occupy, and in the absence of that supply upward motion is not possible. If displacement is forced as, for example, by momentum, forces of restitution and corresponding oscillations would result.

The stratosphere is the only region which presents permanent conditions of stability. Corresponding properties are exhibited in the lower atmosphere from time to time in what are called *inversions* or regions of *negative lapse-rate*; such regions are nearly always found at the ground after clear nights, and generally in the polar regions during winter. They also occur in the free atmosphere from time to time, particularly, for example, in the trade-wind regions of the Atlantic. Such layers of increasing entropy are just as impermeable while they last as the stratosphere itself. We may with some assurance, therefore, regard them as "decks" on the analogy of the deck of a ship. The stratosphere is the main deck of the atmosphere; below it from time to time are temporary decks which act as impermeable layers and effective barriers to the ascent or descent of air.

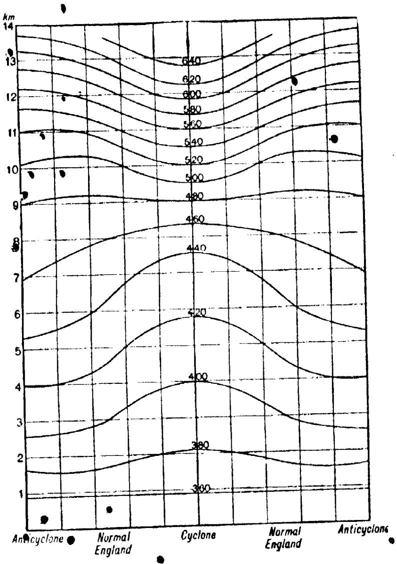


FIG. 10.—Normal Distribution of Realised Entropy in England referred to Conditions of High Pressure (anticyclone), Low Pressure (cyclone), and Normal Pressure (see Table III.).

The realised entropy is calculated by the same formula as *Fig.* 9.

§ (8) THE DISTRIBUTION OF PRESSURE AND THE WINDS CORRESPONDING THEREWITH.—At this part of the subject we make a junction with the dynamics of the atmosphere, which is dealt with in another article (see "Atmosphere, Physics of," §§ (2) *et seq.*).

Pressure in the atmosphere is related to the temperature. The connection between the two is represented with sufficient accuracy for our present purpose by the statical equation of pressure p in a fluid of density ρ at the point where the temperature is T. The equation is

$$dp = -g\rho dz = -\frac{gpdz}{RT},$$

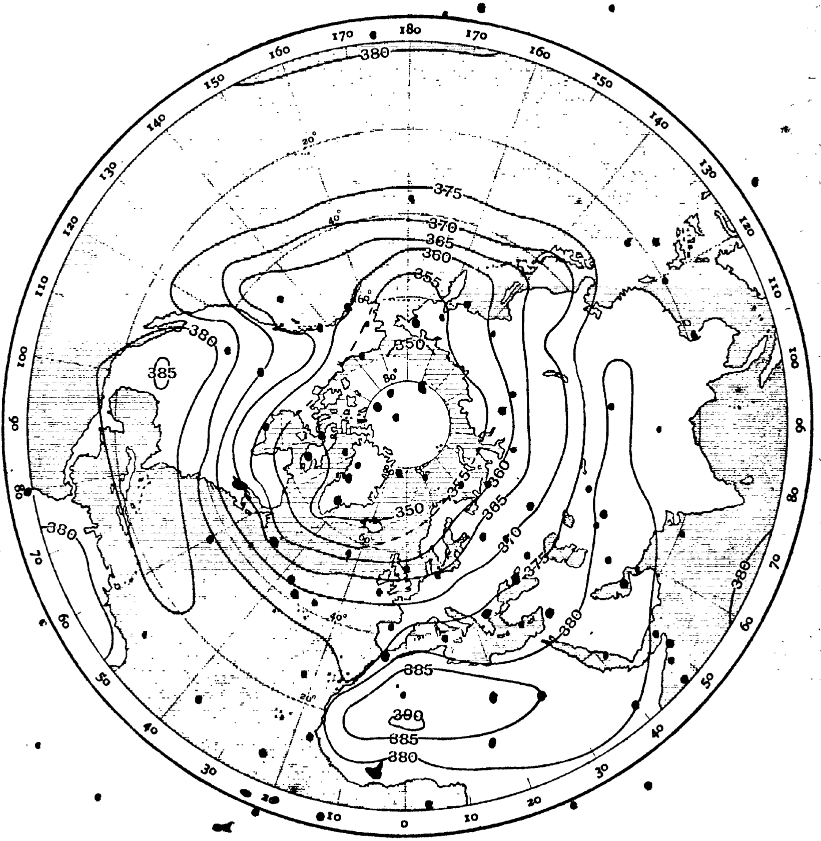
where z is the height co-ordinate measured from below upward.

FIG. 11.—MEAN PRESSURE AT 8 KILOMETRES IN THE NORTHERN HEMISPHERE.

Isobars are drawn for intervals of 5 millibars.

The values are computed from the pressure and temperature at the surface, assuming a lapse-rate of temperature of 5a from 0 to 2 km., 5·5a from 2 to 4 km., 6a from 4 to 6 km., 7a from 6 to 8 km. The variation of gravity with latitude and height is allowed for, but no allowance is made for humidity.

NORTHERN HEMISPHERE



Equivalents.		
mb.	in.	mm.
350	10·34	262·5
355	10·48	266·3
360	10·63	270·0
365	10·78	273·8
370	10·93	277·5

JULY

Equivalents.		
mb.	in.	mm.
375	11·07	281·3
380	11·22	285·0
385	11·37	288·8
390	11·52	292·5
395	11·66	296·3

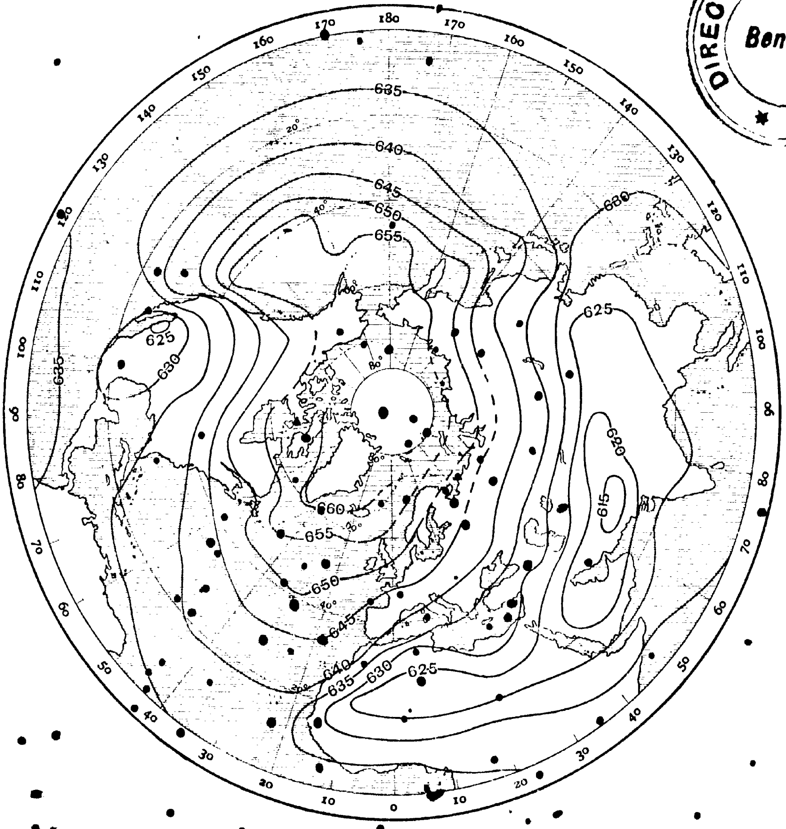
FIG. 12.—THE PRESSURE OF THE LOWER AIR—SEA-LEVEL TO 8 KILOMETRES IN THE NORTHERN HEMISPHERE.

Isobars are drawn for intervals of 5 millibars.

The values are computed from the pressure and temperature at the surface (see *Fig.* 11).

DIRECTOR OF INDUSTRIES Bengal

NORTHERN HEMISPHERE



Equivalents.

mb.	in.	mm.
615	18·16	461·3
620	18·31	465·0
625	18·46	468·8
630	18·60	472·5
635	18·75	476·3

JULY

Equivalents.

mb.	in.	mm.
640	18·90	480·0
645	19·05	483·8
650	19·19	487·5
655	19·34	491·3
660	19·49	495·1

From the surface upwards the pressure decreases with height, the loss of pressure for a given height depending upon the density of the column of air between the surface and the selected height. The chief controlling factor of the density is the temperature, and the next most important factor is the pressure, and only small effect is produced by the water-vapour present. To compare the relative importance of pressure and temperature we may note that the average differences exhibited at the surface in July are for temperature from 311a to 273a, producing about 14 per cent difference in density, and for pressure from 1025 mb. to 996 mb., accounting for 2·5 per cent change in the density. It follows therefrom that as we go upward pressure will diminish more rapidly over cold regions than over hot regions. In so far, therefore, as regions of high pressure are cold compared with adjacent regions of low pressure, their influence will be diminished with height, and in so far as they are warm relative to their surroundings they will become intensified at the higher levels.

Conversely, if a cyclonic area is warmer than its environment it will tend to disappear with height, and in so far as it is colder to be intensified. Cold anticyclonic regions and warm cyclonic regions, in so far as they exist must therefore be regarded as surface phenomena.

This conclusion is amply borne out by observations in the upper air where cyclonic regions are colder than anticyclonic for the same latitude and same time of year. But, as we have seen (Table III.), that state of things has not unlimited extension in the vertical. In the higher regions there is a curious reversal of the conditions obtaining below the tropopause. At the surface there is a gradient of temperature from the equatorial to the polar regions; at 20 kilometres that is completely reversed, the gradient of temperature is from the polar to the equatorial regions. In middle regions a cyclonic area is colder than an anticyclonic area, but at 15 kilometres the relation is opposite. So it comes about that differences of pressure tend to be non-existent at the level of 15 kilometres, and from there downward to 8 kilometres there is a gradient of density from the equatorial region to the polar region; from 8 kilometres to the ground, on the other hand, there is a gradient of density from the polar regions to the equator, which may be highly accentuated by the local cooling of the surface. If the pressure follows the density from 20 kilometres to 8 kilometres there is a gradient of pressure from the equator to the pole, gradually increasing downward. At 8 kilometres there is a level of uniform density, and from there downwards the influence of the reversed gradient of density begins to come in, and may become the dominant factor in some localities near the surface.

Hence we may divide the atmosphere into two layers at the level of 8 kilometres. For the upper layer there is a gradient of density poleward, and for the lower layer a gradient of density equatorward.

Figs. 11 and 12 represent the distributions of pressure which correspond respectively with the upper layer and the lower layer. We call attention to the remarkable similarity in the run of the lines of the two figures though they represent opposite gradients. Superposed, we might imagine that they would approximately annihilate each other. They nearly succeed in doing so, but owing to the indirect influence of land and water we have a surface distribution resulting from a combination of the two (*Fig.* 7) which is quite dissimilar from either of its components.[1]

The distribution of pressure is closely related to the winds. If we could assume that the motion of the winds is a horizontal motion without acceleration of speed, the relation at any point for any moment would be given by the equation

$$\frac{b}{\rho} = 2\omega v \sin\phi \pm \frac{v^2}{E}\cot r,$$

where b is the gradient of pressure transverse to the run of the isobaric line, ρ the density of the air, ω the rotation of the earth, v the velocity of the motion of air which is tangential to the isobar, ϕ the latitude, E the radius of the earth, and r the angular radius of the "small circle" osculating the path of the air. The + sign relates to cyclonic motion and the − sign to anticyclonic motion.

This equation can only be strictly applicable in special circumstances, and those circumstances are obviously absent in the complicated conditions of a travelling cyclone when the angular radius of the path is small. It is also obviously far from exact at the surface, where part of the kinetic energy of the motion is transformed into eddy-motion and dissipated in friction. So much so that the line of motion of the air may theoretically deviate from the isobar to the extent of 45°, and maps sometimes show even greater deviation. Nevertheless observations of wind and pressure at the surface first led to a recognition, in Buys Ballot's law, of some relation between the direction and force of the wind and the direction and separation of the isobars; and that being the case for the surface, where the circumstances are obviously unfavourable, we are justified in assuming that the approximation becomes closer in the upper layers.[2]

---

[1] See Shaw, "The Mechanics of the Atmosphere," *Nature*, July 7, 1904.

[2] The name of Buys Ballot may be offered as a reason for selecting b to represent barometric gradient in place of γ, which is often used for that purpose, but has already been used in this article for the ratio of specific heats.

So far as motion in the upper air is concerned we have not sufficient information to enable us to assign any definite value to the probable deviation of the direction and velocity of the wind from that calculated by the equation, but we know that the range of velocity displayed by the motion of the air in a period measured by hours is generally very small, and the numerical change of velocity per second at any moment can fairly be regarded as negligible in comparison with the instantaneous velocity of the air. It is in fact preferable to assume that for horizontal velocity in the upper air the wind corresponds with the relation expressed by the equation, than to introduce an additional term of which the magnitude is quite unknown, when all the available evidence points to its being very small in all the circumstances to which we can hope to apply the principles of dynamics and thermodynamics with success.[1]

When the direction of movement of the air differs little from the line of a great circle the second term of the equation itself becomes of very small importance, and a provisional relation of wind to pressure can be expressed by the simple formula

$$\frac{b}{\rho} = 2\omega v \sin \phi.$$

This consideration applies in the case of synchronous charts for single epochs; in the case of maps of mean values the mean isobars will correctly represent the resultant effect so far as the vectors of pressure-gradient are concerned, but the velocity deduced from the resultant gradient is not rigorously the actual resultant of the several velocities, because of the dual character of the relation of the velocity to the gradient when the path is curved.

In the course of the formation of the maps of the resultant distribution of temperature, vapour pressure, rainfall, pressure and wind, the peculiar features of any occasion will be merged in the general map, unless the peculiarities are permanently attached to a particular locality. This is notably not so in the important case of travelling cyclones which pass along more or less recognised paths, but are seldom stationary when the hour is the practical unit of time, still less so when the day is the unit, and never when the month is the unit. Consequently we must treat transient cyclonic and anticyclonic conditions separately from the average conditions. Even when we are dealing with average or normal conditions we must expect some deviation between theoretical and observed results, and refrain from carrying the reasoning to details.

§ (9) THE KINETIC ENERGY OF THE ATMOSPHERE.—We may refer again to *Fig.* 11, representing the distribution of pressure at the level of 8000 metres, and consider the velocity corresponding with the distribution of pressure set out therein according to the formula $b/\rho = 2\omega v \sin \phi$. We may note that there is on the one hand a vast circulation from west to east round the poles, and on the other hand a circulation from east to west along the line of the equatorial doldrums is indicated by the isobars near the equator. Between the easterly circulation of the equator and the westerly circulation of the regions of 40° N. we find vast anticyclonic areas over the continents with winds circulating round them.

If we associate the equatorial side of the anticyclonic circulations with the easterly current of the equatorial region, and the polar sides with the westerly current of the polar circulation, we are led on to the idea of dividing the general circulation into two parts, separated, the one from the other, by the ridge-lines of the tropical anticyclones. The one part represents a flow from east to west over the equatorial region, the other a circulation from west to east round the pole.

Calculations from the formula already quoted, supported by observations with pilot-balloons, enable us to make a rough estimate of the kinetic energy which is represented by these circulations. If we take the parallel of 30° as separating the westerly from the easterly circulation, we can look upon the latter as approximately a revolving belt of air 60° wide. Its weight as indicated by the pressure at the surface works out at $2\cdot7 \times 10^{21}$ grammes or $2\cdot7 \times 10^{15}$ metric tons. Taking the mean velocity at 10 metres per second, the energy of the moving mass of air will be

$$\tfrac{1}{2} \times 2\cdot7 \times 10^{21} \times 10^{6} \text{ ergs} = 1\cdot35 \times 10^{27} \text{ ergs},$$

or the energy of 2700 billion tons moving at 10 m/s. or 45 thousand billion foot-tons.

We may next consider the energy of the pair of the revolving polar caps. Their mass is not substantially different from that of the equatorial belt, since their area between 30° and the poles is one-half of the area of the sphere. We may next consider that the moment of momentum of easterly and westerly circulations about the polar axis must be the same, since the motion is caused by internal actions and reactions. The equivalent radius of the moment of momentum will certainly be less than the mean radius of the equatorial belt. Consequently the balance of moment of momentum must be conserved by a greater velocity of the moving air, hence the velocity of air in the westerly circulation ought to be greater than in the easterly circulation, and the kinetic energy of its motion, the mass being the same, should be greater than the energy of the equatorial belt.

---

[1] See "Principia Atmospherica," *Proc. Roy. Soc. Edin.*, 1913, xxxiv. 77.

Hence the whole kinetic energy of the general circulation is of the order of 3×10^{27} ergs.

To this we need to add the energy of the local circulation of cyclones and anticyclones of limited area. Later on in § (26) we have quoted an estimate $1 \cdot 5 \times 10^{24}$ ergs for a depression of no great depth or extent. The figures would be largely exceeded in many cases, and the motions of the atmosphere represent displays of kinetic energy on a gigantic scale.

III. THE AGENCIES AT WORK: RADIATION

§ (10) ANALYSIS OF RADIATION INTO LONG-WAVE AND SHORT-WAVE RADIATION.—The primary agency in the thermodynamic processes is the effect of solar and terrestrial radiation in supplying heat to the atmosphere or removing it therefrom.

The principal effect is related to the surface and is indicated by the maps of the distribution of temperature already cited (*Figs.* 1-4). Therein is noticeable that, as the result of direct absorption of solar radiation at the surface in the equatorial regions, there is not much difference between the *mean values* of temperature of the air over sea and over land, but there is a great difference between them as regards diurnal range of temperature. There is practically no diurnal variation of the temperature of the air over the sea, whereas the diurnal range over the land, increasing from the coast inland, becomes very great in the interior of continents, with extremes as wide apart as 23a for the month of July at Aswan, or 25a at Insalah (lat. 27.17 N., long. 2.27 E.) in July, 26a in March at Jaipur, India, or 26a at Alice Springs, or 30a at Laverton in Central Australia for the month of January 1911.[1] There are corresponding differences in the amount of water-vapour contained in the air, the amount inland being as a rule very much smaller than at the coasts or over the sea. Since water-vapour represents thermal energy in a latent form, which becomes available when condensation occurs, it is necessary to have regard both to the temperature and water-vapour in considering questions of heat and energy in the atmosphere.

The consideration of the thermal effect of the distribution of temperature produced by the absorption of solar radiation or the emission of terrestrial radiation is, however, comparatively simple compared with the effects of radiation from the air itself with its varying amount of water-vapour. The full discussion of that side of the subject involves the consideration of the energy of different wave-lengths and its relation to emission and absorption in an atmosphere of varying composition as regards water-vapour. The variation is of vital importance, because it is the water-vapour rather than the dry air which controls the absorption and radiation of energy by the atmosphere. By way of simplifying the consideration it has become customary to consider radiation as separable into radiation of long wave-length appropriate to water-vapour and the earth, and of short wave-length appropriate to the solar radiation. Solar radiation is diminished by one-tenth in passing through the whole atmosphere, terrestrial radiation by nine-tenths.[2] Thus for terrestrial radiation the atmosphere behaves practically as a black body, and for solar radiation as a transparent body.

The chief phenomenon, which we have to rely upon radiation to explain, is the approximately vertical setting of the isothermal surfaces in the stratosphere. The characteristic of the stratosphere to which the possibility of such an arrangement is attributed is the absence of any convection of the kind which is preceded by statical instability of successive layers in the vertical. The withdrawal of air from under the stratosphere or its heaping up there and the consequent general sinking or elevation of the tropopause and the layers above it may still be allowed, but the other form of convection which would tend towards convective equilibrium cannot exist if the atmosphere remains arranged in isothermal vertical sheets.

§ (11) THE GENERAL BALANCE OF SOLAR AND TERRESTRIAL RADIATION, EMDEN'S EQUATION.—In the absence of convection of the ordinary penetrative character it is to radiation alone that we must look for maintaining the condition of the stratosphere, and for this we are provided with conclusions of a very general character by Humphreys,[3] Gold,[4] and Emden.[5] The last, considering the transference of heat by radiation through the atmosphere, arrives at a curve of variation of temperature with height which is nearly vertical above the level of 10 kilometres.

The reasoning, which is based upon the ultimate equality of gain and loss of heat by solar and terrestrial radiation, is set out in § 28 of F. M. Exner's work on *Dynamische Meteorologie.*[6]

The equation arrived at is

$$\sigma T^4 = \tfrac{1}{2} I_1 \left[1 + mb + \frac{a}{b}(1 - am) \right], \quad . \quad . \quad (1)$$

---

[1] *Réseau Mondial*, 1911, M.O. 207g, pp. 52, 20, 7.

[2] Abbot and Fowle quoted *s.v.* "Atmosphere, Physics of the," by D. Brunt, § (6) (*c*).

[3] *Astrophys. Journ.*, 1909, xxix. 14. *Bull. Mt. Weather Obs.*, 1909, ii.

[4] *Proc. Roy. Soc.*, Series A, 1909, lxxxii. 43.

[5] *Sitz. Ber. Ak. Wiss.*, 1913, p. 55.

[6] F. M. Exner, *Dynamische Meteorologie*, B. G. Teubner, Leipzig, Berlin, 1917.

where T is the temperature on the tercentesimal scale; σ Stefan's constant for the emission of radiation per unit area of a radiating surface; I_1 is the effective intensity of solar radiation per unit area, which is reduced from the full intensity of 2 gramme-calories-per-minute to ·5 gramme-calorie-per-minute to give the mean intensity of solar radiation over the globe throughout the day and night, and further reduced by 37 per cent on account of the dispersal of the solar radiation by reflection or the radiative intensity of the earth's albedo; m is the fraction of the mass of water-vapour in the column of the atmosphere between the layer of temperature T and the top; a is the coefficient of absorption per unit mass of water-vapour for solar radiation, and b the corresponding coefficient for terrestrial radiation.

From that equation Exner gives the following table of relation of temperature and height when the distribution of temperature is controlled by the exchange of radiation under the most general average conditions.

Height in Metres.	Fraction of Water-vapour already traversed by the Solar Radiation.	Temperature required for Steady Condition.	Lapse-rate of Temperature.
		a.	a per 1000 m.
∞	0·00	216	0·0
10,000	0·0215	218	2·3
6,000	0·1	227	4·5
4,190	0·2	236	7·6
3,140	0·3	245	10·7
2,390	0·4	253	12·1
1,810	0·5	260	12·5
1,330	0·6	266	14·7
580	0·8	278	19·0
0	1·0	288	

The nearly vertical line between 10,000 m. and infinity has a lapse of only two degrees, and the temperature at 10,000 m. 218a, compared with a ground temperature of 288a, is in remarkably good agreement with a generalised view of the ascertained average facts for middle latitudes of the northern hemisphere: the corresponding figures for England as set out in Table II. are 289a and 222a in the summer and 270a and 217a in the winter.

In the course of the proof it is assumed that the absorption is due entirely to the water-vapour which the atmosphere contains, and that the solar radiation A_1 at any level going downward is controlled by the equation $dA_1 = -A_1 dm$ or $A_1 = I_1 e^{-am}$; whence, using Abbot and Fowle's value 0·1 for the fractional absorption of the whole atmosphere, $e^{-a} = 0\cdot 9$, and by similar reasoning $e^{-b} = 0\cdot 1$, whence $a = \cdot 105$, $b = 2\cdot 30$.

Relying further upon Hann's equation[1] for the distribution of water-vapour in the atmosphere,

$$e = e_0 \times 10^{-h/6000},$$

where e is the partial pressure of water-vapour at the height h (in metres), and e_0 is the partial pressure of water-vapour at the surface; and further assuming that the partial pressure of water-vapour is separately cumulative as one goes downwards (as it would be in an atmosphere controlled merely by molecular diffusion), so that e_0 represents the whole weight of water-vapour per unit of area at the surface, and e the weight per unit of area at the level h, we thus get $m = 10^{-h/6000}$, and therefore (d) takes the following form for numerical computation of temperature in relation to height:

$$\sigma T^4 = \tfrac{1}{2} \times \cdot 315\left[1 + 2\cdot 30 \times 10^{-h/6000} + \frac{\cdot 105}{2\cdot 30}\left(1 - \cdot 105\,.\,10^{-h/6000}\right)\right].$$

§ (12) THE OUTFLOW OF RADIATION FROM THE EARTH AND THE TEMPERATURE IN RELATION TO THE EQUATION FOR RADIATION.—But the reasoning is of so general a character, involving the averaging of the loss of radiation in the albedo and the averaging of the solar radiation over the whole globe as well as throughout the day and night, that it is difficult to extend its conclusions to the special circumstances of different localities on the earth's surface under similar limitations of a very general character.

We may note that while the argument depends upon the outward stream of radiation from the earth being balanced by the inward stream of radiation from the sun, less than one-sixtieth of the solar radiation comes into the calculation of the absorption of the whole column, *i.e.* ·1 of I_1 or ·0315 g.-cal. per cm.^{2} per min. And in actual practice it varies between one-tenth or more in bright sunshine and zero in the Arctic countries or at night, so that the conditions apply only to an imaginary mean atmosphere. And since the absorption is assumed to depend upon the water-vapour the solar radiation only begins to be practically operative in affecting temperature where there is water-vapour in measurable quantity, and that is comparatively low down in the atmosphere.

We may perhaps put the question in a clearer light by considering the loss of heat by terrestrial radiation independently of its compensation by solar radiation, of which 90 per cent traverses the atmosphere and is taken up by the land or sea or reflected outwards without affecting the body of the atmosphere.

Further, we may perhaps get some insight into the subject by using our knowledge of the temperatures within and beyond the practical limits of the water-atmosphere, to get a measure of the total quantity of water-vapour in different latitudes, instead of assuming that quantity to be expressed by a mean value for the total atmosphere.

Using the same method of reasoning as Exner, and, so far as may be, the same notation, we may proceed as follows: Premising that we have to express a steady flow of heat by radiation outwards from the earth through the atmosphere vertically into space, we may call this flow R. At the extreme limit of water-vapour R remains, passing thence into space as the final result of the process; and at the ground R is represented by the difference between the heat passing by radiation from the solid or liquid surface into the atmosphere and the heat carried as return radiation downwards from the air to the surface. The heat passing from the surface upward by radiation may be expressed as σT_E^4, where σ is Stefan's constant, and T_E the temperature of a "black" body equivalent in radiative power to the surface under consideration.

---

[1] The conditions are discussed and various equations proposed in Hann, *Lehrbuch der Meteorologie*, 1915, 3rd ed. 230.

Let B_1 represent the flow of "earth radiation" upward which at the surface is σT_E^4;

B_2 represent the upward flow by radiation from the atmosphere.

Then $B = B_1 + B_2$ represents the total upward flow at any section of a vertical column of unlimited height 1 cm.^{2} in area of cross section.

Let A represent the downward flow by radiation at the same section.

Then in the steady state

$B - A$ is R, the constant flow outward into space, equal to the difference between σT_E^4 and the flow from the air to the ground.

h = the height of the section under consideration.

Let W = the total quantity of water-vapour in the column;

mW = the amount of water-vapour between the ground ($h=0$) and h.

w = the amount of water-vapour which accumulated in a layer would radiate as a black body.

b = the coefficient of absorption of radiation per unit mass of vapour.

T = the temperature at h on the terrestrial scale.

T_0 = the temperature of the air at the ground.

The radiation from a layer of water-vapour of thickness dh and temperature T with quantity of vapour dm is $(bW/w)\sigma T^4 dm$.

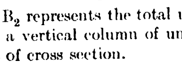


FIG. 13.

Then with the arrangement set out in *Fig.* 13:

$$-dA = -\frac{AbW}{w}dm + \frac{bW}{w}\sigma T^4 dm. \quad . \quad . \quad (1)$$

$$dB = -(B_1 + B_2)\frac{bW}{w}dm + \frac{bW}{w}\sigma T^4 dm. \quad . \quad (2)$$

$$dA + dB = -(B - A)\frac{bW}{w}dm. \quad . \quad . \quad . \quad (3)$$

$$dB - dA = -(B + A)\frac{bW}{w}dm + \frac{2bW}{w}\sigma T^4 dm. \quad . \quad (4)$$

$B - A$ = constant = σT_E^4 − flow from air to ground = R. (5)

From (3), since from (5) $dB - dA = 0$, we get

$$2 . dA = -(B - A)\frac{bW}{w}dm. \quad . \quad . \quad . \quad (6)$$

Integrating (6) from m to the upper limit, where A is 0,

$$2A = (B - A)\frac{bW}{w}(1 - m). \quad . \quad . \quad . \quad (7)$$

From (4)

$$2bW\sigma T^4 dm = (A + B)bW dm,$$

or

$$2\sigma T^4 = A + B = 2A + (B - A)$$

$$= (B - A)\left(1 + \frac{bW}{w}\overline{1 - m}\right). \quad . \quad (8)$$

At the ground $T = T_0$ and $m = 0$,

$$2\sigma T_0^4 = (B - A)\left(1 + \frac{bW}{w}\right).$$

Hence substituting for $B - A$ in (8) the equation between temperature and water-vapour becomes

$$T^4 = \frac{T_0^4}{1 + (bW/w)}\left(1 + \frac{bW}{w}\overline{1 - m}\right),$$

$$\left(\frac{T}{T_0}\right)^4 = \frac{1}{1 + (bW/w)}\left(1 + \frac{bW}{w}\overline{1 - m}\right). \quad . \quad (9)$$

Note.—D. Brunt has pointed out that an equation of this form will take account of "scattering" as well as radiation by augmenting the "loss-coefficient" b, by a constant $\frac{1}{2}s$ where s is coefficient of scattering.

If we suppose the stratosphere to be a region in which the amount of water is so small as to produce no sensible addition to the total mW we may put $m = 1$ in equation (9), substitute for T the normal temperature of the stratosphere in any locality or for any season, and for T_0 the corresponding temperature at the ground, and use equation (9) to determine the value of bW/w, the ratio in which terrestrial radiation is diminished by passing through the water-vapour in the vertical column above it, expressed in terms of the amount required for "black-body" radiation. Results obtained for certain cases are as follows:

Locality and Season.	Temperature at the Ground.	Temperature in the Stratosphere.	bW/w.	Residual Transmission into Space of Unit Radiation from the Ground.
	a.	a.		
Batavia, July .	299	187	5·53	·004
England, July .	289	222	1·87	·154
England, January	276	217	1·62	·198

In computing the curve of relation of temperature and height for radiative equilibrium, F. Exner used for the fractional survival of radiation at the limit of the atmosphere 0·1 as quoted from Abbot and Fowle. From the calculations here given it appears that that value is too great for the equator, where the survival fraction is ·004, and too small for England in summer or winter, where respectively ·154 and ·198 of the radiation from the ground escapes absorption and passes into space. We gather from the figures for bW/w given in the fourth column that the amount of water-vapour in the air above Batavia is five and a half times as much as would be necessary to constitute it a "black body" if concentrated into a single sheet; over England the amount is nearly double the black-body standard in summer, and fifty per cent above black-body standard in winter.

Substituting the values which we have here computed from the temperatures of the stratosphere in the three cases given, we obtain the curves given in *Fig.* 14 to represent the curve of radiative equilibrium for the three cases mentioned. Hann's equation for the variation of water-vapour with height is still assumed as a basis of the calculation.

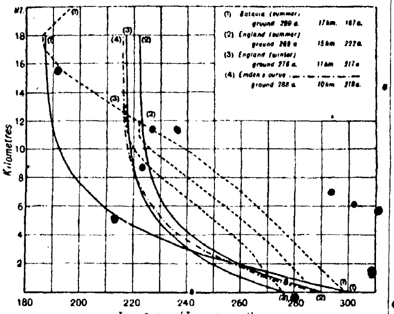


FIG. 14.—Variation of temperature with height in an Atmosphere during steady loss by radiation. The observed temperatures at Batavia, in England (summer), and in England (winter) are shown by dotted lines, and the curves of temperature for steady loss by radiation agreeing with the observations at top and bottom are shown by full lines.

Emden's curve for temperature, with balance of radiation and assumed values for the effect of water-vapour, is shown by a chain line.

To the diagram of the temperatures at successive heights which would correspond with an equilibrium based on radiation alone, the average temperatures as obtained by direct observation have been added; they agree at the ground and in the stratosphere, but between those extremes the actual temperature is largely in excess of the theoretical temperature, which takes no account of solar radiation. The influence of that radiation can be estimated by comparing Exner's curve with those based on the formula here given. Nor is any account taken of the heat communicated to the air by conduction and evaporation of water by contact with the water or ground at the surface and distributed by convection over the region through which convection extends. We note also that the theoretical curve indicates a lapse-rate of temperature above the adiabatic lapse for dry air (10a per 1000 m.) from the ground to 4000 m., so that convection would ensue in any case. The difference of temperature between the theoretical and actual curves (2) amounts to about 25a between the levels of 4000 m. and 6000 m. How the disturbance of the temperature conditions in this sense would affect the distribution of temperature in the stratosphere itself is not apparent, but it is clear that the loss by radiation through the stratosphere would be greatly in excess of that which passes directly from the ground, and that the atmosphere would be losing heat by radiation from the water-vapour throughout the troposphere in addition to that lost by direct radiation from the ground.

§ (13) CONVECTION IN RELATION TO ENVIRONMENT.—The other chief physical process by which thermal changes are effected in the atmosphere is convection. The rudimentary phenomena are familiar as experiments in the physical laboratory, but their application to the atmosphere entails serious complications, because the environment in which the changes take place is not by any means the homogeneous[1] medium which is generally assumed in the explanation of an experiment on thermal convection in the laboratory. Its temperature and pressure, consequently its density and its statical relation to any specimen of air in contact with it, change normally with height on the average, and quite irregularly with height from time to time. Moreover, the air itself, which rises or sinks in consequence of the initial stages of convection, changes its pressure and temperature with the change of height, and the changes for any finite displacement depend upon the uncertain condition whether condensation of the water-vapour sets in or not.

Air warmed to the extent of 1a above its environment at the surface would rise through its environment like air in a laboratory, but as it rose it would cool at the rate of 1a per 100 metres. If the temperature of the environment fell off at the rate of half a degree per 100 metres, the rise would terminate at 200 metres, an elevation which is very high compared with a laboratory enclosure but not of very great importance on the meteorological scale.

It follows that upward convection, as usually understood in physical writings, can only proceed when the existing variation of temperature with height in the environment is greater than the variation which would take place in the air considered if its height were changed in like manner.[1] This is a very serious limitation of natural convective processes.

---

[1] Homogeneous is not a good word for this purpose, as it generally implies merely uniform composition. The term *isoplethic* would be very useful in this connection, as signifying, on the analogy of isothermal, that all the physical quantities—pressure, temperature, and density—had the same value throughout the region considered. But objection has been taken to the use of isopleth, except in a restricted sense, which is not appropriate here.

Thus dry air can only ascend automatically by convection in an atmosphere which is itself in convective equilibrium and has the adiabatic lapse-rate for dry air, and per contra dry air can only descend automatically in a region where there is convective equilibrium of dry air. On the other hand, if condensation begins in the ascending air the further ascent can go on automatically if the lapse-rate of the environment is not less than that of the ascending saturated air.

§ (14) RESILIENCE DUE TO THE DIFFERENCE FROM CONVECTIVE EQUILIBRIUM.—Unless the environment is itself in convective equilibrium for dry air, warmed dry air cannot ascend automatically beyond a certain limit, which can easily be calculated. If saturation supervenes, then the air may continue to rise along the saturation adiabatic (see § (22)) if the lapse-rate in the environment is as great or greater than that of the saturation adiabatic. Hence the continued upward motion depends on the temperature of the environment, and, as we have seen in § (7), the state of the environment from this point of view may be expressed by its realised entropy. We have also noted that from the distribution of entropy arises the capacity of a layer of air to act as a "deck" or ceiling, preventing any vertical motion, and therefore limiting the motion of the atmosphere to horizontal layers. Thus each layer of the atmosphere has a certain capacity for producing forces of restitution in vertical displacement which give it resilience, one of the elastic properties of a solid.

Hence the realised entropy of a layer compared with the layer beneath or above is an index of its resilience; where the variation of realised entropy with height is zero there is no resilience even for dry air, but where the realised entropy increases with height more rapidly than entropy can be supplied to dry air by automatic condensation of water the layer is resilient even to saturated air.

§ (15) PENETRATIVE AND CUMULATIVE CONVECTION.—The process of convection to which we have been referring will naturally be understood to be the penetration of the upper layers by a limited mass of air specifically lighter than its environment, or, on the other hand, the descent of a limited mass of air specifically heavier than its environment. In either case it is gravity which supplies the driving power, in the first case by pushing the lighter air upward and closing in to take its place, and in the second case by the excess weight of the mass itself which displaces the air beneath, leaving the space which it originally occupied to be filled by the environment, with such adjustment of levels as may be necessary.

In the typical experiments, penetrative convection is set up by local heating, and the fluid which replaces that which has ascended, itself becomes heated in turn and ascends; thus the horizontal motion of the fluid on its way to replace the ascended fluid is regarded as part of the penetrative convection, and indeed an index of that convection. It is necessary to point out that this is not always the case in the atmosphere. The air which flows towards the central region of a cyclone does not necessarily form part of a continuous vertical circulation. It may simply accumulate in a gradually increasing mass of air which has no penetrative capacity, so that the progress of the convective action may either be arrested or pass to other parts of the environment. As the mass of air accumulates it may increase in thickness, and physical effects may be produced either in its own mass or in that of the air above it by the elevation due to the gradual accumulation. Such an accumulation of air may be caused dynamically in the atmosphere by the flow of air from high pressure to low pressure across isobars, on account of the fact that the friction of the surface reduces the velocity of the layers near the surface below that which is necessary kinematically to balance the distribution of pressure (see § (8)). It is desirable, therefore, to draw a distinction between the lifting of the air over a large region by the accumulation due to the flow across isobars, and the penetrative convection due to the specific lightness of a limited mass of air. We will call the general lifting due to gradual accumulation *cumulative convection.* The same kind of process takes place in the opposite direction. Air flows away from high pressure to low pressure, but it does not follow that there is anything that can be legitimately called a "descending current" in the central region of the area of high pressure: there may be instead a gradual loss of air or *decumulation* over the whole region which entails a slow settlement and nothing more.

§ (16) THE RELATION OF VERTICAL FLOW TO HORIZONTAL FLOW.—The importance of the distinction between penetrative convection and cumulative or decumulative convection acquires added weight in view of the relation of the horizontal motion across the isobars to the motion in the vertical over the central region. There are three typical distributions of velocity with which any ascertained distribution of velocity at the earth's surface would naturally be compared; the first is that of a uniform current represented by straight isobars, the second a region in which the velocity at any point is directly proportional to the distance of the point from a central axis, and the third a region in which the velocity is inversely proportional to the distance of the point from a central axis.

---

[1] Shaw, *Forecasting Weather,* 1913, p. 175 (Constable & Co.).

If we consider the behaviour of air flowing from " high " to " low " with a certain inclination a of the flow to the line of the isobar which marks the direction of undisturbed flow, we can see that, in the case of straight isobars, the constant inclination a implies that the air is flowing across the field represented by the isobars without any loss or gain, so no upward convection is caused or required. In the second case, in which the velocity may be taken as v_0r, the flow across a depth h of a circular isobar, radius r, $=2\pi v_0 \sin ar^2h$, *i.e.* proportional to the area, the same is true of every other interior isobar and therefore of the ring between any pair. Consequently such a flow requires a uniform loss of air upward over the whole of the area or demands a supply of air from above, equally uniform over the area. On the contrary, when the velocity-law is that of inverse proportionality to the distance or $v=v_0/r$, the flow, with uniform inclination across it, over the isobar radius r is $2\pi v_0 h \sin a$, that is, the same for each radius. Hence across a vertical circulation of that description no relief is required from the upper air, the flow takes place across successive rings without loss or gain.

We cannot, of course, claim that the distributions which we find in a weather map always conform rigorously to the law $v=v_0r$, or $v=v_0/r$, nor that the inclination of wind to isobar is always uniform. But there are cases in which the conditions are sufficiently nearly satisfied to indicate cumulative or decumulative convection. For example, the velocity of air in an anticyclone is often not unfairly expressed as proportional to the distance from the centre; the inclination of the flow to the isobar is not exactly uniform, but sufficiently nearly so for general guidance. We can therefore apply the calculation in such a case.

If we take as an example the anticyclone of March 26, 1907,[1] the radius of the isobar of 30·4 in. is 275 nautical miles, 511 km., the mean rate of outflow at the bottom is estimated from the observations at Faröe, North Shields, Cuxhaven, Paris, Jersey, Brest, Roches Point, and Donaghadee as 1·4 m/s. The volume of outflow (assuming the winds to be along the isobars at 500 m. and the velocity of outflow to vary uniformly from its value at the ground and to be zero at 500 m.) is $\frac{1}{2}\pi \times 1\cdot4 \times 10^{-3} \times 511$ km^3. per second, the area within the isobar of 30·4 in. is $\pi \times (511)^2$ km.^{2}, the volume of the outflow distributed uniformly over the area implies a settlement at the rate of 70/511 cm. or 1·5 mm. per second or 132 m. per day. In like manner the flow of air along the surface from the normal permanent summer anticyclone of the North Atlantic ocean works out at a settlement of 86 m. per day. These calculations assume the anticyclones in each case to be permanent, so that the rate remains uniform. In the case of the " filling up " of a cyclonic depression over the North Sea between August 3 and August 6, 1917, when the pressure difference was assumed to diminish with the flow, the filling up by inflow at the bottom was computed on the same lines to take $2\frac{1}{2}$ days. It may be of interest to note that the water-vapour brought within the area of the cyclone by the inflow would imply a " drizzling rain " which, if distributed uniformly over the area, would amount to ·2 mm. per hour.

Thus these decumulations of air from anticyclones or accumulations of air at the base of cyclones imply a gradual descent or elevation of air over large areas, which may take months for the completion of a journey between the surface and 10 kilometres above it. They are different from penetrative convection, which may represent a current ascending with a velocity of the order of 15 metres per minute (one-tenth of the rate of ascent of pilot-balloons); these would complete the journey of 10 kilometres in 10,000/900, or 11 hours. Thus in a cyclone there may be an ascending current even if the minute or the hour is the unit of time, but in an anticyclone the week or the month must be the unit of time from the point of view of a descending current.

Hence the distinction between cumulative convection and penetrative convection is also the difference between a slow settlement, in which air, unless otherwise disturbed, is left to the incidental changes that may take place in weeks or months, and the phenomena of a descending or ascending current in which air may be conceived of as moving downward or upward under adiabatic conditions.

It is desirable to note that the flow across isobars which is discussed here is a dynamical effect which will take place regardless of the temperature of the air which forms the flow. Temperature only comes in as a disturbing cause when the relations between the surface and the air above it become unstable. We can thus understand that warm air or cold air may travel along the surface for great distances and give rise to notable discontinuities of temperature of the surface which have no counterpart in the upper air.

§ (17) Frictional Convection. Eviction of Air.—One of the effects of penetrative convection which has not hitherto been recognised as a meteorological agency, but is likely to prove of vital importance, is the dragging up of part of its environment by ascending air in consequence of the eddy-motion set up by the discontinuity of motion between the rising air and the environment through which it rises. This is a purely dynamical consequence of the motion in virtue of which the momentum originally belonging to the rising air is distributed over a much larger volume of air formed by the mixing of the rising air with part of the environment and a corresponding loss of kinetic energy.

---

[1] Shaw, *Forecasting Weather*, 1913, p. 284 (Constable & Co.).

In some experiments to ascertain the dimensions of this frictional effect of convection the author has found that a vertical discharge at the rate of 8 litres per minute through an orifice roughly estimated as half a square centimetre, abstracted about 80 litres per minute from a layer of its environment 10 cm. thick.

We use the term "eviction" to denote this effect of penetrative convection.

In the experiment the terminal velocity of the air was about half a metre per second. If all the momentum had been conserved, which is not exactly the case, the initial velocity would work out at 5·5 metres per second. Upward velocities of that order are known to occur in the atmosphere though they are uncommon. The reduction of the original velocity of the jet would reduce the amount of air evicted to an extent not yet ascertained, but the reduction would not be to zero, and consequently the eviction from the environment of air which is in process of penetrative convection must be regarded as of real meteorological significance. Importance must be attached to it because the abstraction of air from any environment in the atmosphere in which there is already some vorticity results in the superposition of the motion of a simple vortex upon the original motion, and the rotation of the earth itself is a sufficient substitute for original vorticity.[1]

Experiment also shows that a corresponding result upon the environment ensues when fine sand is allowed to fall through the air. The amount of air evicted in this case has not been measured, but it is of the same order and numerically larger than that due to the jet of air described.

Thus the penetration of air, whether by air in penetrative convection or by particles of solid as sand, or of liquid in the form of water drops, will produce the eviction of air and cause effects which used, perhaps erroneously, to be attributed to the direct replacement of air by denser air, as in the conventional case of convection in a laboratory experiment.

IV. The Thermodynamic Properties of Dry Air

As the next step in tracing the changes in the thermal state of the air which may be supposed to take part in the maintenance of the general circulation and the local circulations of cyclones and anticyclones, we consider the general equations which represent the thermodynamic properties of air. Peculiar complexity arises in this connection on account of the possible variations in the amount of moisture in the air, and the complexity is the more difficult to deal with in a satisfactory manner because the range of temperatures which have to be considered in dealing with the atmosphere extends from about 350a to about 180a. On the low side this goes 70a below the limit of the classical researches of Regnault upon the saturation pressure of aqueous vapour, and the experimental data are scanty.

The fundamental variable in the thermodynamics of the atmosphere is pressure, because, on the one hand, any motion and especially any variation of height entails a variation of pressure, and, on the other hand, the limiting value of the amount of water-vapour is controlled according to Dalton's law by the partial pressure of the water-vapour when the air is saturated. The other variables are the temperature and the density, or its reciprocal the specific volume.

§ (18) The Characteristic Equation.—The first equation of thermodynamic relationship is the characteristic equation. For *dry air* the relation is expressed, with sufficient accuracy for meteorological purposes, by the equation

$$pv = Rt,$$

where p is the pressure, v the specific volume, t the temperature measured from the zero of the gas thermometer, and R is a constant. If the pressure in millibars be represented by P, specific volume in cubic metres per gramme V, and temperature on the tercentesimal scale T, the value of R becomes $1000/1201 \times 290$, or $2·870 \times 10^{-3}$.

§ (19) The Isentropic Equation.—The second relation is that between pressure and volume under adiabatic conditions, that is when the gas is regarded as enclosed in an environment which prevents it receiving or losing heat by the process of conduction or molecular diffusion or by radiation. For dry air, again, the relation is well known,[2] viz.

$$pv^{\gamma} = f(\phi), \quad . \quad . \quad . \quad (1)$$

where p, v have the same meaning as before, γ is the ratio of the specific heats of air at constant pressure and constant volume respectively, and ϕ is constant so long as the transference of heat is prevented. It follows from the principles of a reversible thermodynamic engine that if amounts of heat H, H′ are required to pass from one adiabatic to another at temperatures T, T′ respectively H/T and H′/T′ are equal; hence if H/T be regarded as a new variable, the *entropy*, it will be constant for all points along each adiabatic, and the change of entropy from one adiabatic to another will be the same at whatever temperature the change may be effected. Hence the line along which H/T is constant is isentropic, and the value of the constant $f(\phi)$ will change in consequence of the change of entropy from one adiabatic or isentropic line to another.

---

[1] Rayleigh, *Proc. Roy. Soc.*, Series A, 1916, xciii. p. 148; D. Brunt, *Proc. Roy. Soc.*, Series A, 1921, xcix. p. 397.

[2] See article "Thermodynamics," Vol. I.

If we suppose the change effected by communicating heat at constant pressure the change of entropy may be computed.

$$\frac{dQ}{T} = c_v\frac{dT}{T} + \frac{Apdv}{T}$$

$$= c_v\frac{dT}{T} + \frac{A}{T}(RdT - vdp)$$

$$= c_p\frac{dT}{T} - \frac{ARdp}{p},$$

$$\int\frac{dQ}{T}(p\ \text{const.}) = c_p \log\left(\frac{T}{T_0}\right). \quad . \quad (2)$$

In order to make use of numerical values we must measure the entropy from a zero corresponding with some standard temperature. In what follows the temperature 200a and the pressure 1000 mb. have been chosen as the datum point for entropy, because dry air conforms with the gaseous laws to that limit or somewhat beyond it, and saturated air at that temperature contains so little moisture that the difference of its behaviour from dry air is of no importance in practice. On second thoughts one would have used the temperature of 100a instead of 200a and so avoided the appearance of negative values in entropy which may have to be dealt with if temperatures go below 200a, as they actually do at high levels over the equator.

With zero entropy at 200a and 1000 mb. we can find the entropy at temperature T_p and pressure 1000 mb.

From equation (2), if dQ/T is denoted by $d\phi$, ϕ being the entropy measured from the standard state, we have

$$\phi = \log_e\left(\frac{T_p}{200}\right)^{c_p}.$$

From temperature T_p and pressure 1000 mb. we can reach a temperature T and pressure p along the dry adiabatic. For the purpose of computation the original equation $pv^\gamma = f(\phi)$ may be transformed by the aid of the characteristic equation into

$$\frac{p}{p_0} = \left(\frac{T}{T_p}\right)^m,$$

where $$m = \frac{\gamma}{\gamma - 1}.$$

For our immediate purpose

$$\frac{p}{1000} = \left(\frac{T}{T_p}\right)^m,$$

hence

$$\log_e p - \log_e 1000 = m\,(\log_e T - \log_e T_p). \quad . \quad (3)$$

Hence the entropy at p, T is the same as the entropy at T_p, 1000 mb., where p and T are connected with 1000 mb. and T_p by the equation

$$\log\frac{T}{T_p} = \frac{\gamma - 1}{\gamma}\log\frac{p}{1000}.$$

T_p is the potential temperature as defined in § (6), hence the entropy of dry air at any pressure and temperature is a simple function of its potential temperature represented by equation (3), and we can use one term or the other as defining the condition of the air.

Having determined the entropy corresponding with a series of isentropics we are now in a position to express pressure and volume in terms of entropy and temperature.

It is perhaps desirable here to point out that entropy is not a physical quantity which is subject to conservation like energy or mass; it is a characteristic quantity like temperature. On a diagram of the properties of dry air referred to temperature and entropy as co-ordinates, the area of a closed curve represents energy. The area between any step of a curve and the line of zero temperature is the energy required to make the step, and the measure of it is the integral $\int Td\phi$. In like manner the energy required to make the step is measured by the area between the curve representing the step and the zero of entropy; in other words, the heat required for the step is $\int\phi dT$. Hence entropy is the rate of change of energy per unit change of temperature just as temperature is the rate of change of energy per unit change of entropy, and thus entropy and temperature are two corresponding identifications of the state of a body—one with respect to the communication of heat at constant temperature, and the other with respect to the communication of energy at constant entropy.

V. Thermodynamic Properties of Moist Air

§ (20) The Characteristic Equation.—When the water-vapour in the air is taken into account the thermodynamic equations have to be changed.

We now consider moist air as composed of X grammes or x kilogrammes of moisture added to one kilogramme of dry air. The value of X for air saturated under various conditions of pressure and temperature is given in the following Table IV.

TABLE IV.—WEIGHT IN GRAMMES (X) OF WATER-VAPOUR IN $(1+x)$ kg. OF SATURATED AIR ACCORDING TO THE FORMULA $X=622e/(p-e)$.

Temperature.	Total Pressure in Millibars.									
	1000.	900.	800.	700.	600.	500.	400.	300.	200.	100.
Tercentesimal Scale.	g.	g.	g.	g.	g.	g.	g.	g.	g.	g.
350	448·9	542·2	684·7	928·5	1442	3225	..	..	..	..
348	390·7	466·6	579·3	763·6	1120	2100	16982	..	..	..
346	341·8	404·4	495·2	638·6	898·8	1517	4857	..	..	..
344	300·3	352·6	427·0	541·0	738·1	1161	2722	..	..	..
342	264·9	308·9	370·5	462·9	616·4	922·6	1832	..	..	..
340	234·3	271·7	323·3	399·1	521·3	751·5	1346	6436	..	..
338	207·7	239·7	283·3	346·3	445·3	623·6	1040	3134	..	..
336	184·5	212·0	249·1	362·0	383·4	524·7	831·2	1998	..	..
334	164·2	187·9	219·7	264·5	332·1	446·1	679·4	1424	..	..
332	146·3	166·9	194·2	232·4	289·1	382·4	564·8	1080	12284	..
330	130·4	148·4	172·1	204·7	252·7	330·0	475·6	851·1	4041	..
328	116·4	132·1	152·6	180·7	221·6	286·3	404·4	688·4	2312	..
326	103·9	117·6	135·5	159·8	195·7	249·4	346·5	567·3	1564	..
324	92·76	104·8	120·4	141·6	171·7	218·0	298·7	474·2	1149	..
322	82·84	93·43	107·1	125·5	151·5	191·1	258·8	400·6	886·2	..
320	73·97	83·28	95·29	111·3	133·9	167·9	225·1	341·2	705·3	..
318	66·03	74·23	84·78	98·81	116·0	147·7	196·3	292·5	573·7	14734
316	58·92	66·15	75·43	87·72	104·8	130·2	171·7	252·1	474·2	3990
314	52·55	58·94	67·09	77·88	92·77	114·8	150·4	218·1	396·8	2191
312	46·83	52·47	59·66	69·13	82·17	101·3	132·0	189·4	335·0	1452
310	41·71	46·69	53·03	61·35	72·76	89·41	115·9	164·8	285·0	1052
308	37·11	41·51	47·09	54·41	64·76	78·93	101·9	143·7	243·7	801·4
306	32·99	36·88	41·79	48·23	57·00	69·68	89·61	125·5	209·4	631·2
304	29·30	32·73	37·06	42·72	50·41	61·46	78·81	109·7	180·5	508·5
302	25·99	29·01	32·83	37·80	44·55	54·24	69·31	95·97	156·0	416·4
300	23·02	25·69	29·05	33·42	39·34	47·81	60·94	83·99	135·1	345·2
298	20·37	22·71	25·67	29·51	34·71	42·12	53·55	73·51	117·2	288·8
296	18·00	20·06	22·66	26·03	30·58	37·06	47·03	64·33	101·8	243·3
294	15·88	17·69	19·97	22·93	26·92	32·58	41·27	56·27	88·40	206·1
292	13·98	15·58	17·58	20·17	23·66	28·61	36·18	49·19	76·82	175·3
290	12·30	13·69	15·45	17·72	20·77	25·09	31·68	42·98	66·77	149·6
288	10·80	12·02	13·55	15·54	18·20	21·97	27·71	37·50	58·00	127·9
286	9·460	10·53	11·87	13·60	15·93	19·21	24·20	32·69	50·36	109·6
284	8·275	9·208	10·38	11·89	13·92	16·77	21·11	28·47	43·70	94·00
282	7·224	8·037	9·056	10·37	12·13	14·62	18·38	24·75	37·88	80·67
280	6·295	7·002	7·888	9·031	10·56	12·72	15·98	21·49	32·80	69·25
278	5·473	6·087	6·910	7·848	9·175	11·04	13·87	18·63	28·36	59·43
276	4·749	5·281	5·947	6·806	7·955	9·570	12·01	16·12	24·49	50·99
274	4·111	4·571	5·147	5·890	6·883	8·277	10·38	13·92	21·11	43·71
272	3·519	3·913	4·406	5·041	5·889	7·080	8·875	11·89	18·01	37·08

270	2·981	3·314	3·731	4·268	4·985	5·992	7·507	10·05	15·20	31·16
268	2·519	2·800	3·152	3·605	4·210	5·058	6·335	8·476	12·80	26·14
266	2·123	2·359	2·656	3·037	3·547	4·260	5·334	7·132	10·76	21·90
264	1·783	1·982	2·231	2·551	2·978	3·578	4·479	5·986	9·022	18·31
262	1·495	1·661	1·870	2·138	2·495	2·997	3·751	5·011	7·547	15·28
260	1·249	1·388	1·562	1·786	2·086	2·503	3·132	4·183	6·296	12·72
258	1·041	1·156	1·301	1·488	1·736	2·085	2·608	3·482	5·238	10·56
256	·864	·960	1·081	1·236	1·442	1·731	2·165	2·890	4·346	8·752
254	·716	·795	·895	1·023	1·194	1·433	1·792	2·392	3·595	7·231
252	·591	·656	·738	·844	·985	1·184	1·479	1·973	2·965	5·958
250	·486	·540	·608	·694	·810	·973	1·216	1·623	2·437	4·894
248	·398	·443	·498	·569	·664	·797	·997	1·330	1·997	4·006
246	·325	·362	·407	·465	·543	·651	·814	1·086	1·631	3·271
244	·265	·294	·331	·379	·442	·530	·663	·884	1·327	2·660
242	·215	·239	·269	·307	·358	·430	·538	·717	1·076	2·156
240	·174	·193	·217	·248	·290	·348	·435	·579	·870	1·742
238	·140	·155	·175	·200	·233	·280	·350	·466	·700	1·402
236	·112	·125	·140	·160	·187	·224	·281	·374	·561	1·124
234	·090	·0995	·112	·128	·149	·179	·224	·299	·448	·897
232	·071	·0792	·0890	·102	·119	·142	·178	·238	·356	·713
230	·056	·0627	·0705	·0806	·0940	·113	·141	·188	·282	·565
228	·0445	·0495	·0557	·0636	·0742	·0891	·111	·148	·223	·446
226	·0349	·0388	·0436	·0499	·0582	·0698	·0873	·116	·175	·349
224	·0273	·0303	·0341	·0390	·0455	·0546	·0683	·0910	·137	·273
222	·0212	·0236	·0265	·0303	·0354	·0424	·0530	·0707	·106	·212
220	·0164	·0182	·0205	·0235	·0274	·0328	·0411	·0547	·0821	·164
218	·0127	·0141	·0159	·0181	·0212	·0254	·0317	·0423	·0635	·127
216	·0097	·0108	·0121	·0139	·0162	·0194	·0243	·0324	·0485	·0971
214	·0074	·0082	·0093	·0106	·0123	·0148	·0185	·0247	·0370	·0740
212	·0057	·0063	·0071	·0081	·0094	·0113	·0142	·0189	·0283	·0566
210	·0042	·0047	·0053	·0060	·0071	·0085	·0106	·0141	·0212	·0423
208	·0032	·0035	·0040	·0045	·0053	·0063	·0079	·0106	·0159	·0317
206	·0023	·0026	·0029	·0033	·0038	·0046	·0058	·0077	·0115	·0230
204	·0017	·0019	·0022	·0025	·0029	·0035	·0044	·0058	·0087	·0174
202	·0013	·0015	·0016	·0019	·0022	·0026	·0033	·0044	·0065	·0131
200	·00093	·0010	·0012	·0013	·0015	·0019	·0023	·0031	·0047	·0093
198	·00068	·00076	·00085	·00098	·0011	·0013	·0017	·0023	·0034	·0068
196	·00049	·00055	·00061	·00070	·00082	·00098	·0012	·0016	·0025	·0049
194	·00035	·00039	·00044	·00051	·00059	·00071	·00089	·0012	·0018	·0035
192	·00025	·00028	·00031	·00036	·00041	·00050	·00062	·00083	·0012	·0025
190	·00018	·00020	·00023	·00026	·00030	·00036	·00045	·00060	·00090	·0018
188	·00012	·00014	·00016	·00018	·00021	·00025	·00031	·00041	·00062	·0012
186	·000087	·000097	·00011	·00012	·00015	·00017	·00022	·00029	·00044	·00087
184	·000060	·000066	·000075	·000085	·00010	·00012	·00015	·00020	·00030	·00060
182	·000040	·000045	·000051	·000058	·000067	·000081	·00010	·00013	·00020	·00040
180	·000027	·000030	·000034	·000039	·000046	·000055	·000068	·000091	·00014	·00027

The characteristic equation will retain the same form for all temperatures above those at which saturation occurs and will be applicable for all higher temperatures, but in consequence of the density under standard conditions being diminished by substituting water-vapour for air the constant R of the equation will be different. When x kilogrammes of water have been added to the one kilogramme of dry air the value of R for the $(1+x)$ kg. for increasing pressure will become R′, where $R' = R(1+x/\epsilon)$, ϵ being the specific gravity of water vapour. The values of R′ for different quantities of water in grammes (X) associated with 1 kilogramme of dry air are given in Table V.

TABLE V

THE "CONSTANT" OF THE CHARACTERISTIC EQUATION FOR UNSATURATED AIR ACCORDING TO THE AMOUNT OF WATER-VAPOUR CARRIED BY ONE KILOGRAMME OF DRY AIR.

X.	R′.*	X.	R′.*
g.		g.	
40	3055	19	2958
39	3050	18	2953
38	3045	17	2948
37	3041	16	2944
36	3036	15	2939
35	3031	14	2935
34	3027	13	2930
33	3022	12	2925
32	3018	11	2921
31	3013	10	2916
30	3008	9	2912
29	3004	8	2907
28	2999	7	2902
27	2995	6	2898
26	2990	5	2893
25	2985	4	2888
24	2981	3	2884
23	2976	2	2879
22	2972	1	2875
21	2967	—	—
20	2962	R	2870

* For pressure in mb. and density in g/m³.

The values of R′ here given may be read as *joules per kilogramme per ten units of temperature.*

§ (21) THE ISENTROPIC EQUATION.—Corresponding with the isentropic equation of dry air there is an entropy equation for moist air, but the amount of water-vapour present constitutes a fourth variable, and moreover the constants involved in that equation are different according as any condensed water-vapour that there may be, or may be due, in view of the physical conditions, takes the form of ice or water. The equations in the various forms required to represent the different physical conditions were first given by H. Hertz,[1] and were subsequently discussed and rearranged by O. Neuhoff.[2] The peculiarity of Neuhoff's treatment is that he deals with a mass consisting of one kilogramme of dry air carrying a quantity x of water-vapour instead of a kilogramme of mixture.

The characteristic equation then becomes $pv = R't$, where $R' = R(1+x/\epsilon)$.

The energy equation takes the form

$$dQ = (c_p + xc_p'')dT - A\left(1+\frac{x}{\epsilon}\right)RT\frac{dp}{p}, \quad . \quad (1)$$

where Q = the amount of energy communicated by transmission as heat;

c_p = specific heat of dry air at constant pressure;

c_p'' = specific heat of water-vapour at constant pressure;

x = the weight of water-vapour in kilogrammes associated with 1 kg. of dry air;

T = temperature;

A = the reciprocal of the dynamical equivalent of heat;

$R(1+x/\epsilon)$ = the constant of the characteristic equation for the air with moisture x;

ϵ = the specific gravity of aqueous vapour = ·622;

p = the pressure.

$$x = \epsilon\frac{e}{p-e} \text{ (where } e \text{ is the vapour pressure).}$$

Putting $dQ = 0$ in equation (1), and integrating, we get

$$\log\frac{p}{p_0} = m_1 \log\frac{T}{T_0},$$

where

$$m_1 = \frac{c_p}{AR}\left(\frac{1 + x(c_p''/c_p)}{1+x/\epsilon}\right) = 3{\cdot}441\left(\frac{1+2{\cdot}023x}{1+1{\cdot}608x}\right).$$

This equation for the adiabatic holds as long as there is no condensation taking place.

Thus the effect of the moisture is merely to change the constant of the adiabatic equation between pressure and temperature; and, as will be seen from the table of the values of X (Table IV.) the difference is generally very small.

The Rain Stage.—When the air is saturated and condensation of water is occurring, a state of things which is conventionally known as the rain-stage, we get water in the liquid form as well as in the gaseous form, and we have to deal with the heat required to raise the temperature of the water and the latent heat of the vapour.

We get a new energy equation,[3] viz.

$$dQ = (c_p + \xi c)dT + Td\left(\frac{xr}{T}\right) - ART\frac{dp'}{p'}, \quad . \quad (2)$$

where $\xi = x + y$, where x refers to water-vapour and y to liquid water;

c = specific heat of liquid water;

r = latent heat of evaporation of water;

p' = the partial pressure of the dry air.

---

[1] *Gesammelte Werke*, 1895, Bd. i. 320; *Met. Zeitsch.*, 1884, Bd. i. 421.

[2] *Mem. Roy. Pruss. Met. Inst.*, 1900, i. 271, tr. by C. Abbé, Smithsonian Miscellaneous Collections, *Mechanics of the Earth's Atmosphere*, 3rd Collection, 1910, ii. No. 4.

[3] Clausius, *Mechanical Theory of Heat*, 1879, p. 156, tr. by W. R. Browne (Macmillan and Co.).

For the adiabatic equation $dQ=0$, hence

$$\log\frac{p'}{p_0'}=\frac{c_p+\xi c}{AR}\log\frac{T}{T_0}+\frac{\log_e 10}{AR}\left(\frac{xr}{T}-\frac{x_0r_0}{T_0}\right)$$

$$=m_{II}\log\frac{T}{T_0}+\frac{\log_e 10}{AR}\left(\frac{xr}{T}-\frac{x_0r_0}{T_0}\right), \quad (3)$$

where

$$m_{II}=\frac{c_p+\xi c}{AR}=\frac{c_p}{AR}\left(1+\frac{c}{c_p}\xi\right)$$

$$=3{\cdot}441(1+4{\cdot}265\xi).$$

When the water which condenses falls away as rain, except for the fraction which remains as cloud, the equation will be altered by having x in place of ξ in the value of m_{II} and the curve represented by the equation has been called a "pseudo-adiabatic" line. It will be more convenient to call it an "irreversible adiabatic," while the curve which assumes the retention of the water is called a "reversible adiabatic."

In these equations values can be assigned for the various constants, and a series of curves corresponding with the various values of ξ can be plotted both for the reversible and the irreversible adiabatics.

For the purposes of this article, in which we propose to deal only with the more general aspects of the thermodynamic processes, the thermodynamic theory might well have stopped with equation (3) and the whole process have been regarded as the condensation of water. We should then have disregarded the amount of heat set free by the freezing of water into ice which occurs when hail is formed; and we should also have neglected the change in the latent heat which is introduced when water-vapour is converted directly into snow and the change in the specific heat of the condensed moisture when ice is produced instead of water. These omissions are not of any serious practical importance, because on the one hand the formation of hail is a comparatively exceptional occurrence depending upon circumstances which are not yet amenable to rigorous dynamical or thermodynamical treatment, and on the other hand the amount of water-vapour in the atmosphere below the freezing-point is so small that the differentiation between the latent heat of vapour from ice as compared with water leads us into niceties which are a long way beyond the limit of the practical application of the equations to the atmosphere. In fact our knowledge of the physical properties of water in the neighbourhood of the freezing-point is altogether on a different plane from that of our capacity to deal with the meteorology of the atmosphere in the regions where such changes occur. As a physical exercise the modification of the equation to deal with the freezing of water and the condensation into ice has some interest, and we therefore reproduce it although its application in detail to the circumstances of the actual atmosphere will not detain us.

The Hail Stage.—The third stage in the gradual process of dynamical cooling of a mixture of air and water-vapour, namely, that which concerns the freezing of the condensed water supposed to have been retained with the cooling air in its ascent, is called by von Bezold the hail stage.

The total water content ξ will now be made up of three parts, x vapour, y water, and z ice, so that $\xi=x+y+z$. The freezing will take place at a constant temperature, and will be consequent upon reduction of pressure and expansion of volume with work on the environment.

The energy equation is

$$dQ=ART_0\frac{dV}{V}+r\,dx-r_e\,dz,$$

where V is the specific volume of the air and r_e is the latent heat of fusion of ice.

Since the process is adiabatic, $dQ=0$ and we get

$$0=\frac{ART_0}{\log_e 10}\log\frac{V}{V_0}+r(x-x_0)-r_e(z-z_0).$$

Since the change is isothermal, $V/V_0=p'_0/p'$ and $x=\epsilon(e/p')$, and as at the beginning $z=0$ and at the end $y=0$ the equation becomes

$$\log p'-\frac{1}{p'}\frac{\log_e 10}{AR}\frac{r+r_e}{T_0}\epsilon e=\log p_0'-\frac{1}{p_0'}\frac{\log_e 10}{ART_0}r\epsilon e$$

$$-\frac{\log_e 10}{ART_0}r_e\xi.$$

Numerical values can now be inserted to determine the change of pressure during the process.

The Snow Stage.—The final stage in the gradual process of dynamical cooling, called by von Bezold the snow stage, deals with the condensation of water-vapour to ice or snow under adiabatic conditions. The equation is similar to that for the rain stage, with the substitution of the specific heat of ice, $c_e=0{\cdot}5$, for that of water and the addition to r of r_e, the latent heat of fusion of ice (79 gramme-centigrade-thermal units).

The differential equation for the adiabatic thus becomes

$$0=(c_p+\xi c_e)dT+Td\left(\frac{x}{T}(r+r_e)\right)-ART\frac{dp'}{p'},$$

and integrates to give

$$\log\frac{p'}{p_0'}=\frac{(c_p+\xi c_e)}{AR}\log\frac{T}{T_0}$$

$$+\frac{\log_e 10}{AR}\left(\frac{x(r+r_e)}{T}-\frac{x_0(r_0+r_e)}{T_0}\right).$$

Writing

$$m_{IV}\text{ for }\frac{c_p+\xi c_e}{AR}\text{ or }3{\cdot}441(1+2{\cdot}105\xi),$$

we get

$$\log\frac{p'}{p_0}=\frac{\log_e 10}{AR}\cdot\frac{r+r_e}{[illegible]}\frac{\epsilon e}{p'}$$

$$-\frac{\log_e 10}{AR}\frac{r_0+r_e}{T_0}\frac{\epsilon e_0}{p'_0}+m_{IV}\log\frac{T}{T_0},$$

from which by substitution the adiabatic curves for air saturated with vapour below the freezing-point can be drawn.

A transcript of the curves obtained by Neuhoff from these equations, with pressure and height, as co-ordinate axes, is given in *Fig.* 15.

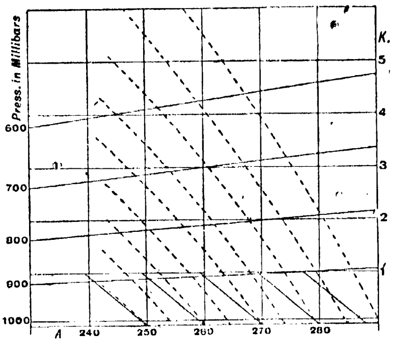


FIG. 15.—Adiabatics for Saturated Air, referred to temperature and height as co-ordinate axes, with lines of pressure in the upper air corresponding with the standard pressure 1013·2 mb. at the surface.

The pressure is shown by full lines crossing the diagram, and the adiabatic lines for saturated air by dotted lines. The short full lines between the ground and the level of 1000 metres show the direction of the adiabatic lines for dry air.

VI. THE MECHANISM OF THERMODYNAMICAL PROCESSES IN THE ATMOSPHERE

§ (22) ENTROPY-TEMPERATURE DIAGRAMS.—The equations which have been adduced in the previous section enable us to follow the changes which must occur when air mixed with a specified amount of moisture is subjected to the changes of pressure incidental to changes of height, or other circumstances, provided that we are able to assume the conditions to be adiabatic. There are two considerations which might be held to invalidate that assumption. One is that the change of pressure is not infrequently due to relative motion between the air "under reference" (to borrow an official phrase) and its environment, and any relative motion between adjacent masses of air implies eddy-motion and consequent mixing; the second is that the air is certainly not completely protected against loss or gain of heat by radiation, and consequently the adiabatic condition cannot be rigorously complied with. The importance of these considerations depends upon the volume of the air under reference in the first case, and the duration of the operations in the second case.

It would require an elaborate investigation to lay down with precision the extent to which the adiabatic condition is invaded in actual circumstances, and in either case it is preferable to assume that the adiabatic condition is rigorously maintained, and trust to dealing with any deviation from that condition as a correction, or otherwise, when the consequences of adiabatic changes have been deduced.

In the classical discussions, pressure (indirectly representing height) and temperature have been selected as the independent variables; adiabatic lines corresponding with specified conditions, as for the dry-air-stage before condensation occurs, the rain-stage when water is being condensed or evaporated, the hail stage when the water suspended mechanically is transformed into ice, or *vice versa*, and the snow stage when there is direct transformation from vapour to ice, or *vice versa*, have been plotted in the diagram. The addition of another set of lines gives the amount of water in the mixture in the form of vapour. The arrangement is very convenient for combining a great deal of information in a single diagram, but if we wish to deal with the subject from the point of view of energy—and that is the chief object of thermodynamical reasoning—the recognised independent variables are pressure and volume of unit-mass, or alternatively the entropy and temperature of unit-mass. With either pair of variables an "indicator-diagram" can be constructed which represents energy by area, in work units in the one case and in heat units in the other. Hence it is desirable to construct, from the information which is supplied by the equations, indicator-diagrams applicable to the various conditions in respect of moisture, to set out the isothermal lines and isentropic lines as referred to co-ordinates representing pressure and volume, and to space consecutive isentropic lines according to equal steps of entropy; or alternatively to set out the isothermal and isobaric lines as referred to co-ordinates representing temperature and entropy. The latter alternative is preferred, because it brings into the open a difficulty which cannot be evaded, and which arises from the changes in the amount of water-vapour in the air in consequence of the falling out of the condensed water.

The difficulty arises in this way: the diagram has to be constructed for a definite mass of working substance, namely, a mixture of air and water, and in order that thermodynamical reasoning may be applicable the processes must be reversible. In the course of the air's history, as soon as water falls out the conditions cease to be satisfied, and further operations apply to a new substance.

There is an advantage in recognising this difficulty, because it requires us to realise that, in dealing with a mixture of which one component is variable, we cannot use language which is appropriate only for a substance of fixed composition. This is particularly noticeable in the case of entropy. For example, if we take the case of air ascending a hill-side and descending on the other side, according to the common explanation of the distribution of temperature in föhn, it starts as a partnership between air and water, each contributing to the entropy of the mixture. On the way one of the partners falls out, and in doing so hands over his store of realised energy to the other, thus leaving the other much more favourably situated in respect of entropy than when he started, and yet having gone through an "adiabatic" process.

In like manner it has been pointed out by Dr. C. W. B. Normand of the Meteorological Office, Simla, in a report recently published,[1] that the process of evaporation from contact with a water surface such as a wet bulb is "adiabatic," because when a steady state has been reached water-vapour is taken up without any communication of heat; the water on the bulb takes its heat of evaporation from the air which carries it away.

We shall do well, therefore, to regard the dry air as the substance that goes through the thermodynamic changes, and the water carried with it as a possible supply of heat. In ordinary circumstances the amount of water-vapour is so small, compared with the air which carries it, that no important error will be introduced if we neglect the other aspects of the effect of water-vapour.

We have therefore transformed the figures derived from the Neuhoff equations, extended to the lower temperatures which occur in the upper air, into entropy-temperature diagrams, the first for dry air and the second for saturated air containing quantities of water-vapour indicated by saturation lines. The work was carried out by Mr. A. W. Lee, M.Sc., of the Imperial College, and included the extension of the tables of thermal constants to cover the required ranges. The results of the computation are given in Table VI.

TABLE VI

SATURATED AIR

Temperatures at specified pressures along lines of specified entropy computed for 1 kg. of dry air saturated with *x* kg. of water-vapour (neglecting the adjustment for the "hail stage" of true reversible adiabatics). The entropy realised at the standard pressure of 1013·2 mb. and at the temperature of the third column is set out in the second column. The figures in each horizontal row are for an adiabatic line.

The entropy is measured from a zero defined by T=200, p=1000 mb.

Entropy.		Pressure in Millibars.										
Total.	Realised at 1013·2 mb.	1013·2.	1000.	900.	800.	700.	600.	500.	400.	300.	200.	100
		Temperatures on the Tercentesimal Scale.										
J/a.	J/a.	a.	a.	a.	a.	a.	a.	a.	a.	a.	a.	a
655	415	303	302·6	299·1	295·2	290·8	285·5	279·0	270·9	260·1	241·0	199·5
622	409	301	300·5	297·0	293·0	288·3	282·8	275·9	267·7	255·8	234·7	193·0
593	402	299	298·5	294·9	290·8	285·9	280·0	272·8	264·3	251·2	228·1	..
566	395	297	296·5	292·8	288·4	283·3	277·3	270·1	260·9	246·5	222·7	..
541	388	295	294·5	290·7	286·2	280·9	274·5	267·1	257·0	241·5	216·7	..
517	381	293	292·4	288·4	283·8	278·3	271·7	264·0	253·0	236·8	211·3	..
496	375	291	290·4	286·3	281·5	275·7	269·2	260·9	249·1	232·0	206·7	..
476	368	289	288·4	284·2	279·2	273·1	266·4	257·5	245·0	227·5	202·7	..
457	361	287	286·4	282·0	276·8	270·7	263·6	254·1	241·1	223·2	198·3	..
439	353	285	284·4	279·7	274·4	268·3	260·7	250·7	237·2	219·2	194·7	..
423	346	283	282·4	277·6	272·1	265·8	257·8	247·3	233·5	215·2	191·3	..
407	339	281	280·4	275·4	269·9	263·2	254·0	244·0	229·8	211·7	..	..
393	332	279	278·4	273·2	267·7	260·7	251·9	240·9	226·6	208·5	..	..
379	325	277	276·4	271·2	265·3	258·0	249·0	237·5	223·4	205·5	..	..
365	318	275	274·4	269·1	262·9	255·5	246·0	234·5	220·2	202·5	..	..
352	310	273	272·4	267·1	260·7	252·9	243·4	231·6	217·4	200·0	..	..
338	303	271	270·3	264·8	258·2	250·2	240·4	228·7	214·4	197·2	..	..
325	295	269	268·3	262·6	255·6	247·5	237·6	225·9	211·7	194·8	..	..
314	288	267	266·3	260·3	253·2	244·9	234·9	223·1	209·0	192·3	..	..
302	280	265	264·2	258·1	250·9	242·4	232·3	220·5	206·8	..	..	..
291	273	263	262·2	255·9	248·6	239·9	229·8	218·1	204·4	..	..	..
280	265	261	260·2	253·7	246·3	237·5	227·4	215·9	202·2	..	..	..
270	257	259	258·2	251·6	244·0	235·3	225·3	213·6	200·3	..	..	..
260	249	257	256·2	249·6	242·0	233·3	223·3	211·6	198·4	..	..	..
250	241	255	254·2	247·4	239·7	230·9	221·0	209·6	196·4	..	..	..
241	233	253	252·1	245·3	237·6	228·7	218·9	207·6	104·6	..	..	..
231	225	251	250·1	243·2	235·4	226·6	216·9	205·5	192·6	..	..	..

Note.—The entropy realised by the dry air at successive stages in consequence of the condensation of water-vapour is indicated in the diagram, *Fig.* 16.

---

[1] *Memoir of the Indian Meteorological Department*, vol. xxiii. part i.

§ (23) INDICATOR-DIAGRAMS FOR SATURATED AIR REFERRED TO "REALISED" ENTROPY AND TEMPERATURE.—In the course of its progress through a cycle of operations the changes in the condition of the air can be traced with approximate accuracy by the aid of two diagrams, one for dry air and the other for saturated air, provided that the conditions of the successive steps can be specified. As working with two diagrams alternately, according as the air is saturated or not, is a cumbrous process, we may use instead an approximate diagram in which the dry air alone is regarded as the working substance and the moisture is regarded merely as a reservoir of latent energy which becomes realised when condensation takes place. In making this approximation there may be some appreciable error in the values of density and other physical conditions at temperatures above 280a, but for the purpose of general consideration of atmospheric changes an approximate diagram of this kind is very convenient. Such a diagram was constructed for the author by Mr. E. V. Newnham of the Meteorological Office in 1917. From Neuhoff's equations, extended to the low pressures and temperatures of the stratosphere, the isentropics of saturated air are there plotted as an entropy-temperature diagram, spacing the lines according to the entropy of dry air (measured from 200a and 1015 mb.), computed from the equation $d\phi=c_p dT/T$, at equal intervals of temperature.

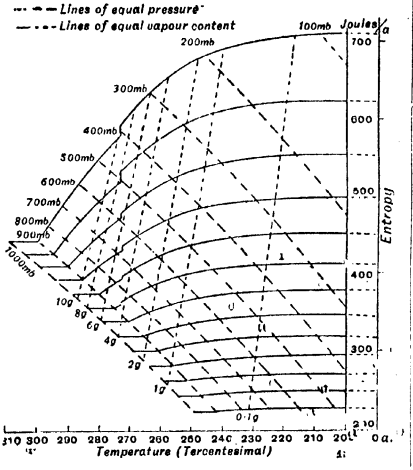


FIG. 16.—Relation between realised Entropy, Temperature, and Pressure of a kilogramme of Dry Air which carries with it sufficient Water-vapour initially for Saturation to be attained when it has risen to a height of 1 kilometre.

The broken lines show the pressure in millibars, and the dotted lines the water-vapour content in grammes.

The latent heat of condensation was allowed for as the condensation took place, and the entropy of the air increased accordingly. In this manner the diagram shown in *Fig.* 16 was constructed. It is clear that on this diagram an adiabatic line for saturated air is not isentropic for dry air, and the full entropy of the mixture of air and vapour is not represented. The entropy shown corresponds only with the realised energy; that which still remains latent is not counted. Hence the use of the term "realised entropy," which is not a very suitable one because it suggests the idea of entropy as a "conservative" quantity, whereas it is no more conservative than temperature.

On this diagram energy is represented by area, along the x-axis by the product ϕdT, and along the y-axis by the product T$d\phi$. If we can sketch out a cycle of operations it is clear that we can obtain values for the energy changes in the course of the cycle, and thence obtain an estimate of the dynamical efficiency of the atmospheric process represented by the cycle.

§ (24) A CYCLE OF OPERATIONS.—With the aid of this diagram we can now try to formulate a cycle through which the atmospheric air may be supposed to pass. We may begin with air saturated at 300a at A (*Fig.* 17), in the environment of equatorial air such as that of Java as set out in Table II. On reference to that table it will be seen that the values of temperature for successive reductions of pressure are less than the temperatures at corresponding pressures or heights in the adiabatic line for saturated air, starting from the same temperature 300a until the height of

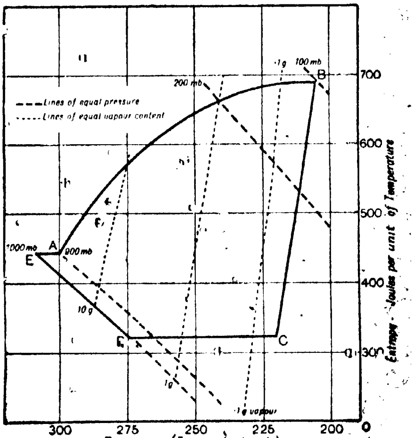


FIG. 17.—Suggested Cycle of Operations for Air saturated at 300a at a height of 1 kilometre.

15 kilometres is reached. Consequently there is nothing unreasonable in assuming that the equatorial air may rise automatically to 15 kilometres or more, represented by the point B, shedding its condensed water on the way. Arrived there at the same temperature as the environment with the water lost, we see that any step downward under adiabatic conditions will be attended by a rise of temperature at the dry adiabatic rate of approximately one degree for 100 metres. The result of direct descent to the surface again may be read off on a diagram for dry air to be 360a. Warming at that rate would immediately bring the temperature of the descending air above that of its environment, consequently descent in any normal atmosphere under adiabatic conditions is out of the question. We must therefore suppose that the next step in its history is for the air to lose its heat, and for that purpose we must appeal to the diagrams of radiation (*Fig.* 14). The loss may presumably take place by air radiating more than it absorbs, however difficult it may be to specify the appropriate conditions. Loss of heat need not necessarily result in diminished temperature; instead of that the air may descend until its temperature is the same as that of its environment, and as the air is originally just under the stratosphere and does not differ in any notable degree from its environment, we must suppose the loss of heat to be general, and to be an incident in the general circulation which in effect carries the air under the stratosphere northward, after some long travel, perhaps first westward and then northward, turning north-eastward, until it is below the stratosphere at some point C where it can find means of descent to the surface at D. The evidence for a direct descent of air through the layers beneath is not very strong except on the cold slopes of the mountainous Arctic and Antarctic lands, such as Greenland and the Antarctic continent, or down the cold slopes of other high mountains during the night hours or in shadow. There is very definite descent of air from above in the case of line-squalls, which are common incidents in the later stages of a cyclonic depression, but we have no definite information as to the height at which the descent commences. Under most favourable conditions, therefore, we may trace the course of the air to its arrival at the surface somewhere about latitude 60° N. or S. at a temperature of 275a. To resume its original condition it has then to find its way over the sea surface, absorbing heat and moisture near the equator, until it is warm enough and moist enough to be saturated again at 300a.

This is the most generalised form of cycle for the atmosphere. Cycles of more limited extent are presumably possible in favourable circumstances. Favourable circumstances are indicated by the existence of penetrative convection, and the evidence of penetrative convection is heavy rainfall over a limited area.

§ (25) THE EFFICIENCY OF THE CYCLE.—The cycle can be represented by the closed figure drawn on the diagram, and the following values can be taken from it:

1. Heat is being absorbed by the air during the passage from D to B through E and A, and its amount may be estimated as the equivalent of the area between DEAB and the line of zero temperature.

2. Heat is being given out by the air during the passage from B to C, and its amount may be estimated as the equivalent of the area between BC and the line of zero temperature.

3. The heat which is converted into work is represented by the area ABCDE.

The approximate values as estimated from the figure are:

Energy taken in:

$$(440-320)292+(690-440)270=102540.$$

Energy given out:

$$(690-320)\times 213=78810.$$

Energy transformed into work = 23730.
Efficiency, 0·23.

§ (26) THE WORK DONE IN THE CYCLE.—The question at once arises as to what form the work of such a cycle takes, and in reply we may point to the fact that the kinetic energy of the general circulation described in II. § (9) is maintained in spite of the losses which take place in consequence of friction with the ground and losses through eddy-motion and ultimately molecular viscosity.

There is also the kinetic energy of cyclonic circulations in the form of tropical revolving storms or of cyclonic depressions of middle latitudes. Meteorological opinion is divided on the subject. Some writers regard the energy of such circulations as derived from the energy of the general circulation, whereas others are disposed to regard the kinetic energy of the more violent winds at least as the expression of the work done in the thermodynamic cycle, of which the first stage is the penetrative convection due to the ascent of either exceptionally warm and moist air in a normal environment, or of normally warm and moist air in an exceptionally unstable environment. There is experimental evidence (referred to in § (17)) to show that penetrative convection results in the mechanical withdrawal of a quantity of air from the environment of the rising air, which may be ten or twelve times the amount of the rising air itself, and such "eviction" in an extended region where there is already some relative motion causes a cyclonic circulation in the environment.

In the latter case the lines of *Fig.* 16 will supply a means of quantitative estimation of the process, and some support for that view is afforded by such events as those which are recorded for the depression formed over the lower part of the North Sea between July 27 and August 3, 1917. Apparently the depression was formed *in situ*, becoming most fully developed after some ups and downs on August 3. Apparently also it filled up *in situ* by August 6. The numerical particulars roughly computed appear to be as follows:

Diameter of depression, 1400 km.

Depth at centre, 10 mb.

Quantity of air removed to allow for the depression, 70,000,000,000,000 kg.

Quantity of water-vapour, 700,000,000,000 kg.

Kinetic energy developed, $1\cdot5 \times 10^{24}$ ergs.

Energy available from the condensation of water, $1\cdot764 \times 10^{25}$ ergs.

Time required to fill up by cumulative transfer near the surface across the bounding isobar, 2½ days.

Water-vapour carried into the area by transference across the boundary equivalent to an average rainfall over the whole area of ·2 mm. per hour.

N. S.

ATMOSPHERIC ELECTRICITY[1]

I. GENERAL ACCOUNT OF THE MAIN FACTS AND PROBLEMS OF ATMOSPHERIC ELECTRICITY

§ (1) FINE WEATHER EFFECTS. (i.) *The Vertical Electrical Force or Potential Gradient.*—Let us suppose that on a fine clear day we make observations on the electrical conditions over the surface of a level field freely exposed to the sky. We shall find that the ground is negatively charged, in other words that there is a force tending to move a positively charged body in the atmosphere downwards, or again that work has to be done in moving a positively charged body upwards. Quantitatively there are three equivalent ways of describing the results of our observations: we may say that there is a certain negative charge σ per sq. cm. of the ground, or a vertical downward electric force $F = 4\pi\sigma$, or that the potential V in the lowest layers of the atmosphere increases with the height, the rate of increase dV/dh being equal to $F = 4\pi\sigma$. It is generally in terms of dV/dh, the potential gradient, measured in volts per metre, that the results of such observations are expressed.

The average magnitude of the positive potential gradient in fine weather is of the order of 100 volts per metre, corresponding to a negative charge on the ground of 3×10^{-4} e.s.u. per sq. cm. or 3 e.s.u. $= 10^{-9}$ coulomb per sq. metre.

The potential gradient at the earth's surface is continually varying; in addition to irregular variations which largely depend on local meteorological conditions, there are well-marked annual and diurnal variations. The average value of the potential gradient and the character of its annual and daily variations differ at different parts of the earth's surface.

The potential does not continue to increase uniformly with increasing height above the ground; the potential gradient soon begins to diminish, and before a height of 10 km. is reached the potential has become almost independent of the height. Thus the potential of the whole upper atmosphere above regions of fine weather is probably less than 1 million volts.

The diminution in the vertical force with increasing height implies that the lower atmosphere is positively charged. The lines of force which start from the negatively charged ground nearly all end (on positively charged dust particles or on positive ions) below a height of 10 kilometres.

(ii.) *The Air-earth Current.*—The normal vertical field of the atmosphere tends to drive positively charged bodies downwards, negatively charged bodies upwards; the motion of such charged bodies under the action of the electric field constitutes an electric current. Positively and negatively charged bodies—ions—are in fact always present in the atmosphere, which has in consequence a varying electric conductivity proportional to the number of ions per c.c., the charge carried by each and their mobility (*i.e.* the velocity with which they move through the air under the action of unit electric force). The conductivity and the sources of the ionisation to which it is due have formed the subject of many investigations.

The current which flows from the atmosphere into unit area of the ground under the influence of the normal potential gradient of fine weather may be found by determining the two factors on which it depends—the potential gradient and the conductivity. It may also be obtained by direct measurement. The average air-earth current is about 2×10^{-16} ampere per sq. cm. = 2 micro-amperes per sq. km. or about 1000 amperes for an area equal to that of the whole surface of the earth.

While the absolute magnitude of the air-earth current as thus determined is small, it is large enough to convey from the atmosphere to the ground in one minute a positive charge of the order of one-tenth of the negative surface charge on the ground. As the ground nevertheless maintains its negative charge there must be some compensating process continually conveying a negative charge to the earth. To find the source of this air-earth current, or to account for the maintenance of the potential difference between the upper atmosphere and the ground, in spite of the conductivity of the intervening air, is one of the main problems of atmospheric electricity.

---

[1] The numbers inserted in the text refer to papers in the list at the end of the article.

§ (2) EFFECTS OF SHOWERS AND THUNDER-STORMS.—The electrical phenomena which are associated with showers and which reach their full development in thunder-storms are very different from those of fine weather. The potential gradient at the ground may be either positive or negative and frequently changes sign The vertical electric force at the ground frequently exceeds 10,000 volts per metre, *i.e.* 100 times the normal fine weather gradient, and its magnitude and direction may change suddenly owing to lightning discharges.

The exchanges of electric charge between the earth and atmosphere are also of much larger magnitude than those which occur in fine weather, and the direction of the current between the earth and atmosphere is continually varying.

There are three ways in which a positive or negative charge may pass during a shower from the atmosphere to the ground. The rain or other form of precipitate may itself be charged and thus carry a convection current, directed downwards or upwards according as its charge is positive or negative. There is evidence that, on the whole, more positive than negative electricity is in this way transferred from the atmosphere to the earth. These convection currents rarely exceed 10^{-13} ampere per sq. cm. or 1 milliampere per sq. km.

In a thunder-storm lightning discharges may pass between a cloud and the ground. These may carry positive electricity from the cloud to the earth or from the earth to the cloud; on the whole the results obtained thus far indicate a greater frequency of the latter kind of discharge. The average quantity discharged in a lightning flash is of the order of 20 coulombs, so that in a storm in which several such discharges occur per minute the average current between the cloud and the earth due to lightning flashes is of the order of 1 ampere.

In addition to lightning discharges continuous conduction currents must traverse the surface of the ground below a thunder-cloud; they have their sign determined by that of the vertical electric force and may thus be either upward or downward. The magnitude of such currents through unit area of the ground may greatly exceed that of the ordinary air-earth current of fine weather; it is likely to exceed it in a ratio much larger than that of the electric fields causing the currents. For not only may there be additional sources of ionisation due to the rain (evaporation of charged drops and splashing at the ground), but the electric field at the ground may itself be strong enough to produce numerous point discharges which together carry a large current. The continuous conduction current between a thunder-cloud and the ground probably exceeds that carried by lightning discharges.

Further observations are required to determine whether the three types of current below shower clouds give, as their resultant effect over the surface of the earth, on the whole a transference of positive or of negative electricity from the atmosphere to the earth; and in the latter case, whether this excess of negative charge is sufficient to compensate approximately for the positive charge carried down by the air-earth current in the regions of fine weather.

§ (3) THE ELECTRICAL CONDITIONS WITHIN A THUNDER-CLOUD.—The magnitude of the electric force near a thunder-cloud or within it must, in order that it may cause a lightning flash, reach values which approach 30,000 volts per cm. This is very large compared with the highest potential gradients observed near the ground; the diminution in the electric force near the surface of the earth may be due to the lines of force either diverging with increasing distance from the charged portion of the cloud from which they start or ending before they reach the ground.

A thunder-cloud may be regarded as an electric machine by the action of which a vertical separation of positive and negative electricity is produced. It is thus essentially bipolar, equal and opposite charges being separated in the upper and lower parts of the cloud in a given time. The charges of the two poles of the cloud are, however, not likely to remain equal, on account of differences in their rates of dissipation.

We may form some estimate of a lower limit to the magnitude of the charges which may accumulate in thunder-clouds by assuming that it is not less than the average quantity discharged in a lightning flash—about 20 coulombs.

The potentials within the charged portions of thunder-clouds probably reach magnitudes of the order of 10^9 volts.

An estimate of the rate of separation of the charges in a thunder-cloud, *i.e.* of the vertical current through the cloud, may be derived from the rate at which the electric field destroyed by a lightning flash is regenerated. This method gives a few amperes as the probable order of magnitude of the vertical current through a thunder-cloud.

The positive or negative charge carried within the cloud from the lower to the upper pole may recombine with that of the lower pole by direct discharge through the cloud. It may, on the other hand, pass to earth by lightning discharge or otherwise and thence to the lower pole. But it is possible that an important part of the current from the upper pole of the cloud may go to the conducting layers of the upper atmosphere, which form with the earth a condenser of considerable capacity. The potential of the upper atmosphere above regions of fine weather is, as we have seen, only of the order of 1 million volts, a value which is small in comparison with the E.M.F. of a thunder-cloud, which is probably 1000 times as great.

§ (4) Effects of Thunder-storms on the Electrical Condition of the Upper Atmosphere.—The conductivity of the air between a thunder-cloud and the upper atmosphere is largely due to ions dragged out of the conducting layers by the action of the electric field of the cloud. This conductivity will be greater if the cloud is of positive polarity (*i.e.* having its upper pole positive), since the negative ions which will under these conditions be dragged out of the conducting layer have much greater mobility than the positive ions which would be dragged out by a cloud of negative polarity. Unless, therefore, clouds of negative polarity greatly exceed in number those of positive polarity, an excess of positive electricity is likely to be transferred from the earth to the upper atmosphere by the action of thunder-clouds. It is possible that it is in this way that the positive potential of the upper atmosphere and hence the normal positive potential gradient of fine weather are maintained.

The frequency of thunder-storms varies greatly over the surface of the earth. It is not easy to form an estimate of their total average frequency for the whole earth, but it is probable that the average number of thunder-clouds in action at a given time exceeds 1000. The effects of thunder-storms and showers on the electrical condition of the upper atmosphere may thus be considerable, and may have to be taken into account not only in considering the atmospheric electricity of fine weather, but also in connection with terrestrial magnetism and auroras.

§ (5) Mechanism of Thunder-clouds.—The mechanism by which the separation of the positive and negative charges in a thunder-cloud is effected has been a matter of much controversy. There is within a thunder-cloud an upward rush of air, and it is generally agreed that the electric field within the cloud is produced by the large drops or hailstones and the smaller particles of the cloud acquiring charges of opposite sign, the charge associated with the cloud particles being carried up by the air stream, while the large drops carrying the charge of opposite sign fall rapidly relatively to the air. As to how the original partition of the positive and negative electricity between the large and small drops is effected there has not been the same agreement; the principal theories are discussed briefly in Part IX.

II. Measurement of the Atmospheric Electric Field

§ (6) Methods of measuring the Field.—The sign and magnitude of the vertical electric force at the earth's surface do not appear to have ever been determined directly in terms of the force exerted on a body which carries a known charge of electricity; the method is by no means an impossible one, especially for the study of very rapid variations in the electric field.

(i.) *By Measurement of the Charge on an Earth-connected Conductor.*—A method which is easier in practice is that in which the sign and magnitude of the charge on a level portion of the ground is observed. The quantity to be measured is small in fine weather—of the order of 3 e.s.u. (electrostatic units of electricity) per sq. metre; the method is more particularly applicable in the study of the intense and rapidly varying fields of thunder-storms. We may increase the quantity to be measured by determining, instead of the charge on a plane at the level of the earth's surface, that on an earth-connected conductor of simpler form which projects above the general surface of the ground. A copper sphere placed at a height of some metres above the ground is convenient for the purpose. The measurement may in this case be equally well regarded as a determination of the atmospheric potential at the height of the centre of the sphere.

The charge induced on an earth-connected sphere, exposed in the free atmosphere, with its centre at a point p, must be such as to bring the potential of the sphere to zero; *i.e.* Q, the charge on the sphere, must be such that $Q/r + V = 0$, where r is the radius of the sphere, and V is the potential at p due to charges other than that on the sphere. The potential V differs from the undisturbed atmospheric potential at the point p by an amount which depends on the charges on the supports of the sphere and on the charges induced on the ground by those of the sphere and its supports. If the sphere is exposed at a height which is large compared with its radius, and if it is supported in a suitable way, the difference between V and the undisturbed atmospheric potential at p is small, and the correction to be applied on this account may be estimated.

The charge induced on the earth-connected conductor, whether this be a "test plate" level with the surface of the ground or a projecting conductor such as the elevated sphere, may be measured by momentarily earthing the conductor while it is exposed, at once shielding it from the earth's electric field (by bringing over it an earthed cover or lowering it into an earthed conducting case), and observing the reading of an electrometer to which the conductor is connected. If the capacity C_1 of the whole conducting system (in the shielded condition) is known, the induced charge on the exposed conductor is at once obtained from the potential v_1 to which the system is raised on shielding; since $v_1 = Q/C_1$.

Measurements of the charge on a conductor, momentarily connected to earth while raised to a known height in the atmosphere and then withdrawn into a closed conducting chamber, were first made by Peltier (1), who appears also to have been the earliest worker on the subject to realise clearly that measurements of the "electricity of the atmosphere" were really measurements of the vertical electric field.

The test plate, sphere, or other conductor may equally well be earthed while in the shielded condition, the electrometer being in this case read immediately after the earth connection has been broken and the conductor has been exposed to the earth's electric field (2), (3), (4). In this case the potential indicated by the electrometer is $v_2 = -Q/C_2$, where C_2 is the capacity of the whole system when the conductor is in the exposed condition, and Q is the charge which would have to be given to the conducting system to bring its potential to zero, *i.e.* it is the charge which would be induced by the electric field on the exposed conductor if earth-connected.

It is in some ways advantageous to use the electrometer as a null instrument and to bring the potential back to zero, after the process of shielding or exposing the test conductor, by giving to the conducting system the necessary charge; this may be supplied and measured by some form of compensator. This charge is obviously equal in magnitude to the charge on the exposed earth-connected test conductor and of the same or opposite sign according as it is the effect of exposing or of shielding which is being neutralised (5), (6). The compensator may conveniently consist of a condenser of variable capacity, of which one of the terminals is connected to the conducting system, while the other is connected to a source of constant potential. A suitable type of capillary electrometer forms a compensator of this kind which is automatic in its action; if one terminal be earthed while the other is connected to a conducting system, the potential of the latter is always automatically brought to zero by displacement of the mercury-sulphuric-acid surfaces, and the amount of this displacement is proportional to the charge given to or removed from the conducting system. When so placed the capillary electrometer simply serves to measure the quantity of electricity which flows along the wire connecting the test conductor with the earth (7), (8).

If the test conductor be kept exposed, any changes in the electric field are accompanied by corresponding changes in the charge on the exposed conductor. The rapidity of the changes which can be followed depends on the nature of the electrometer. With a capillary electrometer or with one of the gold leaf or silvered quartz fibre type, extremely rapid changes, such as are produced in the field by lightning discharges, may be recorded. The conductivity of the air introduces a difficulty in the interpretation of the results obtained when the conductor is kept exposed, for even if the electric field remains constant there will be a gradual change in the reading of the electrometer owing to the flow of electricity from the air into the exposed conductor. This may at once be distinguished from a gradual change in the electric field, since the effect remains even when the field is cut off by shielding the test conductor. If the potential is maintained approximately at zero by a compensator of any form or by having a large capacity attached to the system, then it is only necessary to shield periodically the exposed conductor in order that both the charge on the exposed earthed conductor and the current flowing through it may be separately measured.

(ii.) *By measuring the Charge and Potential of an Exposed Conductor.*—The method of determining the potential at a given height in the atmosphere by measuring the charge on an earth-connected sphere with its centre at that height is a particular case of a more general one. If a sphere be exposed in the atmosphere with its centre at a point at which the undisturbed potential is V, then, if the sphere is small compared with its height above the ground, $V + Q/r = v$, where Q is the charge on the sphere, and v its potential. This holds also for a small conductor of other than the spherical form if c, its capacity, be substituted for r. If both Q and v can be measured we can deduce V; there are two simple cases—one, already considered, in which $v = 0$ and therefore $V = -Q/r$, and a second, in which $Q = 0$ and thus $V = v$.

An uncharged sphere suspended in the atmosphere is at a potential equal to that of the air at the level of its centre, and the same is true of any conductor of which the vertical dimensions are small compared with its height above the ground. To measure the potential of the conductor it is necessary to connect it to one terminal of an electrometer of which the other terminal is earthed. If the electrometer and connecting wires could be made of capacity negligible in comparison with that of the exposed conductor we should have an extremely simple method of measuring the potential at a given height. One of the very earliest methods of measurement employed in the study of atmospheric electricity—that of Beccaria (9), who used a long wire stretched between insulators as his conductor—probably gives an approximation to the required conditions. A long wire stretched horizontally at a height of a few metres over level ground and connected at one end to an electrometer of negligible capacity will, even if initially charged—as, for example, by momentarily earthing it—lose its charge (*i.e.* come to the potential of the air at its own level) in the course of a few minutes as a consequence of atmospheric ionisation. Provided the insulation of the supports is perfect and the capacity of the electrometer is negligible, there is nothing tending to give the wire a charge, and the electrometer, if rapid enough in its movements, will always indicate the potential of the air at the level of the wire, however rapid may be the changes in the electric field. It is impossible to secure perfect insulation of the supports, and the magnitude of the potential indicated by the electrometer will thus fall short of the true potential in the atmosphere at the level of the wire; sudden changes of potential will, however, be correctly recorded even with comparatively poor insulation. On account of imperfect insulation of the supports and of the too great capacity of the electrometer some equaliser of potential or "collector of atmospheric electricity" will generally have to be attached to the wire if it is to remain at the potential of the surrounding air.

(iii.) *By Means of a Collector.*—The method which has been most extensively used in measuring the electrical field consists in keeping a certain portion of a conducting system free from charge—*i.e.* at the same potential as the air near it—by means of an equaliser of potential or collector. The whole conducting system is thus brought to the potential of the air next the collector. For absolute measurements care has to be taken to arrange the collector and the conductor which carries it so that the potential in the air near the collector is disturbed as little as possible by their presence; for comparative measurements of the variations of potential at a given point this condition need not be fulfilled.

Lord Kelvin's water dropper is the form of collector which has been most used at continuously recording stations. A jet of water which escapes from a pipe projecting through the wall of a building, and which is supplied from an insulated cistern within the building, breaks up into drops at the point at which it is desired to measure the potential. The insulated cistern and pipe are connected to the needle of a quadrant electrometer, of which the quadrants are maintained at equal and opposite potentials by connecting them to the terminals of a series of cells of which the middle point is earthed. So long as the potential of the insulated system differs from that of the air near the point where the jet breaks up, the drops carry away a charge proportional to this difference; if the insulation is perfect the potential of the whole conducting system will finally become equal to that of the air at the point where the jet breaks up.

The potential of the insulated system is being raised by the action of the jet at a rate proportional to the radius of the drops and the number breaking away per second and to the difference of potential between the jet and the surrounding air; at the same time it is being lowered, mainly by leakage over the insulators, at a rate proportional to the potential which it has acquired. The resulting rate of change of potential is given by the relation $C\,dv/dt = a(V - v) - bv$, where C is the capacity of the insulated system, a a constant representing the current carried by the drops of the jet for unit difference of potential between it and the surrounding air, and b the leakage when the potential of the system is unity. A steady state is reached when $dv/dt = 0$, *i.e.* when $v = Va/(a + b)$. It is necessary that a should be very large compared with b if the method is to be accurate; if V remains constant v will then finally approximate closely to V.

Let us suppose now that V does not remain constant, but that there is a sudden change in the potential gradient such that V changes suddenly from V_1 to V_2. There will be a simultaneous sudden change in v—the same as would have occurred if the collector had been inoperative and therefore less than $V_2 - V_1$ in a ratio which depends on that of the capacity of the exposed part of the conducting system to the capacity of the whole system. This sudden change in v will be followed by a comparatively slow change of which the rate (if the leakage coefficient b is negligible) is given by $dv/dt = (V_2 - v)a/C$; in most cases a is such that a considerable fraction of a minute will be required for v to become sensibly equal to V_2. Except in the two limiting cases, (1) when—as with the long horizontal wire considered above—the capacity of the portion of the conducting system which is exposed in air at the same potential as that near the collector is very large compared with the whole capacity, and (2) when that ratio is very small, the interpretation of the readings of the electrometer during very rapid variations in the field may be difficult. The comparatively long period of the quadrant electrometer generally used in recording installations will, moreover, of itself prevent very rapid changes in the electric field from being followed.

There are many other equalisers of potential or collectors which have been used besides the water dropper—the flame of a lamp or candle, introduced long ago by Volta, the glowing match or fuse, sprayers (in which the efficiency of the water dropper is increased by dividing the water into very fine drops), and radio-active collectors. Their comparative efficiency and the conditions which must be observed to avoid error in their use have been investigated by Moulin (10).

Continuously recording instruments are almost necessarily contained in a building, and the water jet or other collector is then generally placed to give the potential in the atmosphere within a few feet of the walls of the building. The equipotential surfaces are deformed by the building, which for most purposes may be regarded as a conductor; the situation of the collector is generally such that it gives the potential at a point in an equipotential surface which is more nearly vertical than horizontal. To deduce the values of the potential gradient corresponding to observed values of the potential at this point, we have to do something which is equivalent to identifying at a distance from the building, and over a free surface of level ground, the equipotential surface which passes through the effective part of the collector. This is most conveniently done by making a series of absolute measurements of the potential gradient in the nearest convenient open space while the recording apparatus is at work. A factor is thus obtained by which the recorded values of the potential have to be multiplied to give the potential gradient in the open.

Absolute measurements have generally been made by means of a flame, fuse, or radio-active collector, connected to some portable type of electrometer. To avoid distortion of the equipotential surfaces, in the neighbourhood of the point where the potential is determined, by the observer and his apparatus, the collector should be fixed in the middle of a horizontally stretched wire or at the end of a horizontal conducting rod, the wire or rod being considerably longer than its height above the ground (11), (12).

While the method of measuring the electric field by determining the potential at a point in the air by means of a collector is very convenient, especially in the case of continuously recording apparatus, the methods which depend on measuring the charge on an exposed conductor have undoubted advantages. The rapidity of the changes in the electric field which can be followed with such apparatus is limited only by the speed of action of the electrometer; the insulation difficulties are enormously reduced, since the potential of the whole conducting system employed may be kept as low as we wish, and the apparatus used in this method may be made at the same time to serve for the measurement of the air-earth current.

§ (7) MEASUREMENTS AT A HEIGHT IN THE ATMOSPHERE.—Special difficulties attend the measurement of the electric field in the free atmosphere at a height; such measurements have hitherto been carried out by observers in balloons. We may illustrate the principles involved in measurements of this kind by considering a spherical conductor suspended freely in the atmosphere. If the total charge on the sphere is zero its potential is equal to the undisturbed air potential at the level of its centre; its upper and lower halves carry equal and opposite induced charges proportional to the vertical electric force, which produce equal and opposite effects on the potential at any point in the equatorial plane. The maximum value of the electric force at the highest and lowest points on the surface of the sphere is three times that of the undisturbed electric field; from the sign and surface density of the charges at these portions of the sphere we may determine the sign and magnitude of the vertical atmospheric electric field. If the total charge on the sphere is not zero, this will be indicated by an inequality of the charges of the upper and lower surfaces, and the magnitude of the whole resultant charge may easily be deduced and allowed for in calculating the magnitude of the undisturbed electric field. Instead of the surface density of the charge (or, what is equivalent, the electric force close to the surface of the sphere), the electric force at some distance may be measured. The electric force at any point is the resultant of the undisturbed atmospheric field, of that due to the induced charges on the two halves of the sphere, and of the field due to any total charge which the sphere may carry. The two latter components fall off with the distance according to the inverse cube and inverse square law respectively; a series of measurements of the electric force at different distances, more conveniently vertically below the sphere, will enable these three components to be separately determined. If these distances are large compared with the radius, the two last components are small, and the observed magnitude of the vertical electric force may be readily corrected for them, even if the conductor deviates considerably from the spherical form.

In actual measurements a balloon takes the place of the spherical conductor, and the vertical electric force is deduced from observations of the difference of potential of two collectors suspended at different distances below the balloon.

The distortion of the equipotential surfaces by balloons, airships, and aeroplanes under different conditions has been investigated by means of experiments with models (13).

If a sphere at a considerable height is connected to earth, the total charge upon it will be large compared with the charges which the field would induce on the upper and lower halves of the sphere if its total charge were zero; the resulting charge on the sphere will thus be everywhere of one sign, but the maximum density will be greater above than below. The whole charge Q will be such that $Q/r + V = 0$, where V is the undisturbed air potential at the height of the centre of the sphere. The same general methods as in the previous case will apply, but it would generally be easier, on account of the large value of Q, to measure Q (and thus V) rather than the vertical force dV/dh. A source of uncertainty in the practice of this method (with a captive balloon) is the cable, which, in addition to its direct disturbing effect on the electric field, may introduce, by brush discharges from its surface, charges of unknown amount into the immediate neighbourhood of the region in which the measurements are made.

Still greater difficulties are introduced if it is attempted to insulate the cable of a balloon or kite and bring the whole system to the air potential at the level of the upper end of this conducting system; even in fine weather the potentials to be measured are inconveniently high. Observations by this method have been carried out at Glossop (14). Measurements have also been made of the current down the earthed wire or cable of the kite or balloon; but it is difficult to utilise such measurements for the quantitative study of the atmospheric electric field. The magnitude of the current observed is largely a measure of the rate at which the charge supplied by the cable to the air surrounding its upper portion—by point or brush discharge or by "collectors"—is removed by the wind.

III. The Electric Field in Fine Weather[1]

§ (8) Mean Value of the Potential Gradient at different Places.—Our knowledge of the distribution of the electric charge (*i.e.* of the potential gradient near the ground) over the surface of the globe is very incomplete. Not only are observations on atmospheric electricity completely lacking over very large areas, but, even in regions where data have for long been accumulated at many observatories, the apparatus has been suitable rather for comparative measurements, giving the relative magnitude of the electric field at different times, than for obtaining its absolute magnitude. The determination of the "reduction factor" required in order that we may deduce the true value of the potential gradient in the open is in most cases somewhat uncertain. In recent years very important additions to our knowledge have been made by the various expeditions to the Antarctic, and by the investigations of the Carnegie Institute of Washington on the electric condition of the atmosphere over the oceans.

The potential gradient in fine weather at all parts of the earth's surface for which data are available is almost invariably positive. The mean value of the potential gradient is of the order of 100 volts per metre at all places at which the necessary observations have been made.

So far as data are available, they suggest that the mean potential gradient is higher in middle latitudes than in either the equatorial or the polar regions; the uncertainty regarding the accuracy of the absolute values must, however, be borne in mind.

The potential gradient over the ocean is not markedly different from that over land areas.

Measurements of the vertical electric field at a height by means of balloon observations were first successfully carried out by Le Cadet (15). The results of these observations, which proved that the potential gradient diminishes with height, have been confirmed and extended by later work (16), (17). The potential gradient is already reduced to about $\frac{1}{100}$ of that at the ground at heights considerably less than 10 km.

§ (9) Variations of the Potential Gradient.—Observations made at fixed stations have furnished a large amount of material for the study of the variations of the potential gradient with time; it is, however, only for very limited portions of the earth's surface that we have such data.

Superimposed upon irregular changes, which are probably mainly associated with local meteorological conditions, the records from nearly all observatories show well-marked annual and daily variations. These periodic variations become conspicuous when curves are drawn showing the mean values of the potential recorded for different months of the year or hours of the day; the curves become smoother the longer the period over which regular observations have been continued. In obtaining these curves certain days are excluded as being abnormal, and the form of the curves depends to some extent on the criterion adopted as to what constitutes an undisturbed day.

---

[1] For a fuller account see Chree's article on "Atmospheric Electricity" in the *Encyclopædia Britannica*, 1910.

(i.) *Annual Variations.*—As regards the annual variation, European observations all agree in showing a maximum in midwinter and a minimum in midsummer, the mean potential gradient at the winter maximum being two or three times as high as at the summer minimum. Such evidence as is available goes to show that the annual variation is of the same character, with a maximum in midwinter and a minimum in midsummer throughout middle latitudes in both hemispheres, *i.e.* everywhere outside the tropics and the polar regions. The records of potential obtained at Helwan (Egypt), are exceptional, showing a maximum in midsummer and a minimum in midwinter. The data for mean latitudes in the southern hemisphere are few, but observations made by Berndt (18) at Buenos Aires show an annual variation like that of similar latitudes in the northern hemisphere, the maximum potential gradient being for the month of July (winter) and the minimum for February (summer)—163 and 67 volts per metre respectively.

It is obvious that the annual variation in the tropical regions lying between these two zones of opposite phase must be of a different character. It is doubtful if reliable data from which the annual variation in this region can be deduced have thus far been obtained.

Much more is known concerning the atmospheric electricity of the south polar than of the north polar regions. The various Antarctic expeditions of recent years have all led to similar conclusions as regards the annual variation of the potential gradient. There is a maximum in *summer* (December) and a minimum in *winter* (June); *i.e.* the maxima and minima coincide in time with those of Europe, not with those of middle latitudes in the southern hemisphere (19), (20), (21), (22).

There is no direct evidence for a similar reversal of the phase of the annual variation in the northern hemisphere as we approach the pole from temperate latitudes. At Karasjok (23) (69° 17′ N., 25° 35′ E.) the annual maxima and minima are still in winter and summer respectively, as in lower latitudes. It is, however, quite possible that the annual variation may be reversed nearer the pole.

(ii.) *Daily Variations.*—The daily variation is of a less simple and constant character than the annual variation. In the simplest type there is a minimum in the early morning (4 h.-6 h.) and a maximum in the late afternoon, the maximum value being about twice the minimum. This is the winter (24) type in northern Europe. At Upsala (25), for example, in winter the potential gradient is a minimum at 4 h. (mean = 47 volts per metre), rising gradually to a maximum at 19 h. (mean = 131 volts per metre), and again falling continuously till the early morning minimum is reached. In summer, while the early morning minimum is still the principal one (about 33 volts per metre between 3 h. and 4 h.), there is a second minimum at 14 h., when the potential gradient only slightly exceeds that of the early morning minimum; there are two almost equal maxima of about 70 volts per metre on either side of the afternoon minimum (at 8 h. and 20 h.).

At Kew (12) the afternoon minimum is quite marked even in midwinter, and at midsummer the afternoon minimum is the principal one.

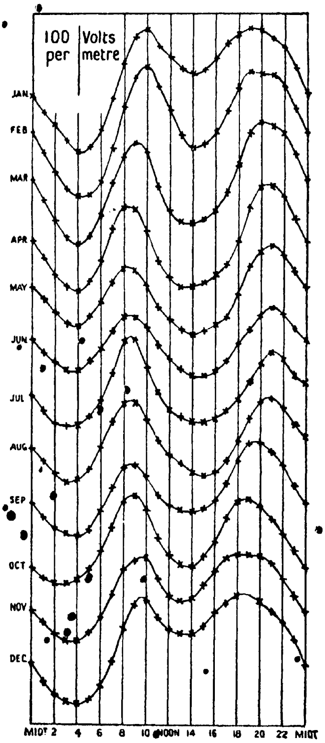


FIG. 1.—Diurnal Inequalities at Kew, 1898-1912 (Chree).

This is at most places the normal course of the daily variation in the summer months; there being thus two maxima and two minima in the course of the 24 hours, the early morning minimum being generally the less marked of the two. At Simla (26) in the month of October the curve of daily variation is of the simple type without any afternoon minimum; in June this minimum is so marked that the potential gradient becomes regularly negative for an hour or two every afternoon. The very low or negative values are associated with great dustiness of the air.

On the Eiffel Tower and at mountain observatories the curve of daily variation, even in summer, is of the simple or winter type, with a single minimum, that of the early morning, and a single maximum, that of the late afternoon. This has led to the winter type of daily variation being generally regarded as the more fundamental; since the afternoon minimum, even in summer, does not apparently extend for more than a few hundred metres above the ground, it is looked upon as a secondary effect, due probably to atmospheric convection. On the other hand, recent observations show that the afternoon minimum is quite a conspicuous feature in the records of observations over the ocean.

In high latitudes the diurnal variation is of the simple type with one maximum and one minimum. This is well shown in the results obtained by recent Antarctic expeditions. While, however, the observations of the Charcot expedition (22) at Petermann Island (65° 10′ S., 66° 34′ W.) showed a minimum between 2 h. and 6 h. and a maximum at 15 h., both the earlier observations made by Bernacchi (19) and the much more complete results obtained by Simpson (20), in McMurdo Sound, agree in showing a maximum in the early morning hours and a minimum in the latter part of the day. Simpson's observations at Cape Evans (77·5° S., 166·5° E.) show a maximum, 104 volts per metre, at 7 h. to 8 h., a minimum of 67 volts per metre at 14 h. to 15 h. He remarks that the daily variation of potential gradient at McMurdo Sound is not only different from that in other parts of the world, but from that in other parts of the Antarctic. He points out that the geographical position of McMurdo Sound is unique among those at which observations in atmospheric electricity have been made in lying between the geographical and magnetic poles.

(iii.) *Non-periodic Variations.*—Even in calm weather and with a clear atmosphere there are minor fluctuations in the records of potential gradient superimposed upon the regular periodic variations. Clouds other than rain clouds do not as a rule produce changes which are large compared with the normal potential gradient. Within a fog, however, the potential gradient generally rises to several times its normal value.

Clouds of dust raised by the wind may produce large changes in the potential gradient, the sign of the effect depending on the nature of the dust. Drifting snow at low temperature—as has been found in all the recent Antarctic expeditions—has very large effects, generally giving very high positive potentials. According to Simpson, the effects of drifting snow may be explained by supposing that when ice crystals strike one another in air the air becomes positively charged, the ice negatively.

IV. ATMOSPHERIC IONISATION AND THE AIR-EARTH CURRENT

§ (10) THE AIR-EARTH CURRENT.—It was shown by Linss (27), in a very important paper published in 1887, that a charged conductor, exposed in the atmosphere under normal conditions, loses a considerable fraction of its charge in the course of a few minutes by conduction through the air. It was later (28) proved that this conducting power of the air is due to the presence of free positive and negative ions which move under the action of the electric field. The normal vertical electric field must, in virtue of the conducting power which the presence of the ions confers on the air, cause a vertical electric current from the atmosphere into the ground. The question of the magnitude of this current per sq. cm. of the ground is of great importance in atmospheric electricity; we may investigate it in more than one way. We may attempt to measure the current directly (29), (5), (6), (30), or we may deduce its magnitude from the results of simultaneous measurements of the potential gradient and of the conductivity of the air (39); we may again measure separately the factors upon which the conductivity depends, *i.e.* the number and mobility of the ions in the air; and finally we may study the sources of atmospheric ionisation and the processes by which the ions are put out of action by recombination with one another or by becoming attached to larger suspended particles.

The vertical conduction current consists of two portions, that carried by the positive ions streaming downwards and that carried by the negative ions moving upwards under the action of the vertical electric force. If F be the vertical electric force, e the ionic charge, k_1, k_2 the mobilities of positive and negative ions, and n_1, n_2 the numbers of positive and negative ions per c.c., then the vertical conduction current per unit area is $\gamma = Fe(k_1 n_1 + k_2 n_2) = F(\lambda_+ + \lambda_-)$, where λ_+, λ_- are the portions of the conductivity due to positive and negative ions respectively. The usual method of determining the vertical current is to measure λ_+ and λ_- at a convenient height, one or two metres above the ground, F being obtained from measurements of the potential at a known height.

If we expose an insulated conducting plate with its upper surface as nearly as possible at the level of the surrounding ground, and keep it at zero potential, a current will flow into the plate from the atmosphere under the influence of the vertical electric force. The vertical current in this case (unless the plate itself emits negative ions) is entirely due to positive ions moving to the plate, and we have $\gamma = Fek_1 n_1 = F\lambda_+$ instead of $F(\lambda_+ + \lambda_-)$ as in the former case.

We cannot assume that F and λ_+ are the same at ground level as at a height of one metre. In the case of still air exposed to ionising radiation between parallel plates the electric force is greater near the surface of the plates than midway between them, while the conduction current is everywhere the same (31), (32). In the atmosphere, however, the effects of the mixing of the air by eddies and convection currents cannot be left out of account (33), (34). The stirring up of the air gives rise to electric convection currents, and the total vertical electric current includes this convection current as well as the conduction current.

Let us assume that there is not a supply of negative ions from the ground, and that the current which traverses the surface of the ground consists only of the conduction current carried by positive ions. Such evidence as is available appears to show that the difference in the vertical electric force at ground level and at a height of one metre is generally very small; *i.e.* the electric charge in the lowest metre of the air is only a very small fraction of the whole positive charge of the atmosphere (12), (35), (36). Let us further assume that as a consequence of eddies or convection currents there is sufficient mixing of the air to make λ_+ approximately the same near the ground as at a height of one metre. Were it not for air currents a positively charged layer would be formed next the ground as a result of the upward flow of negative ions under the action of the field, the layer deprived of negative ions increasing in thickness at the rate Fk_2 and acquiring per second a charge $Fek_2n_2 = F\lambda_-$; this is equal to the negative conduction current or to the excess of the total vertical conduction current in the air over that entering the ground. If there is continual mixing of the air by eddy currents, this positive charge, instead of accumulating near the ground, will be carried up and form an upward convection current of positive electricity sufficient to neutralise exactly the conduction current of negative electricity carried by the upward-moving negative ions. Throughout the region in which mixing is going on, air which is rising from the lower layers will carry a larger excess of positive electricity than descending air.

If the above conditions are satisfied (*i.e.* if there is no escape of negative ions from the ground, and if there is sufficient mixing by eddies or convection currents to prevent any considerable difference in the condition of the air near the ground and at the height at which the conductivity is measured), then the true resultant vertical current is obtained by considering the positive stream of ions only, and is equal to $F\lambda_+$; the negative conduction current $F\lambda_-$ being exactly neutralised by a convection current which is carried by upward streams of positively charged air. According to this view, the resultant air-earth current when deduced from measurements of the potential gradient and conductivity is much more nearly given by $F\lambda_+$ than by $F(\lambda_+ + \lambda_-)$, except when the air near the ground is practically stagnant.

It has been assumed above that no convection current traverses the air-earth surface. According to Ebert's well-known theory, air which diffuses out of the ground, or which escapes as a result of diminution of atmospheric pressure or increase of ground temperature, carries a positive charge sufficient on the average to counterbalance altogether the air-earth conduction current. The Ebert process, as well as any others which may cause a transference of charge between the ground and the lowest layer of the atmosphere, will constitute a convection current not necessarily included in a measurement made by the test plate method, unless the test plate be made in all essential respects identical with the surrounding ground.

The determination of the true resultant air-earth current is obviously a matter of considerable difficulty. Further investigations on the subject are required.

§ (11) Methods of measuring the Conductivity of the Air and Air-earth Current.—In measuring the conductivity we allow a stream of air from the free atmosphere to pass over a charged conductor under such conditions that the ionisation of the air is not appreciably altered by the process. If the surface density of the charge on a portion of the conductor is σ, the electric force at the surface is $4\pi\sigma$, and the current through any small element ds of its surface is $4\pi\sigma\lambda ds$, where λ is the specific conductivity due to ions of opposite sign to the charge on the conductor. The current through the whole surface of the conductor exposed to the stream of air is $4\pi\lambda\int\sigma ds = 4\pi\lambda Q_1$, where Q_1 is the whole charge on the exposed part of the conductor. The current is equal to the rate of loss of charge $-dQ/dt$; we thus have

$$4\pi\lambda = -\frac{1}{Q_1}\frac{dQ}{dt}.$$

This gives the relation (37), (38) between the conductivity due to positive or to negative ions, and the dissipation factor for negative or for positive electricity, *i.e.* the fraction of the charge on an exposed conductor which is lost per second; the dissipation factor is generally expressed in terms of the percentage of the exposed charge which is lost per minute.

(i.) *Gerdien's Method* (39).—The measurements of conductivity are generally made by Gerdien's method, in which the charged conductor is cylindrical and is surrounded by a wider coaxal cylinder kept at zero potential;

a strong current of air is drawn between the cylinders from the atmosphere, and the potential of the inner cylinder is measured by means of an electrometer connected to it. The potential difference between the cylinders should be small and the air current rapid, so that only a negligible fraction of the total number of ions is removed during the passage of the air through the tube. The charge Q_1 on the portion of the conducting system which is exposed to the air stream is given by $Q_1 = c_1 v$, where c_1 is the capacity of the exposed part of the conductor and v its potential; the current or rate of loss of charge $-dQ/dt = -c(dv/dt)$, where c is the capacity of the whole insulated system, so that we have

$$4\pi\lambda = -\frac{c}{c_1}\frac{1}{v}\frac{dv}{dt}.$$

(ii.) *By a Test Plate or Sphere.*—Direct measurements of the air-earth current may be made with the same apparatus as is used in measurements of the potential gradient by the test plate method described in § (6) (i.). The total quantity of electricity which passes from the atmosphere to the test plate during a known time of exposure to the earth's field is at once found from the readings of the measuring instrument before and after the exposure. The shielding of the test plate from the earth's field may be made momentarily at regular intervals, and a record is then obtained of both the potential gradient and the air-earth current. The dissipation factor for negative electricity and the conductivity λ_+ due to positive ions may be deduced from these. Similar observations with a sphere or other conductor raised to a convenient height in the atmosphere are more readily made; they give directly the charge upon and current through an earthed conductor exposed to the earth's field, and from them as dissipation factor and conductivity coefficient may be deduced. The values obtained for the coefficients are found under normal conditions to be independent of the size and nature of the exposed conductor (5), (6). Further experiments are required to test whether they are identical with those holding at ground level at the same time.

§ (12) THE NUMBER AND MOBILITY OF THE IONS.—The conductivity of the air depends, as we have seen, on two factors, the number of the ions and their mobility. The number of the ions of either sign in each c.c. of the air is obtained with apparatus which resembles that used by Gerdien in measuring the conductivities — but the potential difference between the cylinders has to be sufficiently large and the air stream sufficiently slow to ensure that all the ions of sign opposite to its own are caught by the central cylinder; the volume of air which passes must also be measured. The charge gained by the insulated conducting system is then equal to the total charge carried by all the ions (positive or negative according as the inner cylinder is negatively or positively charged) in the air which has passed between the cylinders. By two measurements of this kind with the electric field between the cylinders in opposite directions in the two cases, we obtain $E_+ = n_+ e$ and $E_- = n_- e$, the free positive and negative charges per unit volume of the air, and hence n_+ and n_-, the number of positive and of negative ions per c.c. If measurements of the conductivities λ_+, λ_- are at the same time made, we can deduce the mobility of the ions; for the conductivity is the product of the free charge and the mobility.

The apparatus used is generally that of Ebert (40), who first made measurements of this kind. Some of the sources of error have been pointed out by Swann, who has introduced improved apparatus (4).

The number of positive ions found by the Ebert apparatus generally exceeds that of the negative ions. The difference between the free charges $E_+ = n_+ e$ and $E_- = n e_-$ cannot, however, be taken as giving the resultant volume charge of the air. For, as Langevin (41) has shown, in addition to the ions of mobility about 1 cm. per second for 1 volt per cm., there are present other ions having a mobility amounting to about $\frac{1}{3000}$ part of this; and the number of the "large" ions may greatly exceed that of the "small" ions. It is, however, almost entirely to the small ions that the conductivity of the air is due, the great excess in their mobility much more than compensating for their smaller number.

The large ions are formed by small ions becoming attached to uncharged nuclei or dust particles such as are made visible and counted by Aitken's method of condensing water upon them.

The conductivity of the air depends almost entirely upon the number and mobility of the small ions; their loss of mobility when they become large ions is so great that they cease to contribute appreciably to the conductivity. The number of free ions depends upon the rate at which they are being produced and upon the rate at which they are put out of action—mainly by recombination or by conversion into large ions. If the atmosphere had been dust-free we might have deduced the rate of production of ions q (the number of either sign set free per c.c. per second) from their numbers, using the relation $q = an_+ n_-$, where a is the recombination coefficient, which is known from laboratory measurements; or even from the observed conductivities, since the ions would have known mobilities under given conditions of pressure, temperature, and humidity. The presence of dust particles increases greatly the difficulties of such methods of determining q. It is also impossible to use a direct method of isolating a volume of atmospheric air, and determining the rate at which ions are produced within it, without by so doing altering that rate. We have to use less direct methods.

§ (13) SOURCES OF ATMOSPHERIC IONISATION.—The production of ions in the free air is mainly or entirely due to ionising radiations of different types, α, β,[1] and γ rays from radio-active substances in the earth and in the atmosphere, and possibly also to extremely penetrating rays from the upper atmosphere or from extra-terrestrial sources. The nature and amount of the radio-active substances present in the surface layers of the earth or in the atmosphere may be determined, and their several contributions to the production of ions in the atmosphere estimated.

The radio-active material in the lower atmosphere consists of the radio-active gases—the emanations of radium and thorium—which have diffused out of the ground, and of the disintegration products of these gases. Comparative estimates of the emanation content of the air may be made, as was first done by Elster and Geitel (42), by measuring the radio-activity acquired by a wire which has been exposed in the atmosphere for a definite time while charged to a high negative potential. More exact methods are now in use.

The radio-active substances in the air emit α, β, and γ radiations which all contribute to the ionisation of the air; in addition there is ionisation due to the γ rays emitted by the radio-active materials in the earth. According to Eve (43) the ionisation of the air due to all these radiations amounts over land areas to about 4·3 ions of either sign per c.c. per second. The radio-active content both of the air over large ocean areas and of sea-water itself has been found to be almost negligible in comparison.

There is always a continuous production of ions in air contained in a closed vessel (44), (45). Part of this is due to γ radiation from the ground and from radio-active substances suspended in the surrounding air; but even when the vessel is sufficiently screened to cut off all ordinary γ rays there remains an ionisation of about 4 pairs of ions per c.c. per second in air at atmospheric pressure enclosed in a sealed vessel of copper or zinc (46), (47), (48). Over the ocean the ionisation in such a vessel remains almost the same even when unscreened (11), (49), (4). This residual ionisation which is not cut off by screening may be partly or wholly due to a radiation much more penetrating than ordinary γ radiation; the ionisation will in that case, as with ordinary γ rays, be a secondary effect due mainly to the production of β rays in the walls of the vessel (50), so that the number of ions produced by the radiation in the free atmosphere may amount to a very small fraction of 4 ions per c.c. per second.

Balloon observations have proved that the ionisation in a closed vessel, instead of diminishing with height, as it would if due to radiation from the earth (which would be almost completely absorbed by the lowest kilometre of air), is as great at 1000 metres as at the ground; at heights of a few kilometres it reaches values many times as great (51), (52), (53), (54). These observations appear to prove the existence of a penetrating radiation from the upper atmosphere. Swann attributes the ionisation of the air over the ocean entirely to this penetrating radiation from above (49).

§ (14). RESULTS OF OBSERVATIONS.—The table which follows is taken from the *Report* of the Results of Atmospheric Electric Observations made aboard the *Galilee* (1907–1908) and the *Carnegie* (1909–1916), by L. A. Bauer and W. F. G. Swann (4).

COMPARISON OF LAND AND OCEAN VALUES WITH THE OCEAN VALUES OF CRUISE IV.

Nature of Observations.	n_+.	n_-.	$\frac{n_+}{n_-}$.	λ_+ E.S.U. × 10^{-4}.	λ_-	v_+ $\left(\frac{\text{cm.}}{\text{sec.}} / \frac{\text{volt}}{\text{cm.}}\right)$.	v_-	i. E.S.U. × 10^{-7}.
Mean of land observations obtained by various observers .	737	668	1·23	1·30	1·23	1·08	1·22	6·5
Mean of ocean values for the fourth cruise of the *Carnegie* .	804	677	1·22	1·44	1·19	1·30	1·30	9·5
Mean of former ocean values obtained by various observers .	736	588	1·28	1·44	1·20	..	..	..

(i.) *Land and Sea Observations.*—The difference between the land and ocean mean values of the various atmospheric electrical elements is small. The air over the ocean is comparatively dust-free, and the production of large ions by attachment of small ions to uncharged nuclei is likely to be relatively small. Neglecting the effect of dust particles in putting the ions out of action, Bauer and Swann calculate q, the rate of production of ions over the ocean, from the relation $q = an^2$, n being taken as about 800, and a as having the value obtained in laboratory measurements of the recombination constant. This gives for q the value 1·6, *i.e.* 1·6 pairs of ions must be produced per c.c. per second. On land there would have to be added about 4·5 per c.c. per second due to the known radio-active materials in the ground and atmosphere. But the numbers of free positive and negative ions found per c.c. of air over the land is no greater than over the sea; this is probably due to the action of dust particles in capturing the ions.

(ii.) *Number of Free Ions, Mobility and Conductivity.*—The number of free ions of either sign per c.c. is normally of the order of 500 (very rarely exceeding twice or falling below one half of this value), the positive nearly always exceeding the negative in number.

The mobilities (deduced from the conductivities of the air for positive and for negative ions and the number of ions of each sign per c.c.) generally fall somewhat short of the values obtained in the laboratory; they generally only slightly exceed 1 cm. per second for 1 volt per cm.

The conductivities λ_+, λ_- are of the order of 10^{-4} e.s.u.; the conductivity due to positive ions nearly always exceeds that due to negative ions, the greater number of positive ions more than compensating for their smaller mobility.

The variations in the conductivity of the air are generally in the opposite direction to those of the electric field. Thus the air-earth conduction current, which depends on their product, varies less than either of its factors. The conductivity of the air is, for example, in Europe less in winter than in summer, while the potential gradient is greater in winter. In the Antarctic, where the potential gradient has its maximum in summer and its minimum in winter, the annual variation in the conductivity is, according to Rouch, very marked, and absolutely the inverse of that of the corresponding electric field (21).

The conductivity of the air depends largely upon the clearness of the air, *i.e.* its freedom from dust particles to which the ions can attach themselves.

The average air-earth conduction current of fine weather is of the order of 6×10^{-7} e.s.u. or 2×10^{-16} amp. per sq. cm.

(iii.) *Balloon Observations.*—These observations have shown a very great increase in the conductivity of the air with increasing height. The increase is due partly to its greater freedom from dust and its smaller density and consequent greater mobility and smaller rate of recombination of the ions, but partly also to an increased rate of production of ions at a height (39), (55). The diminution of potential gradient with height is in all probability just sufficient to compensate for the greater conductivity, so that the vertical electric current is the same at a height of several kilometres as near the ground.

V. PROBLEM OF THE ORIGIN AND MAINTENANCE OF THE ELECTRIC FIELD OF FINE WEATHER

§ (15) THE ORIGIN OF THE FIELD.—In the absence of compensating processes the electric field of fine weather would very quickly be destroyed by the air-earth conduction current. The fact that the electric field persists proves that such compensating processes exist and that on the whole the electric field is being regenerated as fast as it is destroyed.

Unless we assume an actual creation of negative electricity or destruction of positive electricity within the earth (56), the total air-earth current over the whole surface of the earth must on the average be zero; *i.e.* the downward flow of positive electricity from the air to the earth which takes place under the action of the normal electric field of fine weather is exactly balanced (if we take the average over any considerable time) by currents, not necessarily conduction currents, in the opposite direction.

The most important of such currents which may possibly be effective are (a) convection currents of charged air, (b) convection currents carried by negatively charged rain or other precipitate, (c) upward conduction currents in regions where the potential gradient is negative, and (d) currents carried by corpuscular radiations.

(i.) *Theories of Ebert and Lenard.*—According to a large class of theories the air in contact with the ground acquires a positive charge, the earth receiving an equal negative charge, *e.g.* by friction, by chemical processes, or as a result of the greater mobility of the negative than of the positive ion. To make such processes effective in creating an electric field in the atmosphere, the air which has acquired a positive charge while in contact with the ground must be continually carried up by convection or eddy currents and be replaced by new air. The energy required to produce the electric field by raising the positive charge against the attraction of the negative charge on the ground is derived from air currents.

The most important of the theories of this class are those of Ebert and of Lenard (57), (58). Ebert supposes that ionised air which diffuses out of the ground, or which escapes as a result of diminishing barometric pressure or rising ground temperature, is charged positively, owing to more negative than positive ions becoming attached to the walls of the capillary air passages in the soil, on account of their greater mobility; air sucked out of the soil is found to be much more ionised than ordinary air, on account of the radio-active emanations with which it is charged. The process, like most of the others of this class, is only effective over land. Lenard has, however, shown that the splashing of salt water, as in the breaking of waves on the surface of the ocean, also gives the air a positive charge.

(ii.) *Convection by Charged Rain.*—Several theories of atmospheric electricity have been based on the assumption that more negative than positive electricity is carried down to the ground by rain. The observations of Simpson at Simla and most subsequent investigations on the subject have, however, shown that rain and other forms of precipitation carry down, on the whole, an excess of positive electricity (see § (22) and § (25)).

(iii.) *Upward Conduction Currents.* — The potential gradient is frequently negative during rain; the vertical conduction current is then from the earth to the air. On account of the very large values of the electric field in showers and thunderstorms and of additional sources of ionisation then in operation, the vertical conduction currents may be very large compared with the normal fine weather air-earth current. Whether the charges carried from the atmosphere to the earth by these currents (and by lightning discharges which may be grouped with them) are preponderatingly positive or negative still remains to be determined (see § (21)).

It is not impossible that, quite independently of precipitation there may be regions where negative potential gradients normally exist and maintain an upward earth-air current.

(iv.) *Corpuscular Radiation.*—The phenomena of the aurora furnish evidence that corpuscular radiations reach our atmosphere from the sun and that they may penetrate to within 100 km. of the earth's surface. The possibility of the negative charge of the earth being maintained by negative, or β rays of sufficient velocity to traverse our atmosphere and reach the earth's surface has frequently been suggested (59), (60). Unless, however, their kinetic energy were enormously great compared with that of the fastest β rays from radio-active substances, their penetrating power would be too small and the ionisation produced in the atmosphere would be too great to make this a possible source of the earth's negative charge. Not only are the rays absorbed by the air, they have also to move against the electric field through a potential difference of about 1 million volts before reaching the earth's surface. On account of the guiding action of the magnetic field, which has also to be taken into account, it is in the neighbourhood of the magnetic poles that the penetrating rays would be most likely to reach the earth's surface.

It is possible that β rays, of which the penetrating power was insufficient to enable them to traverse the lower atmosphere, might become sufficiently concentrated above the region of the magnetic poles to reverse the potential gradient in these regions; a locally reversed earth-air current would thus result, which might be large enough to contribute materially to the maintenance of the earth's negative charge.

It is easier to imagine the existence of a radiation of the γ type which would have sufficient penetrating power to enable it to reach the surface of the earth; such radiation would also be unaffected by the electric and magnetic fields of the earth. Experiments on the ionisation in a closed vessel already give some indications of the existence of a penetrating radiation of this type. Swann (61) has pointed out that a radiation of this kind (started perhaps by the stoppage of very fast β rays in the upper atmosphere) would produce β rays as it traversed the air, these β rays travelling mainly in the same direction as the radiation and thus constituting an upward positive current. The β rays from the lowest layers of the atmosphere would enter the earth and tend to keep up its negative charge.

§ (16) CONSIDERATION OF THE VARIOUS THEORIES.—Much could probably be done towards estimating the relative importance of the various processes by which a transference of electricity between the atmosphere and the ground may take place, by determining the total flow of electricity from all causes through an isolated portion of the earth's surface, *i.e.* by developing the test-plate method of measuring the air-earth current. Observations thus far show, apart from effects of precipitation, that the normal air-earth current flows through such a test-plate even when it is made as nearly as possible the equivalent of an isolated portion of the ground; there is no evidence of a compensating reverse current nor can such a reverse current be detected when the field is cut off. As regards the Lenard effect of the splashing of sea water, apart from other difficulties it is doubtful whether it would compensate for the effect of the charge induced on the spray of a breaking wave by the positive potential gradient.

Insuperable difficulties in the way of accepting theories of the Ebert type have been pointed out by Simpson (62), Gerdien (63), Swann (64), and Schweidler (65). The latter also draws attention to the difficulties of the corpuscular theories.

On the whole, it appears to be likely that the compensating process which prevents the negative charge of the earth from being neutralised by the fine weather conduction current does not act in the fine weather regions where the potential gradient is positive. The return current is in all probability itself also mainly a conduction current flowing in regions where the potential gradient is negative. It is conceivable that such negative potential gradients may exist normally in the immediate neighbourhood of the magnetic poles as a result of negative charges brought from extra-terrestrial sources by corpuscular radiation. It is certain that very intense negative potential gradients exist below cumulo-nimbus clouds and that they produce large upward currents; as, however, do high positive potential gradients with large downward currents. The current which flows between the ground and a cumulo-nimbus cloud is probably large (of the order of 1 ampere), and the number of such clouds acting at a given moment (probably many thousands over the whole earth) is sufficiently great to make it certain that they play an important part in determining the electrical condition of the earth. Whether there is an excess of the upward currents, and whether this is sufficient to compensate for the air-earth currents of fine weather, remains to be determined.

§ (17) Consequences of the High Conductivity of the Upper Air.—We greatly simplify the consideration of the problem of the maintenance and distribution of the electric field if we assume (in accordance with the phenomena of terrestrial magnetism and the aurora and of the propagation of electro-magnetic waves) that the upper atmosphere is sufficiently ionised to possess considerable conducting power. The problem then reduces itself mainly to determining the mechanism by which the potential difference of about one million volts between the conducting upper atmosphere and the ground is maintained. The electrical effects of precipitation, which would, in the absence of the conducting layer, be comparatively local, may now extend to distant regions in which fine weather prevails (5), (64).

Let us for the present assume that the potential of the conducting upper atmosphere which is situated over a considerable area of the earth's surface remains constant. The lines of flow below the conducting layer may be taken to be vertical. If a steady condition is reached the vertical current below the layer of high conductivity will be the same at all levels, and the vertical electric force at different levels will vary inversely as the conductivity. Above the lowest kilometre (*i.e.* above the influence of ionising radiations having their origin in the ground) the conductivity increases with height, and the potential gradient diminishes accordingly.

The potential at a point at a given height above the ground will be increased by any influence which diminishes the conductivity of the air below or increases that of the air above it. The effect of a ground fog in raising the potential gradient is thus readily explained, as was shown long ago by Elster and Geitel (66). A diminution of the conductivity of the lowest layers of the air alone will only slightly diminish the air-earth current, since this depends on the total electrical resistance between the ground and the upper atmosphere; thus there is a tendency for the potential gradient near the ground to vary inversely as the conductivity of the lowest layers. It is possible that the annual variation of the potential gradient may be partly at least explained in this way as due to variations in the conductivity of the air near the ground. The variations in conductivity may result not only from variations in the dustiness of the air, but also from variations in the rate at which ions are being set free. A considerable part of the ionisation of the air near the ground is due to the radio-active emanations of radium and thorium which have diffused out of the ground. Any condition which assists the escape of the emanations, such as high ground temperature, tends to increased conductivity of the air and low potential gradients.

An increase in the conductivity of the air elsewhere than in the lowest layers will increase the air-earth current, and thus the potential gradient near the ground. The early morning minimum has been explained as being due to the ionisation of the upper portions of the atmosphere being then a minimum (67), (68). The afternoon minimum may be due to the effect of atmospheric convection, which then reaches its maximum, in diminishing, by increased dustiness and humidity and production of cloud, the mobility of the ions above the lowest layers. The absence of the afternoon minimum when the potential is observed at a height may be explained as due to the point at which the potential is observed being situated within the air of diminished conductivity; the resistance between this point and the tower or hill-top on which the observations are made being increased relatively as much as that of the air above.

It is unlikely, however, that the annual and diurnal variations of the potential gradient are to be explained as entirely due to variations in the conductivity of the atmosphere below an ionised upper layer, which remains at a constant potential everywhere the same. The conductivity of the upper atmosphere is probably not such as to prevent considerable differences of potential from existing between portions of it at a great distance apart. The distribution of potential within the conducting layer will depend on the situation of the generators which maintain the potential difference between it and the earth.

If we take the view that cumulo-nimbus clouds are the principal sources of the positive potential of the upper atmosphere, then its potential may be expected to be highest over the regions where such clouds are most numerous, *i.e.* in tropical and middle latitudes in the afternoon hours. A factor which may be of importance in the distribution of potential in the upper atmosphere is the greater conductivity of air, at very low pressures, along than across the magnetic lines of force. This will tend to equalise the phase of the diurnal variation of the potential of the upper atmosphere along a magnetic meridian; the maxima and minima occurring at the times when the number of thunderstorms and showers in the neighbourhood of this meridian is greatest and least. This would fit in with the simple winter and high-level type of diurnal variation, and would give a possible explanation of the remarkable results regarding the phase of the diurnal variation in the region between magnetic and geographical south poles to which Simpson has called attention (see § (9) (ii.)).

Again, if there is a fall of potential in the upper atmosphere in passing from the tropical regions to the poles, any increase in the conductivity of the uppermost layers in high latitudes (such as we might expect in summer from increased exposure to ionising solar radiation) will diminish the potential difference between the upper air in polar regions and that of middle latitudes; diminishing the potential in the latter and increasing it in the former. This may be one cause of the annual variation in the potential gradient—which, as we have seen, has its maximum in winter in middle latitudes and (if we can judge from the Antarctic observations alone) in summer in the polar regions.

VI. POTENTIAL GRADIENTS ASSOCIATED WITH SHOWERS AND THUNDERSTORMS

§ (18) THE EFFECT OF CLOUDS AND RAIN. (i.) *Clouds.*—Clouds other than those associated with precipitation do not in general produce conspicuous effects on the potential gradient at the earth's surface. The part played by the great majority of clouds in relation to atmospheric electricity is probably mainly the purely passive one of diminishing the conductivity of the air, and thus increasing the vertical electric force within them and diminishing that above and below them. A very thick cloud layer may reduce the vertical current, and hence the potential gradient at the ground, almost to zero. Time is of course required for the establishment of the steady condition; *i.e.* for the vertical current to supply such charges to the upper and lower boundaries of the cloud as are necessary to make the electric force within it great enough to compensate for the increased resistance, so that the vertical current within the cloud becomes equal to that above and below it.

Ordinary cumulus clouds have very little effect on the potential gradient at the ground. Observation shows that even when great masses of towering cumulus are visible on all sides, the potential gradient is often hardly affected. It is apparently only when some critical condition has been reached, and the cumulus has become a cumulo-nimbus cloud, that it develops any considerable external field. This development of the external field is very rapid, a change in the potential gradient in the neighbourhood of the ground from values of the order of 100 volts per metre to more than 10,000, positive or negative, occurring in a very few minutes.

(ii.) *Rain.*—Rain does not always produce large effects upon the electric field; when the effect is small there is on the whole a lowering of the normal positive potential gradient. Heavy precipitation of any kind generally produces very high, positive or negative, potential gradients, with frequent changes of sign; negative potential gradients on the whole preponderate.

§ (19) MEASUREMENT OF WET WEATHER EFFECTS.—For measuring the very intense and rapidly changing electric fields associated with heavy precipitation and thunderstorms, the ordinary apparatus, which is used for recording potentials by means of a water dropper or other collector, is both too sensitive and too slow in its action; it gives little more than qualitative information. A good deal of additional information might be obtained by reducing sufficiently the sensitiveness and diminishing the period of the electrometer (by using a stronger control) and by using a very efficient collector. But the potential acquired by the collector and the conducting system connected to it will not infrequently be high enough to cause sparking, and no collector is sufficiently rapid in its action to follow the changes which result from the passage of lightning discharges.

Methods which depend on measurements of the charge on an exposed conductor are much more suited for the study of these large and rapidly changing potential gradients.

The fact that a thunderstorm is approaching is first indicated by apparatus of this type by the sudden changes produced in the potential gradient by distant lightning discharges (8). A photographic record of the electrometer readings, obtained while the storm is at a distance of 20 km. or more, generally shows a positive potential gradient not differing much from the normal value; sudden small changes, which may be either positive or negative, indicate the occurrence of lightning discharges. These sudden changes become comparable with the normal potential gradient when the storm is at a distance of from 15 to 20 km. The sudden changes produced in the field by discharges at 10 km. are generally of the order of 500 volts per metre, becoming as great as 10,000 volts per metre or more when the discharges occur at distances of 3 or 4 km. The records obtained thus far show a greater number of instances of discharges causing a positive change of potential gradient than of those causing a negative change.

The vertical electric force at the ground due to a thunder-cloud does not generally much exceed the sudden change which occurs when a lightning discharge passes; very frequently each flash suddenly reverses the sign of the potential gradient, which again within a few seconds recovers its original sign.

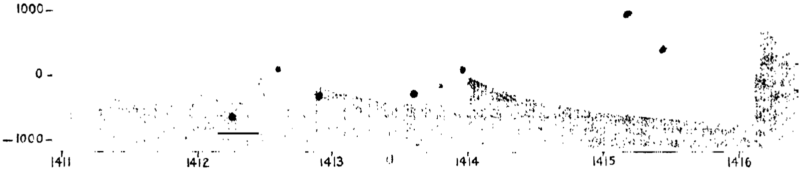


FIG. 2.—Record of Potential Gradient (in volts per metre) at the Solar Physics Observatory, Cambridge, June 13, 1917, 14 h. 11 m. to 14 h. 16 m. 30 s.

After each discharge the electric field tends to return to the value which it had before the discharge, the curve of recovery of the field frequently approaching the exponential form. The initial rate of recovery of the field is generally such that if it had remained uniform the whole field destroyed by the discharge would have been regenerated in a few seconds.

VII. TRANSFERENCE OF ELECTRICITY BETWEEN THE ATMOSPHERE AND THE EARTH IN SHOWERS AND THUNDERSTORMS

§ (20) LIGHTNING.—From the sudden changes which are produced in the electric field at the surface of the earth by the passage of lightning discharges at known distances, we may deduce the electric moments of the discharges; the electric moment being equal to $2QH$, where H is the vertical height through which the quantity of electricity Q has been discharged. For distances great compared with H the electric moment of the discharge is given by FL^3, where F is the sudden change in the vertical force at the ground and L is the distance of the discharge.

The mean value of the electric moments of the lightning discharges thus far studied in this way is of the order of 3×10^{16} e.s.u.-centimetres or 100 coulomb-kilometres; these moments may be positive or negative, but flashes with positive moments (*i.e.* which carry a positive charge upwards) appear to be the more numerous.

Lightning discharges may pass between the upper and lower portions of the thunder-cloud or between the cloud and the ground, or they may pass from the cloud into the surrounding atmosphere without reaching the ground. Discharges from the upper part of a cloud upwards into a clear sky have not infrequently been observed. It is only with discharges between the cloud and the earth that we are at present concerned; such discharges generally extend through a vertical height of one or two kilometres. The average lightning flash between cloud and earth probably discharges a quantity of electricity of the order of 20 coulombs through a height of about 2 km.

In severe storms the average interval between successive lightning flashes may be a few seconds only. If we assume an interval of 20 seconds between successive flashes, each discharging 20 coulombs, the thunder-cloud has to supply 1 coulomb per second, *i.e.* one ampere, to feed the flashes.

§ (21) CONTINUOUS CURRENTS.—The high positive and negative values of the electric field below shower-clouds and thunder-clouds would lead us to expect vertical currents through a given area of the ground which are many times as large as those of fine weather, even if there were no additional sources of ionisation. But there are such additional sources; there are three processes which are likely to provide a supply of ions just in those areas where the potential gradient at the ground is greatest (8). Charged drops falling from a cloud may evaporate completely, producing ions corresponding in number to the charge originally carried by the drops. Again, if heavy rain reaches the ground, the splashing will give rise to ionisation. Lastly, if the potential gradient is strong enough, it will itself cause ionisation by point discharges from exposed pointed conductors.

The last named is probably the most important source of ionisation below thunder-clouds. In some recent experiments made at the Solar Physics Observatory, Cambridge, a positive potential gradient of about 15,000 volts per metre, applied artificially over a field of grass, was found to be sufficient to cause a point discharge of negative electricity from the tips of the blades of grass; a somewhat larger negative potential gradient, about 20,000 volts per metre, had to be applied to cause a measurable positive discharge. For greater potential gradients, positive or negative, the currents increased rapidly and soon reached values exceeding 1 microampere per sq. metre (=1 ampere per sq. km. or 10^{-10} ampere per sq. cm.).

It is probable that potential gradients exceeding the limits required to cause point discharges from grass exist below thunder-clouds, and that the potential gradient at the ground is prevented by such point discharges from exceeding a limit fixed by the current which the thunder-cloud is able to supply. Observations of the sign and magnitude of the potential gradients below thunder-clouds and shower-clouds are evidently much to be desired, since they would afford means of obtaining an estimate of one important component of the air-earth currents associated with such clouds. When a thunder-cloud passes at a comparatively small height above the ground, and when the number of discharging points available is small, the current through such projecting conductors as exist may be comparatively large, and we get visible glow or brush discharges—St. Elmo's fire. The necessary conditions are frequently met with on mountain summits, and the discharges may be intense enough to give considerable illumination and to emit a loud humming sound. The direction of the current (as indicated by the appearance of the discharge) may be either from the ground to the cloud or in the opposite direction.

(22) Electricity of Rain.—The importance of determining the sign and magnitude of the charges carried down to the earth by rain and other forms of precipitation was pointed out by Linss (27); the first measurements were made by Elster and Geitel (69). Observations have now been made in many parts of the world.

The difficulties of such measurements, especially during thunderstorms, are considerable. If we were merely to place an insulated vessel out in the rain and examine the charge which it had gained after a certain time, there would be large spurious effects, due mainly to drops striking the vessel and splashing off. For even uncharged drops would, if they struck the rim of the vessel, carry off some of the charge induced by the electric field. It is difficult to remove this danger (by means of suitably placed screens and diaphragms) without introducing an error of opposite sign, due to rain splashing into the vessel after striking portions of the screening system which are exposed to the earth's field. With the methods which have actually been used the errors due to such causes are probably unimportant.

The earlier observations of Elster and Geitel (69) and of Gerdien (70) led to the conclusion that on the whole more negative than positive electricity was carried to the ground by rain. Simpson (71), however, in a long series of measurements made with self-recording apparatus at Simla, found that a much larger quantity of positive than of negative electricity was brought down to the ground by rain. Similar results have been obtained by nearly all subsequent observers in different parts of the world (72), (73), (74), (75), (76), (77). Schindelhauer (78), however, concluded, from the results of a long series of observations at Potsdam, that there was no excess of positive charge.

Simpson found occasions of positively charged rain to be more than twice as frequent as those of negatively charged rain, and that nearly three times as much positive as negative electricity was brought down by the rain. The currents carried to the ground by rain are in the great majority of cases less than 10^{-13} ampere per sq. cm.; the largest currents observed amounted to nearly 10^{-12} ampere per sq. cm. Positively charged rain becomes relatively more frequent as compared with negative rain the greater the rate of rainfall. Simpson found that rainfalls exceeding 1 mm. in 2 minutes were always positively charged. The charge of rain is generally less than 1 e.s.u. per c.c., but positive and negative charges exceeding 5 e.s.u. per c.c. are not infrequent, and charges approaching 20 e.s.u. per c.c. of rain have been observed.

The convection current carried by precipitation is thus on the whole from the atmosphere into the ground, *i.e.* in the same direction as the air-earth current of fine weather. The density of this convection current per sq. cm. of the surface of the ground is not infrequently 1000 times as great as that of the normal air-earth conduction current, but it very rarely exceeds 10^{-12} ampere per sq. cm.

Of the three kinds of electric current which may accompany precipitation—the convection current carried by rain, the momentary currents of lightning discharges, and continuous currents due to the intense electric fields—it is quite possibly the last which contributes most to the interchange of electricity between the earth and the atmosphere.

VIII. THE THUNDER-CLOUD AS AN ELECTRICAL MACHINE: ITS ELECTRICAL FIELD AND THE SYSTEM OF CURRENTS WHICH IT MAINTAINS.

§ (23) THE E.M.F. AND CURRENT WITHIN THE CLOUD.—A thunder-cloud may be regarded as an electrical machine. It is of interest to try to obtain some estimate of the order of magnitude of the electromotive force developed and of the current which passes through the cloud, and to consider the probable distribution of this current.

The mechanism by which the electromotive force of the cloud is developed will be discussed later. It may be assumed that a vertical separation of positive and negative electricity takes place within the cloud, electricity of one sign being carried down by large drops or hailstones, while a charge of the opposite sign is attached to the smaller cloud particles which are carried up in the ascending air.

If it is within the thunder-cloud that the separation of the positive and negative electricities is effected, equal charges of opposite sign must be developed in the upper and lower parts of the cloud, which is thus essentially bipolar. While, however, the poles of the cloud necessarily develop equal and opposite charges in a given time, they are also losing electricity by the falling out of charged rain, by conduction through the ionised atmosphere, and it may be by lightning discharges. The losses will generally affect the two poles differently, and the upper and lower charges may become very unequal.

Thunder-clouds are generally of great vertical thickness, their bases being generally at a height of from 1 to 2 km., while their summits are generally above 5 km. and may reach to nearly twice that height. There is no difficulty in imagining a relative vertical motion of positive and negative carriers which would result in the top and bottom of the cloud becoming oppositely charged while the greater part of the vertical thickness might be nearly neutral; the centres of the positive and negative charges may be at a considerable vertical distance apart.

We may learn something about the rate at which the vertical separation of positive and negative charges takes place, *i.e.* about the vertical current through the cloud, as well as about the order of magnitude of the potential difference between the poles, from the effect of lightning discharges upon the potential gradient.

We have seen that the quantity of electricity which passes in an ordinary lightning flash is of the order of 20 coulombs. The rate of separation of the positive and negative charges may have to be sufficient to supply many such flashes in one minute, and the average current through the thunder-cloud must in such cases exceed one ampere.

But this does not necessarily represent the whole output of the thunder-cloud; it may supply in addition to or in place of the lightning discharges a considerable continuous current. The way in which the electric field of a distant thunder-cloud is regenerated after a lightning discharge is suggestive in this connection. The curve of recovery of the field is of the form we should expect it to have if, while the rate of separation of the charges in the cloud (*i.e.* the vertical current through the cloud) continues at a more or less constant rate, there is a loss of charge (*i.e.* a return current) which increases continuously as the potential difference between the poles of the cloud increases. The total current or rate of separation of the charges deduced in this way generally greatly exceeds what is required to supply the lightning discharges of the cloud, the average amounting to some amperes.

We may in different ways form some estimate of the linear dimensions of the region in which the charge which passes in an average lightning flash must have been concentrated, and so deduce the order of magnitude of the maximum potential attained within this region. This leads to an estimate of 10^9 volts for the order of magnitude of the E.M.F. of a thunder-cloud.

A shower-cloud which does not produce lightning may be one in which a balance between the separation of the charges and their dissipation by continuous processes is attained before the lightning discharge limit is reached. The magnitude of the E.M.F. and of the current through such a cloud may thus not be greatly less than those associated with a thunder-cloud.

§ (24) DISTRIBUTION OF THE CURRENT SUPPLIED BY THE CLOUD.—A thunder-cloud is situated between two parallel or concentric conductors—the earth and the ionised upper atmosphere—which together form a condenser of high capacity. The existence of this highly ionised layer in the upper atmosphere is indicated by the phenomena of the propagation of electromagnetic waves round the earth as well as by those of terrestrial magnetism. Its height is uncertain, but a knowledge of this is not required for the purpose of estimating the influence of the conducting layers upon the electrical phenomena of thunderstorms; let us assume that the lower limit of the layer of high conductivity is about 50 km. The atmosphere below this limit is also always ionised to a considerable extent, the ionisation diminishing at lower levels; the electrical resistance of the lowest 5 km. of any vertical column of air probably exceeds that of the whole column above this level.

From the sign and magnitude of the potential gradient at places remote from the disturbing influences of shower-clouds, and from the manner in which this falls off with the height, we may conclude that the potential in the upper atmosphere is positive and of the order of one million volts. This is small compared with the E.M.F. of a thunder-cloud.

The sum of the potential differences between the upper pole of the cloud and the high-level conducting layer of the atmosphere, and between the ground and the lower pole, will thus differ relatively very little from that between the two poles of the cloud. Whether then the return current (continuous or discontinuous), by which the charges recombine after being separated within the cloud, passes mainly by the direct route from pole to pole of the cloud, or *via* the upper atmosphere and the earth, will depend on the relative resistances of these two circuits. An increase in the vertical distance between the upper and lower charges will increase the resistance of the short circuit and diminish that of the long circuit for both continuous and disruptive discharges.

The free air at levels above that to which the summits of cumulo-nimbus clouds attain has even normally a conductivity many times as great as that of the lower layers, and this will be greatly increased by the action of the field of the thunder-cloud in dragging ions down from the upper atmosphere. Within the cloud itself (so long as discharges do not occur) the conductivity is likely to be reduced by the entanglement of ions by cloud particles. Below a shower-cloud or thunder-cloud there are the additional sources of ionisation considered in Part VII. It is probable, therefore, that the sum of the resistances above and below the cloud may not be large compared with that within the cloud, and that an important part of the continuous current maintained by a shower-cloud may pass from the ground through the cloud to the upper atmosphere or in the reverse direction.

Consider next the case of lightning discharges. If the cloud is of small vertical thickness the electric field is likely to reach its critical value first within the cloud, and the discharge will be of the nature of a "short-circuit" between the poles; or, if the rate of dissipation of the lower charge (*e.g.* by the falling out of charged rain) has been relatively great, the lightning discharges may be between the upper pole and the ground.

If the vertical distance between the upper and lower charges is considerable, it is quite possible for the critical value of the field to be exceeded at the lower boundary of the high-level conducting layer (on account of the low pressure at this height) before it reaches its critical value within the cloud. There may then be discharge, continuous or disruptive, between the upper atmosphere and the upper surface of the cloud; some forms of "sheet lightning" may possibly be discontinuous discharges originating in this way.

If the potential difference between the upper pole of the cloud and the upper atmosphere is approximately destroyed by a discharge, a potential difference between the lower pole and the ground becomes nearly the same as that between the poles of the cloud. Discharges will tend to occur between the lower pole of the cloud and the ground simultaneously with discharges between the upper pole and the upper atmosphere; the lower charge may, however, have to accumulate for a period occupying several intervals between the upper discharges before it reaches the limit required to cause a lightning discharge to earth.

Increased conduction in the atmosphere above the cloud, whether due to discontinuous discharges or otherwise, tends to diminish the upper charge and thus indirectly to increase the lower charge by diminishing the dissipation current which it receives from the upper pole of the cloud. An increase in the lower charge and a diminution of the upper charge both have the effect of increasing the potential gradient at the ground if it is already in the direction opposed to that within the cloud, or to diminish it if it is in the same direction as that within the cloud.

The conductivity above a thunder-cloud must be largely due to ions dragged out of the conducting upper atmosphere by the electric field of the cloud. The conductivity above a cloud of positive polarity (*i.e.* one of which, the upper pole, is positive) will be greater than that over a cloud of negative polarity, on account of the very much greater mobility of the negative ions than of the positive in dry air at low pressures. And again a discharge from the upper atmosphere to the cloud, owing to the field causing ionisation by collisions, will also occur more readily if it is the negative ions which are dragged out of the conducting layer, *i.e.* if the polarity of the cloud is positive. This excess of conductivity above the cloud of positive polarity will tend to make negative potential gradients predominate below the centres of cumulo-nimbus clouds if we assume that clouds of negative polarity are not greatly more numerous than those of positive polarity. This in turn will lead to the ionisation current in showers, whether it is continuous or passes in lightning discharges, being mainly directed from the earth to the atmosphere.

According to Simpson's theory (71) of the electricity of rain, a thunder-cloud is necessarily of negative polarity; the preponderance of positively charged rain which reaches the ground is in agreement with this theory. But, as we have seen, the convection current which reaches the ground on charged rain-drops is probably a small fraction of the total current and may give no reliable measure of its magnitude and not even indicate its sign correctly. Rain, for example, which was negatively charged when it left the cloud, and which fell through an upward stream of positive ions set free at the ground, might be wholly or partially discharged or even have the sign of its charge reversed before it reached the earth (8). The excess of positively charged rain may indeed be partly a consequence of negative potential gradients being more frequent than positive in heavy showers.

There can be little doubt that the electromotive force of a thunder-cloud must maintain a considerable current—probably of the order of some amperes—between the ground and the upper atmosphere. The average number of thunder-clouds in action at a given time is probably of the order of 1000 at least; and the effect of the much larger number of shower-clouds which do not produce lightning cannot be neglected. It is at least possible that the upward currents due to clouds of positive polarity may more than counterbalance the downward currents due to clouds of negative polarity, and that the excess may be sufficient to maintain the positive potential of the upper atmosphere and compensate for the downward current of fine weather.

IX. Some Processes associated with the Development and Dissipation of the Electric Charges of Thunder-clouds

§ (25) The Movement of Charged Air.—A cloud layer diminishes the conductivity of the air within it by destroying the mobility of the ions which become attached to the cloud particles. Any pre-existing vertical electric field will be intensified within the cloud, until the total vertical current, due to convection and conduction within the cloud, is equal to that above and below the cloud in spite of the smaller conductivity. The rate at which this increased field develops, *i.e.* the rate at which charges of opposite sign accumulate at the upper and lower margins of the cloud layer, will be determined by the magnitude of the vertical electric current.

A vertical electric field arising in this or in any other way within a cloud layer may be intensified by diminishing its horizontal dimensions and thus increasing the density of the upper and lower charges; this reduction of the horizontal dimensions may be brought about by converging winds. A contraction in the horizontal dimensions of a cloud by converging motion of the air will be accompanied by an expansion in the vertical direction and an increased separation of the upper and lower charges. The potential difference between the top and bottom of the cloud will thus be increased by the increase both in the vertical force and in the height through which it extends. The converging air motion will frequently be greater in the upper than in the lower part of the cloud, as is evident when the cloud assumes the cumulus form; the consequent concentration of the upper charge may thus greatly exceed that of the lower.

§ (26) The Fall of Charged Drops and Hailstones.—While the movement of charged air masses as a whole may be a factor in the development of the intense fields of thunder-clouds, it is probably by no means the main one. It has long been regarded as probable that the electrical field of a thunder-cloud is due to large falling drops or hailstones becoming charged with electricity of one sign while the uprising air (or the smaller cloud particles carried up by it) is charged with electricity of the opposite sign. There have been many attempts to explain how this initial separation of positive and negative electricity occurs.

(i.) *Condensation Effects.*—The process of condensation may under certain conditions cause an opposite electrification of the cloud particles which are formed and of the surrounding air. In atmospheric air from which the dust particles have been removed (*e.g.* by the falling out of water drops which have condensed upon them) condensation does not occur until an approximately fourfold supersaturation has been attained; when this stage is reached the negative ions which are continually being set free will serve as nuclei on which water will condense to form drops (79). Positive ions require a considerably higher degree of supersaturation to make water condense upon them, so that they are never likely to come into action in the atmosphere.

It was suggested by Sir J. J. Thomson (80) that this difference between the positive and negative ions might result in a preponderance of negatively charged rain, and thus to the normal positive electrification of the atmosphere. Gerdien (63) based upon it a theory of thunderstorm electricity. There are many reasons against accepting, as a sufficient explanation of the origin of the intense fields of thunder-clouds, this difference in the efficiency of positive and negative ions as nuclei for the condensation of water vapour (59), (81), (82).

With particles of the size of large ions or dust particles the effect of the sign of the electrification on condensation must be very small, and it has never been experimentally demonstrated; it is unlikely that it is of any importance in the atmosphere.

(ii.) *The Electrification of Drops subsequent to their Formation.*—There are several ways in which drops may conceivably acquire a charge after their original formation. It has been suggested (83) that the drops may acquire a negative charge through capturing more negative than positive ions from the surrounding air on account of the greater mobility of the negative ion. Again exposure to radiations of short wave-length (ultra-violet light or γ rays) may cause drops or ice crystals to emit negative electrons and so become positively charged (84); this effect would become most marked at great heights, where the ultra-violet light from the sun is appreciable. There is again the possibility of electrification by friction of water drops with ice crystals or of either of these with dust particles in falling through the air (85).

The intense fields of thunder-clouds arise probably only when large drops or hailstones have formed. Collisions may then occur between the larger and smaller drops on account of the different rates at which they fall relatively to the uprising air; again water drops break up when they have reached such a size (about 5 mm. in diameter) that they fall through the air with a velocity of about 8 metres per second (86).

(iii.) *Simpson's Theory.*—When pure water is allowed to splash or is shaken up with air (58), (87), the water becomes positively charged, the air negatively. Simpson showed (71) that when water drops are broken up in an air current there is electrical separation as in other cases of disruption, the resulting drops being positively electrified while the air contains an excess of negatively charged ions. It is to this positive electrification of the water and negative electrification of the air by disruption of drops, and the subsequent vertical separation of the charges through the relative motion of the suspended water drops and up-rushing air (the latter carrying with it fine cloud particles to which the negative ions attach themselves), that Simpson attributes the development of the intense fields of thunderstorms. On this view all thunder-clouds should be of negative polarity.[1]

(iv.) *Elster and Geitel's Theory.*—Elster and Geitel (88), (89) attribute the electric field of a thunder-cloud mainly to collisions of large and small drops in an already existing electric field. In a cloud in which the electric field is directed upwards, the upper half of a falling drop will be positively charged, the lower negatively. If a droplet carried up in the ascending air stream strikes the lower half of the larger drop and rebounds after making electric contact, it will carry off a negative charge, leaving the larger drop with an excess of positive electricity. The negatively charged droplet will be carried upward relatively to the positively charged drop by the upward air stream. If the electric force were directed upwards as above assumed (*i.e.* if the potential gradient were negative) collisions would lead to the smaller cloud particles becoming negatively charged, the larger drops positively: the former would be carried up relatively to the latter, and there would thus be produced in the region between them an additional field of the same sign as the original field. The process would always result in an originally existing electric field within the cloud being increased in magnitude, and (until compensating processes came into play) at a continually increasing rate. According to this view the thunder-cloud is in effect a large influence electric machine; its polarity will depend on the initial sign of the electric field. In some cases it may be the ordinary normal positive potential gradient that determines the polarity of the cloud, which will then be positive.

The question whether the necessary electrical contact between the large and small drops can occur at the moment of collision without coalescence at the same time occurring is fundamental in relation to Elster and Geitel's theory. The very striking effect of electrification in causing coalescence of water surfaces and the high contact resistance when coalescence does not occur are both well known from the experiments made by the late Lord Rayleigh (90) on jets of water. Simpson regards these as decisive against Elster and Geitel's theory. Further experimental work on the subject is required.

If coalescence of the drops occurs on collision in an electric field, there may be something of the nature of splashing if the smaller drop is not very minute; whether this results in an increase or diminution of the primary field will depend on whether the splash takes place from the under or upper surface of the combined drop (59). In the case of collisions between hailstones, coalescence will not in general take place and the influence effect will act in such a way as to increase the original electric field (89); possibly only wet hailstones would possess the necessary conductivity. Collisions between large hailstones and smaller water drops will again cause splashing from the lower half of the hailstone and thus increase an already existing electric field.

There can be little doubt that the processes suggested by Simpson and by Elster and Geitel are both effective in thunder-clouds; further work is required to determine their relative importance. Other processes, such as those mentioned at the beginning of this section, and in § (25), may be of importance in producing a primary field strong enough to make the Elster and Geitel effect, when splashing and disruption begin, comparable with the Simpson effect.

---

[1] Simpson's explanation of the electrical effects of drifting snow (§ (9) iii.) would, however, seem to imply that snow and hail clouds should be of positive polarity.

§ (27) **Some Effects of the Electrification of Drops.**—The possession of a charge and exposure to an electric field both tend to facilitate the disruption of drops and thus possibly to accelerate the development of the electric field in a thunder-cloud.

Disruption of drops by an intense electric field may be concerned in the production of "false cirrus" at the head of a cumulo-nimbus cloud (8).

The effect of a charge in promoting disruption is probably of considerable importance in the case of evaporating drops. A charged drop in an unsaturated atmosphere is essentially unstable. For the charge required to prevent a drop of given size from evaporating in an atmosphere which is just saturated (91) is identical with the maximum charge consistent with a spherical drop holding together (92). Any diminution in size (such as must result if the atmosphere around the drop is unsaturated) will cause division of the drop; each portion of the drop will again evaporate and in turn divide, the final result probably being the dissipation of the charge into its constituent ions. This process is obviously of importance in connection with the ionisation below a thunder-cloud from which rain is falling and evaporating before it reaches the ground.

C. T. W.

A. General References

F. Exner, "Über die Ursache und die Gesetze der atmosphärischen Elektrizität," *Akad. Wiss. Wien Ber.*, 1886, xciii. 222.

A. B. Chauveau, *Introduction historique et bibliographique à l'étude de l'électricité atmosphérique*, Paris, 1902; *Électricité atmosphérique*, Paris, 1922.

S. Arrhenius, *Lehrbuch der kosmischen Physik*, Leipzig, 1903.

H. Gerdien, "Atmosphärische Elektrizität," *Winkelmanns Handbuch der Physik*, 1905.

A. Gockel, *Die Luftelektrizität*, Leipzig, 1908.

H. Mache and E. v Schweidler, *Die atmosphärische Elektrizität*, Braunschweig, 1909.

C. Chree, "Atmospheric Electricity," *Encyclopædia Britannica*, 1910.

J. Loisel, *Les Orages*, Paris, 1912.

E. v. Schweidler, "Atmosphärische Elektrizität," *Encyclopädie d. mathematischen Wissenschaften*, Leipzig, 1915.

K. Köhler, *Luftelektrizität* (Sammlung Göschen), 2nd Edition, Berlin, 1921.

B. Papers to which Reference is made in the Text

1. A. Peltier, *Ann. de Chimie et de Phys.*, 1842, iv. 385.
2. De Saussure, *Voyage dans les Alpes*, 1786.
3. L. Palmieri, *Leggi ed origine dell' elettricità atmosferica*, Naples, 1882.
4. L. A. Bauer and W. F. C. Swann, *Report of Results of Atmospheric Electric Observations made aboard the Galilee* (1907–8) *and the Carnegie* (1909–16), Carnegie Institution of Washington, 1917.
5. C. T. R. Wilson, *Cambridge Phil. Soc. Proc.*, 1906, xiii. 363.
6. C. T. R. Wilson, *Roy. Soc. Proc.* A, 1908, lxxx. 537.
7. C. T. R. Wilson, *Roy. Soc. Proc.* A, 1916, xcii. 555.
8. C. T. R. Wilson, *Roy. Soc. Phil. Trans.* A, 1920, ccxxi. 73.
9. G. Beccaria, *Della elettricità terrestre atmosferica a cielo sereno*, Turin, 1775.
10. M. Moulin, *Annales de Chimie*, 1907, x. 40.
11. G. C. Simpson and C. S. Wright, *Roy. Soc. Proc.* A, 1911, lxxxv. 175.
12. C. Chree, *Roy. Soc. Phil. Trans.* A, 1915, ccxv. 233.
13. W. Müller, *Ann. d. Phys.*, 1920, lxiii. 585.
14. W. Makower, *Quart. Journ. Meteor. Soc.*, 1910, xxxvi. 121.
15. G. Le Cadet, *Étude du champ électrique de l'atmosphère*, Lyons, 1898.
16. F. Linke, *Gött. Nachr.*, 1904, iii.
17. H. Gerdien, *Gött. Nachr.*, 1904, iii. 277.
18. G. Berndt, *Meteor. Zeits.*, 1913, xlviii. 363.
19. C. T. R. Wilson, *National Antarctic Expedition* (1901–4) *Meteorology*, Part I. p. 519.
20. G. C. Simpson, *British Antarctic Expedition* (1910–13) *Meteorology*, i. 302.
21. J. Rey, *Annuaire de la Soc. Météor. de France*, 1906, liv. 41 and 186.
22. J. Rouch, *Annuaire de la Soc. Météor. de France*, 1911, lix. 117.
23. G. C. Simpson, *Roy. Soc. Phil. Trans.* A, 1905, ccv. 61.
24. A. B. Chauveau, *Étude de la variation diurne de l'électricité atmosphérique*, Paris, 1902.
25. H. Norinder, *K. Svenska Vetenkaps Akad. Handlinger*, 1917–19, lviii. No. 4.
26. G. C. Simpson, "Memoir of the Indian Meteorological Department," *Quart. Journ. Roy. Met. Soc.*, 1914, iv. 12.
27. Linss, *Zeits. f. Meteor.*, 1887, xxii. 345.
28. J. Elster and H. Geitel, *Phys. Zeits.*, 1899, i. 11 and 245.
29. H. Ebert, *Phys. Zeits.*, 1902, iii. 338.
30. G. C. Simpson, *Phil. Mag.*, 1910, xix. 715.
31. J. Zeleny, *Phil. Mag.*, 1898, xlvi. 120.
32. J. J. Thomson, *Conduction of Electricity through Gases*, 1903, p. 68.
33. E. v. Schweidler, *Akad. Wiss. Wien Ber.*, 1908, cxvii. 653.
34. W. F. C. Swann, *Terrestrial Magnetism*, 1913, xviii. 163.
35. A. A. Daunderer, *Über die in den unteren Schichten der Atmosphäre vorhandene freie elektrische Raumladung* (Munich, 1908).
36. H. Norinder, *Researches on the Height Variation of the Atmospheric Electric Potential Gradient in the Lowest Layers of the Air* (Stockholm, 1921).
37. E. Riecke, *Ann. d. Phys.*, 1903, xii. 52.
38. W. F. G. Swann, *Terrestrial Magnetism*, 1914, xix. 23 and 81.
39. H. Gerdien, *Phys. Zeits.*, 1905, vi. 800.
40. H. Ebert, *Phys. Zeits.*, 1902, ii. 662.
41. P. Langevin, *Comptes Rendus*, 1905, cxl. 232.
42. J. Elster and H. Geitel, *Phys. Zeits.*, 1901, ii. 590.
43. A. S. Eve, *Phil. Mag.*, 1911, xxi. 26.
44. H. Geitel, *Phys. Zeits.*, 1900, ii. 116.
45. C. T. R. Wilson, *Cambridge Phil. Proc.*, 1900, xi. 32, and *Roy. Soc. Proc.*, 1901, lxviii. 151.
46. E. Rutherford and H. L. Cooke, *Phys. Rev.*, 1903, xvi. 183.
47. J. C. M'Lennan and E. F. Burton, *Phys. Rev.*, 1903, xvi. 184.
48. C. S. Wright, *Phil. Mag.*, 1901, xvii. 296.
49. W. F. C. Swann, *Proc. 2nd Pan-American Scientific Congress*, 1916.
50. K. M. Downey, *Phys. Rev.*, 1920, xvi. 420.
51. A. Gockel, *Phys. Zeits.*, 1911, xii. 595.
52. V. F. Hess, *Phys. Zeits.*, 1912, xiii. 1084.
53. V. F. Hess, *Phys. Zeits.*, 1913, xiv. 610.
54. W. Kolhörster, *Phys. Zeits.*, 1913, xiv. 1066 and 1153.
55. A. Wigand, *Deutsche physik. Gesellschaft*, Feb. 1914; *Le Radium*, 1914, xi. 204.
56. G. C. Simpson, *Monthly Weather Review*, 1916, xliv. 115.
57. H. Ebert, *Phys. Zeits.*, 1904, v. 135 and 499.
58. P. Lenard, *Ann. d. Phys.*, 1892, xlvi. 584.
59. C. T. R. Wilson, *Nature*, 1903, lxviii. 102.
60. G. C. Simpson, *Nature*, 1904, lxix. 270.
61. W. F. G. Swann, *Journ. Franklin Institute*, Nov. 1919.
62. G. C. Simpson, *Phys. Zeits.*, 1904, v. 325 and 734.
63. H. Gerdien, *Phys. Zeits.*, 1905, vi. 647.
64. W. F. G. Swann, *Terrestr. Magnetism*, 1915, xx. 105.
65. E. v. Schweidler, *Ann. d. Phys.*, 1920, lxiii. 726.
66. Elster and Geitel, *Meteor. Zeits.*, 1900, xvii. 230.
67. O. W. Richardson, *Nature*, April 26, 1906.
68. W. F. G. Swann, *Monthly Weather Review* 1919, xlvii. 453.

69. Elster and Geitel, *Meteor. Zeits.*, 1888, v. 95; *Terrestr. Magnetism*, 1899, iv. 15.
70. H. Gerdien, *Phys. Zeits.*, 1903, iv. 837.
71. G. C. Simpson, *Roy. Soc. Phil. Trans.* A, 1909, ccix. 379.
72. K. W. F. Kohlrausch, *Akad. Wiss. Wien Ber.*, 1909, cxviii. 25.
73. K. Kähler, *Veröff. d. K. Preuss. Meteor. Inst.*, 1909, No. 213.
74. H. Bendorf, *Akad. Wiss. Wien Ber.*, 1910, cxix. 89.
75. A. Baldit, *Annuaire de la Soc. Météor. de France*, 1911, lix. 106.
76. G. Berndt, *Meteor. Zeits.*, 1913, xlviii. 363 (abstract).
77. J. A. M'Clelland and J. J. Nolan, *Irish Acad. Proc.*, 1912, xxix. 81, and 1912, xxx. 61.
78. F. Schindelhauer, *Phys. Zeits.*, 1913, xiv. 1292.
79. C. T. R. Wilson, *Roy. Soc. Phil. Trans.* A, 1899, cxciii. 289.
80. J. J. Thomson, *Phil. Mag.*, 1898, xlvi. 533.
81. G. C. Simpson, *Phil. Mag.*, 1909, xvii. 619.
82. C. T. R. Wilson, *Phil. Mag.*, 1909, xvii. 634.
83. A. Schmauss, *Ann. d. Physik*, 1902, ix. 224.
84. J. Rez, "Sur l'ionisation de l'air par les chutes d'eau," Paris, 1912; *Le Radium*, 1914, xi. 204 (abstract).
85. L. Sohneke, *Meteor. Zeits.*, 1888, v. 413.
86. P. Lenard, *Meteor. Zeits.*, 1904, xxi. 249.
87. Lord Kelvin, *Roy. Soc. Proc.*, 1894, lvii. 335.
88. Elster and Geitel, *Ann. d. Physik*, 1885, xxv. 116 and 121.
89. Elster and Geitel, *Phys. Zeits.*, 1913, xiv. 1287.
90. Lord Rayleigh, *Roy. Soc. Proc.*, 1882, xxxiv. 130.
91. J. J. Thomson, *Conduction of Electricity through Gases*, 1903, p. 150.
92. Lord Rayleigh, *Phil. Mag.*, 1882, xiv. 184.

ATMOSPHERIC RADIATION:

Definition of "effective" radiation. See "Radiation," § (2) (i.).

Measurement of, by thermograph and by radiometer. See *ibid.* § (2) (ii.).

Value of. See *ibid.* §§ (3) (iv.), (4) (i.).

ATMOSPHERIC REFRACTION. See "Trigonometrical Heights," § (9).

AUREOLE: the portion of the *corona* which consists of the inner ring and the bluish inner field between the ring and the luminary. See "Meteorological Optics," § (15) (i.).

AUTOMATIC DIVISION MACHINE. See "Calculating Machines," § (10).

AZIMUTH BY CIRCUMPOLAR STARS. See "Gravity Survey," § (10).

AZIMUTH, DETERMINATION OF, by observation of stars at greatest elongation east or west. See "Latitude, Longitude, and Azimuth, by Observation in the Field," § (3).

B

BABBAGE'S ANALYTICAL ENGINE. See "Calculating Machines," § (2) (iv.).

BABBAGE'S DIFFERENCING ENGINE. See "Calculating Machines," § (2) (iii.).

BALANCE, ANALYTICAL, PERFORMANCE OF A STANDARD TYPE OF, tabulated. See "Balances," § (2) (4).

BALANCE, CONDITIONS AND METHODS OF PRACTICAL USE OF. See "Balances," § (4).

BALANCE, EQUI-ARM, GENERAL THEORY OF STATIC EQUILIBRIUM OF. See "Balances," § (2).

BALANCE, ERRORS AND LIMITATIONS OF THE. See "Balances," § (3).

Causes of changes in the effective length of the arms of a balance. See *ibid.* § (3) (ii.).

Effect of buoyancy of the air on the apparent weight of a body. See *ibid.* § (3) (iii.).

Thermal effects. See *ibid.* § (3) (i.).

BALANCE, GOOD, REQUISITES OF. See "Weighing Machines," § (2).

BALANCE PANS AND SUSPENSION STIRRUPS OF EQUI-ARM BALANCE. See "Balances," § (1) (iv.).

BALANCES

THE balances considered in this article are almost exclusively of the equi-arm type, treated mainly as instruments of precision (see also "Weighing Machines"). The article also discusses the weights used with such balances, together with the methods of determination of the densities of solids, liquids, and gases.

(See § (6) for a discussion of the micro-balance.)

I. THE EQUI-ARM BALANCE

Of all the instruments used in making fundamental measurements, the familiar equi-arm balance is most productive of high accuracy. Such is the precision of this instrument that in many instances it is not used to the full limits of accuracy of which it may be made capable. It is proposed to describe briefly the main features of the balance as generally used by scientific workers, and afterwards to indicate the refinements which may be made if the utmost limit of accuracy is required of the instrument.

§ (1) DETAILS OF CONSTRUCTION.—On account of the general similarity of design of equi-arm balances, the following salient features may be taken as common to all such instruments, independent of the load for which they are used. Essentially (see *Fig.* 1) the balance is an equi-arm lever consisting of a beam, usually of metal (except in micro-balances), which turns about a horizontal central knife-edge as fulcrum, while the loads are applied at parallel horizontal knife-edges at the extremities of the beam.

Arising out of this simple lever, the following items require consideration:

(i.) The design of the beam.

(ii.) The nature of the knife-edges and their attachment to the beam.

(iii.) The suspension of the loads from the terminal knife-edges.

(iv.) The arrestment of the beam and load.

(v.) The methods of measuring the deflection of the beam, when released.

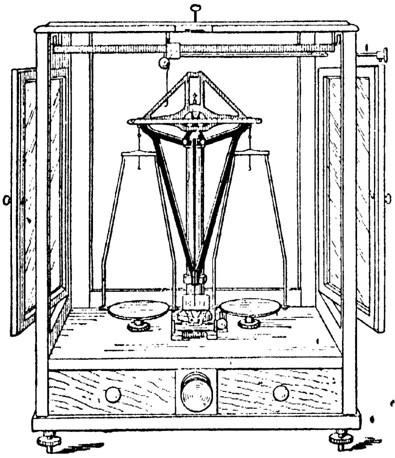


Fig. 1.—A Typical Analytical Balance.

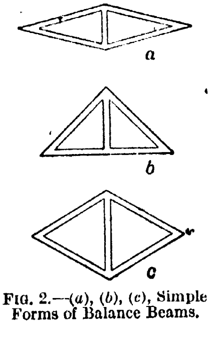


Fig. 2.—(a), (b), (c), Simple Forms of Balance Beams.

(i.) *The Design of the Beam.*—The design of the beam is the fundamental, but by no means the only, factor controlling the accuracy obtainable from the balance, and the first consideration is its shape. Since it is essential, in order to obtain high sensitiveness of swing, that the weight of the beam should be as small as possible, consistent with strength, relatively to the loads weighed from its extremities, the simple forms shown in *Fig.* 2 (*a*), (*b*), and (*c*) have been used.

They are of double triangular shape, and the equilateral form in *Fig.* 2 (*c*) is theoretically ideal for combining strength with lightness.

At first thought, it would appear that a long-beam balance would be preferable to a short-beam one on account of its greater sensitiveness; but it will be shown in § (2) that long beams are inseparably connected with relatively long periods of swing, an important item in general work, if not also in work of special precision. Moreover, given the weight of the beam, the longer its arms the greater the bending of the beam under the load to be weighed. The importance of this will be shown in § (2), when the performance of the balance is considered in connection with the relative positions of the central and terminal knife-edges. Experience has shown that for balances which are required to weigh, in air, loads less than 1 kilogramme, the sides of the triangular frame formation shown in *Fig.* 2 (*a*), (*b*) should, as a first approximation, be of the same order of magnitude, and the beam should be about 6 or 7 inches in length. Other considerations which influence the exact shape of the beam are:

(*a*) The position of the centre of mass of the beam, which in its working condition must lie very close to the central knife-edge. The beam is clearly unstable if its centre of mass lies above the fulcrum knife-edge.

(*b*) The approximate positions of the terminal knife-edges, whether outside or within the triangular framework.

(*c*) The provision of a graduated rider-bar to enable very small adjustments of load to be made with the same rider weight. *Fig.* 3 shows a form of beam which is in use both in England and in other countries.

This general shape is to be recommended as giving a satisfactory working analytical balance for chemists and physicists, in which good sensitiveness is obtained, together with a reasonably quick period of swing. The various modifications of shape of the beam employed by instrument makers can be seen from their catalogues. In some designs the problem of obtaining maximum strength consistent with lightness does not appear to have received sufficient attention.

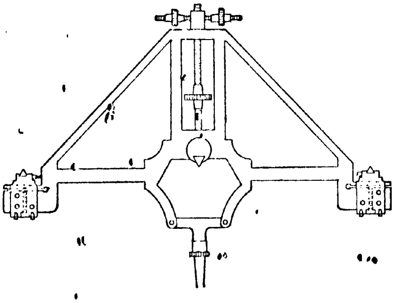


Fig. 3.—Design of Beam of Sensitive Analytical Balance.

Interesting particulars as to the amount of bending of beams of different shapes, under a given load, are shown by Felgentraeger.[1] It is estimated that the vertical depression of the terminal knife-edges for a beam like that shown in *Fig.* 3 lies between 0·01 and 0·001 mm. for the maximum loads used on ordinary analytical balances.

A shape of beam very largely found in balances which are used for loads of 1 kilogramme and upwards is shown in *Fig.* 4.

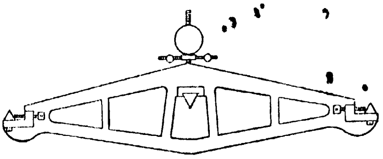


FIG. 4.—General Design of Beam for Heavy Loads.

The material of which balance beams are constructed should be of a low specific gravity, combined with high rigidity. The older balance beams were made of brass or bronze, but the production of aluminium alloys of relatively high rigidity has in recent years led to the more general use of magnalium[2] (or other light alloys) in the construction of beams. Its density is less than one-third that of brass, while there is no sacrifice of rigidity by its introduction. It is desirable that the beam should not tarnish with age. In this respect magnalium is also better than brass, but if considered necessary, the beam should be covered with a protective coating, platinising being the most stable and suitable process.[3]

It is more usual to make the beam in one piece rather than build it up from rods. Most frequently made of cast metal, the beam should be well annealed so as to minimise strains and distortion. Generally speaking, the simpler the form, the more likely the beam is to be free from strains due to variations of temperature during the use of the balance. It is now possible to cut down the weight of a magnalium beam to approximately 100 grammes, or even less, for a balance required to take loads up to 1 kilogramme. In commercial balances, where extreme sensitiveness is not required, the beams are made either of steel or of a copper alloy.

Adjustment of the Centre of Gravity of the Beam.—It need hardly be remarked that the beam should be made as truly symmetrical as possible. The approximate position of its centre of mass can be prearranged by calculation, but when the knife-edges and other attachments to the beam are fixed, it is necessary to have two fine adjustments, viz.:

(*a*) The stabiliser nut, often known as the "gravity bob." This works on a vertical screw thread situated in the plane of symmetry of the beam, and allows of sufficient adjustment of the vertical position of the centre of mass of the beam.

(*b*) The equilibrium nut (or nuts), working on a horizontal screw thread, and enabling the beam to take up a suitable position of equilibrium.

(ii.) *Knife-edges.*—The functions of the three knife-edges of a balance are important.

In the first place the knife-edges must be straight, so that the load may be applied along a straight line. They must also be hard, and not tarnish, nor collect any extraneous deposit.

These conditions are best realised, in all but the largest balances, by the crystal knife-edges made of agate or rock-crystal. Agate knife-edges with agate bearing-planes are now almost universally to be seen in chemical, analytical, and laboratory balances which withstand loads up to a few kilogrammes. Above this load, steel knife-edges have been most frequently used, generally with steel bearing-planes, but sometimes with agate planes. It is desirable in these cases to use the more non-rusting varieties of steel, but in any case, great care should be taken to preserve the cleanness of the knife-edge. The possible influence of extraneous magnetic fields on a beam containing steel knife-edges should not be overlooked when high precision is required.

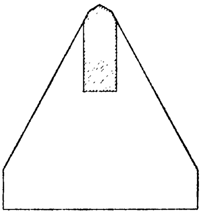


FIG. 5A.—Sectional View of Agate Knife-edge mounted in Brass Triangular Block.

The diagrams (*Figs.* 5A and 5B) show the angular shape of the agate knife-edge as employed in balances nowadays. The actual knife-edge is the intersection of two plane faces inclined at an angle of about 120° as shown.

It should be remarked at once that the knife-edge as employed in practice is only an approximation to the theoretical or mathematical "straight-line" bearing, the degree of approximation depending on the load to

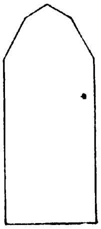


FIG. 5B.—Enlarged Sectional View of Agate Knife-edge before being mounted.

---

[1] *Theorie, Konstruktion und Gebrauch der feineren Hebelwage*, published by Von B. G. Teubner, Leipzig and Berlin.

[2] An alloy consisting approximately of 86 per cent aluminium with 13 per cent magnesium.

[3] Both gilding and lacquering are open to some objection as a protective coating.

be taken by the knife-edge. The virtual bearing part of the knife-edge is, in the limit, a very thin band of surface, the exact location of which is to a small extent variable along the knife-edge, depending on the exact manner of suspension of the load. The consequences of this departure from the "mathematical" knife-edge will be discussed under § (3).

The actual fixing of the knife-edges in the beam requires considerable care, and the best adjustment of their positions demands much patience. Strictly speaking, four conditions have to be satisfied, viz. (1) fixing the central knife-edge square with the plane of the beam; (2), (3), (4) fixing the terminal knife-edges parallel to, equidistant from, and coplanar with the central one. Accordingly, full adjustment should be provided to enable these conditions to be fulfilled.

Usually no adjustment is provided for the central knife-edge, which is dovetailed and cemented into a metal holder which is either part of the beam or else is rigidly fixed to it. The knife-edge on being cemented is set as nearly as possible at right angles to the plane of the beam. The most difficult and tedious part of the adjustment concerns the terminal knife-edges. A typical mode of attachment for English balances is shown in *Fig.* 6.

This form of attachment does not provide a slow vertical adjustment of the knife-edge.

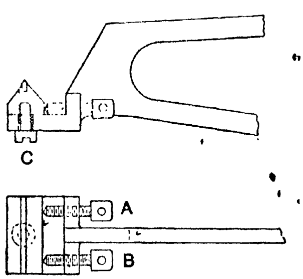


Fig. 6.—Typical Mode of Attachment of Terminal Knife-edges in English Balances.

The effective length of an arm of the balance, as well as the parallelism of the knife-edges in their horizontal plane, can be adjusted by means of the screws, A, B, bearing normally against one side of the knife-edge block, while the knife-edge is held in position by one or more screws, C, passing through a slotted hole in the beam. Equality of arms is determined by the balance itself, and is secured either by weighing two masses which are known to be identically equal, or, if equality of mass is not obtainable, by interchanging the loads on the two pans, and comparing the equilibrium points. Lack of parallelism of the knife-edges may be evident in two ways, either in a horizontal or in a vertical direction, and can be tested by weighing with the load on one arm suspended so as to bear only on one part of the knife-edge. A mass is thus suspended from one end (say the front end) of a terminal knife-edge and counterpoised by a mass suspended from the other arm of the balance. By next suspending the first mass from the other end of the knife-edge, and counterpoising against the mass suspended from the other arm, the lack of parallelism of the knife-edge in the horizontal direction can be detected. Parallelism in the other direction can be tested by noticing whether there is a change of sensitiveness of the balance when the bearing of the suspension is transferred from the front to the back portion of the knife-edge. The effect on the sensitiveness of the balance due to the three knife-edges not being exactly coplanar will be seen in § (2).

An alternative form of attachment of the terminal knife-edges is used on balances made by Sartorius, of Göttingen, and is shown in *Fig.* 7.

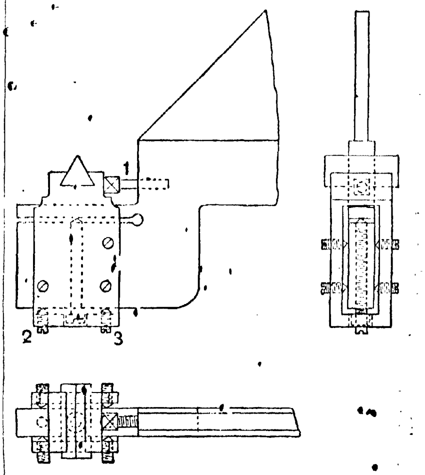


Fig. 7.—Form of Attachment of Terminal Knife-edges made by Sartorius, of Göttingen.

It permits of full rotary and translatory adjustment of the terminal knife-edges of the balance whenever required in the laboratory. The agate knife-edge itself is fixed in a metal casing which can be set in correct position around the end of the beam by means of a number of screws. The scope of the adjustment can be seen from the diagram. In effect the various screws provide both a fine and an extra-fine slow motion. For example, the screw 1 gives a slow-motion adjustment for the length of the arm, while the screws, 2 and 3, can be utilised to give an extra-fine slow motion.

For work of the highest precision with the balance, the provision of these fine adjustments in position of the terminal knife-edges is certainly appreciated. In many balances made for low precision work, all three knife-edges are dovetailed or otherwise fixed to the beam, without means of readjustment.

(iii.) *Balance Pans and Suspension Stirrups.*—If the balance is required to give the highest accuracy, special consideration should be given to the means of suspension of the load. Since each terminal knife-edge is not a mathematically sharp edge, but a blunt or rounded edge in the limit, it is clear that the length of the arm will not be consistent, beyond certain limits, unless the load is applied at the knife-edge in a precisely consistent manner.

In the more familiar types of analytical balance used in this and other countries, the agate plane which bears on a terminal knife-edge is mounted in a metal stirrup which has at its lower end a hook or an eye-hole from which the balance pan hangs. The pivoting of the balance pans is necessary as a preliminary step to secure that the apparent weight exerted by the load shall be independent of its exact position on the pan. If this were not so, the agate plane would not remain horizontally on its knife-edge.

Owing, however, to the finite size of the pivoting, the load is not transmitted through the pivot in exactly the same way each time, and, as a result, the agate plane may rest in a slightly inclined position, which for sake of illustration is exaggerated in the diagram (*Fig.* 8), where W represents the total load on a terminal knife-edge, *i.e.* the resultant weight of the stirrup, the pan, and the load on the pan; R is the normal reaction between the knife-edge and the agate plane; μR is the frictional force between knife-edge and plane.

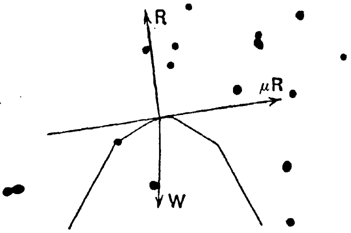


FIG. 8.—Showing the Forces acting at a Terminal Knife-edge when the Agate Bearing-plane is slightly inclined to the Horizontal.

Balances which have their pans suspended by the usual kind of hook-and-eye pivot may be relied upon to maintain a consistency of length of effective arm to within one part in a few million, if other sources of error are absent, provided that the loads are placed reasonably centrally on the pans. For work of the highest precision, increased accuracy may be obtained by inserting, between the stirrup and the pan, either a pair of knife-edges placed transversely or, in the case of small loads, a hollow cone resting on a conical point.

This idea is by no means new, and has been employed in national laboratories more particularly for the accurate maintenance of standards of mass. The platform of a balance pan should be sufficiently stout not to yield appreciably under the load applied.

(iv.) *The Arrestment of the Beam and Pans and Suspension Stirrups.*—In the average analytical balance, three parts require arrestment. The central knife-edge of the beam should be raised a short distance out of contact with its bearing plane, and it is desirable to raise the suspension stirrups a little in order to remove the load on the terminal knife-edges. Further, it is usual to provide an arrestment for the pans so as to facilitate loading and unloading, and to steady them, lest by swinging they should interfere with the oscillations of the beam when released.

The arrestment should enable the beam and stirrups to be raised so that the respective knife-edges are just clear of their agate bearing-planes, the margin of clearance being uniform throughout the length of a knife-edge. (In practice $\frac{1}{10}$ to $\frac{3}{10}$ mm. would be a suitable clearance for the average balance. The amount of clearance at the central knife-edge should be somewhat larger than this.) It is important that the arrestment and release of the beam, etc., should be arranged to be made precisely and consistently each time.

In the first place, the beam and the suspension stirrups should both be fixed definitely in position when arrested. This is well done by the "hole, slot and plane" method of one maker. If an arrangement of this nature is properly adjusted so that the margins of clearance at the knife-edges are uniform when the balance is arrested, and the suspension stirrups ride symmetrically on their respective knife-edges when the balance is released, the chances of inconsistent length of arm due to tilting of the suspension stirrups ought to be minimised as far as the design of the arrestment is concerned.

It should be noted that the majority of English balances do not allow of absolute fixing of the beam and stirrups in arrestment, but provide only "two-point" support, instead of the "hole, slot and plane" support. This means that each item arrested is allowed a certain amount of rotation about the line of the two-point support, with the result that the beam when arrested may take up a position with one of its terminal knife-edges in contact with an agate plane.

The mechanical gear for operating the arrestment is sufficiently familiar to users of the balance to render detailed description unnecessary. A cam operated by a milled wheel outside the balance case raises or lowers a vertical frame which carries the arresting stops or points. The same motion raises or lowers pan-supports, which are in good adjustment when they just touch the under surface of the pans as the arrestment is completed. Sartorius has improved on this simple form of arrestment. Users of his balances will be familiar with the manner in which he has obtained increased consistency and precision of arrestment.

(v.) *Methods of determining the Deflection of the Beam.*—Four methods may be employed to measure the amount of deflection of the beam of a balance. These make use of—

(*a*) The familiar balance pointer, moving over a suitable scale.

(*b*) A microscope or telescope.

(*c*) An optical lever.

(*d*) A combination of mechanical and optical magnification.

(*a*) *The Pointer Method.*—This is the usual method of determining the equilibrium position of a balance when more than ordinary precision is not required. Its operation is too familiar to need description, and it is sufficient to say that by this method the deflection of the beam may be measured with an accuracy of 1′ or 2′ (minutes of arc) without using optical means.

The pointer scale is usually a fixture, and an adjustment is made by means of a small nut on the beam in order that the rest point of the balance, at zero load, should be at the centre of the scale.

M'Dowall[1] has suggested making the pointer scale adjustable to the rest position of the pointer. This may be arranged, if required, so that the adjustment can be made without having to open the balance case.

(*b*) *By Microscope or Telescope* (*without other optical means of magnification*).—In this case the microscope or telescope is focussed on a small scale rigidly attached to the ordinary pointer of the balance, and can be used to read the deflection of the beam to approximately 10 seconds of arc, *i.e.* with a precision of about ten times that in case (*a*).

A telescope is preferable to a microscope, in general, because wherever the precision required is sufficient to justify the use of optical means, it is desirable to read the balance at a distance in order to avoid temperature disturbances due to the nearness of the operator to the balance.

(*c*) *By Optical Lever.*—This method is recommended when balance work of the highest precision is to be done. It is specially suited to the accurate standardisation of weights, and is employed for this purpose at the National Physical Laboratory.

A thin vertical wire, W, is placed in a beam of light from a lantern, L (*Fig.* 9), containing a Pointolite lamp (100 volts 100 watts), and the rays, directed towards the balance, pass through a reflecting prism, P, on to a horizontal plano-concave lens, M, rigidly fixed to the beam of the balance with its concave surface upwards. The plane surface of this lens is silvered so that the beam of light after reflection at this mirror is returned through the reflecting prism and given an exit through the front of the balance, with the result that an image of the wire can be formed at any suitable position, depending chiefly on the focal length of the lens M.

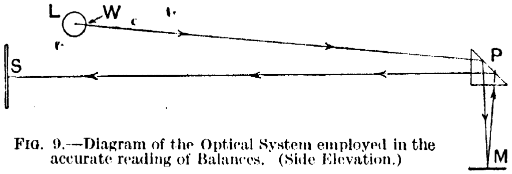


FIG. 9.—Diagram of the Optical System employed in the accurate reading of Balances. (Side Elevation.)

By arranging the focal length of M to be equal to the optical path of the beam of light from W to the lens, the image can be obtained in the same vertical plane as the object-wire W. At the National Physical Laboratory, where a number of balances are fitted with this kind of optical lever as indicator, the object-wire, W, and its image, S, are arranged to be at a distance of about 5½ metres (*i.e.* the length of the room) from the balance. The image S is brought to a focus on a white cardboard scale subdivided in intervals of 2 mm., which gives a convenient spacing for estimating the position of the image with a precision of ¼ mm., corresponding to an angular deflection of the beam equal to 4 seconds. This amount of magnification enables the smaller balances (say, those with load not exceeding a kilogramme) to be read with a precision equivalent to a few millionths of a gramme, which exceeds the accuracy generally obtained with a balance. Consequently this optical method of reading deflections is specially suitable to the examination of the errors inherent in balances.

It may be remarked on referring to *Fig.* 9 that, as an alternative, the lens M may be replaced by a small plane mirror attached to the beam, together with a transparent convex lens of practically the same focal length as before, mounted in front of the reflecting prism. Such a course is preferable in the case of a small assay balance, as it is difficult to obtain a small silvered lens giving sufficiently accurate focal length and good definition, whereas it is easy to obtain a small and very light optically-plane mirror.

---

[1] *Chemical News*, 1906, xciv. 104.

If considered desirable, the lens or mirror at M may be pivoted so as to facilitate adjustment of the position of the image on the scale, but the adjustment may equally well be obtained by providing the reflecting prism with the requisite slow rotations.

In any case it is essential that the components of the optical system employed should be made as perfect as possible if a well-defined image is to be produced at a distance of six yards from the balance. Attention should, therefore, be given to the lens, mirror, reflecting prism, and any plate glass through which the beam of light has to pass, so that their respective surfaces should be finished and polished to a high degree of accuracy.

Note.—Another modification of the method of reading angular deflections by optical lever makes use of a telescope in conjunction with a scale (as object) and a plane mirror which is fixed to the balance-beam and interposed in the optical path from the object scale to the telescope. For comfort in reading, the method illustrated by *Fig.* 9 is preferable, while it also yields ample sensitiveness.

(*d*) *By Combined Optical and Mechanical Magnification* (*special case*).—The following method is applicable in a limited number of special cases only. It deserves to be more generally known and applied for the detection of small motions; and is particularly interesting as it was used by the late Professor Poynting[1] in his classic experiments on the "Determination of the Mean Density of the Earth and the Gravitation Constant." With a load as big as 20 kilogrammes on each arm of his balance, Poynting was able to measure small changes in weight of the order $\frac{1}{3}$ milligramme to within an accuracy of $\pm 0{\cdot}006$ mg., *i.e.* to within one part in three billion of the load.

His method of reading the deflection of the balance beam was to use a double suspension mirror, M (*Fig.* 10), as an optical lever in conjunction with the ordinary balance pointer, P. The mirror was suspended by two threads, one from a fixed point, the other from a point which moved with the swing of the balance pointer. The general arrangement is shown [illegible] *Fig.* 10, but reference should be made to [illegible]ting's[2] account for details of his arrange[illegible] The sensitiveness of this method was so fine that an angular motion of $\frac{1}{100}$ second of arc could be reliably measured.

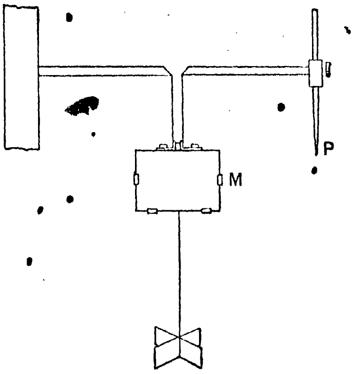


FIG. 10.—Diagram illustrating Poynting's Method of measuring Small Changes in Weight. (Side Elevation.)

Note.—In order to obtain this extreme accuracy, it was necessary that the balance beam should *not* be arrested during the course of a set of readings. In other words, a massive balance, resting in equilibrium on its central knife-edge, was used to measure a small change in weight with great accuracy.

§ (2) GENERAL THEORY OF STATIC EQUILIBRIUM OF THE EQUI-ARM BALANCE.—It is not the intention here to present a detailed theoretical investigation of the performance of a balance which would take into consideration the errors due to imperfect positioning of the three knife-edges. Within the limits of this article full justice could not be done to this subject. It is proposed to give a comparatively simple fundamental analysis of the performance of the balance, and then to consider in a general way the effect of such errors of workmanship or adjustment of the balance as are not covered by the simple theory. This manner of treatment of the subject would cover all ordinary requirements of the balance and also suffice for work of high precision, but in cases where extreme precision is required (as in the maintenance of the national standards of mass) reference should be made to the excellent detailed contribution by Thiesen,[3] entitled "Études sur la balance."

Simple Theory.—The following assumptions will be made in this investigation:

(1) That the three knife-edges are all horizontal, and parallel to one another.

(2) That the effective lengths of the arms of the balance are equal and invariable.

---

[1] *Phil. Trans.*, 1891, clxxxii. 565.

[2] *Loc. cit.*

[3] See *Travaux et Mémoires, Bureau International*, 1886, tome v.

In the diagram (*Fig.* 11) let

D denote the fulcrum knife-edge.
A, B denote the terminal knife-edges.
C denote the mid-point of AB.
S denote the centre of gravity of the beam alone.

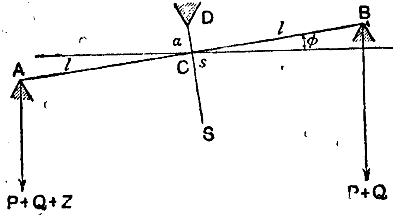


FIG. 11.—Diagram illustrating the Theory of the Balance.

(It is here assumed that DCS is perpendicular to AB. This is done for simplicity, since in practice the distances CD and SD are very small compared with AB.)

Let

l = AC = CB = the length of each arm of the balance.
CD = a.
SD = s.
ϕ = angle of inclination of the beam to the horizontal.
G = mass of beam alone.
Q = mass of each pan with suspension stirrup (supposed the same for left and right pans).
P = load in right pan.
P + z = load in left pan.
g = the acceleration due to gravity.

By taking moments of the vertical forces about the fulcrum D,

$$\mathrm{G}\,.\,s\,.\,\sin\phi+(\mathrm{P}+\mathrm{Q})(l\cos\phi+a\sin\phi)$$
$$=(\mathrm{P}+\mathrm{Q}+z)(l\cos\phi-a\sin\phi),$$

$$\therefore\ \sin\phi[\mathrm{G}s+2a(\mathrm{P}+\mathrm{Q})+za]=l\,.\,z\,.\cos\phi,$$

$$\therefore\ \tan\phi=\frac{lz}{\mathrm{G}s+a[2(\mathrm{P}+\mathrm{Q})+z]}.$$

To within the accuracy with which the sensitiveness of a balance is measured, there is no difference between tan ϕ and ϕ, while z is very small compared with $(\mathrm{P}+\mathrm{Q})$, and may be omitted from the denominator.

The sensitiveness of a balance is defined as the rate of increase of the deflection of the beam for a given additional load on one pan, *i.e.* it is equal to $\partial\phi/\partial z$

$$\text{or sensitiveness}=\frac{l}{\mathrm{G}s+a[2(\mathrm{P}+\mathrm{Q})]}.\qquad(1)$$

If GK² is the moment of inertia of the beam alone about the fulcrum D, K being the radius of gyration, the equation of motion of the whole moving system, when balanced, may be written

$$[2(\mathrm{P}+\mathrm{Q})(a^2+l^2)+\mathrm{GK}^2]\times\frac{\partial^2\phi}{\partial t^2}+f\left(\frac{\partial\phi}{\partial t}\right)$$
$$+g[\mathrm{G}s+2a(\mathrm{P}+\mathrm{Q})]\times\phi=0,$$

where the coefficient of $\partial^2\phi/\partial t^2$ is the moment of inertia of the whole system about the fulcrum, and the expression $f(\partial\phi/\partial t)$ represents the resistance of the air to the motion of the beam. Hence the period t of swing of the balance is

$$2\pi\sqrt{\frac{2(\mathrm{P}+\mathrm{Q})(a^2+l^2)+\mathrm{GK}^2}{g\{\mathrm{G}s+2a(\mathrm{P}+\mathrm{Q})\}}},\qquad(2)$$

that is

$$\frac{2\pi}{\sqrt{lg}}\times\sqrt{\text{Sensitiveness}\times\text{moment of inertia of the moving system}},$$

$$\therefore\ \frac{t^2}{4}=\frac{\text{Sensitiveness}\times\text{moment of inertia of the whole moving system}}{\text{Length of arm of balance}\times\text{length of simple pendulum which beats true seconds}}\qquad(3)$$

The performance of a balance may therefore be summed up as follows, with the help of equations (1) and (2):

(*a*) If the three knife-edges are coplanar under all conditions of loading of the balance, the sensitiveness is independent of the load. This condition is not fully realised in practice owing to the bending of the beam under load.

(*b*) For a beam with its three knife-edges exactly coplanar so that a is zero, increased sensitiveness is obtained by reducing G and s, *i.e.*:

(i.) By making the beam as light as possible consistent with strength, and

(ii.) By diminishing the distance of the centre of mass of the beam from the fulcrum knife-edge.

(Instability of the moving system is, of course, to be expected when the centre of mass lies above the fulcrum.)

(*c*) If the fulcrum knife-edge lies just below the plane through the terminal knife-edges, so that a is negative, the denominator in the expression to the right of equation (1) can be made smaller than it would be if the three knife-edges were coplanar.

Hence increased sensitiveness may be obtained in this way, but greater variation with load is found as compared with the beam which has its knife-edges lying in the same plane.

The increase (or decrease) of sensitiveness with increase of load may be used as a criterion of the position of the central knife-edge with respect to the plane through the terminal ones.

The question as to whether a long beam is preferable to a short one is a compromise between the following considerations.

(i.) For a balance designed to take a given load, an increase in the length of the beam is inevitably accompanied by an increase in its mass. Much depends on the proportionate increase of l and G.

(ii.) For a beam of given mass, the stiffness is considerably diminished by increasing its length. Consequently the ideal condition of obtaining coplanar knife-edges for all loads is less likely to be satisfied by a long beam.

(iii.) Even if the mass of the beam is kept constant, an increase in the length is accompanied by an increase in the moment of inertia of the moving system (*i.e.* of beam and load), and, therefore, by an increased period of swing.

(*Note.*—In the final choice of length of beam, other questions are involved, such as the degree of invariability of the effective length of the balance arm, and the uneven distribution of temperature along the beam.)

Nearly all balances are provided with a stabiliser nut or adjustment for raising or lowering the centre of gravity of the beam. For a given balance beam, increased sensitiveness is only obtained at the expense of increased period of swing. (At the same time a good compromise may be obtained by keeping a relatively quick period of swing and reading the deflection of the beam with greater precision, say, by optical methods.)

The following table indicates the performance of a standard type of sensitive analytical balance by a well-known English maker:

Position of Stabiliser Nut.	Load on each Pan.	Criterion of Sensitiveness of Balance. (Value of 1 Division of White Scale within Balance Case.)	Period of Complete (to and fro) Swing of Balance.
	grm.	grm.	sec.
Low (most stable)	0	0·0008	13
	100	0·0008	21
	200	0·0008	28
Mean	0	0·0005	17
	100	0·0005	28
	200	0·0005	36
High (least stable)	0	0·0001	33
	100	0·0001	53
	200	0·0001	63

Note.—(1) The value given in the third column of the above table is the mass required to be added to one pan of the balance in order to change the rest point by one division of the scale. The smaller the value given, the more sensitive the balance.

(2) The scale referred to above was one of the usual millimetre scales read by a pointer about 10½ in. long. One division thus corresponds to an angular deflection of the beam equal to about $\frac{1}{5}$°.

The above particulars refer to a balance of the type shown in *Fig.* 1 with a beam of light aluminium alloy weighing about 50 grammes. Calculations made from the observed values of the sensitiveness and period of the balance under various loads show that the distance of the fulcrum knife-edge from the plane through the terminal knife-edges is about 0·0002 inch, while the distance of the centre of mass of the moving system below the fulcrum varied from 0·001 to 0·01 inch according as the high or low position of the stabiliser nut was used.

The period of the balance should be regulated, in consideration of the above table, according to the nature of the weighings to be made, and the accuracy required.

Where work of the highest precision is not essential, it is an obvious advantage to use the balance in its condition of quickest period. A to and fro period of 13 seconds is sufficiently long to enable observations to be taken comfortably. The least stable position is not very useful as, owing to great sensitiveness, balance cannot be obtained unless the loads to be compared have very nearly the same weight. A difference in load of 1 milligramme is sufficient in this case to send the pointer off the scale.

§ (3) Errors and Limitations of the Balance.—The high magnification obtainable by present-day optical methods of reading the deflection of the balance beam makes imperfections of the balance all the more prominent; but with the ordinary method of reading by means of a pointer attached to the beam, it is noticeable at times that irregular changes occur in the rest point of the balance even when unloaded. A number of causes, all contributing to a greater or less extent in limiting the performance of a balance, will now be considered.

In the first place it will be assumed that the balance in question is of normal type, say, an analytical balance, without any optical refinements for work of highest precision. Such a balance can readily be obtained with a sensitiveness of ½ or 1 milligramme per division, *i.e.* a change of load of ½ or 1 mg. in one pan causes a deflection of 1 division (usually 1 millimetre) of the scale over which the pointer swings. The user of this balance will be able to read the deflection of the pointer with a precision of $\frac{1}{10}$ division corresponding to $\frac{1}{10}$ mg., or at best $\frac{1}{20}$ mg.

In general this precision in reading the balance will represent the accuracy of weighing, if

(1) The usual buoyancy allowances are correctly applied.

(2) The weights used in the process of weighing are accurate, or their errors from nominal values correctly allowed for (see Part II. § (7) for further reference to standards of mass), provided that due care is taken in checking the rest point of the balance from time to time. This raises a question which

has concerned chemists and analysts for some time.

The rest point (*i.e.* the equilibrium position) of an unloaded balance varies somewhat from time to time, so that during a series of weighings by counterpoise method, *i.e.* with a constant mass on one pan, discrepancies of the order ½ to 1 mg. would occur in the weighings unless the rest point were checked corresponding to known conditions.

These discrepancies are, partly, of a regular nature, though unexplained changes seem also to have troubled balance-workers.[1]

Whether regular or irregular, the errors may be classified under two main headings:

(*a*) Changes in effective length of the arms of the balance brought about by

(1) Slightly different position of contact between a terminal knife-edge and the suspension stirrup.

(2) Thermal expansion of the arm.

(3) Bending of the arm with change of load.

(4) Other small movements in the knife-edge holders.

(*b*) Changes in the atmospheric conditions within the balance case, as affecting the buoyancy of the parts of the balance, more especially the load. These changes are chiefly thermal, but may be influenced by changes in atmospheric pressure and humidity within the balance case.

(i.) *Thermal Effects.*—Consider first the thermal effects on the beam and pans. Unless the distribution of temperature within the balance case is absolutely uniform, the arms of the balance beam, though assumed perfectly symmetrical, will vary in length in a manner which renders exact allowance for it troublesome. A straightforward calculation shows that the total upward force on an arm of the beam due to atmospheric buoyancy is approximately 0·006 gramme for an average sensitive analytical balance, while buoyant force on a suspension pan, including stirrup but no load, is also about 0·006 gramme.

A change in temperature of $\frac{1}{10}$° C. corresponds to a change in air density of 1 part in 3000, if pressure and humidity are constant, and since differential variations in temperature between the left and right arms or pans of the balance do not, in general, exceed one or two tenths of a degree (Centigrade), any change in rest point of the balance due to atmospheric buoyancy must be due to differential buoyant forces on the load only, and these would in the ordinary way be allowed for by making the usual buoyancy correction.

There still remains to be considered the thermal expansivity of the beam. If the material of the beam is of light alloy, *e.g.* magnalium, a coefficient of linear expansion 0·000024 per 1° C. may be assumed.

In weighing a mass approximately equal to 100 grammes, a difference of $\frac{1}{100}$° C. between the temperatures of the arms would correspond to a change of 0·000024 gramme in the apparent weight. This degree of uniformity of temperature is not readily attainable.

It is difficult to measure temperature within a balance case to within ±0°·02 C. with the best mercury thermometers. If the temperature distribution around the beam is steady, the error can be eliminated by double weighing. Very often there is a small but persistent gradient of temperature parallel to the beam, but, in addition, there are fluctuating changes of the order 0°·02 C., which, however, may be considerably diminished by screening the beam from the pans by means of a horizontal partition which divides the balance case into two chambers:

(*a*) The upper one containing the beam: this is kept undisturbed as far as possible from external influences.

(*b*) The lower chamber, containing the suspension pans and load: the temperature distribution within this part of the balance case is more susceptible to fluctuations owing to the practical necessity of opening the case in order to change the load.

Manley[2] has investigated this matter in some detail with the aid of a sensitive bolometer for measuring small temperature changes within the balance case. Particulars are given by him as to the relative amounts of temperature fluctuation in balances with and without protected beams.

There is one other thermal effect which deserves attention. It appears in the existence of a temperature coefficient to the rest point of a balance at a given load. This has probably been noticed by regular workers with the balance, but its explanation is to some extent elusive. The amount of the thermal change per 1° C. is approximately of the order 1 to 10 parts in one million of the load applied, according to the balance. As far as can be judged from the behaviour of several makers' balances, all seem to have an appreciable coefficient, which is not necessarily permanently constant.

Manley[3] has investigated the thermal behaviour of a number of balances, and asserts that the change in rest point due to a uniform change in temperature of the balance cannot be traced to

(*a*) slight differences in the flexures of the two arms of the balance as the temperature varies, nor to

(*b*) different coefficients of thermal expansion of the two arms,

but attributes the phenomenon to small irregularities in the movements of the several groups of screws together with their associated knife-edge blocks.

---

[1] *Chem. Soc. J. Trans.*, 1917, cxi. 1035-1039.

[2] *Phil. Trans.*, 1910, ccx. 387.

[3] *Roy. Soc. Proc.*, 1912, lxxxvi. 591.

It is indeed advisable, owing to inconsistencies in thermal behaviour, that the beams of new balances should be suitably aged, like other precision instruments, before being relied upon to give trustworthy values of the highest order of accuracy.

(ii.) *Further Causes of Changes in the Effective Length of the Arms of a Balance.*—When it is considered that a standard pattern sensitive analytical balance is capable of weighing 100 grammes with a precision of $\frac{1}{10}$ mg., *i.e.* to an accuracy of 1 part in one million, it is readily realised that the relative lengths of the arms of the beam are consistent to this degree of refinement, *i.e.* to within three-millionths of an inch on a 3-inch arm.

As remarked under § (1), the virtual bearing part of a "knife-edge" is not a mathematical line, but a band of finite width the evenness of which depends on the accuracy with which the knife-edge has been ground and polished. In a badly made knife-edge the surface of contact may be irregular instead of being a straight band, and the effective length of the balance arm may be unreliable in consequence.

It is therefore an additional security for high precision weighing if the stirrup can be designed so as to give absolute freedom of suspension of the load at each terminal knife-edge. This has been referred to in § (1). In order to secure sufficient consistency of length of arm for accurate weighing, it is essential that the agate bearing-plane should not vary appreciably in angular position. It is therefore very important that careful attention should be paid to the design of the arrestment, particularly at the terminal knife-edges.

One other source of possible inconsistency of arm is clearly due to the presence of dust or dirt at a knife-edge bearing. This may cause errors of weighing with the best knife-edges, especially if they are not set accurately parallel to the fulcrum knife-edge.

The possibility of unequal bending of the two arms of the beam under load has often been suggested as a cause of error in precision weighing, but in a well-designed balance the total bending of an arm for the maximum load is very small, *e.g.* of the order 0·0002 inch. This amount is too small to produce a considerable shortening of the arm in consequence.

An estimate of the amount of bending can be made by means of the formula (2) in § (2); for if the period of the balance is observed corresponding to several different loads, the vertical distance of the fulcrum knife-edge from the plane through the terminal knife-edges can be calculated from a knowledge of the mass and length of the beam, and the mass of the suspension stirrup and pan.

(iii.) *Effect of Buoyancy of the Air on the Apparent Weight of a Body.*—The necessity for making due allowance for the upward buoyant force of the atmosphere on the body to be weighed is too well known to require special emphasis here; but reference may be made to cases where high accuracy in the application of the buoyancy correction is required. The upward buoyant force on a body is equal to the weight of air displaced by it, *i.e.* equal to its volume × the atmospheric density. Whenever desired, the volume can generally be obtained with sufficient accuracy, but the evaluation of atmospheric density, unless there are experimental means of determining it, requires a knowledge of the three items:

(1) Temperature of the air within the balance case.

(2) Pressure of the air within the balance case.

(3) Humidity of the air within the balance case.

It is usual, in many operations, to assume a specific value of the humidity of the air, and 66⅔ per cent is accepted in some branches of work as a good average value. Table IV., § (19), of air densities, at the end of the article, is based on this assumed value of the humidity. It should be noticed that, given the temperature and pressure, the densities of perfectly dry air and of air saturated with water vapour differ by about 1 part in 200 at ordinary temperatures. In cases where a proportionate accuracy exceeding 1 part in 400 is aimed at, it is desirable to determine the humidity of the air within the balance case. Unless the hygrometer used for this purpose admits of being used within the balance case, it is advisable to remove any drying agent that may be kept inside the case, and measure the humidity outside, assuming that equalisation of humidity has been effected within and without.

§ (4) Conditions and Methods of Use of the Balance in Practice.—It is not intended here to give more than general indications of the conditions and methods attaching to the use of the balance in practice.

From the chemist's point of view an excellent account has been given by Rae [1] and Reilly, with mention of the more usual precautions to be taken in chemical weighing.

(i.) *Setting up a Balance.*—The desirable conditions to be satisfied in choosing a site for a balance are a very steady support in a room with a very steady temperature. The disturbing effects of temperature are probably the causes of most of the difficulties in accurate weighing. When the balance is first set up, assuming that its component parts, and especially the arrestment, appear to be in good working order, the operator should test its performance thoroughly before putting it into general use. An intimate knowledge of the behaviour of the balance is desirable, with a view not only to anticipating or forecasting the errors of weighing, but to adapting the methods of weighing so as best to suit the circumstances.

---

[1] *Chemical News*, 1916, cxiv. 187-189, 200-202.

In order to obtain the highest precision from a given balance it is suggested that the following points should be examined in the initial tests:

(*a*) Sensitiveness and period of the balance for different loads and different positions of the stabiliser nut (gravity bob).

The position chosen for the gravity bob in a particular set of weighings should be that which gives a sensitiveness compatible with the accuracy sought for, together with a suitably short period of swing. Very often it is desirable to sacrifice sensitiveness for the sake of obtaining a quick swing, but the methods of reading may be improved if increased sensitiveness is still required.

It is often useful to be able to adjust the position of the gravity bob quickly and accurately so as to change from one to another predetermined condition of sensitiveness.

(*b*) The degree of equality of the arms of the balance.

(*c*) The consistency of the rest point. This should be tested at several loads (especially at the maximum load taken by the balance) over a considerable period of time. It may happen, more particularly at the maximum load, that the beam when allowed to swing takes a little time to accommodate itself to the strain due to the load. In addition it is often useful to determine initially the effect of temperature on the rest point. This again will vary with the load.

(*d*) The degree of freedom of suspension of the load. While exact centring of the load on the balance pan should not be necessary for general work with balances in which the pan is suspended by the usual "hook and eye" joint, it is well to verify in the first place what errors, if any, result from purposely placing the load out of centre on the pan. This should show up bad workmanship either in the accurate finishing of the knife-edges or in faulty positioning of them. If specially required, the parallelism of the knife-edges may be separately tested as indicated in § (1).

(*e*) Accuracy in the use of the rider. It is here assumed that the graduated bar which holds the rider weight when in use is arranged to be in the plane through the terminal knife-edges. This is not always the case, but unless it is so, the apparent weight of the rider when placed on the rider bar will depend on the inclination of the beam to the horizontal.

The accuracy of graduation of the rider bar should also be tested, especially if it is intended to use relatively large rider weights.

A comparatively recent innovation, which avoids the use of a rider and of fractional weights smaller than 50 mg., has been made by Messrs. Christian Becker & Co., New York, in their chainomatic type of balance. This instrument is of the usual construction of knife-edge balances, with the exception that changes of weight less than 50 mg. are measured by means of a gold chain, with very fine links, hanging from the beam. The change of weight is obtained by varying the effective length of the chain, which hangs in the form of a catenary, with its lower end attached to a vernier, which may be racked upwards or downwards against a scale graduated in milligrammes on a fixed post independent of the balance beam. The position of the vernier (and hence the effective length of the chain) can be adjusted from outside the balance case while the beam is swinging and the case closed. A number of chainomatic balances are in use, particularly in America. For further details reference should be made to Messrs. Becker & Co.'s catalogues.

(ii.) *Methods of Weighing.*—In all work with knife-edge balances, it will be found that the instrument is relatively slow to use. In general, the practice of weighing by the "null" method is followed, *i.e.* the load on one arm of the beam is adjusted so as to obtain zero deflection of the beam, which is usually taken as corresponding to the position of the pointer opposite the central line of the scale. The realisation of the equilibrium position is, of course, facilitated by a prior knowledge of the sensitiveness of the balance in terms of the value of one division of the scale.

Alternatively, if the operator has obtained balance with the pointer reading a few divisions away from the centre of the scale, he may calculate from the sensitiveness of the balance the extra loading on one pan necessary to make the pointer read zero.

This is really a variant in the method of reading the deflection of the beam with the pointer; but apart from this, the following four methods may be used in weighing, according to the precision and other conditions defined by the work on the balance:

(*a*) Single weighing.
(*b*) Counterpoise weighing.
(*c*) Double weighing.
(*d*) "Double-double" weighing.

(*a*) The term "single weighing" implies that the body to be weighed is suspended from one arm and balanced against known weights operating on the other arm of the instrument, and clearly makes an assumption that the arms are equal.

This assumption can always be tested easily by making a "double weighing," *i.e.* by seeing how the weight of the body varies according to the arm from which it is weighed.

It has been found, in general, that the effective arms of a good analytical or chemical balance are usually equal to within 1 part in 100,000, which means that the method of single weighing may be used to give very fair accuracy, other conditions being favourable. It is often sufficient to meet the requirements of certain experimental work where only relative measurements are desired. Analytical and chemical weighings are usually relative only.

(*b*) In cases where it is considered that the equality of the arms cannot be assumed, a choice lies between "counterpoise weighing" and "double weighing." The former is well suited to many kinds of experimental work and is quicker than the latter.

In counterpoise weighing, a constant mass is kept on one pan of the balance, and known weights are used on the other pan in addition to that which has to be determined. Weighing is thus made by substitution, and is independent of the length of the arm of the balance.

Wherever necessary, allowance should be made for the buoyancy of the loads, including that of the counterpoise. Sometimes in accurate work, extending over a period of time too long for constancy of rest point to be obtained, a correction for the temperature coefficient of the balance is needed.

(*c*) In "double weighing," the ordinary single weighing is repeated with the loads interchanged in the pans, the object being to minimise the combined errors due to inequality of arms of the balance beam and inequality of temperature distribution within the balance case.

(*d*) If there is a progressive change of rest point of the balance due either to a gradual increase in temperature or to other causes happening between the two parts of a "double weighing," it is clear that a certain inaccuracy will result, since the two weighings are not symmetrical in point of time with respect to the progressive changes in temperature, etc. Hence in the most accurate work, such as the comparison of standards of mass, the "double weighing" is repeated in the reverse order according to the following scheme, where A and B are the loads compared:

Order of Weighing.	Load on Left Pan.	Load on Right Pan.
1	A	B
2	B	A
3	B	A
4	A	B

If desired, weighings (2) and (3) may be replaced by one weighing.

The above procedure has the advantage that the operator is able to detect changes of zero of the balance and to estimate the corresponding uncertainty of weighing.

(iii.) *High Precision Weighing.*—While the sources of error which are generally associated with the use of balances have already been indicated, it is interesting to notice what degree of accuracy can be obtained with the utmost precautions and the most refined methods. In this connection, reference should be made to the standardisation of mass entrusted to the Bureau International des Poids et Mesures, Sèvres, Paris. A detailed account of the balance, methods of use, and resulting accuracy is given in the publications[1] of the Bureau. It is noteworthy that the various national copies of the International prototype Kilogramme Standard of Mass have been determined with a general precision of ± 0·000007 gramme, *i.e.* to within 7 parts in 1000 million.

In order to obtain this precision, the following precautions were taken in the use of the balance:

(1) The errors due to fluctuation of temperature within the balance case were minimised by providing the balance with gear for loading and unloading the pans from a distance, without opening the balance case, and by reading the balance at a distance. The deflection of the beam was read by optical lever with the operator and telescope at a distance of about 4 metres.

(2) The suspension of the load was made as free as possible by providing what is equivalent to a universal joint between it and each terminal knife-edge. This joint takes the form of a pair of transverse knife-edges, which ensures that the load is applied at one definite point on the arm, *i.e.* the point in the terminal knife-edge vertically in line with the supposed intersection of the additional transverse knife-edges.

Without this precaution the effective length of the arm would not be sufficiently definite for the accuracy required.

§ (5) Vacuum Balances.—In some cases of weighing in air, it is not always easy to determine the actual density of the atmosphere with the precision necessary to determine the buoyancy correction to an accuracy comparable with that of the weighing itself. This is the case in the comparison of standards of mass of widely different densities such as platinum, brass, and quartz. In special work of this kind, and also in the determination of certain fundamental constants, it is often desirable to weigh in a rarefied atmosphere or *in vacuo*, by means of a vacuum balance. Vacuum balances may thus be used in two ways. In the first place, the actual weighing may be carried out in an enclosure where the residual pressure is so low that the buoyancy correction is negligible. This entails the provision of gear for loading and operating the balance from without, particularly in making the final adjustment of load necessary to obtain equilibrium. On the other hand, in the absence of such gear, it is convenient to obtain approximate balance at atmospheric pressure, and then diminish the pressure within the balance case until equilibrium is obtained. In this way it can be arranged for equilibrium to be secured at low pressures of the order 5 cm. of mercury, measured by a mercury manometer gauge connected with the interior of the balance case. The buoyancy correction would then be small enough to admit of accurate determination to within the limit of precision of weighing. Crookes[1] and other investigators have used the balance for weighing in this manner in a rarefied atmosphere, while of the balances designed for weighing *in vacuo*, reference should be made to the vacuum balance[2] constructed by Bunge for the Bureau International at Sèvres. This balance is designed so that the combined operations, viz.

---

[1] See *Travaux et Mémoires*. In this connection see also Conrady, *Proc. Roy. Soc.*, 1922.

(i.) Exact adjustment of equilibrium by the addition of small weights,
(ii.) The release and arrestment of the beam,
(iii.) The reading of the swinging of the beam,
(iv.) The interchanging of the loads on the two pans,

can be performed at a distance of a few metres.

§ (6) MICRO-BALANCES.—Before examining the range and performance of micro-balances, consider first the proportionate accuracy obtainable from balances in relation to the load weighed. It has been shown (§ (4)) that with a balance taking loads up to 100 grammes or 1 kilogramme, a precision of 1 part in 10^8 of the load weighed represents the best accuracy obtainable, under good conditions, with the utmost precautions in weighing. This limiting precision is governed most closely by the degree of uniformity of temperature within the balance case, as affecting the relative lengths of the arms of the balance. With brass and magnalium beams, this limiting accuracy corresponds to a relative consistency of less than $\frac{1}{1000}$° C. in the mean temperatures[3] of the arms of the balance. The larger the balance, the more difficult it is to realise this high degree of consistency of temperature throughout the length of the beam. Consequently increased proportionate accuracy of weighing cannot be expected from the larger balances.

Of the balances with metal beams designed to take small loads not exceeding 1 or 2 grammes, the Assay Balance of L. Oertling, Ltd., may be considered typical. As at present designed, it enables weighings to be performed with a limiting proportionate accuracy lying between 1 part in 10^6 and 1 part in 10^7 of the maximum load, if the balance is read by means of an optical indicator. Greater proportionate accuracy might be secured by redesigning the beam of this balance so as to be lighter in comparison with the load to be weighed, but, generally speaking, little has been done in the direction of obtaining greater absolute accuracy than the millionth of a gramme (or a few ten-millionths) with knife-edge balances having metal beams.

Some progress has, however, been made with the use of quartz for small balance beams, while still maintaining a proportionate accuracy of 1 part in 10^8, or better, in the mass weighed.

Of the earlier micro-balances little will be said here. Reference may be made to the work of Ångström, Nernst,[4] Brill,[5] and others.

A substantial advance was made by Steele[6] and Grant, who designed two micro-balances with beam and fulcrum knife-edge both of quartz. The use of fused quartz as the material of the beam has the following important advantages.

(1) It is light and incorrodible.

(2) Its tensile strength is great, and it is almost free from elastic fatigue.

(3) Its thermal[7] expansibility is very small (about one-twentieth of that of brass or magnalium). This is a valuable property, since it makes for constancy of the effective length of the beam, and also for consistency of rest point.

(4) It is readily obtained in the homogeneous condition, and it is easily manipulated in the oxy-coal-gas flame.

Irregularities of behaviour due to electrification of the beam during handling may be avoided by ionising the air inside the balance case with a small quantity of radioactive substance.

In Steele and Grant's balances, the quartz fulcrum knife-edge was fused in position on the beam. The ordinary scale-pan suspension from terminal knife-edges at the ends of the beam was not used. A counterpoise bulb was rigidly attached to one end of the beam, and balanced at the other end by a suspension pan which hangs from a fine vertical quartz fibre, fused to the beam. This mode of suspension of the load was found to give better results than either the ordinary "knife-edge and plane" or the "point and plane" suspension.

---

[1] *Phil. Trans.*, 1873, clxiii. 277.

[2] *Travaux et Mémoires du Bureau International*, tome 1; also *La Convention du mètre*, by Guillaume, published by Gauthier Villars, Paris.

[3] The coefficient of linear expansion is approximately 0·00002 per 1° C.

[4] *Nachr. K. Ges. Wiss.*, Gottingen, 1902, ii.

[5] *Chem. Soc. Journ.*, 1908, xciii. 1442.

[6] *Roy. Soc. Proc.*, 1909, lxxxii. 580.

[7] The coefficient of linear expansion is of the order 0·000001 per 1° C.

This design of beam and suspension avoids the tedious adjustment, so necessary in the case of ordinary analytical balances, for placing the terminal knife-edges in plane and parallel with the fulcrum knife-edge, and equalising the lengths of the arms.

It should be noticed that in micro-balances with fibre suspension in place of the terminal knife-edge, the effective length of the balance arm varies with the deflection of the beam owing to the lack of perfect flexibility of the fibre, but since the weighing is made from one arm by a null method, no error is introduced on this account.

An important question arises in the calibration of these instruments. If the balances should be intended for weighings in air at current atmospheric pressures only, a set of standards of mass, suitable for the range of balance, would be required. There is clearly a limit to the size of weight which can practically be used.

Even in assay balance work, the smallest weights used, *i.e.* $\frac{1}{2}$ mgrm. (and rarely $\frac{1}{10}$ mgrm.) riders, are very small and easy to lose, though made of light material. Clearly the use of a comprehensive set of small weights is impracticable for a balance which is sensitive to $\frac{1}{100000}$ mgrm. or less.

The use of weights is, however, unnecessary, except in the initial calibration of the balance, since small changes of the load can be balanced by changing the upward buoyant force on the beam and load by varying the pressure within the balance case.

The micro-balance is thus used as a vacuum balance, and the change in weight is measured by the change in density of the air required to restore the balance to its initial position of equilibrium. When the temperature is constant, this change in density is proportional to the change in pressure as measured by a mercury manometer attached to the balance case. For micro-balance work of high precision it is necessary to apply a correction for temperature changes.

It should be observed that the standard of reference with which the weighings are compared is the difference in air-buoyancies of the two arms of the balance under certain recognised conditions of temperature and pressure. Suppose this difference is m grammes. Then any substance not heavier than m grammes can be weighed as follows:

Note the equilibrium point when the scale pan is empty, and read the pressure within the balance case. Place the substance to be weighed on the pan, and adjust the pressure until the same equilibrium position is obtained. The mass to be weighed can then be calculated from a knowledge of the pressures. If the buoyancy difference is not known from volumetric measurements made on the bulb of the beam, it is clear that the balance will need to be calibrated by means of a mass that has been standardised on some other balance, say on an assay balance. In this case the proportionate accuracy given by the micro-balance is limited by that obtainable from the assay balance. Masses larger than m grammes may be weighed on the micro-balance by making a set of weights, in effect counterpoise weights arranged in sizes so that the difference between consecutive sizes is always less than m grammes. Weighings can then be conducted up to a maximum limit equal to the weight of the largest counterpoise, provided this is not too large a load for the balance beam to withstand.

Used in this way, the micro-balance is an excellent means of measuring relative changes of mass with high precision. Steel and Grant have been able to obtain a sensitiveness exceeding one-hundred-thousandth of a milligramme (1×10^{-8} gramme). The absolute accuracy obtained by them was 1×10^{-7} gramme for masses not exceeding one decigramme.

The same type of quartz micro-balance was used with success by Gray[1] and Ramsay in their determination of the density of radium emanation, a sensitiveness of 2×10^{-9} gramme being obtained.

Mention should also be made of the micro-balance used by Aston[2] for the determination of densities of gases of which only small quantities are obtainable. Further reference to this balance will be made in discussing the methods of determining density (§ (18)).

Additional modifications have been made in the design of quartz micro-balances in the direction of omitting the fulcrum knife-edge and suspending the beam from quartz fibres. T. S. Taylor[3] in his determination of the density of helium used a quartz balance similar to that illustrated in perspective in *Fig.* 12.

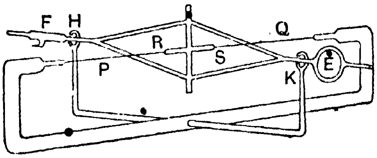


FIG. 12.—Diagram of Quartz Balance used by Taylor.

The balance, which was designed only for the determination of densities, consists essentially of the rhomboidal beam, to which is fused the bulb E and the counterpoise F, and also the rods R, S, perpendicular to the plane of the rhomboid, which in their turn are fused to the horizontal quartz fibres P, Q, on which the beam is mounted as shown. The stops H, K, prevent the balance from producing too great a torsion on the supporting fibres.

---

[1] *Roy. Soc. Proc.*, 1910, lxxxiv. 536.

[2] *Ibid.*, 1913, lxxxix. 442.

[3] *Phys. Rev.*, 1917, x. 653.

Pettersson [1] on the other hand has suspended the beam of his micro-balance by means of two vertical threads. Using a quartz beam of length 10 cm., he has obtained a sensitiveness of 1×10^{-9} gramme.

Working with a still smaller beam of length 5 cm., Pettersson succeeded in obtaining a sensitiveness of 1×10^{-10} gramme with a maximum load of 100 to 200 mgrm.

The field of use of the quartz micro-balance is doubtless limited, and, apart from the work of a few investigators in the determination of densities, little has been done in adapting the instrument to the solution of problems demanding extreme delicacy in the operations of weighing.

II. WEIGHTS

(An introductory account of the fundamental standards of mass will be found in the article "Metrology.")

§ (7) DISTINCTION BETWEEN MASS AND WEIGHT.—Standards of mass are most frequently called "weights," and in this sense this term will be used here. In order to determine mass, the method of weighing is resorted to. The *mass* of any object is unvaried [2] whether it is weighed in air or *in vacuo*. Its *weight in vacuo* is the property actually determined, and is the measure of its mass.

The weight of an object is the force with which gravity acts upon it. Confusion is often caused by the use of the term "weight" to denote "mass," and *vice versa*. The use of such ambiguous or redundant expressions as "apparent mass" or "mass *in vacuo*" should be avoided. Unless the mass of an object can be determined in true units of mass, it is preferable to express it in a definite, unambiguous manner, as, for example, "weight in air (of stated density) when compared with brass weights (of stated density)."

§ (8) MATERIAL FOR MAKING WEIGHTS.—The general desiderata in the choice of material for standards of mass, more particularly fundamental standards, have been outlined in the article "Metrology." [3] Platinum has been chosen in the case of the British Imperial Standard Pound Avoirdupois, while the International Prototype Kilogramme is of platinum-iridium alloy (10 per cent iridium). It is realised that the greatest care has to be taken to avoid changes of mass due to wear resulting even from occasional use, but in other respects platinum, or its iridium alloy, has been found very satisfactory as regards permanence and invariability.

A striking example of this may be seen in the results of the re-comparisons of a number of national copies of the International Prototype Kilogramme, made at the Bureau [4] International des Poids et Mesures, Sèvres, near Paris, during the years 1899–1917. In half the cases examined, the mass of a national standard was found to agree with its initial value at the time of its formal issue, in 1889, to within $\pm 0{\cdot}000010$ gramme, *i.e.* to within 1 part in 10^8. These results bear sound evidence not only of the degree of invariability of the national copies, but also incidentally of the precision of measurement attainable in the use of the equi-arm knife-edge balance.

Referring to weights of less fundamental but more frequent use, the circumstances governing the choice of material depend on a number of conditions. On the whole, it may be said that there is not a very wide range of choice of material of proved reliability. Platinum, on account of its comparative rareness and heavy cost, is almost out of the question except in the case of small weights, *e.g.* fractions of 1 gramme. It is, however, used to considerable advantage in plating weights which are made of corrodible material, but the disadvantages of coating or plating weights will be referred to later.

As a substitute for platinum, no metal or material of high density has yet been found which is obtainable readily and reasonably cheaply, and at the same time suitable as a standard of mass. On the other hand, though there are materials of low density possessing properties which are generally suitable for standards of mass, their use is limited to small weights on account of the comparatively large buoyant force of the atmosphere on them, resulting from their relatively large volumes. Consequently, the materials in general working use as weights are of moderate density. In England, with the exception of very small weights, *i.e.* less than 1 gramme, it is customary to make scientific weights of brass. This material is not ideal as a standard of mass. It tarnishes easily in an ordinary atmosphere (much more so in a chemical laboratory), and, being comparatively soft, loses mass owing to wear resulting from use. These two disadvantages, however, operate in opposite directions. It may be argued that in these circumstances it would be preferable to cover the brass with some protective coating or plating which would render the weight more reliable. Much depends on the exact use to which the weights are to be put, but one English balance maker of high repute prefers to use uncoated polished brass for his scientific weights.

---

[1] *Phys. Soc. Proc.*, 1920, xxxii. 209.

[2] Except in so far that the object may absorb a small mass of air or moisture from the atmosphere, which is not retained by it *in vacuo*.

[3] See "Metrology," Part III. § (6), etc.

[4] *Travaux et Mémoires du Bureau Intl.*, tomes xii., xv., xvi.

Some idea of the amount of variation in mass of uncoated polished brass weights may be obtained from the following values, determined for a number of sets of brass weights by the above-mentioned maker which have been in regular use at the National Physical Laboratory:

Nominal Mass.	Average Range of Variation of Mass in the Course of Two or Three Years measured from the time when the weights were new.
10 kilogrammes . .	± 20 milligrammes
1 kilogramme . . .	± 5 ,,
100 grammes . . .	± 1 ,,
50 grammes downwards to 1 gramme . .	± ½ milligramme (or less)

Note.—It is impossible to give more than approximate figures. The above values relate to brass weights which have been in regular but careful use nearly every week for a number of years, in an atmosphere free from contamination by chemical fumes.

Of other materials, of moderate density, which have been used in preference to brass to serve as laboratory standards of reference, mention may be made of "white bronze," an alloy of copper (50 per cent) and nickel. This alloy admits of a very fine polish, and does not tarnish appreciably. It is not, however, very resistant to wear, but lies between brass and platinum in this respect. A 100 grm. standard of mass in white bronze, which has been in regular use at the National Physical Laboratory for a period of ten years, has lost ½ mgrm. due to wear resulting from frequent and regular usage, in spite of careful lifting with soft-covered forceps. White bronze, however, shows a distinct tendency to absorb moisture.

Another alloy called "baros" has been experimented[1] with at the Bureau International. It is chiefly of nickel, with small proportions of chromium and manganese, and is hard, non-magnetic, and free from blow-holes. An experimental specimen of this alloy was found unaffected by prolonged immersion in water or by moderate heating *in vacuo*. While useful as a working or reference standard, its mass is not sufficiently invariable as a fundamental standard, owing to small decreases which are attributable to a loss of gas occluded in the alloy at the time of casting.

§ (9) Coated Weights.—Protective coatings which have been used for covering weights may be divided into two classes, viz. lacquering and metal-plating. Platinum, gold, and nickel have been used for metal-plating, but the first-named is to be preferred. In the case of gold-plated weights, the soft plating ordinarily used as a surface protection is liable to wear, and if the weights are in frequent use the changes in their values may be considerable.

A platinised brass weight is, however, preferable to an uncoated polished brass one. The platinum plating is deposited electrochemically to a thickness of about 0·002 mm. (0·0001 inch) in this country. It is usual to plate in two or three stages and to polish the weight between the stages. As regards permanence and invariability, platinised brass weights may be taken as approximately similar in behaviour to the white-bronze weights already referred to. In other words, a platinised weight, though suitable as a working or reference standard, is not sufficiently invariable as a fundamental standard of mass.

Lacquer, if used as a protective coating for a weight, should be hard, of moderate thickness, smooth, and not likely to chip. It is well known that lacquers absorb moisture from the air to a variable extent, say from 1 to 5 per cent of the weight of the lacquer, depending on the atmospheric humidity. On an estimate,[2] a 100 grm. lacquered weight may be subject to variations of the order 0·0001 gramme. This value is, of course, only approximate, since much depends on the nature and the method of application of the lacquer. Nitro-cellulose lacquers have been found to absorb about three times as much moisture as those containing shellac. The use of a lacquered weight would probably preclude an accuracy exceeding 0·0001 gramme on 100 grammes, whereas an uncoated polished brass weight would in general maintain a rather bette consistency of mass than this for a period o several months, under good conditions o use.

Since a large weight has a relatively smalle surface area than a small weight of the sam material, it follows that the changes in mas due to variations of surface conditions are o less importance in the case of large weights Commercial weights are generally covered wit a protective coating, either lacquer or suitabl paint, but the reference standards agains which they are ultimately tested may be c brass, platinised, gilded, or even unplated.

Spots that are seen on plated weight are very often due to the presence of sma pores in the metal immediately under th plating, and can best be avoided by using metal that is free from pores. Tobin[1] bronze is recommended[2] by the Bureau of Standards, Washington, as satisfactory in this respect. Besides the usual electro-plating process, there are newer methods of plating, *e.g.* by cathode discharge in vacuum and by metal-spraying, but these have not yet reached a stage of development when they can be definitely recommended.

---

[1] *Travaux et Mémoires Bureau Intl.*, 1917, tome xvi.

[2] See also *Chem. Weekblad*, 1920, xvii. 453.

Amongst the non-metals which may be considered as providing suitable material for weights for certain purposes, mention should be made of the rock-crystal (quartz) weights made by J. Nemetz (Vienna) and Laurent (Paris), of which a few sets are in use in this country. These weights are very constant, but owing to their low density[3] the corrections for the buoyant force of the air are very large when the weights are compared with brass or platinum standards. There may, however, be occasions in the weighing of glass apparatus where the use of quartz weights is an advantage in tending to equalise the buoyant forces on the two arms of the balance.

A number of glass-cased weights have also been made in Austria, though not in this country. These weights consist of glass shells filled with shot, and then sealed. The surface of the shell is easily cleaned, and wears less than that of an average metal weight under ordinary use. They possess the additional advantage in regard to reliability that a slight accident to a metal weight might affect its mass without leaving cause for suspicion, whereas the glass-cased weights would not in general suffer injury unless the injury were so great as to leave no doubt about it.

§ (10) Small Weights. — Small weights from 1 gramme downward are usually of flat sheet-metal, with a corner or a side turned upward so as to facilitate lifting with forceps. They should be of a material sufficiently resistant to oxidation or corrosion not to need plating or coating. Platinum is best suited for weights from 1 grm. to 5 mgrm. Weights of this material smaller than 5 mgrm. are so small that a less dense metal, such as aluminium, is to be preferred. An alloy of palladium and gold has been found to be a good substitute for platinum for fractional weights, and has been used in a large number of sets of high-grade analytical weights. German silver is used occasionally, but not in the higher-grade sets of weights. Aluminium may also be employed in making fractions of 1 gramme, and is to be recommended for sizes from 50 milligrammes downward.

§ (11) Shape and Design of Weights.— Since it is important, in designing a weight, to minimise all possible variations in mass due to changes in surface conditions of the weight (such as those due to tarnishing, wear, or possibly porous or hygroscopic nature of the material), the shape of a weight is usually chosen so as to give a minimum surface consistent with convenience in lifting or moving the weight. Clearly the simpler the shape, the better. Two fundamental standards of mass are either cylindrical, or show but little modification of the simple cylindrical form, and the height of the cylinder is approximately equal to its diameter.[4]

For convenience in lifting them, other scientific weights except fractions of a gramme are usually made with knobs, which may be integral with their respective weights or detachable.

It is advisable, wherever constancy of mass is of the highest importance, to make the weight in one piece with no detachable parts, in spite of the increased labour entailed in adjusting the weight to its nominal size. Chemical and analytical weights, as made in this country, usually have knobs, which can be screwed into the top of each weight, closing a cavity which is used for the purpose of adjusting the weight to its nominal size. The knob should be of suitable shape and size to facilitate lifting the weight with forceps or other weight-lifters, and should be relatively taller in the case of small weights.

Small weights such as fractions of a gramme are usually made of sheet metal, *e.g.* platinum, gold, aluminium, or German silver, except in the case of rider weights, which are made from the same materials in the form of wire.

The shapes just referred to are suitable for use with an equi-arm precision balance. The design of weights for use with other weighing machines, particularly commercial instruments, involves other considerations beyond those already discussed. Commercial weights are often provided with a single adjusting hole closed by a plug which can be sealed by the inspector in charge of the testing of the weight. Specifications for these weights may be obtained from the various national testing institutions where the weights are examined.

§ (12) Adjustment of Weights. — The means of adjustment of weights have already been referred to under § (11). Either a cavity is provided which allows of the addition or subtraction of small fragments in order to make the final mass equal (within limits) to its nominal value, or else the weight is in one piece, and is ground, polished, and lapped in stages until the correct mass is obtained.

---

[1] Of approximate composition: 61 per cent copper, 1 per cent tin, 38 per cent zinc, with a small residuum of other materials.

[2] Circular No. 3 (1918 edition) of the Bureau of Standards.

[3] The density of quartz crystal at 20° C. is 2·65 grm. per c.c.

[4] For a right cylinder of given volume, the surface area is minimum when the height of the cylinder is equal to its diameter.

With the exception of some commercial weights, it is usual in this country to adjust each weight to have a true mass equal to the nominal value associated with it. For example, a brass kilogramme weight is adjusted to have a mass of 1 kilogramme, (*i.e.* equal to that of the International Standard Kilogramme). If, however, the brass kilogramme and the International Standard Kilogramme were compared on a balance, they would not weigh the same unless the weighing were conducted with the balance *in vacuo*. Owing to the greater upward buoyant force of the air on the brass weight, the platinum kilogramme mass weighs heavier, by about 90 mgrm., in air under average conditions of temperature and pressure in the laboratory. In other words, fractional weights to the value of about 90 mgrm. are required on the pan of the balance containing the brass weight in order to give equilibrium.

In adjusting a weight the maker requires some standard—his working standard—with which to compare the new weight on the balance. For the convenience, in the workshop, of avoiding the use of the buoyancy correction, the working standard is of brass, which has presumably been standardised at a national testing laboratory, and its true mass certified. Provided that the mass of the working standard has been adjusted to agree with its nominal value to within satisfactorily small limits, the maker's task is to make new brass weights as nearly as possible replicas of his working standard as far as adjustment of mass is concerned. It is at the national testing laboratory that the difference between the weights in air of the brass and platinum kilogrammes has to be corrected for, and the true mass determined for the brass weight.

The exact nominal mass to which a weight is adjusted is a matter of convention, depending on general usage and convenience. In making a new platinum kilogramme weight which for certain purposes may perchance be required more frequently for weighing against a brass kilogramme than against one of platinum, it may be convenient to adjust the mass of the platinum weight to be accurately 999·91 grm. (*i.e.* 1 kilogramme, less 90 mgrm.) in order that it should weigh the same in air as the brass kilogramme, which has a nominal mass of 1000 grm.

Again, in the case of a set of analytical weights in which the fractions of a gramme are invariably of a different material from that of the larger weights, it is the practice to adjust each weight (whether of brass, platinum, or aluminium, etc.) to have a true mass equal to the nominal value associated with it. Consequently, in a set accurately adjusted in this way, the 1 gramme brass weight, though having the same mass as that of the combination of the platinum fractional weights whose nominal values total 1 gramme, will weigh lighter in air than the composite set.

In both the testing and the adjustment of trade weights, the buoyant force of the air is neglected. This class of weights is adjusted so that the weight in air is equal to that of the brass standard in air, the standard being of true mass (*in vacuo*).

Brass weights of the same nominal value do not have exactly the same volume owing to slight differences in the metal and also (in the case of weights with cavities) in the size of the cavity. Consequently they have not exactly the same buoyancy, but this difference is negligible for commercial purposes, though in accurate work it must be duly allowed for.

§ (13) THE TESTING OF WEIGHTS IN A NATIONAL STANDARDISATION LABORATORY.—In testing a graduated set of weights such as the average set of analytical weights, it is not always necessary to test each weight by direct comparison with a working standard of the same denomination. In general, the various denominations of the set are arranged so that all the weights can be standardised in terms of the head-weight of the set. This is a great convenience, since there is then no need for more than one working standard, *i.e.* that corresponding to the denomination of the largest test-weight. The consequent saving of labour involved in the periodic calibration of working standards is considerable, for in routine testing, when a standard of mass is regularly in use on several occasions each week, it is of the highest importance to avoid errors due to wear and usage.

In commercial weight-testing, the procedure is simplified by the omission of the buoyancy correction. The testing of a weight then becomes a simple comparison between the weight tested and the working standard on a balance or other weighing-machine.

In work of high precision the method of double weighing is used (see § (4) (ii.)). The weights to be compared (for example, a known standard and the weight under test) are weighed in air from each pan of the balance in turn, in order to diminish errors due to the inequality of the arms of the balance beam and unsteady or uneven distribution of temperature within the balance case. For each weighing, the deflection of the beam is measured on a scale previously calibrated by means of known weights.

It is usual, in standardising analytical and other scientific weights at the National Physical Laboratory, to make an allowance for the upward buoyant force of the atmosphere on each weight. This is often essential in comparing weights of the same material, since all weights are not solid throughout. When the weights under comparison are of different material, *e.g.* brass and platinum, the application of the buoyancy correction calls for the highest accuracy in the measurement of the density of the air within the balance case.

Briefly, the process of scientific weight-testing reduces to an accurate measurement of the difference between two nearly equal masses. One mass is then determined in terms of a working standard. The latter in its turn has already been measured in terms of some fundamental standard, which primarily involves comparison with the legalised Imperial or International Standards of Mass.

F. A. G.

III. DENSITY

§ (14) DEFINITIONS.—The *density* of a substance is its mass per unit volume.

The volume of any given mass of a substance varies with its temperature and with the pressure to which it is subjected. In the case of solids this variation is so small that, for many purposes, it need not be taken into account, so far as densities are concerned.

In the case of liquids, the change in volume with temperature is comparatively large, but the compressibility of liquids is in general so small that it suffices simply to specify the temperature of the liquid to which any given density relates.

With gases, however, the volume of a constant mass of gas varies greatly with both its temperature and the pressure to which it is subjected. Consequently both temperature and pressure must be specified in order to secure precision in expressing the density of a gas.

The *specific gravity* of a substance is the ratio of its density to a chosen standard density.

It would appear, therefore, from this definition that the use of the term specific gravity is somewhat superfluous. For if unity (*e.g.* 1 grm. per c.c., or 1 lb. per cu. ft., etc.) were to be taken as the chosen standard density the need for the term specific gravity would disappear, and the simply defined term density would meet all requirements.

There must, therefore, be some reason to account for the widespread use of a term which, viewed from a purely logical standpoint, appears to be unnecessary.

This is doubtless to be found in the fact that the ratio of the densities of two substances can be determined with much greater ease than their absolute densities. The absolute determination of density from direct measurements of mass and volume is indeed so difficult that it has only been carried out to a high degree of accuracy in the case of a single substance, viz., water. The classical determination of the volume of a kilogramme of water carried out at Sèvres[1] was, of course, equivalent to an absolute determination of the density of water in grammes per cubic centimetre. Again, for many purposes the ratio of the densities of substances to the density of some standard substance is as useful as the absolute densities of the substances.

Water is eminently suited to serve as a standard substance for the above purpose, since it is readily obtained in a state of purity, and its density at various temperatures has been accurately determined; also the ratio of the density of a substance to that of water is capable of direct experimental determination to a high degree of accuracy. Consequently water is practically always adopted as the basis to which specific gravities are referred, and the term may therefore be defined alternatively as follows:

The specific gravity of a substance at the temperature t_s relative to water at the temperature t_w is the ratio of the density of the substance at the temperature t_s to the density of the water at the temperature t_w.

It is important to specify both the temperature of the substance and that of the water. For example, the commonly used phrase "specific gravity at 60° F." has really no precise meaning. One assumes that specific gravity at 60° F. relative to water at 60° F. is meant, but this assumption may not in some cases be correct, *e.g.* specific gravity at 60° F. relative to water at its temperature of maximum density may be meant.

A convenient abbreviation for expressing without ambiguity the precise meaning to be attached to a specific gravity is the expression "specific gravity $S_{\frac{t_s}{t_w}}$" or more briefly "$S_{\frac{t_s}{t_w}}$," where t_s is the temperature of the substance, and t_w that of the water to which it is referred. Thus

$$S_{\frac{60^\circ \text{ F.}}{60^\circ \text{ F.}}} = \frac{\text{Density of the substance at } 60^\circ \text{ F.}}{\text{Density of water at } 60^\circ \text{ F.}}$$

The symbol $S_{\frac{t_s}{t_w}}$ has come into extensive use, and its general adoption when dealing with specific gravities would lessen the chances of ambiguity.

It follows from the definition of the litre[2] that specific gravities represented by $S_{\frac{t_s}{4^\circ \text{ C.}}}$ are identical with densities at t_s in grammes per millilitre.[3]

---

[1] See article "Volume, Measurement of," § (2).

[2] See *ibid.*, § (1).

[3] Specific gravities $S_{\frac{t_s}{4^\circ \text{ C.}}}$ are not identical with densities at t_s expressed in grammes per cubic centimetre, as is sometimes stated. The cubic centimetre and the millilitre are not identical, the accepted relation between the two being 1 ml. = 1·000027 c.c.

It follows from the definition of specific gravity that the density of a substance at the temperature t_s may be obtained by multiplying its specific gravity $S_{\frac{t_s}{t_w}}$ by the density of water at the temperature t_w.

The *specific volume* of a substance is the volume occupied by unit mass of the substance.

§ (15) DETERMINATION OF THE DENSITY OF LIQUIDS. (i.) *By Means of a Pyknometer.*—The determination of the weight of a liquid, and also of the weight of water required to fill a pyknometer, affords a ready means of determining the density of the liquid, since the density of water is known to a high degree of accuracy.[1]

The following is an outline of the calculations involved in determining the density of a liquid from observations taken with a pyknometer. The calculation of the specific gravity of the liquid merely involves dividing the determined density by the appropriate density of water.

Let W_P = observed weight[2] in grm. of pyknometer in air of density σ_1 grm./ml.

W_W = observed weight in grm. of pyknometer filled with water of temperature t_W in air of density σ_2 grm./ml.

W_L = observed weight in grm. of pyknometer filled with liquid of temperature t_L in air of density σ_3 grm./ml.

Δ = density of weights in grm./ml.

G = density of glass in grm./ml.

d_W = density of water at t_W in grm./ml.

d_L = density of liquid at t_L in grm./ml.

α = coefficient of cubical expansion of glass in ml. per ml. per ° C.

M_P = mass *in vacuo* of pyknometer.

M_W = mass *in vacuo* of water required to fill pyknometer at temperature t_W.

M_L = mass *in vacuo* of liquid required to fill pyknometer at temperature t_L.

The equations representing the equilibrium obtained in each of the three weighings are

$$W_P\left(1-\frac{\sigma_1}{\Delta}\right)=M_P\left(1-\frac{\sigma_1}{G}\right). \quad . \quad . \qquad (1)$$

$$W_W\left(1-\frac{\sigma_2}{\Delta}\right)=M_W\left(1-\frac{\sigma_2}{d_W}\right)+M_P\left(1-\frac{\sigma_2}{G}\right). \qquad (2)$$

$$W_L\left(1-\frac{\sigma_3}{\Delta}\right)=M_L\left(1-\frac{\sigma_3}{d_L}\right)+M_P\left(1-\frac{\sigma_3}{G}\right). \qquad (3)$$

The mass of liquid required to fill the pyknometer at the temperature t_L is therefore given by

$$M_L=\left[W_L\left(1-\frac{\sigma_3}{\Delta}\right)-M_P\left(1-\frac{\sigma_3}{G}\right)\right]\left(\frac{d_L}{d_L-\sigma_3}\right).$$

The volume of the liquid required to fill the pyknometer at the temperature t_L is

$$\frac{M_W}{d_W}[1+\alpha(t_L-t_W)]$$

$$=\frac{\{W_W[1-(\sigma_2/\Delta)]-M_P[1-(\sigma_2/G)]\}[1+\alpha(t_L-t_W)]}{d_W-\sigma_2}.$$

Hence the density d_L of the liquid at the temperature t_L is given by

$$d_L=\frac{\{W_L[1-(\sigma_3/\Delta)]-M_P[1-(\sigma_3/G)]\}(d_L/\{d_L-\sigma_3\})(d_W-\sigma_2)}{\{W_W[1-(\sigma_2/\Delta)]-M_P[1-(\sigma_2/G)]\}[1+\alpha(t_L-t_W)]}$$

or

$$d_L=\frac{\left\{W_L[1-(\sigma_3/\Delta)]-W_P\dfrac{1-(\sigma_1/\Delta)}{1-(\sigma_1/G)}[1-\sigma_3/G]\right\}(d_W-\sigma_2)}{\left\{W_W[1-(\sigma_2/\Delta)]-W_P\dfrac{1-(\sigma_1/\Delta)}{1-(\sigma_1/G)}[1-(\sigma_2/G)]\right\}[1+\alpha(t_L-t_W)]}+\sigma_3. \qquad (4)$$

Writing

$$\frac{1-(\sigma_3/G)}{1-(\sigma_1/G)} \text{ and } \frac{1-(\sigma_2/G)}{1-(\sigma_1/G)}$$

both equal to unity (by weighing the empty pyknometer immediately before weighing it full of water, and also before weighing it full of liquid, this equality could be closely achieved experimentally), equation (4) reduces to

$$d_L=\frac{\{W_L[1-(\sigma_3/\Delta)]-W_P[1-(\sigma_1/\Delta)]\}(d_W-\sigma_2)}{\{W_W[1-(\sigma_2/\Delta)]-W_P[1-(\sigma_1/\Delta)]\}[1+\alpha(t_L-t_W)]}+\sigma_3, \qquad (5)$$

a close approximation to which is

$$d_L=\left[\frac{W_L-W_P}{W_W-W_P}\right]\left[\frac{d_W-\sigma_2}{[1-\alpha(t_L-t_W)]}\right]+\sigma_3. \qquad (6)$$

If the water content and liquid content are determined at the same temperature,

$$d_L=\frac{W_L-W_P}{W_W-W_P}(d_W-\sigma_2)+\sigma_3. \quad . \quad . \qquad (7)$$

---

[1] P. Chappuis, *Trav. et Mém.*, 1907, xiii. A copy of Chappuis' table of results is given at the end of this article.

[2] Weights are adjusted to have a mass equal to the nominal value marked on them. Hence the "observed weight" referred to above is identical with the mass *in vacuo* of the weights required to produce equilibrium of the balance, *i.e.* assuming the weights to be accurately adjusted. It may also be noted that, strictly speaking, the commonly used term "mass *in vacuo*" is somewhat redundant. The mass of any object is unvaried, whether it is weighed in air or *in vacuo*. The term "mass," therefore, does not really require the qualifying words "*in vacuo*."

If an average value σ is substituted for σ_2 and σ_3 we obtain

$$d_L = d_W \frac{W_L - W_P}{W_W - W_P} + \sigma\left(1 - \frac{W_L - W_P}{W_W - W_P}\right), \quad . \quad (8)$$

and finally, neglecting buoyancy corrections altogether, we obtain the simple approximate relation

$$d_L = d_W \frac{W_L - W_P}{W_W - W_P}. \quad . \quad . \quad . \quad (9)$$

In deciding which of the various equations given above for calculating d_L should be used in any particular case, the degree of accuracy required in the results should be considered in conjunction with the approximations made. Results should not be expressed to a greater number of decimal places than is warranted by the approximations made.[1]

An important limitation on the accuracy of density determinations should be noted. Temperatures measured by means of mercury in glass thermometers, even when the greatest precautions are taken, may differ from the International Hydrogen Scale by 0·01° C. The density of a liquid at any stated temperature is therefore liable to an error corresponding to the change in the density of the liquid for 0·01° C. change in temperature. For example, in the case of water at 20° C. this uncertainty amounts to 2 units in the sixth decimal place, and for absolute alcohol at the same temperature to about 1 unit in the fifth decimal place.

It is clear that careful observations of temperature are of fundamental importance in accurate density determinations. The use of a constant temperature bath[2] to ensure uniformity of temperature when filling a pyknometer is advisable wherever accurate results are desired.

A large variety of pyknometers have been used by different observers. The simple specific gravity bottle with drilled stopper is extensively used. A number of modifications exist, amongst which may be mentioned one provided with a glass cap ground to fit over the outside of the neck and enclosing the drilled stopper.

The Sprengel[3] pyknometer is another well-known form, of which there are many modifications. A notable improvement over the original type is the introduction of a small bulb above the graduation mark on the capillary tube. This allows weighings to be made at temperatures higher than the one at which the pyknometer is filled, the small bulb providing accommodation for the expansion of the liquid in the pyknometer.

A form of pyknometer used by W. R. Bousfield[4] is shown in *Fig.* 13. The two tubes A and B are connected together by the cross tube C, and D is a glass rod which serves as a suspension for the instrument. The capacity of the pyknometer is defined by two graduation marks, one on each of the capillary tubes which form the necks of the tubes A and B.

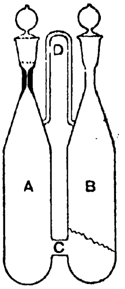


FIG. 13.

References to various other forms of pyknometer are given in the footnote,[5] and a discussion of the best methods of using pyknometers is given by Walter Block, *Z. angew. Chem.*, 1920, xxxiii. 198.

(ii.) *By Means of Sinker Weighings.*—If a glass plummet or sinker, a suitable form of which is shown in *Fig.* 14, is weighed suspended (1) in air, (2) in distilled water, (3) in a liquid, the density of the liquid may be readily determined. The calculation follows exactly the same lines as that given above in the case of the pyknometer, and so need not be repeated.

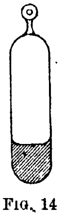


FIG. 14.

The method is particularly well adapted for determining the change in density of a liquid with change in temperature. The liquid under investigation is contained in a vessel which is surrounded by a bath whose temperature can be varied at will, and also controlled so as to remain sensibly constant during any particular observation of the weight of the sinker. The sinker is suspended from one arm of a balance, the latter being mounted directly over the constant temperature bath.

Full details of the apparatus used at the Bureau of Standards for determining the density of ethyl alcohol and water mixtures at various temperatures by the above method are given in the *Bulletin of the Bureau of Standards*, 1913, ix. 371. The sinker method was also used by Plato[6] for sugar solutions, and by Domke[7] for sulphuric acid solutions.

---

[1] For a discussion of various "correction formulae" for the simplification of calculations see Wade and Merriman, *Chem. Soc. Trans.*, 1902, xcv. 2174, and Hartley and Barrett, *ibid.*, 1911, xcix. 1072.

[2] For an account of various forms of thermostatic control see article "Thermostats," Vol. I.

[3] *Chem. Soc. Trans.*, 1873, xxvi. 577.

[4] *Chem. Soc. Trans.*, 1908, xciii. 679.

[5] Perkin, *Chem. Soc. Trans.*, 1884, xlv. 443; *ibid.*, 1896, lxix. 1043; Lunge and Rey, *Z. angew. Chem.*, 1891, p. 165; S. Bosnjakovi, *Z. anal. Chem.*, 1904, xliii. 230; E. Fischer, *Chem. Zeit.*, 1904, xxviii. 359, and *Sitzungsber. K. Akad. Wiss. Berlin*, 1908, p. 542; R. V. Stanford, *Phil. Mag.*, 1905, x. 269; M. Rakusin, *Chem. Zeit.*, 1905, xxix. 1087; E. Sevnagiotto, *Ann. Chim. Applicata*, 1914, i. 198; H. Wustenfeld and Ch. Foehr, *Chem. Zentr.*, 1914, i. 1537; P. B. Davis and L. S. Pratt, *J. Amer. Chem. Soc.*, 1915, xxxvii. 1199; F. Hall, *J. Amer. Chem. Soc.*, 1917, xxxix. 1319; M. Neidle, *J. Amer. Chem. Soc.*, 1917, xxxix. 2387; N. S. Osborne, *Bull. B. of S.*, 1913, ix. 406.

[6] *Abhand. Normal Eichungs Komm.*, 1900, vol. ii.

[7] *Ibid.*, 1904, vol. v.

The Westphal balance is a well-known apparatus for determining densities, which depends upon the variation of the weight of a sinker when suspended in liquids of different densities.

The same principle has been made use of in constructing a recording densimeter.[1] The sinker is suspended by means of a fine wire, the tension on which is balanced partly by a spring and partly by the torsion of a vertical wire which carries a horizontal pointer. Changes in the tension of the wire caused by variations in the density of the liquid in which the sinker is immersed cause corresponding changes in the position of the pointer. The movements of the pointer are recorded on a revolving drum, and so a continuous record is obtained of the variation of the density of the liquid.

(iii.) *By Means of a Hydrometer.*—The hydrometer is a particularly useful instrument for determining the density of a liquid quickly in cases where only moderately accurate results are required. See articles "Hydrometers," "Alcoholometry;" and "Saccharometry."

(iv.) *By Hare's Method.*—An elementary form of apparatus illustrating the principle of the method is shown in *Fig.* 15. The stopcock C being closed, and the pressure in the upper part of the U-tube being less than atmospheric pressure, then the lengths of the columns of liquid A and B will be inversely proportional to the densities of the liquids contained in the left- and right-hand limbs of the U-tube respectively.

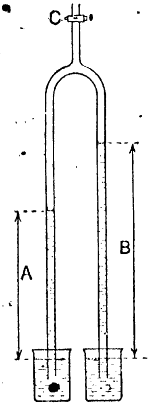


FIG. 15.

A method depending essentially on the above principle has been developed by O. F. Frivold,[2] which is sufficiently sensitive to be used for determining the difference in density between air-free distilled water and distilled water containing air in solution.

(v.) *By Means of Total Immersion Floats.*—A set of hollow glass beads of varying size and mass, known as specific gravity beads, have long been used for approximate determinations of specific gravity. The specific gravity of a liquid can be fixed between limits by noting which bead in the series is the last to float and which is the first to sink. If the liquid happens to be of exactly the same density as one of the beads, then this bead will neither sink nor float, but will remain suspended in equilibrium in the liquid. Very slight changes in the density of the liquid, however, will destroy the equilibrium, and the bead will sink or float as the case may be. The delicate nature of the equilibrium is such that in recent years several methods for determining accurately the density of liquids have been based upon the equilibrium of a totally immersed float.

A. Berget[3] made use of a float totally immersed in the liquid whose density was required, and attached to the upper end of spiral spring of invar, the lower end of the spring being fixed. The float was such that if it had been free it would have floated in the liquid whose density was measured. Consequently there was a tension on the spring, which varied with the density of the liquid in which the apparatus was submerged. The elongation of the spring was measured by means of a cathetometer, the apparatus being calibrated in the first place by means of liquids whose densities were independently determined.

Lamb and Lee[4] used a float which was brought into a definite position by the attraction of an electromagnet on a piece of iron contained in the float. Densities were thus determined to the eighth decimal place.

Ångström[5] used a similar method, his apparatus being shown in *Fig.* 16. Ångström and Petterson[6] used a glass float to which was attached a long chain with 500 very small links of equal weight. When the float is introduced into a liquid whose density is within the range of the instrument, the float sinks and the chain coils up on the bottom of the vessel containing the liquid until a point is reached at which the float is in equilibrium with the surrounding liquid. Observations are taken in a graduated cylinder, the position of the top of the float, which is pointed, being read on the graduated scale etched on the cylinder. The density corresponding to the graduations is determined by a preliminary calibration with liquids of known density. A. L. Thuras[7] used

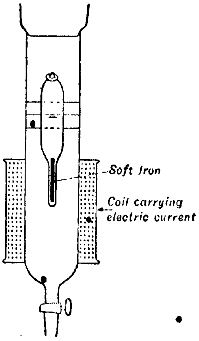


FIG. 16.

---

[1] Cambridge & Paul Scientific Instrument Co.

[2] *Phys. Zeit.*, 1920, xxxix. 529.

[3] *Comptes Rendus*, 1912, cliv. 1294.

[4] *Jour. Amer. Chem. Soc.*, 1913, p. 1666.

[5] *Phys. Rev.*, 1915, v. 249.

[6] *Zeit. Inst.*, 1917, xxxvii. 177.

[7] *Jour. Wash. Acad. Sciences*, 1917, vii. 605.

a totally immersed float for the determination of the density of sea-water on board ship. The float and sample of sea-water were contained in a thin-walled glass tube suspended in a water-bath, the temperature of which could be quickly varied, and when necessary maintained practically constant. The float was such that it floated in sea-water at ordinary temperatures. On raising the temperature of the bath a point was ultimately reached at which the float sank. This temperature was noted. Then the bath was slowly cooled and the temperature at which the float rose to the surface was noted. Then the temperature was raised more slowly than before, and again the temperature at which the float sank was noted. The temperature was again lowered, and so on, successive pairs of temperature readings being obtained, until the difference between a pair of readings was so small that the mean could safely be taken as the equilibrium temperature. The density of the liquid at this temperature is then known, being identical with that of the float, which is carefully standardised once and for all before use. It was possible to locate the equilibrium temperature within ±0·01° C., which corresponded in the case of the samples of sea-water being investigated to a difference of density of less than ±3 units in the sixth decimal place.

VALUES FOR THE DENSITY OF VARIOUS LIQUIDS.—The following references relate to some of the more important liquids whose density has been investigated in detail:

Ethyl Alcohol.—Osborne, McKelvy, and Pearce, *Bull. Bur. of Stds.*, 1913, ix. 327.

Methyl Alcohol.—Doroshevskii and Rozhdestvenskii, *Jour. Russ. Phys. Chem. Soc.*, 1909, xli. 977.

Ammonia.—Lange and Wiernick, *Z. angew. Chem.*, 1889, ii. 181.

Caustic Soda.—Bousfield and Lowry, *Phil. Trans.* A, 1905, cciv. 253.

Mercury.—See tables, p. 131.

Sea-water.—Knudsen, *Hydrographical Tables*, Copenhagen, 1901.

Sugar.—Plato, *Abhand. der K.N.E.K.*, 1904, v. 5.

Sulphuric Acid.—Domke, *Abhand. der K.N.E.K.*, 1900, ii. 140.

Water. See tables, pp. 130, 131.

Tables of density of a large number of liquids are to be found in "Tables Annuelles Internationales de Constantes et Données Numériques," "Recueil de Constantes Physiques," *Société Française de Physique* and *Physikalisch-chemische Tabellen*, Landolt and Bornstein.

TABLE I

DENSITY OF WATER IN GRM. PER ML.[1]

The asterisks imply that the first three figures are those of the line below

°C.	0·0.	0·1.	0·2.	0·3.	0·4.	0·5.	0·6.	0·7.	0·8.	0·9.	Mean Differences.
0	0·9998681	8747	8812	8875	8936	8996	9053	9109	9163	9216	+ 59
1	9267	9315	9363	9408	9452	9494	9534	9573	9610	9645	+ 41
2	9679	9711	9741	9769	9796	9821	9844	9866	9887	9905	+ 24
3	9922	9937	9951	9962	9973	9981	9988	9994	9998	0000*	+ 8
4	1·0000000	9999*	9996*	9992*	9986*	9979*	9970*	9960*	9947*	9934*	− 8
5	0·9999919	9906	9884	9864	9842	9819	9795	9769	9742	9713	− 24
6	9682	9650	9617	9582	9545	9507	9468	9427	9385	9341	− 39
7	9296	9249	9201	9151	9100	9048	8994	8938	8881	8823	− 53
8	8764	8703	8641	8577	8512	8445	8377	8308	8237	8165	− 67
9	8091	8017	7940	7863	7784	7704	7622	7539	7455	7369	− 81
10	7282	7194	7105	7014	6921	6826	6729	6632	6533	6432	− 95
11	6331	6228	6124	6020	5913	5805	5696	5586	5474	5362	− 108
12	5248	5132	5016	4898	4780	4660	4538	4415	4291	4166	− 121
13	4040	3912	3784	3654	3523	3391	3257	3122	2986	2850	− 133
14	2712	2572	2431	2289	2147	2003	1858	1711	1564	1416	− 145
15	1266	1114	0962	0809	0655	0499	0343	0185	0026	9865*	− 156
16	0·9989705	9542	9378	9214	9048	8881	8713	8544	8373	8202	− 168
17	8029	7856	7681	7505	7328	7150	6971	6791	6610	6427	− 178
18	6244	6058	5873	5686	5498	5309	5119	4927	4735	4541	− 190
19	4347	4152	3955	3757	3558	3358	3158	2955	2752	2549	− 200
20	2343	2137	1930	1722	1511	1301	1090	0878	0663	0449	− 211
21	0233	0016	9799*	9580*	9359*	9139*	8917*	8694*	8470*	8245*	− 221
22	0·9978019	7792	7564	7335	7104	6873	6641	6408	6173	5938	− 232
23	5702	5466	5227	4988	4747	4506	4264	4021	3777	3531	− 242
24	3286	3039	2790	2541	2291	2040	1788	1535	1280	1026	− 252

---

[1] P. Chappuis, *Trav. et Mém.*, 1907, tome xiii.

TABLE I (*continued*)

°C.	0·0.	0·1.	0·2.	0·3.	0·4.	0·5.	0·6.	0·7.	0·8.	0·9.	Mean Differences.
25	0770	0513	0255	9997*	9736*	9476*	9214*	8951*	8688*	8423*	261
26	0·9968158	7892	7624	7356	7087	6817	6545	6273	6000	5726	271
27	5451	5176	4898	4620	4342	4062	3782	3500	3218	2935	280
28	2652	2366	2080	1793	1505	1217	0928	0637	0346	0053	- 289
29	0·9959761	9466	9171	8876	8579	8282	7983	7684	7383	7083	- 298
30	6780	6478	6174	5869	5564	5258	4950	4642	4334	4024	- 307
31	3714	3401	3089	2776	2462	2147	1832	1515	1198	0880	- 315
32	0561	0241	9920*	9599*	9276*	8954*	8630*	8304*	7979*	7653*	- 324
33	0·9947325	6997	6668	6333	6007	5676	5345	5011	4678	4343	- 332
34	4007	3671	3335	2997	2659	2318	1978	1638	1296	0953	- 340
35	0610	0267	9922*	9576*	9230*	8883*	8534*	8186*	7837*	7486*	- 347
36	0·9937136	6784	6432	6078	5725	5369	5014	4658	4301	3943	- 355
37	3585	3226	2866	2505	2144	1782	1419	1055	0691	0326	- 362
38	0·9929960	9593	9227	8859	8490	8120	7751	7380	7008	6636	- 370
39	6263	5890	5516	5140	4765	4389	4011	3634	3255	2876	- 377
40	2497	2116	1734	1352	0971	0587	0203	9818*	9433*	9047*	- 384
41	0·9918661	..	..	..	..	..	..	..	..	..	..

DENSITY OF WATER IN GRM. PER ML.[1]

°C.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
40	0·992244	1858	1466	1066	0658	0244	9823*	9395*	8960*	8518*
50	0·988070	7615	7154	6686	6222	5731	5245	4752	4253	3748
60	0·983237	2720	2197	1668	1134	0594	0047	9496*	8939*	8376*
70	0·977808	7234	6655	6071	5481	4886	4285	3679	3068	2452
80	0·971831	1205	0573	9937*	9295*	8649*	7998*	7341*	6680*	6014*
90	0·965343	4668	3987	3302	2612	1918	1218	0514	9806*	9093*
100	0·958375	7653	6926	..	..	..	..	..	..	..

[1] M. Thiesen, *Wiss. Abhand. der Phys. Tech. Reich.*, 1904, Band iv. p. 32.

TABLE II

DENSITY OF MERCURY IN GRM. PER C.C.

°C.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	13·5951$_5$	927	902	878	853	828	804	779	754	730
10	13·5705	680	656	631	607	582	558	533	508	484
20	13·5459	435	410	386	361	337	312	288	263	239
30	13·5214	190	165	141	116	092	067	043	019	994*
40	13·4970	..	..	..	..	..	..	..	..	..

DENSITY OF MERCURY IN GRM. PER C.C.

°C.	0.	10.	20.	30.	40.	50.	60.	70.	80.	90.
0	13·5951$_5$	13·5705	13·5459	13·5214	13·4970	13·4726	13·4483	13·4240	13·3998	13·3756
100	13·3515	13·3274	13·3034	13·2794	13·2554	13·2315	13·2076	13·1838	13·1600	13·1361
200	13·1123	13·0886	13·0648	13·0410	13·0173	12·9935	12·9697	12·9459	12·9221	12·8982
300	12·8743	..	..	..	..	..	..	..	..	..

Note.—Density of mercury at 0° C. = 13·59545 grm. per ml. (Thiesen and Scheel, *Tätigkeitsber. der Phys. Tech. Reich.*, February 1, 1897, and *Zeit. Inst.*, 1898, xviii. 138). Density of mercury at 0° C. = 13·5956 grm. per ml. (Marek, *Trav. et Mém.*, 1883, tome ii. p. D58).

Mean of the above two values = 13·59552 grm. per ml., *i.e.* 13·59515 grm. per c.c., which was the value used as the basis of the above tables.

The following expansion formula was used:

$$V_t = V_0[1 + 10^{-6}\{181·456t + 0·009205t^2 + 0·000006608t^3 + 0·000000067320t^4\}]$$

(*Phys. Soc. Proc.*, 1913–14, xxvi. 96).

§ (16) DETERMINATION OF THE DENSITY OF SOLIDS. (A) By the *Hydrostatic Method, i.e.* by weighing in air and in water (or in other suitable liquids).

(i.) *For a Solid denser than Water and unacted on by Water.*—(If water is unsuitable, some other liquid can often be chosen.) If the solid is first weighed in air, and then weighed[1] suspended, so as to be immersed in distilled water, an approximate value of its density can be obtained from the expression

$$\frac{\text{Weight of solid in air}}{\text{Weight of solid in air} - \text{Weight of solid in water}},$$

in accordance with Archimedes' principle. This value requires correction for the buoyancy of the air and for the deviation, from standard (unit) value, of the water density at the temperature of the weighing.

Let d = the approximate value of the density of the solid as obtained in this way.
D = the true density of the solid.
V = the volume of the solid.
d_W = the density of distilled water at the temperature of the weighing.
σ = the density of the air at the time of the weighing in air.

Then the weight of the solid in air $= V(D-\sigma)$, and the weight of the solid in water $= V(D-d_W)$.

$$\therefore d = \frac{V(D-\sigma)}{V(D-\sigma)-V(D-d_W)} = \frac{D-\sigma}{d_W-\sigma},$$

$$\therefore D = d(d_W-\sigma)+\sigma,$$

i.e. the correction to be applied to the approximate value, d, in order to obtain the true value, D, of the density of the solid, is

$$D - d = d(d_W - \sigma - 1) + \sigma.$$

The values of this correction are given in Table III., corresponding to a number of different temperatures and values of the approximate density.

This table is sufficient to meet cases where the density is required with good precision, *e.g.* to 1 part in 1000, or better, provided of course that the weighings have been made to a corresponding order of accuracy.

In a comparatively limited number of cases where the utmost precision is required, *e.g.* in the determination of physical constants, the density should be worked out from first principles, somewhat on the lines shown in § (15) (i.), where the determination of the density of a liquid by pyknometer is discussed.

In Table I. it is assumed that the temperature of the air is the same as that of the water in which the weighing has been made. All the weights used in obtaining equilibrium have been assumed to be of the same density. In special cases it may be necessary to make separate allowances for departures from these assumptions.

(ii.) *Limitations to the Accuracy of the Hydrostatic Method of determining Density.*—In order to determine the weight of a solid specimen in a liquid, it is usual to suspend it by a thin wire or filament from an arm of the balance. This at once limits the accuracy of the weighing owing to the force exerted by the surface tension of the liquid on the wire. The usual procedure is first to weigh the solid specimen with a definite length of suspension wire immersed in the liquid, and then to weigh the suspension alone, with the same length of wire immersed, the specimen having been removed. The difference between these two weighings gives the weight of the specimen in the liquid, provided that the effect of the surface tension of the liquid on the wire is the same during both weighings.

In practice the angle of contact between the wire and the liquid (whether water or not) is likely to vary, and the total force on the wire due to surface tension is likely to accommodate itself in order to produce equilibrium. There is a small range of load over which equilibrium is obtainable with the balance. In other words, the weight of the specimen in the liquid is indefinite beyond certain small limits depending on the diameter of the wire, the nature of the liquid, and the general cleanness of the wire and liquid.

Since for a given liquid the resultant force on the wire due to surface tension diminishes with the diameter of the latter, it is advisable to use the thinnest wire consistent with bearing the weight of the specimen. For example, a platinum wire of 2 mils[2] (0·05 mm.) diameter is the finest that can be used to withstand loads up to about 50 grammes; but the maximum downward pull on the wire due to the surface tension of the liquid (assumed to be water) is 0·0012 gramme, and the weighing is limited in accuracy by variations (say to the extent of a few tenths of a milligramme) in this force. It is, therefore, clear that beyond a certain limit it is of no avail to increase the sensitiveness of the balance for hydrostatic weighings.

In these circumstances the maximum accuracy is obtained by using as large a solid specimen as possible, since the minimum diameter of available suspension wire varies as the square root of the load, and the surface tension effect becomes relatively less important as the load increases.

Other precautions to be taken in using the hydrostatic method for the determination of the density of solids are not given here in detail. It is recognised, of course, that steps should be taken to remove any gaseous[1] film or bubble which may be trapped between the solid and the liquid. If necessary, the liquid containing the suspended specimen should be put under low pressure for a time so as to remove the entangled air. The use of recently distilled water is to be recommended in this connection.

---

[1] The specimen may either be suspended directly from a wire, or else placed in a suitable vessel or cage which is itself suspended from a wire.

[2] 1 mil = 1/1000 in.

The relative importance of the two errors due to

(1) force of surface tension on the suspension wire,

(2) entanglement of air by the specimen,

depends on the size of the specimen used.

In cases of specimens denser than water, the proportionate accuracy of the resulting density may be taken as equal to the proportionate accuracy of determination of the weight of water displaced by the specimen. This, of course, should be roughly estimated before the experiment is commenced. It would require a specimen whose volume is at least 5 c.c. to ensure that a proportionate accuracy of 1 part in 10,000 is obtained in the final value of its density.

(iii.) *For a Solid which floats in Water (or in the Liquid chosen for the Hydrostatic Weighing).*—If the solid floats on the liquid chosen, a sinker must be attached to it. This does not necessarily increase the number of weighings made, or complicate the working up of the results from the simple formula and the application of the correction table (Table I.). The sinker may be regarded as part of the suspension wire, which is weighed first with, then without, the specimen.

(iv.) *By Nicholson's Hydrometer.*—An alternative plan depending upon the same fundamental principle, but not susceptible of such good accuracy as the foregoing method, is to use a Nicholson's hydrometer. This instrument belongs to the class of hydrometers in which the volume of the instrument immersed is kept constant. It has two cups or pans, a lower one (immersed) and an upper one (not immersed), and can be made to float in water up to a mark on its stem by adding known weights to the upper pan.

The specimen whose density is required is weighed first in air from the upper pan of the hydrometer, and then in water from the lower pan, balance being obtained by varying the known weights on the upper pan until the hydrometer floats up to the reference mark on the stem. From the observations thus made, the weight of water displaced by the solid can be obtained, together with its weight in air. The relative density of the solid can then be calculated.

This instrument can also be used in the case of a specimen less dense than water if the lower pan is suitably shaped, or if a small wire cage is made in which the specimen can be put in order to keep it from floating.

(B) *For a Small Solid Specimen.*—Beyond a certain limiting smallness, the hydrostatic method is clearly impracticable, and methods C, D, and E should be considered according to the nature and amount of specimen at hand. If a large quantity of the specimen is available in the form of fragments, it may be weighed in water by using a suitable receptacle, hanging in the water, from the usual wire suspension. In the case of small fragments and powders, it is not easy to avoid errors due to the adhesion of air to the specimen.

(C) *By the Specific Gravity Bottle.*—(For a solid specimen or for fragments of a solid.) The familiar specific gravity bottle offers a somewhat different method of determining the density of a solid which is only available in small fragments. The form and capacity of the bottle may vary widely according to the special purpose for which it is used. Fundamentally, like the hydrostatic method of (A) (i.), it is the means of obtaining the weight of water displaced by the specimen, and hence its volume.

The specimen, whose weight in air is known, is placed in a clean dry specific gravity bottle, which is then filled up to the top (or to a definite mark) with distilled water or other suitable liquid. After the bottle and its contents are weighed, the bottle is emptied, then filled to the same mark with distilled water, and weighed again. These two weighings give the difference between the weight of the specimen and that of an equal volume of water.

The density of the specimen can then be calculated, the usual corrections being made (see Table III.) for atmospheric buoyancy and for departure of the temperature of the water from standard conditions.

Some precautions are necessary in using this method. Solids, especially in the form of small fragments, filings, or powder, entangle an appreciable quantity of air, which replaces some of the water in the bottle. A considerable[2] proportion of this air may be removed by using a vacuum pump. In order to avoid this source of error, M. Billy[3] has worked with a specific gravity bottle, filled with carbon dioxide, to which he admitted caustic potash solution and the specimen under examination. Any gas entangled by the fragments of the specimen is dissolved by the caustic potash.

The uncertainty in the weight of the bottle containing water, owing to slight evaporation of the liquid during the process of weighing, has been overcome in some types of pyknometer.

---

[1] See Le Chatelier and Bogitch, *Comptes Rendus*, 1916, clxiii. 459.

[2] See Guye and Zacharlades, *Comptes Rendus*, 1909, cxlix. 593.

[3] *Comptes Rendus*, 1913, clvi. 1065.

TABLE III

DENSITY DETERMINATION CORRECTIONS

(For use in the method of weighing a solid in air and in water.)

Note 1.—This table shows the correction to be applied to the approximate value (*d*) of the density of a solid in order to obtain its true density in grammes per millilitre. The correction is additive when its sign is +, and subtractive when its sign is −.

Note 2.—The corrections have been calculated for an atmospheric pressure of 760 mm. of mercury under standard conditions. The values vary to a small extent with the atmospheric pressure, but may be taken as accurate to within ± 2 per cent for any pressure between 740 and 800 mm.

Approximate Density.	Temperature of Water in which the Weighing is made.							
	10° C.	11° C.	12° C.	13° C.	14° C.	15° C.	16° C.	17° C.
0·01	+0·00123	+0·00122	+0·00122	+0·00121	+0·00121	+0·00120	+0·00119	+0·00119
0·10	+0·00109	+0·00108	+0·00106	+0·00105	+0·00103	+0·00101	+0·00099	+0·00097
1·0	−0·00027	−0·00037	−0·00047	−0·00060	−0·00073	−0·00087	−0·00103	−0·00120
2·0	−0·0018	−0·0020	−0·0022	−0·0024	−0·0027	−0·0030	−0·0033	−0·0036
3·0	−0·0033	−0·0036	−0·0039	−0·0042	−0·0046	−0·0051	−0·0055	−0·0060
4·0	−0·0048	−0·0052	−0·0056	−0·0061	−0·0066	−0·0072	−0·0078	−0·0084
5·0	−0·0063	−0·0068	−0·0073	−0·0079	−0·0085	−0·0093	−0·0100	−0·0108
6·0	−0·0079	−0·0084	−0·0090	−0·0097	−0·0105	−0·0114	−0·0123	−0·0133
7·0	−0·0094	−0·0100	−0·0107	−0·0116	−0·0125	−0·0134	−0·0145	−0·0157
8·0	−0·0109	−0·0116	−0·0124	−0·0134	−0·0144	−0·0155	−0·0168	−0·0181
9·0	−0·0124	−0·0132	−0·0142	−0·0152	−0·0164	−0·0176	−0·0190	−0·0205
10·0	−0·0139	−0·0148	−0·0159	−0·0170	−0·0183	−0·0197	−0·0213	−0·0229
11·0	−0·0154	−0·0164	−0·0176	−0·0189	−0·0203	−0·0218	−0·0235	−0·0253
12·0	−0·0170	−0·0181	−0·0193	−0·0207	−0·0222	−0·0239	−0·0257	−0·0277
13·0	−0·0185	−0·0197	−0·0210	−0·0225	−0·0242	−0·0260	−0·0280	−0·0301
14·0	−0·0200	−0·0213	−0·0227	−0·0243	−0·0261	−0·0281	−0·0302	−0·0326
15·0	−0·0215	−0·0229	−0·0244	−0·0262	−0·0281	−0·0302	−0·0325	−0·0350
16·0	−0·0230	−0·0245	−0·0261	−0·0280	−0·0301	−0·0323	−0·0347	−0·0374
17·0	−0·0245	−0·0261	−0·0278	−0·0298	−0·0320	−0·0344	−0·0370	−0·0398
18·0	−0·0261	−0·0277	−0·0295	−0·0317	−0·0340	−0·0365	−0·0392	−0·0422
19·0	−0·0276	−0·0293	−0·0313	−0·0335	−0·0359	−0·0386	−0·0415	−0·0446
20·0	−0·0291	−0·0309	−0·0330	−0·0353	−0·0379	−0·0407	−0·0437	−0·0470
21·0	−0·0306	−0·0325	−0·0347	−0·0371	−0·0398	−0·0428	−0·0460	−0·0494
22·0	−0·0321	−0·0341	−0·0364	−0·0390	−0·0418	−0·0449	−0·0482	−0·0519

Approximate Density.	Temperature of Water in which the Weighing is made.							
	18° C.	19° C.	20° C.	21° C.	22° C.	23° C.	24° C.	25° C.
0·01	+0·00118	+0·00118	+0·00117	+0·00116	+0·00116	+0·00115	+0·00114	+0·00114
0·10	+0·00095	+0·00093	+0·00090	+0·00088	+0·00085	+0·00082	+0·00080	+0·00077
1·0	−0·00138	−0·00157	−0·00177	−0·00198	−0·00220	−0·00243	−0·00268	−0·00293
2·0	−0·0040	−0·0043	−0·0047	−0·0052	−0·0056	−0·0061	−0·0065	−0·0070
3·0	−0·0065	−0·0071	−0·0077	−0·0083	−0·0090	−0·0097	−0·0104	−0·0111
4·0	−0·0091	−0·0099	−0·0107	−0·0115	−0·0124	−0·0133	−0·0143	−0·0152
5·0	−0·0117	−0·0127	−0·0136	−0·0147	−0·0158	−0·0169	−0·0181	−0·0194
6·0	−0·0143	−0·0154	−0·0166	−0·0179	−0·0192	−0·0205	−0·0220	−0·0235
7·0	−0·0169	−0·0182	−0·0196	−0·0210	−0·0226	−0·0242	−0·0258	−0·0276
8·0	−0·0195	−0·0210	−0·0226	−0·0242	−0·0260	−0·0278	−0·0297	−0·0317
9·0	−0·0221	−0·0237	−0·0255	−0·0274	−0·0294	−0·0314	−0·0335	−0·0358
10·0	−0·0247	−0·0265	−0·0285	−0·0306	−0·0327	−0·0350	−0·0374	−0·0399
11·0	−0·0272	−0·0293	−0·0315	−0·0337	−0·0361	−0·0386	−0·0413	−0·0440
12·0	−0·0298	−0·0321	−0·0344	−0·0369	−0·0395	−0·0423	−0·0451	−0·0481
13·0	−0·0324	−0·0348	−0·0374	−0·0401	−0·0429	−0·0459	−0·0490	−0·0522
14·0	−0·0350	−0·0376	−0·0404	−0·0433	−0·0463	−0·0495	−0·0528	−0·0563
15·0	−0·0376	−0·0404	−0·0433	−0·0464	−0·0497	−0·0531	−0·0567	−0·0604
16·0	−0·0402	−0·0431	−0·0463	−0·0496	−0·0531	−0·0567	−0·0605	−0·0645
17·0	−0·0428	−0·0459	−0·0493	−0·0528	−0·0565	−0·0504	−0·0644	−0·0686
18·0	−0·0453	−0·0487	−0·0522	−0·0560	−0·0599	−0·0640	−0·0683	−0·0727
19·0	−0·0479	−0·0515	−0·0552	−0·0591	−0·0633	−0·0676	−0·0721	−0·0768
20·0	−0·0505	−0·0542	−0·0582	−0·0623	−0·0667	−0·0712	−0·0760	−0·0809
21·0	−0·0531	−0·0570	−0·0611	−0·0655	−0·0701	−0·0749	−0·0798	−0·0850
22·0	−0·0557	−0·0598	−0·0641	−0·0687	−0·0735	−0·0785	−0·0837	−0·0892

(D) *By Volumenometer.*—Volumenometers may be considered in two classes, according as they directly or indirectly measure the volume of the solid specimen. The former class comprises graduated vessels such as burettes, flasks, etc., which can be used in some cases in conjunction with a liquid which does not act upon the specimen whose volume is to be measured. The other class of volumenometer depends on measuring the volume of a given enclosure of air by the change of pressure required to decrease (or increase) that volume by a known amount, first with the specimen placed in the enclosure, then with the specimen removed. This class of volumenometer avoids the use of a liquid in contact with the specimen, and therefore provides the only suitable means of determining the density of a solid which, owing to its nature, cannot be immersed in a liquid (*e.g.* a solid specimen which is decomposed by liquid). The same method is applicable to such bodies as cotton or glass wool, provided that the wool is not so closely packed that the equalisation of the air pressures without and within the specimen is hindered.

A number of volumenometers have been described in scientific journals. Among these, reference may be made to the instruments of Bremer,[1] Oberdeck,[2] Zehnder,[3] and Carman.[4] It is clear, however, that volumenometers of this type cannot be used for the determination of the densities of substances which give off water vapour or any gas.

(E) *By Flotation.*—In this method, which is especially suitable for small fragments of certain crystals and minerals, the density is determined from a knowledge of the density of the liquid in which the solid specimen will just float. The range of densities covered by this method is limited by that of the liquid available. Methylene iodide has an approximate density 3·3, and by dilution with benzene, toluene, or xylene, the density of the mixture may be reduced to 0·9. Substances which are attacked by these liquids may be floated in aqueous solutions of potassium mercuric iodide or barium mercuric iodide. The latter salt is not always suitable.

The flotation method is a very sensitive criterion of the density of a small specimen. In practice equilibrium is easily disturbed even by small changes of temperature of the liquid; and if the specimen takes the form of a powder, some of the particles will be found to sink and some to rise when the best adjustment of the density of the liquid has been made.

The liquid mixtures must be chosen so that the solid specimen is unacted upon by it. The accuracy of the flotation method is limited by the adhesion of a thin layer of air to the surface of the specimen. Owing to the small size of each fragment of the specimen, the error due to this cause is relatively larger than it is in other methods of determining density.

The flotation method may also be usefully applied to a number of waxes and fats.

§ (17) DEGREE OF CONSTANCY OF DENSITY OF SUBSTANCES OF NOMINALLY THE SAME MATERIAL.—The density of a specimen may be used as a criterion of the degree of its purity or consistency. In many cases the accuracy of determination of density lies within the limits of consistency of the substance, and has been used in explaining small changes which occur in the internal structure of solids.

For example, the density of a given metal depends, to some extent, on its internal structure. As a general rule, working a metal diminishes its density,[5] while complete annealing increases it.

In another direction[6] Le Chatelier and Wologdine have attributed the very discordant values previously obtained for the density of graphite to the impurity of the specimen, and have succeeded in purifying a number of specimens supplied from widely different sources, obtaining almost identical values for their density.

The consistency of the density of ice has been investigated by Nichols,[7] Vincent,[8] and other workers,[9] who have found differences of density of the order 1 or 2 parts in 1000 between new and old ice, and also between natural and artificial ice. These have been attributed to the effect of dissolved air in the ice.

Porous Bodies.—Porous bodies may be regarded as containing a large number of cells of air or gas, some of which are effectively sealed from communication with the external atmosphere.

The density of a porous body is usually held to mean the average density of the cellular and solid material contained in the specimen, *i.e.* the mass of unit volume of porous substance. Wherever the actual density of the solid material is required, the porous specimen should preferably be reduced to a powder or to small fragments, if good accuracy is to be obtained.

§ (18) DETERMINATION OF THE DENSITIES OF GASES.—The densities of gases, like those of solids or liquids, are usually referred ultimately to the density of water at 4° C. In other words, they are measured in terms of the kilogramme and litre as units.

---

[1] *Rec. trav. chim.*, 1898, xvii. 263, 405.

[2] *Wied. Ann.*, 1899, lxvii.

[3] *Annalen der Physik*, 1903, x. 40; *ibid.*, 1904, xv. 328.

[4] *Phys. Rev.*, 1908, xxvi. 396.

[5] Lowry and Parker, *Chem. Soc. Trans.*, 1915, cvii. 1005.

[6] *Comptes Rendus*, 1908, cxlvi. 49.

[7] *Phys. Rev.*, 1899, viii. 184.

[8] *Phil. Trans.*, 1902, cxcviii. 463.

[9] See summary by Barnes, *Roy. Soc. Canada Trans.*, 1909, iii. 3.

It is often convenient to express the density of a gas relatively in terms of some other gas as standard. Hydrogen was first used as reference standard, being the lightest known gas, but since its density could not be determined with the same proportionate accuracy as that of other gases, oxygen was finally chosen as standard of reference instead.

In determining the mass of unit volume of a gas, or the relative densities of two gases, the conditions of temperature and pressure under which they are confined are of importance owing to the magnitude of their influence on the resulting density. It is preferable to express the density of the gas under standard conditions, *i.e.* at 0° C. and under a pressure of 760 mm. of mercury measured at 0° C. and at standard gravity.[1] As regards measurement of pressure, reference should be made to the article "Barometers and Manometers."

The accuracy of measurement of temperature is equally important, and in work of the highest precision the gas whose density is required is kept at 0° C. while its pressure is being measured.

Methods of Determination : (i.) *By weighing the Gas contained in a Globe of predetermined Capacity.*—This method has been worked out in detail by Rayleigh, Ramsay, Morley, Leduc, and others, and has been found most susceptible of high accuracy provided the amount of gas available is sufficiently large.

Briefly, the experimental method adopted by them is as follows :

A glass globe, consisting of a spherical bulb sealed to a capillary glass stop-cock, is weighed, first when thoroughly evacuated, and then when filled with dry gas under measured temperature and pressure. The evacuation and filling of the globe are repeated several times. These weighings give the mass of a definite volume of gas provided that accurate correction is made for the compressibility of the globe owing to its contraction in volume on being exhausted. (For Rayleigh's method of making this allowance see *Roy. Soc. Proc.*, 1892, l. 460.) The volume of the bulb is determined by weighing it full of water.

Throughout the weighings, the greatest care should be taken to minimise errors which arise from

(1) The large buoyancy of the bulb.

(2) The condensation of moisture on the surface of the bulb.

The best plan is to use, as a counterpoise, a second bulb which is as nearly as possible similar to the first one, both as regards dimensions and material.[2]

This method has been used in the determination of the densities of the principal gases. In this connection, the following references should be consulted :

Rayleigh (for air, oxygen, nitrogen), *Roy. Soc. Proc.*, 1888, xliii. 356 ; 1889, xlv. 425 ; 1892, l. 448 ; 1893, liii. 134.

Leduc (for air, oxygen, nitrogen), *Ann. Chim. Phys.*, 1898, xv. 27 ; *Travaux et Mémoires du Bureau Internatl.*, 1917, tome xvi.

Morley (for oxygen and hydrogen), *Zeits. Phys. Chem.*, 1915, xix. 437.

Germann (for oxygen), *Journ. Phys. Chem.*, 1915, xix. 437.

Ramsay and Travers (for argon and its companions), *Phil. Trans.*, 1901, cxcvii. 47.

(ii.) *By Means of the Micro-balance.*—Although during the last few years the micro-balance, described above in § (6), has been used with striking success to measure the densities of some of the rarer gases, it is by no means a recently devised instrument.

The foregoing method (i.) of weighing a known volume of gas in a glass bulb yields results of very high accuracy so long as that volume is large enough, but in the case of the rarer gases the amount of gas available is so small as to render this method impracticable when high accuracy is required. Since, however, the accuracy of determination of the density of a very small quantity of gas by means of a balance must ultimately be limited by the sensitiveness of that balance, the micro-balance has been looked upon as offering the best chance of obtaining increased sensitiveness. This type of balance operates on the principle that a change in weight is measured by the change in the net[3] buoyant force on the balance due to the gas in which the small balance is suspended, the pressure of the gas being adjustable, and measured by means of a mercury manometer connected with the balance case.

Chemical weights as used with the ordinary balance are replaced by the pressure readings of the manometer used with the micro-balance, allowance being made for temperature changes.

By using the micro-balance in atmospheres of two gases in turn, the relative densities of these gases may be determined, being inversely proportionate to the pressure required to bring the balance to some convenient position of equilibrium, provided the temperature is constant.

The general design of quartz micro-balances has been discussed under § (6).

The densities of the following gases have been measured to a high degree of accuracy by means of the micro-balance: helium by T. S. Taylor, *Phys. Rev.*, 1917, x. 653; radium emanation by Gray and Ramsay, *Roy. Soc. Proc.*, 1910, lxxxiv. 536.

---

[1] Standard gravity is the value of gravity at mean sea-level in latitude 45°. Its value has been accepted as 980·665 cm./(sec.)^2 by the International Conference on Weights and Measures held in Paris, 1913; but see "Atmosphere, Physics of."

[2] The amount of condensation of moisture on glass varies with its composition. Silica would probably be an improvement on glass in this respect.

[3] The beam of the balance is not symmetrical, one arm containing a bulb.

Other balances based on the same principle, but intended for the commercial measurement of the densities of gases, have been designed by Edwards[1] and Arndt.[2]

The method of determining the density of a gas by means of a micro-balance is comparatively quick in practice. Although capable of very high accuracy, it is also convenient to use in cases where only moderate accuracy is required. Aston[3] measured the density of neon to within 1 part in 1000 by means of his quartz micro-balance. Less than a cubic centimetre of the gas was used.

A somewhat similar balance had previously been used by Gray.[4]

(iii.) *Other Methods.*—Of the other methods devised for measuring densities of gases, reference should be made to the ingenious method of Schloesing,[5] which is also suitable when only small quantities of the experimental gas are available. Jaquerod[6] and Tourpaian have used a hydrostatic method of weighing a glass cylinder of known volume in the gas whose density is required.

A method somewhat similar in principle to Hare's method for measuring the relative densities of liquids (§ (15)) has been described by Threlfall.[7] It involves, however, comparatively large apparatus, but can be used to measure relative densities of gases to within 1 part in 2000.

§ (19) DETERMINATION OF THE DENSITIES OF VAPOURS.—The determination of density is an operation which has to be performed much more frequently for a vapour than for a gas.

While it may fairly be claimed that the measurement of the densities of gases has been made to a high degree of precision, the operation of determining the density of a vapour is often attended by considerable experimental difficulties, and is consequently less productive of accuracy.

Present-day methods of measuring the densities of vapours follow the general principles used by Gay-Lussac, Victor Meyer, and Dumas.

(i.) In Gay-Lussac's apparatus a known weight of substance is introduced and vaporised in the vacuum space above the mercury column in a barometer tube which is calibrated so as to read off the volume of the vapour. The temperature of the vapour may usually be taken to be that of the heating[8] jacket surrounding the barometer tube, while the pressure of the vapour can readily be obtained as the difference between the height of the mercury column and that in another barometer tube which contains no vapour. It should be observed that Gay-Lussac's apparatus also admits of determining the compressibility of the vapour.

Ramsay[9] and Steele, in their determination of the vapour densities of carbon compounds, have used an apparatus which is essentially of this form.

It is very important to test the residual pressure in the vacuum space of the tube in which the vapour is to be generated. Serious error may result from the presence of a very small amount of moisture, the full effect of which can be estimated on studying the values of the vapour pressure of water at various temperatures. This precaution should be taken in all methods for the determination of the densities of gases and vapours.

(ii.) In Victor Meyer's method the gas-measuring apparatus in which the volume of the vapour is measured is separate from the tube in which the vapour is generated. The vapour displaces its own volume of air of the same temperature and pressure, expelling it into the gas apparatus, where it is measured at atmospheric temperature and pressure.

High accuracy cannot, however, be obtained by this method.

(iii.) Dumas' method depends on measuring the volume occupied by the vapour of a known mass of substance in a bulb immersed in a suitable bath, but is not frequently used by chemists.

For further details regarding these methods of measuring vapour-densities, reference should be made to the following papers:

Modifications of Gay-Lussac's Method

Thorpe, *Chem. Soc. Trans.*, 1880, xxxvii. 147.

Capstick, *Phil. Trans.*, 1894, clxxxv. 1.

Egerton, *Chem. News*, 1911, civ. 259.

Modifications of Victor Meyer's Method

Biltz and V. Meyer, *Zeits. Phys. Chem.*, 1888, ii. 189; *Chem. News*, 1908, xcvii.

Weiser, *Jour. Phys. Chem.*, 1916, xx. 532.

MacInnes and Kreiling, *Jour. Amer. Chem. Soc.*, 1917, xxxix. 2350.

Modification of Dumas' Method

Schulze, *Physik. Zeits.*, 1913, xiv. 922.

Other Methods

P. Blackman, *Jour. Phys. Chem.*, 1911, xv. 869; *Chem. News*, 1906, xiv. 307.

---

[1] *Tech. Paper, Bureau of Standards*, No. 89, 1917.

[2] *Chem. and Metall. Engineering*, March 15, 1919, p. 291.

[3] *Roy. Soc. Proc.*, 1913, lxxxix. 439.

[4] *Proc. Konink. Akad. Amsterdam*, 1905, vii. 770.

[5] *Comptes Rendus*, 1898, cxxvi. 220, 476, 896.

[6] *Ibid.*, 1910, cli. 666.

[7] *Roy. Soc. Proc.*, 1906, lxxvii. 542.

[8] A liquid of suitable boiling-point is used for heating purposes.

[9] *Phil. Mag.*, 1903, vi. 492.

TABLE IV

DENSITY OF AIR IN GRM. PER LITRE

°C.	720 mm.	725 mm.	730 mm.	735 mm.	740 mm.	745 mm.	750 mm.	755 mm.	760 mm.	765 mm.	770 mm.	775 mm.	780 mm.	785 mm.	790 mm.
5	1·200	1·208	1·217	1·225	1·234	1·242	1·250	1·259	1·267	1·275	1·284	1·292	1·300	1·309	1·317
6	1·196	1·204	1·212	1·221	1·229	1·237	1·246	1·254	1·262	1·271	1·279	1·287	1·296	1·304	1·312
7	1·191	1·200	1·208	1·216	1·224	1·233	1·241	1·249	1·257	1·266	1·274	1·282	1·291	1·299	1·307
8	1·187	1·195	1·203	1·211	1·220	1·228	1·236	1·245	1·253	1·261	1·269	1·278	1·286	1·294	1·302
9	1·182	1·191	1·199	1·207	1·215	1·223	1·232	1·240	1·248	1·256	1·265	1·273	1·281	1·289	1·298
10	1·178	1·186	1·194	1·202	1·211	1·219	1·227	1·235	1·243	1·252	1·260	1·268	1·276	1·284	1·293
11	1·173	1·182	1·190	1·198	1·206	1·214	1·222	1·231	1·239	1·247	1·255	1·263	1·271	1·280	1·288
12	1·169	1·177	1·185	1·193	1·202	1·210	1·218	1·226	1·234	1·242	1·250	1·259	1·267	1·275	1·283
13	1·165	1·173	1·181	1·189	1·197	1·205	1·213	1·221	1·230	1·238	1·246	1·254	1·262	1·270	1·278
14	1·160	1·168	1·176	1·184	1·193	1·201	1·209	1·217	1·225	1·233	1·241	1·249	1·257	1·265	1·274
15	1·156	1·164	1·172	1·180	1·188	1·196	1·204	1·212	1·220	1·228	1·236	1·245	1·253	1·261	1·269
16	1·151	1·160	1·168	1·176	1·184	1·192	1·200	1·208	1·216	1·224	1·232	1·240	1·248	1·256	1·264
17	1·147	1·155	1·163	1·171	1·179	1·187	1·195	1·203	1·211	1·219	1·227	1·235	1·243	1·251	1·259
18	1·143	1·151	1·159	1·167	1·175	1·183	1·191	1·199	1·207	1·215	1·223	1·231	1·239	1·247	1·255
19	1·139	1·146	1·154	1·162	1·170	1·178	1·186	1·194	1·202	1·210	1·218	1·226	1·234	1·242	1·250
20	1·134	1·142	1·150	1·158	1·166	1·174	1·182	1·190	1·198	1·206	1·213	1·221	1·229	1·237	1·245
21	1·130	1·138	1·146	1·154	1·161	1·169	1·177	1·185	1·193	1·201	1·209	1·217	1·225	1·233	1·240
22	1·126	1·133	1·141	1·149	1·157	1·165	1·173	1·181	1·189	1·196	1·204	1·212	1·220	1·228	1·236
23	1·121	1·129	1·137	1·145	1·153	1·161	1·168	1·176	1·184	1·192	1·200	1·208	1·215	1·223	1·231
24	1·117	1·125	1·133	1·140	1·148	1·156	1·164	1·172	1·180	1·187	1·195	1·203	1·211	1·219	1·226
25	1·113	1·120	1·128	1·136	1·144	1·152	1·159	1·167	1·175	1·183	1·191	1·198	1·206	1·214	1·222
26	1·108	1·116	1·124	1·132	1·139	1·147	1·155	1·163	1·171	1·178	1·186	1·194	1·202	1·209	1·217
27	1·104	1·112	1·120	1·127	1·135	1·143	1·151	1·158	1·166	1·174	1·181	1·189	1·197	1·205	1·212
28	1·100	1·108	1·115	1·123	1·131	1·138	1·146	1·154	1·162	1·169	1·177	1·185	1·192	1·200	1·208
29	1·095	1·103	1·111	1·119	1·126	1·134	1·142	1·149	1·157	1·165	1·172	1·180	1·188	1·195	1·203
30	1·091	1·099	1·107	1·114	1·122	1·129	1·137	1·145	1·152	1·160	1·168	1·175	1·183	1·191	1·198

Note.—(1) The values given in the above table relate to air containing 0·04 per cent by volume of CO_2 and two-thirds saturated with water vapour.
(2) The pressures given at the head of the table are millimetres of mercury at 0° C. and standard gravity.
(3) The table was computed from the following relation:

$$\sigma = \frac{1{\cdot}29307p - 0{\cdot}3258h}{760(1+0{\cdot}00367t)},$$

where σ = required air density at t° C. and p mm. pressure.
h = vapour pressure of water at t° C. The values for h were taken from *Wärmetabellen der Phys. Tech. Reich.*, 1919.

The relation is based on the following data:
Density of dry air free from CO_2 = 1·2928 grm. per litre at 0° C. and 760 mm.
" " CO_2 = 1·9768 " " "
Hypothetical density of water vapour = 0·8044 " "

F. A. G.

BALLOONS FOR THE INVESTIGATION OF THE UPPER AIR. See "Air, Investigation of Upper," § (3).
——, KITE. See *ibid.* § (4).
——, PILOT. See *ibid.* § (6).
——, REGISTERING. See *ibid.* § (5).

BALLS, STEEL, elastic compression of, during measurement. See "Gauges," § (16).

BAR. The bar is the dynamical unit of atmospheric pressure, and is equal to 1,000,000 dynes per square centimetre. The pressure of a standard atmosphere is 1,013,193 dynes per square centimetre, so that the bar is approximately the pressure of the standard atmosphere. 1 bar = 29·53 inches = 750·076 mm. at 273° A. in latitude 45°. The bar was approved as a unit of pressure by the Conference of Physicists, Paris, 1900, and was introduced into meteorology by Bjerknes. The unit in use in meteorology is the millibar, one thousandth of a bar. It should be distinguished from the chemical "bar," which is a pressure of one dyne per square centimetre. See also "Barometers and Manometers," § (2) (i.).

BAROGRAPH, ANEROID. See "Barometers and Manometers," § (14).

BAROMETER, ANEROID: an instrument, for the measurement of atmospheric pressures, whose operation depends in principle on the fact that a thin metal disc or membrane responds elastically, to an appreciable degree, to the difference of pressure on its faces. See "Barometers and Manometers," § (10).
- Accuracy of. See *ibid.* § (13) (i.).
- Adjustment and testing of the aneroid mechanism in the works of. See *ibid.* § (12).
- Compensation of, for temperature. See *ibid.* § (11).
- Errors and defects of: "creep" and "hysteresis." See *ibid.* § (13) (ii.).
- Influence of the rate of change of pressure on the calibration of. See *ibid.* § (13) (iii.).
- Limitation of the amount of "creep." See *ibid.* § (13) (v.).
- Vacuum chamber of. See *ibid.* § (10) (ii.).

BAROMETER, FUNDAMENTAL STANDARD. See "Barometers and Manometers," § (9).

BAROMETER, GAUGE-: a barometer which can be put in connection with any artificial gas pressure. See "Barometers and Manometers," § (3) (vi.).

BAROMETER, KEW PATTERN:
- Cistern errors in, tabulated. See "Barometers and Manometers," § (7) (ii.) (*b*), Table IV.
- Temperature correction to. See *ibid.* § (6) (i.) (*b*).
- See also *ibid.* § (3) (iv.).

BAROMETER, MARINE:
- Errors in the use of: pumping of mercury barometers at sea. See "Barometers and Manometers," § (8) (ii.) (*b*).
- Influence of the velocity of a ship on the effective value of gravity acting on the mercury column. See *ibid.* § (8) (ii.) (*c*).
- Kew pattern of. See *ibid.* § (3) (v.) (*b*).

BAROMETER, MERCURY:
- Accuracy and permanence of, as a pressure indicator. See "Barometers and Manometers," § (8).
- Capillary depression of, in tubes of given bores and given values of the meniscus height, tabulated. See *ibid.* § (7) (ii.), Table II.
- Conditions essential to the success of: (*a*) cleanness and dryness of the mercury and tube; (*b*) a good vacuum above the barometric column in the tube. See *ibid.* § (4) (i.).
- Correction of, for gravity. See *ibid.* § (6) (ii.)
- Correction of, for temperature: the Fortin and Siphon types of barometer. See *ibid.* § (6) (i.) (*a*).
- Correction of, for temperature and gravity variations, when graduated in millibars. See *ibid.* § (6) (iii.).
- Correction of standard temperature of, for variation of gravity with height, being diminished by 1° A. for every 520 metres (for a Fortin type barometer) of height above mean sea-level. See *ibid.* § (6) (iii.).
- Errors and defects of. See *ibid.* § (7).
- Errors due to capillary action of the mercury. See *ibid.* § (7) (ii.).
- Errors due to lack of verticality. See *ibid.* § (7) (v.).
- Extent of errors in indications of, due to variation of capillary action in the tube. See *ibid.* § (7) (ii.) (*a*).
- Limitations to the accuracy of, in the measurement of atmospheric pressures: the effect of a wind. See *ibid.* § (8) (ii.) (*a*).
- Method of reading, for ordinary precision. See *ibid.* § (5) (i.).
- Methods of reading, for high precision; methods of setting mercury levels to points. See *ibid.* § (5) (iii.) (*a*). Optical methods. See *ibid.* § (5) (iii.) (*b*).
- Preparation, adjustment, and testing of; preparation of tubes. See *ibid.* § (4) (i.).
- Standard pattern of. See *ibid.* § (9).
- Testing of, comprising—(1) preliminary inspection of barometer to discover errors of design, workmanship, and adjustment; (2) series of comparisons, at current atmospheric pressures, between barometer and a working standard barometer. See *ibid.* § (4) (iv.).
- Types of: not self-recording. See *ibid.* § (3) (vii.) (*a*).

Types of: self-recording. See *ibid.* § (3) (vii.) (*b*).

BAROMETER, MOVABLE SCALE: an instrument in which the zero of the scale is adjusted to the level of the mercury in the cistern. Two designs to be noted are (*a*) the Newman pattern, (*b*) the observatory barometer. See "Barometers and Manometers," § (3) (iii.).

BAROMETER IN PRACTICE. See "Barometers and Manometers," § (15).

BAROMETER TUBES, THE FILLING OF. See "Barometers and Manometers," § (4) (ii.).

BAROMETERS, FORTIN AND KEW, GENERAL ACCURACY OF, tabulated. See "Barometers and Manometers," § (8) (i.), Table VI.

BAROMETERS AND MANOMETERS

THIS article deals only with those instruments which are used for measuring (*a*) such atmospheric pressures as occur in nature, or (*b*) such artificial gas pressures as lie within similar limits, together with instruments which, though barometers in principle, are used to measure in units other than pressures.

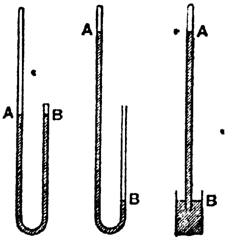
I

§ (1) BROAD PRINCIPLES OF METHODS OF MEASURING PRESSURES.—Pressure is defined as force per unit area, and is naturally measured in terms of weight.[1] In the measurement of air and gas pressures with which this article is concerned the fundamental idea is to balance the pressure against a column of liquid whose weight is known, provided that

(*a*) The height of the column,
(*b*) Its density,
(*c*) The value of gravity acting upon it,

are known.

The principle is familiar, and sufficiently well indicated in the diagrams, *Figs.* 1, 2, and 3, in which the weight of the liquid column between the levels A and B balances the difference of air pressures on the two liquid surfaces, A and B. By arranging for a closed end to the tube above A, with the space above A completely exhausted of all gas and vapour,[2] the liquid column can be used to measure the pressure of the air at the level B, and the manometer of *Fig.* 1 becomes the barometer of *Figs.* 2 and 3. The present-day liquid barometer for measuring atmospheric pressures is in principle essentially as shown in *Figs.* 2 and 3.



FIGS. 1, 2, and 3.

Mercury is practically the only liquid that can be used in barometers that register atmospheric pressures. In addition to having such a large specific gravity that the length of the barometric column is not too cumbersome, its vapour pressure is so small as to be negligible, at all ordinary working temperatures, except in fundamental standard barometers of the highest precision.

Usually in barometry the chief concern is to determine the height of the mercury column, and to use the height as a measure of the pressure, provided that the value is duly corrected to refer to mercury under standard conditions of temperature and gravity (§ (6)).

Accordingly, either the height of the barometric column is measured by a cathetometer, or else a scale is set up near the mercury column as part of the barometer, and suitable means provided for reading the liquid level on the scale. The latter method is usual in the meteorological barometer; the cathetometer is sometimes used in the laboratory when great accuracy is required, and is generally an adjunct of the primary fundamental barometer as used in the various national standardisation laboratories (§ (9)).

Alternatively, a medium which is sensitive to changes of pressure may be used as a barometer. The elastic properties of a thin metal membrane or diaphragm have been used to indicate the difference between the pressures on its two surfaces.

By using a vacuum-box formed of two membranes the present-day aneroid barometer has been evolved, indicating pressures through the medium of the movement of one of the membrances of its vacuum-box, the other being fixed at its centre.

It is customary to use a system of levers to obtain sufficient magnification of the movement of the diaphragm to indicate it on a suitably open scale. This has generally proved satisfactory for all aneroids whether self-recording or not, and is convenient for a portable instrument.

Broadly speaking, the instruments used for atmospheric barometry belong to one or other of the two general types just indicated.

II

§ (2) UNITS OF MEASUREMENT OF PRESSURE. (i.) *The Mercury Unit.*—Since the principal item in the determination of pressure by the mercury barometer is the measurement of the height of the mercury column, it is natural that the scale of this instrument should be graduated in pure length units, *i.e.* in inches or millimetres.

---

[1] See "Pressure, Measurement of," Vol. I.
[2] Except the vapour of the liquid.

The English standard of length being defined at a temperature of 62° F., the inch barometer is designed so that when its temperature is 62° F. the instrument measures true inches of mercury at 62° F. Similarly, since the International Prototype Metre is defined as standard at 0° C., the scale of a metric barometer measures the mercury column in correct millimetres when the temperature of the instrument is 0° C.

The measurement of barometric height is only part of the complete determination of pressure, since the condition of the mercury has not yet been defined. This is done by specifying the mercury to be under standard conditions of temperature and gravity.

Standard gravity is defined as the value of gravity at sea-level in latitude 45°. Its value has been accepted as[1] 980·665 cm. per sec. per sec. by the International Committee on Weights and Measures.

The standard temperature for the mercury has always been taken as 0° C. (32° F.).

It is often customary in practice to regard the expressions "millimetres of mercury," "inches of mercury," as implying that the mercury[2] is under standard conditions, but it is preferable to say so definitely. A metric barometer may be regarded as having a standard temperature 0° C. It can hardly be said that an inch barometer has a standard temperature, for when the mercury is under standard conditions the scale is not, and *vice versa*. There is, however, one temperature, not far removed from 32° F., at which the direct reading of the inch barometer gives true inches of "standard" mercury. This temperature lies between 28° and 32° F., the exact value depending on the type of mercury barometer. It is not of much importance in practical barometry. In general, neither the scale nor the mercury is at its own standard temperature. The indications of the mercury barometer, therefore, need correction for departures from standard conditions. The basis and procedure of application of these corrections is considered in § (6).

The case of an aneroid barometer is rather different. This instrument lends itself to the direct indication of pressures in absolute pressure units. The aneroid, however, can only be calibrated by reference to a barometer of another type, such as the mercurial instrument. Consequently, it has been customary, for the sake of uniformity, to graduate the scales of pressure-reading aneroids in terms of the units used for mercury barometers. Thus we see aneroid dials graduated in "inches" or "millimetres," meaning inches or millimetres of standard mercury.

There is a third unit, an absolute unit of pressure, frequently employed nowadays in the graduation of scales of both mercury and aneroid barometers, particularly meteorological barometers. This is the "bar." It belongs to the C.G.S. system of units, and is defined as the pressure of one[3] million dynes per sq. cm.

In magnitude it is somewhat smaller than the average atmospheric pressure at sea-level. To give a closer conception of the size of this unit the relation between it and the inch and millimetre of "mercury" is given below: It depends on the weight of unit column of standard mercury, and in this connection the following numerical values have been adopted in England:

Density of mercury at 0° C. } = 13·5955 grammes per cub. cm.

Value of "standard" gravity, *i.e.* at mean sea level in latitude 45° } = 980·617[4] cm. per sec. per sec.

With these values it follows that the weight of a cubic cm. of mercury under standard conditions of temperature and gravity is:

$$13{\cdot}5955 \times 980{\cdot}617 = 13332{\cdot}0 \text{ dynes.}$$

Accordingly the following numerical relations have been accepted:

1 millimetre of "mercury" { = 1333·20 dynes per sq. cm.
= 1·33320 millibars.

1 inch of "mercury" { (= 25·4 mm. to within 1 part in 1 million)
= 33863·2 dynes per sq. cm.
= 33·8632 millibars.

Inversely 1 millibar { = 0·750076 mm. of "mercury"
= 0·0295306 in. of "mercury."

If a mercury barometer scale is graduated in millibars its scale divisions will be very nearly equal to $\frac{3}{4}$ mm.

The adoption of the absolute unit for use in practical atmospheric barometry has been a matter for some discussion. It is of practical and scientific importance in meteorological work that regular readings of the atmospheric pressure should be charted, not only over large areas, but over a considerable range of altitude. Excellent progress in this work has been made during the last two decades, and much has been done through international co-operation. One of the principal reasons for the introduction of the millibar unit into the daily weather service of this country is that its use is a step towards the general adoption of an international system of units which would do away with difficulties which have arisen in the past in correlating the "inches of mercury" read by English-speaking countries with the "millimetres of mercury" found in other countries. The substitution for the English unit of the existing metric unit (mm. of mercury) was found not to solve all the difficulties, and in the study of the upper air the millibar has been used by the London Meteorological Office since 1907.

---

[1] At the 15th Conférence Générale des Poids et Mesures, held in 1913, it was decided to retain this value, although the most recent work had resulted in the value 980·621. This decision has an important bearing on the question of accurate determination of pressures in absolute units. It will be referred to again in connection with the correction of mercury barometer readings to standard gravity (see § (6), (ii.)).

[2] From here onwards standard mercury will be referred to as "mercury."

[3] This definition is not entirely universal, as in some spheres of work the "bar" has been understood to be the pressure of 1 dyne per sq. cm.

[4] This value was arrived at before the 15th International Conference made its decision in 1913 to retain the old value 980·665 cm./(sec.)^2 of standard gravity. It has not been thought fit to alter this value for the present. If at any future time a change should be made, the whole of the data should come under review, including the value of the density of mercury, which appears to have been taken too high by a few units in the fourth decimal.

From January 1914 onwards, weather charts in millibars have been regularly published both by the London Meteorological Office and the United States Weather Bureau.

The majority of the barometers used officially in the weather service of this country are now graduated in inches and millibars. This applies to the mercurial and aneroid instruments. The work of graduating existing mercury barometers in millibars is largely a matter of time, and has been in progress since 1914. The general policy has been to preserve the existing inch scales, at least temporarily, and to add a millibar scale to each instrument.

Some doubt was raised, in the first instance, as to the propriety of using the absolute unit for a mercury barometer, since the primary function of this instrument is to measure lengths rather than pressures. The objection was also raised that the nominal millibars of the instrument would only become true millibars by the application of a series of corrections.

Under the then existing unit, the inch barometer read true inches at one temperature only, and required corrections at other temperatures for thermal expansion. The millibar unit introduces gravity as well as temperature into the basis of graduation of the barometer scale, but as the effect of change of gravity on the indications of the mercury barometer at a given pressure is small compared with the thermal effect, it is clear that for any given value of gravity—*i.e.* for a given location of the instrument—there is some temperature at which the barometer reads true millibars.

This temperature is called the fiducial temperature. With this as basis the true pressure is measured by applying a single correction for the departure of the current temperature of the instrument from its fiducial temperature. This principle is in operation to-day in this country, in the absolute determination of atmospheric pressures in millibars for the daily weather service.

(ii.) *The Standard Atmosphere.*—While the use of the millibar unit may be regarded as well established among meteorologists generally, the point of view of the pure physicist still favours the retention of the "millimetre," together with the "standard atmosphere," to which so many reductions are made in gas-pressure work. The standard or normal atmospheric pressure is defined as the pressure due to the weight of 760 mm. of mercury at 0° C. and at sea-level in latitude 45° (*i.e.* 1013·23 millibars).

The meteorologist is inclined to regard his standard atmospheric pressure as

1000 millibars, *i.e.* 1 megadyne per sq. cm., which is equivalent to 750·076 mm. of "mercury."

There are occasions, however, on which he has to make reference to the physicist's standard.

(iii.) *Other Units.*—Apart from the inch, the millimetre, and the millibar, other units are rarely used in pure barometric work. The engineer's unit, the pound per sq. in., is occasionally used for the aneroid barometer described in this article. It belongs rather to the pressure-gauge, which is discussed elsewhere.[1]

Closely allied to barometric work is the measurement of heights in aviation and in surveying. For such purposes aneroid barometers are made with height scales. Mercury barometers would rarely have a height scale unless used as a standard to calibrate such aneroids.

The basis of graduation of height scales is referred to under VI., §§ (16), (17).

III. THE MERCURY BAROMETER

§ (3) TYPES AND DETAILS OF CONSTRUCTION.

—(For barometers of specially high precision see § (9)).

(i.) *The Fortin Type Barometer.* — A general view of a modern Fortin barometer is given in *Fig.* 4.

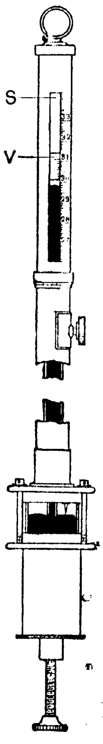


FIG. 4.—Fortin Type Barometer.

The distinctive feature of this type of instrument is the arrangement designed by Fortin for controlling the level of the mercury in the cistern so as to avoid moving the scale which measures the height of the barometric column. Incidentally the design permits the mercury to fill the tube and cistern of the instrument if desired, thus making the barometer easily portable.

---

[1] See "Pressure, Measurement of," Vol. I

(*a*) *Cistern of Instrument.*—A typical Fortin cistern, as made nowadays, is illustrated in *Fig.* 5, both externally and internally.

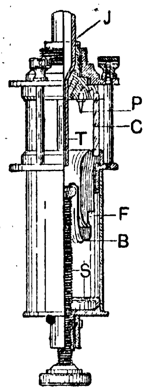


FIG. 5.—Cistern of Fortin Type Barometer.

A flexible bag B, made of suitable soft leather, tied securely to a boxwood frame F, forms the base of the mercury container in the cistern. Immediately below the centre of the bag is an adjusting screw S, which is used to raise the centre of the bag, and so raise the mercury to any desired level in the cistern. The upper end of the screw is pivoted so as not to twist the bag. Direct attachment of the screw adjuster to the bag is not essential, as the weight of the mercury keeps the bag in contact with the end of the adjuster and maintains it extended as far as the adjuster permits.

Some makers, however, do not rely on this, but provide a free joint between the bag and the upper end of the adjuster, together with a stop for preventing the adjuster from being lowered excessively. A glass cylinder C exposes to view the surface of mercury in the cistern, and also the pointer P securely fixed to the roof of the cistern. The end of this pointer serves as the zero of the scale of the instrument, and it is to this point that the level of the mercury must be brought on each occasion of reading the barometric height. It is sometimes called the fiducial point, being the reference or starting-point relied upon in measuring the barometric column. The lower part of the glass barometer tube is also shown. A joint is made between it and the neck of the cistern at J. This may be done by cement, but not necessarily so, as the cistern need only be mercury-tight, not air-tight. The lower end of the tube, technically called the tail-piece, T, consists of a short length of semi-capillary tubing of approximate bore, 1 to 2 millimetres, so as to diminish the chance of air finding its way up the tube. The tail-piece is fused to the main barometer tube, which, in the case of a Fortin type barometer, is usually in one piece of uniform bore.

The construction of the remainder of the cistern can be seen from the diagram, and is generally of brass, but reference should be made to the means provided for ensuring a complete and quick communication of the external atmosphere with the interior of the cistern.

The upper and lower ends of the glass cylinder C rest against leather washers, but the upper of these is not relied upon to communicate with the external atmosphere. This is done by making a porous joint at the neck of the cistern, which is mercury-tight but not air-tight. One maker, at least, relies on a kind of boxwood of such porosity that there is no undue lag in equalising the air pressure outside and inside the cistern. Other kinds of joints are also in use. An alternative arrangement is to make the cistern air-tight, but to insert in its roof a small threaded cock or nipple which can be opened or closed to the outside air. This plan is, in general, not altogether desirable.

(*b*) *Glass Barometer Tube.*—The barometer tube, necessarily closed at the upper end, which is usually about 33 inches above the fiducial point, is in this country usually made of lead glass. It is held vertically at the joint with the neck of the cistern, but other supports or guides are provided between it and its metal sheath, which is of brass, usually of tubular form, also fixed to the neck of the cistern, and terminating at the upper end in a ring from which the whole instrument is suspended. This sheath is suitably slotted in order to admit of viewing the summit of the mercury column in the glass tube.

The general manner of preparation of a glass barometer tube is indicated under § (4).

The bore of the tube depends on the accuracy expected of the barometer, and may vary from ¼ in. to ⅜ in. The most suitable size is between 0·4 in. and 0·5 in.

(*c*) *Barometer Scale.*—It is a convenience to engrave the barometric scale on the metal sheath. This is almost universally done, and incidentally brings the scale as near as practicable to the mercury column measured. The scale is silvered, and the graduation lines blackened for clearness.

Barometers usually have one or two scales according to requirements as to the units in which pressures are to be expressed. When the scales are two in number, they are set one on either side of the slot in the sheath through which the top of the mercurial column is viewed, but the design of the instrument would admit of the addition of a scale in some other unit of pressure, if desired.

The length of graduated scale depends on the station at which the barometer is to be used. At sea-level the maximum range of variation in atmospheric pressure is 31·1 to 27·3 inches of mercury. Consequently for use at stations near sea-level a nominal working range of about 31 to 27 inches (790 to 690 mm., or 1050 to 920 millibars) is sufficient. The scale, however, must be subdivided above the upper limit to a distance dependent on the length of the vernier used in reading fractions of a scale division.

If the barometer is to be used in a mine, the upper limit must be increased, following the increase of atmospheric pressure with the depth of the mine.

The lower limit of the scale is governed by the maximum altitude at which the instrument is to be used. Fortin type barometers are rarely made to read lower than 20 inches, unless made specifically for use in mountaineering and high-altitude surveying. In such cases the instruments are of special dimensions, and are considered as mountain barometers under the sub-heading "Modifications of Fortin Type Instruments" (§ (3), (ii.)).

(*d*) *The Indicator used to read the Top of the Mercury Column in Terms of the Barometer Scale.*—The vernier still remains the usual medium for indicating fractions of a scale subdivision in obtaining the atmospheric pressure by means of the mercury barometer. (See suggested modifications in § (5), (i.)).

The vernier plate V (*Fig.* 4) is a good sliding fit in the slot S in the metal sheath of the barometer. It is attached rigidly to a short length of metal tubing, which is a good sliding fit inside the metal sheath. The lower rim of this short slide, which may now be called the setting slide, is the means of determining the line of sight of an observer in reading the level of the top of the mercury column. Its motion is operated by a rack and pinion, actuated by a milled head placed conveniently at the side of the barometer. The slide is regarded as "set" when the plane through the lower rim of the setting slide appears just to touch the summit of the meniscus of the mercury column, judged by the observer looking through the glass tube against a white background. Unless the illumination of this background by daylight is good, additional means should be provided, giving a diffuse but not a brilliant illumination of the background.

The vernier usually has its zero at approximately the same level as the line of sight in setting, and the reading in terms of the barometer scale can easily be taken, using the vernier to indicate fractions of a subdivision.

The manner of graduating and interpreting a vernier has been set out at some length [1] in publications both of the London Meteorological Office and of the United States Weather Bureau, and will not be described here.

It is sufficient to say that, usually, the vernier of a Fortin type barometer is graduated to read 0·002 inch (0·05 mm., or 0·1 millibar) directly according to the unit used; that is, one subdivision of the vernier corresponds to 0·002 inch.

In the highest grade instrument the vernier may read to 0·001 inch directly, while in some cases, more frequently in other types of mercury barometer, the graduation of the vernier to yield only 0·005 or 0·01 inch is regarded as sufficient.

(*e*) *Attached Thermometer.*—In all but exceptional cases, a mercury-in-glass thermometer is a necessary adjunct to the mercury barometer owing to the relatively high thermal expansibility of the mercury in the barometric column.

The position of the thermometer is a matter of some importance, as it is desirable that the thermometer should register as nearly as possible the mean temperature of the mercury column, yet it is not by any means certain that the barometer will be used in surroundings free from considerable vertical and horizontal gradients of temperature. Accordingly, the best plan, in general, is to mount the thermometer with its bulb as near the centre of the barometric column as practicable. This is customary, the thermometer being mounted in a frame attached to the metal barometer sheath. Under favourable conditions of location of the instrument, this practice may be regarded as sufficiently satisfactory.

Sometimes the bulb of the thermometer is immersed in a tube of mercury of similar diameter to that of the barometer tube in order that the lag of the thermometer should be, as nearly as possible, equal to the thermal lag of the barometric column. Though this practice can be recommended in cases in which the barometer is set up in a room where the temperature is very steady, its object may be defeated by the unsuspected presence of a horizontal gradient of temperature between the thermometer and barometer.

It is not advisable to mount the thermometer in or near the cistern of the barometer.

(*f*) *Procedure in setting up and reading a Fortin Barometer.*—The instrument is here assumed to be in good order. (See § (7) for the consideration of instrumental defects.)

A Fortin type barometer should not be free to swing when read. This would hinder the setting of the mercury to the fiducial point owing to oscillation of the mercury. The instrument, suspended by its ring, should be clamped at the base of its cistern in a vertical position.

In the case of barometers from which an accuracy of 0·002 inch, or better, is expected, care should be taken that the axis is vertical. Since the pointer does not lie in the vertical axis of the instrument, its vertical distance below, say, the 30-inch graduation line of the scale will vary somewhat, the variation depending upon the exactness with which the verticality of the barometer can be reproduced. The error consequently involved is not merely the familiar "cosine" error depending on the angle of tilt of the instrument, but includes an expression which increases with the distance of the pointer from the vertical axis of the barometer. In general, when the barometer is suspended by its ring from a peg, it does not hang in exactly the same way each time it is suspended, nor does it return to exactly the same position after being swung or displaced slightly. The amount of error possible in the barometer reading due to this cause is about 0·001 to 0·002 inch. It is discussed in § (7). Whenever this error is of consequence, the barometer should be set up so as to rotate strictly about a vertical axis. This condition is secured if the pointer, when just in contact with the mercury in the cistern, remains so however the barometer is rotated. Each instrument is usually provided with a means (*e.g.* a ring and three radial screws) for clamping the cistern at the predetermined vertical position. Only in cases where the error due to indefinite verticality of the barometer is known to be small, compared with the accuracy expected of the instrument, should the above convention in setting up a Fortin barometer be ignored.

---

[1] *The Observer's Handbook*, published by H.M. Stationery Office, London; *The Marine Observer's Handbook*, published by H.M. Stationery Office, London; *Barometers and the Measurement of Atmospheric Pressure*, published by the Government Printing Office, Washington.

In order to determine the barometric height from a Fortin barometer, the mercury in the cistern should be raised, by means of the screw provided, so that it just touches the end of the pointer. The observation of exact contact of the point and its image by reflection at the mercury surface is usually a sufficient criterion of a good setting if the mercury is clean, and is normally well judged through a reading lens.

A very small amount of "overlap"[1] in raising the mercury surface can easily be discerned by watching the dimple caused by the end of the pointer dipping in the mercury.

The setting slide at the top of the barometric column is then brought into position as before indicated, and the scale and vernier read.

It is advisable to read the attached thermometer just before making the barometer setting in order to minimise errors due to heat from the observer's body.

In order to obtain the highest accuracy and consistency from the instrument, it is advisable, in all but the largest size barometers, to tap the metal sheath in order to obtain, as far as possible, a consistent shape of the mercury meniscus.

(Errors dependent on the shape of the meniscus are discussed under § (7), (ii.).)

(ii.) *Modifications of the Fortin Type Barometer.*—(*a*) The mountain barometer, as made in this country, is usually of the Fortin type, but with tube and cistern of considerably diminished diameter. In general, the bore of the tube lies between 0·20 and 0·25 inch. Sometimes it is less, though this is not to be recommended, as there is likely to be considerable loss of accuracy in the instrument. The practice, with some manufacturers, of reducing the bore of the tube excessively without reducing the nominal precision with which the vernier reads is misleading.

In practice, a mountain barometer is fitted with gimbals on which it is suspended vertically from a tripod stand. This method of suspension is satisfactory in the circumstances under which the instrument is used.

The average weight of a mountain barometer, excluding tripod, is about 3 lbs.

The scales of mountain barometers are usually graduated down to 18, or even down to 15 inches. There is rarely occasion to use them below this pressure, which corresponds to an approximate altitude of 20,000 feet.

Owing to the length of the graduated scale, two verniers are often provided, one each for the upper and lower half. This is practically a necessity if the "rack and pinion" method of operating the vernier is adhered to.

Some makers dispense with this and fit a single setting slide and vernier designed for a sliding coarse motion and a screw slow motion.

It is noteworthy that the leather bag in the cistern of a mountain barometer is required to withstand a considerable load at high altitudes, say at 20,000 feet, when the instrument is being made portable in the ordinary way by raising the bottom of the bag until the mercury completely fills the cistern and the tube. The pressure on the bag is proportional to the distance from the top of the tube to the top of the barometric column when in atmospheric equilibrium.

(*b*) As an alternative to the use of a leather bag for varying the level of the mercury in the cistern, a piston may be employed, actuated by a milled-head screw at the base of the cistern.

There are not many such instruments in this country.

(iii.) *The Movable Scale Barometer.*—In principle, this instrument differs from the Fortin type in that the zero of its scale, *i.e.* the fiducial pointer, is adjusted to the level of the mercury in the cistern instead of *vice versa*.

Two designs of this type may be noted:

(*a*) The Newman pattern,
(*b*) The Observatory barometer.

The latter is a large pedestal barometer, of massive design, and cannot be classed as a portable barometer.

In the Newman instrument the cistern is divided into an upper and a lower compartment, with an intercommunication port which can be opened or closed at will. In order to make the barometer portable it is carefully inverted, and the port hole closed.

---

[1] Of the order 0·0002 inch in the average barometer.

An instrument of this pattern was in regular use at Kew Observatory as a working standard for the verification of barometers before this work was transferred to Teddington.

(iv.) *The Kew Pattern Barometer.*—The Fortin and other barometers just described all depend on a double setting in giving the atmospheric pressure, *i.e.* it is necessary to make a setting on both the base and the summit of the barometric column.

This procedure can be shortened by the use of a Kew pattern barometer which yields direct indications by means of a single setting on the summit of the column.

If the cistern and the glass tube of the barometer are cylindrical, the change in the level of the mercury in the cistern corresponding to a given pressure change is a definite fraction of the change of level of the summit of the mercury column, and the value of this fraction depends only on the dimensions of the instrument, it being assumed at this stage that the temperature is constant.

It will readily be seen that the movement of the mercury in the tube is always smaller than that which would be obtained if the mercury in the cistern were brought to a fiducial point. Accordingly the scale of the instrument is contracted in order to make its readings comparable with those of a Fortin type barometer. The amount of contraction is not large. In practice, a nominal inch of scale on a Kew pattern barometer rarely measures less than 0·95 true inch.

The majority of barometers made for meteorological use have their cistern of internal diameter about five times that of the tube, corresponding to a scale-contraction-value of 0·96 approximately.

Greater and more permanent accuracy can be obtained by increasing this ratio (see § (7)). In this way a Kew pattern barometer may be made to yield almost as great an accuracy as that of a Fortin type barometer having the same bore of tube.

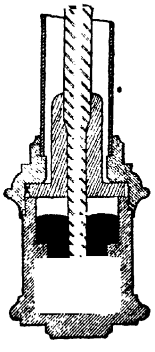


FIG. 6.

A typical small size cistern belonging to a Kew pattern meteorological barometer is shown in *Fig.* 6. It is of iron throughout, and has an internal flange,[1] which not only diminishes the amount of mercury required, but also damps the oscillation of the mercury in the cistern when the barometer is in the horizontal position during transport.

The end of the tail-piece of the glass tube is situated as nearly as possible at the centre of the cistern so that it shall be efficiently sealed by the mercury in all positions of the instrument, whether erect, horizontal, or inverted.

Kew barometers, unless of exceptional size or design, are usually made portable by carefully tilting them[2] until the mercury fills the tube. They may then be carried either horizontally or cistern upward.

The means of atmospheric communication between the interior and exterior of the cistern are similar to those adopted and described for the Fortin type barometer.

The Kew instrument is sometimes referred to as the "fixed cistern barometer." It is now generally understood that the term "Kew pattern" refers to those barometers which have a uniform, contracted scale to compensate for the lack of adjustment made to the mercury level in the cistern by means of a fiducial point.

Technically this compensation is called "Compensation for Capacity."

(v.) *The Kew Pattern Marine Barometer.*—The Kew pattern marine barometer was developed, as its name indicates, at Kew Observatory, in the middle of the nineteenth century, in devising a barometer suitable for use under conditions at sea.

As the oscillation of the mercury was a serious obstacle to the taking of readings aboard ship, the glass barometer tube was constricted so as to oppose the flow of mercury through it. The amount of constriction was arranged to compromise between the error due to oscillation, or "pumping" as it is technically called, and the error due to the lag of the mercury column in following the variations of atmospheric pressure.

The present form of marine barometer tube is shown in *Fig.* 7. It consists of four parts fused together. The cylinder C, through which the top of the barometric column is viewed, is of lead glass carefully selected for uniformity of bore. The constriction S here illustrated is a length of capillary tubing of uniform bore, generally 0·1 mm. to 0·4 mm. according to its length.

In some makers' instruments this tube is replaced by a piece of semi-capillary tubing of approximate bore 2 or 3 mm. This in itself has negligible lagging effect, but the constriction is supplied by drawing it out fine locally. The former alternative is to be preferred, as it is not so likely to lead to choking of the tube as the latter, which is a narrower constriction.

---

[1] This flange is not essential.

[2] In the case of a marine barometer the mercury takes a minute or so to fill the tube owing to its impeded flow through the constricted portion of the tube. The instrument should not be set in the horizontal position until the tube is completely filled.

The air-trap, which is funnel-shaped, is inserted so as to collect at A any air that may rise into the barometer tube from the cistern, and to prevent it from reaching the upper end or vacuum space of the barometer tube. It is of course essential to place this trap below the constriction. Usually a single trap is employed, though a double one may appear necessary in some cases, judging from the amount of air that sometimes finds its way to the top of the tube.

The tail-piece T should be externally cylindrical, since the movement of the mercury level, both in tube and cistern, is expected to be proportional to the pressure-change. Uniformity of tail-piece, however, is of secondary importance compared with that of the cylinder C.

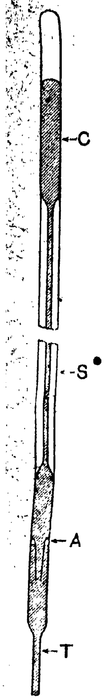


FIG. 7.

Note on Nomenclature.—Mercury barometers designed for use on land may for convenience be classified under the general heading "Station barometers," these instruments being most frequently used at a fixed station. Fortin type barometers belong essentially to this class.

Apart from the constriction of the tube, the Kew marine barometer is similar to the Kew station barometer. As the marine type is not entirely suitable for use on land, the distinction between the two should be carefully drawn. The station barometer is of course useless as a marine barometer.

In order to economise mercury, the station barometer tube is rarely made of uniform bore throughout its length. The lower parts of the tube, though narrowed considerably internally, should not be such as to impede the flow of the mercury, or render the barometric column sluggish in taking up equilibrium with the atmospheric pressure.

(vi.) *The Gauge Barometer.*—This instrument may be regarded as a barometer with a wider sphere of use than that of any type previously described, though differing only in detail. It is understood to mean a barometer which can be put in connection with any artificial gas pressure, and in this article it is considered as covering the ordinary atmospheric pressure range associated with the other barometers described.

The Fortin and Kew meteorological barometers are arranged to measure natural atmospheric pressures only. The gauge barometer measures artificial pressures in addition. It may be of the cistern type, or of siphon pattern. In the former case the cistern is made air-tight, and fitted with a cock or nipple for connection with the pressure to be measured.

If high accuracy is required—*e.g.* for the determination of some physical constants, or for other special work—the diameter of the barometer tube would necessarily be large, say from 0·5 to 1 in., and the mercury levels would be read by suitable accurate micrometers, which might be either optical or mechanical indicators.

For work of good but lower accuracy than the above, the usual vernier indicators or vernier setting-slides are sufficient. In this case it is customary to select a Kew pattern barometer gauge, since it measures pressures by a single setting only. A gauge of this description at the National Physical Laboratory, Teddington, has been found most serviceable in testing aneroid barometers. Its special feature is a large cistern of internal diameter 6 in., which serves to minimise errors due to variation in shape of the mercury meniscus in the cistern (see § (7), (ii.)).

This instrument, which has a tube of ½ in. bore, is graduated from 32 inches downwards with a long range scale, and can be relied upon to yield a precision of ±0·002 in. throughout its range. It is a matter of interest to record that, during its five years' frequent use at the laboratory, no permanent change exceeding 0·001 in. has been detected in its performance.

As an alternative to the Kew type of gauge barometer, the siphon type may be used. A ½-in. tube siphon gauge barometer, read by verniers, would give rather less accuracy than the instrument just referred to. It would be more troublesome to keep in good condition as the mercury in the open limb of the gauge often becomes foul in the course of a few months, and leaves a deposit on the glass tube.

As an advantage, the siphon type admits of being calibrated from first principles by measuring up the barometric column with a cathetometer. The Kew type instrument does not admit of this.

(vii.) *Other Types of Mercury Barometer.* (*a*) *Not Self-recording.*—The Fortin and Kew types of mercury barometer are widely used for accurate meteorological work. Other types, such as the siphon or U-tube barometer, Howson's barometer, the Sympiesometer, etc., will not be considered in detail here. Reference may be made to meteorological publications for a general account of these and other mercurial barometers.

The siphon barometer, though not possessing the practical advantages of a Fortin or a Kew barometer, is of importance since it is used in a somewhat modified and more elaborate form as a high precision fundamental standard barometer, *i.e.* as an instrument designed to admit of accurate determination of pressure from first principles (see § 9)).

(*b*) *The Self-recording Mercury Barometer.*—Self-recording mercury barometers are not easily made portable, and their sphere of usefulness is in general limited to observatories and similar institutions.

When a barometer is made self-recording it is usually at the expense of accuracy. Moreover, mercury barographs are not often compensated against changes of temperature. There are, however, several instruments of widely different design which can be said to attain fair accuracy. One instrument at Kew is arranged so that the summit of the barometric column is directly photographed. Another is based on the principle of a float resting upon the mercury in the open limb of a siphon barometer, and actuating, by means of a lever, a pen which produces a record on a suitable clock-driven drum.

Usually in this type of instrument the accuracy is limited by the friction of the pen and magnifying levers, and as an alternative, electromagnetic devices, actuated by the float, have been introduced to perform the real labour of moving the pen.

Another class of self-recording barometer has been designed on the principle of automatically weighing the barometric tube from the beam of a balance.

An excellent account of several self-recording mercury barometers is given by the United States Weather Bureau.[1] Special reference may be made there to the detailed description of the Marvin compensated barograph.

The tube of this instrument is made in three parts, viz. the U-bend, together with the two limbs, whose lower ends are tapered and ground so as to fit into the U-piece. The purpose of this is to facilitate filling the siphon, and to enable the open limb to be removed at intervals when the surface of the mercury needs cleaning.

Attention has also been given in this instrument to the question of compensating the barometer for changes in its temperature. The difficulties of so compensating the readings of a mercury barograph can best be overcome in the syphon barometer by suitably adjusting the volume of mercury in the instrument, so that the upward motion of the float due to the thermal expansion of the mercury balances the downward motion due to the expansion of the glass tube.

The Microbarograph. — Reference may at this stage be made to an instrument which works on an entirely different principle from the ordinary barograph (whether mercurial or aneroid). It was designed by Shaw and Dines, and records small fluctuations of the atmospheric pressure to which ordinary aneroid, and even mercury, barographs are insensitive. This instrument is referred to elsewhere under the heading of "Meteorological Instruments."

§ (4) THE PREPARATION, ADJUSTMENT, AND TESTING OF MERCURY BAROMETERS. (i.) *Preparation of Tubes.*—The general design of the more usual types of mercury barometer has already been indicated in § (3).

A brief reference should be made, at this stage, to the preparation of the glass barometer tube.

Good quality lead glass is generally used for barometer tubes by instrument makers in this country. Soda-glass is not used; it has been found to exude soda in course of time. The selected tube should be of reasonably uniform bore and circular section. The narrower the tube the greater the precautions taken in its selection, particularly for Kew pattern barometers, where uniformity of bore to within small limits is necessary in order to secure an equi-spaced scale.

The tube having been selected, two conditions may be regarded as essential to the success of the barometer:

(*a*) Cleanness and dryness of the mercury and tube.

(*b*) A good vacuum above the barometric column in the tube.

With regard to (*a*), cleanness is imperative, since the indications of the barometer are dependent upon a reasonable constancy of the capillary depression of the mercury column due to the action of the surface tension at the mercury meniscus in the tube (§ (7), (ii.)).

Mercury admits of preparation in a highly purified state, without great difficulty, owing to the relative ease with which it can be distilled.

Great care should be taken with the cleaning and drying of the tube. In the case of old barometers which require overhauling it is often considered wise not to attempt to clean and fill an old tube a second time. Lack of care in thoroughly drying the interior of the barometer tube may easily lead to the failure of the barometer as a precision instrument. The vapour from a film of water $\frac{1}{1000}$th cubic millimetre in volume, introduced into the vacuum space of the average barometer tube is sufficient to make the instrument read low by an appreciable amount, which can be detected in testing the barometer (§ (7), (i.)).

Closely linked with the question of cleaning and drying the tube is the process of filling it with clean dry mercury, and the method generally adopted is that of boiling the mercury in the tube. It is almost exclusively employed in filling tubes of the usual sizes me in practical barometry. Large tubes ar frequently boiled, but these, when full of hot mercury, are difficult to manipulate, and the risk of accident is consequently considerable.

---

[1] See Circular F of the Instrument Division, entitled *Barometers and the Measurement of Atmospheric Pressure*, published by the Government Printing Office, Washington.

When the tube has been well cleaned and dried prior to being filled, clean mercury is introduced to a depth of a few inches by suitable means, according to the nature of the tube to be filled, and afterwards heated. Air, which is inevitably moist to some extent, is readily trapped between the mercury and the walls of the tube. It can easily be detected by the appearance of the tube, and can be expelled, together with its moisture content, by careful and persistent boiling. Composite tubes, such as those of Kew pattern barometers, which consist of four or more parts fused together, require more attention, as moisture is liable to collect near the joints.

The number of stages required for the complete filling of the tube varies with the size and nature of the tube. Since the boiling of the first portion of mercury introduced into the tube results in the expulsion of most of the air from the tube, and its replacement by mercury vapour, it would appear that the tube could be practically filled in one more stage by dipping its open end under some hot mercury in a reservoir. For the smaller tubes used in Kew barometers the process of filling does not necessitate a large number of stages. Big tubes of uniform bore throughout, such as those used in the construction of the largest Fortin type barometers, would require to be filled and boiled slowly in several stages.

If the tube is clean and dry before the introduction of the mercury, the method of filling by boiling gives satisfactory results, with a good vacuum above the mercury column when the tube is finally set up.

(ii.) *Filling Large Barometer Tubes.* — For primary standard barometers and others which, apart from having large-size tubing, are of less simple design, the method of filling by boiling is impracticable, and the tube is filled under vacuum instead. This method has now been found to give satisfaction, provided a good drying agent is used in conjunction with the vacuum pump while the tube is exhausted. Continuous heating is essential in order to remove the last traces of air which cling to the wall of the tube. There are several varieties of modern vacuum pump which can be used with satisfaction.

(iii.) *Pointing and Adjusting.* — Before the scale of a barometer is ruled with the dividing engine, some test must be made in order to determine the location of a particular graduation line of the scale.

In a Fortin barometer, even if the scale of length is correctly ruled with reference to a zero coinciding with the end of the fiducial pointer, it will be found, in general, that when the top of the barometric column is read against this scale, the resulting indication is not correct within the requisite limits. This is due partly to what may be called an index error (really an error of parallax in transferring from the mercury to the scale), and partly to capillary errors consequent upon the action of the surface tension of the mercury both in the tube and reservoir. These latter errors will be discussed under § (7), (ii.).

Clearly in the case of a Kew barometer it is more difficult, in the absence of a fiducial pointer, to graduate the scale from first principles.

It is therefore the usual practice in the trade to "point" the instrument prior to ruling its scale. In the case of mercury barometers, this is generally done at current atmospheric pressures only, that is, in the neighbourhood of 30 in. (760 mm.). The "pointing" consists in marking off the level of the mercury on the brass tube on which the scale is to be graduated, and observing the corresponding true reading of the atmospheric pressure by means of a previously standardised mercury barometer.

The treatment of a Kew barometer involves, in addition, the determination of the contraction value of the scale. This can be calculated, or found from tables, when the exact sizes of the tube and cistern are known (§ (7), (iv.)).

In general, instrument makers graduate their scales by setting their dividing engines to give the calculated or tabulated amount of scale contraction. Alternatively, the barometer under preparation may be set up in a vacuum chamber and pointed off under artificial pressures within the intended range of the scale. This method is more tedious and does not give better accuracy unless precautions are taken to avoid errors such as those indicated in § (7), (ii.).

In the case of old barometers in which new tubes have to be fitted, pointing is unnecessary since the scale is already ruled.

The Kew barometer can be easily and accurately adjusted to read correctly at current atmospheric pressure by varying the quantity of mercury in the instrument.

Other means, more or less obvious, can be devised for adjusting a repaired Fortin barometer. In this case the amount of adjustment should be small if the instrument was correctly graduated when new.

(iv.) *Testing of Barometers.* — Preliminary tests on a barometer are, of course, necessary in the instrument-maker's workshop: they are made concurrently with the adjustment of the instrument, with the object of securing that the barometer shall be correct within certain prearranged limits before leaving the shop.

Whenever desired, an independent test is made at the National Physical Laboratory. Broadly speaking, the tests on all mercury barometers comprise:

(1) A preliminary inspection of the barometer with a view to finding the more obvious errors of design, workmanship, and adjustment.

(2) A series of comparisons, at current atmospheric pressures, between each "test"

barometer and one of the working standard barometers of the Laboratory, which have been previously calibrated from first principles.

(3) A series of comparisons, made at such artificial pressures as are required, between a "test" and a standard barometer in a vacuum chamber.

These comparisons are usually made by means of a cathetometer (except in the case of gauge barometers), the method being to read the level of the summit of the mercury column of each barometer by telescope, the accuracy of the scale of each instrument being examined separately by cathetometer. It will be observed that in this test at artificial pressures, the barometer may not be read in the same way as it is when in normal use, since the readings taken on it when in the vacuum chamber are not made with the vernier setting slide.

NOTE.—Since barometers which are designed to read atmospheric pressures need not have an air-tight cistern, but only a mercury-tight one, the test at artificial pressures necessitates the enclosure of the whole barometer in a suitable receiver, and thereby increases the difficulty of reading it by its own vernier (or other) attachment in the normal way.

In short, the test at artificial pressures is a relative comparison only, but of high accuracy. It is made absolute by combining the results with those of the test at current atmospheric pressures, which of course are made by vernier.

It is here assumed that the vernier setting slide has the same error of parallax between the mercury column and scale for all positions along the scale; but this assumption is tested independently from time to time, and due allowance made when necessary.

The broad method of test just outlined would vary in detail with the type of barometer. The Kew pattern barometer requires rather more care in testing, since its scale is contracted to an extent depending on the dimensions of the instrument.

In testing the barometer under artificial pressures, therefore, it is necessary to see whether the run of the mercury in the tube corresponds to the amount of uniform contraction of the scale, and is uniform over the working range of pressure of the instrument.

In general the procedure of (*a*) contracting the scale to suit the dimensions of tube and cistern, or (*b*) selecting the tube to suit the scale and cistern, *i.e.* the compensation of the barometer for capacity of the tube and cistern, cannot be done without residual error. An allowance for this lack of compensation has, therefore, to be made, and a "capacity correction" determined and applied as a result of the test.

§ (5) METHODS OF READING MERCURY BAROMETERS.—The methods of reading mercury barometers may be divided into two main classes according as the instrument is intended to yield ordinary or high precision.

By ordinary precision is meant a degree of accuracy lying between 0·001 and 0·01 in. of mercury in the determination of pressure. Mercury barometers which do not reach this degree of accuracy, even under favourable conditions, can hardly be considered as precision instruments.

(i.) *Ordinary Precision.*—It is usual in the case of barometers of ordinary precision for the summit of the mercury column to be read by means of the familiar setting slide with vernier attachment, as described in § (3), (i.), (*d*).

In general, this form of indicator has given satisfaction, though it has its limitations. Even with a slide and vernier of the best workmanship there is a personal element involved in performing the combined operation of "setting" to the top of the mercury column and reading the vernier.

It is, of course, essential to have a good though not excessive illumination of the background, but, given a consistent illumination, it is found that trained observers may disagree to the extent of 0·001 in. in their interpretations of the indications of a good barometer. Consistency of "setting" on the mercury meniscus depends on the limitations of the human eyesight, and one observer may see some light transmitted between the setting slide and the top of the meniscus when another observer cannot. Further, there is not always unanimity in reading a vernier unless its graduation lines are sufficiently fine and accurately ruled.

In addition to the personal element in the use of this type of indicator, there is also a small inconsistency in the error of parallax in transferring from the top of the mercury column to the scale. This is occasioned by imperfections either in the fit of the setting slide itself, or else in the tubular metal barometer sheath along which the slide runs. Tests made on this form of indicator show that the error of parallax, though usually constant at a given point on the scale, is liable to variations of the order 0·002 in. along the scale. Occasionally this estimate is exceeded, though there is very little excuse for excessive errors of this nature.

In the past there has been a tendency to overlook this matter, largely owing to the presence of other instrumental errors of greater magnitude, such as those due to variation of capillary action.

(ii.) *Modifications of the usual Setting Slide and Vernier Arrangement.*—It has on some occasions been considered that the vernier is not the ideal means of reading off fractions of a scale subdivision, especially when in the hands of an unskilled observer.

Attempts have therefore been made to replace the vernier by a micrometer indicator reading directly, on a ring or drum, a convenient fraction of a scale division.

Two types of micrometer deserve mention: they have not yet been developed much beyond the experimental stage. In each case the usual design of setting slide has been retained, with the exception of the replacement of the vernier.

In the first type the micrometer drum is attached at the side of the instrument so as to operate the setting slide through the medium of a rack and pinion. It is essential that both rack and pinion should be accurately cut in order that the vertical motion of the setting slide shall be accurately converted to a uniform rotary movement of the pinion and attached micrometer.

An indicator of this kind was made some years ago by the Cambridge Scientific Instrument Co., Ltd.[1] The micrometer drum was graduated in 100 divisions, each registering $\frac{1}{10}$ millibar (approx. 0·003 in. of mercury). On testing the barometer fitted with this indicator, it was found that with a local exception at the end of the pressure range, the micrometer recorded the vertical run of the setting slide correctly to within half a subdivision, *i.e.* to ± 0·05 mb.

A second type of indicator has been made by Negretti & Zambra, Ltd., in which a micrometer ring, encircling the tubular metal sheath of the barometer a little way below the lower end of the scale, operates the setting slide through a screw-thread of fairly big pitch.

As in the first type of indicator, the micrometer is divided into 100 subdivisions, each equal to $\frac{1}{10}$ millibar, and a complete turn of the micrometer is equivalent to 10 mb.

It is by no means certain that the danger of observational errors is any smaller with a micrometer than with a vernier. Moreover, the micrometer indicator may introduce additional errors to those incurred in the present type of vernier setting slide. The run of the micrometer does not *exactly* correspond with the run of the setting slide, while the latter does not measure *exactly* the run of the mercury in the tube.

A micrometer indicator in which the principle of the present setting slide can be avoided would be welcome.

(iii.) *High Precision.* (*a*) *Methods of setting Mercury Levels to Points or vice versâ.*—The principle of reading mercury levels by pointers is susceptible of great accuracy.

Even the usual kind of pointer found in Fortin type barometers can be employed for setting the mercury-level in the cistern with a consistency of ±0·0001 or 0·0002 in., which is easily sufficient for this kind of instrument. It is possible to see with the unaided eye the dimple caused by a pointer dipping only 0·0002 in. below the mercury surface.

In this connection reference may be made to a paper entitled, "A Note on the Setting of a Mercury Surface to a Required Height,"[2] showing how a consistency of ± 0·00002 in. has been achieved under favourable conditions by observing the reflection in the mercury of a uniform scale of closely spaced lines (about 0·02 in. apart) placed behind the pointer and the mercury. When the mercury is set too high, the formation of a dimple in its surface can easily be detected, if the eye is directed towards the end of the pointer, by a local distortion of the reflected image corresponding to the distortion of the mercury surface at the dimple.

It should be remarked that the attainment of high accuracy by this method requires a good illumination of the mercury surface. Moreover, unless the end of the pointer is fine, it interrupts the viewing of the distortion of the image of the lines when a very small dimple is present.

Alternatively to the foregoing method of pointer-setting, electrical means have been used, though not always with satisfaction, in order to give a criterion of contact of the pointer with a mercury surface.

(*b*) *Optical Methods.*—In barometry of the highest precision, optical methods have in general been employed to read the upper and lower mercury levels.

It is useful, in the first instance, to consider what exactly is measured when the telescope of a cathetometer is focussed on the summit of a mercury column. If the mercury meniscus is brightly illuminated from above, its image in the telescope will scarcely be crisply defined in the neighbourhood of the summit.

The appearance of the image depends to a large extent on the distribution of light and shade over the meniscus.

In order to read the position of a mercury meniscus even to a moderate degree of accuracy by direct focussing with a cathetometer telescope, it is necessary that the illumination should come almost entirely from the back, and be carefully controlled.

In *Fig.* 8 the aperture AB represents the only source of illumination. Of the rays of light which pass through the barometer tube to the eyepiece of the telescope T, only those which are reflected at the surface of the mercury have any influence on the apparent outline of the meniscus as viewed through the telescope.

If BMT represents the path of the limiting ray of light which suffers reflection at M on

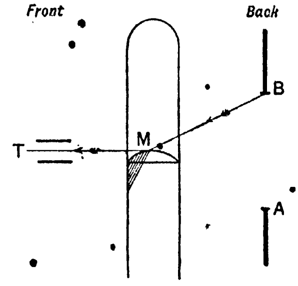


FIG. 8.

---

[1] Now the Cambridge and Paul Instrument Co., Ltd.

[2] *Bureau of Standards Bull.*, 1914, x. 371.

the mercury meniscus before passing into the telescope, it is clear that the portion of the meniscus to the front of M is unilluminated and therefore appears black in the telescope, while the intervening part of the meniscus between M and the true summit appears bright. The boundary between bright and dark parts of the meniscus in the image depends on the height of B above the level of the meniscus top. In any case M appears in the telescope as the summit of the meniscus, and is actually focussed on and measured by the cathetometer.

Accordingly, when there is occasion to read mercury levels in this way by means of telescope and cathetometer, there is always present, to some extent, an error between the apparent and real summit of the meniscus. For a given illumination, *i.e.* a given obliquity of the limiting ray BM, the error increases with the size of the tube. If the inclination of BM does not exceed one part in 100, the resulting error does not exceed 0·01 mm. or 0·0004 in. except for tubes exceeding 1 in. in diameter.

In cases where a large bore of tube (*e.g.* exceeding 1 inch) is essential for other reasons, the method of reading mercury levels by direct focussing with a cathetometer telescope requires modification if an absolute accuracy exceeding 0·01 mm. is required.

Fig. 9 shows the principle of the plan [1] adopted for use with one of the normal barometers at the Bureau International des Poids et Mesures, Sèvres. A collimator OO′ is arranged so as to produce somewhat above the centre of the mercury meniscus whose level is to be read a real image of a horizontal wire F. This image and that formed by reflection at the mercury meniscus are viewed with a micrometer microscope at F_1 and F_2, and the actual summit of the meniscus is taken to be midway between F_1 and F_2.

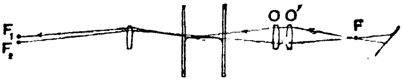


FIG. 9.

This method, which allows of readings being taken to an accuracy of $\pm$ 0·0015 mm., requires some precautions. It is difficult to obtain resulting images which are well defined unless the external and internal surfaces of the glass barometer tube are polished so as to be, as far as possible, optically perfect in the neighbourhood of the mercury surface. The portion of the mercury surface which gives rise to the reflected image should be confined to the more central part of the meniscus, where the curvature is inappreciable.

Other optical errors are involved, but are finally reduced to satisfactorily small limits by reading the upper and lower ends of the barometric column in precisely the same way.

§ (6) REDUCTION OF THE READINGS OF A MERCURY BAROMETER TO STANDARD CONDITIONS.—Since the indications of a mercury barometer are influenced by changes of temperature of the instrument and of gravity acting on the mercury, it is essential that corrections should be made for these changes in order to obtain absolute values of the pressure.

The basis and method of application of these corrections will now be considered.

In determining the temperature allowance, the formulae for inch and metric barometers are slightly different on account of the different temperatures of standardisation of the inch and metric scales.

There is also a fundamental difference in the matter of temperature correction between the Fortin and Kew types of barometer. This difference is relatively small, but has been, to a large extent, overlooked in the past. The investigation of the temperature correction to a Kew barometer will therefore be considered in greater detail.

Moreover, in the case of meteorological barometers graduated in millibars, the procedure of correcting their indications for temperature and gravity has been somewhat modified and simplified.

It will be more convenient to commence with inch and metric barometers, considering first the temperature correction, then the gravity correction. When these units are used, the barometric indications are corrected to refer to mercury at 0° C. (32° F.) and at standard gravity.

(i.) *Correction of the Mercury Barometer for Temperature.* (*a*) *The Fortin and Siphon Types of Barometer.*—For the purpose of making the temperature correction the above types have been selected as representative of those whose scales are graduated in true linear measure, uncontracted.

The temperature coefficient of the Kew barometer, or of any barometer with contracted scale, differs somewhat from that of the Fortin type.

As a first approximation the Fortin coefficient may be used for a Kew instrument, but where good precision is required the Kew barometer should be considered as having a distinctive coefficient after the manner shown in § (6), (i.), (*b*).

The temperature correction to the Fortin and Siphon barometers involves two quantities, viz. the thermal expansibilities of mercury and of the barometer scale.

In the case of the metric barometer, the scale is considered standard at 0° C., which is the temperature to which the mercury has to be corrected.

It can be shown from first principles that the correction formula to be employed is

$$h_t - h_0 = \frac{t(\beta - \alpha)h_t}{1 + t\beta},$$

---

[1] Reproduced from *Travaux et Mémoires du Bureau Intl.*, tome III. p. 22.

where h_t is the indicated barometric height at temperature $t°$ C.,

h_0 is the corresponding barometric height expressed in true millimetres of mercury at 0° C.,

α is the mean coefficient of linear expansion of the scale between 0° and $t°$ C.,

β the mean coefficient of dilatation of mercury between 0° and $t°$ C.

The correction to be applied to the indications of the metric Fortin barometer at $t°$ C. in order to give true millimetres of mercury at 0° C. is therefore

$$-\frac{t(\beta-\alpha)h_t}{1+t\beta},$$

i.e. it is subtractive for all temperatures above 0° C. Under average laboratory conditions of pressure and temperature, *e.g.* at 760 mm. and 16° C., its value is about -2 mm. The expansion of the scale is of secondary magnitude compared with that of the mercury.

The correction formula for the inch Fortin barometer differs from the metric one only by reason of the extra complication involved by the inch scale being standard at 62° F., which is not considered standard temperature for the mercury.

With the same notation as before, except that temperatures t are measured in Fahrenheit degrees, while the expansibilities α and β are also referred to this scale of temperature, and the barometric indications expressed in inches,

$$h_t-h_{32}=\frac{[(\beta-\alpha)(t-32)+30\alpha]}{1+(t-32)\beta}\times h_t.$$

The correction to be applied to the indications of the inch Fortin barometer at $t°$ F. in order to obtain true inches of mercury at 32° F. is

$$-\frac{[(\beta-\alpha)(t-32)+30\alpha]}{1+(t-32)\beta}\times h_t.$$

Under average laboratory conditions of pressure and temperature, the magnitude of this temperature correction is of the order 0·09 in.

The exact value will of course vary with the material of the scale. In the vast majority of barometers brass is used, and accordingly tables have been published giving the values of the temperature corrections to inch and metric barometers, based on accepted values for the thermal expansibilities of mercury and brass. Reference may be made to the *International Meteorological Tables*, or to the *Smithsonian Meteorological Tables* (1918 edition), for the numerical values of the correction, together with the data from which they are derived. For high precision, viz. 0·001 in. or better, the expansibility of the scale of the barometer should be separately determined. This would naturally be done for primary or fundamental mercury standard barometers. In such cases an invar scale would probably be used in preference to a brass one. Occasionally a glass scale is used, or else the scale is engraved on the glass barometer tube. This latter course is not, however, to be recommended for general practice.

(*b*) *The Temperature Correction to a Kew Barometer.*—In considering to what extent the Kew barometer differs from the Fortin barometer with regard to temperature changes, let A and B mark the levels of the mercury in the tube and cistern of a Kew barometer (*Fig.* 10).

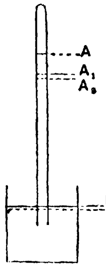


FIG. 10.

Suppose for the moment that the mercury is divided into two portions, viz. that above B and that below B. If each of these portions is cooled down separately from room temperature to 0° C., the upper level will descend in the tube as far as A_1, say, while the lower portion will descend to B_1, leaving a gap in the tube between B and B_1.

In a Fortin barometer the mercury in the cistern would be raised again until it once more occupied its original position B, and the total fall in the tube would be AA_1.

In a Kew barometer the mercury in the tube has to descend a further amount A_1A_2 ($=BB_1$) in order to meet that in the cistern. The total fall in the Kew barometer tube is accordingly AA_2.

Further, in the Fortin instrument the amount AA_1 is measured on a *true*[1] scale, but in the Kew barometer the length AA_2 (already greater than AA_1) is measured on a *contracted* scale, and thereby reads greater still.

It follows that the change in barometer reading corresponding to a given temperature change is appreciably greater for a Kew than for a Fortin barometer.

For want of a better name, the excess of the temperature correction to the Kew barometer over the corresponding correction to the Fortin instrument will be called the "Kew temperature error." An approximate calculation, based on known dimensions of a Kew barometer, shows that the "Kew temperature error" may be as large as 0·008 in. (0·2 mm.) for a 30° F. (17° C.) change of temperature, if the barometer tube is of uniform bore throughout its length.

This is only occasionally the case with a Kew barometer, as the tube is usually narrowed down to a comparatively small bore (about 2 mm.) over the greater portion of its length, whether the instrument is intended for marine or station use.

The effect of narrowing the tube in this way is to diminish the value of the Kew temperature error. This can best be seen from a somewhat different standpoint from that introduced by *Fig.* 10, without separating the thermal effects in the tube and cistern.

---

[1] The scale is a true linear measure, apart from being a barometric measure, at the standard temperature corresponding to the unit of length employed.

The Kew barometer may be considered as a closed reservoir, partly of iron (the cistern) and partly of glass (the tube), containing mercury.

The effect of change of temperature on the instrument taken volumetrically is primarily a relative thermal expansion of a given volume of mercury in a composite reservoir of iron and glass.

If V = total volume of mercury in the Kew barometer under standard conditions,
A = the effective[1] internal area of cross-section of the cistern,
S = the internal area of cross-section of the glass barometer tube at the top of the mercury column,

it follows that the thermal change of zero of the barometer—*i.e.* the thermal change of level of the mercury in the cistern at a given pressure—is

$$\frac{V(\beta-\gamma)dt}{(S+A)},$$

where dt represents the change in temperature,
β is the thermal expansibility of the mercury,
γ is a composite coefficient of expansion intermediate between those of iron and glass.

This change of zero is measured on the barometer scale, which is contracted in the ratio $A/(S+A)$ (§ (7), (iv.)). Hence the Kew temperature error may be expressed as

$$\frac{S+A}{A}\times\frac{V(\beta-\gamma)dt}{(S+A)},$$

that is,

$$\frac{V}{A}(\beta-\gamma)dt.$$

It should be noticed that, for a given change of temperature, the value of the Kew temperature error is independent of the pressure. Consequently the complete correction for temperature to the Kew barometer is of the form

$$(Fh+K)dt,$$

where h is the indicated barometric height,
F is the temperature coefficient of the Fortin barometer,
K is a constant peculiar to the Kew type barometer, depending only on its dimensions and the expansibilities of its parts.

A detailed investigation of the complete formula for the temperature correction of Kew barometers has been made at the National Physical Laboratory, and numerical values have been worked out there for a number of instruments of different dimensions, including large and small cisterns, constricted and unconstricted tubes.

It was found that the value of the "Kew temperature error"—*i.e.* the excess of the Fortin over the Kew temperature correction—is represented to very high accuracy (within ±0·01 mm. in each case for a 20° C. range of temperature) by the expression

$$\frac{V}{A}(\beta-3\eta)dt,$$

where η has the value 0·000010 per 1° C. This value is intended as a composite mean between the coefficients of iron, glass, and brass, the latter material entering into the calculation to a very minor extent.

The exact value of η is relatively unimportant.

(*c*) *Numerical Values of the Kew Temperature Error.*—It should be remarked that the value of V/A is by no means constant for Kew barometers of different dimensions. Even in marine barometers, which may be considered as belonging to a standard pattern, there is appreciable variation. For the average barometer of short pressure range—*e.g.* graduated down to 26 in. (650 mm.)—the value of V/A may be taken to be

$$\tfrac{3}{2}\text{ in. (38 mm.)},$$

and the corresponding Kew temperature error over a range of 30° F. (17° C.) is

$$0{\cdot}0038\text{ in. (0·10 mm.)}.$$

For barometers with longer pressure ranges the value of V/A is usually larger unless the cistern is made exceptionally wide.

As the result of examining a large number of barometers of widely differing dimensions and pressure ranges it was found that the value of V/A may vary between the limits

$$1{\cdot}0\text{ and }3{\cdot}5\text{ in.}$$

Accordingly, in all cases where the Kew temperature error is considered so large that it cannot be neglected, it is advisable to ascertain or estimate the value of V/A. Only in the more frequently used patterns of short-range meteorological barometers can its value be taken as 1·5 in. without separate estimation. This large range of variation of V/A makes it almost impracticable to draw up comprehensive tables of temperature correction to Kew barometers. The distinction between the Fortin and Kew temperature coefficients is not alluded to either in the International or in the *Smithsonian Meteorological Tables.* Attention, however, is drawn to it in the more recent editions of the *Observer's Handbook*, published by the London Meteorological Office, and the appropriate values are used in correcting barometers used on behalf of that office.

The complete formulae for reduction of inch and metric Kew barometric readings to standard temperature, 32° F. (0° C.), are:

Kew Inch Barometer.—

$$h_t-h_{32}=\frac{h_t[(\beta-\alpha)(t-32)+30\alpha]}{1+\beta(t-32)}+\frac{V}{A}(\beta-3\eta)(t-62),$$

measured in inches and Fahrenheit degrees.

Kew Metric Barometer.—

$$h_t-h_0=\frac{h_t(\beta-\alpha)t}{1+\beta t}+\frac{V}{A}(\beta-3\eta)t,$$

measured in millimetres and Centigrade degrees.

(Compare these with the corresponding formulae for the Fortin barometer in § (6), (i.), (*a*).)

(*d*) *Expression of the Kew Temperature Correction as a Simple Coefficient of the Barometer Reading.*—[The following values will be accepted for the thermal expansibilities of brass and mercury in conformity with the International Meteorological Tables.

---

[1] *I.e.* actual internal area of cistern less the external sectional area of glass tail-piece.

$\alpha = 0{\cdot}0000184$ per. 1° C., coefficient of linear expansion of brass.

$\beta = 0{\cdot}0001818$ per 1° C., mean coefficient of dilatation of mercury measured from 0° C. to the temperature t under consideration.

It should be observed that although the above value of β has been accepted as holding for all values of t, to the requisite accuracy, there is a difference between this coefficient, which represents expansion measured from 0° C., and the corresponding coefficient 0·0001812, which represents expansion measured from t° C. (where $t=17$° C.), and is simply $\beta/(1+\beta t)$.

This difference is made apparent by the relatively large expansibility of mercury.]

The immediate purpose of setting out the temperature correction in the form of a simple coefficient is to enable corrections to be made to barometer readings for small changes of temperature. The further use of this coefficient will be apparent when the temperature correction of millibar barometer readings is considered.

In the case of a Fortin type barometer the temperature correction is clearly

$$H(\beta'-\alpha)dt,$$

where H is the indicated barometric height at temperature t,

β' is the expansibility of mercury measured from t,

α is the thermal coefficient for brass,

dt is a small change of temperature.

β', it has been explained, is equal to 0·0001812 per 1° C. for $t=17$°, and is tolerably constant over a range of variation of ±5° from this temperature.

Hence $\beta'-\alpha=0{\cdot}0001628$ per 1° C., corresponding to 0·0000904 per 1° F.

For the Kew barometer the temperature correction becomes

$$H(\beta'-\alpha)dt+\frac{V}{A}(\beta'-3\eta)dt,$$

that is,

$$H\left[(\beta'-\alpha)+\frac{V}{AH}(\beta'-3\eta)\right]dt,$$

so that the expression within the brackets may be considered as a simple coefficient which varies slightly with the barometer reading H.

The value V/A=3/2 in. (38 mm.) may be accepted as applicable to the normal types of short-range meteorological barometer, and an insertion of the thermal expansibilities

$$\alpha=0{\cdot}0000184 \text{ per } 1° \text{ C.},$$
$$\beta'=0{\cdot}0001812 \text{ per } 1° \text{ C.},$$
$$3\eta=0{\cdot}0000200 \text{ per } 1° \text{ C.},$$

leads to the value

$$0{\cdot}0001704 \text{ per } 1° \text{ C. at } 760 \text{ mm.}$$

for the expression within the brackets, which may be regarded as a simple coefficient, almost constant over the short range, 800 – 700 mm.

The corresponding value per 1° F. is

$$0{\cdot}0000946 \text{ at } 30 \text{ in.}$$

(ii.) *Correction of the Mercury Barometer for Gravity.*—(Standard gravity is the value of gravity at mean sea-level in latitude 45°; see § (2).)

If the pressure and temperature are given, the height of the barometric column varies inversely as the value of gravity at the station where the instrument is read; and in order that the barometric height shall be a true relative measure of the pressure it has to be corrected to what it would be at a standard value of gravity. For work of the highest precision, where absolute pressures are required, it is essential that the local value of gravity should be known. In many cases, especially at sea, it is impracticable to determine the value of gravity, even with moderate accuracy. It has therefore been customary to assume the following theoretical relations as representing the variation of gravity with latitude and height above sea-level. They have been calculated [1] from the dimensions and speed of rotation of the earth, treated as an oblate spheroid.

[2] $g_0 = g_{45}(1-0{\cdot}00259\times\cos 2\lambda)$,

$g_h = g_0(1-0{\cdot}000000196\times h)$ if h is expressed in metres

$= g_0(1-0{\cdot}0000000597\times h)$ if h is expressed in feet,

where g_{45} = standard gravity,

g_0 = gravity at sea-level in latitude λ,

g_h = gravity at height h above sea-level in latitude λ.

More recent work [3] on the investigation of gravity shows that the absolute value of gravity at a given station can be computed in the following manner:

(1) Compute g_λ the mean value of gravity at sea-level for the latitude of the station, from the equation:

$$g_\lambda = 980{\cdot}621(1-0{\cdot}002640\times\cos 2\lambda + 0{\cdot}000007\times\cos^2 2\lambda).$$

(2) Correct g_λ for height h above sea-level by the amount:

$C = -0{\cdot}000000314\times h\times g_0 = -0{\cdot}000309\times h$ (where h is in metres)

$= -0{\cdot}000000096\times h\times g_0 = -0{\cdot}000094\times h$ (where h is in feet).

For the highest accuracy, allowances should be made for topography and isostatic [4] compensation, if information is available for making these corrections.

Having determined the value of gravity g_h at a given station, the corrected barometric height at standard gravity, g_{45}, is obtained on multiplication by g_h/g_{45}.

---

[1] See *Travaux et Mémoires, Bureau International des Poids et Mesures*, tome i.

[2] These values are used in the British Meteorological Service and are identical with those given in the International Meteorological Tables. They, however, require revision in view of progress made since the publication of the tables, especially in the development of the theory of the isostatic compensation of the earth's crust.

[3] See Special Publication No. 40 (1917) of the U.S. Coast and Geodetic Survey.

[4] *Loc. cit.*

For stations near London, the correction[1] to be applied to the average barometer reading, 30 in. (760 mm.), to reduce it to standard gravity, is

+0·016 in. (0·4 mm.).

For ordinary stations where mercury barometers are read, the change in gravity with height corresponds to a change on the barometer of about

0·003 in. per 1000 ft. of altitude

(=0·24 mm. (0·3 mb.) per 1000 metres of altitude).

These changes are approximately of the same order as the changes in gravity due to topography and isostatic compensation.

(iii.) *Correction of Mercury Barometers graduated in Millibars for Temperature and Gravity Variations.*—The procedure of correcting mercury barometer readings to standard conditions has been somewhat conveniently modified in the case of instruments with scales graduated in millibars, though it still involves correcting the barometer for variations in the temperature and gravity of the mercury column from standard conditions.

The current method in the case of millibar barometers for the meteorological service may be outlined as follows.

In practice a barometer, though intended to function correctly under certain standard conditions, generally shows a small residual error called the index error, which may be determined by comparison with a known standard barometer. Millibar barometers are designed to read correctly, under standard[2] conditions of gravity, at a temperature of 285° A.

Owing to small permissible instrumental errors in adjustment, etc., the *standard*[3] temperature varies in different instruments by an amount not exceeding 3° A. in general. The exact *standard* temperature can be found by comparison with a standardised instrument.

For stations at sea-level in latitude 45°, the atmospheric pressure may be determined by making a single correction to the reading of the barometer, *i.e.* by correcting it for the departure of its temperature from its *standard* temperature. The temperature correction is equal to

$p \times 0{\cdot}000163$ per 1° A. for a Fortin barometer,

$p \times 0{\cdot}000171$ per 1° A. for a Kew barometer,

where p is the reading of the barometer in millibars.

In general, the barometer is not under conditions of standard gravity, and allowance for this is made by finding a new temperature at which the instrument reads true millibars at its given station. This temperature is analogous to the *standard* temperature, and is referred to as the *fiducial* temperature of the instrument.

The *fiducial* temperature may be obtained from the *standard* temperature by applying the correction shown in the following table:

TABLE I

Latitude.	Correction.	
	Fortin Barometer.	Kew. Barometer.
90° or 0°	± 15·9°A.	± 15·2°A.
85 or 5	± 15·7	± 15·0
80 or 10	± 14·9	± 14·3
75 or 15	± 13·8	± 13·2
70 or 20	± 12·2	± 11·6
65 or 25	± 10·2	± 9·8
60 or 30	± 8·0	± 7·6
55 or 35	± 5·4	± 5·2
50 or 40	± 2·8	± 2·6
45°	0·0	0·0

(For stations in latitudes above 45°, the correction is additive.)

Note.—The above figures for a Fortin barometer are independent of the pressure. Those for the Kew type vary to a small extent with the pressure. They have been calculated for a pressure of 1000 millibars on the assumption that V/A = 1·5 in.

In order to allow for the variation of gravity with height, the *standard* temperature of the instrument further requires to be diminished by

1° A. for every 520 metres[4] of height above mean sea-level.

Once the *fiducial* temperature of the barometer corresponding to a given station is obtained, the true atmospheric pressure, in millibars, at station level can be determined by making the single correction for departure of the instrument temperature from *fiducial* temperature.

It should be remarked that the *standard* temperature of 285° A. is aimed at so as to give for English latitudes a fiducial temperature which is not far removed from the mean temperature of a station, *i.e.* so as to make the uncorrected barometer readings a good approximation to the true pressure.

When the station is fixed, the fiducial temperature is fixed, and the method of obtaining true pressures at that station in millibars by means of a single temperature correction is found very convenient.

For mercury barometers used at sea, the fiducial temperature requires to be determined from time to time, using the foregoing table in conjunction with the measured latitude of the ship.

---

[1] Appropriate Tables of Barometric Corrections for Gravity are to be found in the *International Meteorological Tables* and the *Smithsonian Meteorological Tables.*

[2] British millibar barometers are graduated on the basis of 980·617 as the value of "standard" gravity.

[3] The *standard* temperature for a millibar barometer is the temperature at which the instrument registers true millibars when stationed under standard conditions of gravity.

[4] The value 520 refers to a Fortin type barometer. The equivalent value for a Kew barometer is 540 metres if the value of V/A is taken to be 1·5.

For high altitude work in millibars, the table referred to is only approximate, as it has been calculated to correspond to a pressure of 1000 millibars.

(iv.) *Reduction of Atmospheric Pressures to Datum Level.*—It should be observed that an accurately standardised mercury barometer requires correction only for the temperature and gravity of the mercury in order to obtain the absolute pressure at cistern level. These corrections are inherent in the use of a mercury barometer.

In meteorological work, such as the mapping of isobars for weather forecasting, it is not the cistern-level pressure that is finally required, but the corresponding atmospheric pressure at mean sea-level. Consequently a correction has to be made for the difference in atmospheric pressure between station-level and sea-level. This is not an instrumental correction, but depends on atmospheric conditions together with the height of the station above sea-level.

For meteorological work the requisite correction tables, together with their basis of calculation, are given in the *Observer's Handbook*.[1] (Reference should also be made to § (16) concerning the relation between heights and atmospheric pressures.)

It sometimes happens in laboratory work that the difference of atmospheric pressure is required corresponding to a comparatively small difference in level. The approximate correction factor can be calculated directly from the relative densities of air and mercury.

For stations near sea-level, an altitude difference of 100 ft. corresponds to a pressure difference of 0·110 in. of mercury when the atmospheric temperature is 50° F.

§ (7) THE ERRORS AND DEFECTS OF MERCURY BAROMETERS.—(No attempt is made here to draw a hard and fast distinction between errors and defects. In the majority of cases, the errors considered are inherent in the general design of mercury barometers, and are kept as far as possible at a minimum. There are, on the other hand, errors which appear as such owing to faulty workmanship or design: these may be called avoidable or remediable defects.)

The following paragraphs apply primarily to those barometers which may be considered standard types for general and meteorological use. They are also applicable, in many respects, to high precision mercury barometers, designed and constructed for special purposes, in which case it is the aim of the designer to take special precautions to minimise the respective errors.

(i.) *Defective Vacuum Space.*—In a good barometer, the residual pressure in the vacuum space above the mercury column should not exceed the accuracy usually obtainable from the instrument. This degree of efficiency can be attained, in general, by the ordinary methods of filling barometer tubes by boiling. A defective vacuum is usually due to the presence of air or water vapour. In portable barometers, the presence of air can easily be detected by inspecting the closed end of the barometer tube when inclined so as to be filled with mercury. It should not be tolerated unless the size of the air-bubble is very small. A hard criterion can scarcely be laid down. In a small barometer tube (*e.g.* of ¼ in. bore), a bubble not exceeding 1·3 mm. in diameter when the tube is laid flat may be tolerated even if the highest accuracy obtainable from the instrument is required. For tubes of ½ in. bore, a bubble exceeding 2 mm. in diameter would be considered excessive. These estimates are intended only as an approximate guide. Other considerations also enter into the question. Those accustomed to handling barometers can make a very fair estimate of the air content of a tube from the sound or "clang" which occurs when the mercury is run slowly against the end of the tube, but this criterion is not applicable to tubes (such as those in marine barometers) which have been constricted so as to damp the flow of the mercury through them.

The presence of water vapour above the mercury column in a barometer tube is difficult to detect by a direct test. The maximum possible error due to it is equal to the saturation vapour pressure of water, which at 62° F. is rather more than half an inch of mercury. A comparatively small volume of water is required to saturate the vacuum space. A hundredth of this amount of water will, therefore, produce a vapour pressure approximately equal to 0·005 in., the presence of which can just be detected in the average barometer.

The usual method of testing a mercury barometer for defective vacuum due to the presence of water vapour is to compare its readings at a number of pressures with those of a standard barometer column whose vacuum space is known to be satisfactorily free from air and vapour. These comparisons are usually made by the direct reading of each mercury level with a cathetometer. The presence of air or unsaturated vapour in a given barometer is revealed by an error in the reading of that barometer, which increases inversely as the distance of the mercury level from the closed end of the tube, following Boyle's Law. A good example of this may be seen in *Fig.* 11, which represents the case of an exceptionally defective long-range Kew pattern barometer, which was found to show pronounced signs of water vapour in its tube above the mercury column.

---

[1] Published by the London Meteorological Office.

This case is all the more interesting since it shows a definite discontinuity at C, marking the place

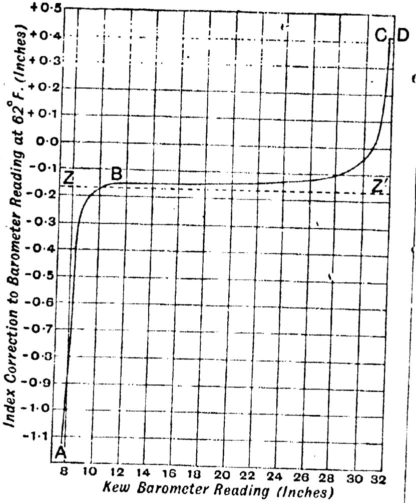


FIG. 11.

where the vacuum space is just saturated with water vapour. For barometric readings between 31·7 and 32·0 in. the vacuum space is saturated with water vapour, provided the temperature remains constant. As a result, the barometric column corresponding to these readings is depressed by a constant amount due to water vapour, the calibration curve being a horizontal straight line CD for this small portion of the range. For readings taken on the instrument below 31·7 in. the water vapour no longer saturates the vacuum space, but exerts a pressure which follows Boyle's Law as the size of the vacuum space changes.

The shape of the portion AB of the calibration curve is caused by the mercury coming into contact with the roof of the cistern, and will be referred to in connection with the errors of incomplete compensation for capacity (see § (7) (iv.)).

If it were possible to eliminate this defect and also remove the water vapour from the tube of this barometer, without otherwise upsetting the adjustment of the instrument, the calibration curve would most probably take the simple form shown by the line ZZ′.

It is wise to reject all barometers that show even small traces of water vapour in their tubes, or to return them to the instrument makers for their tubes to be reboiled or replaced.

(ii.) *Errors due to Capillary Action of the Mercury.*—In view of the considerable effect of capillary action on the accuracy obtainable in mercurial barometry, it is of the utmost importance that the nature and extent of the error due to surface tension should be carefully considered and estimated.

The error is present in more ways than one, but consideration will first be given to the effect of capillary action in the barometer tube due to the surface tension at the mercu meniscus.

Fig. 12 represents the upper portion of barometer tube. In the absence of th phenomenon of surface tension, the surface of the mercury in the tube would be horizontal, as indicated by the line L. Actually, the tension T, acting everywhere throughout the surface layer of the liquid, is equivalent to a resultant force acting along the surface, normal to each element of the ring of contact of the mercury with the glass tube. The resultant tension acting on the mercury column is, therefore, of magnitude $\pi \times d \times T$, acting downwards in a direction inclined to the horizontal at an angle a, say, as shown in the diagram, where d is the internal diameter of the tube.

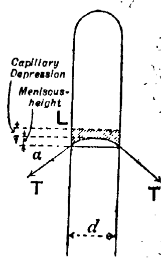


FIG. 12.

Assuming that in the absence of surface tension the position of the top of the barometric column corresponding to a given pressure would be represented by the level L, it follows that the actual position of the summit of the mercury column is determined by the fact that the vertically downward component of the force of surface tension on the barometric column is equal to the weight of the mercury which would have occupied the portion shaded in the diagram, in the absence of surface tension. The linear amount by which the barometric column is lowered owing to the surface tension of the mercury in the tube is known as the capillary depression of the column. It depends largely on the diameter of the tube, and obviously on the value of the angle of contact of the mercury against the glass wall of the tube. In tubes of diameter exceeding 1 inch, the capillary depression is practically negligible.

Even if the value of the surface tension of mercury remains constant, the consistency of the position of the summit of the barometric column in a given tube at given pressure will depend on the amount of variation in the angle of contact.

The angle of contact in a barometer tube is not practically measurable, but since the meniscus-height, *i.e.* the height of the summit of the meniscus above the circle of contact, is determined by a given contact angle, the capillary depression may be looked upon as being a function of the bore of tube and the meniscus-height.

Table II. gives values of the capillary depression corresponding to various sizes of tube and heights of meniscus.

TABLE II

TABLE SHOWING THE CAPILLARY DEPRESSION OF THE MERCURY COLUMN IN BAROMETER AND OTHER GLASS TUBES OF GIVEN BORES AND FOR GIVEN VALUES OF THE MENISCUS-HEIGHT

(Based on 0·444 gramme per centimetre as the value of the surface tension of mercury, *in vacuo*, in contact with glass.)

Bore of Tube.	Height of Meniscus (millimetres).																										Bore of Tube.
	0·1.	Angle.	0·2.	Angle.	0·3.	0·4.	0·5.	Angle.	0·6.	0·7.	0·8.	0·9.	1·0.	Angle.	1·1.	1·2.	1·3.	1·4.	1·5.	Angle.	1·6.	1·7.	1·8.	1·9.	2·0.	Angle.	
mm.	mm.		mm.		mm.	mm.	mm.		mm.	mm.	mm.	mm.	mm.		mm.	mm.	mm.	mm.	mm.		mm.	mm.	mm.	mm.	mm.		mm.
1	5·1	23°	9·1	45°	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
2	1·3	12°	2·5		3·5	4·4	5·2	54°	5·9	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
3	0·5	8°	1·1		1·6	2·1	2·5	39°	2·9	3·2	3·4	3·6	3·7	73°	..	..	..	..	..	..	..	..	..	..	..	..	3
4	0·3	6°	0·6		0·8	1·1	1·4	30°	1·7	1·9	2·1	2·2	2·4	58°	2·5	2·6	..	..	..	..	..	..	..	..	..	..	4
5	0·18	5°	0·36		0·54	0·73	0·90	25°	1·06	1·20	1·34	1·45	1·55	50°	1·66	1·76	..	..	..	..	..	..	..	..	..	..	5
6	0·11	5°	0·23		0·35	0·47	0·59	22°	0·71	0·81	0·91	0·99	1·08	44°	1·15	1·21	1·26	1·30	1·34	66°	1·37	1·40	1·43	1·46	1·48	87°	6
7	0·08	4°	0·16		0·24	0·33	0·41	20°	0·48	0·56	0·63	0·70	0·76	40°	0·81	0·86	0·91	0·96	1·00	60°	1·03	1·06	1·08	1·10	1·12	80°	7
8	0·06	4°	0·12		0·18	0·24	0·30	19°	0·35	0·41	0·46	0·51	0·55	37°	0·59	0·63	0·67	0·70	0·74	56°	0·77	0·80	0·82	0·84	0·85	75°	8
9	0·05	3°	0·09		0·14	0·18	0·23	18°	0·27	0·31	0·35	0·38	0·41	35°	0·44	0·47	0·50	0·53	0·55	53°	0·57	0·59	0·61	0·63	0·64	72°	9
10	0·03	3°	0·06		0·09	0·12	0·15	17°	0·18	0·21	0·24	0·27	0·30	34°	0·33	0·35	0·37	0·40	0·42	51°	0·44	0·46	0·47	0·48	0·49	69°	10
12	0·02	3°	0·03		0·05	0·07	0·08	17°	0·10	0·11	0·13	0·15	0·16	33°	0·18	0·20	0·21	0·22	0·24	49°	0·25	0·26	0·27	0·28	0·29	67°	12
14	0·01	3°	0·02		0·03	0·04	0·05	16°	0·06	0·07	0·08	0·09	0·10	32°	0·10	0·11	0·12	0·13	0·14	49°	0·15	0·16	0·17	0·18	0·19	67°	14
16	0·006	3°	0·013		0·019	0·025	0·031	16°	0·037	0·043	0·049	0·056	0·062	32°	0·068	0·074	0·080	0·086	0·092	49°	0·098	0·104	0·111	0·117	0·122	67°	16
18	0·005	3°	0·009		0·013	0·018	0·022	16°	0·026	0·031	0·035	0·039	0·044	32°	0·048	0·052	0·057	0·061	0·066	49°	0·070	0·074	0·079	0·083	0·088	67°	18
20	0·003	3°	0·006		0·009	0·012	0·015	16°	0·018	0·021	0·025	0·028	0·031	32°	0·034	0·037	0·040	0·043	0·046	49°	0·050	0·053	0·056	0·059	0·062	67°	20
25	0·002	3°	0·003		0·005	0·006	0·007	16°	0·009	0·010	0·012	0·013	0·015	32°	0·016	0·018	0·019	0·021	0·022	49°	0·024	0·025	0·027	0·028	0·030	67°	25
30	0·001	3°	0·001		0·002	0·003	0·003	16°	0·004	0·005	0·005	0·006	0·007	32°	0·007	0·008	0·009	0·009	0·010	49°	0·010	0·011	0·012	0·012	0·013	67°	30
40	..		..		..	..	..		..	..	..	..	0·0001	32°	..	..	..	..	..		..	..	..	..	0·0002	67°	40

Notes.—1. In addition to the capillary depression, information has been given as to the corresponding angle of contact. The angle tabulated represents the inclination of the meniscus to the horizontal at its contact with the glass tube, and is therefore complementary to the angle of contact as usually defined.

2. The values given in the table have been calculated almost entirely from first principles, using the value of the surface tension of mercury *in vacuo* found by Stöckle (*Wied. Annalen*, 1898, lxvi. 499).

Little information is available concerning the surface tension of the mercury in the average barometer tube. Its value is liable to differ from that accepted above by an amount which not only depends on the cleanness of the glass tube and mercury, but is likely to be influenced by small quantities of metal impurities in the mercury.

Values of the capillary depression have been determined experimentally by Mendelejeff and Gutkowski (*Jour. de Phys. Chem. Geo.*, Petersburg, 1877, or *Wied. Beibl.*, 1877). They are considerably lower than the above tabulated values, but do not cover such a large range of sizes of tube.

3. The values given in the table may be taken as generally accurate to the decimal to which they have been rounded off, subject to the assumption of the value indicated for the surface tension of mercury.

4. For tubes of bore less than 1 mm., the shape of the mercury meniscus may, with sufficient accuracy, be assumed spherical.

In practice the scale of a mercury barometer is graduated so as to allow for this capillary depression. This is done by comparison with a standardised instrument, which in its turn has previously been compared with a barometer containing such a large tube (*e.g.* a fundamental standard barometer) that the amount of the capillary depression is negligibly small.

(*a*) *The Extent of Errors in the Indications of Mercury Barometers due to Variation of Capillary Action in the Tube.*—Very little experimental work has been done in the direction of determining the extent of the variation of the value of the surface tension of mercury in the same or in different barometer tubes. Its variation with temperature has been found to be relatively small (Kaye and Laby's *Tables of Physical and Chemical Constants*), but as surface tension phenomena depend to a considerable extent on the cleanness of the mercury and the glass, some variation may be expected in the value of the surface tension.

On the other hand, variations in the meniscus-height in a given tube have been observed, which are too large to be attributable to variations in surface tension, and can only be explained by considerable variations in the angle of contact.

A large number of experimental observations have been made at the National Physical Laboratory on the variation of meniscus-height in barometer tubes of different sizes. From this work the values of the angle of contact corresponding to various meniscus-heights have been estimated by means of tables similar to Table II., working on the assumption of constancy of the value of the surface tension of mercury in barometer tubes as indicated in the table.

As a result, it was found that, on an average for several makers' barometer tubes, the angle of contact of the mercury against the glass was represented by the value, 35°, expressed as the inclination of the meniscus to the horizontal at its contact with the glass tube.[1] (*This value is therefore complementary to the angle of contact as usually defined.*)

In a given barometer tube the average variation of the angle of contact from its mean value was found to be ±8°. This does not include variation with age, but represents variations dependent on the position and cleanness of the mercury in the tube, and also to some extent on the rising or falling condition of the meniscus.

For the sake of example the numerical values for the respective sizes of barometer tubes used in practice are given in Table III.

(*b*) *The Influence of the Shape of the Mercury Surface in the Cistern of a Kew Pattern Barometer on the Accuracy of the Readings.*—While the surface tension of the mercury in the average barometer tube has an important bearing on the resulting accuracy of the instrument, there is also a limitation to the accuracy owing to surface conditions of the mercury in the cistern. As the majority of the Kew pattern barometers made in England have comparatively small cisterns, this source of error is considered here in some detail, not only in investigating the possible inconsistencies of barometer readings at a given time, but in estimating the degree of permanence of the indications of the instrument.

Table III

Internal Diameter of Barometer Tube.	Estimated Absolute Value of the Capillary Depression corresponding to the Average Value of the Angle of Contact (=35° as just defined).	Estimated Average Range of Variation of the Capillary Depression from its Mean Value in a given Tube.
in.	in.	in.
0·20	0·046	± 0·008
·25	·032	± ·006
·30	·023	± ·004
·40	·011	± ·002
·50	·006	± ·001
·60	·003	± ·001
·75	·0015	± ·0005

Note.—In barometer tubes the angle of contact is less susceptible to variation than in tubes open to the air. Tapping a barometer facilitates, but by no means ensures, a consistency in the angle of contact. The absolute values of the capillary depression given in column (2) of the table are admittedly approximate, but may be taken as giving a very fair average for barometer tubes. They have also been verified for some barometers of very small bore in which the capillary depression has been measured directly to a fair degree of accuracy.

The error considered here arises from the variation which occurs in the mercury level in the cistern, even when pressure and temperature are constant. It is due to the varying conditions of angle of contact between the mercury and the cistern, but is not exactly analogous to the variation of capillary depression of the mercury in the barometer tube. Nearly all barometer cisterns are of sufficiently large diameter to make the capillary depression of the mercury in the cistern negligible. A reference to *Fig.* 13 will help to make the nature of the error more easily understood. For the sake of clearness the outlines of the tube and cistern are drawn as simply as possible, and not to scale.

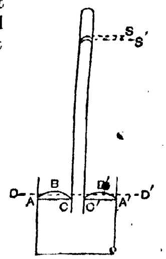


Fig. 13.

The position of the mercury surface in the cistern is shown by the curves ABC C'B'A', representing an average case of a barometer whose cistern is clean. The corresponding position of the mercury in the tube is shown at S. The pressure and temperature are assumed constant for the time. Suppose, now, as a hypothetical limiting case, that the cistern or the mercury in it, or both, become so dirty that the curvature of the meniscus disappears, the surface becoming plane. (This may be an extreme case, but it is not far from what may happen in practice in a cistern to which a moist atmosphere has access.) The level of the mercury surface in the cistern would then fall to DD′, which is defined by the conditions that the volume of the mercury is constant, and the actual height of the barometric column has remained constant.

---

[1] For freshly distilled mercury in air the corresponding contact value would be as high as 45° to 50°.

The flatter the mercury meniscus in the cistern, the greater the tendency of the barometer to read low, as at S′.

The magnitude of the change in the mercury level in the cistern corresponding to a given change in the angle of contact has been calculated from a knowledge of the values of the surface tension and the density of mercury, together with the dimensions of the instrument.

The results are set out in the following table. They are very nearly independent of the size of barometer tube, which is always small compared with the diameter of the cistern.

When the pressure increases, mercury flows from the cistern into the tube, and in order to provide for this flow the mercury meniscus in the cistern becomes flatter until equilibrium is reached between the barometric column and the atmospheric pressure. Owing to friction there is a tendency for the ring of contact of the mercury with the cistern to maintain its position until the meniscus cannot accommodate itself any more to the increase of pressure.

Both mercury menisci in a Kew barometer take part in this accommodation. The meniscus in the tube would, in general, tend to bulge out more in response to the increase in pressure. As long as the meniscus remained stable the ring of contact with the tube would maintain its position, and not move until the friction between the mercury and the glass was overcome.

Tapping the barometer facilitates the formation of the most stable meniscus in tube and cistern. Without tapping it is evident that the barometer loses a certain degree of accuracy depending in magnitude upon its dimensions, since small changes in pressure would, in general, give rise to small changes of shape of the two menisci without yielding the full corresponding change in the reading of the barometer.

These changes are most marked in marine barometers, not only on account of the comparatively small dimensions which are desirable for ships' barometers, but also because the tube is constricted so as to damp the flow of the mercury through it.

Space does not permit of a detailed discussion of the magnitude of these changes which are due to accommodation of the shape of the mercury menisci in the tube and cistern. Experimental tests which

TABLE IV

CISTERN ERRORS IN KEW BAROMETERS

Inclination of Mercury Surface to the Horizontal at its Ring of Contact with the Cistern.	Corresponding Meniscus-height in Cistern.		Zero Error of Kew Pattern Barometer relative to assumed Average Conditions.					
			For Cistern of Internal Diameter.					
			1·0 in.	1·3 in.	1·5 in.	2·0 in.	4·0 in.	6·0 in.
	mm.	in.	in.	in.	in.	in.	in.	in.
90° *	2·6	0·10	$+0\cdot008$[illegible]	$+0\cdot006_4$	$+0\cdot005_2$	$+0\cdot003_9$	$+0\cdot001_8$	$+0\cdot001_3$
67	2·0	0·08	$+0\cdot007_1$	$+0\cdot005_1$	$+0\cdot004_3$	$+0\cdot003_1$	$+0\cdot001_5$	$+0\cdot001_0$
49	1·5	0·06	$+0\cdot002_7$	$+0\cdot002_0$	$+0\cdot001_6$	$+0\cdot001_2$	$+0\cdot000_5$	$+0\cdot000_4$
40	1·25	0·05	$0\cdot000_0$	$0\cdot000_0$	$0\cdot000_0$	$0\cdot000_0$	$0\cdot000_0$	$0\cdot000_0$
32	1·0	0·04	$-0\cdot002_7$	$-0\cdot002_0$	$-0\cdot001_7$	$-0\cdot001_2$	$-0\cdot000_6$	$-0\cdot000_4$
16	0·5	0·02	$-0\cdot008_7$	$-0\cdot006_3$	$-0\cdot005_4$	$-0\cdot003_9$	$-0\cdot001_9$	$-0\cdot001_2$
0	0·0	0·00	$-0\cdot015_5$	$-0\cdot011_3$	$-0\cdot009_6$	$-0\cdot006_9$	$-0\cdot003_4$	$-0\cdot002_2$

(The + sign indicates that the barometer would read too high.) * If stable.

Notes.—1. The fourth row of the above table has been assumed to represent average conditions of contact of mercury in an iron barometer cistern to which the moist atmosphere has access.

2. Values of the angle of contact have been tabulated from 0 to 90°. There is evidence that both these limits are occasionally approached under conditions which obtain in practice.

The foregoing table requires to be supplemented by information as to the amount of variation of the angle of contact of the mercury in the cistern before a satisfactory estimate can be made of the extent of the error due to this inconstancy of barometer zero. The cistern is opaque, and direct observation of the angle of contact is impracticable. There is, it should be noted, a great inducement for the angle of contact to change its value considerably by way of accommodation.

have been made on Kew pattern barometers indicate that unless the instrument is tapped considerable variations of the meniscus-shape occur in the cistern. These are such as to correspond to a change of ±30° in the angle of contact measured from its average value. The effects of these changes can be gauged from Table IV. They have also been measured during the course of tests made on Kew pattern barometers at the National Physical Laboratory prior to making a general rule of tapping the instruments before reading them.[1]

It should be remarked that during an increase of pressure the tendency of a Kew barometer to read low is twofold, *i.e.* the flattening of the cistern meniscus consequent upon mercury flowing into the barometer tube operates in the same direction as the bulging of the meniscus in the tube, due to increase of pressure. Unless the barometer is tapped there will undoubtedly be an appreciable difference between its rising and falling indications.

Tapping very nearly eliminates this difference in the case of Kew barometers with unconstricted tubes. In a marine barometer the effect of tapping is impeded by the fact that the tube is constricted. Consequently the mercury column requires time to take up its new position of equilibrium, and during that time further accommodation of the shape of the menisci takes place. To obtain the full benefit of tapping for a marine barometer it would be necessary to tap more or less continuously for several minutes until the meniscus conditions approached most nearly their average values.

It is found in practice, in the case of laboratory tests on marine barometers, that the difference between the falling and rising indications varies from 0·005 in. to 0·010 in. for ordinary changes of atmospheric pressure. The exact figure depends somewhat on the rate of change of pressure and on the state of cleanness of the instrument.

Very few marine barometers have been made with a tube exceeding ¼ in. in internal diameter. As is to be expected, an increase in the dimensions of the instrument is accompanied by a decrease in the difference between its rising and falling indications.

Occasionally station barometers are found in which the tube is narrowed down sufficiently to make the flow of mercury through it somewhat sluggish. This is not to be recommended.

of (*c*) *Possible Errors in a Fortin Barometer* found*ndent upon the Shape of the Mercury* with ag*us in the Cistern.*—In Fortin barometers position are to variation in the meniscus-shape and also to srn are relatively small, except in condition of the truments of small dimensions.

For the sake of the fiducial pointer is situated the respective sizes he glass tail-piece and the practice are given in Ta mmit of the mercury

(*b*) *The Influence of the* s used in making the *Surface in the Cistern of a Ke*r, and the only *meter on the Accuracy of the Rea*of the cistern the surface tension of the mercu depression. average barometer tube has an ie internal bearing on the resulting accuracy out 1·1 instrument, there is also a limitation to the d

therefore be expected, depending on the shape of the meniscus. It is advisable, before making a setting on a mountain barometer, to raise the mercury in the cistern to the pointer with the object of avoiding a flat mercury surface.

(iii.) *Errors of Temperature.*—(The basis of the method of correcting the readings of a mercury barometer for changes in temperature is shown in § (6) (i.).

Note.—A 1° F. change in temperature corresponds to 0·003 in. on the barometer at normal pressure.

A 1° C. change in temperature corresponds to 0·13 mm. on the barometer at normal pressure.)

These errors may be divided into two classes, according as they are involved in the measurement of the temperature of the barometer, or in the correction of the barometric reading from the observed temperature to a standard temperature.

As far as the temperature of the barometer is concerned two things count—the mercury and the scale. Since the coefficient of expansion of the material (usually brass) on which the scale is ruled is of a lower order of magnitude than that of mercury, the chief aim in the measurement of the temperature of a barometer is to obtain the temperature of the mercury column as accurately as possible by simple methods.

A mercury thermometer is universally used as temperature indicator, except in barometers of the highest precision. Since it cannot be immersed in the mercury of the barometric column the next best thing is usually done, and it is mounted with its bulb very near the barometer tube.

Assuming that its bulb is well screened from heat radiation, either from the operator of the barometer or from other sources, the temperature indicated by the thermometer may differ from that of the barometric column owing to the following two reasons:

(*a*) Horizontal and vertical gradients of temperature in the immediate vicinity of the barometer.

(*b*) Thermal lag of the barometric column relative to the mercury thermometer.

(*a*) Mercury barometers should be set up in a favourable position, away from draughts, doors, or windows, or any source likely to create a temperature gradient. In some rooms which appear favourable the air may become stagnant and stratified, resulting in an appreciable vertical gradient of temperature.

Cases of apparently favourable location of barometers have been known in which temperature differences of nearly 1° F. have been found between the top and bottom of the mercury column. It is therefore clearly advisable to arrange for the bulb of the thermometer to be as nearly as possible midway up the barometric column.

---

[1] For freshly distilled mercury in air the corresponding contact value would be as high as 45° to 50°.

As regards thermal errors due to the presence of a horizontal gradient, a note of warning should be sounded against the practice in some cases, often in the more accurate barometers set up in observatories, of setting up the thermometer with its bulb immersed in mercury in a separate tube, a few inches away from the barometer itself. The object of this, doubtless, is an attempt to obtain a similarity of thermal lag in the barometer and thermometer tubes, but often the error avoided in this way is more than counterbalanced by the error due to the presence of a horizontal gradient of temperature between thermometer and barometer. Thermometers should not be set up in this manner unless it has been verified that the effect of such a gradient of temperature is negligible. Cases have been known in which actual differences of temperature exceeding 1° F. have been found. The prevalent practice of letting the thermometer bulb into a small opening cut into the metal sheath surrounding the barometer tube has much to recommend it. Under good average conditions the temperature of the barometric column can be considered as determined within an accuracy corresponding to ±0·001 in. on the barometer.

(*b*) The magnitude of the error due to the lag of the barometric column in following the temperature indicated by its attached thermometer has been investigated experimentally at the National Physical Laboratory by taking a series of readings, at atmospheric pressure, on barometers of different dimensions immediately after they had been transferred to, and set up in, a hot room at a temperature of about 95° F. (35° C.). At a given pressure, the reading of a correct mercury barometer is approximately $\frac{1}{10}$ in. higher at 95° F. than at the more usual atmospheric temperature 62° F. If therefore a barometer is suddenly transferred from the lower to the higher temperature, a series of readings taken on it at suitable intervals of time, commencing from the time of transference, is all that is needed to obtain an empirical formula for the amount of thermal lag of the barometric column. Actually the barometer may be used as its own thermometer in tracing the law of variation with time of the temperature of the mercury column. In brief, the experimental observations showed that even if the average Fortin or Kew pattern barometer were subject to a steady rise of external temperature of 2° F. (1° C.) per hour, the error due to lag of the barometric column in taking up the temperature indicated by the external attached thermometer would not exceed 0·001 in. (0·02 mm.).

It is, of course, understood that in cases where barometers are moved to new quarters sufficient time should be allowed them to take up fully the surrounding temperature. Half an hour is usually sufficient for this purpose. Incidentally it should be remarked that in measuring pressures by means of a mercury barometer and thermometer the latter should be read first, as it is the more easily influenced by the radiation of heat from the observer's body.

Errors in the Correction of Barometers to a Standard Temperature.—Information for correcting the readings of mercury barometers for changes in their temperature is given in the Meteorological Publications.[1] In some respects the data lack completeness, as no reference is made, either in the International or in the Smithsonian Tables, to the distinction between the Fortin and Kew pattern barometers as regards temperature correction. Further, the data for correcting barometers graduated in the more recent millibar unit do not appear in the two above-mentioned tables; they are, however, given in the more recent publications of the *Observer's Handbook.*

(iv.) *Incomplete Compensation of the Kew Barometer for Capacity.*—Since no zero setting is made in the cistern of a Kew pattern barometer, the instrument has to be compensated for the capacity of the tube and cistern (see § (3), (iv.)).

This is done by correctly correlating the following dimensions:

(1) Amount of uniform contraction C of the spacing of the scale.

(2) The internal diameter d of the barometer tube.

(3) The internal diameter D of the cistern,

and to a small extent the external diameter t of the tail-piece of the tube dipping into the cistern.

With the above notation, it can readily be shown from first principles that

$$C = \frac{D^2 - t^2}{D^2 + d^2 - t^2},$$

giving the spacing of the scale required to suit the given dimensions D, d, and t. In new Kew barometers the problem of graduating the scale is subsequent to the choice of D, d, and t. In instruments undergoing repair, it is the glass tube that usually requires replacement. This has to be chosen so that the above relation is satisfied.

In a new instrument, the question of compensation for capacity is simpler as the dividing engine used in graduating the scale can be set to give with good precision the requisite contraction value. In repairing a barometer, the problem turns largely on the choice of tubing of the requisite bore. Clearly there will be some residual error in the process of compensation of a Kew barometer for "capacity." The error is called the capacity error, and may be defined as the rate at which error is developed along the barometer scale due to imperfect compensation for capacity. In the case of barometers tested at the National Physical Laboratory, it is not allowed to exceed ±0·004 in. per inch of scale, while in long-range instruments a more stringent limit is demanded.

---

[1] *Vide The International Meteorological Tables, The Smithsonian Meteorological Tables, The Observer's Handbook.*

In general, this limit is satisfied by both new and repaired instruments, the new barometers being usually compensated to within half this margin of error. The average capacity error in the case of a repaired barometer corresponds to a precision of about 7 mils[1] in the selection of the internal diameter of the barometer tube.

The average amount of contraction of a Kew barometer scale is 0·96 of full size, which corresponds approximately to a cistern diameter five times that of the glass tube.

In cases where proportionately larger cisterns are employed the contraction value may range up to 0·98, *i.e.* the scale is more openly spaced.

There is a standard size cistern corresponding to a given approximate bore of tube. It is exceptional to depart from this standard size for the sake of obtaining compensation for capacity.

Incidental to the error of capacity compensation is the possibility that the error is not uniform over the working pressure range of the barometer. This may be due to one of the following two causes:

(*a*) Lack of uniformity of bore of tube.

(*b*) Mercury level in cistern beyond the uniform cylindrical portion of the cistern.

As regards (*a*), barometer tubes are carefully selected for uniformity as well as size of bore. It is the exception to meet with a barometer with a tube of such marked conicality as to prejudice the uniformity of compensation for capacity.

As to (*b*), the error may appear in two ways. The mercury either reaches the ceiling of the cistern before the lower working limit of pressure is reached, or else reaches a shoulder or flange[2] at the middle of the cistern before the upper working limit of pressure is reached.

Both defects are occasionally found in the older instruments. The former has also been found in new barometers. Its effect on the readings of the instrument can be seen from *Fig.* 11, which shows the calibration curve for a typical case in which two distinct defects are present, viz.:

(1) Unsatisfactory compensation for capacity over the range 8—11 in., owing to the cistern becoming full of mercury.

(2) Presence of water vapour in the vacuum space above the barometric column (§ (7) (i.)).

With careful designing and dimensioning of the cistern, the lack of compensation for capacity at the ends of the working range of pressure can be avoided.

(v.) *Verticality of Barometers.*—The errors due to lack of verticality of barometers are usually small, but not by any means negligible. Their nature depends on the type of instrument.

In the Fortin-type barometer, when high accuracy is desired, it is essential to mount the instrument so that its axis of rotation is correctly vertical. The necessity for this arises from the fact that the fiducial pointer does not lie in the axis of the instrument. Consequently, unless the axis of rotation is vertical, the reading of the instrument will differ according to the direction in which the barometer faces when the mercury in the cistern is brought to the pointer.

All Fortin barometers tested at the National Physical Laboratory are set up so as to rotate about a vertical axis. This condition is secured if the pointer, when once just in contact with the mercury in the cistern, remains so however the instrument is rotated.

The average Fortin barometer, when suspended by its ring from a peg and allowed to take its own "plumb" position, does not settle down to a sufficiently definite position. Verticality of axis of rotation of the instrument can generally be secured with a little adjustment wherever the barometer is located. Lack of attention to this condition may, as already stated, result in errors of 0·001 to 0·002 in., depending on the distance of the pointer from the axis of the instrument.

The source of this error may, however, be eliminated by re-designing the Fortin barometer so that the fiducial pointer lies in the axis of the instrument.[3]

Kew barometers are usually set up so as to hang plumb, either from gimbals or from a ring, and generally, so long as the instrument is not swinging, the errors due to lack of verticality are negligibly small. At sea swinging is bound to occur.

Syphon barometers need careful adjustment for verticality, especially if the distance between the two limbs is relatively large. Accurate instruments are generally fitted with spirit-levels. In the same way siphon manometers are liable to errors due to tilting. These may be avoided by designing the manometer so that its limbs are arranged concentrically, one within the other.

Barometers of high precision naturally require care in levelling. Errors of verticality are usually dependent on the cathetometer arrangements for referring the mercury levels to the measuring scales.

§ (8) THE ACCURACY AND PERMANENCE OF THE MERCURY BAROMETER AS A PRESSURE INDICATOR. (i.) *Conditions special to the Fortin and Kew Types.*—In view of the foregoing remarks as to the effect of the shape of the mercury surface in the cistern on the resulting accuracy of the Kew barometer, it is clear that in the smaller instruments the Fortin type is rather to be preferred for accuracy and permanence. If a Kew barometer is required to possess the same accuracy as a Fortin barometer with a tube of similar size, it is essential to enlarge its cistern so that the effect of changes of meniscus-shape is inappreciable. This has been done in the case of a Kew pattern gauge barometer used as a standard instrument at the National

---

[1] 1 mil = $\frac{1}{1000}$ in.

[2] This flange is not always present in the cisterns of Kew barometers.

[3] *Roy. Meteorolog. Soc. J.*, 1913, xxxix. 55.

Physical Laboratory. The cistern of this barometer is approximately 6 in. in diameter, and the tube ½ in. Although this instrument has been regularly used in connection with aneroid testing, occasions have been found to compare its readings with those of the Laboratory working standard Fortin barometer at current atmospheric pressures, generally near 30 in. The latter instrument has a tube of ⅜-in. bore.

During the course of the year following the installation of the Kew gauge barometer, 75 comparisons were made, three observers participating at various times. From these 75 comparisons a mean value was found for the error of the Kew barometer. On analysing the readings it was found that 63 per cent (*i.e.* 47 out of 75) individually gave values within 0·0010 in. of the mean, 24 per cent (*i.e.* 18 out of 75) individually gave values within 0·0015 in. of the mean, and 8 per cent (*i.e.* 6 out of 75) individually gave values within 0·0020 in. of the mean, while the departures of the four remaining individual values from the mean were

0·0022 in., 0·0027 in., 0·0023 in., 0·0021 in.

These figures indicate the accuracy which it is possible to obtain from a Kew pattern barometer of sufficiently large dimensions. It should be noted that the vernier of the instrument registered to 0·002 in. directly; but readings were taken on it to closer accuracy with the help of a reading-lens.

As an indication of the degree of permanence of the indications of this instrument, the following figures speak for themselves:

TABLE V

PERMANENCE OF A LARGE KEW BAROMETER

Year.	Mean Error of Barometer relative to the Laboratory Standard.	Year.	Mean Error of Barometer relative to the Laboratory Standard.
	inches.		inches.
1915	+0·0032	1917	+0·0025
1916	+0·0029	1918	+0·0027

The information available concerning the permanence of Kew barometers of smaller dimensions is not so definite owing to the paucity of observational data. It is certain, however, that in course of time the mercury surface in the cistern becomes covered with sediment and loses its former shape to an appreciable extent.

It is not easy to evaluate the accuracy attainable by the more frequently used mercury barometers without making reservations. In giving the estimates shown in the following table, it is understood that the instruments are used in favourable conditions, the indicated accuracy being limited only by instrumental errors, and not by external atmospheric or other conditions:

TABLE VI

GENERAL ACCURACY OF FORTIN AND KEW BAROMETERS.

Internal Diameter of Barometer Tube.	General Accuracy of a Single Reading		
inches.	inches.	millimetres.	millibars.
0·25	0·005	0·12	0·16
0·4	0·002	0·05	0·06
0·5-0·6	0·0015	0·03	0·04

Notes.—1. The above values are applicable almost equally to Fortin and Kew-type barometers. They also indicate the order of magnitude of any permanent shift that may occur in the indications of the instruments (with the exception of any change which takes place when the vacuum space becomes defective).

2. Since the causes which contribute to permanent shift are almost entirely linked up with capillary action, the Fortin barometer may be looked upon as being less susceptible to permanent shift than the corresponding Kew barometer, owing to the greater chance of cistern errors in the latter type of instrument.

The case of special-precision mercury barometers is considered separately in § (9). With the greatest precautions a final accuracy of ±0·005 mm. may be obtained in the measurement of pressure.

(ii.) *Limitations to the Accuracy of the Barometer in the Measurement of Atmospheric Pressures.* (*a*) *The Effect of a Wind.*—The barometer is an instrument which, when there is no wind, measures static atmospheric pressure, *i.e.* the weight of the atmosphere. In the presence of a wind there is superposed on the static atmospheric pressure a pressure due to the velocity of the wind, and equal to $\frac{1}{2}\rho v^2$, where ρ is the density of the air, and v is the velocity of the wind.

The following are the values of this expression corresponding to various wind velocities:

TABLE VII

PRESSURE DUE TO WIND

Wind Velocity (V).		Corresponding Pressure ($\frac{1}{2}\rho v^2$).	
Miles per hour.	Metres per sec.	Inches of Mercury.	Millibars.
5	2·24	0·001	0·03
10	4·47	0·004	0·12
20	8·94	0·014	0·5
30	13·41	0·032	1·1
40	17·88	0·06	2·0
50	22·35	0·09	3·0
80	35·76	0·23	7·8
100	44·70	0·36	12·2

A barometer, whether mercurial or aneroid in type, will, if exposed to the full pressure of the wind, register a value which is in excess of the static pressure by the amount shown in the above table.

In a room whose exterior is exposed to the wind, the air pressure may be greater or less than the static pressure of the external atmosphere. In other words, the wind may produce a pressure or a suctional effect on the air in the room, depending on the nature and position of the doors, windows, cracks, or crevices through which the atmosphere may have access to, or egress from, the room. The amount of pressure or suction produced in the room by the external wind may vary up to a maximum value indicated in the above table. Consequently, the extent of the possible error on a barometer, in its normal use as indicator of static atmospheric pressure, can be estimated.

If two barometers in different buildings are undergoing comparison, allowance should be made for the effect of the wind, which is likely to be troublesome at stations of high altitude.

At sea the wind effect is always present on account of the ship's velocity as well as that of the wind. A full discussion of the resulting errors will not be attempted here. The extent of the errors can be gathered from experimental and observational evidence already published.[1]

It is possible to design a mercury barometer having an air-tight cistern into which is led the static tube of an anemometer head mounted in full exposure to the outside atmosphere.

(*b*) *Other Errors in the use of Marine Barometers. Pumping of Mercury Barometers at Sea.*—In a marine barometer a constricted tube is essential in order to obtain even an approximate reading of the atmospheric pressure, the object of constriction being to counteract as far as possible the errors in the measurement of pressure caused by what is known technically as "pumping" of the mercury column owing to the oscillations of the latter aboard ship.

The following are the chief causes operating to produce "pumping" at sea:

(i.) The existence of a periodic vertical acceleration acting on the mercury column owing to the heaving of the ship.

(ii.) The disturbance of the mercury due to the swinging of the barometer.

(iii.) The effect of the wind on the air pressure in the room where the barometer is hung.

As far as (i.) is concerned, the mercury oscillates about a mean position which should give the true atmospheric pressure at the mean height of the instrument above sea-level.

As regards (ii.), the barometer is mounted on gimbals, and as the instrument swings with the rolling and pitching of the ship, the mercury column oscillates about a mean position which is always higher than the true position corresponding to the atmospheric pressure, since, in general, the barometer is inclined somewhat to the vertical.

The errors due to (iii.) have been referred to separately under (*a*). Other errors at sea of a minor character are not discussed here.

Of the above sources of error, (i.) is generally recognised as the chief cause of "pumping," and while there are non-oscillatory errors inevitably associated with (ii.) and (iii.), it is evident that the oscillations of the barometric column due to (i.), (ii.), and (iii.)—*i.e.* the amplitude of the pumping—can be diminished by damping the oscillations, that is, by constricting the barometer tube more heavily.

The greater the damping, however, the greater the lag of the mercury column in responding to the changes of atmospheric pressure. Hence, constricting the tube is of necessity a compromise between the lag error and the errors due to pumping. This compromise may be arranged to suit average conditions at sea, or else the worst conditions.

It is conceivable that, in existing marine barometers the best compromise may not have been made. For many years past, the constriction of marine barometer tubes has been required to lie between two specified limits. The average tube may be regarded as having a lagging time of roughly four minutes, *i.e.* if the atmospheric pressure is changing at a uniform rate, the barometer at a given time registers the pressure which obtained four minutes before the reading was taken.

In the majority of cases the rate of variation of atmospheric pressure does not exceed 0·030 in. (1 millibar) per hour, *i.e.* 0·0005 in. (0·017 millibar) per minute. The lag error of the average barometer corresponding to this rate, being equal to the change in pressure in four minutes, is therefore 0·002 in. (0·07 millibar).

Occasionally the rate of change of atmospheric pressure may be double or even treble the above estimate, so that the lag error may be correspondingly increased in extreme cases. It is well known that the errors in the determination of atmospheric pressures at sea are much larger[2] than the figures which have been attributed above to lag, but it is difficult to say to what extent they are attributable to pumping, and the consequent uncertainty in reading the instrument, or how much they are due to the influence of the wind on the pressure in the chart-room. Undoubtedly both sources of error have a considerable influence on the accuracy of determination of atmospheric pressure at sea.

(*c*) *Influence of the Velocity of a Ship on the Effective Value of Gravity acting on the Mercury Column of a Marine Barometer.*—When a ship moves in an easterly direction, the net centrifugal force due to the earth's rotation and the ship's motion is increased, so diminishing the effective value of gravity on the barometric column. Consequently the instrument reads too high when the ship is moving eastwards, and too low for westward motion.

---

[1] *Roy. Meteorolog. Soc. J.* xxxiv. 100; *British Assoc. Rep.*, 1919, p. 89.

[2] Gold, *Roy. Meteorolog. Soc. J.* xxxiv. 97.

The decrease in the value of gravity resulting from a ship's easterly velocity of twenty miles per hour (17·4 knots) is approximately 0·08 cm./(sec.)^2 for English latitudes. The mercury barometer therefore reads too high on this account by 0·0025 in. (0·08 millibar).

The existence of the error has been experimentally proved[1] by comparing the readings of a mercury and an aneroid barometer on board ship during a series of short easterly and westerly voyages.

§ (9) FUNDAMENTAL STANDARD BAROMETER.—It will have appeared from the foregoing consideration of the design and errors of mercury barometers that these instruments do not in themselves admit of ready calibration from first principles. It is essential, however, that there should be some standard with which each barometer can be compared and calibrated. This standard should be designed so that all its errors are determinate, and it should give in itself a true value of the pressure. Such an instrument may be called a fundamental or primary standard barometer.

Before describing a typical primary standard barometer, it would be well to consider the main difficulties in the design of the instrument and in the absolute measurement of pressure from first principles.

Errors may arise from the following main sources. These have not only to be reduced to a minimum, but to be made determinate.

(*a*) Imperfect vacuum above the barometric column.

(*b*) Mercury not absolutely pure (deviation from standard density).

(*c*) Inaccuracy of measurement of the temperature of the barometric column.

(*d*) Inaccuracy of measurement of the height of the barometric column, including errors of cathetometry and capillarity.

In consideration of the above errors, it should be observed that, since the best Fortin-type barometers are consistent to 0·001 in. (0·02 mm.) in the measurement of pressure (this being generally regarded as the limiting accuracy which can be expected of instruments of this design), it is desirable that a primary standard barometer should yield an accuracy of a higher order, *e.g.* 0·01 mm. or better.

(*a*) With modern methods, a satisfactory vacuum should be readily attainable. A primary standard barometer should be designed so that the residual pressure in the vacuum space admits of measurement from time to time; for this pressure, though small, or even negligible at the initial setting up of the instrument, is liable to increase with time.

(*b*) To a certain limit, probably corresponding to within ±0·001 in. on the barometer in general, the mercury, as supplied by instrument makers in their Kew and Fortin-type barometers, may be regarded as consistently pure.

Accurate determinations of the density of different specimens of mercury purified by different methods indicate variations of ±0·0001 or even 0·0002 from the mean value, 13·5955 grammes per millilitre, generally accepted as the density of mercury at 0° C. It is evident that the density of mercury should be specially determined for the liquid used in a primary standard barometer. If this differs from the value generally accepted, a correction should be made so that the instrument will yield pressures in terms of mercury of accepted density.

(*c*) Since a change of 1° C. in the temperature of the normal barometric column corresponds to a change of 0·005 in. (0·14 mm.) in its height, the measurement of the temperature of a primary standard barometer requires the most refined methods. It is desirable to limit each of the errors (*a*)-(*d*) to 0·0001 in. (0·002 mm.) at the most. This would necessitate measurement of temperature to within 0·02° C., which may be regarded as very near the limiting accuracy attained under favourable conditions by high-precision mercury thermometers. Under ordinary conditions, the temperature of the mercury in a barometer may vary a few tenths of a degree (° C.) between the top and bottom of the column; and, unless the barometer is arranged for immersion in a constant temperature water bath (which severely limits the design of the instrument), the problem of measuring the mean temperature of the barometric column with the required precision requires considerable care, whether mercury thermometers are used or not.

(*d*) As regards the measurement of the height of the barometric column, the cathetometer measurements can be satisfactorily made in terms of a scale which has previously been calibrated by comparison with a standard of length.

The method of reading the top of a mercury column has been referred to under § (5) (ii.), while the error due to capillary action can be reduced so as to be negligible by making the barometer tube of sufficiently large diameter.[2]

It is usual in primary standards to arrange the barometric column so that the upper and lower mercury surfaces are similar.

In addition to any allowances that have to be made for the errors referred to above under the heads (*a*)-(*d*), a number of relatively small corrections have to be applied before arriving at the final value of the pressure determined by means of a primary standard barometer. A full account of these errors is given in the publications[1] of the Bureau International, where one of the primary standard barometers of the Bureau is described.

---

[1] Duffield, *British Assoc. Rep.*, 1919, p. 92.

[2] See Table II. in § (7).

Briefly, the arrangement of this barometer is, in essentials, like that shown in *Fig.* 14. It is really a combined manometer and barometer, but may be considered here as a standard barometer in duplicate.

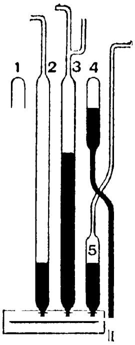


FIG. 14.

Tubes 1 and 2 together form one barometric column with vacuum space at the upper end of 1.

The open end of 2 can be put into communication with the atmosphere or, in fact, any gaseous pressure to be measured.

Tubes 4 and 5 also constitute a barometer, with closed end at 4, and a lower reservoir 5 communicating with the atmosphere through a narrow, bent extension tube.

The tube 3 is a supply reservoir containing mercury which is generally under diminished air pressure. The upper end of this tube may be led to a vacuum pump or temporarily opened to the air according as it is desired to reduce or increase the amount of mercury in each barometer.

Taps are arranged at the lower end of each of the tubes 1, 2, 3, 4, and 5, so that the amount of mercury in the two barometers can be adjusted independently.

This arrangement allows of considerable variation in the volume of the vacuum space above each mercury column, and makes it possible for the residual pressure[2] in each space to be calculated from comparisons made between the two barometers under various conditions of size of vacuum space.

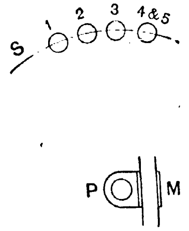


FIG. 15.

The internal diameter of the tubes, excepting the narrower extension parts, is 1·4 in.

Temperatures are determined by means of four high-precision mercury thermometers. For convenience, these are not shown in the diagrams.

Fig. 15 shows, as simply as possible, the general arrangement of the barometer with respect to the cathetometer.

The latter consists of a pair of micrometer microscopes mounted on a vertical pillar P, which can rotate about a vertical axis. The scale of the cathetometer is set up separately at S, so that it lies together with the axes of the tubes on the circumference of a circle with centre at P. Suitable focussing arrangements enable the scale and the axes of the tubes to be brought exactly into the focal plane of the microscopes, so that the height difference between two mercury levels may be measured directly in terms of the scale by rotating the pillar P with its microscopes.

For further particulars reference should be made to the original description published by the Bureau International.

IV. THE ANEROID BAROMETER

§ (10) The aneroid (or non-liquid) barometer is an instrument which, while not susceptible of the same high order of accuracy as a mercury barometer, is a good substitute for the latter under conditions where the use of a mercury barometer is not admissible.

Its operation depends in principle on the fact that a thin metal disc or membrane responds elastically, to an appreciable degree, to the difference of pressure on its faces.[3]

(i.) *Details of Construction.*—*Fig.* 16 shows full details of the mechanism of the aneroid.

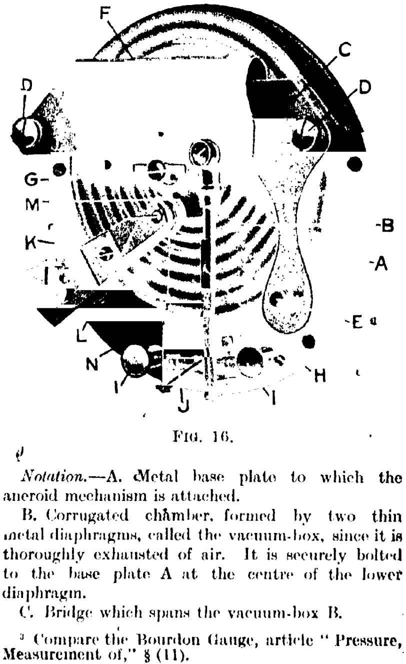


FIG. 16.

Notation.—A. Metal base plate to which the aneroid mechanism is attached.

B. Corrugated chamber, formed by two thin metal diaphragms, called the vacuum-box, since it is thoroughly exhausted of air. It is securely bolted to the base plate A at the centre of the lower diaphragm.

C. Bridge which spans the vacuum-box B.

---

[1] *Travaux et Mémoires*, tome iii.

[2] Modern methods of measuring high vacua may be applied if the upper portion of the barometer tube is suitably designed.

[3] Compare the Bourdon Gauge, article "Pressure, Measurement of," § (11).

DD. Adjusting screws which are used to raise or lower the bridge, thereby altering the tension on the vacuum-box B.

E. Adjusting screw for setting the aneroid to read correctly at current atmospheric pressure. The head of this screw is seen in the back of most aneroids.

It provides a slow motion for raising or lowering the bridge C, thus slightly adjusting the vacuum-box system and therefore the reading of the aneroid.

F. Steel spring which slides in the back of bridge C.

G. Knife-edge (of triangular or square steel rod), passing through the stud of the vacuum-box and coupling the box with the spring F, which tends to open the box by pulling strongly upwards.

The spring and vacuum-box are coupled together by means of the knife-edge G. It should be noticed that as the vacuum-box is similar to a small circular metal box (closely resembling two lids of a tin can soldered together at their edges), it will be sensitive to changes of atmospheric pressure, and, when exhausted of air, will collapse as illustrated by *Figs.* 17 and 18.

One of the functions of the steel spring F is to oppose this tendency of the vacuum-box to collapse. When coupled to the box its tension should be equal to the pressure of the atmosphere on the two diaphragms of the box. Under these circumstances the shape of the box does not return to that shown in *Fig.* 17, because the tension of the spring is applied at the centre of the diaphragm, whereas the atmospheric pressure is uniformly distributed over the face of the diaphragm. The final shape of the vacuum-box will therefore be somewhat like that illustrated in *Fig.* 19, but will depend largely upon the thickness of the metal membrane.

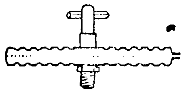


FIG. 17.
Before removal of Air.

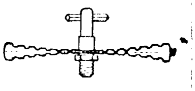


FIG. 18.
After removal of Air.

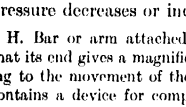


FIG. 19.—After coupling with the Control Spring.

As the under side of the vacuum-box is secured to the base-plate of the aneroid, it is clear that the upper side will move upward or downward with the control spring according as the atmospheric pressure decreases or increases.

H. Bar or arm attached firmly to the spring so that its end gives a magnified movement corresponding to the movement of the vacuum-box. This bar contains a device for compensating the aneroid for errors due to changes in the temperature of the instrument.

I I. Two supports or pillars fitted to base-plate A.

J. Bar, or regulator, working on steel points or pivots passing through the supports I I.

If the illustration be carefully studied it will be noticed that there is a small rod passing from the end of the arm H to the side of the bar or regulator J.

An increase in atmospheric pressure causes the arm H to move downward and the regulator J to rotate outward from the mechanism. An arm is set in an upward direction from the regulator J, which at its upper end magnifies the movement considerably.

K. An arm or cock mounted on an independent pillar fixed to the base-plate.

L. Pin or arbor passing through the end of cock K. The indicating needle of the aneroid (not shown in diagram) is mounted on this pin.

M. Hairspring fitted to pin.

N. Chain of steel one end of which is fitted to the arm passing upward from the regulator J, the other being secured to the pin L.

Concerning the more important parts of the mechanism, the following are further particulars:

(ii.) *Vacuum Chamber.*—This is, in general, made of thin sheet German silver, corrugated in order to produce greater flexibility of the membrane.

The corrugations may be made by stamping or by spinning the membrane in a lathe.

The diameter of the membrane depends on the size of the instrument. It is usually a little more than half that of the dial of the aneroid, and so varies from 1 in. for a small watch-size instrument to about 3 in. for the largest aneroids.

As to the thickness of the metal membrane, there is a rather limited choice for the aneroid maker. Usually, when German silver is employed, 0·004 and 0·006 in. are standard sizes for most aneroids. An increase in the thickness clearly results in a stiffer membrane, *i.e.* there will be less movement of the centre of the membrane corresponding to a given change of pressure. Consequently increased magnification will be required in registering this movement on the dial of the instrument. On the other hand, if a thin membrane (*e.g.* 0·002 in. gauge) is used it will, in general, be unduly distorted on account of its greater flexibility, the control spring operating at the centre of the membrane.

Clearly there is a limit to the thinness of metal used in making up the vacuum chamber. In practice, it is found that German silver, 0·002 in. in thickness, can only be employed in the case of a small watch-size aneroid where the membrane is approximately 1 in. in diameter. Even then it is desirable to restrict its use to a short pressure-range. For longer ranges on a dial of similar size, the requisite magnification can be obtained when a thicker membrane is used.

German silver is not exclusively used in making the membranes for aneroid vacuum chambers. Steel diaphragms have been used with some success in diminishing the defect commonly known as "creep," which occurs to a greater or less degree whenever the membrane is strained by being subjected to large changes of pressure such as are necessarily met with in aeroplane flights.

Some aneroids are still made in this country with steel diaphragms. In at least one type of aneroid, phosphor-bronze is used as diaphragm metal.

(iii.) *Lever System for Mechanical Magnification of the Movement of the Diaphragm of the Vacuum Chamber.*—So far, no mention has been made of the kind of scale to be found on the dial of the aneroid, or of the precise nature of the magnification employed in transferring the movement of the vacuum-box diaphragm to the movement of the indicator on the dial of the instrument.

The exact arrangement of the system of magnifying levers depends on whether the instrument is intended to record pressures or altitudes, or both. Further reference to this point will be made under the subsection dealing with altimeter aneroids (§ (18)).

If the aneroid is intended to record pressures only, it is usual to arrange the subdivisions of the pressure scale on the dial to be more or less uniformly spaced throughout their range. Exact uniformity of spacing is resorted to in cases where a vernier is used with the pressure scale in order to read off fractions of a subdivision, but the use of a vernier with a pressure-reading aneroid is neither usual nor to be recommended in general.

The relation between the amount of movement of the centre of the vacuum-box diaphragm and the pressure-change which causes this movement is very nearly linear, even for a pressure-change from 30 to 15 in. of mercury.

The amount of movement corresponding to a pressure-change of 1 in. of mercury depends upon a number of circumstances, chief of which are:

(*a*) The stiffness of the control spring.

(*b*) The dimensions of the vacuum-box, including the thickness of the diaphragms.

(*c*) The material of the diaphragms, etc.

A fair average estimate in the case of a single box of two membranes would be 0·005 in. movement for a pressure-change of 1 in. of mercury. This movement is indicated on a pressure scale in which a nominal inch of mercury usually measures at least a linear inch (sometimes more, according to the openness of the scale). Hence, a magnification of the order 200 or more has to be obtained.

Roughly speaking, a twentyfold magnification is easily obtained by the lever system shown in *Fig.* 16, between the vacuum-box and the metallic chain. A further magnification of twenty may be obtained in transferring the motion of the chain to that of the pointer on the scale, using the wheel and axle principle. The magnification is, generally speaking, tolerably uniform throughout the range of the pressure scale.

Variation of magnification with the position of the pointer on the dial can be obtained by suitable arrangement of the angular positions of the shorter levers (see § (18)).

Of course, increased or diminished magnification throughout the range of pressure may be obtained by suitable alterations of the lengths of the levers.

§ (11) COMPENSATION OF THE ANEROID FOR TEMPERATURE.—The effects of temperature on the mechanism of an aneroid barometer are sufficiently marked to necessitate compensation in aneroids generally. They are twofold in character, for in addition to the thermal expansion of the aneroid mechanism, particularly the diaphragms, there is a thermal change in the values of the elastic moduli of the material of the vacuum-box system; and the result of an increase in temperature is to make the diaphragms of the box approach one another.

Commercially, an aneroid is called "compensated" if it has some device in it which will make the reading independent of temperature at such pressures as occur at sea-level (*i.e.* at about 30 in.). This, however, does not necessarily amount to complete thermal compensation of the instrument. If the scale of a "compensated" aneroid is correctly graduated at a given temperature for all pressures, it will not necessarily be correct at another temperature for all pressures. The existence of a temperature coefficient to the scale value of an aneroid should not, therefore, be overlooked. In the majority of cases in which the aneroid has been compensated at a constant sea-level pressure this coefficient is small, but not necessarily so.

Two methods are employed in practice to compensate aneroids for temperature changes. The better and sounder plan is to make the long arm of the lever system of two different metals, viz. brass and iron firmly brazed together.

Actually, this arm is chiefly of brass, but a length of iron is inserted in the upper side. Owing to the unequal thermal expansion of the two metals (the expansivity of brass being greater than that of iron) the effect of an increase in temperature is to bend the bimetallic lever, making it slightly concave-upwards, *i.e.* towards the dial. This opposes the thermal expansion of the diaphragms, counteracting the tendency of the diaphragms to approach one another under increase of temperature. With this method of compensation, the vacuum-box must be thoroughly exhausted. Mathematical investigation shows that satisfactory compensation over a large range of pressure and temperature is not likely to be approached unless the box is thoroughly exhausted.

On the other hand, it is a fairly general practice among instrument-makers, while using the bimetallic device for indicating aneroids, to adopt the plan of leaving a little air in the vacuum-box system of self-recording aneroids. These latter instruments usually have a number of diaphragm-boxes in series (up to six or eight) for the sake of increased sensitiveness. In one or two of these a little air is left at quite low pressure, in order to oppose the thermal changes in the vacuum-box, the remaining boxes being thoroughly exhausted. Such compensation is limited to a comparatively small range of temperature.

Experiments made on diaphragms of a number of different metal alloys indicate that the change of stiffness with temperature of an aneroid diaphragm-box is small for German silver, and distinctly larger for steel and phosphor-bronze.

In practice, it is found that even in aneroid mechanisms made as far as possible similar in dimensions and material, the thermal changes in each instrument requires individual compensation.

§ (12) The Adjustment and Testing of the Aneroid Mechanism in the Works. Although, generally speaking, each maker has a standard size of mechanism for each type of aneroid according to the pressure range and the size of the dial, it is necessary to regulate each aneroid separately in order to obtain the appropriate stiffness of the spring-controlled vacuum-box and the requisite magnification of the lever system. The need for this arises from the fact that diaphragms of the same thickness, diameter, and mode of corrugation, apparently similar, are not sufficiently alike in their elastic behaviour to enable aneroid mechanisms to be absolutely interchangeable without the precaution of further adjustment and testing.

Consequently, in fitting together the component parts of the mechanism, means of adjustment are provided in at least three places:

(*a*) At the legs of the bridge which supports the control spring, for the adjustment of the tension on the vacuum-box.

(*b*) At the long arm of the lever system, enabling the length of this arm to be modified.

(*c*) At the regulator, where the turning moment of the regulator can be modified by a slow adjustment screw.

(In addition, it is sometimes found useful to modify the effective length of the long arm of the regulator.)

It should be noticed that the whole of the adjustment of the lever system cannot be thrown on to the regulator, because the function of the latter is to control not only the magnitude of the magnification, but also its rate of variation with change of pressure.

It is this latter provision which restricts the range of rotation of the regulator. (This subject will be referred to again in connection with the magnification mechanism of the Barograph (see § (18).)

In connection with the calibration of aneroids, an instrument may be fitted with a truly uniform scale, or else the openness of the scale may vary with the pressure. The former is a little more difficult, as the angular position of the regulator requires more careful adjustment.

In all cases a reference standard barometer is required in order to identify the indications of the aneroid, and for this purpose a mercury barometer of the Kew pattern gauge type (described in § (3) (v.)) is generally used.

It is recommended that a thoroughly accurate mercury barometer should be selected for this work. It should have a comparatively large bore tube (0·4 in. or larger), and for preference a cistern of large diameter. Such a mercury barometer would make a thoroughly reliable reference standard which, with ordinary care, would require little attention for upkeep in the course of some years.

It is usual in the works to "point" off a few marks on the dial of the aneroid and find the pressures given by the mercury gauge corresponding to the "point" marks on the dial.[1] The scale on the dial may then be ruled by reference to these "point" marks, using a dividing engine.

§ (13) Accuracy, Errors, and Defects of Aneroids. (i.) *Accuracy.*—The accuracy obtainable from an aneroid barometer in the measurement of pressure is of a lower order than that given by the average mercury barometer. Although an aneroid is sensitive to quite small changes of pressure, its reading cannot be relied upon to give the absolute value of the pressure to a precision shown by one subdivision of the scale of the instrument (0·01 in. in short range aneroids to 0·05 in. for long ranges).

The utility of the aneroid as an absolute pressure indicator is limited by gradual and appreciable changes in the internal structure of the metal of the vacuum-box, and there is room for progress in the direction of stabilising the diaphragm with a view to securing greater permanence in the accuracy of the absolute readings.

---

[1] For this purpose the aneroid is placed in an air-tight chamber in which the pressure can be varied artificially.

The most justifiable use of the present-day aneroid is as a relative pressure indicator. Under good conditions, an accuracy of ±0·01 in., or even better, can be secured with an aneroid in the measurement of differences of pressure extending to a few inches of mercury; but wherever an aneroid is used to give absolute values, it is highly advisable to compare it, at suitable intervals of time, with a reliable mercury barometer, in order to obtain information as to the magnitude of the gradual or secular changes in the aneroid.

In spite of this disadvantage, there is considerable scope for the use of an aneroid as a precision instrument.

The mercury barometer has its limitations. Although it is unrivalled as a reliable pressure indicator at land stations, its accuracy is reduced aboard ship by the oscillations of the ship and other causes, while its use in aircraft is largely impracticable.

Consequently the aneroid, on account of its portability and general convenience, is indispensable on aircraft, and of considerable use, not only at sea, but for survey and other work on land.

(ii.) *Errors and Defects—"Creep" and Hysteresis.*—Most important of all the defects to which the aneroid is susceptible is that generally known as "creep."

It will assist in the appreciation of this defect if a passing reference is made to *Figs.* 17, 18, and 19, in which the vacuum-box is shown in three stages of development:

(*a*) Before being exhausted of air.

(*b*) Immediately after exhaustion.

(*c*) After coupling up to the control spring.

In (*a*) there is no tension in the membrane, while in (*b*) the diaphragms are clearly deflected. In (*c*), although the tension of the control spring is arranged to balance the total atmospheric pressure on the vacuum-box at normal pressure (*i.e.* 30 in.), the diaphragms are still somewhat in a state of strain, since the tension of the control spring is applied at the centre of the upper diaphragm, while the atmospheric pressure acts uniformly over the surface of the diaphragm. Even at 30 in. there may be a tendency for the internal structure of the metal of the vacuum-box to readjust itself gradually under the action of the tension in the diaphragms, but when the atmospheric pressure is diminished considerably below 30 inches, with corresponding movement of the diaphragms, the tension in the latter will be considerably increased. Consequently the diaphragms, though responding to the diminution of pressure, do not remain steady at the new pressure, but show a further small but gradual change in the same direction, in the course of time, while the same pressure is maintained. This change, or "creep," as it is technically called, may be represented as the accommodation of the internal structure of the metal diaphragm under shearing stress. Commencing with the application of the shearing stress, it may last days or even weeks before it becomes inappreciable.

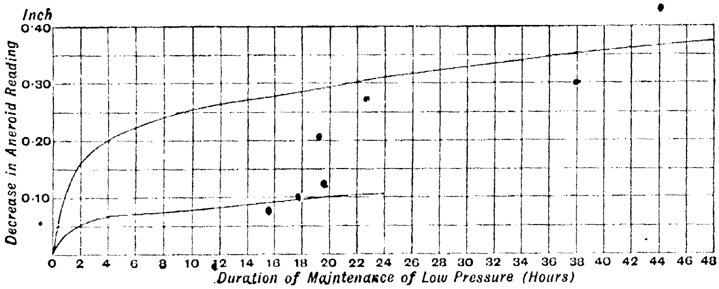


FIG. 20.—Curves illustrating "Creep" at Low Pressure.

To give a concrete example, the preceding curves (*Fig.* 20) represent the variation of the reading of an aneroid barometer with time while maintained under a constant low pressure. They may be considered typical of the average aneroid.

The lower and upper curves correspond to the pressures 22 and 14 in. of mercury respectively. In both cases the "creep" was measured from the time when the aneroid was submitted to the given low pressure.

It will be observed that the "creep" follows at least approximately an exponential law of variation with the time.

Other conditions being equal, the amount of "creep" will be increased:

(*a*) The further the pressure is diminished.

(*b*) The thinner the diaphragm.

(The rate of diminution of the pressure also has an influence on the "creep," which is likely to vary in diaphragms of different material.)

On the other hand, if the pressure is restored to its initial value, say to 30 in., the aneroid will first read lower than initially by a quantity of the same order of magnitude as the total amount of "creep." A recovery by a similar gradual process of accommodation of the internal structure then takes place, the curve of recovery being somewhat similar to that given in *Fig.* 20 inverted. If the pressure had been brought to some intermediate value instead of being restored to 30 in., only a proportion of the loss due to "creep" at the lowest pressure would be recovered, and the aneroid would be compromised between a recovery due to a partial restoration of pressure and a possible further "creep" due to its maintenance at a pressure below 30 in.

(iii.) *Influence of the Rate of Change of Pressure on the Calibration of an Aneroid.*[1] From the foregoing reasoning it follows that, since an aneroid "creeps" to greater or less extent at any diminished pressure, the graduation of the pressure scale involves a consideration of the rate of change of pressure. It is clear that in the case of mountain aneroids the rate of change of pressure, or climb, will be very slow, while for aircraft aneroids the rate is usually very quick.

In the workshop it is desirable to calibrate aneroids following a quick rate of artificial change of pressure such as would be convenient and economical with respect to time.

At the Government testing institutions, where aneroids are examined, the rate of change of pressure may be slow or quick according to the use to which the instrument is intended to be put in practice.

(iv.) *Example of Influence of Time Factor on the Aneroid Reading at Low Pressure.* The reading of an average aneroid, graduated from 30 to 23 in., is approximately 0·05 in. higher at 23 in. following a very quick diminution of pressure from 30 to 23 in. (*e.g.* lasting only one or two minutes) than after a very slow diminution (lasting about one hour).

For an average aneroid of range 30 to 16 in., the corresponding discrepancy at 16 in. would be of the order $\frac{1}{10}$ in.

Incidentally, it is desirable not to confuse the expressions "lag" and "creep" as applied to aneroids. The term "lag," as generally understood in its broadest sense, implies that the instrument under consideration does not immediately and fully respond to the changes it is intended to indicate.

An aneroid barometer certainly responds quickly to the changes of pressure to which it is subjected. Whether the change is large or small, the instrument responds almost entirely within a minute of completion of the change. Under circumstances of very rapid change of pressure, such as occur to an aeroplane when the pilot is making a very rapid descent, it is not expected that the instrument will be free from lag.

On the other hand, as *Fig.* 20 illustrates, an aneroid starts "creeping" to a greater or less extent when put under a given diminished pressure. The exponential shape of the curves may suggest a time lag indicating that the instrument may be approaching the true condition of deformation of the vacuum-box corresponding to the diminished pressure. Actually, however, the aneroid has quickly and fully responded to the diminution of pressure, following the elastic laws of deformation, but owing to excessive straining of the diaphragm near its surface, and particularly near the centre, where it is pulled out by the control spring, changes in the internal structure of the diaphragm metal take place beyond the ordinarily recognised elastic laws, and give rise to the phenomena designated as "creep."

It may also be asked, What differences are found between the falling and rising readings of the aneroid at any given intermediate pressure, if the lowest pressure is maintained for only five minutes or so, before the return to initial pressure is commenced? *Fig.* 21 shows curves illustrating the average difference found between the errors of the scale for diminishing and increasing pressures in the course of laboratory tests on aneroids.

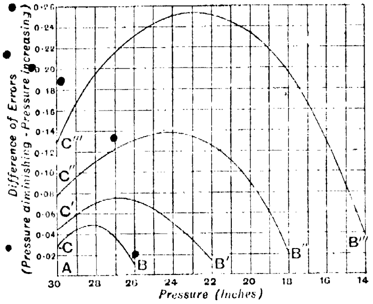


FIG. 21.—Curves illustrating the Amount of Hysteresis in the Average Aneroid when submitted to a Pressure Cycle.

(Compiled from particulars of aneroid tests at the National Physical Laboratory, and arranged to represent a mean of the various sizes in which aneroids are made.)

Though illustrating "creep," they are also known as hysteresis curves.

In the above diagram it has been assumed that the pressure scale of the aneroid has been correctly graduated with pressure diminishing in accordance with the most usual rate of test at the laboratory (*i.e.* 1 in. per 5 minutes).

The curved lines BC, B'C', B''C'', and B'''C''' represent the errors with pressure increasing from 26, 22, 18, and 14, in. respectively, the residual errors at 30 in. corresponding to the four different pressure cycles being AC′, AC″, AC‴, and AC⁗.

---

[1] This particularly concerns the long-range instruments. The effect on the calibration of short-range aneroids is not very marked.

Further examples might be cited of the behaviour of aneroids under diminished pressures, but all would be illustrative of what has already been described as "creep."

(v.) *Limitation of the Amount of "Creep" in an Aneroid.*—Various attempts have been made to reduce the errors consequent upon "creep."

During the Great War a large number of aneroids were made not only for barometric purposes, but as height indicators—*i.e.* altimeters—in aircraft (to which reference will be made in § (18) (iv.)), and the question of reduction of "creep" was given considerable attention both before and during the war. As a result it was shown that the source of the "creep" lay almost entirely in the vacuum-box, a negligible proportion being due to the control spring.

Two methods are feasible for diminishing the creep:

(*a*) Judicious selection and treatment of the diaphragm metal for making the vacuum-box.

(*b*) Judicious control of the stiffness of the elastic system, including both spring and vacuum-box.

The former was attempted with some success by the use of a vacuum-box with diaphragms of steel, tempered glass-hard.

The latter method depends upon the following consideration:

The source of the bulk of the error due to "creep" has been located in the vacuum-box, not in the control spring. If the spring is made stiffer, so that its response to a given force is smaller, the combination of the spring and vacuum-box is stiffer. Hence, as the movement of the vacuum-box in response to a given pressure-change is smaller, the shearing stress in the diaphragms is relatively smaller corresponding to that pressure-change, and consequently the "creep" is smaller.

This has been expressed mathematically by Hersey.[1] This theorem is very important from several points of view, and can be made the basis of a mathematical analysis of the performance of an aneroid in terms of the features of the component parts of its mechanism. Starting with the definition that the stiffness of a body, or of a system of bodies, is the ratio of the force applied to the deflection produced, the theorem states that if S is the stiffness of a coupled system consisting of two component parts whose stiffnesses are S_1 and S_2, the relation

$$S = \lambda(S_1 + S_2)$$

is satisfied, where λ is a dimensionless constant characteristic of the component to which the external force is applied, being unity if applied at the coupling. By differentiation

$$dS = \lambda(dS_1 + dS_2) = \frac{S}{S_1 + S_2}(dS_1 + dS_2).$$

Hence the fractional change in the stiffness of the coupled system, in terms of the fractional changes in the stiffnesses of the components, due to any cause whatever, is

$$\frac{dS}{S} = \eta\frac{dS_1}{S_1} + (1 - \eta)\frac{dS_2}{S_2},$$

where η denotes $\dfrac{S_1}{S_1 + S_2}$.

By applying this relation, and considering the fractional variations dS/S, etc., to be due to "creep" alone, it appears that a vacuum-box having in itself a given "creep" dS_1/S_1 would, if coupled to a perfectly elastic steel spring (*i.e.* with $dS_2/S_2 = 0$) twice as stiff, give a system with only one-third the "creep" of the vacuum-box alone.

Note.—The magnification given by the lever system would need to be correspondingly increased, since in this method the reduction in "creep" is obtained by sacrificing sensitiveness of movement of the diaphragms.

(A further reference to the influence of "creep" upon the performance of present-day aneroids will be given under the heading "Altimeters" (see § (18) (iv.).)

(vi.) *Errors due to Friction of Mechanism, etc.*—In linking together the component parts of an aneroid mechanism, a compromise must be made between errors or inconsistencies in the reading of the instrument due to friction and those due to backlash consequent upon loose jointing of the lever system, metallic chain, etc. A hair-spring is fitted to the pin which carries the indicating needle in order to take up this backlash.

In a good aneroid these mechanical errors are quite small. The extent of their magnitude can be seen by tapping the aneroid from different sides and holding it in a variety of positions. In general, the indications of an aneroid vary slightly according as the instrument is read in the horizontal or in the vertical position. This difference depends upon the way in which the parts of the aneroid are balanced.

One important defect which prevents a more extended use of the aneroid in practice is the susceptibility of the instrument to slow and usually progressive changes, in course of time, at ordinary atmospheric pressures. In the case of aneroids that have been subjected repeatedly to low pressures, such a progressive change is not surprising.

With meteorological aneroids used only for sea-level pressures, *i.e.* for the range 31 to 27 in., it would be expected that once the aneroid were set to read correctly by comparison with a standard mercury barometer it would not need readjustment for some time. Such, however, is not the case, as the diversity of readings of many a household aneroid barometer will suggest. Aneroids take some time to settle down to a comparatively stable state after being first put into use.

---

[1] *J. Wash. Acad. Sci.*, 1916, vi. No. 16.

It is wise, therefore, not to rely implicitly on the aneroid for absolute readings, but rather for differences of pressure. In many cases there are occasions for comparing its readings with independent values of the atmospheric pressure. Where the instrument is found to read increasingly low, a leakage of air into the vacuum-box may be suspected. Nowadays this trouble happens but rarely.

§ (14) THE ANEROID BAROGRAPH.—(For Mercury Barograph see § (3) (vi.).)

Fig. 22 shows a normal pattern aneroid barograph, or self-recording barometer.

The type illustrated is of short range, suitable for meteorological purposes at or near sea-level, where the range of variation of atmospheric pressure is but a few inches of mercury.

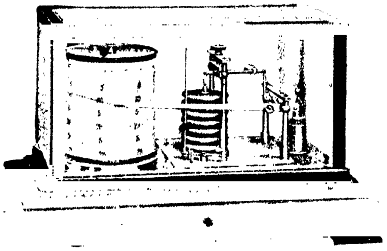


FIG. 22.

It will be seen from the illustration that the control spring of the type usually fitted to an indicating aneroid is absent. It is general practice in making barographs to insert a suitable control spring inside each vacuum-box.

Broadly speaking, the mechanism of the indicating aneroid is retained, except that the metallic chain operating the indicating needle is replaced by a long pen lever, which traces out the record on a uniformly revolving drum, driven by clockwork.

When a barometer is made self-recording, it is usually at the expense of accuracy, which is limited by the friction of the pen and sometimes by a lack of balance of the component parts of the mechanism. In good barographs these disadvantages are satisfactorily small, and do not outweigh the advantages offered by a self-recording instrument.

In the field of aeronautics there is a demand for a self-recording aneroid for certain classes of work. In several of the height records attempted by pilots during the last decade, the estimation of the height attained by aircraft has been based on sealed barographs which were tested in the laboratory and checked at the conclusion of the flight.

On account of progress in aviation, attention has been given to the long-range barograph. Its calibration in the workshop presents somewhat more difficulty than the short-range instrument. Beyond a certain limiting pressure, generally in the neighbourhood of 15·12 in., but dependent on the dimensions of the vacuum-box, the movement of the metal diaphragms loses consistency, and departs from the general linear law of deflection with pressure which is obtained at higher pressure.

It is natural that the accuracy of an aneroid barograph should deteriorate with increasing range. On the other hand, the graduation lines of the chart are less openly spaced than in the short-range instrument.

The recording mechanism of the average barograph may leave much to be desired, if the instrument is used on aircraft, owing to vibration and also to backlash in its parts. Experiments have been made on barographs in which these errors have been diminished by making the record on a different principle. In one case an instrument was developed which recorded photographically by means of a narrow beam of light reflected from a small mirror actuated by the vacuum-box system of the barograph, the record being traced on sensitised paper on the revolving drum.

This is but one example of the possibilities of development of the barograph, which in its present form has scarcely reached a condition of finality such as aviators would desire.

V

§ (15) THE BAROMETER IN PRACTICE.—With the exception of meteorological work, there are, in general, different fields of use for the aneroid and the mercury barometer.

Among the many uses to which barometers are put nowadays, the following should be mentioned as being the most important:

(i.) *Weather Forecasting.*—Systematic use of barometers and other instruments is made by the Meteorological Office, which is responsible for the collection of information required for the purpose of weather forecasting. Instrumental observations are made on a comprehensive basis in order to ascertain the distribution of atmospheric pressure, not only over the British Isles but, in conjunction with other countries, over the Northern Hemisphere generally.

This involves barometric readings taken both on land and at sea. In the British Isles there are a number of officially recognised stations which daily, or in many cases more frequently, send their observational data to the Meteorological Office. At sea, observations are made on the atmospheric pressure by means of a mercurial marine barometer of the type described in § (3) (iv.).

(The limitations to the accuracy of

mercury barometer in the measurement of atmospheric pressure are discussed in § (8) (ii.). Some of these factors refer equally to aneroid barometers, but the latter are not so generally used as the former in connection with official weather forecasting.)

An outstanding advance has been made during the past decade in the investigation of the meteorology of the upper atmosphere. This has been done by sending up small registering balloons, each supplied with a set of recording instruments called a meteorograph. The instruments are exceedingly light and of compact design, and are referred to elsewhere.[1]

(ii.) *Surveying.*—Considerable use is made of the aneroid barometer for surveying purposes, but care should be taken not to overrate the performance of this instrument. Accurate results can be obtained from the aneroid under suitable conditions, but if it is used under extended ranges of pressure or height, then it follows that much care is needed in dealing with the errors due to instrumental and atmospheric conditions (see Part VI. § (16) on the "Determination of Heights by the Barometer").

The most justifiable use of the aneroid for survey work lies in the evaluation of heights intermediate between two given contours which have been accurately determined by other methods (*e.g.* by means of a theodolite). This requires relative measures of height only, and for such interpolation work the surveying aneroid is usually provided with a uniformly spaced scale of heights in addition to its pressure scale.

The accuracy obtainable in determining the height of a given station by reference to two known stations depends on a number of circumstances. For interpolation over ranges up to about 3000 ft. above sea-level, it is possible to obtain an accuracy of ± 10 feet, using an aneroid of the best quality and sensitiveness, in which errors of friction and backlash in the mechanism have been satisfactorily minimised. Such an accuracy would represent the best attainable limit under special circumstances. In surveying by interpolation at high altitude, the final errors are considerably larger.

It is highly desirable that the instrument should be well compensated for changes in its temperature. In a number of cases the accuracy obtainable in this survey work is limited by the incomplete thermal compensation of the aneroids employed.

pa... (iii.) *Aviation.*—The basis of the determina-

...f the barometer is

$S=\lambda(S_1+S_2)$ ...ogether with

is satisfied, where λ is a dimensionless const... al baro-

characteristic of the component to which the external

not

(iv.) *Physical and Chemical Determinations.*—For these purposes the mercury barometer is ordinarily used, since the barometric precision demanded generally exceeds that obtainable from an aneroid barometer. There are occasions, however, on which an aneroid may more readily be used, though with sacrifice of accuracy.

The use of the barometer is incidental to many operations of a physical or chemical nature, some involving high accuracy, others only a moderative accuracy. In special cases, such as the determination of some physical constants, and in certain measurements and standardisations of a fundamental nature, the barometric factor is of high importance, and sometimes limits the final accuracy of the determination.

In many cases the work requires the use of a manometer, or a manometer in conjunction with a barometer. Further reference to this point is made in Part VII. § (19) on "Manometers."

Suffice it to mention but two such instances where high precision is required of the barometer.

The determination of the boiling-point on a thermometer is dependent on the measurement of atmospheric pressure. A change of 0·27 mm. in the pressure corresponds under normal conditions to a change of 0·01° C. in the boiling-point. An accuracy of 0·01° C. is often demanded in calibrating thermometers, and in the determination of the primary fundamental thermometer standards a still higher order of precision is required.[2]

Similar barometric accuracy is demanded in the maintenance of precision standards of mass, for the reason that the fundamental and legal standards are of platinum, a metal which is too costly for sub-standards of mass, which are usually of considerably less dense material. Consequently, in comparing the sub-standards with the fundamental ones, high precision is required in the evaluation of the buoyancy correction, and this incidentally demands high barometric accuracy for the determination of air density.

VI

§ (16) THE DETERMINATION OF HEIGHTS BY THE BAROMETER.—In climbing from one level to another, whether on land or in the air, it is found that the pressure, temperature, and density of the atmosphere as a general rule decrease.

As the variation of pressure with height is the most considerable of these three quantities, the barometer has, in consequence, been used as a means whereby atmospheric heights can be obtained. In aircraft, where a knowledge of the height attained in flight is obviously useful, the barometer offers practically the only method of determination. There are other methods for determining the height of aircraft from without. These have been successfully developed during the war.

---

[1] *J. Wash. Acad. Sci.*, 1916, vi. No. 16.

[2] See "Thermometry," § (3) (v.) Vol. I.

(i.) *Fundamental Formulae.*—The determination of heights by a portable barometer depends, in the first instance, upon a knowledge of the relation between heights and pressures. A theoretical investigation of this relation will be found in the article "Atmosphere, Physics of" (§ (1).

It is there shown that the assumption that the temperature is constant and equal to T° absolute leads to the formula

$$H = \frac{RT}{g \log_{10} e}(\log_{10} p_1 - \log_{10} p_2),$$

which at sea-level in latitude 45° becomes in feet

$$H = 221 \cdot 1T(\log_{10} p_1 - \log_{10} p_2),$$

and in metres

$$H = 67 \cdot 4T(\log_{10} p_1 - \log_{10} p_2),$$

T being measured in absolute Centigrade degrees. If the temperature decrease uniformly with the height according to the law

$$T = T_1 - \beta H,$$

where T_1 is the temperature at zero height,

then
$$\frac{\beta H}{T_1} = 1 - \left(\frac{p_2}{p_1}\right)^{\frac{R\beta}{g}},$$

and this, assuming a fall of 6·5° C. per kilometre, gives us the value for H in metres:

$$H = 153 \cdot 85 \times T_1 \left\{ 1 - \left(\frac{p_2}{p_1}\right)^{\cdot 1904} \right\}.$$

Though the logarithmic formula given above is based on the assumption of an isothermal atmosphere, it can be shown by observations to give high accuracy in the determination of heights, provided the assumed isothermal temperature T is taken equal to the mean atmospheric temperature over the height considered.

Numerical values of the height-difference between two hypothetical stations have been worked out from these two formulae for several particular cases, including large and small variations of pressure and temperature, taken at random with sufficient range to test the formulae severely. In general, the two formulae yield height-values which are in agreement to within 1 part in 1000, provided that the mean value of the atmospheric temperature between the two stations is employed in the isothermal formula, while a uniform gradient of temperature defined by the temperatures at the two stations is assumed in using the other formula. In no case does the discrepancy exceed 1 part in 500.

It is, however, clearly impossible, owing to the extensive vertical gradient of atmospheric temperature, to select a single value to represent accurately the mean atmospheric temperature for all heights. Consequently, although the isothermal formulae (with properly selected temperature) may be justifiably used as an accurate means of determining the difference in height between two levels at which the pressure and temperature are known, any table of corresponding heights and pressures based on the isothermal formula, with one fixed value of the atmospheric temperature for all heights, can, at the best, be only approximate.

In the past, the isothermal type of formula has generally been used in the determination of heights by the barometer. It received international sanction some years ago, and was incorporated in the most recent (1890) edition of the International Meteorological Tables.

Effect of Variation of Gravity[1] *with Altitude.* In the formulae arrived at so far, gravity has been treated as constant throughout the range of altitude considered. The effect of taking into account its small variation with altitude in deducing the height formula would naturally be to alter the shape of the formula and unduly complicate it. Sufficient accuracy is obtained by assuming gravity constant and equal to the mean value over the range of height considered.

(ii.) *The Old International Meteorological Formula for computing Heights from Pressures.*—The formula given below is, in the main,[2] that adopted by the International Meteorological Committee in their tables for the determination of heights by the barometer:

Notation.—h_1 = Altitude of lower station in metres,
h_2 = Altitude of upper station in metres,
T = Mean atmospheric temperature between the two stations expressed in absolute degrees,
T_0 = Absolute temperature corresponding to the melting-point of ice,
ϕ = Mean pressure of aqueous vapour in the air column between the stations (supposed in the same vertical line),
η = Mean pressure of the air in the column between the two stations,
λ = Latitude of the stations,
p_1 = Atmospheric pressure at lower station,
p_2 = Atmospheric pressure at upper station,
} expressed in the same units.[3]

---

[1] The value of gravity at a height of 10 kilometres is less than that at sea-level in the same latitude by 1 part in 300.

[2] See *Tables Météorologiques Internationales*, published by Gauthier-Villars, Paris, in 1890.

[3] If pressures are obtained by means of the mercury barometer, the readings of the instrument should be corrected to the same temperature and gravity.

Formula :

$$h_2 - h_1 = 18400 \times \frac{T}{T_0} \times \left(\frac{1}{1 - 0{\cdot}378 \times (\phi/\eta)}\right)$$

$$\times (1 + 0{\cdot}00259 \cos 2\lambda) \times \left(1 + \frac{h_1 + h_2}{6371104}\right) \log_{10} \frac{p_1}{p_2}.$$

This formula consists of six factors, which are designated as[1]

A, B, C, D, E, F

for the sake of further explanation, so that the formula may be written

$$h_2 - h_1 = A \times B \times C \times D \times E \times F.$$

(A.) The factor A, equal to 18400 for altitudes measured in metres, is computed from the expression

$$\frac{T_0 \times R}{\mu \times g_{45}}$$

(§ (16) (i.)), including the elementary logarithmic formula and barometric constant without refinements for gravity and humidity,

where R is the gas constant for dry air with normal content of carbon dioxide.

$\mu = \log_{10} e$.

g_{45} is the value of gravity at mean sea-level in latitude 45°.

(B.) B, equal to T/T_0, is the temperature factor.

It is obviously very important, since the altitude varies directly as the mean absolute temperature of the air column. For accurate determination of heights by the barometer every possible effort should be made to obtain the value of T as accurately as possible.

The factors, C, D, and E are relatively less important than A, B, and F.

Their combined effects do not, in general, exceed 1 per cent of the calculated value of the altitude.

(C.) *Humidity Factor.*—The density of aqueous vapour is 0·622 relative to air. A given volume of moist air weighs less than the same volume of dry air at similar temperature and pressure. Hence, for a given pressure difference between two stations, the altitude is greater the greater the humidity of the air.

The factor C, therefore, always exceeds the limiting value 1 for dry air.

Consider an elementary volume of dry air of unit mass. If we substitute for it the same volume of moist air, of humidity k, at the same pressure and temperature, we get a resulting mass

$$1 - k + k \times 0{\cdot}622 = 1 - k \times 0{\cdot}378.$$

In the International Meteorological Tables, k is written as ϕ/η,

where ϕ = mean pressure of aqueous vapour } between the two stations,
η = mean pressure of the air }

and the correction factor C for humidity is

$$\frac{1}{1 - 0{\cdot}378(\phi/\eta)}.$$

The correction for humidity can only be made approximately. According to the tables, its maximum effect does not exceed 0·5 per cent of the calculated altitude unless the mean air-column temperature T exceeds 283° A.

(D.) In the determination of the barometric constant A, standard gravity (g_{45}) was assumed. For the measurement of heights in another latitude λ, the constant A must be multiplied by

$$\frac{g_{45}}{g}, \text{ i.e. by } (1 + 0{\cdot}00259 \cos 2\lambda).$$

This factor does not influence the calculated altitude by more than 0·3 per cent. For English latitudes the effect is 0·1 per cent, the corrective factor D being less than unity.

(E.) *Corrective Factor for Variation of Gravity with Height above Mean Sea-level.*—

If g_0 = value of gravity at sea-level in a given latitude,
g_{h_1} = value of gravity at height h_1 in a given latitude,
g_{h_2} = value of gravity at height h_2 in a given latitude,
R = radius of earth in metres,

$$g_{h_1} = g_0 \left(\frac{R}{R + h_1}\right)^2 = g_0 \left(1 - \frac{2h_1}{R}\right)$$

with sufficient accuracy, and similarly

$$g_{h_2} = g_0 \left(1 - \frac{2h_2}{R}\right);$$

therefore the mean value of gravity between heights h_1 and h_2

$$= g_0 \left(1 - \frac{h_1 + h_2}{R}\right);$$

and the corrective factor

$$D = \left(1 + \frac{h_1 + h_2}{6371104}\right),$$

R being taken as 6371104 metres. The effect of this factor is about 5 ft. on altitudes up to 10,000 ft.

(F.) The formula is logarithmic on account of the assumption that the temperature of the air column is constant between the two stations. (Gravity is also assumed constant.)

The justification of these assumptions has already been considered in § (16) (i.).

If heights are expressed in feet, the value of the constant corresponding to 18400 becomes 60368.

Further, if a mean atmospheric temperature

---

[1] The corrective factors B and E given above are expressed in a somewhat different form in the published tables. For simplicity, most of the numerical terms given in the International Tables are retained, though more recent determinations of physical and geodetic constants have resulted in slightly different values.

50° F. is assumed, as has usually been the practice in England, in graduating the scales of height indicators, the international formula becomes

$$h_2 - h_1 = 62579 \log_{10}\frac{p_1}{p_2} \text{ (feet)},$$

as a simple modification without introducing such refinements as the corrections for gravity and humidity.

(iii.) *The Measurement of Heights in Practice.*—Provided the barometer does its part faithfully, the determination of heights by this means is an operation which depends primarily on a knowledge of the distribution of temperature throughout the atmosphere, and secondarily on the humidity of the air. The latter factor is usually neglected.

The atmospheric temperature at any given height is so variable that it is impossible to lay down a numerical relation between heights and pressures which is more than approximately true in general.

(*a*) *Basis of Height Scales in England.*—In adapting the barometer to measure altitudes by means of a height scale instead of (or in addition to) a pressure scale, some numerical relation must be chosen as the basis of conversion from pressures to heights. The relation first adopted was given by the late Sir George Airy, Astronomer Royal, in 1867, and is still known as Airy's Table.[1]

It served as the basis of graduation of the height scales of aneroid barometers used for surveying or mountaineering purposes. Mercury barometers were not used in this way for direct indication in height units.

Airy's table was confined to a range of heights of 0 to 12,000 feet, and was based on the supposition that the mean atmospheric temperature between any two given heights was 50° F. No information was given (*loc. cit.*) as to the values of the constants from which the table was calculated, but its basis can easily be shown to be the logarithmic formula:

$$h_2 - h_1 = 62759 \times \log_{10}\frac{p_1}{p_2} \text{ (feet)}.$$

It appears that Airy, in using the logarithmic constant 62759, assumed an average value of the humidity of the atmosphere, since the heights given by his table are about 0·3 per cent higher than they would have been on the assumption of a dry atmosphere.

The predominating influence of atmospheric temperature on the accuracy of height determination was recognised by Airy, who added to his table instructions for correcting the height for deviation of the observed temperature from the assumed value, 50° F.

With the development of aviation came a demand for height-measuring instruments. Clearly, a mercury barometer was impracticable, and accordingly the aneroid barometer was used, its pressure scale being replaced by a scale of heights, from which it became known as an altimeter aneroid. Though at first, in the more experimental stages, the altimeter scale may have been based on Airy's table, a rather different standard basis of graduation was adopted by the Government for English altimeters. This is shown in the following table:

TABLE VIII

BASIS OF GRADUATION OF ENGLISH ALTIMETER ANEROIDS

Pressure. (Inches of Mercury.)	Corresponding Altitude. (Feet.)	Pressure. (Inches of Mercury.)	Corresponding Altitude. (Feet.)
33·390	−3000*	17·863	14,000
32·183	−2000	17·217	15,000
31·021	−1000	16·596	16,000
29·900	0	15·996	17,000
28·820	+1000†	15·418	18,000
27 779	2000	14·861	19,000
26·775	3000	14·324	20,000
25·807	4000	13·807	21,000
24·875	5000	13·308	22,000
23·977	6000	12·827	23,000
23·111	7000	12·364	24,000
22·276	8000	11·917	25,000
21·471	9000	11·487	26,000
20·695	10,000	11·072	27,000
19·948	11,000	10·672	28,000
19·227	12,000	10·286	29,000
18·533	13,000	9·915	30,000

* Descent. † Ascent.

Notes.—1. The mean temperature of the atmosphere over any given range of altitude has been taken as 50° F. (10° C.).

2. The zero of altitudes has been arranged to correspond to the pressure 29·900 in.

3. Pressures are given in terms of inches of mercury at the same temperature and gravity.

4. The above table is based on the formula:

$$H_2 - H_1 = 60368 \times \frac{T}{273} \times \log_{10}\frac{p_1}{p_2}$$

$$\left(= 62579 \times \log_{10}\frac{p_1}{p_2}\right),$$

where T, the mean atmospheric temperature between two given stations, has been taken as 283° abs. (50° F.),

p_1 p_2 being the pressures } at the lower and upper
H_1 H_2 being the altitudes } stations respectively,

T being expressed in absolute degrees.

This is the simplest approximation to the complete formula adopted in the old *International Meteorological Tables.*

5. The above table should not be confused with Airy's Table of Altitudes which is still generally used in graduating surveying aneroids. Airy's table makes an allowance for the moisture content of the atmosphere, and therefore gives altitudes which are greater than those shown above by about 3 feet per 1000 feet.

(*b*) *Basis of Graduation of Altimeter Scales in Other Countries.*—In the United States the mean atmospheric temperature, 50° F., has also been taken as the basis of graduation of height scales. The formula adopted there is not, however, quite the same as in this country, since a different allowance has been made for atmospheric humidity

---

[1] *Proc. Brit. Met. Soc.* iii. 406.

In European countries where the metric system is in vogue, it has been customary to graduate height scales on the supposition of a mean atmospheric temperature 0° C. This basis cannot, however, be said to represent universal practice on the Continent.

§ (17) ATMOSPHERIC CONDITIONS. (i.) *Errors of the Readings of Altimeters due to Variations of Temperature of the Air.*—Of all sources of errors to which the indications of altimeters, in general, are susceptible, whether instrumental or atmospheric, the temperature of the atmosphere gives the most trouble, not so much on account of its variation with height as by reason of its variation at a given height.

Reference to the altitude formula in § (16) (i.) shows that the true height corresponding to a given pressure change is proportional to the mean atmospheric temperature over the height considered, expressed in absolute degrees. In view of the fact that the annual mean temperature of the atmosphere at sea-level differs but little from 50° F., it is clear that the higher the altitude, the further does the mean atmospheric temperature depart from 50° F. Consequently the percentage error as well as the absolute error in the uncorrected indication of the altimeter increases with the height.

In consequence of the actual temperature being generally lower than the assumed value, the indications of the altimeter are usually too high. The pioneer work done by W. H. Dines in the investigation of the upper atmosphere has enabled satisfactory estimates to be made of the probable errors of assumed atmospheric temperatures at various heights. Working from this information, Dobson[1] has calculated the probable
mo- -rors of the indications of a standard Air
air a- -e altimeter[2] due to variations in atmo-
for a give-
the altitude -mperature alone. The diagram (*Fig.*
the air. from Dobson's Report, and shows
The factor C, the of this error, at a given height,
value 1 for dry air. probable frequency with which
Consider an element n given limits.
mass. If we substitute urves lie almost entirely
of moist air, of humidity *k*, a of zero error, *i.e.* in
temperature, we get a resulting g to too high an

$$1-k+k\times0{\cdot}622=1-k\times0$$

the point that
n heights and
atmospheric
atory but

Having regard to the fact that as a general rule the aviator has no opportunity to make adjustments of his altimeter or to apply corrections to its readings whilst in flight, it is desirable for the scale of the instrument to be designed so that the uncorrected readings may be as accurate as possible. This condition is secured by making it conform to average conditions.

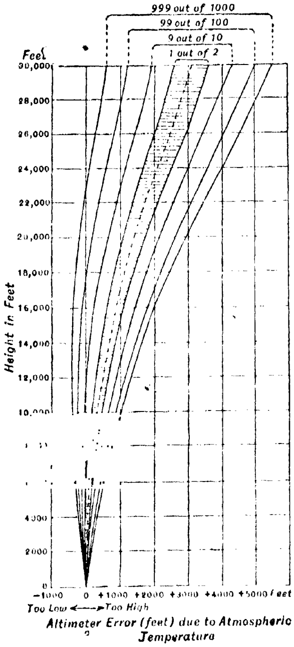


FIG. 23.

By choosing the relation which has been found empirically to represent actual atmospheric conditions on the average, and making it the basis of graduation of height scales, the altimeter could be improved in so far that, on the average, its indications would be correct, though individual readings would err on either side of the correct value according to the atmospheric conditions.

For the sake of comparison, *Fig.* 24 (also taken from Dobson's Report) shows for various heights the probable distribution of errors in the indications of a hypothetical altimeter with its scale graduated on the basis of the average corresponding values of height and pressure, as found empirically by Dines (see Table IX.). It will be observed that while the range of error at a given height is the same in *Figs.* 23 and 24, the average error in *Fig.* 24 is zero.

---

[1] The corrective factors B and E given a expressed in a somewhat different form in the lished tables. For simplicity, most of the numei terms given in the International Tables are retaine ittee though more recent determinations of physical an geodetic constants have resulted in slightly different values.

TABLE IX

TEMPERATURE IN DEGREES ABSOLUTE CENTIGRADE, PRESSURE IN MILLIBARS, AND DENSITY IN GRAMMES PER CUBIC METRE AT DIFFERENT LEVELS

Height in Kilometres.	England, S.E.			Europe.			Canada.			Equator.		
	T. a.	P. mb.	D. g/m³.	T. a.	P. mb.	D. g/m³.	T. a.	P. mb.	D. g/m³.	T. a.	P. mb.	D. g/m³.
20	219	55	87	219	55	87	214	54	88	193	53	91
19	219	64	102	219	64	102	215	63	102	193	63	113
18	219	75	119	219	75	119	214	74	121	193	75	135
17	219	88	139	219	88	139	211	87	144	193	90	162
16	219	102	162	219	102	162	211	102	169	195	107	191
15	219	120	191	219	120	191	211	120	198	198	128	225
14	219	140	223	219	140	223	212	142	233	203	152	261
13	219	164	261	219	164	261	214	167	268	211	178	294
12	219	192	305	218	192	307	216	195	314	219	209	331
11	220	224	355	219	225	358	219	228	365	227	244	374
10	222	261	409	222	262	411	223	266	415	235	283	419
9	228	303	463	227	305	467	229	309	470	243	327	469
8	234	352	524	233	353	528	236	358	528	251	376	522
7	241	407	589	241	408	592	243	413	592	258	430	581
6	248	469	658	248	470	661	251	475	662	265	491	645
5	255	538	735	255	538	735	258	543	733	272	558	714
4	262	615	819	261	614	819	264	618	815	279	632	789
3	268	699	909	267	699	913	270	703	905	285	713	871
2	273	795	1014	272	794	1017	275	798	1011	290	803	968
1	278	900	1128	277	899	1128	278	903	1134	295	903	1067
0	282	1014	1253	281	1014	1258	282	1017	1258	300	1012	1174

The figures for Canada above 15 km. are somewhat doubtful, and for the Equator very doubtful, owing to paucity of observations.

For further information, see the *Computer's Handbook*, also *Geophysical Memoirs*, No. 13, published by the Meteorological Office, London.

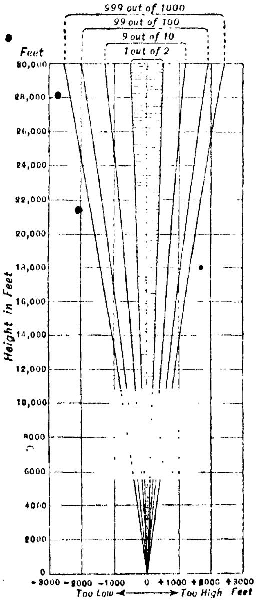
On this hypothetical basis the probable error of the indications of the altimeter due solely to variations of atmospheric temperature would be :

Feet.	Feet.	Metres.	Metres.
± 150	at 10,000 altitude	± 45	at 3000 altitude
± 320	at 20,000 ,,	± 95	at 6000 ,,
± 500	at 30,000 ,,	± 150	at 9000 ,,

Note.—The chance that the probable error will be exceeded is equal to the chance that it will not be exceeded.

While the adoption of such a new scale would doubtless improve the average accuracy of the indications of the altimeter, the utmost accuracy in the case of individual readings can only be obtained by correcting the indications in accordance with the particular distribution of atmospheric temperature on the occasion of reading. This would involve the use of a thermometer in conjunction with the altimeter, together with a computer or some means of making the correction for temperature.

In a number of cases in practice, only approximate heights are required, and the existing altimeter aneroid meets their requirements. In special branches of aircraft work greater accuracy is needed, and an altimeter graduated on the empirical height-pressure relation would be welcomed, even though in



Altimeter Error (feet) due to Atmospheric Temperature

FIG. 24.

cases where exceptional accuracy is required further correction would have to be made for variation of atmospheric temperature from the average.

(ii.) *The "Standard" Atmosphere.*—While it is recognised that the time has come for the scales of new altimeters to conform with average atmospheric conditions, it is not yet agreed how best to represent the average atmosphere. International agreement on a common basis would be desirable wherever practicable. For England the average atmospheric conditions of temperature differ but slightly from those for Middle Europe, but for countries in low latitudes a different basis would be called for.

In choosing the best basis of graduation, either a table of empirical values of corresponding heights and pressures or an algebraic formula may be used. As far as accuracy is concerned there is little to choose between the alternatives, since a simple formula can be made to express the average facts with sufficient accuracy. A formula, however, has distinct advantages for interpolations, and has been suggested as a basis of altimeter scales in preference to a table.

Two such formulae have already been put forward, and have received international sanction. The first is Soreau's formula—

$$z = (15320 + 8{\cdot}65\mathrm{P} - 0{\cdot}0055\mathrm{P}^2)\log_{10}\frac{760}{\mathrm{P}},$$

where z is the required height in metres, and P the pressure in millimetres of mercury.

This formula [1] has been adopted by the Fédération Aéronautique Internationale in 1919, and is the basis of tables issued by that body and accepted by the Royal Aero Club of Great Britain.

Toussaint,[2] on the other hand, has suggested the exponential formula—

$$\frac{\mathrm{P}z}{\mathrm{P}_0} = \left(\frac{288 - 0{\cdot}0065z}{288}\right)^{5{\cdot}256},$$

where Pz, like P$_0$, is the pressure in millimetres corresponding to z metres height; and

P$_0$ is the pressure at the zero of heights.

This formula is based on the linear law of vertical gradient of atmospheric temperature given in § (16) and defined by

$$\mathrm{T} = 288 - 0{\cdot}0065z,$$

where T is the temperature, in absolute degrees, at height z metres, the temperature at sea-level being taken as 288° abs., for if we write

$$\mathrm{T} = \mathrm{T}_0 - \beta z,$$

we find

$$\frac{p}{p_0} = \left(\frac{\mathrm{T}_0 - \beta z}{\mathrm{T}_0}\right)^{\frac{g}{\mathrm{R}\beta}},$$

which, on substituting the numerical value, gives Toussaint's formula.

Both Soreau's and Toussaint's formulae make no pretensions to the highest accuracy, but are designed to represent the average conditions in the simplest way. F. J. W. Whipple [3] has discussed them in detail, and has suggested a somewhat modified Toussaint formula—

$$\mathrm{P} = 1015\frac{(285 - 0{\cdot}0065z)^{5{\cdot}256}}{285},$$

as being the best to employ as basis in the graduation of altimeters. (*Note.*—In this formula the pressure P is measured in millibars, the value at sea-level being taken as 1015 mb.)

The whole matter is at present under consideration both by the Meteorology and Navigation Sub-Committee of the Aeronautical Research Committee, and by the British Engineering Standards Association. At the time of writing no new specification has been issued for the graduation of altimeter scales. Although it seems possible that a new altimeter scale may be established in the near future for the general use of aviators, special cases may arise where a knowledge of the altitude is required with greater accuracy than that given directly by a new altimeter scale. In such cases an appropriate correction should be applied for the deviation of actual conditions from average conditions. Alternatively, the altimeter may be replaced, or supplemented, by an aneroid barometer, and the height calculated from readings of the pressure together with any temperature observations that may have been taken during the flight. Whipple (*loc. cit.*) has given tables for this purpose.

The basis of graduation of altimeter scales has also been considered at some length by the Air Ministry in a report [4] entitled "The Measurement of Height by Aneroid Altimeter." Instrumental methods of correcting the scale readings of the present-day altimeter for changes of atmospheric temperature are also discussed in this report.

§ (18) INSTRUMENTAL CONSIDERATIONS. (i.) *Adaptation of the Aneroid Mechanism to a Uniform Scale of Heights.*—*Fig.* 25 shows the dial view of a normal altimeter aneroid accepted by the Government for use in the Air Service. The instrument has an adjustable zero, *i.e.* the dial is capable of rotation by thumbscrew so that the zero of heights may be brought opposite the indicator needle at the commencement of a flight. At the same time the dial is stiff enough not to be adversely affected by vibration of the aircraft.

Because the zero is adjustable, the graduations on the dial must be uniformly spaced, otherwise the altimeter would give inconsistent values of the height difference.

---

[1] *Comptes Rendus*, Dec. 1919, clxix. 1023.

[2] *Met.* 59, *Advisory Committee for Aeronautics*, Dec. 1919.

[3] *Met.* 71, *Aeronautical Research Committee.*

[4] *Met.* 104 (1921), *Aeronautical Research Committee.*

The arrangement of the lever mechanism of an aneroid so as to yield an equi-spaced scale

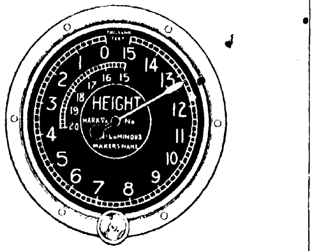


FIG. 25.

of heights can be satisfactorily performed in the average instrument. Its principle is the same fundamentally for both indicating and self-recording altimeters, and can be understood on reference to the diagram, *Fig.* 26, which shows in a simple form, in elevation, the lever system of magnification usually adopted in altigraphs.

FE shows the pen lever, F being the pen-point. E represents the end view of what is

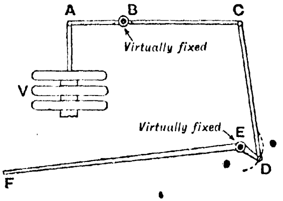


FIG. 26.—Lever Mechanism of Altigraph (not drawn to scale).

known as the regulator in aneroids, DE being an arm of the regulator, linked up through the arms CD and ABC to the vacuum-boxes V. A, C, and D are pin-joints which move with the operation of the mechanism. B and E are fixed, except that usually an independent slow motion of B upwards and downwards is provided by a thumbscrew, for the purpose of adjusting the pen to read zero altitude when required.

Note.—The arm ABC is free to turn about B, while the pen lever FE and the arm ED are free to turn together about E, the angle FED being constant.

The response of the vacuum-box to changes of pressure imparts a vertical motion to A (and therefore to C) very nearly proportional to the change in pressure. The amount of movement of A and C in the horizontal direction is small. For all changes of pressure over a considerable range, equal increments of pressure impart equal amounts of rotation to the arm ABC, but will not generally correspond to equal angular movements of the arm ED.

If ED is horizontal and its range of movement small, approximately equal angular movements will be obtained corresponding to given equal pressure changes, and consequently a uniformly spaced scale of pressures can be obtained. This is the case in the barograph illustrated in *Fig.* 22. The approximately horizontal position of the regulator arm (corresponding to DE) in this illustration should be noted.

Usually the arm DE is short, and its angular movement often ranges over 30° or 40°. Hence, on account of the relatively large movement of D in circular motion, the rate of angular movement of DE with change of pressure will increase considerably as DE passes from the horizontal to the vertical position. In this way twofold or even threefold magnification can be obtained at diminished pressures compared with the corresponding magnification at the initial pressure. A study of a table of sines or cosines will give an impression of the scope of variation of magnification by this means. It should be observed that on the basis of graduation used for height scales, a change of atmospheric pressure of 1 inch of mercury corresponds to about

900 feet at ground level (*i.e.* at about 30 inches pressure),
1350 feet at 11,000 feet altitude (*i.e.* at about 20 inches pressure),
2700 feet at 30,000 feet altitude (*i.e.* at about 10 inches pressure).

In short, the height equivalent of 1 inch of mercury varies inversely as the pressure, whereas the variation of magnification of the lever mechanism is largely a cosine function. Although it is easy to select two angular positions of DE giving the correct relative magnification corresponding to the initial and maximum height, the magnification obtained in intervening positions may not correspond sufficiently with that required by the height-pressure relation. Generally speaking, this arrangement of the system of magnifying levers has been worked out satisfactorily in the trade, though largely by method of trial and error, both for altigraphs and altimeters. In the latter case, the lever system is illustrated by *Fig.* 27. The arrangement of the magnification is largely dependent on the length and angular position of the arm DE.

In cases where the circular motion of a single arm through a considerable range does not give the desired results at all parts of the scale, some improvement may be obtained by combining the variation of magnification produced by two such angular movements.

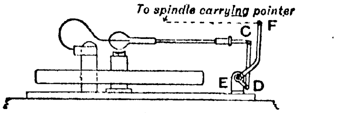


FIG. 27.—Altimeter Mechanism (not drawn to scale).

The most satisfactory way to solve the magnification problem for a number of aneroids of different ranges and sizes is to make a large-scale model of the lever mechanism and find experimentally the requisite lengths of levers and their angular positions corresponding to the specific problems to be solved. A model lever mechanism, say twenty-five times the normal aneroid dimensions with arms of adjustable lengths, might well be used not only as an illustration, but as an accurate measure of the relation between the final aneroid reading and the amount of movement of the vacuum-box.

(ii.) *Note on the Adjustment of the Zeros of Altigraphs and Altimeters.*—In the type of lever mechanism illustrated in *Fig.* 26, it should be noticed that when the altigraph indicates zero height the angular position of the arm DE and the magnifying power of the mechanism are fixed. In order to satisfy the logarithmic height-pressure relation, the magnification must vary inversely as the pressure, *i.e.* the magnifications of the mechanism at the pressures 29, 30, and 31 in. must be $\frac{1}{29} : \frac{1}{30} : \frac{1}{31}$. Hence strictly the zero of altitudes on the chart can correspond to only one pressure, and a definite convention should be adopted for setting such instruments without violating this principle. This is a point which seems to have been overlooked in the designing of some altigraphs. The arrangement whereby the initial height reading cannot be adjusted without altering the angular position of the mechanism is unsound in principle. It would be desirable to mark opposite the height zero of an altigraph the corresponding pressure for which the instrument is adjusted.

In the case of an indicating altimeter aneroid, zero reading is obtained at the commencement of a flight by rotating the dial. This does not interfere with the magnifying power of the mechanism. Consequently the accuracy of the altimeter readings should be independent of the actual pressure at the starting-level, to within small limits in a carefully made instrument.

(iii.) *Effect of Changes in the Internal Structure of the Metal of the Vacuum-box.*—In a perfect aneroid the exact relative positions of the component parts of its mechanism at a given fixed pressure would be permanently fixed.

The vacuum-box is not, however, a perfectly elastic structure. Small departures from its initial condition take place, which in the course of time accumulate. As a result, the position of the regulator arm DE of the mechanism (*Fig.* 27) is not wholly permanent for a given pressure, and a small and probably progressive change of magnification takes place in course of time. This secular change is more important in an altimeter than in an aneroid barometer. In the latter, the magnification is usually at least approximately uniform over the working range of pressure, whereas it is arranged to vary up to two- or even three-fold in the altimeter. It is usual in specifying altimeter aneroids to arrange that the indicator needle shall be vertical (the dial being vertical) when the atmospheric pressure is 29·90 in. of mercury.

In this way altimeters in which considerable secular changes have taken place since their manufacture can be detected and returned to the instrument makers for readjustment and overhaul.

In the case of aneroid barometers, the effect of a secular change on the relative accuracy of the scale values is not so marked, and the reading may be regulated from time to time by means of the setting screw at the base of the instrument after comparison with a suitable standard, say a reliable mercury barometer.

(iv.) *Errors due to "Creep" or Hysteresis.*—(The nature of these errors has already been discussed for aneroid barometers. See § (13).)

The British Government specifies a certain standard of quality of aneroid mechanism in respect of hysteresis. The criterion of this quality is the average difference (or else the individual differences) between the errors of the rising and falling indications of the altimeter at each 1000 feet of altitude. The following table shows the maximum values permitted. They refer to hypothetical flights made with altimeter aneroids under laboratory conditions of test, *i.e.* at a given rate of ascent and descent, without spending any considerable time at the summit of flight.

TABLE X

Nominal Range of Altimeter.	Maximum Permissible Value of the Average Difference between the Errors of the Rising and Falling Indications of the Altimeter at each 1000 Feet of Altitude.
Feet.	Feet.
0-10,000	+140
0-20,000	+360
0-30,000	+720

Note.—In comparing the performances of altimeter and barometer aneroids it should be remembered that the height-equivalent of a given pressure change is about three times as large at 30,000 ft. altitude as at sea-level.

The above figures represent average values over the whole range of flight indicated. On return to starting-level, the residual error is smaller than this average.

The following figures correspond to the same standard of quality of the aneroid mechanism as that indicated by Table X.

Table XI

Range of Flight. (No protracted Stay at Summit.)	Error of Altimeter, immediately on return to the Starting-level.
Feet.	Feet.
0-10,000	+ 90
0-20,000	+160
0-30,000	+250

Notes.—1. The above figures refer to laboratory conditions of test in which the pressure at 0 ft. altitude is the same at the commencement and completion of the flight. They may be taken as representing the maximum residual hysteresis effect on return to starting-level after a steady flight, without protracted stay at the summit, or steep descent to ground.

2. In the course of a prolonged stay—*e.g.* one hour or more—at the summit of flight, the indications of the altimeter would undergo a considerable amount of gradual creep, part of which is present temporarily on return to starting-level.

3. Tests made on standard-pattern altimeters of different makes indicate that the residual error on return to starting-level after a flight to 10,000 ft. altitude may be as large as

150 ft., corresponding to a stay of 1 hour at 10,000 ft.
200 ft. corresponding to a stay of 1 day at 10,000 ft.

It should be observed that an altimeter nearly always reads too high on return to starting-level; and in view of the above figures, the instrument cannot be used to give accurate heights near ground-level in order to assist the air-pilot in landing. In fact, the pilot would already have reached land before his altimeter indicated a return to starting-level.

Even if the residual error were small, or if an allowance for it were practicable, the pilot would require to know the difference between his starting- and landing-levels, and also the change of ground-level pressure during flight, before the altimeter could be relied upon to assist him in landing.

(v.) *Other Instrumental Errors.*—(See also § (13) under "Aneroid Barometers.")

The indications of altimeters are subject to some uncertainty owing to the vibration of aircraft. The error involved is likely to vary with the type of machine flown, and it is difficult to specify for an altimeter a vibration test that will cover all errors of this nature that may occur during flight.

(vi.) *Mercury Altimeter Gauges.*—In general, aneroids are tested and calibrated by comparison with a standard mercury gauge. In the case of altimeter aneroids for aircraft purposes, which are usually made without a scale of pressure, the calibration work has been much facilitated by making mercury standards graduated in heights following the same height-pressure relation as that laid down for the aneroid instruments. Such a mercury standard is essentially a "station" instrument—*i.e.* it measures hypothetical heights—and is not itself moved from one height to another.

In the workshop and test laboratory, altimeter aneroids are set up inside a vacuum chamber to which a standard mercury altimeter gauge is connected, and checked by means of comparison readings made at various artificial pressures.

The mercury altimeters suffer from the disadvantage that their scale divisions are of necessity not equi-spaced. Consequently a vernier cannot be used to read off fractions of a subdivision. This has to be done by estimation, or else the pressure must be slowly adjusted so that the mercury is exactly opposite a scale division mark.

On the other hand, these mercury altimeters give correct hypothetical height differences independently of the temperature at which they are used, for owing to the logarithmic nature of the height scale the change in height indication, corresponding to a given change of temperature of the instrument, is the same for all heights. If the design of such an instrument permitted of its being used for mountain climbing, it would be necessary to correct the mercury column at each observational station to a standard or reference temperature before heights could be read off.

VII. Manometers

(Excluding high-pressure manometers, micromanometers, and gauges for measuring high vacua, which are referred to in the articles on "Pressure, Measurement of," Vol. I., and "Meteorological Instruments" in the present volume.)

§ (19).—A manometer may be regarded as an instrument for measuring the difference between two pressures which occur simultaneously, and may be either naturally or artificially controlled. The barometer, as generally understood, is a particular case of a manometer in which one of the two pressures is zero.

In manometry, as in barometry, the fundamental hydrostatic principle in general use is that of balancing the gas or vapour pressures against the weight of a column of liquid whose height is taken as a measure of the difference of the two pressures acting at the ends of the liquid column.

All liquid manometers of the range included in the scope of this article are of the U-tube (siphon) type, or some simple modification of it. Aneroid manometers—*i.e.* those instruments which are not of the liquid type—are dependent on liquid manometers for their standardisation.

Provided that the density of the liquid

employed is known, and also the local value of gravity, accurate manometry is a question of accurate cathetometry in the measurement of the height of the liquid column.

In a large number of cases, the magnitudes of the pressure differences to be measured by the manometer are considerably smaller than the normal atmospheric pressure, so that there is a choice of liquids in manometry, though mercury is generally the most suitable.

There is a wide and varied field for the use of manometers. Chemical and physical conditions governing the measurement of gas and vapour pressures often limit the design of the instrument and the nature of the manometric liquid used. In a few cases the use of liquid is inadmissible, and an aneroid manometer becomes essential.

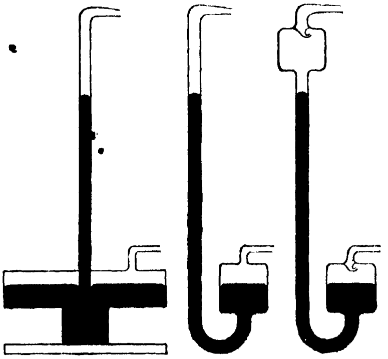
A variety of types of manometer will be considered, commencing with the simpler and more fundamental designs.

(i.) *The Simple* U*-tube* (*Siphon*) *Manometer.*—The simple siphon manometer is so familiar that it hardly needs discussion in this article, except as regards its use when measurements of high precision are required (§ (20)).

With the exception of certain modified designs used for small pressure differences, the siphon type of instrument suffers from the drawback that the liquid-level has to be read in *each* limb of the siphon before the pressure is obtained. Consequently, unless the pressure is either stationary, or varying slowly and uniformly with the time, a limit is set to the resulting accuracy owing to changes of pressure which may take place during the interval between the reading of the two liquid-levels, especially if a thermostat is not employed. The form of the siphon manometer has been modified so that only one liquid level need be read, the scale being calibrated, at static pressures or under conditions approaching static as nearly as possible, by means of a reference standard mercury manometer.

(ii.) *Modifications of the Simple Manometer, using One Liquid only.*—*Figs.* 28, 29, and 30 illustrate some slightly modified instruments in which either mercury or other suitable liquid may be used. In form they are similar to some of the types of mercury barometer described for measuring atmospheric pressures, except that the manometer tube is open at the top. They can be used to measure pressure differences up to 800 mm., or more, without becoming cumbersome to manipulate.

All three types shown correspond in principle to the Kew pattern barometer (§ (3) (iv.)), *i.e.* only one meniscus-level is read in each instrument, the reservoir being made cylindrical so that the change in level of the liquid in the tube is proportionate to the change in pressure to be measured. Each instrument, therefore, requires a uniformly spaced scale, contracted to an amount depending on the relative dimensions of the reservoir and tube. By making the diameter of the reservoir very large compared with that of the tube, the movement of the liquid in the index tube is very nearly 1 in. per 1 in. change of pressure, whereas in the simple siphon manometer with uniform bore of tube throughout, a change in pressure of 1 in. would result in a change of half an inch in the liquid level in each limb of the instrument.



FIGS. 28, 29, and 30.

With mercury as the manometric liquid, the instruments shown in *Figs.* 28-30 could be used to give a general accuracy of ± 0·05 mm., if the tube is of ½-in. bore, provided the reservoir is of sufficiently large diameter (see remarks under § (7) (ii.) (*b*)). It would be necessary to calibrate these instruments by comparison with a suitable reference standard manometer.

Of the three types illustrated, errors of levelling are minimised in the first, where the tube and reservoir are co-axial. This instrument may be fitted with a scale and vernier in much the same way as a meteorological mercury barometer, but would not give indications near zero owing to the nature of the reservoir. The other two instruments could be graduated down to zero, but would require careful levelling. The third manometer (*Fig.* 30) embodies a modification, suggested by Menard.[1] There is a trap at each end which prevents the escape of liquid when the instrument is inverted. Menard has made the liquid traps with narrow orifices so as to impede the flow of the air or gas through them, thus damping the oscillations of the liquid consequent upon sudden pressure changes; but this object may be attained in all three instruments by suitably constricting the lower part of the tube. Whenever constriction is resorted to, great care should be taken to keep the instrument clean so that the constricted portion does not become stopped up with dirt or sediment.

---

[1] *Comptes Rendus*, 1920, clxxi. 1129.

If the liquid used in the manometer is not mercury, its level is determined by reading the bottom of the meniscus (instead of the top when mercury is used). In reading liquid-levels the meniscus should be suitably shaded so as to appear black against a white background.

(iii.) *Modifications of the Siphon Manometer for Measuring Small Pressure Differences.*—In the direction approaching micromanometry the simple siphon manometer has been modified so as to measure small pressure differences—*e.g.* from 0 to 30 mm. of mercury—with high precision. The design is based on the principle of the tilting U-tube developed by Lord Rayleigh[1] and others. In its original form this instrument contained two fiducial points, one in each axis of the two limbs of the gauge, and was mounted so as to admit of being tilted by a measured amount in the vertical plane containing the two points.

By tilting the manometer and adjusting the amount of mercury in it, simultaneous contact can be made between the mercury in each limb of the instrument and the respective points.

From the measured inclination, together with the previously determined distance between the two fiducial points, the difference of level of the two liquid surfaces can be obtained.

Provided that the two points are sufficiently far apart, the accuracy of this type of manometer is limited only by the accuracy of setting a liquid level to a point (see § (5) (ii.), in connection with the same problem in barometry), for the amount of tilt given to the instrument is capable of far better proportionate accuracy.

Rayleigh's tilting mercury manometer was proved to measure with an accuracy of $\frac{1}{2000}$ of a millimetre of mercury, pressure differences ranging from 0 to 1·5 mm.

The tilting manometer is by no means limited to this small range. Scheel and Heuse[2] have modified the instrument so as to measure pressure differences up to 5 mm. of mercury with the same accuracy, viz. $\frac{1}{2000}$ mm., and in another instrument (*loc. cit.*) have succeeded in raising the upper limit of measurement to 30 mm. without loss of proportionate accuracy.

The principle of the tilting liquid manometer has been widely adopted, especially for accurate work in the measurement of air-speeds in aerodynamic laboratories. Instruments designed for this purpose are rather to be regarded as micromanometers. For this work the use of fiducial points as level indicators is tedious, and an exceedingly useful sensitive indicator has been developed by using the movement of the meniscus separating two liquids in a manometer as a means of balancing the pressure difference between the limbs of the instrument.[3]

(iv.) *Other Precision Liquid Manometers.*—The accurate use of a manometer is of the utmost importance in many experiments of a fundamental nature in the determination of physical constants. In some cases the final accuracy of such experiments is limited by the precision obtainable from the manometer. Owing to different conditions regulating the use of manometers in these cases, *e.g.* magnitude of pressure range, temperature of manometer, etc., it is impracticable here to give a detailed account of the various manometers used and the method of reading them. The following is just one of many examples of important work involving the use of a manometer with high precision.

(*a*) *Manometer with two Liquids moving in Limbs of Different Diameter.*—*Fig.* 31 illustrates the main outline of a manometer used by Jaeger,[4] in which pressure differences were measured with an accuracy of ± 0·005 mm. of mercury.

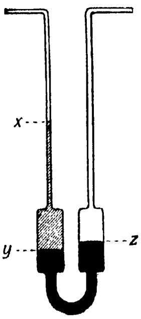


FIG. 31.

The instrument illustrated has a range of 30 mm. of mercury, but admits of extension to measure higher pressures. The primary object of Jaeger in using this type of manometer was to avoid reading two liquid levels, as in the simple mercury siphon barometer, a procedure which may be very troublesome with a varying pressure.

Octane and mercury were used with this manometer, the specific gravity of the latter liquid being twenty times that of the former. The use of mercury was dictated by chemical considerations. Actually, as will be seen later, the presence of a second liquid does not improve the sensitiveness of the manometer.

Let A = the area of cross-section of each cylindrical bulb.

a = the area of cross-section of the index tube.

$$\left.\begin{array}{l} x = \text{the level of the lighter liquid in the index tube} \\ y = \text{the level of the heavier liquid in the left bulb} \\ z = \text{the level of the heavier liquid in the right bulb} \end{array}\right\} \text{measured above any given reference level.}$$

ρ_1 = the density of the lighter liquid.

ρ_2 = the density of the heavier liquid.

---

[1] *Roy. Soc. Phil. Trans.* A, 1901, cxcvi. 205.

[2] *Zeits. Instrumentenk.*, 1909, xxix. 344-349.

[3] "Pressure, Measurement of," Vol. I.

[4] *Roy. Acad. Amsterdam Proc.*, 1914, xvii. 12.

The pressure difference P measured by the manometer is

$$(x-y)\rho_1-(z-y)\rho_2,$$

and by differentiation

$$dP=(dx-dy)\rho_1-(dz-dy)\rho_2.$$

Assuming constant temperature, the constancy of volume of the two liquids in the manometer leads to

$$adx=Ady=-Adz,$$

$$\therefore \frac{dP}{dx}=\left(1-\frac{a}{A}\right)\rho_1+\frac{2a}{A}\rho_2.$$

Case I.—When the manometer contains one liquid only, *e.g.* the lighter one

$$\frac{dP}{dx}=\left(1+\frac{a}{A}\right)\rho_1.$$

The multiplication factor of the instrument is then the reciprocal of the right-hand side of this equation, *i.e.*

$$\frac{A}{a+A}\times\frac{\rho_2}{\rho_1},$$

if pressures are measured in units of mercury.

Case II.—For two liquids in the manometer, the multiplication factor of the instrument corresponding to the movement of the lighter liquid in the index tube is

$$\frac{1}{(1-(a/A))\rho_1/\rho_2+2a/A}.$$

As an example, the ratio ρ_2/ρ_1 may be taken as approximately 20 for mercury and octane, while if the bore of the index tube is 2 mm., and the bore of each bulb is 40 mm.,

$$\frac{a}{A}=\left(\frac{1}{20}\right)^2=\frac{1}{400},$$

and the value of the multiplication factor in Case II. becomes 18·22. In Case I. the corresponding factor is 19·95.

For the two liquids referred to, the multiplication factor can never exceed 20, neither can it be less than 10, the minimum value corresponding to a simple U-tube of uniform bore throughout.

For further details of the working of this type of manometer, together with the method and accuracy of reading the index level, reference should be made to Jaeger's paper.

The length, 600 mm., of the index tube permits of pressure differences up to 30 mm. of mercury being used.

By increasing the length of the bulbs, the manometer can be modified so as to measure larger pressure differences without, however, increasing the range of measurement, which is limited by the length of the index tube. In this case the amount of liquid in the instrument has to be arranged so that the octane just begins to rise in the index tube when the pressure to be measured is approaching attainment. This idea may be extended to give a manometer of tolerably large pressure range, with high magnification locally, *e.g.* at one end of the range.

The manometer should be used by preference in a thermostat bath. Failing this, the indications of the instrument should be corrected for changes in its temperature. The instrument may be calibrated either from first principles or by comparison under static conditions with a standard mercury manometer.

(v.) *The Barometer and Manometer used in Conjunction.*—In some cases where it is desired to measure the pressure of a gas or vapour in an enclosure, it is impracticable to do so by direct connection of the enclosure with a barometer, though a manometer may be employed to give the difference between the atmospheric pressure and that in the enclosure. In such cases the manometer used is supplemented by barometric readings of the atmospheric pressure. Owing to the frequent use of a mercury barometer for laboratory work, together with the fact that an accuracy of ±0·05 mm. can be obtained from it under ordinary conditions (±0·03 mm. in the best cases of vernier-read instruments), it is frequently convenient to use a mercury barometer in conjunction with a manometer which measures relatively small differences of pressure, often with a liquid of low specific gravity.

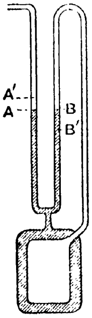


FIG. 32.

An interesting type of manometer which may also be used as a barometer is shown in *Fig.* 32.

The inner bulb contains air at approximately atmospheric pressure, and is surrounded by the manometer liquid, which may be pure sulphuric acid or some suitable light liquid with a low vapour pressure. The ingenious part of the instrument is the method by which the level of the liquid in the right-hand limb of the manometer is made independent of changes of temperature of the instrument by the device of balancing the thermal expansion of the liquid against the thermal increase of pressure of the air enclosed in the bulb. This can be done without unduly restricting the choice of liquid or making the dimensions of the instrument unwieldy.

Let A and B represent the level of the liquid in the manometer tubes when the air pressure p in the bulb is equal to the outer atmospheric pressure, the temperature being t. Suppose that the tubes forming the limbs of the manometer are cylindrical and equal in bore.

Let v = the volume of liquid in the instrument. If the external atmospheric pressure is assumed constant for the present, the increments dp, dv, corresponding to the thermal change dt, are given by

$$dp = p\alpha dt$$
$$= (x + y)g\rho,$$

where x and y are the supposed changes, AA′, BB′, in the liquid levels in the uncompensated manometer; α is the gas coefficient, 0·00367; $g\rho$ is the weight of unit volume of manometric liquid.

$$dv = V\beta dt$$
$$= (x - y)S,$$

where β is coefficient of dilatation of the liquid relative to that of the containing vessel, and S is the sectional area of the liquid column.

Hence $$\frac{p\alpha}{v\beta} = \frac{(x + y)g\rho}{(x - y)S}.$$

If p is measured in terms of the height of the manometer liquid and v is measured in terms of the equivalent length of manometer tube,

$$\frac{p\alpha}{v\beta} = \frac{x + y}{x - y}.$$

If $p\alpha = v\beta$, the dimensions of the instrument can be chosen so that y vanishes, *i.e.* the level B is independent of temperature.

As an example:

$\alpha = 0{\cdot}00367$ (for air),
$\beta = 0{\cdot}00057$ (for pure sulphuric acid),
$p = 30$ in. of mercury,
$= 222$ in. of pure sulphuric acid (density 1·84),
whence $v = 1430$ (measured in linear inches of manometer tubing).

This condition allows a fair amount of scope in choosing the size of the manometer, and the following approximate dimensions would be found suitable:

Diameter of inner cylindrical bulb	= 1½ in.
Diameter of outer cylindrical bulb	= 2½ in.
Length of outer cylindrical bulb	= from 3 to 4 in.
Diameter of manometer tubing	= $\frac{1}{10}$ in.

A manometer tube of this bore (2½ mm.) would be convenient to use in practice, since the variations in capillary action of the acid in it should be negligibly small. (*Note.*—The capillary elevation is about 4 mm.)

If reasonably accurate thermal compensation has been obtained, the open limb of the instrument may be calibrated by comparison with a standard mercury barometer. For a change in atmospheric pressure equivalent to 1 in. of sulphuric acid, the liquid levels in the manometer will be changed by ½ in. —very nearly—since the effect of external pressure changes on the presence of the enclosed air is of a secondary order.

The foregoing instrument is one of several types of sulphuric acid manometer, many of which have been used by Callendar[1] in his work on air thermometry. In some cases a manometer of this kind is used in order to avoid the more troublesome method of working from first principles with a mercury manometer.

For work of the highest accuracy, the reader is referred to § (9) for an account of a fundamental standard mercurial instrument which is virtually a combined barometer-manometer (see *Fig.* 11).

(vi.) *Non-liquid Manometers.*—Although, in general, mercury is the liquid best suited for manometry, it is impracticable to use a liquid at all in some cases, such as the measurement of pressures of vapours which attack mercury and other manometric liquids, or measurements in which the experiment requires the manometer to be at a high temperature, or measurements made on aircraft.

Consequently, manometers have been designed on aneroid principles, using the deflection of a thin membrane (not necessarily of metal) as the means of indicating pressures.

For the statical determination of vapour pressures of many pure substances, and also the study of chemical equilibria, a glass or quartz manometer is almost indispensable.

Jackson[1] has described three modifications of a glass manometer of a compact nature which can be used at high temperatures inside chemical apparatus. These consist of one or more thin plano-convex or concavo-convex bulbs about 1½ to 2 cm. in diameter (see *Fig.* 33), which change their shape when pressure is applied inside or out. The extent of this elastic deformation is measured by the angular movement of a fine glass pointer, which forms a continuation of the bulb and is drawn out in the blowing operation.

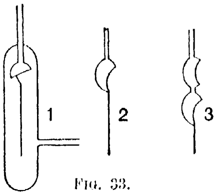


FIG. 33.

This manometer is somewhat similar in principle to the familiar aneroid barometer, but is simpler, both in general construction and in the manner of magnifying and reading the elastic deformation of the bulb. In one of these manometers, Jackson used a pointer about 20 cm. long, the end of which was read with a microscope. The zero of the instrument was found to be unaltered after the manometer had been used under a pressure difference of 150 mm. of mercury; neither was it affected by slow changes of temperature up to 300° C.

Other manometers in glass or quartz have also been developed, both in this country by Gibson[2] and others, and abroad by Bodenstein,[3] Preuner,[4] etc.

---

[1] See, for example, *Phil. Trans.* A, 1887, p. 171; *Phil. Trans.* A, 1891, p. 126; *Roy. Soc. Proc.*, 1892, l. 247.

[1] *Chem. Soc. J. Trans.*, 1911, xcix. (Part I.), 1066.

[2] *Roy. Soc. Edinburgh Proc.*, 1913, xxxiii. 1.

[3] *Zeits. f. Elektrochem.*, 1909, xv. 244.

[4] *Zeits. f. physikal. Chemie*, 1913, lxxxi. 129.

While the foregoing forms of glass and quartz manometers are indispensable for use at high temperatures with gases and vapours which preclude other types of manometers, the metal membrane manometer also has a sphere of use, and is capable of yielding an accuracy of $\frac{1}{4}$ to $\frac{1}{2}$ mm. of mercury under favourable conditions, provided the range of measurement is not too large and the instrument is calibrated under the conditions in which it is used. The instrument is very similar in construction to the familiar aneroid barometer, except that the diaphragm-box of the latter is no longer a vacuum-box in the manometer.

For short pressure ranges, the diaphragm metal can be made very thin (about 0·1 mm.) for the sake of increased sensitiveness.

(*a*) *Use of the Metal Diaphragm Manometer as a Sphygmomanometer to measure Blood Pressures.*—An extremely light and convenient form of aneroid manometer is shown full size in *Fig.* 34. Essentially it consists of a diaphragm-box, the interior of which is not exhausted as in the case of the ordinary aneroid barometer, but is submitted to pressure applied at the nozzle of the instrument. The dial is graduated in millimetres of mercury up to 300 mm., and indicates the excess of pressure within the box over the atmospheric pressure outside it.

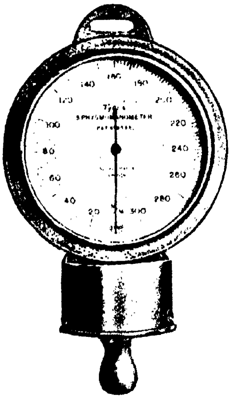


FIG. 34.

For medical diagnosis, the instrument is used in conjunction with a rubber bag which is wrapped around the arm of the patient above the elbow directly over the brachial artery. The manometer is then attached to one of two rubber tubes leading from the bag, while an inflating bulb, with valve, is attached to the other. This apparatus is used with or without a stethoscope for the determination of the maximal and minimal pressures in the arterial system.

Originally a mercury gauge was used as sphygmomanometer, but this has been superseded by the more convenient and robust aneroid instrument.

§ (20) METHODS OF READING MANOMETERS, AND THE RESULTING ACCURACY OBTAINABLE. (See also the corresponding section in § (5) referring to barometers.)—To a large extent the methods adopted in reading manometers are much the same as those applicable to barometers. They are summed up briefly under the following headings. Owing to the somewhat wider scope in the design and use of a manometer, greater variety in methods of reading it may be expected.

(*a*) *High-precision Manometry.*—When high precision is required, the use of a cathetometer is in general essential, and the remarks on the accurate reading of mercury levels by cathetometer (see § (5) (ii.)) are equally relevant here. A number of fundamental investigations into physical constants have been carried out at the Bureau International des Poids et Mesures at Sèvres, by Chappuis, Leduc, etc., and some detailed accounts of the manometers used in those experiments appear in the publications of the Bureau.[1]

In this connection reference may be made to the following experimental determinations:

(1) The fundamental determination of the boiling-point of sulphur.

(2) The dilatability of mercury (hydrostatic method).

(3) The mass of a litre of air under standard conditions.

In all these experiments, the method of reading mercury levels generally requires the application of a number of small, but by no means negligible corrections, especially when the levels are optically measured. Under good conditions the final accuracy obtained in the measurement by cathetometer of pressure differences up to 1000 mm. of mercury is of the order ± 0·005 mm.

Special Case of a High-precision Manometer designed for Use with a Constant Head of Liquid.—In many operations requiring high precision, the pressure in an enclosed system may be chosen at will, provided that it admits of accurate measurement.

For example, in the determination of the densities of gases, considerable time may be gained by working with a gas under the pressure of a given head of liquid. Rayleigh[2] adopted this plan in one of his manometers. The guiding principle was to use a steel rod to determine, and also measure, the distance between the upper and lower mercury surfaces, arranged so as to have the same vertical axis.

The rod contains two fiducial points, the distance between which can be accurately measured, before using the manometer, by comparison with a suitable standard of length. This distance determines the head of mercury in the manometer, and a suitable slow adjustment of the volume and level of the mercury in the manometer tube enables both levels to be brought just into contact with the upper and lower points on the measuring rod. The head of mercury being known, the corresponding pressure can soon be calculated after taking into consideration the temperature of the mercury. The manometer may be surrounded by a water bath if this is thought necessary for the accurate measurement of the mean temperature of the liquid column.

---

[1] See *Travaux et Mémoires Bur. Int.* tome xvi. and earlier volumes.

[2] *Roy. Soc. Proc.*, 1893, liii. 135.

In considering the accuracy obtainable in the measurement of pressure by this means, the chief limitation is imposed by difficulties of temperature measurement. Under good conditions an accuracy of ±0·02° C. should be obtained for the mean temperature of the mercury, corresponding to ±0·002 mm. of mercury in a column of 760 mm. The distance between the fiducial points may also be measured to within ±0·002 mm., and a final accuracy within ±0·005 mm. should be obtainable.

It should be remarked that this manometer avoids the uncertainty, experienced in working with optically read instruments, due to irregular refraction by the walls of the manometer tube.

(*b*) *Medium-precision Manometry.*—The method of reading by means of a scale and vernier slide, after the manner of the meteorological mercury barometer, is to be recommended, when a large number of readings have to be taken on a mercury manometer, if the utmost precision is not required.

It would be advisable to calibrate the instrument, either from first principles by means of a cathetometer, or else by comparison with a fundamental precision manometer.

The accuracy generally given by a calibrated vernier-read manometer is of the order 0·03 to 0·05 mm.

In many cases, especially with liquids of low specific gravity, sufficient accuracy is obtained by reading the position of the centre of the liquid meniscus against an accurate scale, either ruled on the glass manometer tube or placed conveniently behind it. Attention should be paid to the illumination of the meniscus and to the nature of the background against which it is observed.

(*c*) *An Optical Lever Manometer for Measuring Small Pressure Differences.*—A mercury manometer with a delicate and sensitive optical indicator has been designed by Shrader[1] and Ryder, and is shown diagrammatically in *Fig.* 35. Apart from the indicator, the manometer is the familiar U-tube of large diameter (5 cm. or upwards). The optical indicator consists of a light mirror M, rigidly attached to a lever L, which rests on two knife-edges fixed to the wall of the glass tube.

The lever L has a small glass bead B fused to its end to act as a float.

The principle of the method of reading hardly needs any further explanation. A beam of light incident upon the mirror from the direction I is reflected so as to give an image at a suitable distance on a scale S.

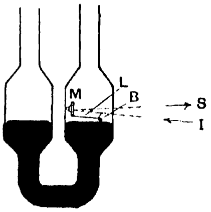


FIG. 35.

Shrader's manometer has a working range extending from 1/1000 mm. to 3 or 4 mm. of mercury. One of its special features is its quick response to changes of pressure. It can also be made photographically self-recording, yielding accurate records of pressure changes due to such phenomena as vaporisation, freezing, diffusion, etc.

§ (21) MANOSTATS.—It is frequently desirable in chemical experiments to control the pressure within the apparatus. This is especially the case in fractional distillation, and in the determination of vapour pressures and boiling-points. For this purpose, some form of manostat or pressure regulator is indispensable.

The chief requirements of a manostat are sufficient sensitiveness, adaptability over a wide range of pressure, and ability to maintain pressures above or below the normal atmospheric pressure.

The majority of manostats are designed on the broad principle that whenever the pressure departs from the desired constant value, the manometer itself, through the change in liquid level in one of its limbs, automatically actuates either a supply or removal of gas to or from the apparatus or else some equivalent means of restoring the desired pressure. This may be achieved by thermal control, in which case the manostat is similar in operation to a very familiar type of thermostat. It is illustrated in *Fig.* 36. The enclosure E, which is to be kept at constant pressure, is heated by a gas burner, the supply of gas being led in through the narrow orifice O.

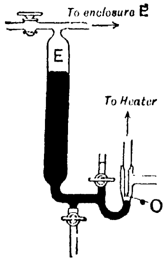


FIG. 36.

The heater prevents the pressure within the enclosure from falling below the desired constant value, while if an excess of pressure exists, the mercury in the manometer partially covers the orifice O, thus temporarily diminishing the heat supplied. The stop-cocks allow of initial adjustment to the desired pressure conditions. This manostat may be designed to cover a large range of pressures, both above and below atmospheric pressure. It can of course be adapted to electrical heating.

---

[1] *Phys. Rev.*, 1919, xiii. 321.

Its use is, however, limited to temperatures above that of the laboratory.

Of the types of manostat in which constancy of pressure is secured by the supply or removal of air from the enclosure, some depend on an electromagnet for actuating a valve or cock leading to the source of supply, the electrical circuit being completed automatically by the mercury in one of the manometer tubes. Electrically controlled manostats have been designed and developed by a number of investigators, including Beckmann[1] and Liesche, Smits,[2] Speranski,[3] and Drucker.[4]

Alternatively a manostat may be devised so as to admit air in much the same way as the supply of coal-gas is controlled in a mercury thermostat (or in the manostat of *Fig.* 36). An instrument has been designed on this principle by Wade and Merriman,[5] using the analogous device of an air-inlet passage which is automatically left uncovered by the mercury of a manometer when the pressure falls below a limiting value. This manostat may be arranged for pressures above or below the normal atmospheric pressure, *e.g.* ranging from 0 to 1500 mm. of mercury or more. It is capable of maintaining constancy of pressure to within ± 0·2 mm. from the mean. This performance is dependent on the dimensions of the manostat, for which particulars are given by Wade and Merriman. The actual pressure in this type of manostat is subject to fluctuations in the atmospheric pressure.

For further details of a number of manostats reference may be made to Arndt's *Handbuch der physikalisch-chemischen Technik*.[6]

§ (22) APPLICATION OF THE BAROMETER OR MANOMETER TO MEASURE IN UNITS OTHER THAN PRESSURES. (i.) *The Measurement of Gravity at Sea with a Mercury Manometer.*—In general, the method of balancing an air pressure hydrostatically against a column of liquid is used with the object of measuring the height of the liquid column, and so determining the pressure absolutely through the adoption of known values of the density of the liquid and of local gravity. A modification of this principle to measure gravity at sea has been employed by Duffield, and is fully described in the article "Gravity Survey."

(ii.) *The Measurement of Velocity with a Manometer.*—The principles underlying the measurement of wind velocity are set out in the article on "Meteorological Instruments." Briefly, if the difference between the static and total air pressures can be measured, the velocity of the wind is given by the formula

$$p = kV^2,$$

where p is the difference between the static and total air pressures, *i.e.* the dynamic pressure,
V is the wind velocity,
k is a constant depending on the units of measurement.

An "anemometer head" ensures that the true static and total pressures are operating on the manometer used to measure the pressure difference between them. The manometer may be of the liquid or aneroid type, and can be arranged to read air speeds directly if graduated in accordance with the formula

$$V = \text{constant} \times \sqrt{p}.$$

Air-speed indicators for use on aeroplanes are of the aneroid type, in which metal or rubber diaphragms have been employed by instrument makers.

Silver diaphragm boxes have been used with success in sensitive instruments. Owing to the nature of the formula connecting air speeds and pressures, it is impracticable to obtain anything approaching a uniformly spaced scale of air speeds.

Some of the current patterns of air-speed indicators used by the Government register up to about 170 miles per hour, corresponding to which the actual pressure difference in the manometer is about 14 in. of water.

Although this value is low compared with the pressure differences operating on the diaphragms of altimeter aneroids, defects such as hysteresis and "creep" are by no means negligible in air-speed indicators.

(iii.) *The Measurement of Heights by the Barometer.*—(See Part VI. of this article.)

F. A. G.

BAROS: the name given to an alloy, experimented with at the Bureau International, for use as a material for making weights; composed chiefly of nickel, with small proportions of chromium and manganese, but not sufficiently invariable for a fundamental standard. See "Balances," § (8).

BAROTHERMOGRAPH: a self-recording instrument which records the temperature as a function of the pressure. See "Meteorological Instruments," § (38).

---

[1] Beckmann and Liesche, *Zeits. f. physikal. Chemie*, lxxxviii. 13; also lxxix. 565.

[2] Smits, *Zeits. f. physikal. Chemie*, xxxlii. 39.

[3] Speranski, *Zeits. f. physikal. Chemie*, lxxxiv. 160.

[4] Drucker, *Zeits. f. physikal. Chemie*, lxxiv. 612.

[5] Wade and Merriman, *Chem. Soc. Trans.* xcix. 984.

[6] Published by Von. F. Enke, Stuttgart.

Bates Saccharometer. See "Hydrometers," § (9).

Baumé's Hydrometer, used as saccharometer. See "Saccharometry," § (9).

Beam of Equi-arm Balance:
Design of. See "Balances," § (1) (i.).
Methods of determining the deflection of. See *ibid.* § (1) (vi.).

Beam-scales. See "Weighing Machines," § (1).

Benoit, Fabry, and Perot: determination of metre in terms of wave-lengths. See "Line Standards," § (7) (ii.).

Beranger Balance. See "Weighing Machines," § (3).

Berne, Universal Comparator at, description of. See "Comparators," § (7).

Bishop's Ring: a reddish-brown ring, of inner radius 12° and outer radius 22°, observed round the sun. It is named after its first observer. See "Meteorological Optics," § (15) (iv.).

Board of Trade Standards. See "Metrology," IX. § (13).

Bolometer as applied to Solar Investigations. See "Radiant Heat and its Spectrum Distribution," § (18).

Bourdon-tube Thermograph. See "Meteorological Instruments," § (9) (iii.).

Boyer Recorder Tachometer. See "Meters," § (12).

Brass Measures of Capacity. See "Volume, Measurements of," § (21).

Briggs or National U.S.A. Standard Pipe Thread. See "Gauges," § (53).

British Association (B.A.) Standard Thread. See "Gauges," § (45).
Table of sizes. See *ibid.* § (57).

British Standard Fine Thread (B.S.F.), table of sizes. See "Gauges," § (56).

British Standard Pipe Threads (B.S.P.), table of sizes. See "Gauges," § (58).

British Standard Whitworth Threads (B.S.W.), table of sizes. See "Gauges," § (55).

British Units of Volume. See "Volume, Measurements of," § (3).

Brocken Spectre. When the shadow cast by the observer's head on a bank of fog or mist is surrounded by a series of coloured rings—glories—the phenomenon is known as the Brocken Spectre. See "Meteorological Optics," § (15) (iii.).

Brunner Comparator, description. See "Comparators," § (5).

Brunsviga Calculators. See "Calculating Machines," § (6).

Burettes. See "Volume, Measurements of," § (18).

Buttress Standard Thread. See "Gauges," § (51).

Buys Ballot's Law. This law states that in the northern hemisphere pressure decreases from right to left of an observer who stands with his back to the wind. The reverse holds in the southern hemisphere. The explanation of the law lies in the fact that the rotation of the earth tends to deflect any body in motion on its surface towards the right in the northern hemisphere. The wind blows in such a direction that this tendency is counteracted by the pressure. At heights removed from the effect of surface turbulence the wind must therefore blow along the isobars. Near the ground on account of frictional effects the wind is partly checked and blows into the region of low pressure, but the deviation from the isobar is never great enough to vitiate the law as stated above.

— C —

CALCULATING MACHINES

§ (1) **Historical Sketch.**—Aids to calculation may be tabular, graphical, or mechanical. Among the mechanical aids are the calculating machines. These employ toothed gearing, and perform the operations of arithmetic. As they are arithmetical they differ from the more purely mathematical instruments, such as planimeters, integrometers, integraphs, harmonic analysers, and the like, while as they employ toothed gearing they are distinct from the slide rules.

In attempting, however, to present even the briefest sketch of the development of the calculator, reference should be made to some devices, which are not, strictly speaking, calculating machines, such as the abacus, and the numbering rods of Napier.

So long as Roman numerals were used, computation was no easy matter, and some form of reckoner soon became a necessity. In early classic days there was the dust board, a tray filled with damp sand, which was scored by the fingers into squares, like a draughtboard. These squares could be punched by the fingers with dots, to facilitate counting. Then pebbles (*calculi*) would be used, and later a more permanent form of reckoner was made by stringing beads on wires. In this way was formed the abacus, an instrument which has played an important part in the practical or commercial arithmetic not only of the ancient and mediaeval worlds, but also in the Far East at the present day.

The arithmetic of mediaeval Europe consisted of two parts, theoretical and practical. The former involved much that was mystic, and discussed the ratios and properties of numbers. It was the work of the Greeks, transmitted, imperfectly at first, through the writings of Boethius. The latter, the practical arithmetic, dealt with calculation, and employed the abacus.

(i.) *The Abacus.*—In modern Europe the abacus is only used for teaching infants to count. In China and Japan, however, it is an important calculating device. These nations no doubt obtained it originally from India, into which country it had been introduced probably by some of the Semitic races, who were the great traders of the ancient world.

Fig. 1 shows a Chinese or Japanese abacus. On any selected wire each bead of the five-group represents unity, while each bead of the two-group represents five. Hence a decimal notation is obtained. *Fig.* 2 shows the number 37408 or any decimal variation of this sequence of digits. The operations of addition and subtraction are self-evident. Those for multiplication and division are effected by learning a multiplication and division table.

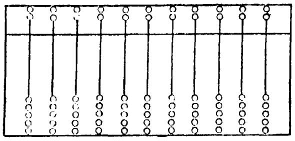


FIG. 1.

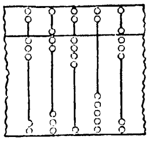
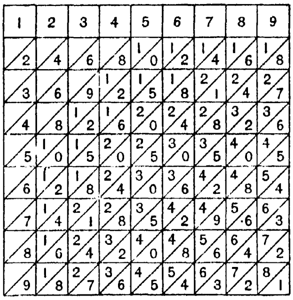


FIG. 2.

(ii.) *Napier's Bones.*—The difficulties of computation in the Middle Ages were so great that the subject was almost beyond the powers of those who were not skilled mathematicians. Later, in 1617, Napier invented his system of numbering rods or "bones," which gave a more or less mechanical method of effecting multiplication. Each rod consists of the multiplication table for one of the numerals arranged in a vertical column as in *Fig.* 3.

As an example, suppose that it be required to multiply 4185 by 752. Arrange the rods 4, 1, 8, and 5 in order (*Fig.* 4), setting on the right the rod which contains the numbers 1 to 9. Then the line of partial product 4185 × 2 is shown by the second row, *i.e.* the numerals are, in succession, reading from right to left, 0, 1+6, 1+2, 0+8, *i.e.* 8370. In the same way the partial products for 5 and 7 are given by the fifth and seventh rows respectively, viz. 20925 and 29295. Adding these lines of partial product, as in ordinary multiplication, we have 3147120. It will be observed that by the use of Napier's rods the labour of multiplication is replaced by the simpler process of addition.

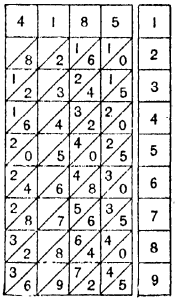
Napier's rods were of great practical value. To save time in arrangement the numbers were, later on, inscribed on parallel cylinders, which, by revolving, could be set rapidly to the required factors. So good is this simple device that it can hold its own as an inexpensive calculator even at the present day.



1	2	3	4	5	6	7	8	9
2	4	6	8	1/0	1/2	1/4	1/6	1/8
3	6	9	1/2	1/5	1/8	2/1	2/4	2/7
4	8	1/2	1/6	2/0	2/4	2/8	3/2	3/6
5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5
6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4
7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3
8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2
9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1

FIG. 3.

(iii.) *Pascal's Machine.*—The first true calculating machine was made by Blaise Pascal in 1652. It was constructed to help his father in his departmental work. Its use was chiefly for adding sums of money. It forms the type from which the modern arithmometers may trace their descent. Number wheels on the upper face of the machine are turned to effect additions. The model worked with a peg instead of a handle, somewhat like the Brical cash counter. Pin-wheels of the simplest form were used where now there would be bevel gearing.



4	1	8	5	1
8	2	1/6	1/0	2
1/2	3	2/4	1/5	3
1/6	4	3/2	2/0	4
2/0	5	4/0	2/5	5
2/4	6	4/8	3/0	6
2/8	7	5/6	3/5	7
3/2	8	6/4	4/0	8
3/6	9	7/2	4/5	9

FIG. 4.

(iv.) *Other Early Machines.*—The next important advance was the invention by Leibnitz of a machine containing a type of stepped reckoner. This in an improved form is the counting device employed in modern arithmometers. Leibnitz had the idea in 1671, and his earliest model was made in 1694. At an earlier date a British calculating machine had been brought out by Sir Samuel Moreland. This was dedicated to Charles II. in 1666. It was for adding sums of money, and its carrying device was like that of a modern speed counter. Another British machine was made by James Bullock in 1775 for Earl Stanhope, and an improved form was brought out in 1777.

Other machines were made during the

eighteenth century by Lépine, 1725; Leupold, 1727; Boistissaudeau, 1730; Gersten, 1735; and Pereire, 1750. The stepped reckoner was used by Hahn, 1774, and Müller, 1783.

§ (2) EARLY MODERN MACHINES. (i.) *The Thomas Arithmometer.*—The first machine which could be called a commercial success was an arithmometer made by C. X. Thomas, of Colmar, Alsace, in 1820, and improved by Payen. It was manufactured extensively about the middle of the century, and is still much used. Most modern arithmometers differ little from it in general construction, though they have been modified considerably in details, particularly with a view to automatic working, as in the "Madas" and "Fournier." The work of the original firm is still carried on by M. A. Darras (Paris), who makes the "Thomas" in a modern form, in sizes ranging from twelve to twenty figures.

(ii.) *Direct Multiplying Machines.*—The first *direct* multiplying machine was invented in 1889 by Léon Bollée. He introduced the use of tongue-pieces whose lengths represented numbers. This has been incorporated in the design of the "Millionaire," and is its most interesting feature.

The first keyboard machine was as late as 1861.

(iii.) *Babbage's Differencing Engine.*—Before proceeding to examine the modern calculating machines a brief reference may be made to the differencing engine, and the analytical engine, of Charles Babbage.

The differencing engine was for the mechanical construction of mathematical tables, such, for example, as occur in the *Nautical Almanac.* The principle on which it worked is easy to understand. Suppose we have the tabulated values of any function arranged in a column. We can form in the usual way the column of first differences, *i.e.* by subtracting successive pairs. By treating this column in the same way we can form the column of second differences, and from it the column of third differences. Conversely, then, if instead we had been given the column of third differences, and the leading value of the second and first differences and of the function, we could, by adding, reproduce the columns, first of second differences, then of first differences, and finally of the function. Many tables may be constructed in this way, given a column of differences, which is usually fairly constant over a wide range. Babbage's differencing engine, which was designed to give for each turn of the handle corresponding values in the various difference columns, and also the value of the function, all by straightforward additions, was accordingly a glorified adding machine.

(iv.) *Babbage's Analytical Engine.*—The aim of the analytical engine was to carry out any sequence of operations with any given numbers, and so in fact to evaluate any algebraical formula. It consisted of three parts—the mill where the operations were performed, the store where the results were placed ready for use, and the controlling mechanism which directed the operations. The latter was of the nature of a Jacquard apparatus. It was worked by an arrangement of two sets of perforated cards—one for the mill and one for the store—which determined the operations, and specified their order. In this way numbers set up in the store were transferred to the mill, operated on there as required, and the values so obtained were returned to the store, till the final result should be obtained.

Babbage began his analytical engine in 1833, but by the time of his death in 1871 only a comparatively small part had been finished.

§ (3) MODERN ARITHMETIC MACHINES. *Counting Devices.*—The fundamental operation in a calculating machine is addition, since from this the others can be derived. Mechanical addition involves two distinct processes, the summation of the digits, and the "carrying" from one denomination to another. The summation of the digits is effected by one of five mechanisms: (*a*) the stepped reckoner; (*b*) the variable toothed wheel; (*c*) the circular rocking segment; (*d*) the reciprocating rack; (*e*) the double crown wheel. These classify calculating machines into different groups according to the mechanism used.

The operation of "carrying" is mechanically the most difficult to perform, and the mechanism which controls it is the most delicate part of the machine. In its simplest form it is illustrated by the speed counter (*Fig.* 5), in which the dials are removed to show the working. Here, as the 9 in the units dial changes to 10, the 0 appears in the units dial in succession to the 9, while at that moment the projecting tooth on the units wheel engages with the tens counting wheel, and moves it on by one step or unit. In calculating machines, however, the carrying is much more complicated, and a more elaborate mechanism is required.

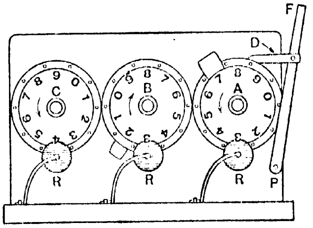


FIG. 5.—A Speed Counter.

§ (4) THE ARITHMOMETERS.—The arithmometers form the oldest group of calculating machines. The earliest successful one was the Thomas already mentioned. Modern arithmometers resemble it closely, but embody various minor improvements as well as some important additions.

(i.) *Addition and Multiplication: General Account.*—The top of an Archimedes machine, one of the class, is shown in *Fig.* 6. It is in two parts; the upper portion, with its two rows of figures, can slide to right or left. On the lower portion are shown a series of vertical slots with the digits 0—9 arranged by their sides. Pointers or markers slide in these slots and can be placed opposite any digit required. As figured the markers, counting from the right, are opposite to the digits 3, 5, 5, 1; these digits appear in the row below and read 1553. This is mainly for convenience. The machine is worked by the handle seen on the right, and the mechanism, which will be explained shortly, is such that one turn of the handle, when, as at present, the machine is set for addition, transfers these digits from the markers to the result dials in the row immediately above. The result is shown in this row. A second turn transfers the same digits a second time to this row, and so on for a third or any succeeding number of turns. In this manner the number 1553 is added once for each turn of the handle to the figures in the top row: we obtain in effect the result of multiplying 1553 by 1, 2, 3, etc. The top row merely shows the number of times the handle has been turned, *i.e.* it registers the multiplier.

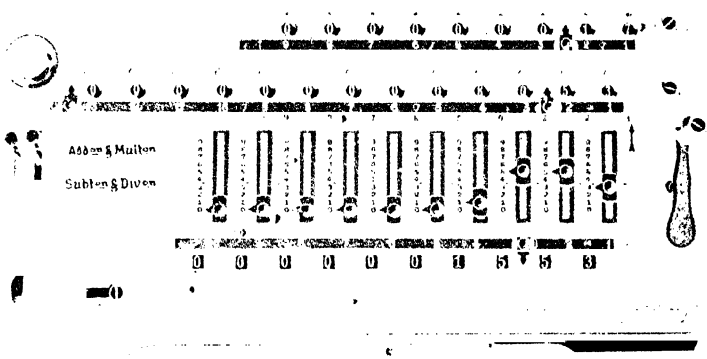


Fig. 6.—Archimedes.

Multiplication, thus, is performed by a repeated process of addition; the bottom row shows the multiplicand, the top row the multiplier, and the central row the product.

The slide already referred to enables us to obtain the result more rapidly; the mechanism, when the handle is rotated, transfers a number from the marker to that place in the result row which is immediately above it; if we move the slide one place to the right we bring the tens' place in the product above the units in the markers, the hundreds above the tens, and so on; thus in this position a turn of the handle adds ten times the multiplicand to the product, *i.e.* multiplies by ten. Thus to multiply by 235 we should push the slide to the left as in the figure, turn the handle 5 times, thus multiplying by 5, then push the slide one place to the right and turn 3 times, thus multiplying by 30, then push the slide one place further on and turn 2 times to multiply by 200.

The process may be shown thus:

$$\begin{array}{rl}
\begin{array}{r}1553\times 5=\\ \textit{Five turns}\end{array} & \left\{\begin{array}{r}1553\\1553\\1553\\1553\\1553\end{array}\right.\\
\begin{array}{r}1553\times 30=\\ \textit{Three turns}\end{array} & \left\{\begin{array}{r}1553\\1553\\1553\end{array}\right.\\
\begin{array}{r}1553\times 200=\\ \textit{Two turns}\end{array} & \left\{\begin{array}{r}1553\\1553\end{array}\right.\\
 & \overline{}\\
1553\times 235= & 364955
\end{array}$$

(ii.) *The Stepped Reckoner.*—The above description applies in general terms to a number of machines, but the adding device—the mechanism, that is, whereby the digits are transferred from the markers to the result dials—differs. In the Thomas and Archimedes machines the stepped reckoner is used. This device, which is shown at A in *Fig.* 7, is placed below each of the marker slots; thus in the machine illustrated there are ten stepped reckoners. Each of these consists of a circular cylinder which, by turning the crank handle, can be made to rotate about an axis parallel to the length of the slots. The outer surface of the cylinder is machined away, leaving on part of the circumference nine radial projections or steps of gear-tooth form and of varying axial length; these engage a small pinion or gear wheel, causing it to rotate as the cylinder is turned. The pinion is shown at B′ in *Fig.* 7. The axis of this gear wheel carries at one end a bevel wheel (*i*, *Fig.* 7) which, when the sliding carriage is down, engages with a second bevel wheel, shown at *d′*, the numbering wheel; to this the figures giving the result are attached, and these appear in turn on the corresponding dial as the wheel rotates; the motion of the stepped reckoner is thus transferred to the result dial. The projections or steps on the reckoner vary in length parallel to its axis, as shown in the figure, increasing gradually from the first to the ninth; the pinion which engages with them slides on a square axis parallel to that of the cylinder, and by altering its position on this axis the number of steps which will engage with it, and hence the angle through which it turns for one turn of the handle, is varied: this alteration is effected by moving the marker to which the pinion is attached; when the marker is at zero the reckoner can be turned without moving the pinion; no steps engage. As the marker is moved to one, two, etc., one, two . . . steps engage the pinion, which is thus moved through the angle corresponding to one, two . . teeth; the motion is transmitted by the bevels to the result dials and the digits 1, 2, 3, etc., appear in turn. Thus if the marker be set to 7 and the handle turned, seven teeth engage; the numbering wheel turns through seven places and 7 appears on the dial: if the marker be now set to 2 and the handle turned, two teeth engage, the result dial is moved through two more places, and 9 appears.

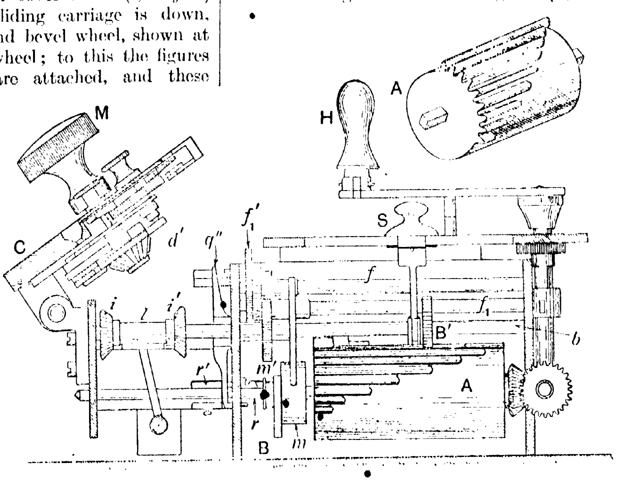


FIG. 7.—End Elevation of the Thomas de Colmar Arithmometer (sliding carriage shown raised).

H, handle; S, marker; A, stepped reckoner; B′, counting pinion on square axle *b*, at the other end of which are the bevel wheels *i* and *i′* on a sleeve *l*, giving the adding or subtracting rotations to the bevel wheel *d′*, which works the figure disc seen below the dial. M is a milled head which works the effacer. The other letters show the carrying, numbering, and effacing mechanisms.

(iii.) *Carrying.*—The chief mechanical difficulty is in the carrying; when this is about to occur—*i.e.* when the 9 on the result dial is about to change to 0—an extra single tooth, sliding on the next higher axis, is pushed into position by a lever and stud device on the axis of lower denomination; this moves the result dial of next higher denomination through one place and thus effects the carrying.

(iv.) *Subtraction and Division.*—As addition is performed by causing the numbering wheels to turn in the positive direction, so subtraction can be carried out if we can make the numbering wheels rotate negatively; the positive rotation is given to the numbering wheel *d′* by a bevel wheel *i* which engages it at one end of a diameter; a second bevel *i′* engaging it at the opposite end of the same diameter will give the negative rotation. These two bevels are connected by a sleeve *l* sliding on the square axis, their distance apart being greater than the diameter of the numbering wheel. By means of a stud either can be brought into action and a positive or negative rotation given to the numbering wheel. To perform subtraction then the total is placed on the result dials, the amount to be subtracted on the markers, the studs being set to subtraction, and the handle is turned once; the answer appears on the result dials.

Division is effected by a process of repeated subtraction. The machine being set for subtraction, the dividend is set on the extreme left of the result dials and is followed by a series of zeros; the divisor is set on the markers and the slide moved to the right until the left-hand figure of the dividend is above[1] or one place to the left of the highest figure of the divisor. Thus the part of the dividend which is to be operated on is greater than the divisor. The handle is turned once, thus subtracting the divisor, as in long division, from the higher figures of the dividend; if the remainder shown on the result dial is still greater than the divisor, the operation is repeated, and this is continued until the remainder is less than the divisor; the number of times this has been done appears on the top dial and is the first figure of the quotient. The slide is moved one place to the left and the process repeated, thus obtaining on the top dials the second figure of the quotient. This is continued until all the figures are found.

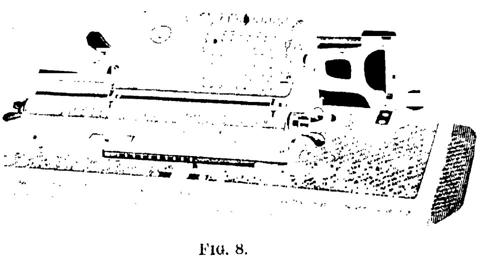


Fig. 8.

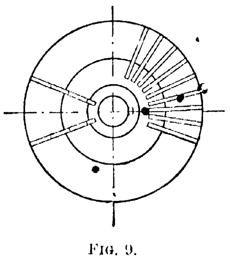


Fig. 9.

An arrangement is usually added for effacing rapidly the figures in the view-holes, thus resetting all dials to zero.

§ (5) Other Arithmometers.—Among the older arithmometers may be mentioned the Thomas and the Tate, while among the newest are the Archimedes, Colt, Layton, Tim, Unitas, and Madas. The last is referred to among the automatic division machines.

A recent development of the arithmometer is the Fournier Calculator. It is an arithmometer fitted with a compact and distinctive keyboard, whose feature is that the depression of a key depresses all in its column of lower value, while the highest value alone is registered. It also multiplies by a single turn of the crank for each figure in the multiplier. In order to accomplish this, energy is stored in springs during the inactive portion of the turn of the reckoner, and this is sufficient to finish the operation. This arithmometer was designed to render all the operations of arithmetic as nearly automatic as possible.

§ (6) The Dactyle or Brunsviga Group.—The idea of a wheel with movable pins or teeth was described by Poleni of Venice in 1709. A Brunsviga machine employing this device is shown in *Fig.* 8. Additions and subtractions are obtained by the convenient method of turning the handle forward or backward, *i.e.* positively or negatively.

The adding device is a wheel consisting of two parts—a disc (*Fig.* 9) and a cover (*Fig.* 10). In the disc are slots in which lie steel fingers or teeth (*Fig.* 11). A projection on each of these fits into a groove in the cover-disc. The act of setting a number, say six, by the marker rotates the cover of the disc, and in so doing forces the projections of six of the fingers to slide along the groove from the inner portion *a*, past the crossing *b*, to the outer portion of the groove. This makes six teeth project from the wheel. As there is a counting pinion ready to engage with this, a positive rotation of the operating handle advances the counting wheel by six units, and so adds this number to any already on the result dials. A negative rotation of the handle would subtract it. Carrying is performed by sliding a movable tooth into position when required.

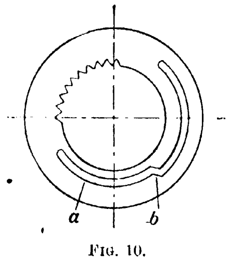


Fig. 10.

In this way any numbers may be set by the markers, and added or subtracted as desired by positive or negative turns of the handle. As in the arithmometer, multiplication is obtained by successive additions, and division by successive subtractions.

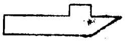


Fig. 11.

The dials are set to zero by an effacer worked by turning the butterfly nuts shown in the figure.

Other machines of this group are the Triumphator, Zeetzmann, Muldivo, and Marchant.

§ (7) Adding Machines.—Adding and listing machines are employed widely in business, and are worked most conveniently by a keyboard. A machine worked by keys was first put on the London market in 1861 by V. Schilt. The fundamental device employed in adding machines is the toothed sector, or sectorial rack.

---

[1] Above if equal to or greater than the divisor, one place to the left if less.

Reference to the diagram (*Fig.* 12) shows that when a key, such as number 5, is depressed, it moves, by means of a bell-crank lever F, a stop wire, and pulls the end inwards to the bottom of the slot. This limits and specifies the travel of the toothed sector B. This toothed rack B is carried by the pivoted lever A, and moves between the guide plates D. On depressing the key the rack descends through the specified number of teeth—five in this case—and the corresponding type for printing, carried at the other end of the pivoted lever A, comes into position, and is struck by a hammer and printed. During the descent of the rack the counting wheel C is thrown out of gear, but on the ascent it engages, and so is turned through the number of units specified by the key. The number set by the key is thus transferred to the registering device, and so is added to the total. If then a set of racks and counting wheels be arranged in series, and possess the necessary carrying apparatus, an adding machine is formed, and the registering wheels will show at any time the sum of all the numbers which have been set on the keyboard. An illustration of the machine with its cover removed is given in *Fig.* 13.

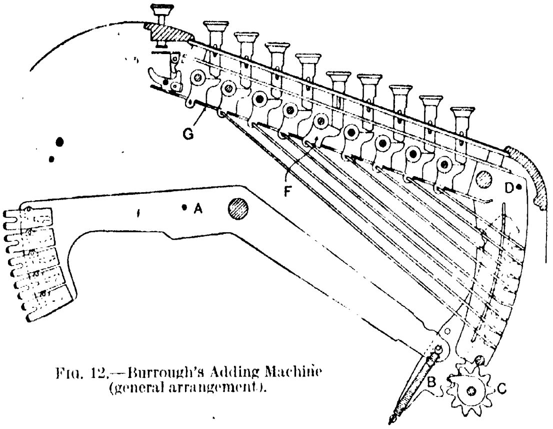


FIG. 12.—Burrough's Adding Machine (general arrangement).

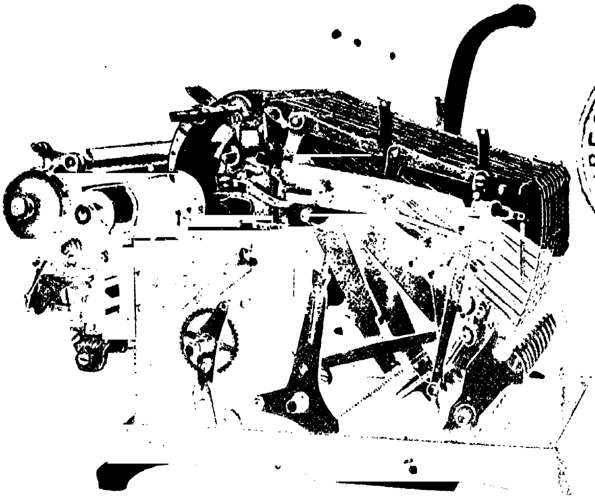


FIG. 13.—Burrough's Adding Machine (cover removed).

DIRECTOR OF IN
Bengal

In the smaller machines, such as the small Burroughs adding machines and the Comptometers, the sum is shown at once on the result dials. In the large Burroughs listing machines the sum is to be printed. This necessitates, in addition to the printing device, a totalising key, whose depression, after an extra stroke is given to the working handle, prints the sum, and sets the numbering wheels all to zero, *i.e.* clears the machine.

The principle of the carrying device is to make the next higher rack rise through the distance of an extra tooth. This carries an extra unit to its number wheel when the rack ascends.

These machines are energised through springs, to prevent the mechanism from being strained, and the large Burroughs machines have also a dashpot.

Subtraction may be carried out by adding the complementary number. The latter may be defined by saying that any number plus its complement forms some whole multiple of ten. Thus 7 and 3 are complementary. For subtraction the complementary numbers are marked on the keys in small type. In many cases it is more convenient to take the complement with respect to 9 instead of 10, and in that case a special mechanism adds unity to the result. Thus, instead of subtracting 764 from 876, and obtaining 112, we may write

$$764 = 1000 - (235 + 1).$$

$$\text{Thus } 876 - 764 = 876 + 235 + 1 - 1000$$
$$= 1112 - 1000 = 112.$$

We thus add 235 + 1, the complementary figure to 876, and reject the highest digit.

Multiplications and divisions are accomplished as repeated additions and subtractions respectively, but the "typist" touch for doing this rapidly requires a certain amount of training.

§ (8) CALCULATOR TYPEWRITERS.—Adding mechanisms or totalisers are now attachable to certain typewriters, so that typing, listing, and adding may be carried out on the same machine. Among calculating typewriters may be mentioned the Hammond, Monarch, Wahl, Smith, and Underwood.

The Elliott-Fisher adding machine is a further development of this idea. It has a standard keyboard which traverses the frame of the machine, and moves over a large sheet of paper. Adding registers are clamped on a bar of the frame wherever a column of figures is required on the paper. The keyboard, when passing these registers, engages them with studs, and so totalises each column of figures in the sheet.

§ (9) THE MILLIONAIRE MACHINE.—The Millionaire was patented by O. Steiger, and is manufactured by H. Egli in Zurich. It performs the operations of arithmetic, but is specially devised for multiplication. This it does not accomplish by repeated additions, but by a single turn of the crank for each digit of the multiplier. It forms accordingly a class by itself.

The most striking feature is a set of nine tongue plates or multiplication pieces. These represent by their lengths the multiplication table. Thus the tongues on the plate 8 are of lengths 8, 16, 24. . . . A figure such as 24 is represented by two tongues, one of length 4 in the digits place, the other of length 2 in the tens place. A tongue plate is shown in *Fig.* 14; the light bars represent the units, the dark bars the tens. Suppose that a number, set by the marker, is to be multiplied by 8. The multiplication lever is set to 8, which places tongue plate 8 in the position for working. Straight racks, forming part of the counting device, rest against the tongue plates, and are displaced by them in working to an extent determined by the lengths of the tongues. This displacement of the racks moves the counting wheels of the recorder through the corresponding number of steps or units, and transfers these values to the result dials, so giving the result of the multiplication. For addition and subtraction the multiplication lever is set to unity. Division is simplified by a table of reciprocals. The Millionaire is often fitted with a keyboard for office work.

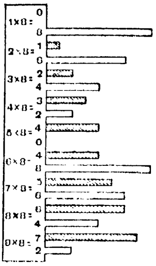


FIG. 14.—The Millionaire. Tongue Plate.

§ (10) AUTOMATIC DIVISION MACHINES.—Division, though the most troublesome of the simpler rules, is easy to carry out on a calculating machine. The operation, however, is usually not automatic. Two machines, the Mercedes-Euklid and the Madas, while performing all the arithmetical operations, also carry out automatic division.

(i.) *The Mercedes-Euklid.*—This machine was designed by Ch. Hamann of Friedenau, Berlin. Its adding mechanism employs the straight rack and pinion. In this case there are ten parallel racks, one to correspond with each of the horizontal rows of figures 0, 1 . . . 9 extending from slot to slot. The displacements of these racks, which must be proportional to their digits, are obtained by the simple device of a proportional lever. This is worked by the handle, and each rack gets a displacement proportional to its number. Thus the counting pinions, set by the markers, register displacements of the corresponding number of units, and convey these to the recorders. The subtracting device is particularly ingenious. The proportional lever is then pivoted at the other end, and the pinions get the complementary displacements. This addition of the complements of the number is the same, with the exception of a unit, as subtracting. In carrying out automatic division the machine approaches the value of the quotient through upper and lower limits successively, subtracting and adding till the result is obtained.

(ii.) *The Madas.*—This machine is an arithmometer, and as such uses the stepped reckoner. It is worked in the usual fashion. The mechanism for division is modified in such a way that, when the sequence of subtractions for obtaining a figure of the quotient has been completed, the carriage moves automatically one space to the left. Hence continuing to turn the handle in the same sense produces the successive figures of the quotient.

§ (11) The Monroe Machine.—In its original form this was the invention of Mr. F. S. Baldwin (Conn.). It possesses a keyboard in front, like a comptometer, and a sliding carriage at the back, like an arithmometer, while the working recalls the horizontal handle of the Brunsviga.

The adding device is simple and ingenious. It is a wheel consisting of two opposed parts, like crown wheels, which can be made to approach or recede from each other. On one of these discs are five equal pins, and on the other four stepped pins. These pins engage the counting wheel. They project, alternately, into the space between the discs, so that the closed wheel looks like a lantern pinion, or a very flat squirrel-cage rotor. If a number less than five (say three) be set on the keyboard, the discs move so that the counting wheel will engage with three pins of the stepped disc. If a number greater than five (say eight) be set, the discs close so that five teeth of the one disc and three of the stepped disc will gear with the counting wheel and operate it. This is analogous to the use of the stepped reckoner in the arithmometer.

The working of the Monroe is somewhat like that of the ordinary arithmometer, except that the numbers are transferred rapidly to and from the keyboard by a positive or negative turn of the horizontal handle. Such machines, with a long sliding carriage, possess the advantage that one can work different parts of a calculation at the opposite ends of the carriage, and thereafter combine them. This is specially the case with the Monroe, as one of its features is the speed with which it can combine the operations of arithmetic, as in the evaluation of algebraic formulae.

The Amco is a smaller machine on similar principles to the Monroe.

§ (12) Tabulating and Sorting Engines.—Some large commercial firms use special modifications of engines like the census machine. These are controlled by a Jacquard arrangement, operated by punched cards. Any group of details may be picked out and sorted in one machine, and summed and tabulated in another. These engines are larger and more elaborate than the machines already discussed. The information, which is usually representable by figures, requires to be punched systematically by holes in the cards. This takes time, but once it is done the information is dealt with by the machines at an extraordinarily rapid rate. Two of the best known of these sets of engines are the Holerith and the Powers. The former works electrically, and the latter mechanically.

See "Le Calcul mécanique" (*Encyclo. Scient.*), Paris, 1911; *Napier Tercentenary Handbook*, Royal Soc., Edinburgh, 1914; *Trans. Inst. Eng. and Ship. in Scotland*, Glasgow, 1919-20, tr. xiii.; *Bulletin de la Soc. d'Encouragement pour l'Industrie National*, 1920, tome 132, No. 5.

E. M. H.

Calibration of Indicators for Metrological Observations. See "Metrology," § (32) (ii.).

Calibration of Scale: successive stages, method of taking observations in subdividing comparator, computation of results, and theory underlying same. See "Comparators," § (12).

Callendar Radiation Recorder. See "Meteorological Instruments," § (31).

Camera Obscura: as used for observing clouds and for measuring the direction of motion and the speed-height ratio. See "Meteorological Instruments," § (34).

Campbell-Stokes Sunshine Recorder. See "Meteorological Instruments," § (24). See also "Sunshine Recorders," "Radiant Heat and its Spectrum Distribution," § (1).

Cavendish Experiment for the determination of the constant of gravitation by a torsion method of measuring the attraction of leaden balls for one another. See "Earth, Density of the," § (2) (i.).

Centrolinead. See "Draughting Devices," p. 260.

Check Gauge, Definition of. See "Metrology," § (19).

Chronograph: recorder of subdivisions of a second. See "Clocks and Time-keeping," § (15).

Circular Calculators. See "Draughting Devices," p. 261.

Clairaut's Theorem. See "Gravity Survey," § (13).

"Class" of Fit: definition of term. See "Metrology," § (29) (i.) (*a*).

Clearance Fits: definition of term. See "Metrology," § (29) (ii.) (*a*).

Climate, effect of radiation on. See "Radiation," § (4) (ii.).

Clinograph. See "Draughting Devices," p. 262.

Clocks:

- Controlled systems of. See "Clocks and Time-keeping," § (18).
- Historical. See *ibid.* § (2).
- Standard of actual performance. See *ibid.* § (19).

CLOCKS AND TIME-KEEPING

§ (1) THE MEASUREMENT OF TIME. — Time enters as an element in all natural events even more universally than do Space and Mass, but whereas different lengths and masses may be compared, under proper precautions, with the same identical fiducial standards, the Warden of the Standards is unable to produce from his presses a standard Hour or Minute for verification, and Time can only be measured by the repetition of a process. Measurement is therefore bound up with the theory of the process selected, and so is liable to adjustment, should the theory be varied. The main process obviously presented as suitable and convenient is the rotation of the Earth, though others have been proposed for standards, especially such as involve the transmission of light, as being even more uniform and fundamental in character. Apart from the rotation of the Earth, the natural clocks offered by astronomy are the revolutions of the Moon, the Planets, and the Satellites of Jupiter. All these are deeply involved with difficulties of theory or observation and can only be used in a last resort for confirmation of suspected changes. The art of time-keeping is the distribution by subdivision in convenient form of the standard process. This is the subject of the present article. Before proceeding directly to it, it will be convenient to register a few points regarding the definition and determination of the Day in this place.

The rotation of the earth is an extremely good standard of uniform motion, but not a perfect one. It is certain dynamically that it is liable to change. Contraction of the body must accelerate it. Tidal friction must retard it. Both act continuously in one sense and therefore cannot be negligible. It would take us too far into astronomical details to pursue the points more fully, but it may be said that the effect of the variation can now be identified pretty certainly with residual terms showing themselves as an apparent displacement of the moon and of the planets, otherwise unexplained.

In defining the rotation of the earth it is necessary to make clear what is meant by fixity of direction. This is a matter both delicate and elaborate. The direction of very remote stars must be taken as absolutely fixed. The distances are so great that no velocity we could ascribe to an individual star would produce a sensible deviation, and in the mass the result would be null in an absolute sense. Passing to the brighter and less distant stars upon which daily observations must depend, it is generally agreed that these show systematic motions in streams, so that an origin of direction based upon a stream to which the sun did not belong would be a moving origin. The amount is such that it would correct by a fraction of a second of time the duration of a century. It would be merged with variations in the actual rotation of the earth and could not be separated out observationally. But practically, we use this basic reference to the mass of stars only indirectly. Lists or catalogues of stars made after full discussion show the positions of each relative to the mass. Of these several hundreds are suitable for time observations. Their positions relative to one another and to the whole sphere is pretty certain down to 0·01 sec. or 0·02 sec. A full observation for time in an observatory will rest on the mean of the determinations of the moment of passage of, say, about 10 of these stars across the local meridian. This might reasonably be expected to be reliable within 0·01 sec. But in fact individual observations stand out from the group and one night's determination from another by at least five times that amount. This discrepancy must be shared between the clock with which they are compared, the chronographic system of recording its indications, faults in measuring the position of the telescope with respect to the meridian and the horizon, personality of the observers, and, finally, to deviations of the stars from their mean places due to atmospheric causes. The uncertainty adds to the difficulty of testing the going of a clock, inasmuch as its error cannot be assigned with certainty within ± 0·05 sec. upon any given day, and owing to weather must sometimes go undetermined for many days together. This point will be dealt with later.

The first standard of time, then, is the sidereal day, and this is reckoned from the zero of the star catalogues. This zero point is not itself one of the stars; it is not a fixed direction as defined above, but possessed of a certain defined motion. This so-called First Point of Aries, which constitutes the natural, indeed the inevitable, zero for star places, is one of the intersections of the plane of the ecliptic with that of the equator. Both planes are subject to movements which are accepted as determined from prolonged observation and elaborate theory. At the epoch 1900 the regression of the First Point of Aries along the equator was 50·26″ annually, or 0·138″ per day, with respect to the mass of the stars. Sidereal time at any moment measures the angle about the Pole by which the First Point of Aries has been carried past the meridian of the place, at the rate of 24 hours to one complete revolution or sidereal day. The sidereal day is divided into hours, minutes, and seconds in the usual way.

Mean Solar Time, upon which all civil work depends, is never determined directly, but is calculated from the observed sidereal time. The link is made by the *Theory* and *Tables* *of the Sun* for the time being current, at present those of Newcomb.[1] These tables give the position with respect to the First Point of Aries of an imaginary body called the Apparent Mean Sun, which represents a smoothed mean position of the sun, taking account of the whole mass of observations on which the *Tables* are based. Just as the First Point of Aries gives the sidereal time at any place, so the Apparent Mean Sun gives the local mean time. For daily use its position with respect to the First Point of Aries is calculated and shown in the *Nautical Almanac*, under the double form of "sidereal time at mean noon," and "mean time of transit of the First Point of Aries." The mean sun progresses along the equator at the rate of one complete revolution with respect to the First Point of Aries in a "tropical year" of 365·2422 mean solar days. The same interval is equal to 366·2422 sidereal days. The mean solar day being divided into hours, minutes, and seconds in a similar manner to the sidereal day, it follows that

1 sidereal day = 23 h. 56 m. 4·09 s. solar,
1 mean solar day = 24 h. 3 m. 56·56 s. sidereal,

and

1 sidereal d., h., or m. = 1 − ·002730 mean solar d., h., or m.,
1 mean solar d., h., or m. = 1 + ·002738 sidereal d., h., or m.

From these relations between the units, and the relative position of the two origins at the beginning of the day in question, as given in the *Nautical Almanac*, local mean solar time is calculated from the observed local sidereal time. The conversion, a little troublesome for a detached case, can be made very easy and rapid for systematic serial work. It is however unusual to employ purely local mean time. Based upon Greenwich as the prime or zero meridian, the world is partitioned by convention into zones of one hour or one half-hour in width, at the margins uniting which the mean time employed is changed abruptly. Thus the Observatories of Paris and Edinburgh both use the mean time of Greenwich, calculating it by applying to their local mean time found, as described above, their adopted respective longitudes or differences from the Greenwich meridian. The Observatory of Washington follows the same process, but allows a further 5 hours for difference of zone. The boundaries of the time zones generally speaking follow straight meridians over the ocean, but occasionally deviate to one side or another on land so as to make them coincident where possible with national or provincial boundaries. Their positions as accepted by the British, French, and Italian Admiralties are shown on the Admiralty publication, *The World Time-Zone Chart*, 1919.

A further convention is required to fix the "date line," or line at which the date changes when crossed from east to west or west to east. It follows generally the 180th meridian (12 h.) from Greenwich, the day of the week and day of the month being one day more advanced upon the western side of the line than upon the eastern side; but it deviates about half-an-hour to the east so as to include the East Cape of Siberia with the mainland of Asia, then half-an-hour to the west to include the Aleutian Islands with the American continent, and again moves half-an-hour to the east in south latitudes so as to bring Fiji and Chatham Island within the area reckoning the later date.

§ (2) HISTORICAL.—The problem of making a clock—that is, the provision of a secondary process for estimating time, by which the Day might be correctly subdivided to any degree and the results consistently distributed—has presented numerous difficulties which are not all overcome at the present day. Omitting primitive devices like water-clocks and sand-glasses, and the more scientific sun-dial, it is to be remarked that the literature and remains of early time-keepers are very scanty. The art of wheel work and its adjuncts was developed early and, it would seem, was carried to achievements of the highest ingenuity at the time of Hero of Alexandria (*circa* 100 B.C.) in the form of self-acting models and toys. In the mediaeval period of our own epoch a parallel standard was reached in carillons and other musical-boxes, and in moving images chiming the hours. But time-keeping was secondary to the externals, as it is in most clocks to-day. This part was treated merely as a tradesman's production and was altered or discarded without respect. It is probable that it did not deserve much respect from those who had to use it for time-keeping and were not skilled enough to adjust it to its best performance.

The two earliest clocks known, and still in going order, are in the South Kensington Museum—one from Glastonbury Abbey, of date 1325, and the other from Dover Castle, of date 1348. A similar clock, made by De Vick or Wieck, a German craftsman, for Charles V. in 1370, and restored more than once, is in the Palais de Justice in Paris. All originally belong to the period anterior to pendulums. The alternate detention and release of the wheel work was provided by a crown wheel, engaging successively two pallets on a verge; the verge was pivoted parallel to the face of the crown wheel; when the wheel was released by throwing out one pallet, immediately the other pallet was thrown in on the opposite side, bringing the train to

---

[1] *Publications of the American Ephemeris*, vi. part i.

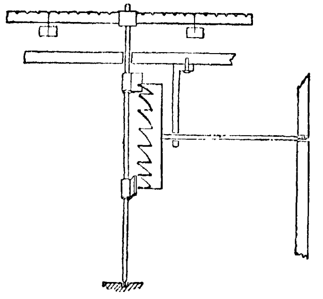


FIG. 1.

rest until the second pallet was thrown out in turn in the opposite direction. Thus the problem of preventing the train from racing, and of maintaining a rocking motion in the verge, was solved, but the sole time-keeping principle of this appliance was equality of impulse, very faultily realised in early work. The rate could be adjusted by shifting pendent weights along a foliot or cross-bar fastened at right angles to the vertical verge and supplying a variable moment of inertia for its rocking motion. To control the time-keeping by introducing a pendulum which should permit or prohibit the release was the practical adaptation by Christian Huygens (1629-1695) of Galileo's discovery of the isochronism of the pendulum. His *Horologium* (1658) and *Horologium Oscillatorium* (1673) show drawings and details of the arrangement of wheels and numbers of teeth, with crown and verge escapement carried to a crutch which maintains but is controlled by a pendulum. There is further the device of continuous cord and double ratchet wheels, known as Huygens' going barrel, by which the weight can be wound up without disturbing the going, and if a chiming train is added a single weight can be made to drive both trains. Huygens' study of the theory led him to mathematical developments of much beauty and some importance. First, there was the proof that for perfect isochronism the pendulum bob should swing not in a circle, but along a cycloid, with the geometrical description of the manner of producing such a motion by means of cycloidal cheeks against which the flexible suspension of the bob should bear. In practice any such device is pernicious and introduces errors in excess of what it is supposed to remove. His theory of the motion of the pendulum led him also to lay down the foundations of rigid dynamics, including a version of the principle of *vis viva*, before Newton had put any dynamical argument upon a regular basis.

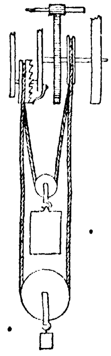


FIG. 2.

The failure of Huygens' device of cycloidal guides illustrates the fact that for clocks the theorist must work side by side with the craftsman. We owe to an unsurpassed craftsman, George Graham (1674-1751), the next two great contributions. Both of these have only been definitely superseded in the present day. The first is a pendulum compensated as to its length for changes of temperature by making the rod of steel and the bob a vessel containing mercury. The latter expanding more rapidly than the steel, nullifies the expansion of the latter if its amount is duly adjusted, which may be done by calculation, or finally by trial. His other invention is the dead beat escapement, described more particularly below. Until the introduction of Riefler's escapement, which left the pendulum entirely free at the ends of its swing, and with the exception of a very small number like Dent's (Airy's) at Greenwich, and Tiede II. at Berlin, all the best clocks in the world still use Graham's escapement. To John Harrison (1692-1776), the next in the fine school of eighteenth-century English clockmakers, we owe the introduction of the spring maintenance, by which the pull of the weight does not act upon the train of wheels directly, but through a spring. The spring is prevented from releasing its tension during the winding by a ratchet wheel the click of which takes off from the clock frame (see § (7) below). This replaces Huygens' device in key-wound clocks. Harrison's other invention of a compensated pendulum with a "gridiron" of parallel steel and brass rods, so connected that the former expand downwards and the latter upwards and the total lengths are adjusted to suitable proportions (17 : 11), is a clumsy device, very inferior to the mercurial pendulum. An equivalent was, however, realised later in a good form by Dent, in which drawn tubes of zinc and steel enclosing the steel rod effect the purpose of the gridiron.

Of all writings on the clock, none exceeds in importance Airy's theorem[1] that an irregularity of impulse at the ends of the swing changes the epoch of the pendulum, that is to say, its time-keeping directly, while an irregularity in the middle of its swing changes only the amplitude of its arc. But mention should be made of the fact that it was partly the problem of air resistance to a pendulum rod and its bob that occupied Stokes in his researches on the motions of cylinders and spheres through viscous fluids, illustrating the fertile stimulus to mathematical research supplied by a concrete problem.

---

[1] *Camb. Phil. Trans.*, 1827, iii.

At the present day effort is directed to realising, by a better escapement, Airy's condition for equivalence to a free pendulum. The manner and degree in which this is attained is described more particularly below. The cord and drum with their accessories are abolished, and the clock rewinds itself by an electric contact at suitable intervals. Sometimes the dial and counting train is completely separated from the pendulum and its maintenance, forming a secondary "slave clock" actuated by electric current sent by the simplified primary. Compensation of the pendulum is much simplified by the use of invar in place of steel for the rod. The barometric error is met by enclosing the movement in a sealed case from which a partial exhaustion of the air serves as a final regulation of rate. And finally, in reliance on the principle of the "double zero," for the best work the clock theoretically compensated against changes of temperature is kept under circumstances where the temperature is constant.

§ (3) CONSTRUCTION OF THE PENDULUM.—The chief elements to consider in the construction of the pendulum are the method of compensation for changes of temperature, the support or head, and the form of the bob. Compensation for changes of barometric pressure is sometimes also made by an attached feature in a method given below, but is usually dealt with by sealing the case. Compensation for temperature changes is now almost universally made by the use of invar for the rod, as described below, but as there are many excellent pendulums in use with other compensations, these will be first referred to. One desirable condition for finer work is that the compensating device should be distributed as far as possible over the length of the rod, so as to counteract effects of stratification of the air, a thing very liable to occur if the case is sealed and the circumstances kept uniform, as in a special cellar. The gridiron pendulum, which meets this condition fairly well, is condemned because of its liability to twist under expansion and behave irregularly. The zinc and steel pendulum ((*a*) *Fig.* 3), in which a zinc tube is carried up from the lower end of the rod, and is enclosed in a steel tube, pendent from its upper end and carrying the bob, is not liable to this objection, but the tubes with their layers of air form too effective a temperature shield of the parts which they enclose. The old-fashioned form of mercury compensation, (*b*), in which a steel rod ended in attachment to a roomy stirrup which carried a tray upon which a glass vessel containing mercury rested, shows under experiment a much increased air resistance, and what is not so obviously to be expected, a much increased barometric error. A better form of the same construction, (*c*), employed by Dent, enclosed the mercury in a steel vessel of cylindrical form with rounded edges, into the top of which the pendulum rod was screwed. Both are open to the objection that the whole compensation is concentrated at the lower end, and hence takes place erroneously if the air is stratified. Moreover, the thin rod takes up its change of temperature more quickly than does the massive bob and its contents. These objections were in large measure met by Riefler (before the use of invar) by making the rod a steel tube, 16 mm. internal diameter, and partially filling it with mercury. The bob was then screwed to the rod; it was supported at its centre of mass so that its expansion became a secondary matter. Some examples are given (1894) to show the excellence with which the compensation has been effected with this device in one of his clocks. Guillaume's invention of invar has, however, abolished the use of the steel rod and mercury compensation. The expansion of invar is anything from one-twelfth that of steel down to zero. Hence a brass tube or collar, pinned at its lower extremity to the invar rod, and 4 or 5 cm. in length or less, will effect the compensation. The bob, usually of type metal, is carried with its centre of mass resting on the upper end of this collar; or the collar may be dispensed with, and the bob carried on a pin through the rod from a point below its centre of mass. The construction

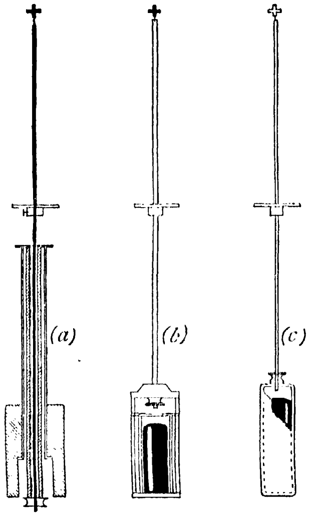


FIG. 3.

is here extremely simple and regular. There remain, however, the criticisms that the brass compensator on the inner part of the bob is somewhat shielded against ready communication of heat, and the whole compensation is concentrated at one end. To meet the latter objection Riefler cuts his pendulum in the middle, uniting the halves by a sleeve in which is the compensating device. The calculation of the lengths follows from the formulae given below. It will be noted that the change of the radius of gyration of the bob under expansion plays a part in it. This was first taken into account by Riefler. In spite of the theoretical care devoted to the compensation of pendulums, there are obscurities in their performance when serving enclosed clocks or exposed to sudden changes of temperature. More experimental study is wanted. It would be easily practicable to make the compensation adjustable in such a manner that it could be set by trial after the clock was completed. One such method is shown below in King's pendulum.

A lens-shaped form of bob has been fitted by some good makers, but a cylindrical one is usually preferred, and among cylinders one that approaches more or less to the form of a sphere because the sphere is the figure of least surface for a given content of mass, and so may be supposed to reduce skin friction. Exactly what way air resistance attacks the pendulum has not been fully worked out, but it can be verified that even trifling projections on the bob increase it materially. The edges of the cylinder should therefore be bevelled away to avoid production of eddies, and the surface given as high a polish as it will take. Type metal will take such a polish very well.

The head of the pendulum and its method of suspension require description. Experiments show that a pendulum, supported by steel knife edges rocking upon agate, maintains its arc at least as well as the best spring suspension so long as the edges are in good order. It is used as an accessory feature on Riefler's construction, as will be described below, but otherwise it has generally been rejected on the score that the knife edges change with wear, although its features are well known in connection with the construction of balances, and it presents some distinct advantages. Among these, the only motion geometrically possible is uniplanar, and any desired variation of the arc-equation or circular error could be introduced in a form not open to objection by suitably adjusting the cross-sections of the knife edges and of the bed on which they rock.

Turning to the spring suspensions actually practised, there is considerable difference in the lengths and strengths of springs employed. The lengths range from 1 cm. or more to less than half that amount. Even springs 2 mm. in length perform well. The spring is sometimes in one piece, sometimes of a pair. The purpose of using a pair is to avoid possible buckling when the spring is gripped by the upper and lower jambs. But if its use results in tolerating a lie of the two springs which is not coplanar, nothing is gained. It is clasped above and below between jambs, very carefully gaged in respect to parallelism, and then pinned or screwed together, taking care to avoid any suspicion of buckling. The lower jambs are arranged to hook with a good fit into the upper extremity of the rod, and through the upper jambs the cross head passes and is secured by binding screws. This cross head is usually made circular in section and the pendulum allowed to take up its own position with respect to the vertical, but Riefler provides for the same end by passing two small cone-ended screws through the cross head upon the points of which the pendulum rests. The clock is adjusted for beat by turning these screws. A pendulum, by King, whose clock is further described, *Fig.* 4 (*b*), presents features worth remark. A plate is fixed at the bottom of the bob, through which a screw bolt passes which is fixed to the end of the inner rod. Coarse adjustment for rate is made by turning the bob, and the expansion of the bob upwards more than suffices to compensate the expansion of the rod (which it appears in this specimen is sensibly zero). The head is T-shaped, 2½ inches broad by 3¾ long, and through its shaft the upper end of the inner rod passes freely. A number of holes pierce the shaft of the T and also the inner rod, and a single pin passed through one of them supports the pendulum while leaving it free to find its direction vertically. The springs support the T; they are 2 inches apart and 1½ inches long. They are not pinned to the T or to the fixed cock, but bear upon them with rounded cheeks. By shifting the supporting pin to different holes we have a means of

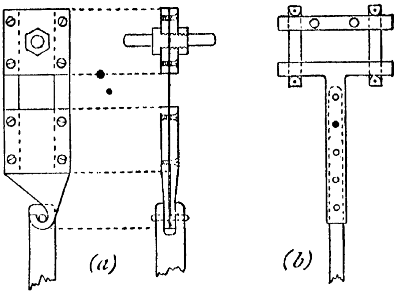


FIG. 4.

altering at will the temperature compensation without otherwise altering the length of the pendulum.

I add here the necessary formulae bearing upon some of the cases referred to above.

§ (4) PERIOD OF FREE PENDULUM.—If $2\pi/n$ is the period, and l the length of the simple equivalent pendulum, so that $n=(l/g)^{\frac{1}{2}}$,[1] the length l is dependent on the variations of g. For the seconds pendulum the period is 2 seconds, so that $l=g/\pi^2$. Helmert's formula for g,

$$g=978{\cdot}03+5{\cdot}18\sin^2\phi-{\cdot}00031\mathrm{H}\ \text{cm./sec.}^2,$$

where ϕ is the latitude and H the altitude in metres above sea-level, agrees with observation generally within about ± 0·02 cm/sec.², subject to some anomalous cases of excess. The corresponding formula for l is

$$l=99{\cdot}095+0{\cdot}524\sin^2\phi-{\cdot}000031\mathrm{H}\ \text{cm};$$

Reducing to a table, as regards the latitude correction we have

ϕ.	l.	ϕ.	l.
	cm.		cm.
0°	99·095	50°	99·402
10	·111	60	·488
20	·156	70	·558
30	·226	80	·603
40	·311	90	·619

The derivation for the formula for n for free oscillations *in vacuo* of a pendulum supported by a spring of given strength is given in *Proc. R. Soc. Edin.*, 1918, xxxviii. 85; very approximately it runs

$$\{(d+\tfrac{1}{2}s)^2+k^2\}n^2=g\{d+s\lambda^{-1}\coth\lambda\}$$

where s is the length of the spring, d the distance from its point of attachment to the centroid of the pendulum, k the radius of gyration about the centroid, and λ depends on the spring, viz.

$$\lambda=s\sqrt{(\mathrm{M}g/\mathrm{E})},\quad \mathrm{E}=\tfrac{1}{12}qbc^3,$$

where b, c are the breadth and thickness of the spring, and q, Young's modulus for steel, say $q=2{\cdot}0\times10^{12}$ dynes/cm.² λ is large for a weak spring and diminishes for a strong one. In the usual case, for a spring which is short and comparatively weak, we may take

$$l=d+s(1-\lambda^{-1}\coth\lambda)+\frac{k^2}{d}.$$

With specimen pendulums in common use, λ may be found to have values between, say, 1 and 2, and consequently the coefficient of s to range from about −·32 to +·48. Geometrically most of the curvature of the spring takes place close to the upper jambs, the greater part of its length is nearly flat, and there is a second increase of curvature close to the attachment to the rod, due chiefly to the gyrational inertia of the bob. Under maintenance, the behaviour of the spring is different. For example, if an impulse is given in the vertical position, and above the centroid, the initial form of the spring will contain an inflexion. Such a difference will make the maintained rate of the pendulum different from the free rate.

The oscillations of a pendulum are only isochronous for indefinitely small arcs. Taking the case of the simple pendulum executing vibrations of semi-amplitude α, the equation of motion is

$$l\left(\frac{d\theta}{dt}\right)^2=2g(\cos\theta-\cos\alpha),$$

when the time taken, the position (θ), and the vertical position is

$$t=\tfrac{1}{2}\left(\frac{l}{g}\right)^{\frac{1}{2}}\int_0^\theta d\theta\left(\sin^2\frac{\alpha}{2}-\sin^2\frac{\theta}{2}\right)^{-\frac{1}{2}},$$

or writing $\sin\frac{\theta}{2}=\sin\frac{\alpha}{2}\sin\psi$, $l=\frac{g}{\pi^2}$,

we have

$$t=\frac{\psi}{\pi}\left[1+\frac{1^2}{2^2}\sin^2\frac{\alpha}{2}+\frac{1^2.3^2}{2^2.4^2}\sin^4\frac{\alpha}{2}+\ldots\right]$$

$$-\frac{\sin2\psi}{\pi}\left[\frac{1}{8}\sin^2\frac{\alpha}{2}+\frac{3}{32}\sin^4\frac{\alpha}{2}+\ldots\right]+\ldots$$

$\psi=\pi$ gives the half period corresponding to the full excursion; $t-1$ increases with α; multiplied by 86,400 we have the losing daily rate of a simple seconds pendulum swinging with semi-arc α. This is the "Circular Error." The effect upon time-keeping, owing to this cause and arising from a change from one arc to another, is found by the difference of the rates assigned for the two arcs. The following table is extracted from *Proc. R. Soc. Edin.*, 1918, xxxviii. 169.

α.	Daily Rate.		α.	Daily Rate.	
	secs.			secs.	
					1417
0′	·000		160′	11·699	
		46			1508
10	·046		170	13·207	
		137			1599
20	·183		180	14·806	
		228			1692
30	·411		190	16·498	
		320			1783
40	·731		200	18·281	
		411			1874
50	1·142		210	20·155	
		503			1965
60	1·645		220	22·120	
		594			2057
70	2·239		230	24·177	
		685			2150
80	2·924		240	26·327	
		778			2241
90	3·702		250	28·568	
		868			2331
100	4·570		260	30·899	
		959			2423
110	5·529		270	33·322	
		1051			2515
120	6·580		280	35·837	
		1143			2606
130	7·723		290	38·443	
		1234			2697
140	8·957		300	41·140	
		1325			
150	10·282		..	..	

---

[1] See "Simple Harmonic Motion," Vol. I.

The change of daily rate for change of arc may also be taken from the formula

$$\Delta T=10^{-3}\times 0^{s}\cdot 914\times a\Delta a,$$

where a, Δa are expressed in minutes.

Under the heading of Observed Performance of Clocks, some examples will be given later of comparison of these numbers with observation. As a rule, search in the rates for exhibited circular error is unsuccessful. A number of causes may contribute to this. In the first place the variations of arc offered for examination are small and variable. If Airy's condition (below, § (7)) is not satisfied, the escapement contributes an arc-term to the rate which is merged in circular error. Finally there is the question how closely the equation of motion may be identified with the form

$$l\left(\frac{d\theta}{dt}\right)^2=2rg(\cos\theta-\cos a),$$

from which the theoretical circular error is derived. For ordinary arcs the difference between this and a right-hand member of the form $g(a^2-\theta^2)$ which would give strictly synchronous motion is excessively slight. Circular motion corresponds to a lift of the bob equal to $l(1-\cos\theta)=\frac{1}{2}l\theta^2-\frac{1}{24}l\theta^4\ldots$ For a semi-arc of 100′, in a seconds pendulum, these two terms amount respectively to 0·42 mm and $0{\cdot}30\cdot10^{-4}$ mm. The presence or absence of this latter term will affect the rate by 4·57 sec. per day.

It is very desirable that the knife-edge suspension of a pendulum, so much simpler geometrically than the spring, should be investigated more fully. Riefler's construction shows that it may be used successfully in the finest clocks. By giving the edges and the bed on which they roll specific curvatures any desired coefficient can be given to the term in θ^4. But the effect to be produced is so excessively small that the best results would probably be got by directing attention to the production of true uniform surfaces for the bed and edges, and then keeping the arc of oscillation nearly constant, without attempting to give specified curvatures, defined in advance.

As regards the compensation of the length against expansion by rise of temperature, the condition is that the length of the equivalent simple pendulum should be unchanged, viz.

$$l=d+s\mu+\frac{k^2}{d},$$

where μ stands for the factor $1-\lambda^{-1}\coth\lambda$.

In this expression d is the distance of the centroid below the point of attachment of the spring; it will consist of a number of length terms, each multiplied by a coefficient representing a fixed factor of mass. Hence if d is made up of a number of lengths d_x, and m_x stands for the associated mass-factor, while Δd_x . . . represents the linear expansion per 1° C., the condition to be satisfied is

$$0=\Sigma m_x d_x \Delta d_x\left(1-\frac{k^2}{d^2}\right)+\mu s\Delta s+\frac{2k\Delta k}{d}.$$

In this expression those terms which represent masses supported from below, as rising collars of brass or zinc, columns of mercury, etc., must be reckoned with a negative coefficient. There are substantial uncertainties attaching to this calculation; the possible causes for which will have been gathered from the foregoing pages. As a result, some of the finest clocks are not found to hold their rate under changes of temperature. An approximate calculation, together with a construction which allows adjustment by trial after the clock is going, would produce more reliable results than are at present shown.

§ (5) ADJUSTMENT OF RATE OF THE PENDULUM.—There are three means available for adjusting the rate: (1) by threading the extremity of the rod with a screw, upon which the bob may be turned in order to raise or depress it; (2) by adding weights upon a tray fastened usually half-way up the length of the rod, and (3) by altering the barometric pressure within the sealed clock case, as described in the next section. There are also magnetic methods that will be dealt with under the heading of Controlled Clocks.

With regard to (1) from the period equation

$$T=\frac{\pi}{n}=\pi\sqrt{(l/g)}$$

we have

$$\frac{\delta T}{T}=\frac{1}{2}\frac{\delta l}{l}.$$

Hence the change in length which produces a change of rate of 1 sec. per day is $l/43200=$ 0·023 mm., or say ·001 inch, a convenient number to remember. As regards (2), if M is the mass of the pendulum, and a small mass ΔM is added at a distance h below the point of suspension, variation of the formula for l on p. 207 above gives

$$\delta l=\frac{\Delta M}{M}h\left[\frac{h}{d}-1-\frac{k^2}{d^2}\right].$$

The maximum value of the factor of $-\Delta M/M$ is given by $h=\frac{1}{2}d(1+k^2/d^2)$, and is $\frac{1}{4}d(1+k^2/d^2)^2$. The tray is accordingly usually placed about half-way down the rod; but the rule is unimportant; thus $h=\frac{1}{4}d$, for example, will only reduce the factor to about $3/16d$.

For a pendulum of 7 kg., a gram added on a tray half-way down the rod gives a gain of rate of 1·55 sec. per day.

§ (6) AIR RESISTANCE AND BUOYANCY.—The presence of air about the pendulum declares itself in two features: it is the principal cause of the decay of motion, and when the air pressure varies, there is a change in the rate of the clock, known as the barometric equation. Taking the latter first, theoretical researches have, as so often in physical questions, proved of greater interest mathematically than practically by pointing to a field that was worth exploring. They do, however, give a rough indication of what effect to look for, and the numerical results actually experienced are not outside what may be derived from reasonable qualification of the strict data. If ρ is the density of the bob, and σ the density of air, buoyancy would have the effect of diminishing the effective weight of the bob in the ratio $1 - \sigma/\rho$. At the same time, if the motion is taken as parallel to that of a solid though a frictionless incompressible fluid moving irrotationally, the presence of the fluid would increase the inertia of the bob in the ratio $1 + k\sigma/\rho$, where the factor k ranges from the value $\frac{1}{2}$ for a spherical bob to 1 for an elongated form approaching that of a thin rod. Taking both factors together gravity would be effectively diminished in the ratio $1 - (k+1)\sigma/\rho$. Having regard to the period equation, $2\mathrm{T} = 2\pi/n = 2\pi\sqrt{(l/g)}$, we have $\delta\mathrm{T}/\mathrm{T} = -\frac{1}{2}\delta g/g = +\frac{1}{2}(k+1)\,\Delta\sigma/\rho$, where $\Delta\sigma$ represents the variation in density of the air. Hence if the clock is running true with the barometer at 30 inches, and ρ is taken at the density of lead and k at its least value, $\frac{1}{2}$, a rise of barometer produces a losing rate per day of 0·26 sec. per 1 inch, or 0·010 sec. per 1 mm. This is for a pendulum moving in the open or in a very large container; but in fact there is often quite a small clearance between the case of the clock and the bob in motion; the air currents set up, which can be traced by hanging pieces of gold leaf within the case, are of a regular reciprocating kind and must modify the result numerically. It is easy to make some estimate of the degree in which they modify it by considering one sphere executing small oscillations within an outer fixed sphere. The result might add one-fiftieth to the value assigned above. Much more material is the interpretation of the "density of the bob." It may easily be supposed that as the figure is not in fact a smooth sphere but has at least the projection of the rod and usually others, it will carry with it a certain volume of air which must be added to its own volume in assigning its effective density. The inference is that the figure above will be a minimum, and the actual value must be ascertained by experiment in each case. It will be seen from the numerical particulars given below (§ (19)) that the equation determined experimentally may be as much as 0·72 sec. per 1 inch. But in assigning it to its cause there are further considerations to be applied. An increase of pressure is followed by a diminution of the arc of swing. The amount of diminution depends upon the construction of the case, the air pressure within it, and character of the maintenance, but may be taken generally at about 30″ in semi-arc for 1 inch of the barometer. This variation of arc will show itself in the rate, through the arc-rate equation dealt with below. So far as the arc-rate equation consists of the calculated circular error, this would tell in a sense contrary to the barometric equation, making the clock show a gaining rate with the decreased arc due to an increased pressure, and so far annulling the barometric equation. But there is again the question of how far the motion really follows circular motion, and further what is the amount and sign of the escapement error which combines with the circular error to give the entire arc-rate equation. All these points require further study.

When the clock is enclosed in an air-tight case the barometric equation supplies the method generally used for altering the rate of the clock at will. If the pressure within the case is kept normally a few inches below atmospheric pressure, a correction of rate of +0·5 sec. per day is easily available in a form that ensures no disturbance of the pendulum when applying it.

If the clock case is not kept air-tight the barometric equation may be corrected by an attachment proposed by Robinson of Armagh (*Mem. R.A.S.*, 1831, v. 125), in which a mercurial barometer of suitable bore is fastened to the rod of the pendulum (see *Fig.* 5). Increasing pressure diminishing effective gravity in the bob induces a losing rate, but at the same time by transferring mercury from the bottom to the top of the barometer shortens the effective length of the pendulum, and the two causes can be made to cancel one another. A barometer tube of 1·5 mm. diameter would, for a rise of 1 mm. of the barometer, effect a transfer of ·012 gr. to the top, say at a distance $\frac{1}{4}$ of the length of the rod from the point of suspension, and this would correct a barometric error of 0·010 sec. per mm. for a pendulum of 10 kg. It could be adjusted to an exact balance with the observed barometric equation by raising or lowering the point of attachment to the rod.

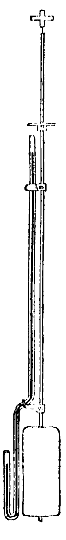


FIG. 5.

In respect to the frictional effect of air in reducing the motion of the pendulum, existing theory is of little service. The equations which give, say, the steady motion of a sphere through a viscous fluid depend upon assumptions that the inertia terms are negligible in comparison with the frictional terms, and this requires that either the size of the sphere or the velocities considered should be extremely minute. If the resistance proportionate to its velocity thus offered to a sphere of 10 cm. diameter is calculated, it would indicate a coefficient, corresponding to κ/n used below (§ (7)), of about one-fiftieth of that experimentally found present.

In default of a clear guide as to a theoretical form around which observed results may be shaped, it is necessary to build up an empirical equation from observations. In actual clocks the pendulum is subject to loss of energy not solely by air resistance but also by the work of releasing the escapement. Experiments show that for the best escapements the energy robbed from the pendulum for this purpose may be about the tenth part of that required to overcome air resistance under ordinary circumstances of atmospheric pressure and with an arc of about 3°. Leaving this part aside, the resistance to a free pendulum may be examined by the rate of decay of its arc. To obtain good determinations some precautions are wanted. First, it may be said that no consistent results will be got without closing the case of the clock and protecting from all air currents except those which the pendulum itself sets up. The reading is made with a reading microscope carrying an eyepiece scale and viewing a scale, divided say to 10′, carried by the pendulum. The carried scale is not visible while in motion but comes into view for an instant at the end of each swing. With a little practice the readings can be made exact to as small a quantity as 2″. It is then found that the logarithmic rate of decrease of energy per second may be written

$$-\frac{\delta E}{E}=\kappa=P+Q\left(\frac{a'}{100'}\right)^2,$$

where P and Q are quantities which vary with the barometric pressure, and a is the semi-arc, in minutes, executed by the pendulum. For a pendulum with a bob 7 kg. in mass and cylindrical form, 12 cm. high and 9 cm. diam., swinging in a cylindrical case about 30 cm. diameter, the values may be given

$$10^4\times P=\cdot 75+\cdot 63\times(\text{barom./30 inches})$$

$$10^4\times Q=\quad+\cdot 27\times(\quad\textit{ibid.}\quad).$$

In consequence, a semi-arc of 100′ will diminish to about 75′ in an hour. For a more massive pendulum the value of P may be found substantially diminished, but will always contain a term which is independent of the air density.

The unit of time is 1 sec., so that $\pi/n=1$. Thus, for example, for different semi-arcs for this pendulum

a.	$\kappa/n\times 10^4$	
	const. term.	bar. term.
0′	·24	·20
50′	·24	·23
100′	·24	·29

§ (7) THE ESCAPEMENT AND MAINTENANCE.—The first function of the pendulum is to permit the periodical escape of the counting train, and its subsequent locking after advancing one step; and associated with this is the function of the train, as maintenance, to give an impulse to the pendulum that shall restore the motion it has lost between two releases. In a modern precision clock the counting function of the train is quite subsidiary; it should be quite detached from the action on the pendulum; it may indeed be performed by means of an external slave clock controlled by the master clock through an electric signal. We shall therefore treat the matter primarily from the point of view of maintenance. In describing below a certain number of forms of maintenance actually in use, criticism of them will be more intelligible if we first examine theoretically the conditions for correct maintenance.

These conditions are contained in a theorem of which the original form is due to Airy.[1] If $x=a\cos(n't+\epsilon)$ represents a steady (maintained) oscillation of the pendulum, then any irregularity of the maintenance may produce changes Δa in the semi-arc, $\Delta\epsilon$ in the epoch or phase, supposing n' prescribed by the construction. A change in the phase is a change in the time-keeping, since it affects the moment at which the pendulum reaches a prescribed position, say the vertical position. A change in the arc will affect the time-keeping indirectly through the rate, by the circular error or whatever function of the arc replaces this in any actual case. Then writing x as a solution of the equation

$$\ddot{x}+\kappa\dot{x}+n^2x=R,$$

where R stands for the maintaining force divided by a mass-coefficient, the theorem in question runs

$$\Delta a=\frac{1}{n'}\int\Delta R\cos\tau\,dt,$$

$$\Delta\epsilon=-\frac{1}{n'}\int a^{-1}\Delta R\sin\tau\,dt,\quad[\tau=n't+\epsilon].$$

The condition for fixed epoch, $\Delta\epsilon=0$, is usually compressed into the statement that if the impulse is made at the bottom of the swing, *i.e.* at the instant when $\tau=0$, any variation of it represented by ΔR will produce no effect on the time-keeping. But this is an imperfect version of the fact. The impulse may vary in any way on either side of the bottom of the swing, provided only that it is symmetrically distributed. Failing such symmetry each impulse produces a change of epoch, and the time-keeping will vary if the intensity of the impulse varies. An immediate consequence is very important. In nearly all clocks (before Riefler's construction) the pendulum is connected to the train through a crutch. The crutch is a rocking piece carrying on one side the anchor or escapement proper, and on the other communicating with the pendulum, usually by means of a fork which encloses the pendulum and is moved to and fro by it, and in return conveys the impulse of the train to it. It is not clear of the pendulum at any part of the swing. It constitutes, therefore, a second pendulum of an entirely different natural period. Under no circumstances could the pendulum proper be free at the end of its swing, because it must either carry the crutch forward to that point or hold it back from going beyond it. Any jar, even the residual vibration shown by analysis of the vibrations of the ground and walls, would be conveyed to the pendulum through the crutch at the point of the swing where it could affect the time-keeping.[1] There can be little doubt that the abolition of the crutch is the most important element in the time-keeping of Riefler's clocks.

---

[1] *Camb. Phil. Soc.*, 1827, iii. 105.

The actual maintenance will as a rule consist of, first, a loss of energy by the pendulum while it unlocks the maintenance, followed by an impulse which may be variable in intensity and may last for a finite part of the swing. It is periodic and discontinuous, and therefore its natural expression will in all cases be in terms of a Fourier series, of which the fundamental period is $2T = 2\pi/n'$, say two seconds. Of such a series the only terms that matter are the first, say $A \cos n't + B \sin n't$, if we take $\epsilon_0 = 0$ to correspond to the vertical position; it is nearly obvious that these will far outweigh in effect their overtones, and it may be verified strictly that such is the case. Airy's condition is simply, $B = 0$. For any given form of maintenance A, B are immediately determined by the equations

$$\frac{\pi}{2}A = n'\int_0^T R \cos \tau dt, \quad \frac{\pi}{2}B = n'\int_0^T R \sin \tau dt;$$

but in many cases it will suffice to make a graphical analysis, by plotting the discontinuous impulse and running a sine curve through it by eye, when the value of B may be picked up by identifying the result with the form

$$(A^2 + B^2)^{\frac{1}{2}} \cos \left(n't - \tan^{-1}\frac{B}{A}\right).$$

An illustration will be given under the description of the Graham dead-beat escapement, below.

The maintained periodic motion of a pendulum presents one most essential difference from the cases of approximately synchronous motion met with elsewhere and leading to large amplitudes of vibration for small intensities of force. In the case of the clock pendulum the period of the impulse is strictly identical with the natural period of the pendulum. Some difficulty might be anticipated in solving the equations; in fact the case is very simple, but it differs in form from cases of mere syntony. Write

$$\ddot{x} + \kappa\dot{x} + n^2x = A \cos n't + B \sin n't,$$

where $2\pi/n'$ is the true period of the solution, then we have the exact equations

$$x = \frac{A}{\kappa n'} \sin (n't + \epsilon_0),$$

where $$\frac{n'}{n} = \left(1 + \frac{B^2}{A^2}\frac{\kappa^2}{n^2}\right)^{\frac{1}{2}} - \frac{1}{2}\frac{B\kappa}{An}.$$

It is often stated that the frictional forces in a clock affect the period of oscillation of the pendulum only to the second order. This is an inference from the expression for the complementary function $C exp(-\frac{1}{2}\kappa t) \cos (\nu t + a)$, where $\nu^2 = n^2 - \frac{1}{4}\kappa^2$. We see that this statement is erroneous. The value of κ/n will modify the period to the first order, unless $B = 0$. Any variation in the value of κ/n, as by thickening of the oil, or an unsymmetrical irregularity of whatever kind, will under the same circumstances show itself in the time-keeping. The condition $B = 0$ may be read as a specification for symmetrical distribution of the impulse about the lowest point, showing that if Airy's condition is violated, variations of other circumstances may find their way into the rate, affecting it in proportion to the first order of their magnitude.

I shall now proceed to a brief description of the most important actual escapements. It may be premised that these descriptions are not meant for craftsmen to work from. Such descriptions are given in more technical works to which reference may be made. The points to which the following remarks are addressed, are first, the intention of the appliances used, and secondly, a criticism of them as they affect the time-keeping.

§ (8) DEAD-BEAT ESCAPEMENT.—The going train of wheels ends in a scape wheel of 30 pointed teeth, undercut so that they bear only with their points. Attached to the crutch, and therefore rocking to and fro with the pendulum, is the anchor which can just permit the scape wheel to pass between its pallets, but in actual movement holds it up alternately on one side and the other. The anchor will enclose a space of, say, seven and a half teeth. The acting faces of the pallets are sloped at their extremities so that when the swing of the pendulum brings this point under the tooth of the scape wheel the tooth runs down this slope under the action of the train, giving the pendulum an impulse in doing so. Preceding the sloped face is the "dead" face of the pallet. This is curved, to a centre at the pivot of the anchor, so that while the tooth of the scape wheel is in contact with this face, sliding up or down with respect to it, no work is done either way apart from friction. The pieces are set out so that the middle of the impulse takes place in the vertical position of the pendulum. When the impulse is past the train is free from control and would "race" if the pallet on the other side were not now in position to restrain it. The scape-wheel tooth drops on this pallet, just beyond the sloped face, and is locked on the dead face. By the swing of the pendulum the dead face is carried past the motionless tooth in contact with it up to the end of the swing and back again, until finally the sloped face coming beneath the tooth the latter repeats the operation of impulse and escape. The scape wheel is made of brass, thin and well hardened by hammering; the anchor of steel and the pallets preferably of sapphire. The teeth of the scape wheel and the pallets are oiled. This escapement performs so well that though it must now be considered superseded for the finest work, only an insignificant proportion of the best clocks are at the present day fitted with any other. The theoretical features of the escapement may be clearly seen by making the graphical analysis of the forces called out by the method referred to in § (7). Lay out t along the axis, and denote by the points B, A, B′, A′ the lowest position, the complete excursion to the right, the return to the lowest position, and the complete half excursion to the left, respectively. Suppose the impulse lasts for 1° on either side of the lowest position, and the "supplementary arc" or angle performed after the impulse ceases is ½°. With these numbers the impulse extends for a time 0·23 sec. on either side of B or B′, and the pallet friction begins soon after the impulse is over and is in the opposite sense up to the position A ($t = 0{\cdot}50$ sec.), after which it is reversed and so lasts up to $t = 0{\cdot}77$ sec., when the new escape and impulse commence. Taking impulse and pallet friction separately and supposing each constant in magnitude, the diagrams will run as in *Fig.* 7.

---

[1] Airy himself was a bad offender in this respect. The crutch of the standard sidereal clock at Greenwich constructed for him by Dent, with his chronometer escapement, is particularly massive.

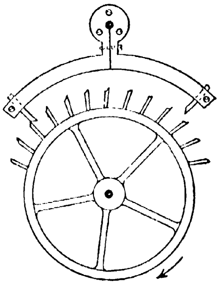


FIG. 6.

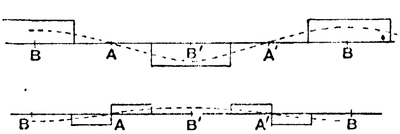


FIG. 7.

The dotted curves show the sine curves of period 2 sec., corresponding to the first harmonics, which best represent the given forces. It will be seen that the impulse may be completely symmetrical about the lowest point of swing, and the pallet friction only fail to be so in so far as the drop of the tooth does not come exactly on the point where the slope of the pallet begins. But again any irregularity of the friction, especially in the neighbourhood of A, A′ at the ends of swing, will shift sensibly the zero of the second curve, corresponding to a change of epoch.

The contact of the tooth with the pallet at the end of the swing, when the pendulum is most sensitive to disturbance of its epoch, is the weakest point of the construction, as of all constructions where the pendulum is linked with the traditional crutch and fork, which must always be made a little loose, and oiled, and is so liable to variation of action by thickening of the oil. A much improved form employed by Mr. E. T. Cottingham may be applied with advantage. The crutch is made slightly overweighted on one side and so bears constantly on one side of the pendulum, and the sole connection between them is a double-ended needle, which may act in jewel cups. This greatly diminishes, though it does not abolish, the disadvantages of the crutch.

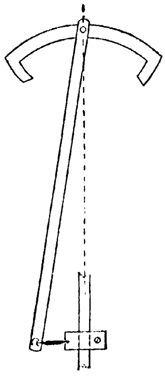


FIG. 8.

§ (9) GRAVITY ESCAPEMENTS.—Every weight-driven clock depends upon gravity, but the name gravity escapement is reserved for the class in which the maintenance of motion is effected by a small weight which is picked up by the pendulum in its swing, carried to the end of its arc and back again, being relinquished at a point lower than that at which it was picked up; the train resets the weight in its first position ready to be picked up again by the pendulum on its return. The great feature of this construction is that the impulse on the pendulum is independent of the force in the train, and therefore it is particularly well suited to turret clocks where the reserve of force must be large in order to drive the hands in all weathers, and hence would be liable, with the dead-beat Graham escapement, to cause the arc to vary widely. The most celebrated and best tested of the numerous designs is Beckett's (Lord Grimthorpe's) Double Three-Legged Gravity Escapement, as fitted to the great clock at Westminster. The impulse weights are two pendent brackets pivoted near the top of the suspension spring, on either side of it; each carries a locking stop for holding up the train, and a pallet for receiving the lift from the train in the form of a projecting arm. The scape wheel of the train is triple, consisting of two three-legged pieces and between them three lift pins. To guard against possibility of racing, a large fly is attached to its arbor. The pins are in the plane of the brackets, and the three-legged pieces outside them, in the planes of their respective stops. The figure shows the pendulum moving to the right and just about to take over the right-hand bracket which is in its upper position, as may be gathered from the position of the leg or tooth upon its stop. When this bracket is lifted the leg escapes, the pin lifts the left-hand bracket, and then the leg of the alternate piece locks upon the stop of this bracket, leaving it ready for release when the pendulum returns and makes its swing to the left.

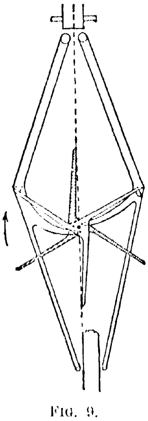


FIG. 9.

To adapt a gravity maintenance to high-precision time-keeping is an inviting problem that has often been approached with more or less complete success. Among designs actually put into operation may be mentioned Tiede II. in Berlin,[1] Gill's clock at the Cape Observatory, and E T. Cottingham's at Edinburgh Observatory.[2] The last is extraordinarily simple, containing only two moving parts besides the pendulum. The impulse is given once in two seconds. The stops for the crutch or gravity bracket are reset by an electro-magnet actuated by a contact made and broken by the pendulum when it picks up or drops the crutch. This feature requires careful treatment, but it possesses one remarkable advantage, viz., that the signals represent absolutely the motion of the pendulum and contain no adventitious element such as is inseparable from an indirect contact.

All these constructions are open to two objections. The weight of the impulse piece must be considerable in proportion to the energy it has to convey, since it is only the difference of its excursion upwards and downwards that tells; and as it must not be picked up near the end of the excursion it has to be done when the pendulum is moving with considerable speed, and therefore with an impulsive stroke that results in vibrations.

§ (10) THE CHRONOMETER ESCAPEMENT.—In the chronometer escapement, as finally developed for marine chronometers by Arnold and Earnshaw, the balance wheel carries a pin or tooth that displaces a detent which holds the train locked. As soon as the detent is displaced the train escapes and gives an impulse to the balance wheel. On the return of the balance the pin must pass the same point, with the detent in its locking position. This is effected by making the tip of detent easily flexible in one direction but stiff in the opposite direction; the detent being prolonged to a "horn," which is a light steel lever, against which, and projecting slightly beyond it, is fastened a weak spring made of a strip of gold. In one direction the pin can readily thrust this gold spring aside, in the contrary one it thrusts it against the "horn" and releases the detent.

The best-known application of this escapement to clockwork is the Sidereal Standard Clock at Greenwich, constructed by Dent in 1872, and up to the present day so employed. The plan is described in a crude diagram by Airy,[3] reproduced by Grimthorpe. The actual workmanship of the clock is extremely fine. The principle is exactly that of the chronometer, substituting for the balance wheel the to-and-fro motion of the crutch, an arm attached to which releases the detent and receives from the train an impulse on the pallet. A great advantage of a construction involving unilateral impulse once every two seconds is that it permits the adjustment of the moment of impulse to any chosen phase of the motion of the pendulum, and therefore the satisfaction of Airy's condition for no disturbance of the phase. There is also the absence of dead friction, but this is in large degree nullified by the use of the crutch, the objections to which have been given already. A neater form than Airy's for carrying out the same idea is shown by Grimthorpe,[4] who at the same time recommends discarding the crutch, if it were practicable. But a much better application of the chronometer escapement has been invented and constructed by Mr. W. F. King and is in operation at the Royal Observatory, Edinburgh (*Fig.* 10). The impulse is given at the foot of the pendulum, which allows condensation upon a much smaller are. It is given horizontally, upon a stone pallet, once every two seconds. This pallet with the releasing horn and its gold spring are attached to a "skate" carried at the end of the pendulum rod, and adjustable laterally to satisfy Airy's condition. The impulse wheel has 5 teeth; its force is conveyed to it not directly by the train, but by a hair-spring which connects it with the scape wheel—much on the principle of Harrison's maintenance for ordinary clocks. The scape wheel, also of 5 teeth, is driven by the train. Both impulse wheel and scape wheel are on the same vertical axis, the arbor of the impulse wheel being pivoted in a recess at the top of that of the scape wheel. The escapement operates as follows. The pendulum releases the detent of the impulse wheel; the latter strikes the pallet and proceeds to release the detent of the scape wheel, and then locks itself upon its own detent. The scape wheel moves forward one step and also locks itself. It can, if required, be made to operate an electric signal in doing so. The impulse wheel is set, say, one complete turn in advance of the scape wheel; when the pendulum releases it, it degrades $\frac{1}{5}$ turn, but the movement of the scape wheel that follows sets it up again. An impulse wheel as light almost as watch-work easily keeps a pendulum of about 14 kg. up to any ordinary arc of, say, 2° or 3°. The pendulum is absolutely free except for the brief period of passing, release, and impulse. In this form the construction seems very near theoretical perfection.

---

[1] Foerster, *Astron. Nachr.*, 1878, xci. No. 2182.

[2] *Proc. Roy. Soc. Edin.*, 1918, xxxviii. part i. p. 83.

[3] *Camb. Trans.*, 1827, iii. 105.

[4] *Clocks, Watches, Bells*, 8th ed., 1903, 106.

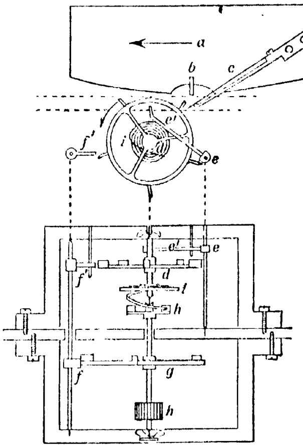


FIG. 10.

§ (11) CUNNYNGHAME'S ESCAPEMENT.—The escapement (*Fig.* 11) devised and made by Sir Henry Cunnynghame represents the weight-driven clock reduced to its limiting simplicity. The impulse is given once in 2 sec., a passing spring, as in the chronometer escapement, opening a small click which then lets fall a pivoted impulse arm which gives an impulse to a roller attached either to a crutch or directly to the pendulum. When the impulse is over the arm falls further and closes the circuit of an electromagnet which resets it upon its click in readiness for the next release. The set of the escapement laterally with regard to the vertical position of the pendulum allows of satisfying Airy's condition.

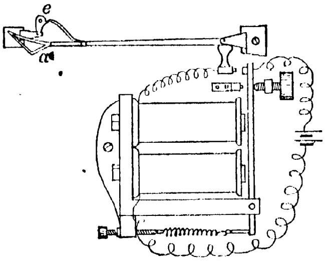


FIG. 11.

§ (12) THE SYNCHRONOME ESCAPEMENT.—This escapement (*Fig.* 12) described by Mr. W. H. Shortt (Patent 9527, 1915) carries out a variation of the intention of Cunnynghame's construction, with so many individual features as to constitute a different type. The pendulum is connected by a link with the crutch, which carries, first, a jewel which effects the release, and below it a small roller upon which the

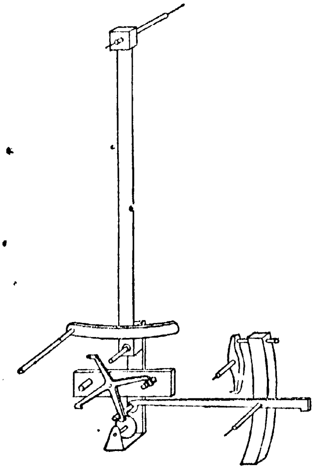


FIG. 12.

impulse is given. The stone strikes to one side or the other an X-shaped piece, releasing from its lower extremity the impulse arm which rested upon it. When released, this arm descends upon the roller and conveys the impulse as the roller is carried away by the pendulum. The impulse arm then moves on and releases from its click a second heavy arm, which falls and, in doing so, resets the impulse arm on the X in readiness for the next release. The heavy arm now moves on and makes an electric contact, which in turn sets it back upon its click and also serves as means for actuating a counting dial or slave clock, or working a relay for operating a chronograph. The impulse is conveyed to the pendulum without any vibration, and variations of arc are in some degree self-compensating since a lower arc and slower movement of the pendulum keeps the roller under the impulse piece for a larger part of its fall. Airy's condition is, however, not satisfied. The use of the link and crutch are open to the same objection as elsewhere, and an improvement can be made by adopting the connection by means of a double-ended needle referred to above (under Graham's escapement), or even by abolishing the crutch altogether and giving the impulses a little off the axis of the pendulum.

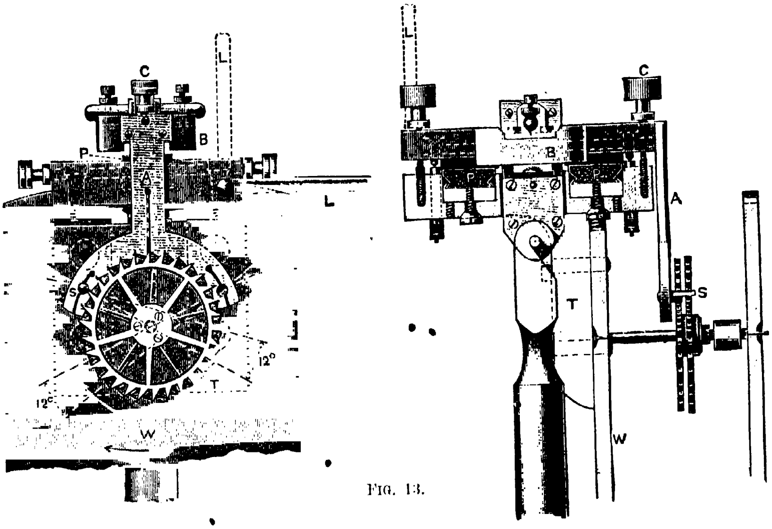


FIG. 13.

§ (13) RIEFLER'S ESCAPEMENT.—This is the boldest and most original of all the devices, as it is so far the one from which the best results have been obtained; but it is so peculiar that one cannot but feel it will ultimately be superseded by something simpler. The pendulum is hung by a spring to a cross head as usual; the cross head rests upon a cock which is itself not fixed but is pivoted upon knife edges, the line of which is supposed to follow closely the axis in the spring, about which the pendulum would turn. The same cock carries the anchor. The train locks the anchor in its vertical position, and when this is the case the pendulum turns upon its spring, suspension being entirely free at the ends of its swing; as it returns from its completed excursion to the vertical position, the spring becomes straight and ceases to hold the pallet of the anchor up against the scape-wheel tooth which was locked upon it; the escape takes place, the pendulum and anchor with the cock rocking upon the knife edges with the spring straight until the pallet on the opposite side is brought against the impulse slope of the scape wheel. The latter has also started to move forward under the weight on the train; it engages the pallet and forces the anchor back to the vertical position and locks upon it. In doing so it bends the spring, and this supplies the energy required for maintenance. The cycle then repeats itself. The weight is a small mass pivoted on the axis of the Third Wheel (next the scape wheel) and engaging a ratchet wheel parallel to this wheel. Every thirty-eight or forty seconds it is thrown upwards to a fresh position on this ratchet by the operation of an electromagnet.

The use of knife edges, generally discarded by other experience, is a surprising feature of this construction, as are also the movement of the actual cock on which the pendulum rests and the change over in the course of the swing from pivoting upon knife edges to turning by bending the suspension spring. Their success in use would seem to say that much the most important condition for good time-keeping is the complete freedom of the pendulum at the ends of its excursion. But it should be noted too that the energy is conveyed without any shock or vibration to the pendulum. Riefler retains train and dial, but their work is almost completely detached from the action on the pendulum.

§ (14) RECORDING TIME.—The pendulum, maintained in motion, divides the day into 86,400 parts. These parts require, on the one side, to be counted off as whole seconds, minutes, and hours, and, on the other side, the second requires to be subdivided to any desired degree. The series of toothed wheels which effect the counting is known as the train; it is customarily used at the same time for conveying the impulse to the pendulum through the scape wheel, and the locking of the latter after each escape guarantees that the train advances only one step for each swing of the pendulum. Or, again, if the clock is arranged so that an electric current is sent at each beat of the pendulum, the counting may be effected apart from the maintenance by making these currents actuate the escape of a sub-train or dial; or, again, they may control the escapes of a slave clock otherwise driven.

Consider first the simple train. The traditional construction (for a weight-driven clock) consists of four wheels, besides a subsidiary attachment for counting the hours. The magnification of motion which can be conveniently attained with wheel-work is, say, about 8 times. Thus if the scape wheel turns in 1 minute, the Third Wheel next to it turns in 7½ minutes, the Centre Wheel in 60 minutes, while the Great Wheel, upon which the weight acts, may turn in, say, 12 hours, but its period is immaterial. Then in front of the plate and behind the dial two more pairs are wanted to count the 24 (or 12) hours. The first may reduce the motion 6 (or 3) times and the second 4 times more; all this part is friction-tight upon the clock movement only, to allow of setting without disturbing the movement. Or the gear may be dispensed with for a 12-hour clock and the hour hand driven directly off the Great Wheel, recalling that in standard clocks it is usual to have the hour hand and minute hand upon separate axes.

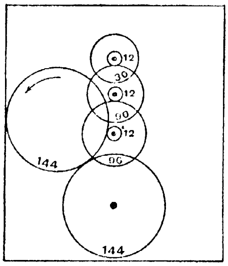


FIG. 14.

The weight is supported by a silk cord or gut, which is hung from a fixed point and doubled through a pulley, then passed directly, or over an idle drum, to the drum of the Great Wheel. The latter does not connect with the train directly but through a spring, which it keeps in a state of compression. This spring bears upon the spokes of the toothed wheel which is on the same arbor as the drum. The toothed wheel is united with a ratchet wheel, which takes off from a click mounted on the clock-plate. When the weight is removed from action during the process of winding, the ratchet wheel prevents the spring from becoming relaxed, and the movement is kept going. This is Harrison's maintenance.

The number of teeth in the train depends upon the number selected for the pinions. If these are, say, 12, the numbers would run as in the figure. If the leaves of the pinions are 8 or 16, proportionate changes in the number of teeth of the wheels must be made. As a rule the finest work would have pinions of 16, though not invariably, Riefler employing 12.

Consider the above construction from the point of view of time-keeping. Apart from the escapement, the errors of which have been considered above, the demand on the train is that the force in the maintenance should be constant. The construction shows the force on the teeth of the scape wheel about one thousand times reduced from the acting weight, putting friction aside and supposing the scape wheel about the same diameter as the drum. But in fact the friction of the gear, and that of starting the whole train from rest each second, consume nearly the whole of the energy derived from the weight, as will be seen by a short calculation. The energy of the pendulum being $Mgl(1-\cos a)$, this gives for a pendulum of 7 kg. $2{\cdot}3\times10^5$ ergs $=1{\cdot}7\times10^{-2}$ ft.-pds., for an arc $2a=3°$. When the pendulum swings freely unmaintained the arc would fall to about ¾ of its original value in an hour, or about one-half the energy would be lost, so that the maintenance must supply about 10^5 ergs, or $0{\cdot}8\times10^{-2}$ ft.-pds. per hour. A fine clock might be driven by a weight of, say, 1 kg. falling through a metre in the course of a week, or 6×10^5 ergs $=4\times10^{-2}$ ft.-pds. per hour. But as a rule the weight is made considerably greater than this. So that stiffness in the cord, the pulley work, irregularities in the wheel cutting, stickiness in the oil, and dead friction in starting the train from rest at each escape consume at least nine-tenths of the force. Any irregularity in these factors will find its way to the scape wheel and maintenance, and will show itself either by increasing the arc, or in dead friction during locking. It is evident, therefore, that a train of wheels, driven from the slow-moving end, is a most undesirable feature in precision clocks. The counting train must be preserved in some form, but it may be entirely separated from the maintenance of the master pendulum. It should, however, be remarked that in the form in which Riefler's clocks preserve the counting train, it is open to no serious objection. The driving is off the Third Wheel, by a small weight which is made to spring up a ratchet, every 38 or 40 seconds, and the force conveyed to the pendulum is a geometrically measured bending of the suspension spring.

As remarked already, the large reserve of force which is necessary for turret clocks makes an essential for good going the use of an escapement, such as the gravity escapement, in which variations of force on the scape wheel cannot produce a change in the arc of the pendulum.

Another serious objection is the weight. As sometimes constructed it might be regarded as a second pendulum hanging from the same cock. When it reaches the level of the true pendulum, the period becomes the same, and sympathetic oscillations are sometimes excited which will modify the true period as indicated in the theory of the double pendulum. Moreover, its presence in the neighbourhood is liable to modify the air resistance.

For any subdivision of the second the pendulum itself is inapplicable and the ear is of little service. In spacing the interval between the sounds of two beats, clock-makers have the practice of counting, say, thirds of seconds, 123123 . . . or fourths, or otherwise, until they are satisfied that they are in agreement with the signals they are comparing. But most clock sounds are blunt, or double, and the highest practice will not reach what can be easily secured with a chronograph, provided the clock makes automatically an electric contact. When such a signal is sent out, as remarked above, the train and dial work may be dispensed with and the counting effected by control of a slave clock or otherwise.

With regard to such a contact several precautions require to be observed. In the first place, it must not foul if run continuously every second for several years, so that it may not be necessary to disturb or stop the clock prematurely in order to attend to it. This is effected, first, by keeping the current as small as possible, that is, enough only to work a relay or control. For such a purpose 15-20 milliamps. suffices and this should not be exceeded. In the second place, the current density should be reduced by making the areas of true contact amply large. Where suitable two large faces of hard carbon smoothed almost to a polish will be found to work well, but as a rule the best form is a platinum nose striking on a platinum face. It will hammer itself a suitable bed. If the bed is not fixed but held to its place by a spring which yields a little under the blow, there is a slight rubbing of the surfaces in contact which is an advantage, but this is not essential. Further the usual devices for suppressing the spark that occurs on break should be applied, namely a condenser bridging the gap and a non-inductive shunt as a byepass, either connected in parallel with the electromagnet or bridging the gap. One or other may be found sufficient but both are preferable. The condensers sold for telephone work, if 1 or 2 mfd., are suitable for the purpose, and the resistance of the shunt may be found from trial, but may generally be taken at, say, 5 times the resistance of the inductance.

When the contact is not required for continual use, but can be attended to on each occasion, mercury may be used. A platinum point carried at the tip of the pendulum passes at each swing through a bead of mercury. The difficulty of presenting a clean surface is met by the device shown. A little fresh mercury from the beaker is poured into the cup, the foul surface overflows and is replaced from below, and the overflow is caught in the receptacle beneath. A contact of this sort may be used, for example, with a heavy pendulum and no maintenance, to send 1 sec. signals, sensibly, for a couple of hours. In all questions of fouling contacts it will be remembered that it is the positive pole of the contact that suffers, becoming burned and pitted by the negative ions.

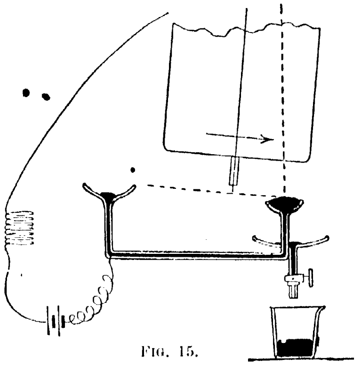


FIG. 15.

In actuating the contact from the movement the point requiring attention is to avoid prejudicing the time-keeping. This is attended to in various ways: (1) Contact directly from the pendulum has the great advantage that the signals sent out are truly spaced seconds, without any accidental and variable addition, such as the use of any secondary device will introduce. It entails, of course, an impulse on the pendulum, which must be regarded as variable, but this will not affect the time-keeping, if occurring in the vertical position. Such a case is that of the bead of mercury contact just described. A fitting suitable for permanent work is a platinum contact, as described above, actuated by a small roller, *e.g.* a watch balance, carried on the pendulum and striking each swing against a stone carried by the lower spring of the contact. The stroke throws this piece up clear of the wheel, and into contact with its opposing spring. A pair of set screws regulates a suitable position.

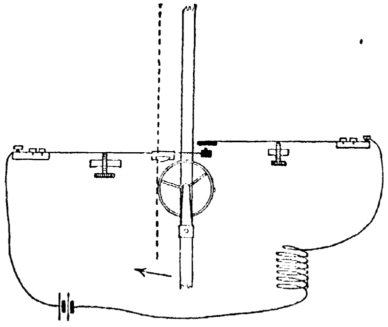


FIG. 16.

In Cottingham's clock the current was carried through the pendulum and out through the gravity arm which it picked up. Some difficulty was found in keeping the contact faces in good order. Otherwise from the point of view of time-keeping the arrangement is not open to criticism. In these forms the zeroes of the minutes must be marked by a supplementary device in the counting dial, a suitable form for which is given in § (14).

If the contact is not made directly through the pendulum the signals will differ from true seconds by accidental amounts representing the inequalities of working of the secondary appliance which is used. These amounts may run to about ±·003 sec., but generally not more. This is the case with the Synchronome clock, where the pendulum releases the impulse arm, the latter releases the remontoire, and the remontoire finally makes the contact which resets it and also serves as signal. It is also the case with Riefler's contact. Here a wheel of 30 teeth or slopes on the scape-wheel arbor —one of which is filed away to mark the zero—alternately lifts out of contact an arm carrying a face which rests against a nose, and lets it fall back into contact with the nose. The contact is on and off alternately for periods of about an entire second. In all the other forms the duration of the contact is brief, it may be about 1/20 second. A form of contact which may be fitted to a scape-wheel arbor is a tilt hammer form similar in principle to that described above for direct pendulum work, except that it is actuated by a wheel of 60 teeth (minus one) fixed upon the arbor. These teeth striking the stone make the contact in the same way as the roller already described. The set of their contact requires to be carefully adjusted and should be provided with a double slide for the purpose. The stroke should take place between the impulse and the fall of the scape-wheel tooth on the dead face of the pallet. During this interval (which is very short) the train and scape-wheel are completely detached from the pendulum, and the going could only be affected by the restraint the contact makes, in so far as it changes the point at which the tooth encounters the dead face.

In connection with contacts it may be remarked that a mercury and platinum contact which would spark actively and foul quickly if open will remain permanently clean if covered with paraffin, but this is usually not practicable for clock work.

When it is desired to mark the zeroes of the minutes by omitting one second-signal in sixty, a device that may usually be applied is the following. A light wheel of brass with flat rim is fixed on the scape-wheel arbor; one-sixtieth of the circumference is cut out and replaced by ivory or any non-conductor. The current is led through this wheel by a light trailer, and out again by a second trailer on the arbor. The advantage of this construction is that the friction required for its operation is uniform.

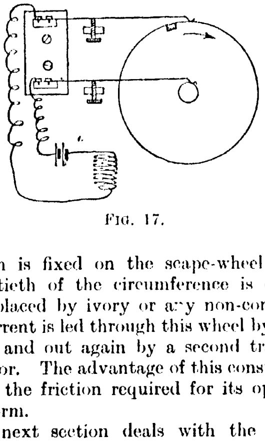


FIG. 17.

The next section deals with the chronographic systems of recording the signals provided upon a tape or otherwise, and a later one upon the use of the signals for actuating controlled clocks, whether mere dials or slave clocks.

§ (15) Chronographs.—Just as we introduce the pendulum in order to subdivide the day, so we must introduce another subsidiary mechanism in order to subdivide the second, and parallel questions will arise as to its reliability, though not so far to the same degree. Attempts to subdivide the second by the senses, without the use of a mechanism, are very unreliable. It is not clear what they depend upon. Certainly the ear is very easily misled by interfering sounds. Co-operation of two senses, as tapping seconds with the hand while following the beat of the clock either by eye or ear, are still less to be trusted, or, again, timing by a stop watch. Apart from the fact that the signal taken cannot be very brief or it will not be perceived at all, the senses are not co-ordinated strictly, nor is the lag between their operation constant, even in the same individual. Such methods should never be trusted to an accuracy beyond 0·1 sec., and they will show large occasional variations in excess of this.

To get the best results from sense perception, we note coincidences of two beats which overtake one another. When two sounds are sharp, single, and exactly like one another, a separation between them of as small a matter as 0·01 sec. can be perceived by ear. A slight thickening of the sound as they fall out of agreement is immediately sensible. This is the standard method of taking mean time. The error of the sidereal clock being taken from the stars, a coincidence of the beat of the sidereal clock and mean time clock is observed. When both clocks are going correctly this recurs every 365 (or 366) seconds. Both times must be noted, but this is easily done by noting how many seconds the sidereal dial is going to be behind the mean time dial at its next coincidence, and then attending merely to the sounds. When the coincidence is judged to occur, a glance at the one dial will then suffice to tell the coincident seconds on both. But difficulty arises from the beats being seldom identical and never single. If listened to closely, the three necessary elements of escape, impulse, and locking may usually be heard distinct from one another, or blurred together, and this confuses the perception. It will be found particularly difficult to compare one tick in a telephone receiver with another outside it, as is necessary if the clocks are not side by side. In all cases where it is possible to do so, reliance should be reserved for some self-recording mechanism.

There are numerous devices for giving very short intervals of time, familiar in photographic work, like the focal-plane shutter, in which no co-ordination with the clock and time-keepers is wanted. These are not considered here. What we are engaged upon is marking specific fractions of specific seconds, taken from the clock. For this purpose they fall under two heads: (1) Chronographs[1] giving records that may be read to 0·01 sec., but not beyond; (2) Microchronographs, for smaller intervals.

The former class works mechanically and requires to be robust, the moving parts possessing substantial inertia. Hence it cannot be operated directly by the small clock current (15 milliamps.) but requires the introduction of a relay. The best form for such a relay is a slave clock controlled by the primary clock, and carrying a contact maker on its pendulum, such as has been described above. When this is not available, the ordinary polarised relay of the Post Office type will be found to work steadily with less than this current and to give good results. The current required on the secondary circuit may be about 0·3 amp. The contacts will require occasional cleaning and should not be difficult to get at. It must be remembered that any type of relay introduces a possibly variable lag; the determination of lag will be considered later.

The principle of the chronograph is to measure time by the help of uniform motion, as in rotating a drum, or feeding out a fillet of paper. Every second the clock marks the paper, and therefore to read correctly the time of an intermediate mark, the going need only be smooth over that second; as a matter of experience, even when the driving is below its best, this standard is very easily attained to 0·01 sec. It is, moreover, easily verified.

In the drum chronograph the drum revolves once a minute, and concurrently the marking pens progress alongside it, by means of a screw, marking spirals of fixed pitch upon it. One of these pens marks the seconds, the end of the minute being shown by omission of a mark, and another records the signal which it is required to time. When the motion is regular the marks for given seconds in different minutes will lie along straight lines, and the counting of minutes is easily performed; moreover, the original sheets are convenient to preserve. More compact in construction is the fillet chronograph in which the marks are received upon a paper tape, which is fed out uniformly. It is also more convenient for occasional use, whereas a whole hour's record sheet must be detached at once from the drum; the run is verified and the seconds counted by passing the tape across a board marked by a taper scale of 60 lines, which slope towards one another to allow for fitting to the momentary rate of driving. The fillet is less suitable to preserve and consult than is a sheet from a drum. On either of these systems the actual marking of the seconds and signals may be done either by pens or by prickers. The actuating piece is an electromagnet, made live by the current from the relay or otherwise. When a pen is used, it may either trail continuously, being drawn aside sharply at the signal, or it may strike the paper and mark a dot. The dot is preferable for accuracy. A much sharper dot is made by employing a pricker in place of a pen, but the construction of the arm and head requires alteration, otherwise the pricker will hold up the run of the paper. A correct construction is shown in the figure: *a* is the arm moved by the signal; the needle *b* is carried by a piece *c*, pivoted at *d* in a way that allows no lateral motion, and provided with a tail or counterpoise *e* which normally holds the piece against a stop *f*; *g* is the drum which carries the paper fillet. With a pricking chronograph well constructed, coincidences of mean time and sidereal clock can be read usually with no uncertainty to 2 secs., that is, the time may be compared to 0·005 sec. The following represent an ordinary test of such a chronograph, to verify that it is in good order, by taking successive hourly coincidences of a solar clock (R) with two sidereal clocks (Σ_1, Σ_2). It will be remembered that the comparison requires a calculated conversion, the figures for which in the *Nautical Almanac* only run to 0·01 sec.

---

[1] See "Watches and Chronometers, Rating of."

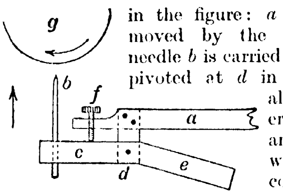


FIG. 18.

1920.		$\Sigma_1 - R$.			$\Sigma_2 - R$.		
		Observed.	Calculated.	Δ.	Observed.	Calculated.	Δ.
	hrs.	secs.	secs.		secs.	secs.	
June 14	19	20·88	20·87		31·99	31·98	
				3			3
,,	20	·84	·84		32·00	32·01	
				3			4
,,	21	·81	·81		·05	·05	
				3			4
,,	22	·78	·78		·09	·09	
				2			4
,,	23	·77	·76		·14	·13	
				3			3
June 15	0	·73	·73		·14	·16	
				3			3
,,	1	·70	·70		·19	·19	
				3			3
,,	2	·67	·67		·22	·22	
				3			3
,,	3	·64	·64		·25	·25	
				2			3
,,	4	20·63	20·62		32·28	32·28	

The column "calculated" represents a smooth run. Comparison of observed and calculated for $\Sigma_1 - R$ indicates no fault; that for $\Sigma_2 - R$ is less satisfactory, but points to irregularity in the clocks as much as in the chronograph, within the period of comparison. Chronographs have been constructed which print to 0·01 sec. the time of a signal. One of these, at Paris Observatory, is described in *Bulletin Astronomique*, xxii. 270. Another was designed by Professor G. W. Hough, of Dearborn Observatory. It contained two driving clocks, one marking minutes and seconds and moving forward step by step and as the clock signal was sent; the marking was made by figures engraved in relief upon two wheels over which a paper fillet and inking ribbon were passed. The third wheel, marked with the figures 00, 02, . . . 98 for the hundredths of seconds, alongside the two former, was driven independently and ran continuously, controlled first by a centrifugal governor set so as to always run a shade fast, and finally by a brake or check, opened each second by the clock signal and connected through a spring with the arbor of the going wheel. The signal to be recorded actuated a large electromagnet, which threw down three hammers that impressed the paper on the type wheels. The stroke was very rapid so as not to hold up the wheels. The instrument could be verified by making the clock print its own signal; the outcome being a small correction due to be applied to the readings of any other signal. In the case of a duplicate instrument used at Durham Observatory, these verifications showed completely satisfactory performance.

All methods with mechanical chronographs introduce lag, owing to their inertia, and are, moreover, limited in accuracy to about 0·01 sec. For finer work, and for the important necessity of measuring lag with precision, other devices must be employed.

One of these is the method of the time vernier, by which a free pendulum is loaded so as to gain, say, 1 beat in 50 upon each of two separate pendulums, the lag of whose beats upon one another it is desired to find. By taking repeated determinations of the coincidence of the vernier beats with each of the other two in succession, their separation can be found. But often there may be a great difference between the sounds, that will make any determination by ear to 0·01 sec. uncertain.

This method is employed very successfully in the W.T. "scientific" time signals sent out from the Eiffel Tower. The two beats to be compared are those of one's own clock with that of the Observatory of Paris, the correction of the latter being given. The wireless signals are the vernier gain about 1 in 50; the coincidences are taken by the observer for his own clock, and at Paris for the Paris clock; each coincidence alone allows only the determination of the respective clock time of (say) the beginning of the vernier signal; but as Paris immediately makes this calculation and issues the result by W.T., the observer's calculation by comparison will give his lag upon the Paris clock, as corrected for its adopted error and rate.

Two other devices for measuring the lag of two clock beats are described in a paper by Gen. Ferrié, *M.N.*, May 1920.

Unquestionably the best method for fine works, allowing also comparisons with a clock, which are free from uncertainty to 0·001 sec., is an adaptation of the Einthoven galvanometer, or oscillograph, to continuous time work.[1] The virtue of this instrument is that the faintest currents can be made to yield mechanical effects such as the rotation of a mirror, because the work called for is drawn from the strong superposed magnetic field. Moreover, the instrument, properly constructed, is absolutely dead beat and free from lag as between one signal and another. The original signals of the clocks may therefore be used; indeed it is possible to measure the inequalities of the signal seconds corresponding, say, in Riefler's clock, to inequalities in the teeth of the scape wheel, and to recognise the accidental uncertainties of signals not taken directly off the pendulum. Moreover, all lags may be measured without personality or other uncertainty by arranging a contact to be made just at the point which it is desired to record. For this class of work an absolutely reliable sub-process converting time into recorded movement is essential. Any mechanical drive, such as a motor with Helmholtz governor, cannot be trusted without verification. Such a motor is used for moving the photographic film or paper past the recording spot of light, but the sub-standard actually relied upon is a vibrating tongue or a tuning-fork. A tongue, beating 0·1 sec., may be run unmaintained and will continue to make good occultations of a slit for, say, two minutes with complete uniformity.

As a specimen of results obtained from the microchronograph the following may be quoted, relative to the lag of the tick produced by the pendulum of a controlled clock, upon the signals sent out by the master clock Riefler (*l.c.* p. 609). The signals from Riefler for individual seconds are unequally spaced and require an allowance which was separately determined:

No. M. Second.	Lag of Tick.	
	Uncorrected.	Corrected.
	secs.	secs.
1	+ ·147	+ ·147
3	·153	·146
5	·156	·145
7	·154	·145
9	·158	·146
11	·157	·146
13	·150	·147
15	·149	·147

The lag determined in this way on different days shows variations, some of which were deliberately produced by reducing the controlling current from 10 to 6 milliamps.

No.	Lag.	No.	Lag.	No.	Lag.
	secs.		secs.		secs.
A	+ ·168	F	+ ·160	K	+ ·155
B	·164	G	·154	L	·172 *
C	·160	H	·159	M	·146
D	·160	I	·193 *	N	·145
E	·157	J	·159	..	..

* Controlling current reduced.

It cannot be doubted that the audion valve has an important future in timing work and in maintenance, as in so many other regions. Its perfect response to stimulus, freedom from lag and power of amplification point to it as an ideal relay when certain difficulties are overcome. The chief of these is the reduction of the natural high period by introducing sufficient inductance and capacity into the circuit so as to make it synchronous, or nearly, with the slow movements it is convenient to employ for time measurement. M. H. Abraham has, however, illustrated it by an experiment so simple that he employed it in his lectures at the Sorbonne.

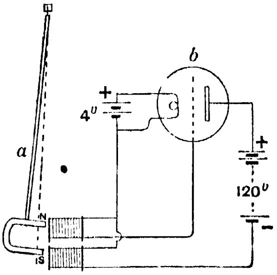


FIG. 19.

The pendulum, about 50 cm., carries a bob in the form of a horseshoe magnet, of about 300 gr.

---

[1] Cf. Sampson, *Monthly Notices R.A.S.*, June 1918.

The poles swing into the cores to two solenoids, wound to as high a resistance and as many turns as practicable, say several thousand ohms. One of these solenoids is in the grid circuit, and the other in the plate circuit. The former is excited by N pole of the magnet, and responds by exciting the circuit which comprises the other, which in turn acts upon the S pole, the whole constituting a *negative resistance*; the pendulum will gradually set itself in motion, and maintain itself energetically.

A very similar arrangement has been used by Mr. F. E. Smith for maintaining the vibrations of a tuning-fork. Ordinary methods of maintenance, as by the make and break employed, *e.g.* for electric bells, spoil the fork as an absolute time register, because the impulse acts towards the end of the excursions. If the poles of a large permanent magnet are shod with soft iron projections, directed towards one another and these wound as cores to solenoids, which are connected to the lamp circuit as indicated, a tuning-fork set vibrating between them can be maintained in motion which stands the most searching examination for regularity. With suitable inductances and capacities, frequencies as low as 30 per second are manageable.

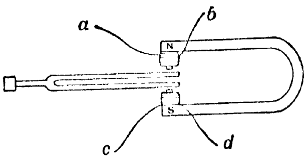


FIG. 20.

§ (16) THE ARC OF OSCILLATION.—The arc of oscillation plays a secondary part only in the time-keeping, but it is important enough to require express treatment, in addition to the numerous remarks already made regarding it.

Following the theorem of § (2) on the effect of an irregularity in the impulse upon semi-arc and epoch of oscillation:

$$\Delta a = \frac{1}{n'}\int \Delta R \cos \tau dt,$$

$$\Delta\epsilon = -\frac{1}{n'}\int a^{-1}\Delta R \sin \tau dt, \quad [\tau = n't + \epsilon_0],$$

we see that the conditions for constant arc and for fixity of epoch are independent and to some degree complementary, the latter requiring in ΔR symmetry about $\tau = 0$, and the former symmetry about $\tau = \pi/2$. It is therefore not possible to assign the going of the clock by examining the arc alone, since this could not reveal any change of epoch, but it requires to be studied because a change in arc will produce a change in rate owing to lack of perfect isochronism in the pendulum. It is, moreover, the only external feature of the going that is open to measurement.

The direct effect of variation of arc upon the time-keeping may be written

$$\Delta T = Ca\Delta a,$$

where C is a constant; if ΔT stands for seconds per day, and a, Δa are measured in minutes of arc, then as given in p. 208 above, for pure circular error, $C = 10^{-3} \times 0{\cdot}914$ sec. But this cannot be counted upon, and for each particular clock the coefficient C would require to be determined by experiment.

We notice that for a given change of arc, Δa, the variation of rate increases in proportion to a; but $a\Delta a$ varies as the increment of energy. Hence for a given increment of energy, the variation of rate is the same whether a is large or small. It is therefore not clear whether the time-keeping would be better for a small arc or a larger one.

Referring to the equation shown on p. 211, $x = A/\kappa n' \sin(n't + \epsilon_0)$, we see that $a = A/\kappa n'$, and the arc can vary either in proportion to variation of the impulse, or inversely as the frictional coefficient κ. The commonest cause for producing the latter is change of barometric pressure. When the barometer rises the semi-arc diminishes. For an arc of about 100′, experiments have shown, for Riefler's clock, $\Delta a = -3{\cdot}1''$ per mm. ($-77''$ per 1 inch), and for the Synchronome clock, $\Delta a = -1{\cdot}2''$ per mm. ($-31''$ per 1 inch). The difference is due to the escapement of the latter clock, which automatically compensates in part for a falling arc. These changes produce a change of rate through the circular error, or its representative $\Delta T = Ca\Delta a$, and thus, *pro tanto*, compensate the barometric equation; the observed change of rate for change of barometer is the residual when they are deducted. The observed values for the two clocks are $+{\cdot}41$ sec. per 1 inch, and $+{\cdot}37$ sec. per 1 inch. If we calculate from the theoretical circular error, the numbers above would give respectively ·12 sec. and ·04 sec., which would require to be added to what was observed, in order to get the pure effect of the barometer. It has been suggested that an arc could be chosen, so that the observed barometric effect was completely annulled by associated change in circular error; but the proposal is evidently illusory, since the required arc is too large.

Variation of arc may also affect the rate in another way. If the coefficient B of p. 211, which measures the phase by which Airy's condition fails to be fulfilled, is dependent upon the arc, as it may be, any variation in the latter will produce a change in B, and therefore in $2\pi/n'$, the period. But there are no observational data at present available on this point.

The traditional method of reading the arc by means of a fixed scale, with a pointer carried by the pendulum, is rough, though by using an eye glass and taking precautions against parallax it can be made good to about 0·3′. A reading microscope can be made to reach about ten times this precision, giving consistent readings to 2″. A suitable arrangement is shown in the sketch. The scale is carried on the pendulum, and its excursions are noted on the focal scale carried in the microscope. This microscope is placed by Riefler inside the case, but it is better outside it. No difficulty will be found in obtaining a sufficiently good image of the vertical lines of the pendulum scale given through the unprepared glass cylinders in which some clocks are encased, but of course a good plane window is better.

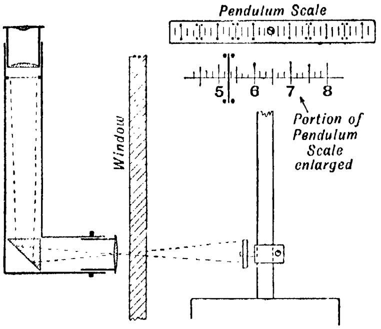


FIG. 21.

When read in this manner, the variation of semi-arc in the best clocks, kept under circumstances of constant pressure and temperature, will be found to reach a range of about ±15″ within a week, and exceptionally twice as much, the full semi-arc being taken at about 100′.

A convenient position for the scale may be 86 cm. from the point of suspension of the pendulum; a pendulum scale then divided at spaces of 2·5 mm. shows 10′ intervals. Taking unit magnification at the object-glass of the microscope, the eyepiece scale would show 1′ divisions when ruled to 0·25 mm., and, in proportion, less for closer ruling. Only the variations of arc are read in the microscope; coarse readings to 10′ or so are made by eye directly.

§ (17) THE DOUBLE PENDULUM.—The mutual influence of two pendulums upon one another was first observed by John Ellicott.[1] Two clocks, identical in pattern, with 23-lb. pendulums, and capable of moving with arc of 3° under a weight of 3 lb., were made to exhibit the now familiar phenomena of beats by setting the cases near one another upon a floor, with a light post just tight enough to support its own weight between them. When set going with normal arcs, the one arc would increase to as much as 5° and the other diminish until the movement ceased to escape, but would afterwards increase again at the expense of the first, and the going of the train would be resumed. By starting the two clocks with large arcs, both were kept going continuously for several days. Disconnected, No. 2 gained on No. 1, 1 min. 36 secs. per day. Connected, they moved together, No. 1 gaining 1 min. 17 secs. per day on its original rate, and No. 2 losing 19 secs.

It is evident that corresponding effects must show themselves, in proper degree, wherever two vibrating systems have an elastic connection. In particular, the rate of a pendulum will be dependent upon the stiffness and the mode of fixture of the cock that carries it. The effect of suspension, deliberately varied, upon the going of a chronometer, has been given with admirable clearness by Lord Kelvin (*Popular Lectures*, ii. p. 360). The discussion below follows nearly upon his lines.

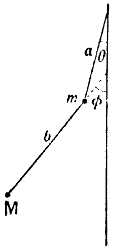


FIG. 22.

Take as model of the system a particle M suspended by a thread of length b from a particle m, which is suspended from a fixed point by a thread of length a. Let θ, ϕ be the inclinations of the threads a, b to the vertical; then for the execution of small vibrations the kinetic and potential energies, T, V, are given, say, by

$$2\mathrm{T} = \lambda\dot{\theta}^2 + 2\mu\dot{\theta}\dot{\phi} + \nu\dot{\phi}^2,$$

$$2\mathrm{V} = \pi\theta^2 \qquad\qquad + \sigma\phi^2,$$

where

$$\lambda = (m+\mathrm{M})a^2, \quad \mu = \mathrm{M}ab, \quad \nu = \mathrm{M}b^2,$$

$$\pi = (m+\mathrm{M})ag, \qquad\qquad \sigma = \mathrm{M}bg.$$

Here the special character of the model considered is expressed by the relation $\lambda/\mu = \pi/\sigma$, and if this relation is not used the conclusions will be quite general. The coefficient of $\theta\phi$ is absent from the expression for V owing to the choice of datum from which ϕ is measured.

Let $\theta = \mathrm{A}\sin pt$, $\phi = \mathrm{B}\sin pt$ be a principal mode of vibration; the characteristic of a principal mode is that the magnitudes of θ, ϕ bear a fixed ratio to one another throughout the motion. If A : B is positive, their coordinates θ, ϕ move in the same direction; if it is negative, their directions are opposite.

---

[1] *Phil. Trans.*, 1739, xli. 126-135.

Then we have the equations

$$A[-\lambda p^2 + \pi] - B\mu p^2 = 0,$$

$$-A\mu p^2 + B[-\nu p^2 + \sigma] = 0,$$

giving the two principal modes corresponding to the roots of

$$(\lambda p^2 - \pi)(\nu p^2 - \sigma) - \mu^2 p^4 = 0,$$

the associated ratio A : B for each mode following by substituting the value of p^2 in either of the equations above. Let

$$p_0{}^2 = \frac{\pi}{\lambda}, \quad p_1{}^2 = \frac{\nu}{\sigma},$$

so that p_0, p_1 are the frequencies of the separate systems in which (M, b) or (m, a) are respectively suppressed.

Then the equation for the frequencies p is

$$(p^2 - p_0{}^2)(p^2 - p_1{}^2) - \frac{p^4\mu^2}{\lambda\nu} = 0.$$

This shows that the two roots p^2 are *outside* the values $p_0{}^2$, $p_1{}^2$; the greater value of p^2 exceeding the greater of them, and the smaller value falling below the less.

For the greater value of p^2, corresponding to the more rapid mode of vibration, the ratio A : B is negative, and the two co-ordinates move in opposite senses; for the less value of p^2 they move in the same sense.

The quantity $\mu^2/\lambda\nu$ in the concrete example or model from which we started depends only on the masses and is equal to M/(M + m). It may be taken as ranging from zero up to unity according as we approach the limits when m and M respectively dominate the construction. The ordinary case of a pendulum suspended from stiff, massive cock would be parallel to M/m small, and $p_0{}^2$ large.

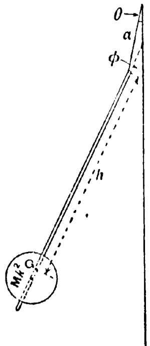


FIG. 23.

The case of a massive pendulum, whose rotational inertia must be considered, suspended by a thread or weak spring (a) attached at a distance h above its centroid, gives

$$\lambda = Ma^2, \quad \mu = Mah, \quad \nu = M(h^2 + k^2),$$

$$\pi = Mga, \qquad \sigma = Mgh,$$

so that

$$\frac{\mu^2}{\lambda\nu} = \frac{h^2}{h^2 + k^2},$$

which will approach the value 1.

The case discussed by Lord Kelvin, of a chronometer suspended with dial horizontal by a pair of threads of length l is given by

$$\lambda = MK^2 + mk^2, \quad \mu = mk^2, \quad \nu = mk^2,$$

$$\pi = \frac{(M + m)gr^2}{l}, \qquad \sigma = S,$$

in which MK², mk^2 represent respectively the moments of inertia of the chronometer and of its balance wheel, 2r is the distance between the points of suspension (the threads being vertical), and S depends upon the balance spring; the two co-ordinates are the angle of rotation of the chronometer, and of its balance relative to the chronometer, respectively.

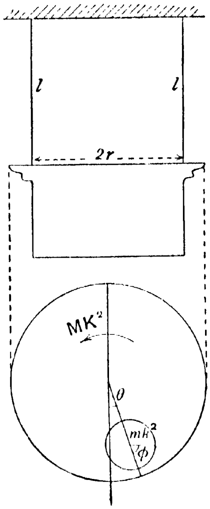


FIG. 24.

Here

$$\mu^2/\lambda\nu = mk^2/(MK^2 + mk^2),$$

and is a small quantity. Consider first the second case, with rotational inertia and a weak suspension. Here

$$\mu^2/\lambda\nu \doteq 1,$$

and the two roots are

$$p^2 \doteq \frac{p_0{}^2 + p_1{}^2}{1 - \mu^2/\lambda\nu}$$

and

$$p^2 \doteq \frac{p_0{}^2 p_1{}^2}{p_0{}^2 + p_1{}^2},$$

where

$$p_0{}^2 = \frac{a}{g}, \quad p_1{}^2 = \frac{h^2 + k^2}{hg}.$$

The first of these will represent a very rapid oscillation of rod and thread suspension in opposite senses; the second, giving $p^2 \doteq g/(a + h + k^2/h)$ will be recognised as the characteristic frequency of the pendulum.

Now take the case $\mu^2/\lambda\nu$ small. The two roots are

$$p^2 \doteq p_0{}^2\left[1 + \frac{\mu^2}{\lambda\nu}\frac{p_0{}^2}{p_0{}^2 - p_1{}^2}\right]$$

and

$$p^2 \doteq p_1{}^2\left[1 + \frac{\mu^2}{\lambda\nu}\frac{p_1{}^2}{p_1{}^2 - p_0{}^2}\right],$$

provided $p_0{}^2 - p_1{}^2$ is not small. The two independent natural periods are approached, but always from beyond, not from between them. If the cock from which the pendulum is hung is not perfectly stiff, that is, if p_0^2 is large but not infinite, the frequency that is recognised as that of the pendulum will be given by $p^2 = p_1^2[1 - \mu^2/\lambda\nu\ p_1^2/p_0^2]$, and therefore the clock will lose if the support becomes less rigid. There will also be a rapid vibration approaching the frequency of the cock, $p^2 = p_0^2[1 + \mu^2/\lambda\nu]$.

If the suspension is made less stiff, as, for example, by hanging a chronometer by two cords, the lengths of which are increased (or their points of suspension brought nearer one another), then p_0^2 may be thereby diminished until it approaches p_1^2 and passes it. In the critical case $p_0^2 = p_1^2$, the two values are

$$p^2 = \frac{p_1^2}{1 + (\mu^2/\lambda\nu)^{\frac{1}{2}}} = p_1^2\left[1 - \left(\frac{\mu^2}{\lambda\nu}\right)^{\frac{1}{2}}\right]$$

and

$$p^2 = \frac{p_1^2}{1 - (\mu^2/\lambda\nu)^{\frac{1}{2}}} = p_1^2\left[1 + \left(\frac{\mu^2}{\lambda\nu}\right)^{\frac{1}{2}}\right].$$

It will be recognised that the departure of p^2 from the natural period p_1^2 is considerably greater than before, because the square root of the small quantity $\mu^2/\lambda\nu$ is greater than that quantity itself. Hence the balance of the suspended chronometer will lose more and more (but always at a finite rate) as the length of suspension increases up to the critical point of equality of periods. When this point is passed, the roles of p_0^2, p_1^2 are interchanged. The period proper to the balance changes discontinuously to $p_1^2[1 + (\mu^2/\lambda\nu)^{\frac{1}{2}}]$, representing a gain of rate equal to its former loss. Continuing the extension of the suspension, this gain becomes less and less, until p_0^2 being now separate from p_1^2 and much smaller than it, the frequency settles down to $p^2 = p_1^2(1 + \mu^2/\lambda\nu)$. From the beginning up to just before the critical point of equality of periods, while $p_0^2 > p_1^2$, we have A : B positive, or the balance wheel and the whole chronometer are executing oscillations in the same sense; just after this point and beyond it, their oscillations are executed in opposite senses. The greatest ratio of A : B is found at the critical point and is equal to $+ 2(\nu/\lambda)^{\frac{1}{2}}$.

The foregoing examples will serve as guides and parallels in a number of cases, though it must be noted that the case observed by Ellicott does not fall exactly under them. It is somewhat more complicated. It will be noted that the common rate taken up by his two clocks when going together was not outside their independent rates but between them.

§ (18) CONTROLLED SYSTEMS OF CLOCKS.—In view of the unexplained and sudden changes of rate which clocks are liable to assume, a number of independent time-keepers of equal grade may be, at present, the best means of marking and eliminating irregularities. But for most purposes independence and diversity are a nuisance. Hence the effective control by a master pendulum of a system of "slave" clocks has a great and growing importance. The points of interest fall under separate heads, according as we are dealing on the one hand with the highest class of time-keeping, and on the other with systems for ordinary use, where reliability within its prescribed scope and the minimum of attention are the desiderata.

Considering first systems of high precision, the use of a slave clock to replace the counting train relieves the pendulum and maintenance of an unnecessary complication, besides permitting the master pendulum to be kept, as it should be, remote from traffic, while its indications are shown simultaneously in as many other positions as may be desired. The means by which this must be effected is the signal current of say 12 - 20 milliamps. sent out by the master pendulum. The points to attend to, after reliable working is secured, are the amount, and the constancy, of the lag between the signal current and the indications of the slave clock.

The very simple system of control used, *e.g.*, by Riefler is quite satisfactory. The slave clock is any weight-driven clock of good character. To the bottom of the pendulum an arm is attached, carrying an armature, which passes at the end of the excursion near an electromagnet, set in an adjustable position upon the side of the case and made live whenever the signal current passes. At the end of the excursion the phase of the pendulum is sensitive to disturbance. A suitable electromagnet would be wound, say, 50 ohms with S.W.G. 35 copper wire. The pendulum adjusts itself and never separates from the signal. If the signal is on for one whole second and off for the next the set taken is approximately such that the end of the excursion which carries the armature over the electromagnet agrees with the middle of the second during which the signal is on. Hence the vertical portion of the pendulum will agree with the moment of the signal beginning. Some measured experiments made on this point and quoted above in relation to the microchronograph show further that variation in the current introduces a variation in the datum; but not of a degree which will

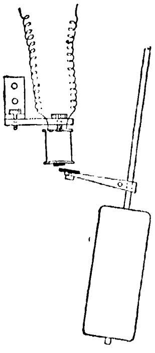


FIG. 25.

generally disturb the purpose to which a controlled clock may be put—of which the most exacting is making contacts for a chronograph. It will be seen that there should under careful circumstances be no variation reaching 0·01 sec.

If the signal sent out by the master pendulum is of the type which lasts for, say, 0·05 sec. and occurs every second, the simple system of a single electromagnet may still be successfully applied, but it is necessary to take more care to have a well-rated, well-made slave clock, because a small deviation from agreement will place the pendulum in a position where the current has no useful effect; it may then pass right through its proper position and come under the influence of the signal on the alternate second, and this process may repeat itself, the clock keeping its own rate on the average subject to large periodic fluctuations representing the control. Further, the set taken by the pendulum when under circumstances that give control in agreement with the period of the signal, may show marked differences according as the slave clock has an independent rate which is gaining or losing on the signal rate. It should be noted that when the slave clock does not keep the signal rate, it will keep a different rate of its own according as the signal is passing or not passing. The subject is not sufficiently explored, but it may be taken as certain that wherever a controlled system or relay is used for transmitting the signals of a standard clock, the regularity of its performance will require close supervision. A necessity for any control is a reliable signal current. If the current is interrupted for a period, the pendulum of the slave clock may fall out of phase, and when the current comes on again its effect may be such as to throw it further out in place of restoring it. The pendulum may stop altogether, and restart later on, working itself slowly up under the feeble impulses of the inductance.

The signal current may also be used for actuating a mere dial, possessing no pendulum or weight of its own, and with the sole function of counting the seconds. An electromagnet moves an armature and so pushes the movement forward one step each second. As the current is small and comes on with a jerk, and there is nothing like a continuously moving pendulum to carry the movement through a failure, while on the other hand the movement must be sufficiently locked to prevent an impulse shifting the finger two seconds in place of one, it is clear that the requirements are severe. Such dials should not be used where a controlled clock with its own weight can be made available for the service. It may be mentioned, however, that a construction, fitted by the Synchronome Company, following closely the lines of their commercial dials described below, has been found to work well and give on the whole very little trouble.

In the controlled systems adopted for civil use, there is no interest at all attaching to intervals and lags of less amount than one second; indeed the most satisfactory systems have settled down to the employment of half-minute steps, the controlled clocks standing still from one half-minute to the next.

The pioneers of electric control for civil purposes were Wheatstone and Alexander Bain. About 1840 each applied electromagnetic induction to drive a clock. Bain, employing the feeble current derived from two metallic plates buried in the earth, succeeded in maintaining his pendulum in motion and using its energy to drive the movement. Soon after R. L. Jones, station-master at Chester, applied Bain's system to the control of one pendulum by another. Early in the 'seventies the system was revived and improved by James Ritchie & Son, of Edinburgh. It is still working successfully, and is used, for example, to control the circuit, upwards of four miles in length, operated by a master clock at the Royal Observatory, Edinburgh, and performing the public time service of the city.

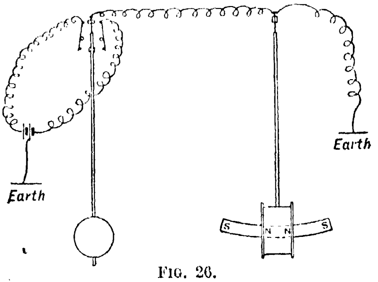


FIG. 26.

At each controlled clock the pendulum bob is a solenoid wound upon a brass frame pierced by a large central aperture. The wires to and from the solenoid pass up the pendulum rod, and out by the suspension springs, which are insulated from one another, the jambs that clip them being made of ebonite. Through the aperture of the bob a rod passes covered with permanent bar magnets, the N-poles facing one another and separated by an interval about equal to the length of the solenoid. The pendulum swings to and fro, enclosing this rod with its magnets. The master clock is arranged so as to reverse at each semi-swing of its pendulum the current which it sends through the line. In consequence the pendulum of the slave clock is alternately attracted to one side and the other. If sufficient energy is put into a slave pendulum it may be made to actuate its dial step by step without the assistance of a weight. The change of polarity of the current was originally secured by providing the master clock with long weak contact springs with which the pendulum makes contact alternately as it swings to and fro; the poles of a battery, the middle of which is connected to earth, lead to these springs; the line leads away from the pendulum and is connected to earth after passing through all the slave pendulums in series. As remarked above, when the circumstances are fairly uniform the system works without objection as a control. The current used is about 20 milliamps. But it is open to several criticisms. The contact springs rubbing the pendulum of the master clock at the end of its excursion are prejudicial to its time-keeping, and the current they supply is not steady. The same purposes may be served by using a Post Office polarised relay, joining the line to the tongue and the + and − battery connections to the two contact faces. This relay is actuated by a signal from the master clock which is alternately on and off for a whole second. The construction of the slave pendulums and their permanent magnets is also open to objection. The position of the solenoid with respect to these magnets is unstable, and the pendulum is apt to screw or strike the bar until steady movement is established. Further, if the arc is prolonged, the attraction by the N. pole is changed to repulsion by the S. pole, and some confusion of the control may result.

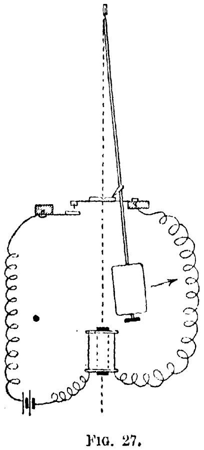


FIG. 27.

To the class of electric winding or electric maintenance rather than electric control belongs the device of the Foucault-Hipp butterfly. On the pendulum is carried by a pivot a small trailer; this is drawn to and fro across the upper of two spring arms which close an electric circuit when pressed together. There is a projection or bank attached to this arm, and when the arc of the pendulum is normal the trailer is drawn past this bank at each extremity. But if the arc falls to such a degree that the trailer fails to clear the bank, the return stroke jams it in a recess and makes the contact. This makes active an electromagnet placed below the bob, and the adjustments are such that the bob is attracted and the arc increased.

By this device the pendulum can be kept moving at approximately the same arc, and employed to find energy for the movement of dial work, in spite of the widest variations in the resistance to such movement, due, for example, to exposure to wind. It is so employed successfully by the Standard Time Company. It should be remarked, however, that neither the stroke of the trailer nor the pull of the electromagnet can be given to the pendulum in its vertical position, and therefore the time-keeping must suffer each time the arc is restored.

A form of occasional control, applied when desired to correct an ascertained error, is used by the Admiralty to bring independent clocks into agreement with a given signal. At the passage of the signal a free pendulum is released from a click which was holding it aside, and begins to swing freely. It swings about the same axis as the pendulum of the clock which it is desired to correct. When the signal passes the two pendulums, being found out of phase with one another, the clock pendulum is brought back into phase by temporarily increasing or diminishing effective gravity by making live two fixed solenoids, which then attract or repel the poles of a permanent magnet attached to the pendulum. As soon as the phases are brought into agreement, the current is turned off and the clock thereupon takes up its natural rate again. One solenoid is on each side of the pendulum and the poles of the permanent magnet pass just above them. The solenoids are made without core, to guard against residual magnetism.

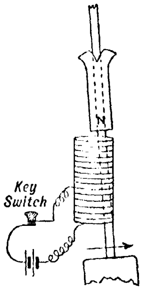


FIG. 28.

The direct use of the electric current to maintain the motion by attracting the pendulum is generally regarded with disfavour. It is found better to employ the current indirectly to stretch a spring, or to lift a weight, and to allow this to work upon the pendulum. But this matter belongs rather to the subject of maintenance than control, and under that head the systems employed by Riefler, Sir H. Cunnynghame and W. H. Shortt have been described above.

The same is true to some extent of the master clocks specially designed to control systems of slave clocks; they are independent movers and require to be judged as such. But as they are designed with control as the aim, and their features are modified accordingly, they are described here.

The maintenance for master clocks employed by the Synchronome Company, in its final form, is virtually the same as Sir Henry Cunnynghame's escapement, described above, except that the impulse is not applied at the bottom of the pendulum and operates only once in thirty seconds. During thirty seconds the pendulum is completely free, except for the work of rotating a count wheel, after which a contact is made which releases a gravity arm that restores the motion of the pendulum, and at the same time provides a signal by which any number of dials are kept in step, with the counting of the master pendulum moving half a minute at each step. The work done by the pendulum, and on the pendulum, is arranged to fall while it passes its vertical position. The importance of such a system is in proportion to its practical success, and certain points deserve examination. The first of these is the character of the contact. The gravity arm presses on the armature and ensures a good contact until it is reset. Sparking is suppressed as usual by non-inductive shunts. The length of contact may be varied and must last until the most sluggish of the dials has operated. The whole operation may be examined and recorded by means of the oscillograph. Some specimens are appended, which show the circumstances of the original make and growth of current, the effect upon it of the entry of a series of dials; others showing the difference in character when the current is taken directly off lighting mains interposing only a lamp resistance, and several other variations, are given in a paper "Modern Electric Time Service," by F. Hope Jones (*Proc. Inst. Elec. Engineering*, Feb. 1910, vol. xlv.). The growth of the current is quite free from "splash"; the necessary amount is about 0·3 amp., and a duration of contact of about 0·05 sec. gives an easy margin of safety. The actual consumption of current is a negligible quantity. Extensive systems, for example, can be driven for some years upon a set of dry cells, and the greater part of the loss of energy of the cells is then due to leakage, not to work.

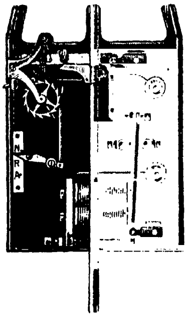


FIG. 29.

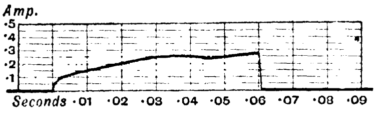


FIG. 30.

A feature deserving mention is the automatic signal supplied when the battery power is not equal to its work of replacing the gravity arm upon its click. In such a case the pendulum upon its return swing finds the roller in the lower position and assists the magnet to lift it. When it does so, it can be arranged that the contact it makes closes a circuit and causes a lamp to flash. Or a prolonged current may be made to operate the stroke of a sluggish bell which pays no attention to one which lasts only the twentieth of a second. Finally if the battery refuses its work, the pendulum will come to rest in a position which holds the contact open, and, for instance, will not completely exhaust an accumulator which only wants recharging.

The system of control fitted by Messrs. Gent & Co., Leicester, embodies almost identical features.

The master pendulum, like every other independent clock, will require occasional adjustment to time. This is done by pulling the count wheel round if the clock is slow, or by causing the engaging arm of the pendulum to ride clear of the count wheel if the clock is fast. Adjustments of less than two seconds in the error can only be made, as in any other pendulum, by touching the pendulum with the hand.

When an automatic standard time signal is available it can be used to correct such a clock, but regard must be had to the fact that the correction must be made to the rate of the pendulum, while the available indication is the error, or integral of the fault of rate. Any form of automatic correction will result merely in rocking the error to and fro. If, for example, it takes the simple form of placing a small weight upon the tray in case the standard signal shows the clock to be slow, this weight must be great enough to meet at least errors due to the barometric equation, or, say, to produce not less than a change of 0·5 sec. in a day in the error; this may be too much, and at the best performance the error ascertained by the standard signal will be alternately positive and negative. The only cure is to make the standard signals more frequent. In case they are sent once an hour, the amount of rocking is negligible; we approach the case of continuous control. No better result can be attained by attempting to feed in automatically changes of rate which will slowly efface a large error. If the clock is slow and the error positive, the automatic arrangement continues to correct the rate until it becomes negative; the error is still positive, and it goes on correcting the rate more and more, until when the error is effaced the clock is left with a substantial negative rate, which will carry the clock as much fast as before it was slow before it in turn is effaced. A more primitive device disregards the rate of the clock and corrects the error by attaching a pin to the minute-hand, and causing the signal to close a clip upon it, by which it is brought forward or backward to the exact hour. But this hardly deserves the name of a control.

Passing to the system of distributing time to a series of dials, the master clock makes available once every half-minute a current such as is indicated, say, by the oscillograms shown above. The rate of growth of this current depends upon the self-inductance of the circuit, for given voltage. A gradual operation is desired and consequently the whole system is put in series. In this way upwards of 300 dials are operated off a single master clock. There is no theoretical limit to the number, and the faults that occur are chiefly those incidental to wiring systems, to residual magnetism of armatures, and other details of such a kind. The physical and indeed the practical problem are perfectly determinate and may be considered as solved. The wheel work required for each dial consists of a star-wheel of 120 teeth to the arbor of which the minute-hand is attached, together with a pair of gear-wheels operated by it, the second of which carries the hour-hand. For the largest turret dials, say 6 feet or more in diameter, all this is contained in a little box 4 or 5 inches square, which hardly projects beyond the boss of the hands and leaves the rest of the dial empty for illumination.

The requirements for moving the star-wheel every half-minute are satisfied with too much variety of detail to illustrate all the constructions, but present certain common features. The movement conveyed by the electromagnet to its armature is too sudden to be impressed directly on the wheel work. The movement is stored in a spring, and when the armature is released, the latter replaces the armature in its original place and pushes the hands round. The rest is arrangement for securing provision of a backstop on the one hand to prevent the hands regreding, and the prevention of tripping on the other, *i.e.* the wheel moving forward two spaces for a single impulse. Inspection of the diagram shows sufficiently how these requirements are secured. The actual system illustrated is the Synchronome. The plan adopted in their second-by-second dial referred to above is almost identical, except for added gear-wheels, the numbers of teeth, and lighter construction. The operation of the armature is easily seen; it is pivoted at D and works upon the star-wheel through the click E, against the end of which the spring F bears. The stop H limits the forward movement which this click can convey to the wheel. The backstop is J, a square pin in an arm G, pivoted at K. G also contains a stout pin I, which prevents the click from picking up more than one tooth when the armature comes over.

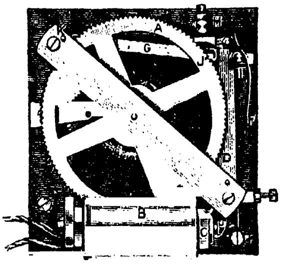


FIG. 31.

§ (19) ACTUAL PERFORMANCE OF CLOCKS.—There is more obscurity in stating what is the standard of actual performance that has been got from the best clocks than might perhaps be anticipated. Changes due to hygrometric conditions of the atmosphere may be completely set aside by making the case air-tight. Barometric equation may be dealt with by the same device with equal completeness, though it is not manifest, when the barometer is changed in order to change the rate, that the change of rate taken up agrees completely with anticipation.[1]

Elimination of change of rate due to changing temperature is by no means certainly secured by the methods of compensation actually practised. Riefler[2] collects published determinations for a number of the best clocks, which range from +0·06 sec. per day per 1° C. to −0·04 sec., the value for Riefler No. 1, with the mercury compensation described above, p. 205, being practically zero. For the clock Riefler No. 258, at the Royal Observatory, Edinburgh, the coefficient is +0·05 sec. But the question is confused by problems of stratification of the air. Where practicable, the clock should be kept at constant temperature.

---

[1] But cf. Tisseraud, *Bulletin Astronomique*, 1896, p. 255, where a variation, equal apparently to the full barometric equation, is followed in a sealed clock. But it may be suspected that there was some undetected leak, for the clock does not appear to have been furnished with any manometer to prove the constancy of the pressure within the case.

[2] *Die Präcisions-Uhren*, Munich, 1894, p. 14.

The following data may be given to illustrate behaviour under change of barometer; the observations were made at the Royal Observatory, Edinburgh.

CLOCK. RIEFLER No. 258. (1917.)

Clock enclosed in 11-inch cylinder. Cylindrical pendulum bob 15 cm. × 8 cm. diam.

Experiment.	Change in Bar. Press.	Change in Rate per Day.	Change in Semi-arc.
	mm.	sec.	″
1	−14	−·19	+55
2	+14	+·22	−42
3	+15	+·25	−60
4	−15	−·26	+65
5	+14	+·22	−39
6	−15	−·26	+21
7	+15	+·26	−40
8	−14	−·21	+32
9	+14	+·25	−56
10	−14	−·23	+34

Mean 1 mm. = +·0163 sec. = −3·1″
(1 inch = +·40 sec. = −77″)

SYNCHRONOME No. 1

Clock enclosed in 9-inch cylinder. Cylindrical bob 10 cm. × 8 cm. diam.

	Experiment.	Change in Bar. Press.	Change in Rate per Day.	Change in Semi-arc.
		mm.	sec.	″
1917	1	+2·30	+·83	..
	2	+ ·97	+·41	..
	3	+1·12	+·45	..
	4	+1·02	+·37	..
	5	+1·00	+·32	..
	Mean	1 inch = +0·38 sec. (1 mm. = +·0150 sec.)		
1918	1	+2·04	+·66	− 30
	2	+1·01	+·35	− 20
	3	+1·22	+·69	− 106
	4	+1·53	+·31	− 63
	5	+2·02	+·67	− 34
	Mean	1 inch = +0·34 sec. = −32″ (1 mm. = +·0130 sec. = −1·3″)		

DENT. No. 1506

Cylindrical bob 9 in. × 2½ in. diam.
Clock enclosed in rectangular case, 14 in. wide × 8 in. deep.

Mean of numerous experiments } 1 inch = +0·40 sec.
(1 mm. = +·016 sec.)

FRODSHAM. No. 1030

Mercury in glass, with large stirrup, 11 in. × 4 in.
Clock enclosed in rectangular case, 14 in. wide × 8 in. deep.

Mean of numerous experiments } 1 inch = +0·72 sec.
(1 mm. = +·029 sec.)

In the same clock, by numerous experiments, removing glass front without altering pressure caused gain of rate of −0·4 sec. per day.

The foregoing numbers represent the joint effect of barometric change proper plus the effect on rate owing to consequent effect upon arc. The latter cannot be separated unless the arc-rate equation can be experimentally determined. It may not be assumed that it is, substantially, equal to the theoretical circular error. The following numbers, however, present an example of such equality. The clock is King No. 2, of which the escapement has been described above; by setting up the remontoire so that the impulse wheel is a different number of steps (equal to fifths of a complete turn) in advance of the scape wheel, different impulses can be given and the arc varied permanently within wide limits.

Semi-arc	55′	66′	71′
Observed rate	0·60 sec.	1·32 sec.	1·60 sec.
Difference	+·72 sec.	+·28 sec.	
Cf. theoretical circular error	+·61 sec.	+·31 sec.	

The observations are means of several determinations.

With regard to the actual performances of clocks in their essential function of time-keeping, though a large amount of material has been published, it is not very easy to draw definite limiting conclusions from it. The ideal clock would keep permanently a constant rate, when reduced to constant circumstances. No clock does this, and again, the means of testing whether it does so or not are themselves marked by sensible errors. Interpreting a series of recorded errors becomes a problem of plotting. It is much to be desired that simple impartial principles should be accepted for plotting, smoothing, and interpreting observed errors. This would cover the questions of (1) the admissible amount of the accidental errors of observation; (2) whether in changing rate the change is gradual or sudden, and by what coefficients; (3) in either case, to assign the lengths of time over which a formula with given constants expresses the rate. These should be settled on a basis that permits continuous application to the performance of the clock, between say two cleanings, and not mere consideration of some specimen run. The simplest and therefore the best supposition with respect to changing rate, if it is admissible, is a dead smooth rate broken by sudden changes, the graph of the error being thus a broken rectilinear figure. This would throw any fluctuations of error of short period in among the errors of observation, and together they would represent the departures between observation and the rectilinear graph. There would then be three elements to assign: (1) the succession of *base* *rates*, as they may be called, being the slopes of the successive portions of the rectilinear graph; (2) the number of days over which each base rate holds, and (3) the mean departures, say the *erratics*, between observation and the graph. If the circumstances are changed, as, for example, by regulating the clock by means of the barometric pressure, the base rate would of course be the rate exhibited plus the correction to standard pressure. An analysis upon these lines is made of the going of the clock Riefler No. 258 at the Royal Observatory, Edinburgh, and the extracts above illustrate the result. They start after closing the clock after a cleaning.

Riefler No. 258. (1916)

No.	Base Rate per Day.	Term.	Duration.	Mean Erratic.	No. of Observations.
	secs.		days	secs.	
		Mar. 27			
1	- ·255		19	·040	12
		Apr. 15			
2	- ·234		7	..	..
		Apr. 22			
3	- ·161		6	·040	3
		Apr. 28			
4	- ·128		12	·060	2
		May 10			
5	- ·117		20	·040	11
		May 30			
6	- ·142		28	·036	10
		June 27			
7	- ·186		24	·020	8
		July 21			
8	- ·146		13	·020	8
		Aug. 3			
9	- ·099		17	·020	2
		Aug. 20			
10	- ·081		17	·027	3
		Sept. 7			
11	- ·102		18	·031	8
		Sept. 25			
12	- ·117		14	·057	6
		Oct. 9			
13	- ·141		22	·035	13
		Oct. 31			
14	- ·170		8	·035	4
		Nov. 8			
15	- ·157		23	·056	5
		Dec. 1			
16	- ·108		13	·013	3
		Dec. 14			
17	- ·093		14	·023	6
		Dec. 28			
18	- ·092		9	·020	3
		Jan. 6			
Means (omitting Nos. 1, 2)	- ·122	..	16	·033	6

The changes in the base rate that are revealed are themselves a smooth undulation. This may be taken as a fair example of the highest performance. For ordinary clocks so stringent an analysis is quite inapplicable, the graph of error when plotted to ·01 sec. being obviously curved and sinuous in short periods.

R. A. S.

Cloud:
- Diurnal variation of. See "Atmosphere, Physics of," § (20).
- Measurement of, at night. See "Meteorological Instruments," § (26)
- Wave-like form of. See "Atmosphere, Physics of," § (17).

CLOUDS:
- Albedo of. See "Meteorological Optics," § (16) (iv.).
- Instruments for measuring motion of. See "Meteorological Instruments," VII., §§ (33), etc.
  - (1) Camera obscura. See *ibid.* § (34).
  - (2) Darwin-Hill mirror. See *ibid.* § (35).
  - (3) Nephoscopes. See *ibid.* § (33).
- Iridescent. See *ibid.* § (15) (ii.).
- Opacity of. See *ibid.* § (16) (i.).
- Rain, effect on electric field of atmosphere. See "Atmospheric Electricity," § (18).
- Translucence of. See "Meteorological Optics," § (16) (iii.).

COEFFICIENTS OF EXPANSION OF LINE STANDARDS: method of measuring in comparator. See "Comparators," § (2).

COL. See "Atmosphere, Physics of," § (18).

COMBINATION OF OBSERVATIONS. See "Observations, The Combination of," § (5).

COMPARATOR FOR SLIP GAUGES, 4″ capacity: calibration of, to accuracy of one millionth of an inch. See "Metrology," IX. § (32) (ii.).

COMPARATORS

REFERENCE to the article "Line Standards of Length" will show that it is necessary in the first instance to determine with extreme accuracy the length of a line standard, and subsequently, from time to time, to check the result so as to detect any possible change. This necessity is met by an apparatus called a comparator, by means of which, as its name implies, the lengths of two standards may be compared one with the other. There are many types of comparators, but all are designed to the same end, viz. the measurement of the small difference between the lengths of two standards. One of these standards being of known length, the length of the other may thereby be determined.

In order to appreciate fully how the necessary accuracy is attained, it is proposed to describe in detail a typical comparator, how it is used, the precautions necessary before and during the course of observations, together with some account of the manner in which the results are finally computed. This will be followed by short descriptions of other comparators.

§ (1) ONE-METRE COMPARATOR. (i.) *Description.*—The instrument at the National Physical Laboratory was constructed by the Société Genevoise, and is designed to accommodate standards up to 40 inches in length; it is therefore suitable for standards of the usual type, either a yard or a metre in length. Any such camparator consists essentially of a tank or bath suitably fitted to receive the bars, and, for viewing the lines, two microscopes so mounted that they can be rigidly fixed in any desired position; the whole being carried on a cast-iron bed which rests in turn on a massive concrete block.

(*a*) *Housing.*—The whole apparatus is housed in a room which has a double roof and no outside walls, and which is suitably heated and ventilated, hence the variations in temperature of the room over a long period are only slight. The steadiness of the room temperature is a considerable factor in maintaining a steady temperature in the apparatus.

(*b*) *Foundations.*—The concrete block rests in a brick-lined pit or well, the bottom of which is about three feet below the level of the floor of the room, and is separated from the walls of the pit by an air-space, thus isolating it from vibrations and other disturbances that may be set up in the floor of the room. The portion extending above the floor is encased in stout woodwork, which is secured to the floor without touching the concrete or any part of the apparatus. This is clearly seen in *Figs.* 1 and 2, as well as the other parts about to be described.

(*c*) *Base.*—The cast-iron bed, by means of four stout levelling screws, rests on brass plates let into the top of the concrete. Its front and lower portion, about 4 feet long and 18 inches wide, carries the tank, the weight of which is supported entirely by two machined strips, one at each end. The rear portion is extended upwards along its whole length, and carries a girder slightly overhanging the lower portion of the bed. The whole is made hollow, and the rear portion is relieved in the usual way, so as to reduce its weight without affecting its rigidity. The top surface of the girder forms a carefully machined bed, which carries two slides from which project, over the tank and at right angles to the length of the girder, two strong supporting arms for the microscopes. The slides can be adjusted in relative position along the girder, while the arms are adjustable horizontally in a direction at right angles to the direction of movement of the slide. The arms are provided at their ends with split collars, which hold the microscopes in a vertical position at any desired height above the tank. Thus, each microscope is adjustable in position in three azimuths, and the movable parts can be securely clamped, once the desired positions are attained.

(*d*) *Tank.*—The tank consists of two troughs made of sheet copper, disposed one within the other in such a way that they nowhere come into direct contact, the upper edges of each lying in the same horizontal plane. The space between the walls and bottoms of the troughs is filled with water, which, by surrounding the inner trough on five sides, serves as a protective lagging against radiation, and thus assists in stabilising and maintaining the temperature of the inner trough. The water may also be heated or cooled in order to vary the temperature. The thin copper walls of the trough, by virtue of their high conductivity, naturally assist in attaining, with rapidity, the desired temperature conditions. Also with a view to preventing radiation into or from the outer trough, the walls of the latter are encased with thick oaken planks.

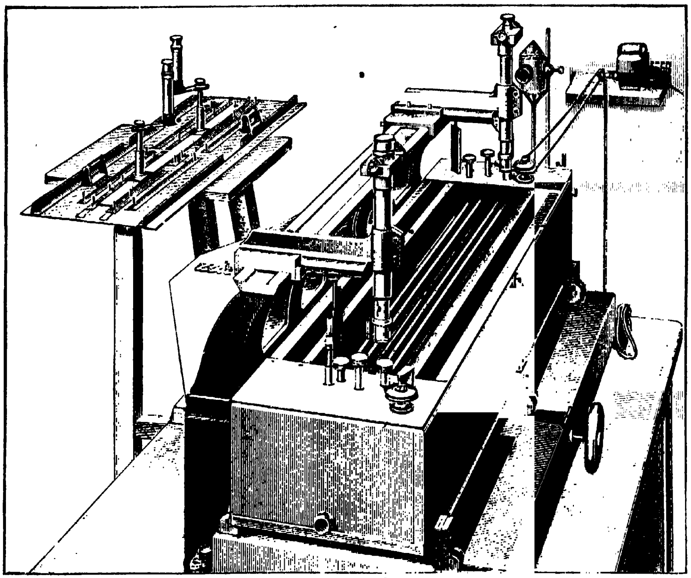


FIG. 1.—The One-metre Comparator at the National Physical Laboratory. (Cover removed, showing two bars in position.)

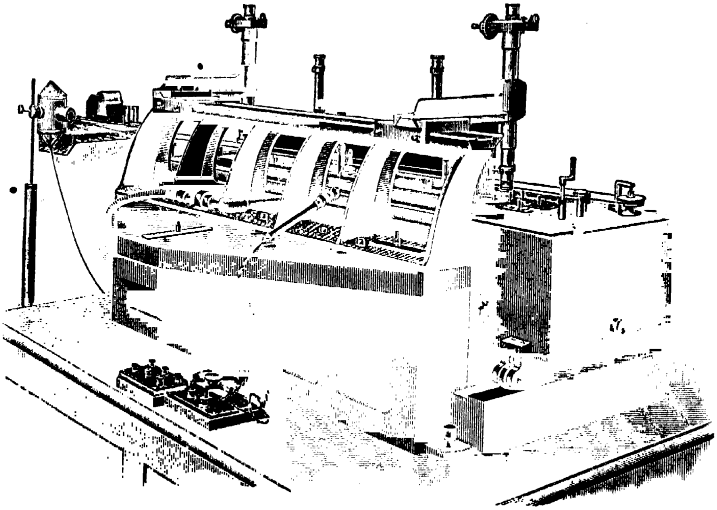


FIG. 2.—The One-metre Comparator at the National Physical Laboratory. (Rear view.)

(*e*) *Carriage.*—The whole tank is mounted on a cast-iron platform with four small wheels which run on the machined strips at each end of the lower portion of the bed, and its motion to and fro is controlled by a wheel in front, which when turned actuates a worm gear placed under the centre of the platform. The direction of motion is controlled by two small guide wheels attached to the tank, fore and aft, and running in a V groove across the centre of the iron base. The whole weight of the tank is borne entirely on the four wheels of the tank and not any of it on the guide wheels (see remarks in (iii.) (*b*)).

(*f*) *Girders.*—The inner trough is provided with two girders whose function is to carry the bars when they are under observation. Each is fitted with three levelling screws, two at one end and one at the other, and these rest on small platforms or steps at each end of the trough. The points of support, *i.e.* the ends of the screws, fit into the usual slot on one platform and the hole and plane on the other, thus allowing for expansion or contraction without constraint. The platforms can be adjusted slightly in position by means of screws which project above the top of the tank at either end. These adjustments, together with the movement of the tank as a whole, enable one to alter the positions of the girders in three directions mutually at right angles, that is, up and down, to and fro, and longitudinally. Girders, steps, and adjusting mechanism inside the trough are made of gun-metal to avoid rust. Each girder carries, transversely, two small rollers on which the bars rest, and which can be clamped to the girders in positions which ensure that the standards shall be supported at the Airy points. In use, the inner tank is usually filled with distilled water, which is preferable to ordinary tap water; the latter, if used, is apt to deposit on the surface of the standards a thin opaque film which is not easily removed. Both inner and outer troughs are provided with small stirrers which are worked by a small electric motor on the wall near by. The outer tank is connected by pipes to hot and cold water supply, and is provided with a drainage pipe.

(*g*) *Microscopes.* — The microscopes, of moderate power, are fitted with micrometer screw eyepieces, and the magnification is adjusted so that one division of the drum corresponds to a micron, tenths of a micron being read by estimation. The eyepiece is fitted with the usual comb, or rough scale, each tooth of which corresponds to one revolution of the screw, *i.e.* 100 μ. There are three pairs of transverse cross-wires or spider lines, broad, medium, or fine, together with one longitudinal line across the centre of the field of view at right angles to the pairs of lines. Every fifth recess of the comb is cut a little deeper than the rest, and one of the deeper cuts approximately locates the centre of the field of view. Arbitrary values may be given to the divisions of the comb, and it is usual with these particular instruments to call the centre one 1000 μ, the others being numbered accordingly from left to right in the field of view (*Fig.* 3).

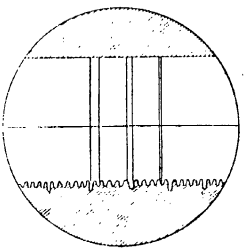


FIG. 3.

(*h*) *Illumination.*—The illumination of the lines of the standards is by vertical reflection from the graduated surface. The source of light is a distant Pointolite lamp, placed so that the light beam is in the direction of the length of the comparator. Its position, several feet away, ensures that it has no disturbing heating effect on the bars or any part of the apparatus. The light from the lamp is collimated and falls on a vertical mirror fixed under the supporting arm of the microscope. The mirror, placed at an angle of 45° to the direction of the beam, reflects the light through an angle of 90° towards a hole in the tube of the microscope just above the objective. Here it falls on the under surface of a cover slip tilted at 45°, so that the light is reflected vertically downwards. The light is finally reflected back from the horizontal surface of the bar vertically upwards to the eyepiece.

(*i*) *Thermometers.*—The temperature of the inner trough is determined by two or more precision mercurial thermometers, which are supported on metal crutches attached to the girders so that they lie near to, and at about the same mean level as, the bars. Their positions are chosen so that their readings will eventually give the mean temperature of the water or other surrounding medium.

(*j*) *Cover.*—The tank is provided with a movable metal cover which is slotted along its length so that the thermometers or any portion of the bar may be readily viewed. Varying lengths of metal strips cover these slots, and by suitably rearranging these, any desired small portion of the slot may be left uncovered for viewing the lines. Attached to the lid are low-power microscopes for reading the thermometers.

(ii.) *General Considerations, and Sources of Error.*—It should be stated here that the foregoing description relates to a typical comparator, which is shortly to be replaced by a more modern and elaborate machine, but that the remarks which follow are quite general, and are applicable, not only to this, but to all comparators.

(*a*) *Microscopes.*—The micrometer microscopes of a comparator should be the best procurable, otherwise they may prove a fruitful source of error. Even the best, however, give rise to errors, which need careful investigation before the instruments are used.

Errors may arise from numerous causes. Change of temperature, by altering the length of the tube, by varying the constant of the lenses, or by expanding the micrometer screw, may give rise to variations in the magnification, but as a rule these effects are negligibly small. Errors sometimes occur through inaccurate focussing, arising from objectives which are not aplanatic. The micrometer screw may possess progressive and periodic errors, or an error in the nominal value of a division. Periodic errors arise from such defects as variation in pitch, the abutments of the screw not being at right angles to the axis of rotation, eccentric drum or irregular divisions of the drum, etc. The undesirable presence of foreign matter in the thread or bearings of the screw may cause additional errors, but these can, with care, be avoided as a rule.

The most abundant source of error is the screw, and it is necessary to calibrate it with great care. To measure each error separately is a laborious, if not an impossible task, but there exists a simple and effective method of measuring their cumulative effect. This consists of calibrating one microscope with the aid of the other, and is, very briefly, as follows.[1]

The eyepiece of the microscope to be measured is removed, thus exposing to view the cross-wires or spider lines, and the microscope is then mounted so that these cross-wires, suitably illuminated, may be viewed by the other microscope vertically above it. The latter is fixed in a carriage so that it can move parallel to the axis of the micrometer screw.

The method consists in measuring successive nominally equal lengths of the micrometer screw of the lower microscope by the micrometer screw of the upper one, the same portion of the latter being used each time for each length of the former. For example, suppose every nominal 100 μ length is to be examined, *i.e.* a length corresponding to one turn of the drum. The drum of the lower one is set to zero, and a reading is taken by the upper microscope on the cross-wire of the lower one. The drum is then advanced one turn, and another reading on the cross-wire taken. Suppose these readings are a_1 and a_2. The upper microscope is now translated bodily so that it again reads a_1, or as near to it as it is possible to get, on the cross-wire below, which is not moved in the meantime. The drum is again rotated 100 divisions, and a reading obtained on the cross-wire. The reading will very closely approximate to a_2. This process is repeated as often as necessary. Certain errors of setting may arise, and these may be cumulative, but this effect can be counteracted by repeating the calibration with, say, 500 divisions, *i.e.* 5 turns, or even larger intervals. It is useful, though not necessary, to alter the magnification in this case, by changing the objective of the upper microscope, so that as long a length as possible of the screw of the upper one is used.

Of course, shorter intervals than the 100 divisions can be measured, say every 10 divisions, and it is probably then safe to interpolate for errors over the intervening points.

All the readings thus obtained can by a simple graphical method be translated into corrections, which are tabulated for further use.

The method thus described is simple and accurate. Any errors that the screw of the upper microscope may have do not affect the result, since the same portion of the screw, within very small limits, is used each time.

It should be realised, however, that where there are moving parts such as obtain in a micrometer screw, changes take place which lead to gradual changes in the errors, and that it is necessary, therefore, to revise the calibrations from time to time. It is sufficient, as a rule, to check at frequent intervals the value of a division by comparing the readings directly against a divided millimetre of known calibration.

As the N.P.L. microscopes have very small screw errors, it is the practice to use the figure obtained by comparing with a divided millimetre as a mean error to be applied to the result. It is expressed as a percentage, and is usually small, being of the order of 0·1 per cent or less. In order to minimise the chance of error due to possible defects of the lenses, it is also usual to limit the run of the screw by confining the observations to a small portion of the centre of the field of view. A run of 200 μ is seldom exceeded where the best class of work is involved, and usually it is confined to within half this amount.

(*b*) *Temperature.*—The accurate measurement of temperature is by no means the least of the difficulties that arise in comparator work, and the very best precision thermometers are therefore necessary. These should have large capacity bulbs, and narrow, straight, uniform bores, so that the scales are not only even, but are sufficiently "open" to enable one to make readings easily to well within $0^{\circ}\cdot01$ C. The total range of the scales should be limited to that over which observations are likely to be made. The thermometer must be most carefully calibrated, and the corrections thus obtained so adjusted that the resulting temperatures are given on the hydrogen scale. It is sometimes necessary to carry out the calibrations with the thermometers in a vertical position, but in the comparator work they are used in a horizontal position, with the consequence that the reading will be higher than in the vertical position, necessitating a corresponding correction, which is proportional to the length of the mercury column. This correction can be obtained by direct comparison in the horizontal position at various temperatures, with a thermometer whose corresponding correction is already known. It can, however, be found directly, if each thermometer be provided near the upper end with an expansion chamber and a boiling-point mark. Comparison of the readings at the boiling-point in both horizontal and vertical positions will readily give the required "vertical to horizontal" correction.

---

[1] *Bull. of B. of S.* x. 375.

As is well known, the zero reading of a thermometer is subject to fluctuations which depend on its previous history and treatment, and which constitute a serious source of error. It is therefore necessary to check this point at frequent intervals, and to adjust accordingly the calibration curve obtained as above. It is probable that the correction for the change in zero will not be so well known at any time as the relative scale calibration.

But, however carefully the calibration be made and applied, it is seldom possible, even under the best conditions, for the temperatures obtained with a mercurial thermometer to be reliable to a greater accuracy than 0°·01 C., and uncertainties of 0°·02 C., or even more, sometimes arise. When it is pointed out that an error in temperature greater than 0°·01 C. may give rise to an error in the final result greater than the probable error due to any other cause, as, for example, the error of observation, the importance of accurate thermometry should be thoroughly appreciated.

Owing to the limitations of the mercurial thermometer, it is probable that any further refinement in this direction will arise from the employment of other and more accurate means of making the measurements, such as, say, an electrical resistance method.

(*c*) *Other Sources of Error.*—In addition to the microscopes and thermometers, there are other sources of error too numerous to mention in detail. Some of them will be apparent from a study of the way in which the apparatus is set up and used, and their incidence and size will depend on the skill and care employed in manipulating the apparatus.

(iii.) (*a*) *Setting up and manipulating the Apparatus.*—The two bars which are to be compared, say two metre standards, are placed in the inner trough, one on each girder, and supported by the rollers symmetrically at the Airy points, and with a sufficiency of water covering them. If one or both of the bars be liable to rust or other damage, the comparison may be made in air, or if necessary some other liquid may be used, such as paraffin.

The microscopes are next adjusted so that they are at the same level, and this can be done by focussing each in turn on a particular spot, say near the centre of one of the bars. They are next placed approximately at a metre apart, so that the zero and the 100 cm. lines appear in or near the centres of the fields of view, and parallel to the cross-wires of the eyepieces. Both bars should be observed in fixing the microscope so that the best central position may be obtained. A horizontal line cutting the axes of the two microscopes should then be parallel to the axis of the bars. The microscopes, then rigidly clamped in position, are kept so during the course of the subsequent set of observations.

It may be necessary at this point to adjust the level of the girder by means of the end screws, so that the light is reflected perpendicularly from the surface of the scale and passes vertically through the microscope. The lines are best viewed by diffused light, which is attained by attaching to the microscope, so as to intercept the light, a piece of fine ground glass.

The thermometers may now be placed in position on their supports and arranged so that their scales can be easily read from above. The surface of the water is next skimmed with blotting-paper to remove all foreign matter that might distort the image of the line, and so give an error in reading. It is necessary to repeat this at frequent and convenient intervals, even when the utmost care is taken with regard to cleanliness. The cover is now placed in position, and the whole allowed to settle down to a uniform temperature, assistance being given in this process by frequent stirring of the water in both troughs.

(*b*) *Observations.*—When it is found that the temperature is steady and reasonably uniform, the tank is moved so that the rear bar is brought under observation. Certain adjustments are necessary before a reading can be taken. The lines are first of all very carefully focussed by raising or lowering the girder; the bar is next aligned, that is, the girder is moved transversely so that the longitudinal wire in the eyepiece appears to be mid-way between the two longitudinal lines of the bar; and finally the two defining lines are adjusted by moving the tank longitudinally, so that they appear symmetrically disposed in the respective fields of view.

The bar is now in correct position for making a micrometer setting. Choosing one of the pairs of cross-wire, say the medium, these are moved across the field until the defining line appears mid-way between them. The reading of the micrometer on each line is noted in the observation book. *Fig.* 4 shows the appearance of the spider lines and those on the bar, when a correct setting has been made, the heavy lines being those on the bar, and the lighter those in the eyepiece.

The tank is now bodily moved backwards so as to bring the other bar under observation, and the whole process repeated. Several observations are made in this way alternately on the bars, the final one being on the bar which was observed first.

Temperatures are carefully read at the beginning and at the end of the set, as well as at convenient intervals in between, and at the same time pauses are made for stirring the water so as to ensure maintenance of uniformity of temperature. It should be noted that in reading the thermometers, parallax errors may occur due to either a lack of uprightness of the microscopes or to the thermometers not being quite horizontal. The latter fault can readily be avoided with a little care at the commencement. The former fault is brought about by imperfect mountings, and in the type of microscope used at the N.P.L. the errors are counteracted to some extent by rotating the microscopes through nearly 180° half-way through a set of observations.

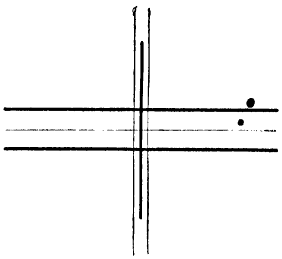


FIG. 4.

Fig 5 shows an actual set of observations taken in this way on two line standards, and indicates at the same time how corrections are applied and the results obtained. It will be noticed that the temperature changes slightly during the course of observations, and it is probable that all the parts of the apparatus have changed temperature in a similar way. This may bring about a slight expansion or contraction of the girder carrying the microscopes, and therefore a small alteration in the distance between the microscopes. Theoretically, this distance should remain constant, but if observations are taken alternately on the bars in the manner described, the error due to such change, which in any case is very small, is automatically cut out.

In a similar manner the personal equation of the observer is eliminated. Errors due to this cause are usually the same on all the readings, and annul one another when differences are taken.

The two bars having been compared in this way, their positions relative to one another are changed, and another comparison made. This change may be effected by turning either bar end to end, or by exchanging them on the girders, or by a combination of both. It is thus possible to arrange the bars in eight different ways, and consequently to obtain eight different sets of comparisons All this may seem unnecessary, but it is possible for errors to arise owing to the position of a bar. For example, the appearance of a line may be slightly different when the bar is turned end to end, with a consequent effect on the micrometer setting.

That an exchange on the girders is necessary is best exemplified by recording the discovery of a serious fault in the design of the N.P.L. comparator. Observations were taken in the usual way on two bars, but it was found that their exchange on the girders gave rise to a difference of 3 μ or 4 μ in the result, an error out of all proportion to the observational errors. The cause of this was looked for and was located in the tank guide wheels, which were found to be bearing heavily in the V groove. The centre of the cast-iron bed was therefore bearing some of the weight of the tank, and suffered a slight distortion which was transmitted to the girder carrying the microscopes. The distortion varied with the position of the tank, and the ultimate effect was to alter the distance between the microscopes when the tank was moved backwards. The obvious remedy for such a fault is to have the girder and the tank on quite separate beds, so that one is not affected by any change in the other. But it could not be done with the apparatus in question, and the difficulty was obviated to a great extent by first relieving the pressure on the guide wheels, and secondly by making an alteration in the mounting of the girder. A heavy triangular cast-iron slab was inserted between the girder and bed, and arrangement made to support it freely at three points only, the addition of a right-angled bracket to the bed being necessary for carrying one point of support.

(iv.) *Computation.*—Returning to the comparisons of the bars in eight different positions, it is obvious that they have been all obtained at different temperatures, and before the results can be compared it is necessary to correct them, so that they all appear as if taken at the same temperature, by applying the known expansion coefficients. The best temperature for this purpose is the approximate mean temperature of all the observations. The mean value of the results can then be determined, and hence from the "known" bar the length of the other one can be found.

The direct comparison of two bars by the method just described gives sufficiently accurate results for many purposes, the accuracy being probably of the order of within one in a million; but it is sometimes necessary to obtain a greater accuracy than this, as for instance in the standardisation of certain bars which are used as standards at the N.P.L. and at other similar institutions. This increased accuracy is attained by a method which involves the complete intercomparison of several bars at one time. The manner of computing the results provides not only the most probable value of the bars, but also a measure of the accuracy of the work done.

Any number of bars, from three upwards, may be taken, the greater the number the greater as a rule the probable accuracy obtainable; but it is best to limit it to five or six, as if more be used the slight advantage gained in the results is discounted by the disproportionate extra amount of labour and time involved. Further, if at least five or six bars be taken, it is possible to reduce the number of observations by one half, since it is sufficient in such a case to obtain only four sets, instead of the complete eight sets, for each pair of bars.

For……………………………………

Comparison of ……Ni……(E)

With Standard……Ni.43%……No. (C)

Standard A B ~~C D~~ Position A C……100…… B D……0……

Bar C D ~~A B~~ Position C A……100…… D B……0……

Left | A……Ni……B | C……Ni.43%……D | Right

Date ……18.10.20.

Observers ……L.O.C.J.……

	Thermometers		Left Microscope No. ……A……		Right Microscope No. ……E……	
	Front Nos.	Back Nos.	Cross Wires—Narrow, Medium, Wide Mean Value of 1 Division—0·07%		Cross Wires—Narrow, Medium, Wide Mean Value of 1 Division……—……	
	32053	32055	A	C	B	D
Stir	12·790	12·840				
			937·6		1040·6	
				995·8		1003·8
			956·4		1060·5	
Stir	12·777	12·835				
				1017·1		1025·6
			978·2		1082·3	
				1037·3		1045·7
Stir	12·772	12·822				
			898·6		1104·0	
				1054·6		1064·5
			1018·5		1123·4	
Stir	12·772	12·822				
Mean Readings	12·778	12·830	977·86 (a)	1026·20 (c)	1082·16 (b)	1034·90 (d)
Differences			−48·34 (a—c)		+47·26 (b—d)	
Corrections to give Deg. C. (Hyd.) and Microns	0·000	−0·048	+0·03		0·00	
Corrected Results	12·778	12·782	−48·31 (a—c)		+47·26 (b—d)	
Mean Temperature ……12·780……			Difference (AB minus CD) −95·57 (a—c)−(b—d)			

Remarks—(Experimental conditions; Nature of graduations; etc.……………………

FIG. 5.

Each bar having been compared with all the others in turn, by the method just described, the results (the mean of 4 or 8 sets, as the case may be, for each pair) reduced to a common temperature are set down in rows and columns to form a square, such as is illustrated by *Fig.* 6. The six bars involved in this particular square are denoted by the letters A, B, etc., which, it will be seen, are placed at the heads of the columns and the beginnings of the rows. The observed differences (all expressed at 15° C.) are the figures shown in upright type in the centres of the smaller squares. Any particular result is readily associated with the bars from which it is derived by reading the letters which denote the column and row in which the result is placed. For example, −24·09 is obtained by a comparison of B and D, and is completely interpreted: length of B − length of D = − 24·09 μ. Similarly + 88·14 should be read: length of E − length of B = + 88·14 μ. Care should be taken in expressing the difference between two bars in the correct way, the difference always being equal to (bar denoted by column) − (bar denoted by row). It will be noticed that each result appears twice, but with opposite signs, and a moment's thought will explain why this is so. Six small squares, except for the diagonal through them, are left blank, since there are obviously no results to be placed in them.

Having explained the building up of the square, it remains to show its purpose. The six columns are added together, the separate sums (S) forming another row. The total of these sums should be zero, since the total involves pairs of equal value and of opposite signs. Each sum is now divided by six, the number of bars, thus giving a row M of mean values for the columns. From these mean values we can now *calculate* the *most probable* values for each sub-square by a process of simple subtraction. Taking examples:

Length of A − length of B = mean of column A − mean of column B = + 92·57 − (− 92·75) = + 185·32 μ.

Length of B − length of E = (− 92·75) − (− 4·68) = − 88·07 μ.

In this way calculated values are obtained for all the pairs of bars, and these are inserted in the squares (italicised figures) for comparison with the observed results. The residuals, *i.e.* the difference between the observed and calculated values, are also entered, and serve to give an idea of the accuracy with which the observed results have been obtained. It will be noted that the calculated values and residuals are shown in half the square only; it is obvious that to insert them in the other half would be only needless repetition.

	A	B	C	D	E	F
A		-185·34	-12·49	-161·22	-97·10	-99·27
B	*+185·32* +185·34 +·02		+172·75	+24·09	+88·14	+86·18
C	*+12·37* +12·49 +·12	*-172·95* -172·75 +·20		-149·17	-84·93	-86·86
D	*+161·21* +161·22 +·01	*-24·11* -24·09 +·02	*+148·84* +149·17 +·33		+63·72	+61·80
E	*+97·25* +97·10 -·15	*-88·07* -88·14 -·07	*+84·88* +84·93 +·05	*-63·96* -63·72 +·26		-2·11
F	*+99·28* +99·27 -·01	*-86·04* -86·18 -·14	*+86·91* +86·86 -·05	*-61·93* -61·80 +·13	*+2·03* +2·11 +·08	
at 15° C. S	+555·42	-556·50	+481·22	-411·82	-28·06	-40·26
at 15° C. M	+92·57	-92·75	+80·20	-68·64	-4·68	-6·71
Corrⁿ. to 0° C.	-189·07	-17·14	-119·86	-18·35	-188·50	-5·95
at 0° C.	-96·50	-109·89	-39·66	-86·99	-193·18	-12·66
add	+73·40	+73·40	+73·40	+73·40	+73·40	+73·40
	-23·10	-36·49	+33·74	-13·59	-119·78	+60·74

FIG. 6.

The method thus described of computing the most probable from the observed values is an exceedingly simple operation, yet it satisfies the law of least squares, which in other circumstances is much more troublesome to apply. The calculated results, which are the basis of the finally accepted values of the length of the bars, are probably correct to 1 in 5,000,000, and may on occasion be accurate to 1 in 10,000,000, that is 0·1 μ per metre.

The final step consists in obtaining the absolute lengths of the bars from the mean values M, which so far are only relative, but this cannot be done unless the absolute length of one of the bars is already known by means of some previous determination. But, before doing this, it is necessary to decide at what temperature the final absolute lengths are to be stated. The actual temperature chosen may depend on circumstances, but with standard metres, and standard yard bars, the temperatures at which these are defined, viz. 0° C. and 62° F., are usually selected. The relative values M of the six bars that have just been discussed are stated at 15° C., and since they are metre standards, the values are usually expressed at 0° C., and utilising the predetermined coefficients of thermal expansion, corrections are applied to the means M. The next two rows, *Fig.* 6, will make this step clear, but the values thus stated at 0° C. are still relative only. Bar A is taken as the "known" standard, with a length at 0° C. of 1 metre − 23·10 μ.

The figure −23·10 is substituted for +96·50, and the others are amended accordingly, thus giving the absolute lengths of all of them at 0° C., *e.g.* C's equation to scale is 1 metre + 33·74 μ, or 1·00003374 metres.

It may be interesting to add that A, B, etc., are six standards actually in use at the N.P.L. A and B are made of pure nickel, B and E of invar, C of 43 per cent nickel steel, and E of silica. *Fig.* 6 gives the results of their actual intercomparison during the month of October 1920. Bar A had just previously been compared direct with a copy of the International Prototype Metre at Sèvres, and hence the reason for selecting it as the basis of the determination of the lengths of the others.

§ (2) Coefficients of Expansion.—Thermal expansibilities are so indissolubly associated with all line standard work that it is necessary to be able to measure with the highest possible accuracy the coefficients of expansions of the various bars used. Means of doing this over normal ranges of temperature are provided by the N.P.L. comparator and other comparators of similar design. The method employed is entirely analogous to that of comparing the lengths of two bars.

Two standards are placed in the inner trough in the usual way, one of them having at the commencement a known coefficient of expansion, from which that of the other may be derived. The lengths of the two standards are compared at various temperatures varying from about 0° C. to 35° C. The various differences obtained are plotted against temperature, and the points will as a rule be found to be on a nearly straight line. If the mean coefficient only is required over the range of temperature used, it will be sufficient to draw the straight line passing through or near the points, and obtain the coefficient accordingly. But the most accurate line standard work demands something better than this, and as the expansibilities almost invariably follow a quadratic law of expansion and not a linear one, it is necessary to find the quadratic or B term of the expansion equation $L^\theta = L_0(1 + a\theta + \beta\theta^2)$, and this is best computed with the help of the law of least squares.

The temperature of the standards is varied by means of hot water or ice placed in the outer trough, and where both a and β are to be determined, it is best to take as many points as possible on the curve, seven at least.

It is evident that this method can be employed for determining the expansibility of any material which can be fashioned into a rod or bar, and which can be made to carry two reference lines, one near each end. The distance between these two lines can be compared at different temperatures with a like distance on a line standard whose coefficient of expansion is known, in the manner just described. The lines can be ruled on the polished surface of the rod in the case of metals and like substances, but with other materials it is necessary to insert metal plugs at each end for this purpose. With some materials the water as a medium must be dispensed with. For example, steel rusts in it, brick absorbs it, etc.

It may be replaced by some other liquid, such as paraffin, which does not affect steel (say), or the determination may be carried out in air. In the latter case the result will not of course be so reliable as a similar result obtained with a liquid medium.

§ (3) Double-tank Comparator.—While the comparative method of determining coefficients of expansion in a single-tank comparator meets most cases that arise, it becomes necessary at some time or other to find by an absolute method the expansibility of a bar. Resort is therefore made to a comparator fitted with two separate tanks. Such a comparator, as used at the International Bureau, is illustrated in *Fig.* 7, from which its main features will be evident. The two tanks are on a common platform, with sufficient space between them to prevent the heat of one affecting the other when they are at different temperatures. The platform,

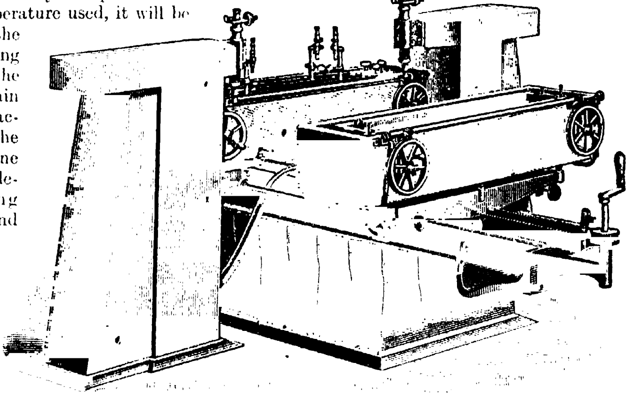


Fig. 7.—Double-tank Comparator. (Made by La Société Genevoise.)

with the tanks, can be moved along rails, so as to bring each tank in turn under the microscopes, which are mounted on two separate pillars on either side of the platform.

Other comparators of this type, the Universal Comparator at Berne, and the four-metre comparator of the Indian Geodetic Survey, are more fully described later, and reference should be made to them (§§ (6) and (8)). The method of using the double-tank comparator for obtaining absolute coefficients is briefly as follows. The bar whose expansibility is to be found is placed in one of the tanks, and a reference bar (not necessarily of the same material) is placed in the other. The temperature of the reference bar throughout the series of observations is kept constant, preferably with the assistance of a thermostat. The temperature of the other bar is varied in the usual way, and its length at different temperatures compared with the unvarying length of the reference bar. The differences in length obtained obviously give absolute expansion, from which the absolute coefficient is determined. It is thus seen that the method of taking observations is quite analogous to the comparative method employed with the single-tank comparator, and differs only from it in that one bar (the reference) is kept at a constant temperature, and therefore of unvarying length.

§ (4) TUTTON WAVE-LENGTH COMPARATOR.—This comparator is so designed that it is possible to determine the difference between the lengths of two standards by two different methods. The instrument may be used in exactly the same way as the N.P.L. comparator, by alternately observing the lines of each bar with the help of two rigidly fixed micrometer microscopes; or it may call to its aid the distinguishing feature of the apparatus—a special optical arrangement which enables the difference in the lengths of two standards to be measured in terms of wave-lengths of monochromatic light. This optical device consists chiefly of the Tutton interferometer[1] for producing fringes or bands, together with a suitable source of light, means of selecting and directing the particular kind of light required and of viewing the bands, and a mechanical arrangement for producing slow and steady relative motion between the interfering surfaces.

"The essence of the interferometer is that homogeneous light, of a definite wave-length corresponding to a single spectrum line—isolated with the aid of a constant-deviation prism from the spectrum derived from a cadmium or hydrogen Geissler tube, or a mercury lamp—is directed by an auto-collimation method, ensuring identity of the path of the incident or reflected rays, normally on two plane surfaces, arranged close to each other and nearly, but not absolutely, parallel; the two reflected rays give rise, by their interference, to rectilinear dark interference bands on a brilliantly illuminated background in the colour corresponding to the selected wave-length.

"In the instrument now described, one of these two reflecting surfaces concerned in the production of the interference bands is carried by, and moves absolutely with, one of the two microscopes employed to focus the fiducial marks, or 'defining lines,' determinative of the length of the standard, the other being absolutely fixed. The movement of either of the surfaces with respect to the other causes the interference bands to move, and the extent of movement of the surface is equal to half the wave-length of the light employed for every interference band that moves past a reference mark carried by the fixed surface. The movement of the microscope parallel to itself and to the length of the standard bar is thus measured by counting the number of bands, and the initial and final fractions of a band which are observed to pass the reference spot during the movement, and multiplying that number by the half wave-length of light radiation used in the production of the bands. It is only necessary, therefore, in order to compare the lengths of two bars, (1) to place the bar of known length, say, the Imperial Standard Yard, under the two microscopes so that the two defining lines are adjusted in each case between the pair of parallel spider lines carried by each of the micrometer eyepieces; (2) to replace the standard by the copy to be tested, so that the defining line near one end is similarly adjusted under the corresponding microscope, then, if the other defining mark is not also automatically adjusted under the second microscope which carries the interferometer glass surface, as it should be if it is an exact copy, (3) to traverse that microscope until it is so adjusted, and (4) to observe and count the number of interference bands which move past the reference spot during the process. The product of this number into half the wave-length of the light used to produce the bands thus obviously affords the difference between the two lengths included between the defining marks on the two bars."[2]

Such an optical arrangement depends for its success on the exceptional accuracy of finish of certain mechanical details. It is necessary in traversing the microscope that it should move truly parallel to itself, *i.e.* without any rotational motion throughout the whole distance of its traverse, and this condition can only be fulfilled by ensuring that the various surfaces which are in sliding contact are lapped extremely truly, a task obviously calling for the highest mechanical skill. Also, the motion of the microscope, and hence the passage of the bands across the field of view, must be under perfect control, and this is effected by a special slow-motion screw. That these mechanical details are satisfactory is shown by the fact that the bands move across the field of view with perfect steadiness, without rotation in the field of view, without alteration in width, and with no vibration, and when the actuating screw is stopped they cease moving immediately.

---

[1] *Phil. Trans.* A, 1898, cxci. 324.

[2] Tutton, *Phil. Trans.* A, 1910, ccx. 1.

In the following paragraphs a brief description is given of the main details of the apparatus.

The Tutton Comparator (*Fig.* 8) is in use at the Standards Department of the Board of Trade, Old Palace Yard, Westminster, where it is housed in a room in the basement. The floor of the room is 10 feet below the street level, and the actual foundations for the instrument commence 4 feet lower still. The foundations, which are "isolated" from the surrounding floor and earth, consist of two concrete blocks carrying two stone blocks, which in turn support the apparatus.

Since it is necessary to control the temperature during the course of observations, which may at times be rather protracted, the room is provided with a number of electric heaters, which are controlled by means of a thermostat. The thick stone walls and the double doors assist by reducing the amount of radiation in or out of the room. The constant temperature of 62° F. is thus maintained, day and night, throughout the year.

The greater part of the apparatus is carried on the larger of the two stone blocks, the smaller and lower block carrying the interferometer-telescope pedestal. The large block, on its upper surface, carries a V and plane bed 6½ feet long. In front of the block, and forming a step 7½ inches wide, is another surface, nearly 8 inches lower than and parallel to the top surface, and this also carries a V and plane bed. Both beds are identical except in width. They lie on iron plinths securely bolted to the stone block, are made of close-grained cast-iron so as to secure homogeneity, are machined and lapped very accurately, and are so secured to the plinths as to be under the least possible strain.

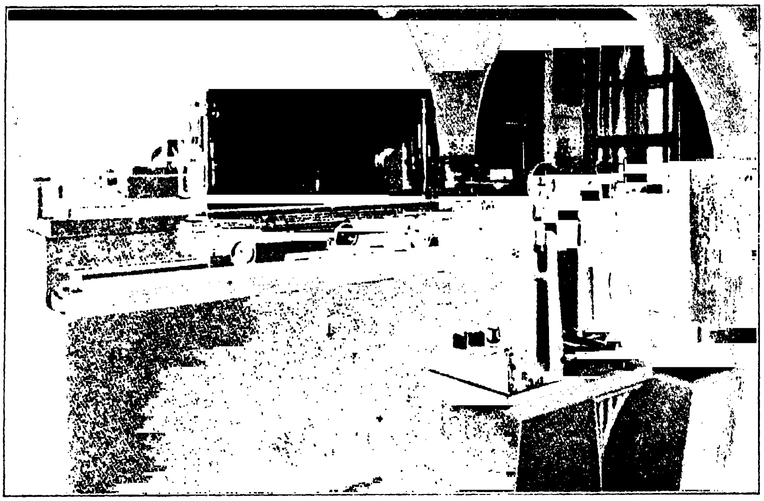


FIG. 8.—General View of Tutton Wave-length Comparator.

The lower bed carries a special table, on which are placed the standards to be compared, and which are capable of various adjustments, both quick and fine, which are needed to bring the lines of the standards into correct position in the field of view of the microscopes. Some of these adjustments are of a novel nature, and provide a smooth and easy motion devoid of jerkiness.

The upper bed carries two sliding steel blocks, which can be secured to the former in any desired position. Each block is finished so that the upper surface provides a V and plane bed for the support of a steel slab to which is rigidly attached a microscope overhanging the table on the lower bed. The motion of the slab over a block is controlled by the special slow-motion screw already referred to, the mechanism being briefly as follows. In a cylindrical hole, bored through the length of the block parallel to the direction motion of the block, slides a well-fitting phosphor-bronze rod, which carries a projecting piece engaging through a slot in the upper part of the block with the slab above. The rod, and therefore the slab, is moved by means of a fine screw, fifty threads to the inch, fitting into one end of it. The end of the screw which projects from the block carries a worm-wheel of 100 teeth, which engages with an endless screw at right angles to it. This endless screw is worked through a flexible shaft by means of the control wheel mounted on the small stone block in front of the instrument and near to hand for use by the observer viewing the bands in the telescope. One turn of the control wheel, which is graduated on its circumference, corresponds to the passage of fifteen interference bands across the field of view, corresponding to a movement of the microscope of ·005 mm. The fine screw is also provided with a milled head for turning it independently of the control wheel, together with a divided drum for taking measurements of the traverse.

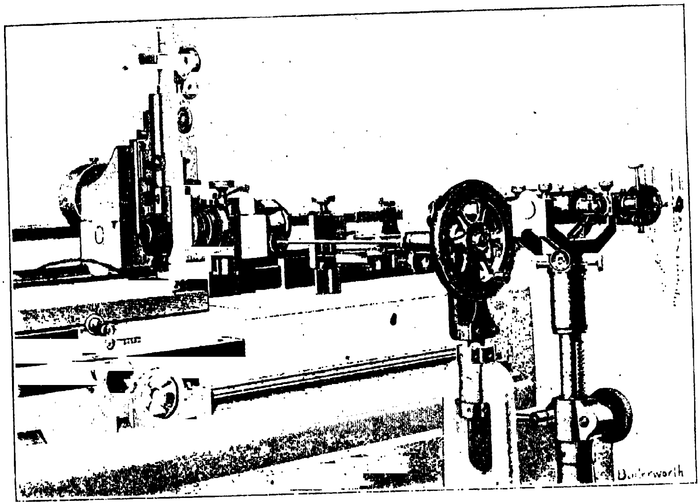


FIG. 9.—Central Part of Tutton Wave-length Comparator, showing Interferometer.

As already mentioned, the parts referred to in the foregoing paragraphs are those calling for such refinements in accuracy of the moving parts (the straightness and flatness of the bed, etc.), and rendering possible the use of the interferometer.

The microscopes, specially made by Messrs. Beck, are fitted with various adjustments so that they can be set up with their optical axes vertical and parallel. Each is counterpoised by a leaden weight on the opposite side of the steel slab. They are fitted with double-motion cobweb micrometer eyepieces, and the magnification can be varied according to the nature of the lines under observation. Using a $\frac{2}{3}$-in. objective, and eyepieces No. 1 and No. 2, magnifications of 150 and 280 are obtained. Either of these arrangements of lenses is suitable when bars like the Imperial Standard Yard are being observed, since it permits of focussing on to the lines at the bottoms of the cylindrical holes and at the same time provides sufficient clearance for the bars to be traversed under the microscope.

With a $\frac{1}{15}$-in. dry objective, magnifications of about 1600 and 3000 are attained, suitable for viewing such fine lines as the Grayson rulings. Illumination of the microscope is by means of a distant Pointolite lamp, the rays from which are first filtered through a copper acetate solution, thus giving a greenish blue light.

The interferometer (*Fig.* 9) consists of an auto-collimating telescope, a dispersing apparatus, and three truly plane glass plates, two of which are responsible for the formation of the fringes.

The telescope is fixed by the side of the control wheel previously mentioned, with its axis at right angles to the run of the bed of the comparator, and is carried on a rigid pedestal resting on the lower and smaller stone block. The pedestal has adjustments by means of which the telescope can be moved in three directions mutually at right angles.

The source of the monochromatic light is a cadmium, hydrogen, or neon vacuum tube fixed by the side of the telescope.

The dispersing apparatus rests on an adjustable horizontal divided circle attached to the upper bed of the machine, and serves to select the required radiation. The dispersion may be effected either by two refracting prisms, or by a Hilger constant deviation prism. The latter has proved to be very satisfactory in practice ; it has the advantage that, in order to change from one kind of radiation to another, only one operation is necessary, viz. the rotation of the prism.

Of the three optical plates or discs (*Fig.* 10), one, g_3, is of black glass, and the other two, g_1 and g_2, of clear glass. The black disc, optically polished on one side and ground on the other, is suitably mounted on the microscope just above the objective and so that a line normal to its surface is parallel to the run of the bed. The two clear glass discs, polished truly on both sides, are in the form of wedges with 35-minute angles, and are identical. They are supported in a suitable holder G, so that they are very nearly parallel to one another and to the black glass disc, but so placed that the thin end of one is opposite the thick end of the other. The holder G is fitted to, and can slide along, the front of the upper bed, between the microscope and the dispersion prism, and can be rigidly fixed for use in any desired position. All three discs are fitted with adjusting screws, c, h, and k, so that the various polished surfaces may be set correctly with respect to one another.

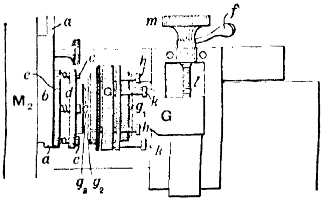


FIG. 10.

Interference takes place between the polished front surface of the black glass g_3 and the nearer surface of the disc g_2 immediately opposite, and the angle between the two interfering surfaces is so arranged that the fringes appear vertical in the field of view of the telescope. Reflection from the back surface of g_2 is eliminated by its 35-minute inclination to the front surface. But this same inclination of the two surfaces gives rise to a slight dispersion effect, the correction of which is brought about by the introduction of the third or countervailing disc g_1. The bands of course move across the field of view when the distance between the interfering surfaces is varied, and this, as is noted earlier, is brought about by the movement of the microscope.

It has already been noted that the apparatus can be used with or without the interferometer, but whichever method is employed for comparing two standards, the comparator has the advantage that the conditions existing during a set of observations can be repeated at a later time, mainly because the temperature is under control.

The control of the bands is, as a rule, a matter of care and patience, and is frequently a very tedious process when the fringes are numerous. The number of bands observed, however, depends, at the commencement and at the finish of the counting, on the way in which the central axes of the defining lines on the standards are estimated in the microscopes. If the lines are wide or, worse still, have irregular edges, the operation of judging the centres of the lines is a difficult process, and it is probable even that different observers may make different estimations, with a corresponding difference in the total number of bands counted. The coarseness of the lines is therefore a drawback to the method, and may give rise to errors out of all proportion to the accuracy with which measurements can be made with the interferometer. Some idea of the coarseness of the lines may be gathered from the fact that the width of those on the imperial standard yard corresponds to the passage of forty-five interference bands past a reference spot, and those on the platino-iridium yard to fifteen bands. The remedy appears to be to produce standards with lines the width of which is comparable with a wave-length, coupled with an increased power in the microscope, but this point has been discussed elsewhere.[1]

§ (5) THE BRUNNER COMPARATOR.—The Brunner Comparator,[2] *Fig.* 11, at the Bureau International des Poids et Mesures, is employed only in the comparison of standards one metre in length. In principle it is the same as the N.P.L. comparator, and differs from it mainly in the manner in which the microscopes are mounted, these being supported solidly by two massive pillars, one on either side of the rest of the apparatus, which is supported on a separate block. The tank, fitted with girder and rollers for supporting the bars under observation, is fixed to a carriage which in turn rests on a cast-iron base. By means of wheels attached to the carriage, the latter (and therefore the tank) can be moved along the bed, to and fro under the microscope, or can be moved entirely from under the microscope to render the interior of the tank more accessible. The carriage is fitted with an adjustment whereby the four screws on which the tank rests can be simultaneously turned, and the tank therefore bodily moved vertically.

---

[1] "Line Standards," § (1) (vi.).

[2] *Trav. et Mém. de B.I.P.M.* vii.

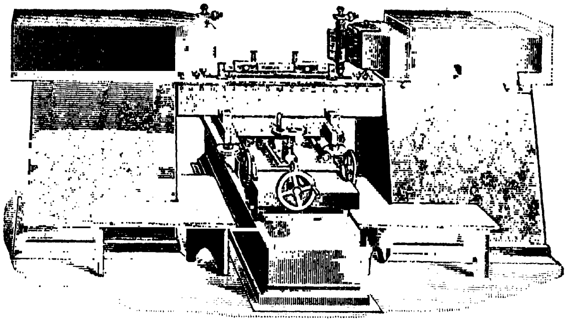


FIG. 11.—The One-metre Comparator at the Bureau International des Poids et Mesures. (Made by Brunner frères.)

§ (6) THE UNIVERSAL COMPARATOR AT BERNE.—This comparator (*Fig.* 12), made by the Société Genevoise, is in use at the Bureau Fédéral des Poids et Mesures at Berne. Its design is such that all the determinations required in connection with line standards or scales may be made by means of the one piece of apparatus. Thus, using only one of the two tanks, it is possible (*a*) to compare any two similar standards having any length between the limits 0·1 metre and 1·25 metre, and (*b*) to determine coefficients of expansion by the comparative methods. Employing both tanks, (*c*) absolute coefficients of expansions can be measured. Further, by removing one of the tanks and replacing it by a specially made bed provided with a longitudinal movement, it is possible (*d*) to calibrate the subdivisions of a scale, and (*e*) to compare short scales (*i.e.* those which cannot be determined by the method indicated in (*a*) above). For method of carrying out (*d*) and (*e*) see "Subdividing Comparator" (§ (9)).

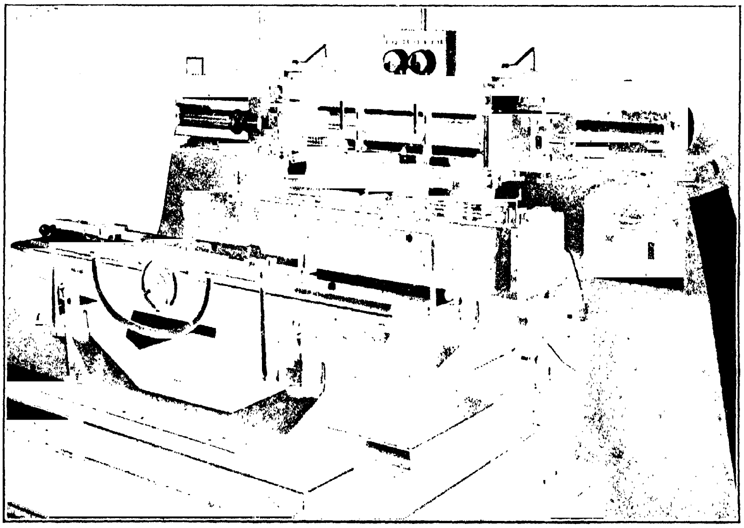


FIG. 12.—The Universal Comparator of the Bureau Fédéral des Poids et Mesures at Berne. (Made by La Société Genevoise.)

The main features of the apparatus are given below and will be readily followed by means of the illustration. The microscopes are carried on a strong cast-iron box-girder, which can be filled with water for the purpose of preventing abrupt changes in its temperature. The girder is supported at the ends on two separate massive pillars, and any longitudinal constraint in it is obviated by means of three balls which provide the actual points of supports, and which rest in the usual hole slot and plane. One vertical side of the girder is provided with two horizontal guide-rails, to which the microscopes are fixed, and along which they may be moved as desired. The microscopes can be fixed at any desired distance apart between the minimum 0·1 metre and the maximum 1·25 metre, and a scale fixed to the slide facilitates the operation of setting them at the required distance apart. Each microscope is fitted with all the necessary adjustments, and illumination is provided by small electric glow lamps placed near the objective, the current being fed through two special horizontal insulated rods attached to the girder, contact being made with these by means of brushes attached to the microscope slide. The microscopes are fitted with objectives giving magnifications of 60 and 100 diameters.

One of the tanks is fitted with two troughs, the inner one being fitted up in the usual manner for receiving two standards or scales, and provided with the usual adjustments in three azimuths. Provision is made for the thermometers, and for stirrers worked by a small outside motor. This is the tank used for ordinary comparisons of length and for the determination of coefficients of expansion ((*a*) and (*b*) above).

The other tank is a single one and contains one girder only with its adjusting gear. In conjunction with the other tank, it is used for the determination of absolute coefficients of expansion ((*c*) above).

Both tanks will, as already gathered, accommodate specimens up to 1·25 metre in length. Placed with a sufficient distance between them, they are carried on special cast-iron supports, which are bolted to a cast-iron frame which rests on four rollers. These rollers can be moved along two rails on the main bed underneath, one rail being an inverted V shape in section, the other being flat, the wheels being shaped to fit accordingly. Side play in the wheels is eliminated. The frame or truck, and therefore the tanks, can be moved to and fro under the microscopes by means of a special screw working in a nut attached to the truck, power for the purpose being supplied by a small electric motor. This provides for great displacements, but fine adjustments can be made by means of two small fly-wheels set in motion by hand and rotating the nut only.

The single tank can be removed and replaced by a light but rigid cast-iron bed, on which can slide longitudinally a plate provided with various adjustments, and with supports for a bar having any length up to 1·1 metre. Two small plates are also provided for use with two small scales, and each has its own vertical adjustment so that any difference in the thickness of the scales may be allowed for when bringing their graduated surfaces into the same plane. The sliding movement is made by means of a rack and pinion actuated by a hand wheel which is provided with a tangent screw for fine adjustments. A wooden cover protects scales and bed against temperature changes. This portion of the apparatus is used for calibrating scales and for comparing short scales ((*d*) and (*e*) above), and *Fig.* 12 shows it in position with the cover removed.

The observer, when using any portion of the apparatus, stands on a movable platform which follows the movement of the truck. This platform surrounds the double tank, and enables the observer to place himself either between the tanks for reading on either of them, or on the outer side of the double tank.

§ (7) FOUR-METRE COMPARATORS. — The four-metre standard, as mentioned elsewhere,[1] provides the link between the ultimate standard length, the metre, and the longer standards in the form of tapes and wires, and it is necessary, therefore, to provide a special comparator in which the four-metre standard may be compared with the metre standard. Such a comparator is only a large edition of the smaller variety, and is manipulated in the same way.

(i.) *The N.P.L. Four-metre Comparator* (*Fig.* 13) is constructed on the same principle as the one-metre comparator. The double tank is provided with two girders supported at the ends, and capable of adjustments in three azimuths. It is supported at either end on two concrete pillars, and is moved to and fro on four wheels, two at each end, each pair running in a V groove in a plate attached to the concrete block. The bed for the microscope supports rests on a concrete block, just over twelve feet long, and placed immediately behind the tank. There are five microscopes for viewing the five lines defining the metre lengths on the four-metre bar. Illumination, etc., are as in the N.P.L. one-metre comparator. An electric motor under the tank works two rotary pumps, which circulate separately the water in the two tanks.

Method.—In order to carry out the comparison, the longer standard is placed on one of the girders, with its supporting rollers at the Airy points. The metre standard, also supported at its Airy points, is placed on the other girder opposite one of the metre lengths of the longer standard, say the first. These two-metre lengths are then compared in precisely the same way as two similar lengths in the one-metre comparator. The metre bar is then turned end to end and the comparison repeated. Next, the shorter bar is moved until it is opposite the second metre length of the longer standard, and two comparisons again made. The process is then repeated with the third and fourth lengths. The next step consists in turning the longer bar end to end, and obtaining further pairs of observations on the metre length.

---

[1] "Line Standards," § (3).

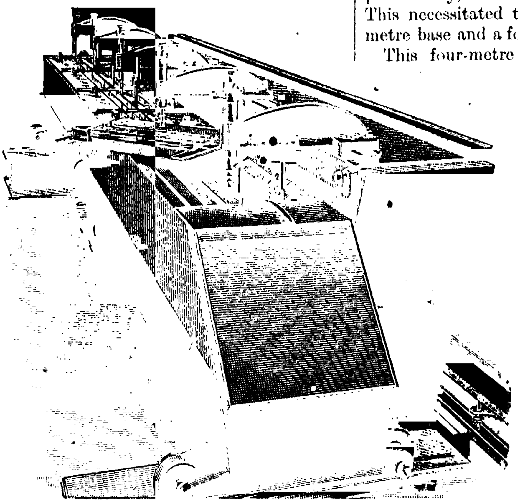


FIG. 13.—The Four-metre Comparator of the National Physical Laboratory.

After this the bars are exchanged on their girders, and similar sets of observations made.

Thus each metre length of the four-metre standard is compared eight times with the metre standard. When all the results have been corrected to the same temperature—and this correction is almost negligible for bars of the same material—the mean of each eight sets can be found. Then the sum of the four results will give the total error of the four-metre length, the accuracy of the determination being probably equal to that obtaining with the known length of the metre standard, against which it has been compared.

(ii.) *Coefficients of Expansion.*—The N.P.L. four-metre comparator can of course be used for determining coefficients of expansion of standards or materials that cannot be accommodated in a one-metre comparator, a comparative method being necessarily employed.

§ (8) THE FOUR-METRE COMPARATOR OF THE INDIAN GEODETIC SURVEY.—Just previous to the commencement of the great war, the Indian Government considered the question of bringing the equipment of the Geodetic Survey up to date, and with the assistance of Sir David Gill (who was appointed consulting engineer), the N.P.L., and the International Bureau, it was decided to acquire a set of invar tapes and wires, and, for checking these periodically, a number of four-metre standards. This necessitated the provision also of a 24-metre base and a four-metre comparator.

This four-metre comparator is illustrated in *Fig.* 14, and embodies many features due to the late Sir David Gill. It is primarily intended for the determination of the lengths of the four-metre standards against predetermined one-metre standards, but of course can be used also for the comparison of any two similar lengths up to four metres, and for the determination of coefficients of expansion, either absolutely using both tanks, or comparatively using one tank only. The way in which the apparatus is manipulated and the manner of taking observations have already been described in connection with other comparators, and it only remains here to give some account of its construction and working.

The construction of the comparator was entrusted to the Cambridge Scientific Instrument Co., and was set up and tested at the India Office Store in London before being despatched to India.

The microscopes MM are carried on a cast-iron box-girder or bridge G, which is supported at its ends by two masonry pillars. The girder is well lagged and filled with water, which by convection and its high specific heat tends to maintain the girder at a uniform and constant temperature, and therefore reducing to a minimum any change in the length of the girder and in the distance between the microscopes. In order to prevent strains due to temperature, the girder is mounted on three Hoffman balls, two at one end K, and one at the other end L. Of those at K, one rests between two coned seatings and the other between two parallel plates, while the ball at L rests in two parallel opposing V grooves. Along the upper and lower edges of one long vertical side of the girder are blunt V-section-shaped guide-rails to which may be clamped, and along which may be moved, the two microscope carriers. The latter are made of cast-iron, are roughly triangular in shape with the apex downwards, and are provided at the corners of the triangle with "claws" for engaging with the guide-rails, the bottom one being provided with a clamp. The two Zeiss micrometer microscopes are arranged to give magnifications of 15 or 25, and are provided with adjustments for setting them vertical, parallel, and at the same height. Illumination is secured by either a small glow-lamp near to the microscope in each case and used only in the preliminary observations, or by a distant source of light, used when observations are made. The microscopes can be set apart at any distance varying between ·17 metre and 4 metres.

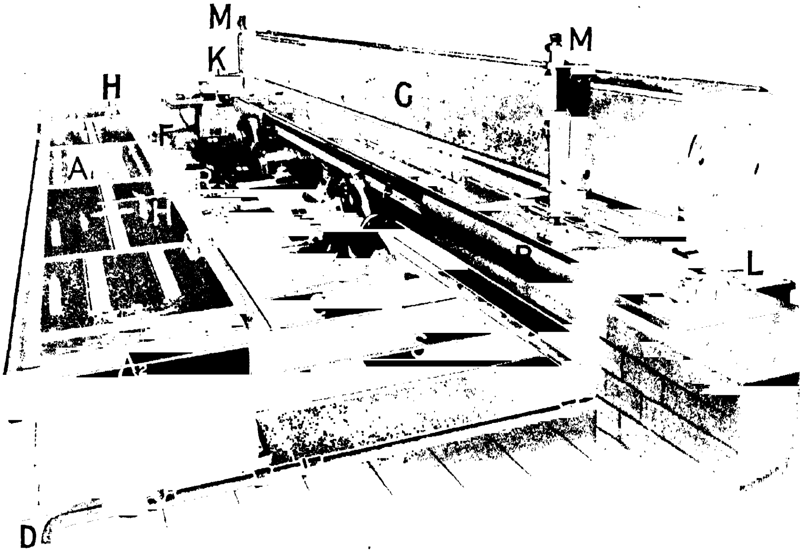


Fig. 11.—The Four-metre Comparator of the Indian Geodetic Survey. (Made by the Cambridge Scientific Instrument Co.)

On rails S below the bridge is the carriage C for the tanks A and B and their various accessories. The rails are about 4½ feet apart, one of them, acting as a guide rail, is bevelled along its top, while the other is flat-topped, the two wheels working on the former being flanged, and the two working on the latter being barrel-shaped. The carriage—a framework of girders and tie-rods—is traversed under the bridge and microscopes by means of a square-threaded screw-shaft, centrally placed between the rails. This screw moves in a block coupled to the carriage, and is worked either by hand or by means of a ½-horse-power motor. The carriage, when near the end of its run in either direction, comes into contact with and operates a tumbler switch, thus cutting off the current of the motor and preventing any over-running of the carriage.

Of the two tanks provided, one A is double—that is, it consists of an outer and inner trough, A_2 and A_1 respectively—and is fitted to accommodate two standards. The other tank B is a single one, and can accommodate one bar only at a time. The troughs of both tanks are made of copper, and each tank is lagged with felt and teak boards. The inner tank is supported at the Airy points of the four-metre bar. The gun-metal girders of both tanks are of I section, and are supported at the Airy points by means of gun-metal saddles, which are provided with means of adjustment so that the girders may be given slight movements, up and down, to and fro, and longitudinally. These adjustments are operated by hand by means of vertical spindles which project through the covers of the tanks. The covers, five in number for each tank, are slotted longitudinally for viewing the bar or the thermometers, and also transversely at metre intervals corresponding to the metre intervals of a 4-metre bar. Attached to each girder at intervals are crutches for supporting the mercury thermometers. Each trough is filled with water, the bars in the troughs A and B being thereby totally immersed. Each has its own system of circulating the water by means of rotary pumps driven by motors. The water, drawn out by one pipe, passes through the pump and is returned to the trough by another pipe, the whole of the water being completely circulated every 1½ minute, and equalisation of temperature thereby maintained. The three pumps and motors are carried on a platform placed between the two tanks.

Placed in the water circuit of the outer tank are an electric heater for raising the temperature of the water, and a thermostat R for maintaining it at any desired temperature, both being carried on the platform between the tanks. The electric heater consists of eight heating coils, carried in seven narrow flat pockets placed vertically, and communicating with a common chamber at the top, two of the heaters being in one pocket. Pockets and chamber are filled with oil. The whole is in a box, and the water is made to circulate through the box and past the pockets, baffle plates fixed between the pockets helping to direct the flow and bring the water into intimate contact with the heating units. Of the two heaters in one pocket, one is connected with the thermostat, the other one, together with the remaining six, is connected with the 100-volt circuit, and any or all of them can be put into service by means of switches on the lid of the box. The thermostat consists of a number of tubes, connected with a short length of glass capillary tube projecting through the lid of the thermostat, and the whole filled with mercury, thus forming a thermometer "bulb." The capillary tube ends in a short open-ended funnel. Just above the mercury surface is a platinum needle, the distance of whose free end from the top surface of the mercury can be controlled by a micrometer screw. The height of the mercury surface is also adjustable by another micrometer screw working on the diaphragm of a cell connected with the thermometer bulb.

To raise the temperature of the water, most or all of the heaters are put into action until the desired temperature is nearly attained. Certain of the coils are then cut out, leaving in operation, in addition to the thermostat coil, only those necessary to compensate nearly, but not quite, for the loss of heat due to radiation. The thermostat coil more than makes up any loss of heat, and the temperature continues to rise until the rising mercury column meets the platinum wire (the distance between them having been previously adjusted) and so closes a circuit containing a battery which operates a relay F, which, in its turn, cuts off the thermostat coil. The temperature then falls slightly until contact between the mercury and platinum is broken and the thermostat coil again called into action. This cycle of operations is repeated so long as it is required, and by its means any temperature can be maintained to about 0·01° C. for an indefinite period.

If it is desired to reduce the temperature of the water, it may be circulated through a cooling tank, and back through the heater and thermostat (both out of action, of course) to the trough.

The microscopes for reading the thermometers are of two types. One is of the ordinary kind, mounted on a tripod, and calls for no further comment. The other kind is designed to overcome the difficulty of reading the temperature when the tank containing the thermometer is underneath the bridge, and when the ordinary type cannot obviously be used. The microscopes are therefore bent, as the diagram indicates (*Fig.* 15), the path of a vertical ray of light through the objective O being twice deflected at right angles by two total reflection prisms into the eyepiece E. To balance the weight of this horizontal arm a counterpoise C is added.

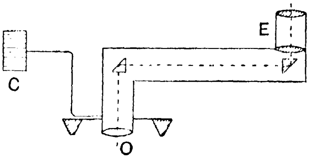


FIG. 15.

In order to eliminate the effect of surface disturbances of the water, the apparatus is provided with immersion glasses which consist of a piece of plain glass set at the bottom end of a short brass tube carried by a small stand with three studs or feet. When in use the immersion glass is placed in the opening of the tank over the graduation line, the three feet resting on the tank-cover, and the glass below the surface of the water. (See sketch, *Fig.* 16.) To prevent the possibility of dust and air-bubbles adhering to the glass if the device is placed directly in the water, a special holder in the form of a saucer with a bent handle is provided. The saucer is filled with clean water, and the immersion glass carefully placed in it. The two together are placed in the water, and when the immersion glass is seated in its place the holder is withdrawn. The use of such an immersion glass will give good results only when the glass has flat and parallel surfaces, and is placed in the water horizontally, thereby preventing any relative displacement of the image of the line.[1]

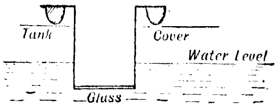


FIG. 16.

---

[1] See *Engineering*, Aug. 20 and 27, 1915, c. 179, 209.

§ (9) LONGITUDINAL OR SUBDIVIDING COMPARATOR.—(i.) This apparatus, as its name implies, is used mainly for examining the lengths of the subdivisions of a standard or scale, and so obtaining the amount by which each deviates from its nominal length. It is so designed that nominally equal subdivisions of a standard or scale may be compared one with another with great facility, and then with the help of the over-all length of the scale, which has been obtained by means of one of the transverse comparators already described, their individual lengths may be determined. In other words, a calibration of the scale is obtained, and provides therefore a selection of shorter standards that may be used for the direct measurement of corresponding lengths on other scales.

The comparator can also be used for the comparison of a bar of unknown length with one whose length is known, provided the apparatus will accommodate the two placed end to end, and it becomes the essential instrument for this purpose when the two lengths are too short for observation in a transverse comparator. It will be realised that with the latter apparatus a certain minimum observable length is imposed upon it by the limiting relative positions of the microscopes when, by reason of the space they and their supports occupy, they cannot be brought any closer together. This minimum distance is usually about one decimetre, *i.e.* about four inches, and hence the necessity for employing other means for determining lengths shorter than this. The same limitation of distance exists of course between the microscopes of the subdividing comparator, and it will be made clear in what follows how the latter overcomes the apparent difficulty of dealing with the shorter lengths.

(ii.) *N. P. L. Subdividing Comparator. Description.*—A brief description of the comparator in use at the N.P.L. will suffice to indicate the general features of such an apparatus.

This comparator (*Fig.* 17), in common with others of the type, is simple in both construction and manipulation. It consists essentially of a main bed, a carriage which can be moved along the bed, and two microscopes suitably mounted on another bed for observing the lines. The whole is supported on three concrete blocks which rest on the bottom of a brick-lined pit, and which are isolated from the walls of the latter by an air-space. Two of the blocks, about 18 in. × 12 in. square horizontal section, are set about four feet apart, and serve to support securely the ends of the main bed which stretches from one to the other. The other block of rectangular horizontal section, about 4 ft. × 1 ft., is situated between these, and so that it is immediately behind the bed. It carries on its upper surface, which is sufficiently elevated for the purpose, the bed for the microscope mountings. The main bed, about 8½ ft. long and 4 in. wide, is made of cast-iron, and is of sufficiently stout construction to support the comparatively light carriage, which can slide along the V and plane of the bed, the direction of motion being parallel to the length of the bed, and hence the term *longitudinal* in contradistinction to that of the *transverse* type of comparator previously referred to. The carriage consists of a flat iron plate, about 5 ft. long and 6 in. wide, to the end of which are bolted cast-iron uprights to which are secured the supports or steps for the single girder employed for carrying the scale or scales to be observed. This girder is supplied with adjustments similar to those on other comparators, and is also provided with the usual rollers, which can be fixed at the Airy points of a scale. The girder and supports are enclosed in a wooden box of which the iron plate forms the base. The lid is made in two pieces which, when in place, leave a

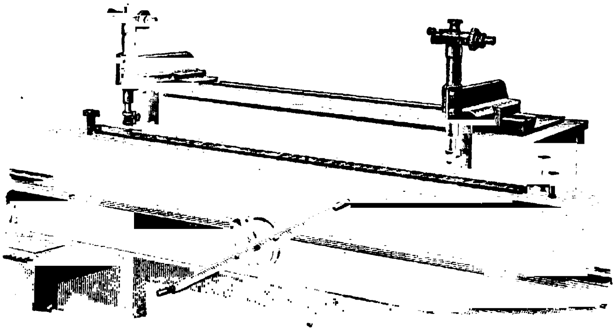


FIG. 17.—The Longitudinal or Subdividing Comparator at the National Physical Laboratory.

slot through which the whole scale can be viewed; but in use the slot is usually covered with a series of small blocks of wood which can readily be moved so as to permit of any line being viewed without exposing the whole scale. The underneath side of the carriage is provided with a rack which, with a pinion working in it, and connected to a hand wheel in front, provides means of traversing the carriage to and fro. The microscopes are the same type as those used on the N.P.L. transverse comparator, are mounted in the same manner, and are provided with the same system of illumination. A thermometer placed inside the box completes the equipment.

§ (10) THE LONGITUDINAL COMPARATOR, illustrated in *Fig.* 18,[1] at the Bureau International des Poids et Mesures, was designed by MM. Benoit and Guillaume expressly for the purpose of comparing standard decimetre scales. It consists of a cast-steel base in the form of a trough supported by three adjusting screws. Its internal dimensions are 1·2 m. × ·23 m. In the bottom of the trough is a V and plane bed supporting a carriage 65 cm. long which can slide along the bed, and to which motion is imparted by means of a rack attached to the near side of the carriage, and a pinion worked by a hand wheel in front of and outside the trough. This wheel is graduated along its outside edge into 100 parts, which, as the wheel is turned, indicate the number of millimetres through which the carriage is moved. Fine adjustments of the carriage are obtained by a long lever which can be clamped to the pinion spindle by means of a screw, but which otherwise is left free and out of action.

Resting on two thick strips of nickel placed along the top edges of the trough are two microscopes, which can be fixed in position by means of binding screws. The microscopes are so mounted that they can be adjusted easily in three azimuths, and are illuminated in the usual way.

The carriage supports a plate, mounted on three adjusting screws for levelling purposes, and this again, when necessary, supports two smaller plates, each adjustable in the same way. The smaller plates are specially adapted for supporting decimetre standards.

The trough is provided with a suitable sectional cover, which is placed in position when the apparatus is set up and ready for use, in order to assist in obtaining a uniform and steady temperature inside the trough.

As compared with the N.P.L. comparator described above, this apparatus is limited in use owing to the limited displacement of the carriage, but its utility can be increased by the addition of sheet-iron extensions at each end of the trough, whereby the latter is lengthened 32 cm. each way.

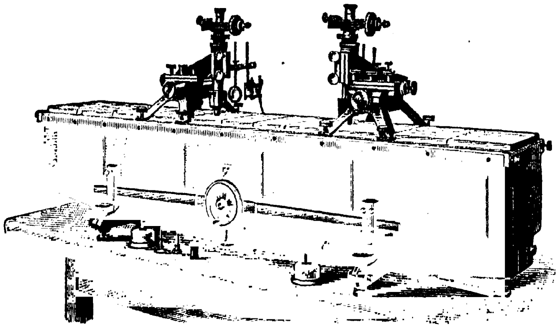


FIG. 18.—The Longitudinal or Subdividing Comparator at the Bureau International des Poids et Mesures. (Made by MM. Bariquand et Marre.)

§ (11) CALIBRATION OF SCALE. — (i.) The complete calibration of a scale is usually carried out in a number of successive stages, involving a corresponding number of successive subdivisions. This is best understood by considering actual cases, as, for example, the calibrations of the various subdivisions of a metre scale or of a yard scale.

In the case of a metre scale it is usual to commence by (*a*) calibrating the decimetre divisions, and this is done by comparing each one with all the others in as many ways as possible. Since the microscopes can be placed one decimetre apart, the method of carrying this out is analogous to that of intercomparing the same number of line standards in the transverse comparator, as will presently be clear. The second step consists in (*b*) calibrating the centimetres, and since the microscopes cannot in this case be placed so close together as to be only one centimetre apart, a modified method, differing from that for (*a*), is employed in carrying out the comparison. It consists, briefly, in comparing the centimetres in any one decimetre division with the centimetres in another, but *not adjacent*, decimetre division, and obviously five such pairs of decimetres in all can be dealt with in this way. The method of doing this is explained below. The third step consists in (*c*) calibrating the millimetres in each centimetre length by a method similar to that for (*b*). If the scale is further subdivided, the process can of course be continued.

---

[1] *Trav. et Mém. B.I.P.M.* xiii. E.

In a similar manner a yard scale can be dealt with in stages, such as (*a*) divisions each 6 in. long, (*b*) inch divisions, (*c*) tenths of an inch divisions, etc. Alternatively, (*a*) divisions each 9 in. long, (*b*) inch divisions, (*c*) tenths of an inch, etc. Other ways of arranging the steps will suggest themselves.

In the case of lengths other than yards or metres, similar stages of calibration can be devised, each varying according to the nature and extent of the subdivisions and of the calibration required. In most cases, where the scale is long enough, the first step (*a*) usually involves subdivisions, each not less than the minimum possible distance between the microscopes. Successive stages include lengths shorter than this. Further, it is better to limit the number of subdivisions compared at any one time to not more than ten. A larger number may be taken, but an increase in the number always involves a much greater increase in the amount of observations to be taken, the latter being always as the square of the number of divisions.

(ii.) *Method.*—The method of using the N.P.L. subdividing comparator for determining the calibration of a scale is explained in the following paragraphs, the actual example taken being the calibration of a metre scale into (*a*) decimetres, (*b*) centimetres, (*c*) millimetres.

(*a*) *Decimetres.*—The scale is placed in position on the girder of the comparator, with the supporting rollers as usual at the Airy points, and with, say, the zero end of the scale towards the left of the observer. The microscopes, previously adjusted for height, etc., are next fixed so that their axes are approximately one decimetre apart. This can be done by viewing the defining lines of one of the decimetres, and adjusting the microscopes until the lines both appear in the centres of the fields of view. The carriage is moved so that the first decimetre of the scale is in correct position beneath the microscopes (the various adjustments for this and subsequent settings being exactly the same as those in connection with the N.P.L. transverse comparator), with the lines in the centres of the fields of view. The cross-wires are set on the lines in the usual way and the readings noted. The carriage is next moved to the left, so that the second decimetre is in position beneath the microscopes, and the left-hand line so arranged that the reading in the microscope is, as nearly as possible, the same as that given by it when observing the first decimetre. The right-hand microscope then gives a reading differing by only a few microns from its previous one. Having taken readings and noted them, the third decimetre is observed in like manner, and so on up to the last decimetre.

The series of readings is now repeated in the reverse order (the end to end position of the scale remaining unchanged), thus giving two readings on each interval.

From the readings taken in the manner just described, the differences between successive intervals are readily found, that is, the difference between the first and the second, the second and the third, etc., nine in all.

The precaution, whereby approximately the same readings are obtained with each setting of the microscopes, is a very necessary one, since it ensures that only a very small portion of the run, merely a small fraction of a turn, of the micrometer screw is used, and that, in consequence, the error introduced by the screw is entirely negligible.

These results are independent of the temperature (assumed steady) at which the observations are made, as a little thought will clearly show. The material on which the scale is ruled is homogeneous, and hence expands uniformly throughout its length. The difference between two successive nominally equal lengths is as a rule only a matter of a few microns, and consequently this difference is not measurably altered by any expansion or contraction of the scale. If, however, the temperature alters during the short time elapsing between the measurement of two successive intervals, an error may be introduced into the subsequently obtained difference, and if this change of temperature is continuous over a whole series of readings the error becomes cumulative. In practice it is found that the occurrence of such a change of temperature can rarely be avoided, but as the rate of change is as a rule slow and quite steady, the error can be eliminated by taking readings in both directions of the scale as already indicated. A skilled observer, however, runs little risk in this way, as he will take the whole of the readings in the course of a few minutes. Moreover, if there is likely to be a rapid change of temperature, he will postpone his observations to a more favourable opportunity.

Having compared the decimetres singly,

the next step is to alter the positions of the microscopes so that they are now two decimetres apart, and to take readings on every such interval, following the same order as with the single decimetre intervals, and observing the same precautions. The first interval so observed involves the first and second decimetres, while the second interval involves the second and third decimetres. The second decimetre being common to both intervals, the difference between the intervals therefore gives the difference between the first decimetre and the third decimetre. Dealing with all the successive intervals in this way, we thus get a series of comparisons, or differences, between alternate decimetres, the first and third, the second and fourth, etc., there being eight in all.

Similarly, with the microscopes set three decimetres apart, the difference between the first and fourth, the second and fifth, etc., may be found, seven in all, and obviously the process can be repeated until the microscopes are nine decimetres apart, when the difference between the first and last decimetres is obtained.

Finally, the scale may be reversed on the girder of the comparator, and the whole of the observations repeated. The results obtained will be opposite in sign to those previously found, but otherwise should approximately agree. The mean of the two sets will obviously be taken.

The whole process thus described results in the complete intercomparison of the decimetres, each one in turn with all the others. This is entirely analogous to the complete intercomparison of ten standard lengths (see § (1) (iv.)), and the likeness goes further in that the final values of the decimetre intervals, or rather the amounts by which they differ from exact tenths of the nominal metre length, are computed in precisely the same way by means of a square. Such a square is shown in *Fig.* 19, the entries being the results of observations made, in exactly the manner just described, on an invar bar at the N.P.L. The square is interpreted in exactly the same way as *Fig.* 6, *e.g.* interval 1/2 - interval 5/6 = +5·22 μ, or second decimetre − sixth decimetre = +5·22 μ. The various differences should be readily identified; for example, the nine results given in the small squares adjacent to and to the left of the diagonal line are the differences found with the microscopes one decimetre apart; the next diagonal line of results to the left of this gives the differences obtained with the microscopes two decimetres apart; while the bottom left-hand corner gives the difference between the first and last decimetres. The results are repeated in the other half of the square to the right of the diagonal, but with the signs changed. All entries having been made in the square, each column is added, and the sums entered at the bottom. The sum of these totals should be zero. Each total is now divided by ten, the number of decimetre intervals, giving the means M as shown. The results taken in order are the errors of the respective decimetres as indicated at the heads of the columns of the square.

In the calibration thus obtained, the over-all lengths of the scale (that is, the metre length) has been taken as the unit of measurement; or, expressed otherwise, the over-all length has been taken "as correct," that is, having no end errors. But it is obvious that, although the over-all length may be exactly one metre at a particular temperature, it cannot be so at any other temperature, and in considering the absolute length of a subdivision this over-all error must be taken into account. Suppose x microns is this over-all error at a certain temperature T, or, in other words, let the equation to scale be

$$\text{over-all length} = 1 \text{ metre} + x \text{ microns.}$$

If the scale were correctly divided, each nominal decimetre length would be $\frac{1}{10}$ metre + $x/10$ microns. Hence, in order to express correctly the absolute length of each decimetre at a temperature T, it is necessary to add $x/10$ microns to the error of each decimetre. x may of course be positive or negative. More generally, if d_1, d_2, . . . d_{10} be the errors of the decimetres taken in order, on the assumption that the over-all length of the metre is correct, then the absolute errors of the lengths of the decimetres at temperature T are

$$d_1 + \frac{x}{10},\ d_2 + \frac{x}{10},\ \ldots\ d_{10} + \frac{x}{10}.$$

Further remarks on this will be found in section (iii.) (*q.v.*).

(*b*) *Centimetres.*—The method of dealing with intervals shorter than one decimetre, that is, shorter than the minimum possible distance apart of the microscopes, is described below.

Each decimetre is subdivided into ten centimetres, thus forming ten groups of centimetres. To commence with, two non-adjacent groups are selected, and the centimetres of one compared with the centimetres of the other. This done, other pairs of groups may be dealt with similarly. The choice of the most suitable pair to commence with depends on the extent to which the calibration is to be carried and the purpose for which it is required. If the calibration is not to be carried farther than the centimetres, that is, if lengths involving only exact multiples of a centimetre are required, then it is not necessary to calibrate the whole of the centimetres; it will be sufficient to determine only those in the first and last decimetres. But if the calibration is to be carried as far as the millimetres, then it is necessary to determine most of the centimetres, though it is usually found convenient to determine all. In the latter case, the best way of selecting pairs of groups for comparison is to take the first with the sixth, the second with the seventh, and so on, finishing with the fifth and tenth. Subsequently, the calibration of the first ten and the last ten millimetres will then usually suffice for all purposes that are likely to arise;

fractions of a millimetre can be measured by means of a reading microscope. As an illustration of the method, the remarks which follow are confined to the consideration of the manner of comparing the first and last decimetres.

	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/10
0/1		+1·22	-3·85	+0·45	-2·37	-4·20	+0·82	-1·27	+0·22	-4·27
1/2	*-1·16* -1·22		-4·97	-0·50	-2·75	-5 22	-0·57	-2·67	-1·55	-5·47
2/3	*+3·93* +3·85	*+5·09* +4·97		+4·50	+1·92	-0·02	+4·12	+2·95	+3·90	-0·17
3/4	*-0·66* -0·45	*+0·52* +0·50	*-4·57* -4·50		-2·70	-4·90	-0·15	-1·70	-0·75	-5·07
4/5	*+2·08* +2·37	*+3·24* +2·75	*-1·85* -1·92	*+2·72* +2·70		-1·67	+2·77	+0·42	+2·22	-2·12
5/6	*+3·97* +4·20	*+5·13* +5·22	*+0·04* +0·02	*+4·61* +4·90	*+1·89* +1·67		+4·10	+2·75	+4·22	-0·67
6/7	*-0·55* -0·82	*+0·61* +0·57	*-4·48* -4·12	*+0·09* +0·15	*-2·63* -2·77	*-4·52* -4·10		-1·50	-1·20	-5·02
7/8	*+1·31* +1·27	*+2·47* +2·67	*-2·62* -2·95	*+1·95* +1·70	*-0·77* -0·42	*-2·66* -2·75	*+1·86* +1·50		+0·85	-2·10
8/9	*+0·22* -0·22	*+1·38* +1·55	*-3·71* -3·90	*+0·86* +0·75	*-1·86* -2·22	*-3·75* -4·22	*+0·77* +1·20	*-1·09* -0·85		-3·22
9/10	*+4·14* +4·27	*+5·30* +5·47	*+0·21* +0·17	*+4·78* +5·07	*+2·06* +2·12	*+0·17* +0·67	*+4·69* +5·02	*+2·83* +2·10	*+3·92* +3·22	
S	+13·25	+24·92	-26·02	+19·72	-7·52	-26·41	+18·81	+0·23	+11·13	-28·11
M	+1·33	+2·49	-2·60	+1·97	-0·75	-2·64	+1·88	+0·02	+1·11	-2·81
Successive *Addition*	0/1 = +1·33	0/2 = +3·82	0/3 = +1·22	0/4 = +3·19	0/5 = +2·44	0/6 = -0·20	0/7 = +1·68	0/8 = +1·70	0/9 = +2·81	0/10 = 0·00

FIG. 19.

The microscopes are first of all fixed at a distance of 90 cm. apart, and readings taken successively on the intervals 0/90, 1/91, . . . 10/100, and also in the reverse direction, precisely as with the observations on the decimetres, and with the same precautions. From the readings a series of differences is obtained, and a little thought will show that these are respectively the differences between the 1st and 91st, the 2nd and 92nd, the 3rd and 93rd, etc., centimetres, ten in all. Next the microscopes are placed 91 cm. apart, and a further series obtained giving the difference between the 1st and 92nd, the 2nd and 93rd, the 3rd and 94th, etc., centimetres, nine in all. With the microscopes 92 cm. apart, a similar series of eight differences is obtained, and the process is repeated until the microscopes are 99 cm. apart, giving one difference only, that between the 1st and the 100th centimetre.

Next the microscopes are placed successively 89, 88, 87, . . . 81 cm. apart, and from the readings further differences are obtained.

With the bar turned end to end, the whole of the foregoing observations may be repeated, if such a course is thought desirable or necessary.

The results obtained, 100 in all, represent the comparison of each centimetre in turn of the first group with all the centimetres of the other group, and in order to find the errors of each centimetre in each group the hundred differences are entered in a square, as illustrated in *Fig.* 20. The entries are the results of observations on the same invar bar which is involved in *Fig.* 19. The square presents obvious points of difference from previous ones considered. The columns and rows have different designations, the former representing the centimetres in the first group, while the latter represents those of the last group. Also, each small square contains an independent entry, *i.e.* each entry occurs only once (cf. previous squares), and further, both columns and rows are summed and meaned. The actual computation of the final results is a somewhat complicated matter, and in order to understand how this is done it is necessary to consider more generally the constitution of the square.

Let d_1 and d_{10} be the errors of the first and tenth decimetres, assuming the over-all error of the metre to be zero. Let $c_1, c_2, \ldots c_{10}, c_{91}, \ldots c_{100}$ be the errors of the centimetres as denoted by the numbers suffixed to the letters. Then $c_1+c_2+\ldots+c_{10}=d_1$, and $c_{91}+\ldots+c_{100}=d_{10}$. A square (*Fig.* 21) can now be built up similar to *Fig.* 20.

In the explanation which follows, *Figs.* 20 and 21 should be constantly compared in order to discover the significance of each entry. In *Fig.* 21 the equations given in the rows and columns marked sums and means will give all the desired information.

First, however, it will readily be seen that

$$\sum_{1}^{10} S = \sum_{91}^{100} S = 10(d_1 - d_{10}),$$

and that therefore

$$\sum_{1}^{10} M = \sum_{91}^{100} M = (d_1 - d_{10}). \quad . \quad . \quad (1)$$

That is, sum of means of columns = sum of means of rows = difference between the two decimetres. This result is important as it is required for some of the calculations. Further,

$$M_1 = c_1 - \tfrac{1}{10}d_{10},$$

$$\left.\begin{array}{lr} \text{or} & c_1 = M_1 + \tfrac{1}{10}d_{10}, \\ \text{similarly} & c_2 = M_2 + \tfrac{1}{10}d_{10}, \\ & \cdot \quad \cdot \quad \cdot \quad \cdot \\ \text{and} & c_{10} = M_{10} + \tfrac{1}{10}d_{10}. \end{array}\right\} \quad . \quad . \quad . \quad (2)$$

In like manner

$$M_{91} = -c_{91} + \tfrac{1}{10}d_1,$$

$$\left.\begin{array}{lr} \text{or} & c_{91} = -M_{91} + \tfrac{1}{10}d_1, \\ \text{and} & c_{92} = -M_{92} + \tfrac{1}{10}d_1, \\ & \cdot \quad \cdot \quad \cdot \quad \cdot \\ \text{and} & c_{100} = -M_{100} + \tfrac{1}{10}d_1. \end{array}\right\} \quad . \quad . \quad . \quad (3)$$

	$^0/_1$	$^1/_2$	$^2/_3$	$^3/_4$	$^4/_5$	$^5/_6$	$^6/_7$	$^7/_8$	$^8/_9$	$^9/_{10}$	S	.M	Add $-\frac{1}{10}d_1+0{\cdot}03_2$ $=-0{\cdot}10_1$	*Successive Addition & signs changed*	*Add* $+2{\cdot}81(=-d_{10})$
$^{90}/_{91}$	-0·61 -0·35	-2·25 -1·57	-0·25 -0·32	-0·35 -0·22	+1·30 +1·17	-1·01 -1·02	-1·30 -1·22	-0·63 -0·92	-0·54 -0·80	-1·35 -1·75	-7·00	-0·70$_0$	-0·80$_1$	$^{90}/_{91}$ =+0·80	$^{0}/_{91}$ =+3·61
$^{91}/_{92}$	+2·56 +2·97	+0·92 +0·57	+2·92 +2·60	+2·82 +2·82	+4·47 +4·17	+2·16 +2·40	+1·87 +1·70	+2·54 +2·42	+2·63 +3·02	+1·82 +2·05	+24·72	+2·47$_2$	+2·37$_1$	$^{90}/_{92}$ =-1·57	$^{0}/_{92}$ =+1·24
$^{92}/_{93}$	+0·50 +0·30	-1·14 -0·76	+0·86 +0·42	+0·76 +0·90	+2·41 +2·45	+0·10 +0·40	-0·19 -0·80	+0·48 +0·60	+0·57 +0·50	-0·24 +0·07	+4·08	+0·40$_8$	+0·30$_7$	$^{90}/_{93}$ =-1·88	$^{0}/_{93}$ =+0·93
$^{93}/_{94}$	+0·03 +0·02	-1·61 -1·95	+0·39 +0·40	+0·29 +0·57	+1·94 +1·80	-0·37 -0·35	-0·66 -0·40	+0·01 +0·42	+0·10 +0·10	-0·71 -1·17	-0·56	-0·05$_6$	-0·15$_7$	$^{90}/_{94}$ =-1·72	$^{0}/_{94}$ =+1·09
$^{94}/_{95}$	+0·14 +0·52	-1·50 -1·77	+0·50 +0·87	+0·40 +0·25	+2·05 +2·10	-0·26 -0·27	-0·55 -0·57	+0·12 -0·47	+0·21 +0·45	-0·60 -0·60	+0·51	+0·05$_1$	-0·05$_0$	$^{90}/_{95}$ =-1·67	$^{0}/_{95}$ =+1·14
$^{95}/_{96}$	+0·74 +0·40	-0·90 -1·37	+1·10 +1·47	+1·00 +1·42	+2·65 +2·72	+0·34 +0·42	+0·05 -0·17	+0·72 +0·87	+0·81 +0·75	+0·00 +0·02	+6·53	+0·65$_3$	+0·55$_2$	$^{90}/_{96}$ =-2·22	$^{0}/_{96}$ =+0·59
$^{96}/_{97}$	-0·16 -0·12	-1·80 -1·55	+0·20 0·00	+0·10 -0·20	+1·75 +1·75	-0·56 -0·67	-0·85 -0·85	-0·18 +0·02	-0·09 +0·22	-0·90 -1·07	-2·47	0·24$_7$	-0·34$_8$	$^{90}/_{97}$ =-1·87	$^{0}/_{97}$ =+0·94
$^{97}/_{98}$	+1·07 +0·72	-0·57 -0·50	+1·43 +1·45	+1·33 +1·17	+2·98 +3·20	+0·67 +0·45	+0·38 +0·97	+1·05 +1·10	+1·14 +1·07	+0·33 +0·17	+9·80	+0·98$_0$	+0·87$_9$	$^{90}/_{98}$ =-2·75	$^{0}/_{98}$ =+0·06
$^{98}/_{99}$	+0·09 +0·37	-1·73 -1·77	+0·27 +0·40	+0·17 -0·17	+1·82 +2·10	-0·49 -0·75	-0·78 -1·00	-0·11 -0·35	-0·02 -0·27	-0·83 -0·40	-1·84	-0·18$_4$	-0·28$_5$	$^{90}/_{99}$ =-2·47	$^{0}/_{99}$ =+0·34
$^{99}/_{100}$	+0·54 -0·10	-1·10 -1·00	+0·90 +1·05	+0·80 +0·77	+2·45 +2·37	+0·14 +0·10	0·15 +0·17	+0·52 +0·77	+0·61 +0·35	-0·20 -0·02	+4·46	+0·44$_6$	+0·34$_5$	$^{90}/_{100}$ =-2·81	$^{0}/_{100}$ =0·00
S	+4·73	-11·67	+8·34	+7·31	+23·83	+0·71	-2·17	+4·46	+5·39	-2·70	+38·23				
M	+0·47$_3$	-1·16$_7$	+0·83$_4$	+0·73$_1$	+2·38$_3$	+0·07$_1$	-0·21$_7$	+0·44$_6$	+0·53$_9$	-0·27$_0$		+3·82$_3$			
Add $\frac{1}{10}d_{10}+0{\cdot}03_2$ $=-0{\cdot}24_1$	+0·22$_4$	-1·41$_6$	+0·58$_5$	+0·48$_2$	+2·13$_4$	-0·17$_8$	0·46$_6$	+0·19$_7$	+0·29$_0$	-0·51$_9$		+0·38$_2$			
Successive Addition	$^0/_1$ =+0·22	$^0/_2$ =-1·19	$^0/_3$ =-0·61	$^0/_4$ =-0·13	$^0/_5$ =+2·01	$^0/_6$ =+1·83	$^0/_7$ =+1·37	$^0/_8$ =+1·56	$^0/_9$ =+1·85	$^0/_{10}$ =+1·33					

FIG. 20.

Equations (2) and (3) therefore give directly the required errors of the centimetres in the two groups. They involve, however, a knowledge of the individual errors of d_1 and d_{10}, and these have been previously determined from an intercomparison of the decimetres, such as is given by the square of *Fig.* 19. Further, a square such as *Fig.* 19 will also give the value of $d_1 - d_{10}$, which should agree with the value given by (1) above.

Also, equations (2) and (3) are of assistance in determining the calculated value of the difference between any two centimetres; for example:

$$c_1 - c_{91} = (M_1 + \tfrac{1}{10}d_{10}) - (-M_{91} + \tfrac{1}{10}d_1)$$

$$\left.\begin{aligned} &= (M_1 + M_{91}) - \tfrac{1}{10}(d_1 - d_{10}), \\ \text{similarly}\quad c_3 - c_{95} &= (M_3 + M_{95}) - \tfrac{1}{10}(d_1 - d_{10}), \\ \text{and}\quad c_9 - c_{100} &= (M_9 + M_{100}) - \tfrac{1}{10}(d_1 - d_{10}), \\ &\text{etc.} \end{aligned}\right\} \quad . \quad (4)$$

It will be noticed that there is a common term $-\tfrac{1}{10}(d_1 - d_{10})$ to each of these equations, and further that the three terms on the right-hand side of the equations can all be read directly from the square, *Fig.* 21.

	0/1	1/2	2/3	------	9/10	Sums	Means
90/91	$C_1 - C_{91}$	$C_2 - C_{91}$	$C_3 - C_{91}$	------	$C_{10} - C_{91}$	$S_{91} = -10C_{91} + d_1$	$M_{91} = -C_{91} + \frac{1}{10}d_1$
91/92	$C_1 - C_{92}$	$C_2 - C_{92}$	$C_3 - C_{92}$	------	$C_{10} - C_{92}$	$S_{92} = -10C_{92} + d_1$	$M_{92} = -C_{92} + \frac{1}{10}d_1$
92/93	$C_1 - C_{93}$	$C_2 - C_{93}$	$C_3 - C_{93}$	------	$C_{10} - C_{93}$	$S_{93} = -10C_{93} + d_1$	$M_{93} = -C_{93} + \frac{1}{10}d_1$
---	----	----	----	------	-----	---------	---------
---	----	----	----	------	-----	---------	---------
---	----	----	----	------	-----	---------	---------
99/100	$C_1 - C_{100}$	$C_2 - C_{100}$	$C_3 - C_{100}$	------	$C_{10} - C_{100}$	$S_{100} = -10C_{100} + d_1$	$M_{100} = -C_{100} + \frac{1}{10}d_1$
Sums Means	$S_1 = 10C_1 - d_{10}$ $M_1 = C_1 - \frac{1}{10}d_{10}$	$S_2 = 10C_2 - d_{10}$ $M_2 = C_2 - \frac{1}{10}d_{10}$	$S_3 = 10C_3 - d_{10}$ $M_3 = C_3 - \frac{1}{10}d_{10}$	------	$S_{10} = 10C_{10} - d_{10}$ $M_{10} = C_{10} - \frac{1}{10}d_{10}$	$10(d_1 - d_{10})$	$d_1 - d_{10}$ $\frac{1}{10}(d_1 - d_{10})$

FIG. 21.

Returning now to the square of *Fig.* 20; giving the results of actual observations, the above equations can be applied. Thus from (1)

$$d_1 - d_{10} = +3{\cdot}82\,\mu, \quad \text{and} \quad \tfrac{1}{10}(d_1 - d_{10}) = +0{\cdot}38_2\,\mu.$$

The value of $d_1 - d_{10}$ obtained from the square of *Fig.* 19, giving the results of the intercomparison of the individual decimetres, is $+4{\cdot}14\,\mu$, differing only from the foregoing by $0{\cdot}32\,\mu$; this is reasonable agreement.

Again, using equations (4), the calculated values can be obtained. For example:

$$0/1 - 90/91 = c_1 - c_{91}$$
$$= +0{\cdot}47 - 0{\cdot}70 - (+0{\cdot}38) = -0{\cdot}61\,\mu,$$

$$2/3 - 94/95 = c_3 - c_{95}$$
$$= +0{\cdot}83 + 0{\cdot}05 - (+0{\cdot}38) = +0{\cdot}50\,\mu.$$

These are inserted in their correct squares, and the residuals can then be found by simple subtraction.

In calculating the actual errors of the centimetres, the errors of d_1 and d_{10} have been taken from the square of *Fig.* 19 on the assumption that the over-all length of the metre is correct (a correction for any error on the metre length can be applied afterwards), *i.e.* $d_1 = +1{\cdot}33\,\mu$, $d_{10} = -2{\cdot}81\,\mu$. Applying equations (2) and (3) and using these values of d_1 and d_{10}, the errors of the centimetre intervals are thus found. For example:

$$0/1 = c_1 - M_1 + \tfrac{1}{10}d_{10}$$
$$= +0{\cdot}47 + \tfrac{1}{10}(-2{\cdot}81) = +0{\cdot}19\,\mu,$$

$$6/7 = c_7 - M_7 + \tfrac{1}{10}d_{10}$$
$$= -0{\cdot}22 + \tfrac{1}{10}(-2{\cdot}81) = -0{\cdot}50\,\mu,$$

$$90/91 = c_{91} = -M_{91} + \tfrac{1}{10}d_1$$
$$= -(-0{\cdot}70) + \tfrac{1}{10}(+1{\cdot}33) = +0{\cdot}83\,\mu,$$

etc.

These results are subject to a small correction. If the errors of the centimetres in any one decimetre be added together, their sum should be equal to the error of the decimetre itself. It is found, however, that there is usually a small difference, a residual error, between the two, and that a slight adjustment of the centimetre errors is necessary. For example, the sum of the centimetre errors of the first ten centimetres (*Fig.* 20) is $+1{\cdot}01\,\mu$, while the error of the first decimetre, as given by *Fig.* 19, is $+1{\cdot}33\,\mu$, a difference of $+0{\cdot}32\,\mu$, and the centimetre errors are adjusted by adding $\tfrac{1}{10}$ of the error to each of them, *i.e.* $0{\cdot}03_2\,\mu$. See *Fig.* 20 for final results.

As with the decimetre calibration, it now remains to adjust these errors for any error on the over-all length. As before, let x be the over-all error of the metre length at a temperature T and c_1, c_2, etc., be the errors of the centimetres as found above. Then the actual errors of the centimetres at temperature T are $c_1 + x/100$, $c_2 + x/100$, . . . etc. Further remarks on this are given in section (iii.) (*q.v.*).

(*c*) *Millimetres.*—The millimetres are calibrated in exactly the same way as the centimetres, that is, the millimetres contained in any one centimetre are compared with the millimetres in any other centimetre. In choosing the millimetres, due regard must be of course paid to the distance between the microscopes. It is usual to choose the first and last centimetres.

(iii.) *Final Form of Calibration.*—The calibration derived by the method just described gives the actual length of each subdivision. It is, however, more convenient to convert the calibration into another form, which enables one to read directly the error on the nominal distance of any one of the subdividing lines from the zero line. Take, for example, the errors on the decimetre, d_1, d_2, . . . d_{10}, on the usual assumption that the metre length is correct. Let 0/1, 0/2, 0/3, etc., represent the distance of each decimetre subdividing line from the zero line, y_1, y_2, y_3, etc., the errors of these lengths.

Then,

$$\left.\begin{array}{ll} \text{error of } & 0/1 = y_1 = d_1 \\ \quad ,, & 0/2 = y_2 = d_1 + d_2 \\ \quad ,, & 0/3 = y_3 = d_1 + d_2 + d_3 \\ \quad ,, & 0/10 = y_{10} - d_1 + d_2 + \ldots + d_{10} \end{array}\right\} = 0, \quad (5)$$

(since the error of the whole length is assumed zero).

The convenience of this method of expressing the calibration is that the error on the nominal distance between any two lines on the bar can be obtained by the simple subtraction of two quantities taken from the calibration figures. For example, the five-decimetre length $2/7 = y_7 - y_2$, as the equations (5) above readily show.

Thus, the calibration of the decimetres of the square of *Fig.* 19 may be expressed in this manner, as given in the last line of the square, the figures, of course, still not taking into account the over-all length of the metre.

Other subdivisions, such as the centimetres, can be expressed in like manner.[1]

(iv.) *The Constants of a Scale.*—It is evident from what has been said that the calibration of a scale is completely determined at any temperature T by a knowledge of:

(*a*) The errors of the subdivisions, on the assumption that the over-all length is correct;

(*b*) The error of the over-all length at a stated temperature (0° C. for the metric scales, and 62° F. for scales of the British system).

(*c*) The coefficient of expansion of the scale.

Of these "constants" (*b*) is sometimes variable, as, for example, with an invar scale, and it is on this account that (*a*) and (*b*) are always expressed separately. It has been seen in an earlier section that (*b*) must be measured from time to time to determine the extent of the change (if any), and hence it is always possible, from the rate of change, to infer the value of (*b*) at any time.

§ (12) MEASUREMENT OF SHORT LENGTHS.—In the introduction to the subdividing comparator it was mentioned that the instrument was the essential one for the measurement or comparison of short scales or lengths which could not be determined otherwise. The scale to be measured is placed end to end with a "known" scale, and both secured, without constraint, so that there is no relative movement of one with respect to the other during observations which are made on them. A calibrated yard or metre is the most convenient known scale to employ, a suitable interval, nominally equal to the unknown scale, being chosen. The two lengths are then compared in exactly the same way as two decimetre lengths are compared, and the difference between them thus found, and consequently the length of the unknown scale determined.

It must be pointed out, however, that if the two scales compared are of different material, that is, have different expansibilities, it is necessary to know with accuracy the common temperature of the scales, and a correction applied accordingly. But if the two scales are of the same material and presumably, therefore, have the same expansibility, an exact knowledge of the temperature is not necessary, since the difference between the two lengths concerned will be constant for all temperatures, but it is obviously important that they should be at the same temperature. W. H. J.

COMPARATORS FOR TESTING GAUGES. See "Gauges," Section VI. § (80), etc.

COMPASSES, PROPORTIONAL. See "Draughting Devices," p. 262.

COMPASSES, TRIANGULAR. See "Draughting Devices," p. 262.

COMPENSATED METER FOR SPECIFIC GRAVITY CHANGES OF THE GAS. See "Meters for Measurement of Coal Gas and Air," § (3) (ii.).

COMPUTATION OF RESULTS obtained by intercomparison of a number of line-standards, method of. See "Comparators," §§ (1), (4).

CONDENSATION, transference of heat by, in the atmosphere. See "Radiation," § (3) (iv.).

CONDITIONED OBSERVATIONS: a class of observations where all systems of values are not equally possible, owing to the existence of conditions which must be exactly satisfied. See "Observations, The Combination of," § (6).

Combination of. See *ibid.* § (6).

CONDUCTION, transfer of heat by, in the atmosphere. See "Atmosphere, Physics of," § (6).

---

[1] See Guillaume, "L'étallonage des échelles divisées," *Trav. et Mém.* xiii. E.

Constants for Dry and Saturated Air. See "Atmosphere, Thermodynamics of the," § (2).

Constants of a Scale. See "Comparators," § (12) (iv.).

Constraint, "geometric" and "engineering" methods of, as used in mechanism. See "Metrology," § (34) (ii.).

Method of locating three feet of tripod. See *ibid.* § (34) (ii.) (*b*).

Use of balls and rollers in mechanism. See *ibid.* § (34) (iii.).

Use of "slippers" in mechanism. See *ibid.* § (34) (ii.) (*b*).

Continents and Islands, different values for *g* for. See "Gravity Survey," § (14).

Continuous Flow Water Meter. See "Meters for Measurement of Liquids," § (3).

Contraction of Alcohol and Water Mixtures. See "Alcoholometry," § (3).

Convection, transference of heat by, in the atmosphere. See "Radiation," § (3) (iv.).

Convection in the Atmosphere:

Absence of, in the stratosphere. See "Atmosphere, Thermodynamics of the," §§ (5), (10).

Causes of. See "Atmosphere, Physics of," § (22).

Conditions for, in the atmosphere. See "Atmosphere, Thermodynamics of the," §§ (5), (13).

Cumulative, definition of. See *ibid.* § (15) *et seq.*

Effect on lapse-rate of temperature. See "Atmosphere, Physics of," § (6).

Frictional effect of. See "Atmosphere, Thermodynamics of the," § (17).

Penetrative, definition of. See *ibid.* § (15) *et seq.*

Relation of, to the environment. See *ibid.* §§ (13), (14).

Relation of, to horizontal flow. See *ibid.* § (16).

Convective Equilibrium in the Atmosphere, definition of. See "Atmosphere, Thermodynamics of the," §§ (6), (13).

Conversion Factors, Imperial to Metric Units:

American. See "Metrology," § (15) (ii.).

British. See *ibid.* § (15) (i.).

Core Diameter of Screw, definition. See "Metrology," § (23) (i.).

Corona: a coloured ring or series of rings surrounding the sun or moon. See "Meteorological Optics," § (15) (i.).

Correction Tables for Volumetric Glassware. The observed weight in gms. in air of a quantity of water is numerically not very different from its volume in cubic centimetres. Hence simple tables of corrections may be prepared which serve to convert observed weights in air into volumes. See "Volume, Measurements of," § (7).

Correlation Coefficient as applied to Sunspots and Mean Temperature. See "Radiation," § (1).

Cotidal Chart: a chart by means of which, in certain regions, the travel of the tidal wave along the coast may be studied. See "Tides and Tide-prediction," § (7).

Cotidal Line. A line drawn through all the points on the surface of the sea at which the high water following full or change of moon occurs at the same hour of Greenwich time. See "Tides and Tide-prediction," § (7).

Counter Machines. See article "Weighing Machines," § (3).

"Creep" of Aneroid Barometer, Error due to: located in the vacuum-box and mathematically expressed by Hersey. See "Barometers and Manometers," § (13) (v.).

Crest of Screw Thread, definition. See "Metrology," § (23) (i.).

Crystals, Ice, in the Atmosphere:

Degrees of freedom of. See "Meteorological Optics," § (19).

Forms of. See *ibid.* § (18).

Phenomena due to. See *ibid.* §§ (17), (20), (21), (22).

Cubic Centimetre: the C.G.S. unit of volume. For the relation between the cubic centimetre and the litre, see "Volume, Measurements of," § (2).

Cubic and other Equations. Graphical methods of solution. See "Nomography," § (12).

Cycle Engineers' Standard Thread (C.E.I.). See "Gauges," § (52).

Cycle of Operations for Atmospheric Air. See "Atmosphere, Thermodynamics of the," § (24) and *Fig.* 17.

Efficiency of. See *ibid.* § (25).

Work done in. See *ibid.* § (26).

Cyclone. A cyclone or depression is a region in which the atmospheric pressure is lower than in the surrounding regions. In the northern hemisphere the winds blow round the depression in a counter-clockwise direction. At the approach of a depression the sky becomes overcast at first with fairly high cloud, which becomes lower and finally brings rain. The rain is very heavy in the north-east quadrant, but steady rain is usual in the south-east quadrant. The western portion of the depression is usually a region of detached cloud, with frequent showers. The passage of the trough line, or line drawn through the centre at right angles to the direction of motion, marks the lowest pressure attained at any particular point. Its passage is accompanied by heavy squalls of rain, and a rapid change of wind from S.W. to W. or N.W. The temperature is usually fairly high in the front of the depression, but with the passage of the trough line there is a rapid drop in temperature due to the arrival of a current of air from more northerly directions.

Accumulation of air in. See "Atmosphere, Thermodynamics of the," §§ (15), (16).

Characteristics of. See "Atmosphere, Physics of," §§ (18), (20).

Distribution of pressure, temperature, and density, and height of the tropopause in. See "Atmosphere, Thermodynamics of the," § (5), Table III.

Distribution of realised entropy in. See *ibid.* § (6), *Fig.* 10.

Kinetic energy of. See *ibid.* §§ (9), (26).

Persistence of. See "Atmosphere, Physics of," § (15).

Rate of filling up of. See "Atmosphere, Thermodynamics of the," § (16).

Theories of. See "Atmosphere, Physics of," §§ (15), (21).

Vertical extension of. See "Atmosphere, Thermodynamics of the, § (7).

Wind-velocity in. See "Atmosphere, Physics of," § (9).

CYCLOSTROPHIC WIND. If in equations (1) and (2) of § (9), article "Atmosphere, Physics of the," the deviating force due to the earth's rotation is neglected, the gradient of pressure is balanced entirely by the centrifugal force. The wind velocity necessary to maintain this balance is called the cyclostrophic wind. In very low latitudes the deviating force due to the earth's rotation is negligible on account of the smallness of the factor $\sin \phi$, and the cyclostrophic wind is then the best approximation to the gradient wind.

CYMOGRAPH. See "Draughting Devices," p. 263.

— D —

Dactyle Calculators. See "Calculating Machines," § (6).

Dalton's Law of Vapour Pressure. See "Humidity," I.

Daniell's Dew-point Hygrometer. See "Humidity," II. (1).

Darwin-Hill Mirror. An instrument for measuring the direction of motion and velocity of aircraft, clouds, shell-bursts, etc. See "Meteorological Instruments," § (35).

Day, Division of. See "Clocks and Time-keeping," § (14).

Day, Sidereal, as standard of time. See "Clocks and Time-keeping," § (1).

Density. The density of any substance is the mass of unit volume, and is measured in grammes per cubic centimetre, or in pounds per cubic foot. The term specific gravity is occasionally used to denote the density of a substance relative to that of water.

1 g./c.c. = 62·43 lb./c. ft.
1 lb./c. ft. = ·01602 g./c.c.

(i.) *Density of Water.*—Water has its maximum density at 3·98° C. when pressure is 760 mm., at other pressures the temperature of maximum density is given by the formula $t_m = 3{\cdot}98 - {\cdot}0225(p - 1)$, where p is measured in atmospheres.[1]

The density of pure water under one atmosphere for different temperatures is as follows:

Temperature a.	Density g./c.c.
268	·99930
273	·99987
277	1·0000
293	·99823
323	·9881
373	·9584

(ii.) *Density of mercury* at the normal freezing-point of water = 13·5955 g./c.c.

(iii.) *Density of Dry Air.*—The density of dry air varies with pressure and temperature according to the formula

$$\rho = \rho_0 \cdot \frac{p}{p_0} \cdot \frac{T_0}{T}.$$

For dry air free from CO_2 Regnault obtained the value $\rho_0 = 1292{\cdot}78$ g./m.³ for $p_0 = 760$ mm., $T_0 = 273$, which gives

$$\rho = 348{\cdot}321\frac{p}{T},$$

where p is measured in millibars. The addition of 0·04 per cent CO_2 increases the value of ρ_0 by 0·021 per cent, and the formula becomes

$$\rho = 348{\cdot}394\frac{p}{T},$$

i.e. $\rho = 1201$ g./m.³ approximately at 1000 mb. and 290 a, or approximately ·0808 lb. per cubic foot at 30 inches and 32° Fahr.

(iv.) *Density of Damp Air.*—The density of damp air may be obtained from the density of dry air by means of the formula

$$\rho = \frac{\rho_D(p - 0{\cdot}378e)}{p},$$

---

[1] Kay and Laby, *Physical and Chemical Constants*, 1918, p. 22.

where ρ_D is the density of dry air,
p is the total pressure,
e is the vapour pressure.

Hence $\rho = 348{\cdot}394\dfrac{p - 0{\cdot}378e}{T}$.

See Vol. I., "Measurement, Units of."

Of dry air. See "Atmosphere, Thermodynamics of the," § (2).

Of water-vapour, distribution of, in the upper air. See *ibid.* § (5).

Measurement of, in the upper air by Dobson's barothermograph. See "Meteorological Instruments," § (38).

Numerical value of. See "Atmosphere, Thermodynamics of the," § (2), Table I.

Density of the Atmosphere:

Distribution of, in cyclones and anticyclones. See "Atmosphere, Thermodynamics of the," § (5), Table III.

Gradient of, in the upper air. See *ibid.* § (8).

Relative effect of pressure and temperature on. See *ibid.* § (8).

Variation with height. See "Atmosphere, Physics of," § (7).

Density of a Gas, Determination of. See "Balances," § (18).

Density of a Liquid, Determination of. See "Balances," § (15).

Density of a Solid, Determination of. See "Balances," § (16).

Density of a Substance: a term used to denote the mass per unit volume of the substance. See "Balances," § (14).

Density of a Vapour, Determination of. See "Balances," § (19).

Density Determinations: corrections for the temperature of the water in which the weighing is made, tabulated. See "Balances," § (16) (i.), Table I.

Density Hydrometer. See "Hydrometers," §§ (4) and (7).

Depression. See "Atmosphere, Physics of," §§ (18), (20). See also "Cyclone."

Depression Range-finder, Range-finding by. See "Trigonometrical Heights," § (8).

Descartes. Theory of the rainbow. See "Meteorological Optics," § (14).

Dew-point:

Definition of. See "Humidity," II.

Determination of. See *ibid.* II. § (1).

Distribution of, over the northern hemisphere. See "Atmosphere, Thermodynamics of the," *Fig.* 5.

Dew-point Hygrometers:

Crova's. See "Humidity," II. § (2).

Daniell's. See *ibid.* II. § (1) *et seq.*

Regnault's. See *ibid.*

Effect of wind on. See *ibid.* II. § (2).

Theory of. See *ibid.* II. § (3).

Diagrams: forms of screw thread for use with projection apparatus. See "Gauges," § (69) (v.).

Dial Surface Gauge. See "Gauges," § (91).

Diaphragm Meter. See "Meters for Measurement of Coal Gas and Air," § (4).

Diffraction of Light in the Atmosphere: colours of the sky due to. See "Meteorological Optics," § (12).

Diffraction Phenomena in the Atmosphere (coronas, iridescent clouds, glories, etc.). See "Meteorological Optics," § (15).

Diffusion. Theory of wet-bulb thermometer. See "Humidity," II. § (5) (i.).

Diffusion of Heat by Eddy-motion. See "Atmosphere, Physics of," § (13).

Dine's Pressure-tube Anemometer. See "Meteorological Instruments," § (20) (ii.).

Discharge Coefficient: dependence on diameter of orifice and head producing discharge. See "Meters for Measurement of Coal Gas and Air," § (4) (ii.).

Displacement Distance Measurers. See "Meters for Measurement of Liquids," § (1).

Displacement Methods of determining Volumes. See "Volume, Measurements of," § (8).

Distance Measurers. See "Draughting Devices," p. 263.

Dividing Engine for ruling Line Standards and Fine Scales: general description and method of use. See "Line Standards," § (2).

Double-tank Comparator: description and outline of method of using for determination of absolute coefficient of expansion. See "Comparators," § (3).

DRAUGHTING DEVICES

Centrolinead

This is used for ruling radial lines in perspective drawing to obviate the use of long straight edges, etc. By its use the centre of radiation, or vanishing point as it is known in perspective drawing, may be any distance outside the drawing, but the instrument when at work requires no more space than is required for the drawing.

The instrument was invented by Peter Nicholson, and consists of a long straight edge to which are fitted two short arms at one end. The arms are fixed by milled set screws in quadrant slots cut in a metal plate attached to the end of the straight edge, and are arranged to clamp at any angle to each other and to the straight edge. The axis or centre of rotation of the quadrant is in line with the ruling edge of the straight edge.

The centrolinead is guided in its radial movement by two pins or studs, set in a position according to the distance of the vanishing point from the drawing. The method of use is as follows:

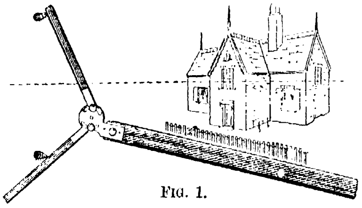


FIG. 1.

The horizontal or normal line for the perspective drawing is ruled, and perpendicular to it, at the extreme edge of the board, another line is drawn at either the left or right hand edge according to the assumed direction of the vanishing point. On the perpendicular, and equidistant on each side of the normal line, the studs or pins are fixed a suitable distance apart. The centrolinead is now set with the straight edge along the horizontal line and the short arms adjusted to touch the pins, and locked. The angle between the arms to suit the assumed distance of the vanishing point is found by trial, *i.e.* by moving the straight edge about, at the same time keeping the arms in contact with, and sliding against, the pins, and judging whether the angle of the lines drawn by the straight edge suit the perspective to be drawn. Should the vanishing point appear to be too near, the angle between the short arms should be increased or *vice versa*.

The figure shows the instrument in use when the vanishing point is to the left-hand of the drawing. When in use for a right-hand vanishing point the arms and the straight edge are disconnected, and the quadrant re-fixed with the reverse side upwards. It is preferable, however, to have two instruments set as required.

To adjust the instrument quickly and accurately is really a matter of experience.

The angle for the setting of the arms may be found by calculation if the position of the vanishing point is definitely known and the distance between the studs settled.

A simple form of centrolinead can be made by forming the straight edge and the arms in one piece, the adjustment in this case being obtained by the position of the guide pins.

CIRCULAR CALCULATORS

In principle these instruments are similar to that of the flat slide rule. The scales, however, instead of being straight, are arranged concentrically on dials at the front and back of the instrument, and are protected by glass discs mounted in a metal frame. The design of the instrument follows closely that of a watch, the size being approximately the same.

The Boucher Calculator (*Fig.* 2) has a movable dial at the front, rotatable by the milled stem at the top, and a fixed dial at the back.

The fine needle pointers, which are set in line back and front, are rotated by the milled head at the side. A fixed pointer is attached to the case, and extends over the four scales on the dials.

Reading from the centre the scales on the front dial are as follows:

The first and second concentric scales represent square roots of the numbers on the third scale. The third scale is the ordinary logarithmic scale. The fourth, or outer scale, is the sine scale, and is marked with angles of which the natural sines can be read off on the third scale.

The scales on the back, reading from the centre, are first, second, and third cube roots of numbers given on the third scale on the front dial, and are located by means of the rotating pointers. The outer ring is evenly divided and gives logarithms.

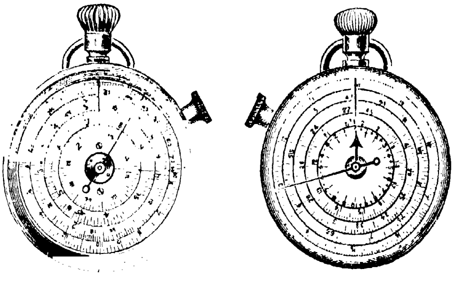

Front. Back.

FIG. 2.

The working of the calculator is similar in principle to the ordinary slide rule. It will be seen, however, that only one logarithmic scale is provided which does not move relatively to a fixed scale as in the case of the slide rule. The addition and subtraction of scales in this case is performed by fixing an arc length by the pointers, which represents the distance equivalent to a fixed scale, and adding or subtracting from this by the movable scale. For example, to multiply 2 × 4, the 2 on the ordinary log scale is set against the fixed pointer, and the movable pointer set to the 1 of the scale, thus giving an arc length of 2 between the pointers. To complete the calculation we simply add a length 1 – 4 to this, thus move the dial backwards until 4 is under the movable pointer, and read 8 the result under the fixed pointer. Division is accomplished by subtracting in a similar way.

The back dial in the figure has a scale and pointer at the centre. This is an addition which is arranged on the Stanley Boucher calculator for finding the position of the decimal point, by automatically indicating the number of digits to be added or deducted from the result.

Another type of circular calculator is the Halden Calculex. This type is provided with fixed and movable logarithmic scales similar to the flat slide rule. These scales are arranged concentrically on the dial, and are similar. The outer scale is fixed, and the inner one can be rotated by central milled buttons at the back and front of the instrument. The protecting glasses on each side rotate in the outer frame, and are marked with hair lines on their inner surface, thus serving the purpose of a cursor.

CLINOGRAPH

The clinograph is more or less an adjustable set-square, as shown in *Fig.* 3, with one of the sides made movable by means of a stiff hinge placed at the apex. The movable side can be adjusted to any desired angle.

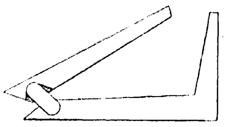


FIG. 3.

COMPASSES, PROPORTIONAL

Proportional compasses are used for dividing, reducing, or enlarging in any given proportion. The instrument is made in brass or electrum, and consists of two flat pieces of metal with a short point at each end as shown, each identical in shape, and cut away at the centre to form a dove-tailed slot. Fitted to the dovetails and sliding throughout their length is a split die provided with clamping screw which forms the axis of the instrument and locks the legs in position. The legs are kept in register when closed by a small projection on one leg which fits into a corresponding slot in the other leg while the adjustment of the axis is made. For setting the instrument suitable scales are marked, along the edges of the slots, and an index line is provided on the sliding die. The scales, usually four, are marked Lines, Circles, Plans, and Solids.

The scale of lines is used for enlarging or reducing drawings and the figures on the scale represent the proportions. Thus if the index is set to division 4 the proportion of the long points to the short points will be as 4 is to 1.

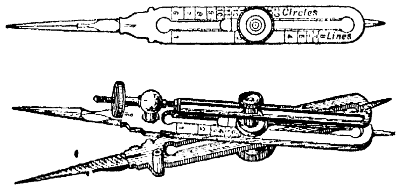


FIG. 4.

The scale of circles is used for dividing the circumference of a circle into equal parts up to 20. If the index is set to 12 the short points will divide the circumference of a circle of the radius given by the long points into 12 equal parts.

The scale of plans is used to reduce or enlarge the area of a plan. For example, if the slide is set to the division 3 the area of a circle struck by the long points will be three times the area of one struck by the shorter points.

The scale of solids is used in cases where enlargement and reduction on drawings is required to be in proportion to the cubical contents of the articles drawn. For example, to make a drawing of a tank to give a cubical capacity of six times that of one already drawn the slide would be set to 6 and dimensions taken from the existing drawing with the short points would be set off on the larger drawing with the long points.

COMPASSES, TRIANGULAR

A simple three-legged compass made in the form of a pair of dividers with an extra leg hinged on the centre pin of the head joint. The extra leg moves in any direction horizon-

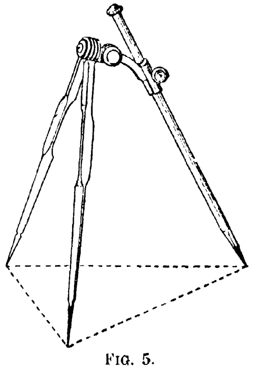


FIG. 5.

tally or vertically. The leg also has a clamping screw by which its length can be adjusted. This instrument is used for copying plans and drawings and is useful for testing the accuracy of copies of plans, etc.

CYMOGRAPH

An instrument for copying the profiles of mouldings or carvings, and sometimes used for copying portions of drawings full size.

The instrument is attached to a small drawing board by adjustable clamps (see *Fig.* 6).

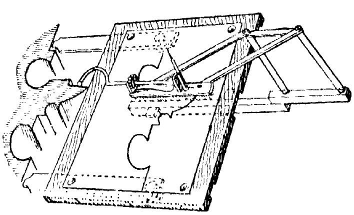


FIG. 6.

By an arrangement of two pivoted parallelograms, a platform, carrying the copying stylus and marking pencil, is controlled to give a parallel motion over the whole surface of the board.

The stylus, pencil, and parallelogram pivots are in line on the platform.

To enable undercuts or intricate profiles to be produced the stylus arm is made in the form of a bow, and is held in any desired position by a pawl engaging a ratchet wheel. The rotation takes place about the axis of the stylus ball.

DISTANCE MEASURERS

(i.) *The Opisometer* (*Fig.* 7).—A small instrument for measuring distance on maps, and is constructed as follows:

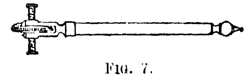


FIG. 7.

A small wheel with a milled edge is mounted on a screwed spindle, and the latter is carried by a fork in which it is free to slide, but not to rotate, the wheel being guided between the arms of the fork as shown. On each end of the screwed spindle are collars which act as stops. From one of the arms a pointer is fixed, and a line on the face of the wheel forms a zero mark.

The action is as follows: The milled wheel is rotated until one of the stops is brought against the fork, and the pointer against the zero mark on the wheel. The wheel with the handle in a vertical position is run over the map along the distance it is required to measure. This distance may then be found by running the wheel the reverse way along a suitable scale until the collar is back against the stop, and the pointer at zero. The distance travelled along the scale represents the distance on the map. The instrument is sometimes calibrated by means of a scale along the screw, and a divided wheel, so that the use of a scale is unnecessary.

(ii.) *The Rotameter* (*Fig.* 8).—This instrument is used for the same purpose as the opisometer described above, but it is more compact in form, and gives a direct reading on a dial. As will be seen in the figure, it consists of a small tracing wheel which, by means of clockwork or suitable gearing, moves the pointer on the dial, thus registering the distance travelled by the wheel over the map. It is made to read inches and $\frac{1}{16}$ths up to 40 inches, or by means of two hands, inches and $\frac{1}{8}$ths up to 25 feet.

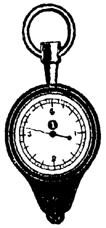


FIG. 8.

DRAUGHTING MACHINES

The Parallel-arm Machine.—This apparatus is designed to dispense with the use of the tee-square, set squares, scale and protractor, and is clamped to the drawing board, either at the left-hand top corner or, in some designs, in the middle. There are several types of this apparatus, all more or less following the same principle. A special type of machine is shown in *Fig.* 9, and its principle of construction is as follows: By means of two pivoted parallelograms a hinged arm is formed, one end of which is connected by a clamp to the top of the drawing board: the other end carries a combined type of square and protractor which is free to move throughout the surface of the board, the ruling edges of the square at the same time maintaining a parallel motion. The parallel motion is obtained by forming a crosshead at the elbow of the arm which rigidly connects the inner or centre short sides

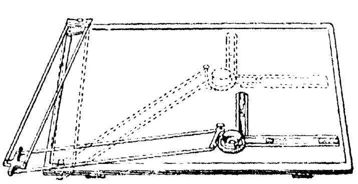


FIG. 9.

of the parallelogram at right angles to each other as shown in the figure.

The square is made up of two graduated scales set at 90°, which are detachable and fit into a stock which is pivoted and forms a protractor graduated in degrees. By this means the angular position of the square is controlled and it can be clamped in any position. It has a range of 90° clockwise and 45° in an anticlockwise direction. In some machines the protractor has a range of the complete circle. For rapidly setting standard angles such as 30°, 45°, 60°, and 90° an automatic spring lock is provided. The protractor is sometimes fitted with a vernier reading to minutes.

The standard set of graduated scales, which are interchangeable in the stock, consist of two long and two short scales covering 3, $1\frac{1}{2}$, $1\frac{3}{4}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$ and $\frac{1}{8}$ inches to the foot, and a 24-inch straight edge for inking purposes.

A horizontal hinge is provided at the clamped end so that the apparatus can be lifted clear of the board.

Draughtsman's Curves

French or irregular curves are made in thin pear-wood, vulcanite, or celluloid, and are used for ornamental designs, rounding angles, graph curves, oblique projections, etc. There are numerous shapes; some of typical shape are shown in *Fig.* 10.

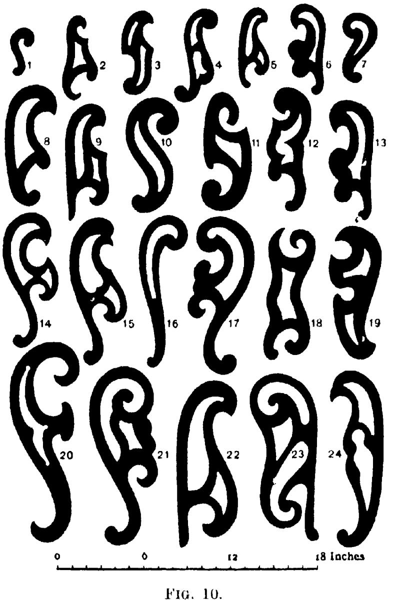


Fig. 10.

Architectural curves are similar to the above, but the profile is more irregular, and arranged to suit mouldings, arches, and the small-scale curves usual in architectural drawings. Typical examples are shown in *Fig.* 11. Radius or corner curves are made of thin plate, and provide a rapid means of rounding off small corners on drawings. The curves are made to standard radii, and merge into a straight line, so that a smooth finish to the drawn curves can be made. Similar curves for standard radii of larger dimensions are also made.

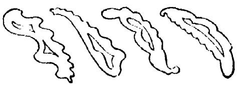


Fig. 11.

(i.) *Mathematical Curves* (*Fig.* 12).—A useful set of curves of this class have been calculated and designed by Mr. J. W. Brookes, and are made in transparent celluloid. The set of five consist of the following:

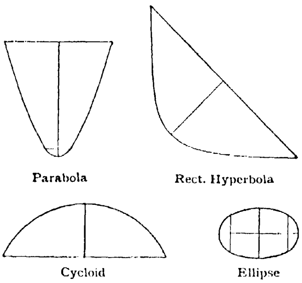


Fig. 12.

Parabola. Axis, focus, and latus rectum marked; equation $y = x^2$, unit 1 inch.

Hyperbola (*rectangular*). Axis marked; equation $xy = 1$ inch.

Cubic curve. Equation $y = x^3$.

Ellipse. Axes and foci marked. Major axis 3″, minor axis 2″.

Cycloid. Roulette of circle 2″ diameter; central ordinate marked.

(ii.) *Flexible Curves.*—Several forms of this type of curve have been designed by Mr. J. W. Brookes. The simple type shown in *Fig.* 13 consists of a thin steel strip controlled by a stiff link work attached to a number of small

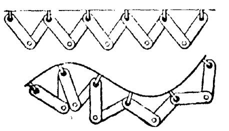


Fig. 13.

rings fixed to one side of the strip as shown. This type is also made in a form in which the curvature of the strip is controlled by the fingers instead of the links. Attached to one face of the strip are a number of eyelets into which the tips of the fingers can be placed to hold the curve to the desired shape (see *Fig.* 14).

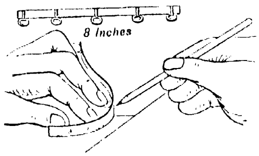


FIG. 14.

(iii.) *Copying Curve* (*Fig.* 15).—A curve of this type, also designed by Mr. Brookes, has a steel strip, similar to the above designs, controlled by several equally spaced rods sliding in suitable guides fixed to a straight rod.

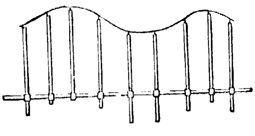


FIG. 15.

(iv.) *Bow Curve* (*Fig.* 16).—This curve, designed by Mr. W. F. Stanley, consists of a length of wood of rectangular section fitted with a set-screw at the centre, and two sliding links, one at each end. A small spline of lancewood is threaded through the links as shown in *Fig.* 16, and is bent to the required curvature by means of the set-screw. Splines of various lengths and sections may be used according to the curvature required.

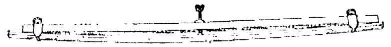


FIG. 16.

(v.) *Railway Curves* (*Fig.* 17) or curves of standard radii are made of strips of pearwood, sycamore, cardboard, or celluloid, about 1½″ wide. They are made in sets, the full set of 100 curves covering radii from 1 to 250 inches, the length of the largest radius being about 18 inches. The outer and inner edges are made to the same radius.

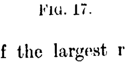
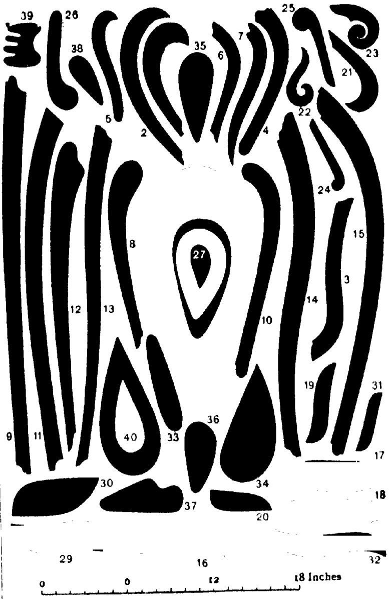


FIG. 17.

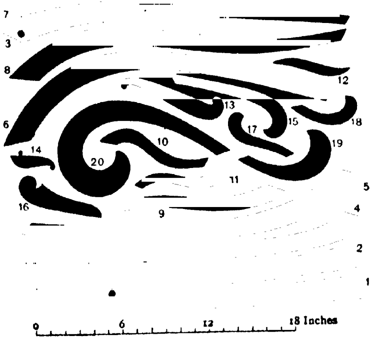
Some types are made with a short straight portion at one end, and the tangent point indicated by a line.

(vi.) *Ship and Yacht Curves* (*Figs.* 18 and 19).—These are made in special shapes for forming curves peculiar to ships and yachts. They are carefully designed, and in most cases the contour is a mathematically generated form of the ellipse, hyperbola, or parabola. The set usually consists of about forty curves, but as many as eighty different kinds are made.

Ship Curves.



Yacht Curves.



FIGS. 18 and 19.

CURVE RADIATOR (*Fig.* 20).—A simple device for finding the centre or radii of curves, such as railway curves, etc. It is made of vulcanite plate, and is used by setting the edges A and B to the curve and drawing a line along the edge C. The line thus drawn is radial to the curve. The intersection of two or more radial lines gives the centre of the curve. The device may be used on either side of the curve.

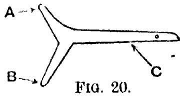


FIG. 20.

Drawing Scales

One of the most essential features in a drawing scale is accuracy. The material from which a scale is made is therefore important, and it should be carefully selected, bearing in mind the effects due to atmospheric changes, temperature, etc. The latter may be neglected except in very special cases, but ordinarily the physical qualities of the material and atmospheric effects must be taken into account if reliability is to be obtained.

The materials usually employed are steel, ivory, boxwood, vulcanite, and cardboard. Steel scales can be relied upon to retain their accuracy, but for drawing purposes they are not considered desirable owing to the reflecting surface, and hence the difficulty in reading the divisions; the weight is also objectionable. Ivory is excellent as regards wearing qualities, definition of marking, and cleanliness, but unfortunately it gradually contracts and continues to do so for a long period. It is also very expensive. Boxwood is more generally used, and if carefully selected and well seasoned is quite satisfactory for ordinary drawing purposes. Boxwood, however, is rather soft and fine markings are difficult to maintain. To overcome this difficulty boxwood scales are now often provided with white opaque celluloid edges attached by means of a tongue and groove, or moulded and glued on to the woodwork. Vulcanite is seldom used as it is particularly susceptible to atmospheric influences, but for use where accuracy is not important a vulcanite scale with white divisions tends to diminish eye-strain when working by artificial light. Cardboard scales, when specially treated and varnished, are satisfactory for ordinary rough drafting, and, being flexible, the laying off of dimensions with rapidity is facilitated. They are, however, difficult to keep clean and wear out quickly. Scales are made in several sectional forms; the more or less standard sections are shown in *Fig.* 21.

A thin edge is desirable if the dimensions are to be laid off with ease and without inaccuracies caused by parallax effects. The flat scale possesses advantages in this direction as it lies perfectly flat on the drawing. The oval type is usually held in a tipped position when in use and consequently is easily lifted from the drawing. This section is also more adapted for marking on both sides.

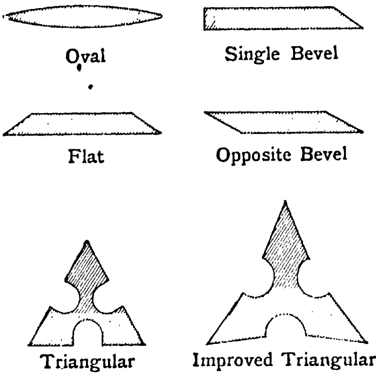

Oval
Single Bevel
Flat
Opposite Bevel
Triangular
Improved Triangular

FIG. 21.

The triangular scale is not so much in general use on account of its clumsiness. It is also expensive to manufacture, but it possesses three thin edges which lie flat on the drawing, and as each edge is marked on both sides six independent scales can be arranged. This type of scale is also made in "electrum" or steel and is formed by three thin pointed blades set radially at 120° to each other.

In cases where one scale only is required on each rule, thus avoiding mistakes often made through choosing the wrong scale when several are marked on one rule, the "single bevel" type is used. As an alternative to reduce the number of scales required by half, the "opposite bevel" type may be used. This type has the advantage, as in the case of the oval type, that when lying flat on the drawing it can easily be picked up.

The dividing or marking of a scale is now almost entirely done by means of a dividing engine, and this method is essential if uniform accuracy is to be maintained throughout the length, although in the earlier days marking was generally hand work. There are a considerable number of different forms of marking, and each is individual to the class of work for which the scale is required. The engineer, architect, surveyor each have their particular forms of marking. The engineer's scale is usually made 12″, 18″, or 24″ long, and subdivided duodecimally, decimally, or into eighths and sixteenths of an inch.

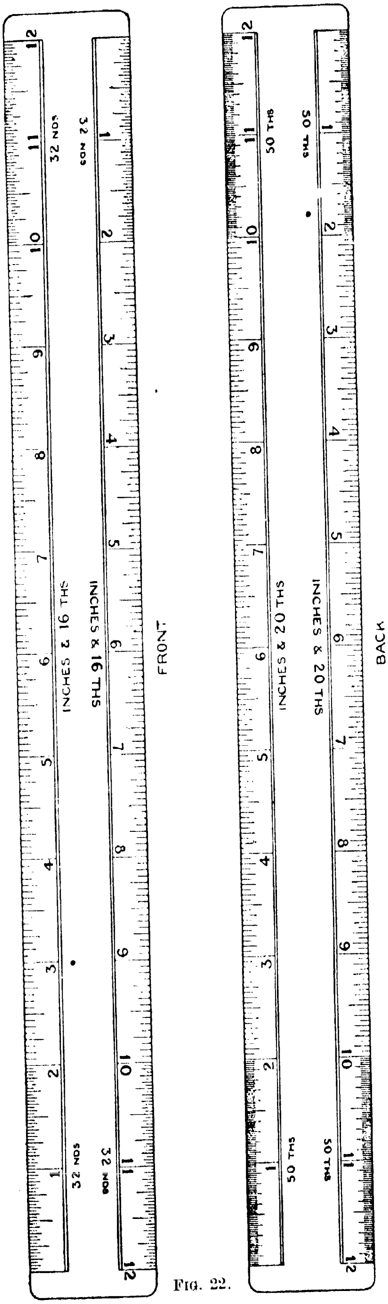

32 NDS
INCHES & 16 THS
FRONT
50 THS
INCHES & 20 THS
BACK

FIG. 22.

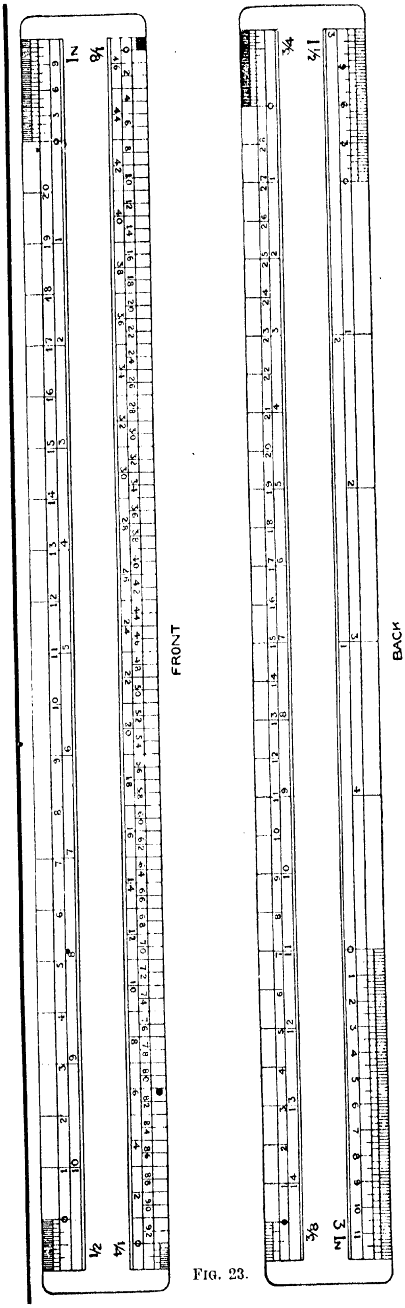

1 IN
3/8
1/2
1/4
FRONT
3/4
1 1/2
3/8
3 IN
BACK

FIG. 23.

The style of dividing preferred is a matter of choice. Some prefer the open divided type with subdivisions at the ends only, and others the fully divided type with subdivisions marked throughout the length of the scale. The figuring also varies. Some scales are figured from one end only, either from left to right or *vice versa*: others are figured in both directions, so that the figures can be easily read independently of the way in which the scale is placed on the drawing. Again, some are figured from the centre to each end, and are known as the "bisecting back" type.

Engineers as a rule prefer both edges on each face of the scale divided alike, so that if when in use the scale is accidentally turned round mistakes do not occur. With this system, of course, several scales are necessary to cover the range of proportions, but this is preferable to crowding on one scale a large variety of divisions which have always to be carefully examined to make sure that the right set is being used.

Mechanical engineers often prefer for general work a simple fully divided scale marked into inches and sixteenths throughout the length of both edges on one side, and on the other side similarly divided into inches and twentieths.

The necessary reduction or enlarging in the case of scale drawing is then done mentally. It is convenient, however, to have each end for, say, two inches again subdivided, *i.e.* one side into thirty-seconds and the other into fiftieths. A scale of this type is shown in *Fig.* 22. For ordinary small and medium-sized mechanical drawings when the inch is the unit, the scale described above will conveniently cover drawings of the following proportions: 2/1, 1/2, 1/4, 1/8. In large size mechanical and architectural drawings when the unit is the foot, it is, of course, more or less necessary to have a scale for each of the proportions, *i.e.* 6″, 3″, 1½″ and ¾″ to the foot. The subdivisions in this case are in twelfths. Standard scales for engineers and architects can be obtained in proportions as follows: $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 inch to the foot; also 1½ and 3 inches to the foot.

A special scale (see *Fig.* 23) known as the "Armstrong," for use in technical schools and colleges, is made oval in section. The four edges contain "eight scales," viz. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and 1 inch to the foot, $\frac{3}{8}$, $\frac{3}{4}$, 1½, and 3 inches to the foot, and the scales are open divided and subdivided at each end as shown.

Some scales intended to be more or less of a universal character have the whole available space on both sides occupied with scales, and in some cases nearly twenty different scales are provided. They are obviously inconvenient for general work as some of the scales do not lie along an edge and stepping off dimensions with dividers is therefore necessary. The close arrangement adopted is confusing, and liability of error is increased by choosing the wrong scale. Scales of this type are still made for builders and architects.

(i.) *Chain Scales.*—These, which are used by civil engineers and surveyors, are usually made in the flat section, and are fully divided, the bevelled edges only being marked. One bevel is usually divided into chains and the other into feet. Sixty-six divisions on the feet edge are made equal to 100 divisions on the chain edge. Thus conversions can easily be made.

Six chain scales constitute a set for general work, each having an identification number according to the divisions per inch; thus a 40 scale will have 40 divisions per inch. The numbers of a set are 10, 20, 30, 40, 50, and 60, although, of course, scales up to 100 are used for topographical surveys. Offset scales used with the chain scale for plotting are made from 2″ to 6″ long according to the length of the chain scale. The offset is marked similarly to the scale with which it is used, and the ends are accurately square with the bevelled edges. For a typical example of scale and offset see *Fig.* 24.

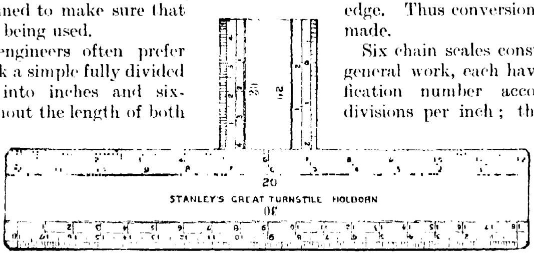


FIG. 24.

(ii.) *Diagonal Scales*, although often marked on the back of a type of rectangular protractor supplied in cases of drawing instruments, are seldom used for general work.

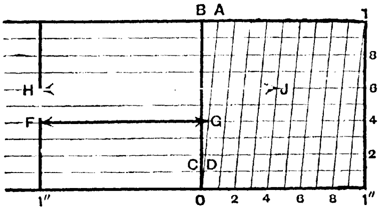


FIG. 25.

Fig. 25 shows the form of the marking on a full-size diagonal scale with which it is possible to obtain dimensions with dividers varying by ·01 inch, provided, of course, that the marking is sufficiently fine and accurate. The scale is divided vertically into 10 equal spaces by lines drawn horizontally. Crossing these lines perpendicularly the unit divisions (in this case 1″) are set out. The top and bottom lines of the end division are also divided horizontally into 10 equal parts by sloping parallel lines, the slope being 1/10 in the full width of the scale. Thus the first division on the top is connected by an oblique line to the 0 division on the bottom line, the second division on to the top line to the first division on the bottom line, and so on.

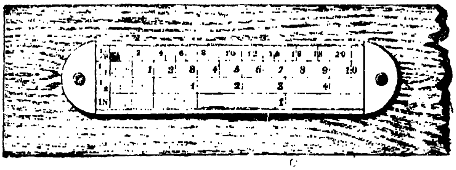


FIG. 26.

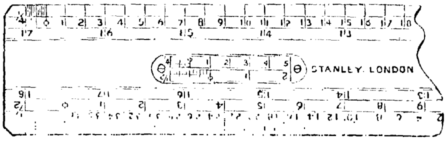


FIG. 27.

The principle of the scale is as follows:

Consider the triangle OAB. The distance OC is 1/10 OB. As OA and OB are straight lines the distance CD must equal 1/10 of BA, but BA is ·01″. Therefore CD = ·001″. The tenths on the scale are read off by figures on the bottom horizontal line and hundredths by the figures on the vertical line at the end. For example, it will be seen that the dimension FG = 1·04 inch and the dimension HJ = 1·46.

(iii.) *The Compass Scale* (*Fig.* 26) is a small open-divided scale usually 2″ or 3″ long, in brass, electrum, or ivory, made for the purpose of setting dividers, compasses, or spring bows, and thus save the drawing scale from the injury of the compass points. It is sometimes attached to the tee-square, but it is more convenient to have it sunk into the drawing scale and marked with corresponding divisions (see *Fig.* 27).

DRAWING TABLES

(i.) *Adjustable Tables.*—These are made to a variety of designs. The general principle, however, is more or less similar and provides a means by which the drawing-board can be fixed at any desired angle from horizontal to vertical by an arrangement of the clamps, quadrants, and levers. Trays and ledges are suitably arranged for the accommodation of pencils, instruments, etc. The tee-square or sliding straight edge is designed to move freely and yet retain its position when at rest without slipping. Where the straight edge is used pulleys and cords are arranged to control the parallel movement, but in the case of the tee-square some simple quick-acting clamp is provided

(ii.) *The Stanley Drawing-table* (*Figs.* 28A, 28B).—This consists of a balanced drawing-board which may be inclined at any angle between the horizontal and vertical, and is clamped rigidly in position. Height adjustment is also provided so that the operator may either work in a sitting or standing position.

The tee-square may be clamped instantly in any position and is always held

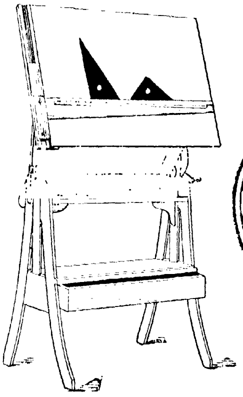


FIG. 28A.—Board raised.

DIRECTOR OF INDUSTRIES Bengal

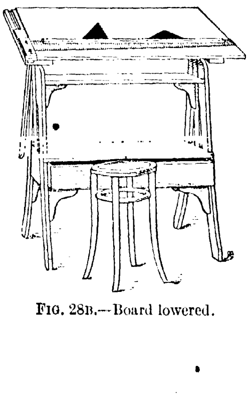


FIG. 28B.—Board lowered.

close to the surface of the paper. A ledge on the blade acts as a rest for instruments, pencils, etc., when the board is set to a steep incline. The board is readily detachable and can be replaced by another when required.

(iii.) *Perspective Drawing-board* (*Fig.* 29).—This board, which is fitted with an adjustable stand, is, as its name implies, specially designed for perspective drawing. It is more or less of the standard pattern, fitted with a tee-square, running in a groove, and provided with a clamp. On both the extreme right and left hand edges of the board are grooves into which adjustable studs are fitted for use with two centrolineads. The centrolineads are designed to suit the board, and are each fitted with a hinged ruling arm so that either rule, when not in use, can be moved clear of the board.

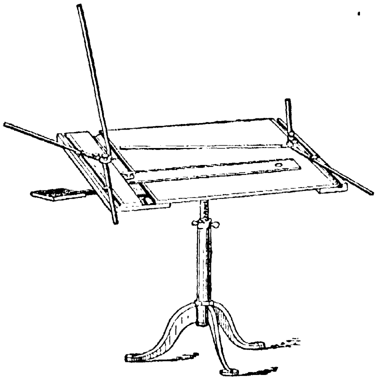


FIG. 29.

For a detailed description of the centrolinead and its method of use, see above under heading "Centrolinead."

(iv.) *Tracing Frame* (*Fig.* 30).—Used for copying drawings when the copy is required on thick paper and with minimum injury to the drawing to be copied. It consists of a simple framework carrying a sheet of plate glass which is hinged to a sub-frame at base, and by means of movable struts the inclination of the glass can be adjusted to suit the direction of the light.

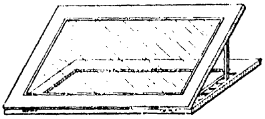


FIG. 30.

The drawing to be copied, with the plain paper above, is pinned to the frame over the glass, and the frame adjusted to receive a maximum amount of light through the glass and paper. By the aid of a good light the tracing can be very accurately done. A reflector is sometimes used if the paper is thick and a more concentrated light is required on the glass.

ECCENTROLINEAD

A simple instrument for drawing eccentroradial lines from a given centre: for example, the taper arms of a wheel or the teeth of milling cutters and other uses in mechanical drawing. In its simplest form it consists of a small ivory rule about 5″ long, to which is pivoted a small arm carrying a needle-point. The pivot is provided with a locking screw. The needle-point swings radially about the end of the rule and can be fixed at any desired angle to the right or left of the centre of the rule. Its position is indicated on a scale. In use, when set to the required angle, the needle-point is put in the centre of the wheel to be drawn and one side of each of the arms drawn. The needle-arm is then thrown over to the reverse side of the centre of the rule, and the remaining side of the arms drawn.

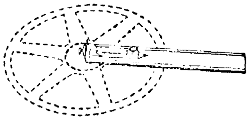


FIG. 31.

ELLIPTICAL TRAMMEL

(i.) With the aid of this instrument large ellipses can be easily and accurately drawn. The type shown is designed more particularly for sizes above 5″ minor axes. The bed is constructed in the form of a cross, with short needle-points protruding from the under side for fixing the instrument to the paper. Through the centre of the arms, on the upper side, are two vee-slots

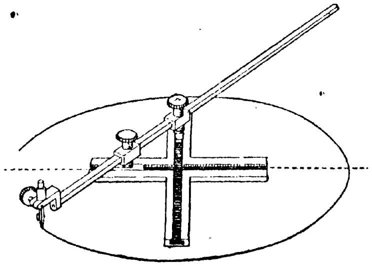


FIG. 32.

crossing each other at right angles. Each slot is fitted with a slide carrying a vertically pivoted swivelling head through which a rectangular trammel rod is made to slide freely, or be locked in any position with a milled set-screw. On the end of the rod a fitting is fixed to receive the scribing pen or pencil. To draw an ellipse, the instrument is used as follows: Two lines are drawn at right angles on the paper and the major and minor axes marked off. The instrument is then placed in position with the centre lines of the tee-slots normal to the lines drawn, and is fixed by pressing the needle-points into the paper. With the set-screws in the slides free, the trammel rod is brought into line with the major axis, and the pencil-point set to the axis mark. The minor axis slide now being in the centre of the cross is locked to the rod by the set-screw. (It should be noted that the major axis slide is between the scribing point and the minor axis slide.) In a similar way the major axis slide is set by moving the rod through 90° so that the pencil can be set to the minor axis mark, and the rod locked with the set-screw to the major axis slide. The slides are now both locked to the rod, and the ellipse may be drawn by moving the pencil carefully round the instrument, allowing the pair of slides to control its path.

The action of the trammel depends on the property of an ellipse, that if, from a point on the minor axis as centre, a circle be described having the major semi-axis as radius, and the centre be joined to the point of intersection of the circle and ellipse, then the intercept on this line between the point of intersection and the major axis is equal to the minor semi-axis of the curve.

(ii.) *The Semi-elliptic Trammel* (*Fig.* 33).—This is similar in principle and action to the instrument described above, but with it much smaller ellipses can be drawn, and it has a much larger range over all sizes. It has the disadvantage, however, that only half of the ellipse can be drawn with one setting. The slides are arranged in different planes, and being independent a smoother working action is obtained. The minor axis slot is at the top, similar to the elliptic trammel, but the major axis slot is cut in a vertical face along the front of the instrument. In setting the instrument the vertical face can be set along the line of the major axis, enabling a much easier and more accurate setting to be obtained.

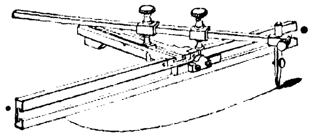


Fig. 33.

(iii.) *Elliptograph* (*Fig.* 34).—This instrument is designed to produce ellipses of small size with one setting, and has a range from ¼″ to 6″ major axis. The principle of its action is a combination of a rotary and a rectilinear motion.

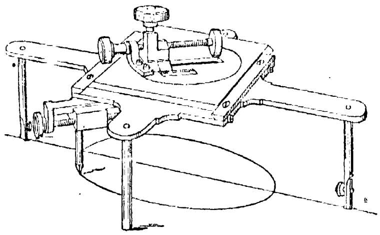


Fig. 34.

The base is in the form of a table on four legs. Two of the legs are fitted with needle-points for locating the axis of the instrument along the major axis line of the ellipse to be drawn. In line with the needle-points a long slot is cut in the centre of the table and fitted with a slide. Across the table at right angles to the slot a rectangular plate is arranged to slide in dovetailed guides, and the plate has a large circular hole in its centre into which is fitted a disc free to rotate in the plate. The disc has an adjustable centre in the form of a slide fitted in a slot across its diameter, and provided with an index and a scale, as shown, for setting the eccentricity of the centre. This centre is arranged to work coaxially with the centre of the slide in the slot of the table beneath, and directly through their axes a square vertical rod passes, to the bottom of which a bar carrying an adjustable pencil is attached. By turning the milled head at the top of the vertical rod the ellipse is described, and by lifting the milled head the pencil is removed from the paper. The pencil is adjusted radially by means of the horizontal milled head shown in the figure, and a scale and index are provided.

To set the instrument for a given ellipse, the pencil is adjusted in its slide to the required minor axis, setting of course by the scale. On the disc above the difference between the major and minor axes is set to the scale on the disc.

It should be noted that the bar carrying the pencil is in line with the slot and slide in the disc.

After the above adjustments, the instrument is set in position by the needle-points along the major axis line drawn on the paper, and by rotating the milled head referred to above the ellipse is produced to the required dimensions.

It will be seen that during the operation the rectangular plate moves to and fro in its transverse slides as the disc rotates, and that the disc merely forms a crank to which the pencil is attached, and is controlled in its movement by the major axis slot in the bed beneath.

Hudson's Horse-power Computing Rule

A form of slide rule for calculating the proportions of steam engines. It is fitted with two slides movable in either direction. It is made in cardboard or boxwood. It gives without any calculation—

(1) The I.H.P. from the usual data.

(2) The size of the engine for any given power.

(3) Piston speed.

(4) Ratio of cylinders of compound engines.

(5) The mean pressure for any cut-off can be found from the initial pressure.

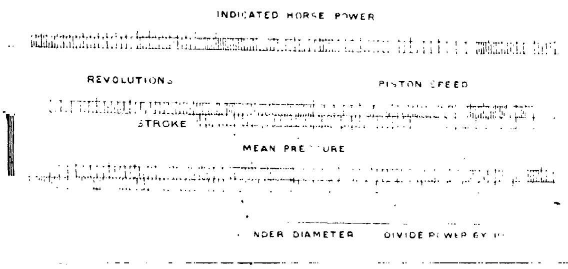


Fig. 35.

Isograph

A form of adjustable protractor used similarly to the ordinary set-square for setting off any angle. It is similar in construction to the carpenter's two-foot single folding-rule: the laminated hinge in the case of the isograph is graduated in degrees to form a protractor, and arranged so that the two straight edges can be set to any inclination between 0 and 180 degrees.

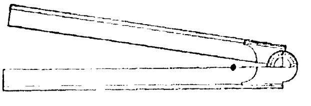


Fig. 36.

Laths for Curve Drawing

(i.) *The Parabola.*—This is a simple apparatus by which parabolic curves of given dimensions may be drawn with reasonable accuracy. It consists of a lath and a tee-square.

The lath has one edge which is true and straight, and the square is similar to an ordinary tee-square, but is flush on both sides; the top of the stock is square with the blade edge.

In producing a given parabola the axis and base lines are drawn and the focus and base points marked. The lath is set in a position parallel to the base line either along the directrix or in a position to leave sufficient space for the curve to be drawn.

The stock of the square is placed against the lath with the edge of the blade intersecting the base line at the base point as shown. A cord is then stretched from a pin fixed in the focus point and attached to the blade of the square at the base point. The attachment is made by drawing the cord through a hole drilled transversely through the blade at the required place, and fixed by means of a round taper wedge. Several holes of suitable pitch are usually provided.

The parabola is drawn with a pencil, commencing at the base point by holding the cord with the pencil-point against the blade edge and at the same time allowing the square to slide along the straight edge towards the axial line. The length of the cord is equal to the distance between the base line and the directrix. Hence clearly the distance of the

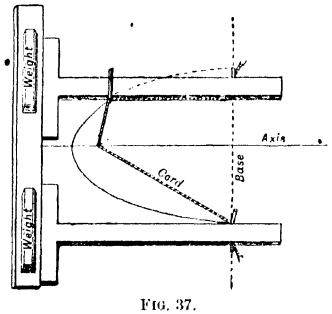


Fig. 37.

pencil from the focus is equal to its distance from the directrix, the fundamental property of a parabola. When one half of the parabola is completed the other half is drawn by turning the square over and repeating the operation. The lath is held in position by weights. The illustration shows the square in two positions

(ii.) *The Hyperbola* (*Fig.* 38).—The hyperbola may be drawn in a similar way to the parabola method described above. In this case a simple lath is used. A centre, at one end, in line with the edge is provided by means of a hole drilled in a metal projection. Along the edge of the lath transverse holes are drilled, and a wedge is provided for fixing the cord. The method of use is as follows :

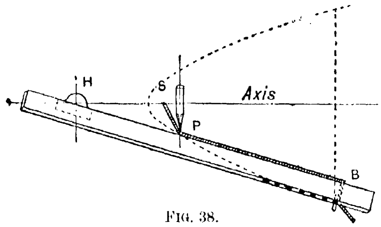


FIG. 38.

The axes and base line of the hyperbola are drawn and the focus S, his second focus H, and the base points marked. The lath is centred at the second focus H point by means of a pin passing through the centre hole referred to above, thus forming a fulcrum. The lath is then set so that the true edge intersects the base line at the base point. From a pin fixed at the focus S a cord is attached to the edge of the lath and fixed by means of a wedge to one of the transverse holes at B referred to above.

The semi-hyperbola is drawn by running the pencil along the edge of the lath towards the apex and at the same time keeping the cord between the pencil-point and the lath, allowing the latter to swing until in line with the axis. For we have

$$HP + PB = \text{a constant,}$$

and

$$SP + BP = \text{a constant.}$$

Thus $HP - SP =$ a constant, which is the fundamental property of the hyperbola.

The hyperbola is completed by turning the lath over and repeating the method on the other side of the axis.

LOG-LOG RULES

The Log-log rule is similar to the ordinary slide rule, but is made wider to accommodate additional scales which enable powers and roots to be determined by one operation. The principle involved was first used by Dr. Roget in 1815, but not until recent years has the system been successfully applied to the slide rule.

There are several types of log-log rules made, but one of the earliest of this type was designed by Professor Perry, F.R.S., and is known as the " Perry Rule " (described below). The special log-log scales are generally arranged along the upper and lower edges of the rule, and are used in conjunction with the ordinary scales. The log-log scale is graduated in divisions proportional to the logarithm of the logarithm of the numbers indicated along the scales, and is used for calculating $a^n = x$. The logarithmic method of calculating $a^n = x$ is $\log a \times n = \log x$.

By again taking logs,

$$\log(\log a) + \log n = \log(\log x).$$

It is now evident that given a suitable log-log scale on the rule this calculation can be accomplished by simply adding n by the slide and reading x on the log-log scale.

(i.) *The Perry Rule* (*Fig.* 39).—In the Perry rule the log-log scales are used in conjunction with scale B of the slide, and extend along the upper and lower edges of the rule. The upper scale, known as E, has a range of numbers from 1·1 to 10,000, and the lower scale, $-E$ or E^{-1}, a range from ·0001 to 0·91, enabling numbers less than unity to be easily dealt with. The scales are reciprocal one with the other. Thus, ·5 will be found on scale E^{-1} opposite 2 on scale E.

In using the rule to find a^n, a is simply multiplied by n, using the scales E and B. Thus, set the unit of slide B opposite a on E and read the answer on E opposite n on

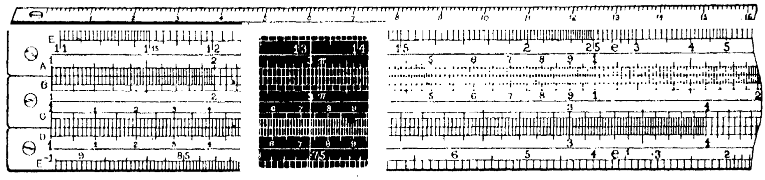


FIG. 39.

B. The index n may be positive or negative. The result for a negative index is read off on the reciprocal scale because $a^{-n} = 1/a^n$, that is, a^{-n} is the reciprocal of a^n. To find a^{-n}, if a is greater than unity we multiply a on the scale E by n on the slide and read the answer on scale E^{-1}, but if less than unity multiply a on E^{-1} by n on B and read the answer on E.

In using the log-log scales it should be noted that the values of the marking should be taken as given (*e.g.* 1·5 is definitely 1·5, and cannot be taken as 1500 or ·0015, etc., as with the ordinary scales).

(ii.) *The Faber Log-log Rule* (*Fig.* 40).—In this rule the two log-log scales are arranged in a similar way to the Perry rule, but the upper scale has a range from 1·1 to 2·9, and the lower scale from 2·9 to 100,000. These scales are used in conjunction with the C scale of the slide. When using on the right-hand end of the slide a special unit line marked W is used as the unit point. This rule is also provided with two special scales on the stock beneath the slide, one for calculating efficiency of dynamos, effective horse-power, etc., and the other for loss of potential, current strength, etc.

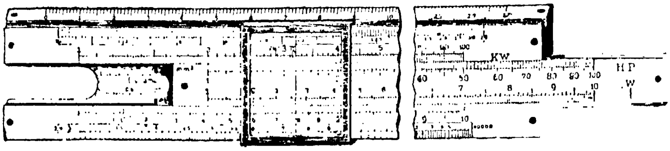


FIG. 40.

PARALLEL RULES

The simple type (*Fig.* 41) consists of two similar rules, one with a bevelled edge, pivoted together by two metal links as shown in the illustration. The pivot centres on each of the rules should be equidistant. The centres of the links also should be equal if accurate parallel lines are to be produced.

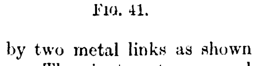


FIG. 41.

The range of this instrument is limited by the swing of the links. To increase the range another pair of links connecting another rule on the opposite side may be added. When arranged with two sets of links parallel lines directly above the base can be drawn without sliding the base along sideways.

Parallel rules are often used for section lining on engineers' drawings, and adjustable devices are sometimes fitted to limit the movement of the bevelled blade to the desired pitch of the sectioning. The method of use is as follows:

The rule is closed and a line drawn along the bevelled edge. The bevelled edge is then extended out to the limit allowed and another line drawn. The base of the rule is now closed up and the bevelled rule again extended and another line drawn. It will be seen that by opening and closing after each line is drawn a series of equidistant parallel lines are produced.

Set squares serve the purpose of this type of rule equally well, with the result that the parallel rule is not often used by draughtsmen.

The rolling type of parallel rule (*Fig.* 42) is far superior to the link type described above, and is used more particularly by civil engineers and architects for ruling parallel lines on large drawings when it is difficult to use a tee-square, and also when parallel lines in various directions are required. With this type of rule it is possible to rule parallel easily and rapidly in any direction directly in line with the base line.

The rule in its simple form consists of a fairly stout rectangular bevelled rule mounted on two wheels of equal diameter which project slightly below the lower surface. The wheels are mounted solidly on a long axle running on bearings fixed to the rule, and are protected above by a semicircular cover. These rules are made in ebony with brass and electrum fittings, and the sizes range from 6″ to 24″ long, and sizes up to 36″ are sometimes made. Weight within reason is an advantage. The lighter types are liable to slip on smooth paper and for this reason the rules are sometimes made throughout in gun-metal or electrum.

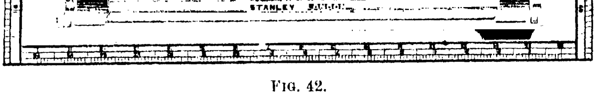


FIG. 42.

PROTRACTORS

The simple type (see *Fig.* 43) consists of a circular plate of electrum, vulcanite, or celluloid, the circumference of which is bevelled to a thin edge and divided into 360° and half degrees. The marking is figured every 10°. The circular plate is cut away near the centre, leaving a cross-bar one edge of which is diametral, and bevelled to a thin edge. Central along the bevelled edge a line is marked which represents the centre or setting point of the protractor.

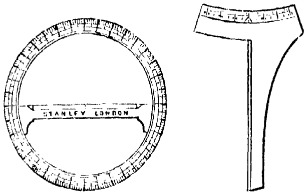


FIG. 43.

The transparent celluloid type is usually made solid, and is marked and divided on the underside, thus eliminating errors due to parallax. The setting point merely consists of a small hole pricked through or cross-lines on the underside. To obtain more accurate results a small separate arm marked with a vernier is used (see *Fig.* 43). With this attachment, marking off to minutes may be accomplished, and the bevelled edge of the arm enables a portion of the radial line to be drawn.

Another type of circular protractor (see *Fig.* 44), made in brass or electrum, has the vernier scale and arm arranged to rotate radially about a hollow centre bearing supported by four radial arms. The setting point in this case is marked on a small circular piece of glass or celluloid let into the centre of the hollow bearing and concentric with it. The setting point is sometimes located by a small circle as well as the cross-lines, but in this case the lines join the circle but do not cross it. With the aid of the lines it is possible to judge the centre of the circle more easily. Marking off directly from the radial arm often leads to inaccuracies if the thickness of the pencil-point is not accounted for. A more accurate means as an alternative is provided by forming the end of the arm to accommodate a small hole through which a pricker can be used. The arm is sometimes made in the form of a spring, with a needle-point at the end. Thus by slight pressure with the fingers the required position is located by a puncture point on the paper.

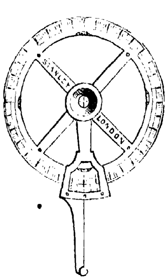


FIG. 44.

The radial setting points on the type of protractor just described are engraved at 45° on bevels arranged on the inner circumference of the divided circle. Needle-points are also provided at the outer ends of the arms for fixing the protractor in position on the paper.

A more elaborate type of protractor shown in *Fig.* 45 is used more particularly for theodolite plotting. This instrument is fitted with two arms in conjunction with the vernier, these being fitted with needle-points hinged and controlled by a spring. The needle-points are arranged diametrically opposite each other, thus giving two punctures as well as the centre mark through which the angular lines can be drawn.

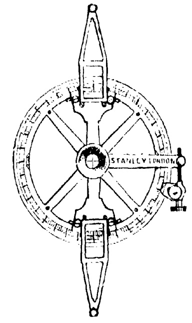


FIG. 45.

SLIDE RULES

The slide rule provides a simple mechanical means by the aid of which arithmetical, algebraic, and trigonometric calculations can be easily and rapidly performed. Its operation is based on the properties of logarithms, and it may almost be regarded as a table of logarithms in scale form. In slide rule calculations the system of logarithms is used more or less unconsciously by the operator, and it is not necessary for the operator to understand the principle of logarithms, as the figures representing the value of the divisions marked on the scales and used in the calculations are not logarithmic: it is the spacing of the divisions only which bears relation to logarithms.

Slide rules depend for their action on the law that

$$\log xy = \log x + \log y.$$

The lengths of the divisions of the rule are proportional to the logarithms of the corresponding numbers, and numbers are multiplied by adding their logarithms.

(i.) *The Logarithmic Slide.*—The straight logarithmic scale was invented by Edmund Gunter in 1620, and with it calculations were made by the aid of dividers. After this various types of logarithmic slide rules with one or more sliding scales were invented, but it was not until 1850 that Mannheim designed the standard British rule used at the present time. This type of rule, also known as the "Gravêt" rule (see *Fig.* 46), consists of three parts, the body, the central slide, and the cursor. The body and the slide are each marked with two scales, and it will be seen that when the slide is in its normal position the adjacent scales of the slide and body are alike.

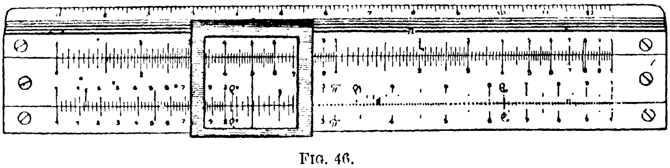


FIG. 46.

The cursor is a small metal sliding frame containing a glass on which a hair line is drawn and is used for the purpose of co-relating divisions of one scale with those of another.

For reference purposes the scales on the front face of the rule are generally referred to by the letters A, B, C, and D (see *Fig.* 47). Scales are also marked on the back of the slide and are lettered T and S. These scales are used in conjunction with the others for trigonometrical computations, T for tangents, and S for sines. Another scale marked L, for obtaining the mantissa of logarithms, is often to be found in the centre of the slide between scales T and S.

The divisions of the scales A and B, C and D, are proportional to the common logarithms of the numbers with which they are marked. Thus, for example, the distance from the left-hand unit to the figure 3 does not represent 3 of any specific unit, but is proportional to the logarithm of 3, and if measured on scales C or D of a true[1] 10″ rule the distance would be 4·771″, *i.e.* the log of 3 in the proportion 10/1. Similarly the distance of 2 will be the logarithm of $2 \times 10 = \cdot 3010 \times 10 = 3\cdot 01$. It will be seen, therefore, that the scales can be readily marked off from a table of logarithms.

Scales A or B are marked off from logarithms in the proportion 5/1, making them half the size of scales C and D. The divisions on the scales are arranged decimally, and by simple observation the value of any subdivision is found, and by interpolating, the value of readings between two subdivisions can be obtained.

The value of the commencing figures of the rule and slide may either be taken as 1 as indicated, or ·1, 10, 100, 1000, provided, of course, the values of the other figures are altered in the same proportion; if 1 is taken as 10, then 2 will be 20, and so on.

When calculating with the slide rule, beginners often have difficulty in locating the position of the decimal point in their results. It is possible to formulate rules for this purpose, but unless they are memorised and carefully applied they cause delay. Experienced users of the instrument usually calculate throughout, regarding all the figures as whole numbers, and fix the position of the decimal point in the result by inspection or by a rough mental calculation.

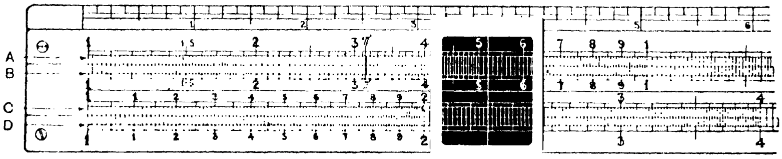


FIG. 47.

Sometimes, however, when an expression contains a number of figures involving zeros and decimals it is desirable to rearrange the decimal points so that each quantity contains one whole number only, multiplying in each case, of course, by 10 to some power positive or negative according to the number of places and direction the decimal points are moved. For example:

$$\frac{268\cdot 3 \times \cdot 000675}{\cdot 03 \times 8006\cdot 1} = \frac{2\cdot 683 \times 10^{2} \times 6\cdot 75 \times 10^{-4}}{3 \times 10^{-2} \times 8\cdot 0061 \times 10^{3}}$$

$$= \frac{2\cdot 683 \times 6\cdot 75 \times 10^{-2}}{3 \times 8\cdot 0061 \times 10} = \frac{2\cdot 683 \times 6\cdot 75 \times 10^{-3}}{3 \times 8\cdot 0061}.$$

This method not only assists in finding the decimal point, but simplifies the calculation to a great extent.

A considerable amount of practice is necessary with the slide rule to obtain results with rapidity and precision, and also for rapidly settling the position of the decimal point. Some general methods of calculation are explained below.

---

[1] The length of a scale or rule usually described as 10″ will generally be found to be 25 centimetres.

(ii.) *Squares and Square Roots.*—If the scales A and D are examined it will be seen that the numbers on A are the squares of the numbers directly below on D. The use of the cursor enables the coincidence of the divisions on the two scales to be more easily noted. If the cursor is set to coincide with a given number on D, the square of the number can be read off on A. In extracting square roots this process is, of course, reversed.

(iii.) *Multiplication and Division.*—In using logarithms, multiplication of two or more numbers is effected by adding the logarithms of the factors, and division by subtracting the logarithms of the factors. This is precisely the principle attaching to the use of the slide rule for these calculations, the addition or subtraction in this case being performed by the slide scale, since the linear dimensions of the scales as described above are proportional to the logarithms of the numbers marked on the scales. For example, to multiply 2·5 by 6, set the unit of the slide opposite 2·5 on the scale A, and against 6 on the slide read the product 15 on A. For division the operation is *reversed*. Thus, to divide 6 by 2·5, set the 2·5 on the slide against 6 on A, and read the quotient 2·4 on A against the unit of the slide.

It will be observed that by setting the unit of the slide against a given number on A the products of the number multiplied by any number on the slide can be read off directly on scale A.

The lower scales, C of the slide, and D of the rule, may be used in the same way as A and B for calculation, and they are sometimes preferred as the divisions are more open. It should be noted, however, when using these scales, or when continued multiplication is performed on the upper scales, that it is necessary to use the right-hand unit if the result of the calculation cannot be found on the scale, or that during a calculation it may sometimes be necessary to transfer from the unit at the one end to the unit at the other.

(iv.) *Proportion.*—This is simply a combination of multiplication and division. For example:

$$4:2::7:x=\frac{4}{2}=\frac{7}{x}.\ \therefore x=\frac{2\times 7}{4}.$$

Using the rule, set 4 on B opposite 2 on A, and opposite 7 on B read the answer 3·5 on A. Now that the slide is set, any proportion of the ratio 4 : 2 can be read off along the scale.

(v.) *Reciprocals.*—The reciprocal of a number n being $1/n$, the principle already described for division can be applied. For example, to find the reciprocal of 16, set 16 on C opposite the left-hand unit on D, and read opposite the right-hand unit of C the answer ·0625 on D. Or, alternatively, set the left-hand unit of the C opposite 16 on D, and opposite the right-hand unit of D read the reciprocal ·0625 on C.

Another method, by which all reciprocals can be read off directly without moving the slide, is to reverse the slide so that scale C is above scale B. Then, by setting the end units of the slide and rule in coincidence, the reciprocals of the numbers on scale D can be read directly on the inverted scale C by using the cursor. For example, in line with 4 on D will be seen ·25 on C. The numbers on C are, of course, read as decimals.

If the rule is examined with the slide set as described, it will be seen that the length of slide representing the reciprocal is added to the length of scale D representing the number, and since the reciprocal of a number multiplied by the number is equal to 1 the two lengths of the scale must be equal to 1 or 10, *i.e.* the total length of the rule.

(vi.) *Cubes and Cube Roots.*—The cube of a number is obtained by finding the square as previously described, and multiplying it by the number itself; or, as follows, which is similar, set the left-hand unit of the slide opposite the number on D, and opposite the number on B read the cube root on A.

A simple alternative method is to invert the slide so that scale B is adjacent to scale D. Find the number of which the cube is required on each of these scales, and set the divisions to coincide, then opposite the unit of the slide the cube will be found on scale A.

The cube root is obtained by locating the number on A with the cursor and adjusting the slide so that two numbers of the same value appear at the same time, one on the scale B under the cursor, and the other on scale D under the left-hand unit of scale C. Simply the reverse of the cube method.

An alternative method by inverting the slide as described above is perhaps more simple. Set the left- or right-hand unit opposite the number on A; look along the scales B and D, and find two numbers of the same value which coincide. The number found is the cube root. For example, to find the cube root of 64, invert the slide, set the unit of the slide (the right-hand one in this case) opposite 64; by glancing along the two lower scales it will be seen that divisions marked 4 coincide; 4 is then the cube root.

(vii.) *The Fourth Power and Fourth Root.*—The former is found by squaring the square root, and the latter by finding the square root of the square root.

(viii.) *Logarithms, Sines, and Tangents.*—On the reverse side of the slide will be found three scales. The upper one, marked S, gives values of sines; the middle one, marked L, the mantissa of logarithms, and the lower one, marked T, tangents.

The scale L, it will be seen, is divided into 10 equal parts from left to right, and subdivided into 50ths, and represents the mantissa of logarithms of the numbers found on scale D, to which they can be related by means of the lower reference line in the right-hand gap at the back of the rule. Thus, to find the logarithm of 4, set the left-hand unit of the slide opposite 4 on scale D, turn the rule over, and opposite the line in the gap read ·602 on scale L. This number is the decimal part or mantissa of the logarithm. The characteristic according to the rules of logarithms must of course be added.

(ix.) *Sines.*—By the use of the scale S the numerical value of the sine can be obtained for a given angle, or conversely the angle from the value of the sine. To find sine 30°, set 30 on scale S opposite the mark in the gap, turn the rule over, and opposite the right-hand unit of A read ·5 on B.

An alternative method enabling sines of angles to be read off direct is accomplished by turning the slide over so that sine scale lies along the scale A with the end units in coincidence. It will then be seen that values of the sines are given on A opposite the angles on the sine scales, and *vice versa.*

(x.) *Tangents.*—Tangents may be found in a similar way to sines, described above, by using the left-hand gap and the scale marked T. For example, to find the tangent of 30°, set the 30 on scale T opposite the line in the gap, turn the rule over, and read the value ·577 opposite the left-hand unit of D on scale C.

Powers and roots other than those of the simple forms already described can be calculated with the slide rule by the application of the ordinary logarithmic principles for power and roots. Thus logarithmically, to find a^n, multiply the log of a by n and delogarise the product, and find the number of the logarithm obtained. Similarly, to find $\sqrt[n]{a - x}$, proceed as above by dividing by n instead of multiplying. This process can be worked out on the slide rule by using the L scale in conjunction with scales C and D. It involves, however, several re-settings. Rules are now made, however, with additional scales by means of which power and root calculations can be made with one setting of the slide only (see "Log-log Rules.")

(xi.) *Cubing Rules* (*Fig.* 48).—The Reitz rule is an example of this type and has, in addition to the usual scales, A, B, C, and D, a special scale arranged along the top edge above scale A, which is equivalent to three times the length of scale D, thus enabling cube and cube roots to be read directly by means of the cursor, using scale D similarly to the way in which it is used with scale A for squares and square roots.

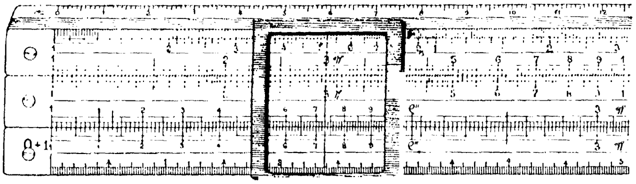


FIG. 48.

A scale, similar to the L scale on the back of the ordinary slide, is provided along the lower edge which gives the mantissa of the logarithms of the numbers on scale D.

(xii.) *Fuller's Slide Rule* (*Fig.* 49).—This rule is in cylindrical form, and the logarithmic scale is marked helically on the surface of the cylinder, and is equal to a straight slide 83 feet long. Multiplication, division, involution, and evolution can be performed with great accuracy, the results being read off to four places of decimals.

Referring to *Fig.* 49, it will be seen that the cylinder A, on which the scale is marked, is mounted on a sleeve to which is fixed a handle B and pointer C. The cylinder is free to rotate about or move up and down the sleeve. Fitting and sliding in the sleeve at the opposite end to the handle is a plunger carrying a pointer D. The readings are obtained by means of the pointers C and D. For example, to calculate $5 \times 8 \div 2$, set the pointer C to 5 on the spiral, then move the pointer D to the denominator 2, slide the cylinder until 8 appears under the pointer D, and read the answer on the cylinder under the pointer C.

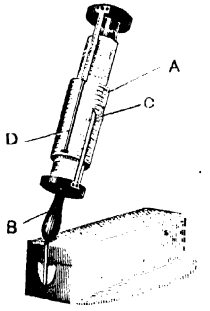


FIG. 49.

THE THACHERS RULE (*Fig.* 50).—This intrument consists of a cylindrical core which its into an open framework made up of twenty triangular bars as shown in the figure. The core is marked with forty longitudinal scales. The right half of each scale is similar to the left portion of the scale in advance.

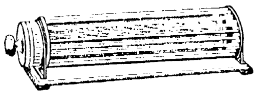


FIG. 50.

The lower edges of the triangular bars are marked with scales similar to the core, and on the upper edges scales of square roots are marked.

The triangular bars are fixed to rings which rotate in the supports at each end. The core is free to rotate and move longitudinally in the frame. Thus the scale on the bars can be brought in view, and any line on the slide brought opposite any line on the frame.

SPLINES AND WEIGHTS

These are used for drawing large irregular curves, more particularly of the type met with in ship designs.

The splines, or battens, as they are sometimes called, are long square strips of lancewood of varying sizes in lengths from 18″ to 8 feet, and from $\frac{1}{8}''$ to $\frac{3}{8}''$ square. A set is usually made up of about 75 sizes. The splines are also made to taper uniformly throughout their length or from the centre to each end, or *vice versa*. The weights are rectangular blocks of lead cased in mahogany or covered

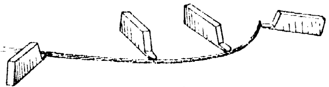


FIG. 51.

with leather. The base at one end has a pointed tongue with which the splines are pressed into the desired shapes. A. T.

DRAUGHTING MACHINES. See "Draughting Devices," p. 263.

DRAUGHTSMAN'S CURVES. See "Draughting Devices," p. 264.

DRAWING SCALES. See "Draughting Devices," p. 266.

DRAWING TABLES. See "Draughting Devices," p. 269.

DRAWINGS:
- Figuring of tolerances. See "Metrology," X. § (37) (iii.).
- Marking of. See *ibid.* X. § (37).
- Method of indicating accumulation of tolerances. See *ibid.* X. § (37) (iv.).

DROPS OF WATER, METEOROLOGICAL EFFECTS OF. See "Meteorological Optics," §§ (14)-(16).

DUFFIELD'S METHOD FOR MEASUREMENT OF GRAVITY AT SEA. See "Gravity Survey," § (5) (ii.) (*b*).

DYNAMICAL THEORY OF TIDES. See "Tides, and Tide Prediction," § (6).

— E —

EARTH, THE MEAN DENSITY OF THE

§ (1) THE CONSTANT OF GRAVITATION.—While geologists and others may have some kind of parochial interest in knowing that the earth contains a little more than $5\frac{1}{2}$ times as much matter as an equal volume of distilled water of its normal density, the real interest of the physicist is directed not towards this figure at all, but to the fundamental constant of universal gravitation of which the figure is a mode of expression. Thus, under the laws of gravitation as formulated by Newton, the orbits of planets round the sun or of satellites round their respective planets, or rather the common centre of gravity of each system, give the data from which the relative masses of the planets and sun may be determined, but no amount of astronomical enquiry will fill in the one unknown, the absolute mass of any one, or what comes to the same thing, the value of the constant G in Newton's equation:

$$\text{Force of attraction} = G\frac{Mm}{d^2},$$

where M and m are the respective masses of a pair of bodies, small compared to their distance apart or spherical in form, and d is the distance between their centres all expressed in consistent units. In order to find this constant G, experiment must be made in which M, m, d and the force of attraction can all be ascertained. Then knowing the acceleration of a falling body at the earth's surface, or of the moon towards the earth, and taking account in the former case of the corrections due to the elliptic form and centrifugal force and the distance to the earth's centre, the total mass and consequent mean density follow by calculation, and so, as stated above, this is a mode of expression for the constant of gravitation.

(i.) *Mountain Observations.*—There are two distinct ways of approach to the experimental solution of the problem. That practised first by Bouguer on Chimborazo, Maskelyne and Hutton at Schiehallion, James at Arthur's Seat, and others, depends upon the supposition that an isolated mountain may be treated as an excrescence on an otherwise homogeneous earth, or if not homogeneous and smooth, other local variations are as far as possible allowed for. Then a zenith instrument, set first to the north and then to the south, will have the direction of the vertical disturbed by the attraction of the mountain as a large independent body, making the apparent difference of latitude greater than that found by surveying. Data so obtained would, if the method were suitable, provide the means whereby with much calculation the mass of the earth and of the mountain could be compared, and so from a study of the mountain the mass of the earth could be ascertained. The trouble is, however, that the study of a mountain at the best leaves some doubt as to its geometrical and mass constant, and the supposition of a homogeneous earth or, what is sufficient, of an earth composed of homogeneous concentric shells is not justified. Mountains may be supported in whole or in part by material of lower density below than that on either side, giving rise to a kind of hydrostatic equilibrium and not requiring other support. In such case the attraction due to the mountain may be in part balanced by a diminished local attraction below. Besides this the homogeneous shell structure irrespective of the mountain may not exist. It follows therefore that, knowing the constant of gravitation from laboratory experiments, experimental comparison between a part of the earth and the rest of the earth may be used to get some idea of local density, but it is of little use to find the mean density or the constant of gravitation, and it manifestly cannot be used for finding both local and mean density. For this reason it appears to the writer unnecessary to enter into more details of these experiments.

(ii.) *Colliery Experiments.*—Another class of experiment is best known in relation to Airy's researches at the Harton Colliery in 1854, and repeated by Von Sterneck with an invariable pendulum at Pribram in 1883 and at Freiberg in 1885. In these the value of g is found by pendulum experiments at the surface and again at a considerable depth in the mine. At the surface the whole earth attracts the pendulum at the whole radius for distance; in the mine the spherical shell exterior to the pendulum has no action on the pendulum if homogeneous but the pendulum is attracted as by a smaller earth at a smaller distance. If the whole earth were homogeneous in density the attraction would be less in the ratio of the two diameters, but as the density of the interior of the earth is much greater than that of the surface, more is gained by being nearer the smaller and denser inner sphere than is lost by the elimination of the outer shell. It is evident that if the homogeneous shell structure could be relied upon, experiments of the kind would lead to good determinations of the mean density owing to the great accuracy with which the period of a pendulum can be determined, but the same general uncertainty as to local variations of density detract from the value of such measurements, and they will not be described further. They are discussed in Poynting's monograph[1] and by Burgess.[2]

For the same reason the experiments of Carlini at Mont Cenis, 1821, and of Mendenhall at Fujiyama, 1880, in which the period of an invariable pendulum is observed at the summit and foot of the mountain, are only mentioned.

(iii.) *Laboratory Experiments.*—The second way of approaching the problem, now generally considered that most likely to give good results, is—on paper—the simple and obvious one of observing in the laboratory the attraction f between two masses at a known distance apart. If these are homogeneous spheres of masses M, m and the distance between their centres is r, G is found at once from the equation

$$G=\frac{fr^2}{Mm},$$

i.e. if all the units employed are consistent. The difficulty of measuring this force, however, depends upon its extremely small value. Taking all the apparatus of the physical laboratory in which small forces or couples are measured, such as galvanometers, electrometers, which are generally considered delicate instruments, no precautions whatever are taken to allow for or avoid changes of distance between the moving and fixed parts resulting from a deflection and the consequent changes in the gravitational attractions, simply because these are immeasurably small in comparison with the forces or couples which the instruments are designed to measure.

In all cases where these forces have been measured in the laboratory some form of balance has been used. If the axis of rotation is vertical a torsion balance, if horizontal a true gravity balance, and in the latter case the arm of the balance may be horizontal as in an ordinary balance or vertical as in a metronome.

§ (2) The Torsion Balance.—The first laboratory instrument ever made was designed and constructed by the Rev. John Michell, who realised the value and appears to have been the inventor of the torsion method of balancing very small couples, but he did not live to complete it. Some years later Cavendish[3] altered and completed the Michell apparatus and made the experiment which bears his name. It is interesting to note that Newton discussed the possibility of carrying out both the mountain and laboratory types of observation, but by a curious numerical blunder concluded that the movements to be observed would be vastly smaller than as a fact they are. If he had not made this mistake he might well have applied his experimental aptitude to the problem. He, however, divined that the mean density was likely to be between 5 and 6.

---

[1] *The Mean Density of the Earth*, Charles Griffin & Co., Ltd.

[2] *Phys. Rev.*, 1902, xiv.

[3] *Roy. Soc. Phil. Trans.*, 1798.

(i.) *Cavendish.*—In the apparatus of Cavendish a light deal lever 6 feet long, stiffened with wire ties and carrying suspended at its ends lead balls 2 inches in diameter, was suspended by a fine silvered copper wire 39¼ inches long within a wooden case. After trial with wires of different degrees of fineness one was selected which gave a period of 7 minutes. The ends of the lever were observed by telescopes in the walls of the building. The larger masses were balls of lead 12 inches in diameter suspended from a turn-table under the ceiling, and turned round from one position close to the sides of the box, where their attractions tended to turn the lever in one direction, to the other position where the attractions were reversed, tending to turn the lever in the opposite direction. The whole angle between the two positions of rest so found was double the angle due to the attraction in either case. A determination of the period of oscillation combined with the ascertainable constants of the apparatus give all the information that is needed to find the actual attraction and G the constant of gravitation. Cavendish fully realised the importance of uniformity of temperature and the disturbances to be expected from convection currents, and he made a number of tests directly bearing on these disturbances. Cavendish obtained twenty-nine results, from which he deduced a mean value for the density of the earth Δ :

$$\Delta = 5{\cdot}448 \pm {\cdot}033.$$

The corresponding value for the constant of gravitation in C.G.S. units is

$$G = 6{\cdot}754 \times 10^{-8}.$$

(ii.) *Reich and Baily.*—The Cavendish experiment has been repeated several times, first by Reich[1] and Baily,[2] but no improvements of note were introduced until Cornu and Baille[3] made their investigations. They reduced the size of the Cavendish apparatus, using a lever consisting of an aluminium tube only 50 centimetres long, carrying at its ends balls of copper of 109 grammes each. A torsion wire of annealed silver 4·15 metres long was used together with a mirror, telescope, and scale for reading the movements, and the period of oscillation was 408 seconds. The attracting masses were of mercury aspirated from one pair of hollow cast-iron spheres to another pair so as to double the deflection. The spheres of mercury so used were 12 centimetres in diameter. By this means uncertainty as to uniformity of density in the interior of the large masses was entirely avoided and the movement of the mercury gave no disturbance to the apparatus. Cornu and Baille discussed the effect of linear dimensions on sensibility, and showed that in apparatus in all respects similar a change of linear dimensions in the ratio of $1 : n$ will increase the couple due to the attraction in the ratio of $1 : n^5$, but that if the torsion wire is so proportioned as to maintain the same period the torsional rigidity will also increase in the ratio of $1 : n^5$, so that the deflection will be unaltered. Their value for Δ was 5·56 and the corresponding G is $6{\cdot}618 \times 10^{-8}$.

(iii.) *Boys.*—The Cavendish experiment was very greatly modified by the present writer,[4] whose new instrument the quartz fibre (*q.v.* Vol. III.) is so eminently adapted for measuring the infinitesimally small forces due to the mutual attraction of masses of moderate dimensions. He showed that while with similar apparatus of dimensions $n : 1$, the angle of deflection is the same if the period is the same, yet it is possible with very small apparatus to use for large masses balls enormously greater in proportion than any that had been contemplated or were practicable with large apparatus; *e.g.* with a diameter more than four times the length of the beam, instead of one-sixth of this as in the apparatus of Cavendish. Then as such balls would act on the balls at the more remote end of the lever to a nearly equal extent in the opposite direction, he suspended these at a different level and so removed each small ball to a great extent from the adverse attraction of the large ball on the other side. With the proportions ultimately decided upon for the whole apparatus he calculated the azimuth of the vertical plane containing the large balls with respect to that containing the small, which should give the maximum deflection, and used them in that position where not only is the deflection greatest and the exact determination of the azimuth of the least importance, but the period of oscillation is least affected by the gravitational attraction. He further arranged that the heavy fixed parts of the apparatus should be made in the form essentially of figures of revolution about the axis of suspension, so that mutual gravitation between these parts and the suspension should have no component in the direction of motion.

---

[1] *Comptes Rendus*, 1837.

[2] *Roy. Astr. Soc. Memoirs*, 1843.

[3] *Comptes Rendus*, 1873 and 1878.

[4] *Roy. Soc. Proc.*, 1889, xlvi.; *Roy. Soc. Phil. Trans.*, 1895, p. 186; and *Rapports du Congrès Internat. de Phys. Paris*, 1900, iii.

Also the whole of the suspension which was composed of metal was either pure gold or was electro-gilt, and the interior of the tube in which the suspension moved was also electro-gilt and polished. Thus no forces due to contact electricity could exist, and even if they did they would have no components in the direction of motion. The lever and mirror were the same piece of glass about 23 millimetres long by 6 wide and about $\frac{1}{2}$ a millimetre thick, in the ends of which sharp V grooves had been cut; the small gold balls were suspended by quartz fibres lying in these grooves and hanging from hooks above. Thus the azimuth of the plane containing the gold balls was accurately determined by the mirror, in which a scale of 50ths of an inch ($\frac{1}{2}$ millimetres very nearly), at a distance of 7000 millimetres, was read by means of a large telescope. Thus one division on the scale represented an angle of deflection of $\frac{1}{25,000}$, and a tenth of a division, which could be observed with certainty, an angle of $\frac{1}{250,000}$ or $\frac{3}{4}$ second of arc. The large balls were made of lead cast and then compressed under hydraulic pressure in a strong accurately turned cast-iron mould 108 millimetres in diameter; each weighed 7407 grammes. Any vacuum cavities, had they been formed, would have been squeezed up before the superfluous lead was ejected in the form of wire, and uniformity of density was in this way assured. The small balls were of pure gold compressed in hollow steel hemispheres of three sizes, and they weighed 1·30, 2·65, and 3·98 grammes. The horizontal distance between the large balls was 15 centimetres, and this also was the vertical distance between the pairs of balls at the two levels. The angle giving the maximum couple was 65°. The complete period of oscillation of the mirror and small balls varied between 186 and 246 seconds, depending upon the pair of gold balls chosen and quartz fibre used for suspension. These were 432 millimetres long and about $\frac{1}{40}$ millimetre in diameter. The distance apart of the quartz fibres by which the gold balls were suspended, as also of the wires by which the lead balls were suspended, was determined by the aid of a special instrument called an optical compass in which two microscopes were made to see successively the fibres or wires and a very finely divided micrometer scale which had been verified both at the Standards Office and in Professor Viriamu Jones's Whitworth measuring machine.

The apparatus was set up by kind permission of Professor Clifton in a vault under the Clarendon laboratory at Oxford, and in its construction and in its surroundings was screened from temperature disturbances by multiple concentric metallic screens, double wood housing, and curtains to an extent that might seem superfluous. Convection currents were, however, by this means cut down to such an extent on good observing nights as not to disturb the motion by $\frac{1}{10}$ of a division on the scale, which corresponds to an air movement past the balls of 1 inch a fortnight.

Nine experiments in all were made: the first four before the complete screening and other improvements in details had been provided; the last in the winter, in a hurried visit of one night to Oxford. The remaining four, which were consecutive, were made under favourable conditions, and in these experiments every variation of weight and of distance and position, and of quartz fibre, was made. These four results were $\Delta = 5·5291$, 5·5268, 5·5306, 5·5269, from which without any regard to least squares, but with a recollection of the behaviour of the apparatus on the several occasions, the writer concluded that 5·527 was the best value to be derived from the experiments. The other values obtained under less favourable conditions were 5·5213, 5·5167, 5·5159, 5·5189, and 5·5172. These had but slight influence in arriving at the result given above. The corresponding value of G is $6·658 \times 10^{-8}$.

(iv.) *Braun.*—At the same time that the investigation last described was being made, Dr. Carl Braun,[1] S.J., designed and constructed with his own hands an apparatus which he set up in an enclosed corner of his room in the monastery at Mariaschein, a beautiful spot in the mountains of Bohemia. Dr. Braun used a lever 246 millimetres long, from the ends of which were suspended balls of 54 grammes. The larger balls, weighing 9146 grammes each, ranged between 38 and 43 centimetres from centre to centre. All four balls were at the same level. A torsion wire 840 millimetres long was used. Dr. Braun introduced a new feature in that the moving system was carried within a tall glass bell making an air-tight joint with a thick plate-glass base, and he maintained a vacuum of 4 millimetres of mercury under the bell. The mirror, carried centrally below the lever, was inclined at 45° to the vertical, and a horizontal scale below was read by reflection from the inclined mirror by means of a telescope. Dr. Braun had great trouble with the want of perfect elasticity and creeping of his zero, as others have had before him, a trouble entirely overcome by the use of the quartz fibre. The writer therefore paid a visit to Mariaschein, taking with him a number of quartz fibres of suitable dimensions. This visit, happening to be on Friday, Michaelmas Day, their feast day, and a fast day, was in other respects full of interest. Dr. Braun, already advanced in years, did not live to make further observations. The results obtained by him were $\Delta = 5{\cdot}527 \pm {\cdot}001$ and $G = 6{\cdot}658 \times 10^{-8}$, identical with that last given.

---

[1] *Denkschr. Akad. Wiss. Wien, Math.-Natur. Kl.*, 1896, lxiv., and *Nature*, 1897, lvi.

(v.) *Eötvös.*—Of all physicists who have interested themselves in this problem, the contributions of the late Baron Eötvös[1] are the most remarkable for the inventiveness and skill with which he has handled the balance, whether of torsion or supported on knife-edges; for with the torsion balance he not only determined G, but observed differential attraction on the balls at the two ends of his lever, and with a slowly rotating gravity balance he has demonstrated the rotation of the earth. When using the torsion balance to find G he employed the method, used by Reich, of observing the period of oscillation when the attracting exterior masses were in the same plane as the lever, so as in effect to add to the stability given by the torsion wire, and again in a transverse position when they acted in the opposite sense. He was able with this device to observe the differential attractions of terraces and of tidal water. The levers used by Eötvös ranged between 250 and 340 millimetres in length, and the weights at the ends were of 30 grammes. The lever was supported by a very long and fine wire of platinum which had been kept stretched for a month by a weight nearly equal to the breaking weight. The lever was suspended in a box made with double walls of thick brass, and the space in the vertical sense was restricted to the utmost so as to avoid convection currents as far as possible. The results obtained were:

$$\Delta = 5{\cdot}53 + {\cdot}01 \text{ and } G = 6{\cdot}65 \times 10^{-8}.$$

(vi.) *Burgess.*—Burgess,[2] working in the Sorbonne, devised a method intended to increase the angle of deviation possible by relieving the torsion fibre of quartz of a great proportion of the weight of the suspended system by making use of the principle of flotation. For this purpose the lever carried below it a metallic cylinder wholly immersed in mercury, being connected thereto by a narrow neck which pierced the capillary surface. The disturbances due to capillarity were greatly reduced by a thin layer of dilute sulphuric acid. The lever had obviously to be taken down outside the vessel of mercury to a point low enough to give stability. The lever was 12 centimetres long, and carried suspended from its ends lead balls of 2 kilogrammes each at the same level. The large balls were of lead and weighed 10 kilogrammes each. The quartz torsion fibre had no more than 5 to 10 grammes to support, and so could be made so fine as to allow of very large deflections. The result obtained was $\Delta = 5{\cdot}55$, $G = 6{\cdot}64 \times 10^{-8}$. Burgess found the accuracy of the experiment limited by uncertainty of deflection, due, no doubt, to the action of the liquids. He did not determine the torsional rigidity of the quartz fibre by observations in the gravity apparatus, but independently by noting the time of oscillation of cylinders of such proportions as to have the moment of inertia the same about any axis and of different weights, so as to cover the range of stretching force applied to the fibre. This is, in the writer's opinion, an important improvement. On the other hand, the increase in the angle of deflection, rendered possible by so much buoyancy, seems to the writer to be undesirable if it entails, as it appears to do, interference with the elastic perfection of the quartz fibre, and unnecessary if optical means of observation have not been pushed to the furthest limit of accuracy possible.

§ (3) TEMPERATURE VARIATIONS.—Poynting and Grey[3] have made use of a torsion balance somewhat on the lines of that used by the present writer, in which an attempt was made to detect, if it existed, any variation of the gravitation due to the direction of the axis of crystallisation in balls of quartz. For this purpose the larger fixed balls were kept in rotation so as to turn synchronously with the period of the lever. If then there were any variation a swing would slowly be developed, but none was detected, and these observers concluded that there can be no variation of as much as $\frac{1}{10000}$ of the whole attraction.

Dr. P. E. Shaw[4] has made an attempt to connect the value of G with the temperature of the larger body, from which, if such a connection were definitely established, the relative masses of the sun and the planets, derived from the periodic times of the planets round the sun and of the satellites round their primaries in the usual way, would not be correctly deduced. Dr. Shaw used an apparatus based upon the general design of the present writer, but so arranged that the large masses could be heated and cooled so far as possible without disturbance of the delicate suspension. In these experiments Dr. Shaw believed he found a temperature coefficient of $+1{\cdot}2 \times 10^{-5}$ per degree centigrade, corrected in the later paper to $+1{\cdot}3 \times 10^{-5}$. This is a matter of such profound importance that its reality can only be accepted on the clearest proof. In no other way does gravity show any regard to any kind of interference, as do the other physical agencies, and it is not surprising that in more carefully conducted experiments[5] Dr. Shaw has failed to detect even this small effect, but is now satisfied that any temperature coefficient, if it exists, cannot exceed 2×10^{-6} per 1° C.; thus the conclusion that there is such a temperature coefficient can hardly be accepted.

---

[1] *Wied. Annalen*, 1896.

[2] *Comptes Rendus*, 1899, cxxix., and *Phys. Rev.*, 1902, 14.

[3] *Roy. Soc. Phil. Trans.*, 1899, cxcii.

[4] *Ibid.*, 1916, and *Phys. Soc. Proc.*, 1916–17.

[5] Roy. Soc., Abstract at Meeting, March 23, 1922.

It should be mentioned, however, that Professor Hicks[1] found that the results of Baily arranged in order of temperature gave a small steady fall of Δ, *i.e.* rise of G, with an increasing temperature.

§ (4) THE HORIZONTAL BALANCE BEAM. (i.) *Von Jolly.*—The earliest of the experiments with a horizontal balance beam resting on knife-edges was made by Von Jolly[2] in 1878. He set up a balance in a tower in the University of Munich, from the arms of which he suspended two balls, each 5 kilogrammes in weight. These could be hung either directly under the balance or 20 metres lower down. If the ball on one side were in the high position and that on the other in the low, or if either ball were shifted from one position to the other, the change in distance of 20 metres from the centre of the earth made a marked difference in its apparent weight. The observed change of 29·967 milligrammes differed from the calculated amount of 33·05 milligrammes, and this was attributed to local high ground. Then a ball of lead 1 metre in diameter and weighing 5·772 tonnes was built up under one of the balls in the lower position. The increased apparent weight due to this was ·589 milligramme. Great difficulty was experienced from convection currents, and much trouble was taken to reduce these. Even so, observations were only possible when the outside temperature was fairly uniform. The result was:

$$\Delta = 5{\cdot}692, \text{ or } G = 6{\cdot}465 \times 10^{-8}.$$

(ii.) *Richarz.*—Richarz and Krigar-Menzel[3] used a similar balance to the last except that the high and low positions of the suspended balls, each of nearly 1 kilogramme, was a little over 2 metres only. After the effect of changing the two balls in position, one up and the other down, had been determined, this being 1·2453 milligramme, a cubical block of lead of 100 tonnes was built up in the space between the two positions, with fine passages for the suspending wires. After the block was in position the effect on the change of weight was more than sufficient to counteract the increased attraction of the earth on the lower ball, and $-{\cdot}1211$ milligramme was observed. The arithmetical sum of these two figures, 1·3664 milligramme, is therefore the effect of the lead block, and this is four times its attraction on any one of the balls. The result calculated from very many experiments was:

$$\Delta = 5{\cdot}505, \quad G = 6{\cdot}684.$$

The apparatus was set up in an earth-covered casemate in the citadel of Spandau.

(iii.) *Poynting.*—Professor Poynting[5] used the "common balance" with remarkable success in his well-known experiments, made in the basement of Mason's College, Birmingham. A very fine Oertling bullion balance was carried on girders, and below it on the floor was a heavy turn-table. From the arms of the balance were suspended balls of 21½ kilogrammes (each about), and on the turn-table a ball weighing 153½ kilogrammes (about) was so supported that it could be brought directly under either one of the suspended balls. Most refined means of observing the balance from a room overhead were provided, and of course all the distances were capable of exact measurement. When the experiment was made certain anomalies appeared, which were ultimately traced to the effect of moving so great a weight as 153 kilogrammes through a distance of 1¼ metre. This had the effect of bending the ground and tilting the whole building, and with it the girders and balance case, and thus, even if the beam had not moved at all, relatively to the building it would have done so, and a spurious deflection would have been observed. This difficulty was remedied by extending the turn-table and making it carry a smaller weight of 76½ kilogrammes, which, therefore, was far enough away from the other suspended ball, and sufficiently oblique in the direction of its attraction to produce a relatively small counter effect, which, however, was allowed for. The result obtained was:

$$\Delta = 5{\cdot}4934, \quad G = 6{\cdot}6984 \times 10^{-8}.$$

Wilsing[5] employed a metronome form of balance with a beam 1 metre long, carrying at its two ends brass spheres, each of 540 grammes, or lead spheres of 745 grammes, and a central knife-edge. The attracting masses were cast-iron cylinders, each weighing 325 kilogrammes, suspended by a wire rope passing over an overhead pulley. These were so arranged that one cylinder was opposite the upper ball on one side while the other was opposite the other ball on the other side. Then on turning the pulley the cylinders came into position to reverse the previous effect, and so the total change represented four times the attraction of one cylinder on one ball. Wilsing's final result was:

$$\Delta = 5{\cdot}579, \quad G = 6{\cdot}596 \times 10^{-8}.$$

All methods of finding Δ or G which depend upon a balance moving in a vertical plane suffer very much from inevitable convection currents and from expansion of the arms of the balance. The knife-edge, wonderfully perfect as it is, is, however, also a disturbing factor. Then again the settlement of dust on the moving parts may give rise to slow progressive changes. From these disturbances the torsion balance is almost perfectly free so that it is certainly preferable. Still, in determining the value of a constant of such importance every method should be employed, and if a "common balance" is ever used again it is likely that the superlative micro-balance, shown at a meeting of the Physical Society in 1920[1] by Dr. Pettersson, would be the best form. In this the balance beam is made of fused quartz, and it is suspended by quartz fibres, as also are the weights from its ends. Such a beam and weights in a vacuous case would be free from friction, expansion of arms, convection currents, and deposition of dust, and, as Dr. Pettersson has shown, it can be made with a very small degree of stability. Very large weights could be employed as attracting weights. While this is possible, it does not seem to be within the bounds of possibility to obtain a period comparable in length with that easily attained by torsional methods, and in the writer's opinion an apparatus on the lines of his own, but on a larger scale so as to reduce the damping effects of the viscosity of the air, would be most likely to give improved results. If it were possible to employ a vacuum also it would be an advantage. The prior determination of the rigidity of the fibre—or prior and subsequent—by the method of Burgess would eliminate the difficulties attached to the method of vibration of the actual suspension used for the deflections, and the combination of this with the Boys general design and the Braun vacuum, if possible, seems now to offer the best chance of obtaining increased accuracy.

---

[1] *Cambridge Phil. Soc. Proc.*, 1884, v. 2.

[2] *Abh. d. k. Bayer. Akad. d. Wiss.* 14 Bd. 2 Abth., and *Wiedemann's Ann.*, 1881, xiv.

[3] *Sitz. der Berl. Akad.*, 1884, and *Wied. Ann.*, 1885, xxiv.

[4] *Roy. Soc. Phil. Trans.* A, 1891, clxxxii.

[5] *Publ. Astr. Phys. Observ. Potsdam*, 23, vi 3.

§ (5) SUMMARY OF RESULTS.[2]—

METHODS USING PART OF THE EARTH

Date.	Name.	Place.	Δ.
1749.	Bouguer	Chimborazo	?
1775.	Maskelyne and Hutton	Schiehallion	4·71, 4·45
1821.	Carlini	Mont Cenis	4·77, 4·95 4·84
1854.	Airy	Harton Colliery	6·57, 5·48
1855.	James and Clarke	Arthur's Seat	5·32
1865.	Pechmann	Gerold	6·13
1880.	Mendenhall	Fujiyama	5·77
1883.	Von Sterneck	Pribram	4·77
1885.	Von Sterneck	Freiberg	6·77
1887.	Preston	Haleakala	5·57
1892.	Preston	Maunakea	5·13

TORSION BALANCE

1798.	Cavendish	5·45
1837.	Reich	5·49
1852.	Reich	5·583
1843.	Baily	5·67 5·55
1878.	Cornu and Baille	5·50 5·56
1895.	Boys	5·527
1896.	Braun	5·527
1896.	Eötvös	5·53
1901.	Burgess	5·55

CHEMICAL BALANCE

1881.	Von Jolly	5·692
1891.	Poynting	5·493
1898.	Richarz and Krigar-Menzel	5·505

METRONOME BALANCE

1889.	Wilsing	5·55

Burgess, after considering all that had been done up to 1902, concluded that $5{\cdot}5247 \pm {\cdot}0013$ most nearly represented the value of Δ and $6{\cdot}6607 \times 10^{-8}$ the value of G.

C. V. B.

EARTH, RADIATION OF THE:

Equivalent temperature of. See "Radiation," § (4) (i.).

Measurement of. See *ibid.* § (3) (iii.).

Wave-length of maximum energy of. See *ibid.* § (2) (ii.).

EARTHQUAKES, KNOTT'S THEORETICAL WORK ON, tabulated results of. See "Earthquakes and Earthquake Waves," § (8).

EARTHQUAKES AND EARTHQUAKE WAVES

§ (1) MEASUREMENT OF EARTHQUAKES: SEISMOLOGY.—In ordinary language an earthquake is any sudden palpable movement of the ground, occasioning at times great havoc and destruction. The origin of the shock is undoubtedly underneath the surface; and its existence implies an overstraining of the material composing what is generally known as the crust of the earth. The word "crust" has survived the theory which suggested the term; for it is doubtful if in these days any one who has given real thought to the subject believes that the earth consists of a liquid interior covered by a solid shell or crust. The word is convenient, however, when it is used to indicate the heterogeneous surface layer of the earth which is more or less in a state of stress and strain.

In scientific language the earthquake is of much wider significance. It not only includes the shakings which are felt by mankind, but also those far-spreading impalpable tremors which, by means of delicate instruments, can be detected and measured far beyond the region where the shock is felt. It was the gradual evolution of the seismograph and seismometer as delicate instruments for automatically recording these unfelt tremors which created the modern science of seismology.

---

[1] *Phys. Soc. Proc.*, xxxii.

[2] *The Mean Density of the Earth*, Poynting, and *Physical Review*, 1902, 14.

§ (2) **Milne Seismograph.**—As early as 1883, Professor John Milne, then resident in Japan, predicted that "every large earthquake might be with proper appliances recorded at any point on the land surface of the globe." Six years later a record obtained at Potsdam on a very delicate horizontal pendulum, which Von Rebeur Paschwitz had set up for measuring the gravitation action of the moon, was identified with an earthquake whose origin lay near Japan. This observation marked a new era in the development of the science of earthquakes, and in this development no one took a greater share than John Milne, the pioneer of modern seismology. His annual reports to the British Association on Earthquake Phenomena in Japan from 1881 to 1895, followed by his Reports on Seismological Investigation from 1893 to his death in 1913, give in rugged detail the stages through which knowledge widened and deepened and speculation ripened into theory.

In 1895 Milne left Japan and established his own seismological observatory at Shide in the Isle of Wight, and in 1897 he was able, through the agency of the Seismological Committee of the British Association and with the help of the Foreign and Colonial Offices, to inaugurate a scheme whereby a special type of instrument was installed in many localities all over the globe. The observatory at Shide became the central office of a world-wide seismic survey; and by 1901 the main facts regarding the propagation through and round the earth of earthquake tremors were established, mainly through the labours of Milne himself.

No seismometer[1] has been constructed which can give a measurable record of the whole displacement of the ground. The seismologist is compelled to use at least three instruments, each of which gives on a moving sheet of paper as a time graph one component only of the displacement—or of the velocity in the case of the Galitzin instruments. Two of these are generally of the type known as the horizontal pendulum, indicating two components of the horizontal displacement, generally in directions north–south and east–west respectively. The third is designed so as to indicate the vertical displacement. Unfortunately the great majority of stations are still provided with instruments which measure the horizontal components only.

§ (3) **Seismograph Records.**—Records obtained on delicate seismographs at stations far distant from the epicentre of a strong earthquake have an unmistakable characteristic appearance. This had to some extent been already recognised on records obtained on the less sensitive forms of seismograph designed for recording palpable movements of the ground in earthquake countries like Japan. There had been seen on these what Milne called preliminary tremors followed by the large movements constituting the main shock. These tremors were in the early days ascribed to elastic compressional vibrations (like sound waves in air) running ahead of vibratory waves of slower transmission.

Now in the ordinary theory of elastic vibrations in extended isotropic solid bodies two types of vibration had long been recognised as transmissible through the substance with different speeds of propagation. These may be distinguished as the Compressional Wave and the Distortional Wave, sometimes also called the longitudinal and transverse waves in recognition of the fact that the vibrations in the compressional wave take place mainly in the direction of propagation of the wave, whereas in the distortional wave the vibrations take place in the wave-front perpendicular to the direction of propagation. The speed of propagation depends in each case upon a definite elastic modulus and the density of the material. More exactly, the squares of the speeds of propagation of the compressional and distortional waves are given respectively by the expressions

$$\frac{k+4n/3}{\rho} \text{ and } \frac{n}{\rho},$$

where k is the incompressibility or resistance to compression, n the rigidity or resistance to shear, and ρ the density of the material.

When the characteristic form of earthquake records as consisting of preliminary tremors and large waves was recognised, the explanation which first suggested itself was that the preliminary tremors were the compressional waves, and the more slowly following movement the distortional waves. But Milne's early studies of distant earthquake records[2] brought out the interesting result that although the large waves were transmitted from the earthquake origin to various distant stations in times which were nearly proportional to the arcual distances of these stations from the epicentre, this was obviously not true of the first preliminary tremors. In 1901, R. D. Oldham pointed out that the preliminary tremor could be separated into two distinct parts, which he distinguished as the first and second preliminary tremors. These are now called the Primary and Secondary Waves, and are symbolised by the letters P and S. On every good seismogram they are clearly recognised (see *Fig.* 1).

---

[1] See "Seismometry."

[2] See *B.A. Report* for 1900.

§ (4) The Primary and Secondary Waves.—Thus a satisfactory seismogram obtained on a delicate instrument at a distance from the origin of the shock consists of a prolonged series of complicated oscillations traced upon a moving strip of paper, and giving, therefore, the time graph of a component of the displacement of the ground; but in spite of the complexity of the graph the practised eye recognises three markedly distinct portions. The primary waves P enter with sharp rapid vibrations, on which after a certain interval there is superposed with similar abrupt beginning the secondary waves S. The abrupt rapid character of the graph gradually changes as time goes on, until the large maximum movement is reached which is characterised by smoother oscillations of distinctly longer period and greater amplitude. After the maximum movement has been reached the disturbance gradually dies away, lasting in many cases for several hours.

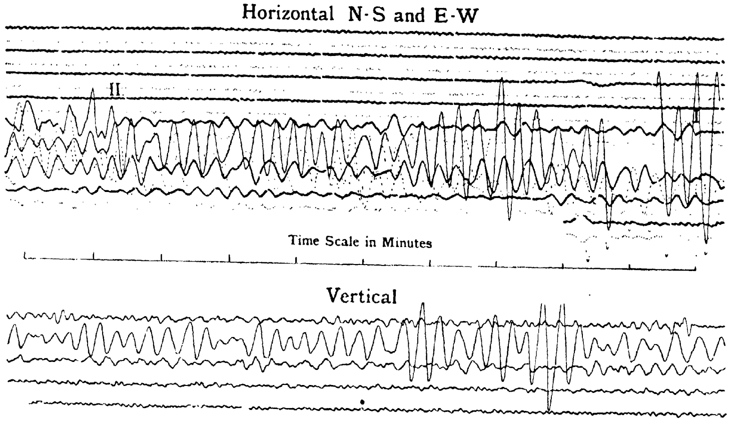


Fig. 1.—Reduced, by kind permission of the Meteorological Office (Air Ministry), from seismograms of the earthquake of December 21, 1917, as given by the Galitzin instruments at Eskdalemuir Observatory. The full line record gives the N-S component, the dotted record the E-W. I. marks the advent of the Primary, II. of the Secondary Waves, nearly nine minutes later. The part shown in each record is a limited portion of the whole, each successive line being about thirty minutes later than the similar line immediately above it. In these later portions the oscillatory nature of the long waves is clearly indicated in both the Horizontal and Vertical records.

It is now universally admitted that the beginning of the primary waves marks the advent of the compressional wave coming from the origin of the disturbance. The sudden appearance of the secondary wave marks the advent of the distortional wave. The earthquake shock has meanwhile set the whole region into vibrations, so that the surface at the epicentre is thrown into more or less violent movements which pass outwards in all directions over the surface of the earth. These surface waves form the long-period wave portion of the seismogram; and later investigations have fully established Milne's early conclusion that they pass round the earth with a definite velocity. An exact determination of this velocity is difficult, owing to the uncertainty in identifying the same maximum phase in seismograms obtained at different stations. With very large earthquakes, however, it has been found possible to observe on the same seismogram a second succession of long-period waves reaching the station after passing through the antipodes of the epicentre. From such an observation Galitzin estimated the surface velocity of the long waves at 3·53 kilometres (2·2 miles) per second.

In his earliest endeavours to locate earthquake origins from the seismographic record, Milne made use of the interval of time by which the first preliminary tremors outran the long-period waves;[1] but after the existence of the secondary waves was recognised he was able to substitute for the comparatively vague advent of the maximum long-period waves the more clearly marked instant at which the secondary waves made their appearance. With the improvement of instrumental records and the steady accumulation of observations from all over the globe, the time graphs as originally constructed by Oldham (1901) and Milne[2] underwent gradual corrections; and, in 1907, Wiechert and Zöppritz, from a careful discussion of a number of well-defined earthquakes, constructed a set of time graphs which have served as the basis for all important work since then. The following table and diagram (*Fig.* 2) will suffice to indicate the general character of the data. Distances from the epicentre are given in degrees of arc, and the average times of arrival of the primary, secondary, and long-period waves measured from the time of occurrence of the epicentral shock are given in minutes.

---

[1] *B.A. Reports*, 1898.

[2] *Ibid.*, 1902.

Arcual Distance.	Times of Transit in Minutes of			Difference Times.
	Primary. P.	Secondary. S.	Long-period. L.	S – P.
15°	3·65	6·53	7·9	2·88
30	6·47	11·57	15·7	5·10
45	8·65	15·25	23·6	6·60
60	10·20	18·38	31·5	8·18
75	11·82	21·43	39·4	9·61
90	13·27	24·23	47·2	10·97
105	14·57	26·70	55·1	12·13
120	15·70	28·82	63·0	13·12

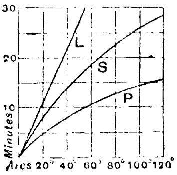


FIG. 2.

The times for L, the long-period wave of maximum amplitude, are calculated from Galitzin's estimate of their velocity round the earth. They have been traced to much farther distances than the highest value tabulated here; but the primary and secondary waves have not been clearly discriminated in records obtained at distances greater than about 120°. These numbers are averages; and it is certain that the data derived from the seismograms produced by any particular earthquake would frequently deviate from the means by as much as a tenth of a minute.

§ (5) LOCATING AN EARTHQUAKE.—When a good record is obtained at any station the interval of time (S – P) between the advents of the primary and secondary waves is immediately measurable, and the approximate distance of the epicentre deduced by reference to an extended table of the kind just given. The earthquake origin is therefore situated close to the circle drawn on the earth's surface with its centre at the observer's station and the arcual radius deduced as above. The whole process was very simply carried out by Milne by means of a globe and a closely fitting arc with the degrees and corresponding times marked on it. By laying this arc with its one end at the Isle of Wight and sweeping it round in close contact with the globe, he saw at a glance the various possible positions of the epicentre as being those points on the globe which lay at the observed time-distance along the arc. For most purposes this rapid method is sufficiently accurate, but when greater accuracy is required the necessary calculations must be carried out by means of spherical trigonometry.

If the station is provided with two seismographs, one giving the north-south component and the other the east-west component of the horizontal displacement, and if the initial stage of the primary wave is well marked on both, it may be possible to calculate the azimuth of the motion, that is, the position of the vertical plane containing the first longitudinal displacement. If this plane coincides with the plane containing the epicentre and the ray, the epicentre will lie on either of the two points of intersection of the plane with the circle already supposed drawn on the earth's surface. This method was first applied with success by Galitzin, whose perfected forms of seismograph provided records capable of being interpreted in this way. The simultaneous record of the vertical motion seismograph may then enable the observer to fix definitely which of the two possible points of intersection just mentioned is the epicentre. This ideal interpretation of the three simultaneous records obtained at any one locality is in most cases rendered nugatory by the imperfections of the instrumental records, or the disturbing effect of the so-called tremors, whose presence on the seismograph frequently masks the first appearance of the primary waves, or the influence of the heterogeneity of the crust of the earth in throwing the displacement out of the azimuthal plane of the issuing ray.

When Galitzin's method is not applicable recourse must be had to the method used so effectively by Milne in localising earthquake origins over the earth's surface. Any one seismogram showing clearly the advents of the primary and secondary waves determines one circle on the earth on which the epicentre must lie. The seismographic record of the same earthquake obtained at a second station will give another circle passing through the epicentre. These will intersect in two points, one of which must be near the epicentre. The record from a third station will provide a third circle, also passing near the epicentre, which will then be determined as the approximate meeting-point of these three circles.

This is the one universal method of determining all kinds of earthquake origins, and the only way of determining those which lie below the bed of the ocean. In the *B.A. Reports* from 1900 to 1913, Milne, working along these lines, has chronicled an immense amount of detail regarding earthquake origins and the main localities in which they are distributed.

§ (6) Waves on an Elastic Solid.—In 1885 Lord Rayleigh showed that, at the plane boundary of an otherwise infinite elastic isotropic solid, surface waves of a peculiar type would exist, having a speed of propagation somewhat less than that of the distortional wave, and having the normal displacement at the surface about a half greater than the displacement parallel to the surface. The latter penetrates only a short distance downwards into the substance, practically disappearing for the case in which Poisson's ratio is one-quarter at a depth of one-sixth of a wave-length. Now the long-period earthquake surface waves travel with a speed of 3·53 kilometres per second, and their period varies between 10 and 20 seconds. The wave-lengths corresponding to these periods are 35·3 and 70·6 kilometres respectively. If the long-period waves were pure Rayleigh waves started by the disturbance at the epicentre, the component displacement parallel to the surface would vanish at a depth of from 6 to 12 kilometres. There is no doubt, however, that actually in any earthquake there will be displacements parallel to the surface at much greater depths; and, moreover, observation shows that the tangential displacements associated with surface vibrations due to earthquakes are much greater than the normal or vertical displacements. Rayleigh waves are no doubt present, but they are intermingled with body waves passing directly from the earthquake origin, or from the epicentral region. In general those body waves, which are represented by the primary and secondary waves of the seismogram impinging internally on the surface of the earth, or on the surface which separates the more homogeneous nucleus from the heterogeneous crust, will each start the three types of secondary waves, namely, reflected waves of the compressional and distortional types, and surface waves of the Rayleigh type. When we further bear in mind that the original disturbance cannot be strictly instantaneous, but must have extended over a finite and measurable interval, there is no difficulty in accounting in a general way for the continued and complex vibrations which are propagated through and round the earth as the result of a cataclysmic disturbance occurring in the earth's crust.

The remarkable thing is that in passing from the homogeneous nucleus to the heterogeneous crust the primary and secondary waves should to such a degree preserve their characteristic and distinctive properties. These facts seem to indicate that the heterogeneous crust is of comparatively small thickness, and that further information of the interior of the earth is to be derived from a study of the transmission of the compressional and distortional waves which crop out at the surface as the primary and secondary tremors. The data are the times of transit of each of these types of wave motion to given distances from the epicentre of the earthquake shock. The problem is to find the paths or rays by which each type passes and the velocities of propagation at different depths in the earth.

§ (7) Waves in the Interior of the Earth.—The times of transit show that the waves are not transmitted along the surface. They are transmitted through the earth by brachistochronic paths in accordance with the recognised principles of wave-motion. R. von Kövesligethy[1] and M. P. Rudzki[2] were among the first to form definitely the brachistochronic equations suitable to elastic waves passing through the earth from a source of disturbance close to the surface on the assumption that the speed of propagation was a function of the distance from the earth's centre. To get an integrable form, the Hungarian mathematician virtually assumed that the elastic moduli were constant throughout and that the speed of the wave at any point depended only on the density. Rudzki was content to give the mathematical theory without any practical application to the problem of earthquakes. At that time, indeed, the data of observation were too scanty to permit of any accurate discussion.

Let the earth's radius be taken as unity, the centre of the earth as origin, and the radius passing through the epicentre as axis of reference, then it is found by the usual brachistochronic methods that the polar co-ordinates r, θ are connected by the equation

$$\theta = \pm p\int\frac{dr}{r\sqrt{r^2/v^2 - p^2}} + \text{constant}, \quad . \quad (1)$$

where v, the speed of propagation at position r, is measured in terms of the unit "earth-radius per minute," and where p is a time-parameter which is constant for each given ray but varies as we pass from ray to ray. The value of p for the ray which emerges at arcual distance a from the epicentre is equal to $d\mathrm{T}/da$, where T is the observed time of transit. This time-parameter p which defines the ray is therefore known from the data of observation; but, since its values depend on the first differences of the times of transit from point to point, they must be subject to greater uncertainties than are the original time determinations.

In the above expression the value of v as a function of r is quite unknown, and therefore no direct evaluation of the integral is possible.

---

[1] *Math. u. Naturw. Berichte aus Ungarn*, 1897 xiii.

[2] Gerland's *Beiträge zur Geophysik*, 1898, iii.

Tentative solutions may, however, be obtained in a variety of ways. Thus Benndorf,[1] by a geometrical synthetic method, worked out a law connecting speed of propagation with depth, starting from the angles of emergence as measured by Schlüter. In a paper communicated to the Royal Society of Edinburgh, Knott[2] showed that with $v^2=V^2(1-qr^2)$ through the surface shell from $r=1$ to $r=0{\cdot}9$, and with the speed continuing constant to all lower depths, a solution could be obtained in fair agreement with the data of observation. This paper also contained a method for estimating the law of distribution of seismic energy over the surface of the earth due to a single earthquake.

To the same category of tentative solutions belongs the method described and graphically worked out by E. Wiechert in the first of a series of important memoirs which appeared in the *Nachrichten von der königlichen Gesellschaft der Wissenschaften zu Göttingen* between the years 1907 and 1914. In Memoir I. Wiechert gives a full discussion of recognised elastic theory in a form suitable for seismological applications, and draws attention to the phenomena of the reflected waves at the earth's surface (see § (10)). A solution of the brachistochronic equation is obtained by supposing the earth to consist of a central core of constant velocity of wave propagation surrounded by two spherical concentric layers within each of which the rays have constant curvature. This is equivalent to supposing that, a and b being constant in each layer, but varying from layer to layer, the speed of propagation v can be expressed in the form $v=a+br^2$. The solution is carried out by an elegant geometrical method. In Memoir II. which formed the second part of the whole paper, Zoeppritz worked out the time graphs afresh from the best available material; and his tables have formed the foundation of all later seismological work.

Already in 1907, however, G. Herglotz[3] had shown that the equation belonged to a type of integral equation which had been solved by Abel. The transformation leads to an equation which Wiechert and his coadjutors made use of in the third and following memoirs referred to above, though not to its full extent.

More recently, however, a complete solution has been obtained by direct numerical calculation based only on the data of observation, without any assumption of a relation between v and r^2 or use of any indirect method leading to such a relation.[4]

Returning to equation (1) integrate half along the ray from $r=1$ to $r=z$, and $\theta=0$ to $\theta=\alpha/2$; and there results

$$\tfrac{1}{2}\alpha=p\int_z^1\frac{dr}{r\sqrt{r^2/v^2-p^2}}. \quad . \quad . \quad (2)$$

Now p is given as a tabulated function of $\alpha/2$; and we may suppose the relation inverted and $\alpha/2$ given as a tabulated function of p, say $\alpha/2=f(p)$. By the substitution $\eta=r/v$ equation (2) may be put in the form

$$f(p)=p\int_{\frac{z}{v}}^{\frac{1}{V}}\frac{d\eta}{\sqrt{\eta^2-p^2}}\frac{\partial}{\partial\eta}(\log r) \quad . \quad . \quad (2')$$

an integral equation of the type solved by Abel, V being the value of v at the surface $r=1$. The required solution is

$$\log r=-\frac{2}{\pi}\int_\eta^{\frac{1}{V}}\frac{f(p)dp}{\sqrt{p^2-\eta^2}}, \quad . \quad . \quad (3)$$

in which p has become the variable and the known function $f(p)$ is under the integral sign. The solution may now be effected by quadratures, thus: (1) Tabulate the values of $f(p)$ for a series of sufficiently close descending values of p at equal intervals dp. (2) Choose any one for the lower limit value η, and tabulate the values of $\sqrt{(p^2-\eta^2)}$ corresponding to the successive values of p. (3) Calculate the value of the expression under the integral sign for every value of p from η to the highest value. This gives the logarithm of the distance (r) of the mid point or vertex of the ray for which the parameter p has the chosen value η. Thus η and the corresponding r are known, and $v=r/\eta$ is the speed of propagation of the wave at the distance r from the centre of the earth. Also to every η there corresponds an arcual distance α giving the limits of the ray whose vertex is at distance r from the centre of the earth.

§ (8) Calculations of Velocities and Rays.—The following table gives a few of the corresponding sets of values of η (or p), α, r, R, and v, R being the distance in kilometres from the earth's centre, and r the corresponding value in fractions of the earth's radius:

	Parameter of Ray.	Arcual Range of Ray.	Distance of Vertex of Ray.		Speed of Wave at this Distance. km./sec.
	η or p.	α.	r.	R.	V.
Primary wave.	888	0°	1	6378	7·18
	774	13·4	0·971	6194	8·00
	544	30·0	·880	5612	10·32
	430	43·0	·802	5115	11·90
	372	53·0	·752	4794	12·89
	355	74·0	·741	4534	12·77
	315	95·0	·634	4044	12·84
	258	122·0	·512	3268	12·67
Secondary wave.	1604	0	1	6378	3·98
	1375	13·7	0·969	6183	4·50
	974	30·6	·881	5618	5·77
	802	40·0	·818	5216	6·50
	716	53·4	·773	4929	6·88
	688	71·0	·739	4716	6·85
	573	97·0	·614	3920	6·84
	458	117·0	·492	3139	6·85

The corresponding graphs giving v in terms of R are shown in *Fig.* 3.

The following empirical linear relations worked out by least squares for the tabulated values give results for v in terms of the depth ($D=6378-R$) well within the errors of observation:

---

[1] See *Mitt. d. Erdbeben-Kommission d. Kais. Akad. d. Wissensch. in Wien*, 1906, xxix.

[2] See *Proc. R.S.E.*, 1907–8, xxviii.

[3] *Physikalische Zeitschrift*, viii. 145; also subsequently pointed out by H. Bateman, *Phil. Mag.*, April 1910.

[4] See Knott, "Propagation of Earthquake Waves through the Earth," *Proc. Roy. Soc. Edin.*, 1918–19.

Speed in Terms of Depth.	Between Limits of Depth Equal to	
$7·27+0·0040\ D$	128 and 668	Primary wave
$7·86+0·0032\ D$	758 and 1588	
$4·1+0·00208\ D$	78 and 678	Secondary wave
$4·39+0·00181\ D$	758 and 1338	

In accordance with equation (1) the path or ray depends on the integral

$$p\int\frac{dr}{r\sqrt{r^2/v^2-p^2}}=p\int\frac{dr}{r\sqrt{\eta^2-p^2}}.$$

The radius r having been obtained as a tabulated function of η, the latter may be obtained as a function of r tabulated for successive equal increments dr. Choosing any value of η as the value p for the lower limit, we may work out by a process exactly similar to that already used successive points on the ray defined by the chosen value of p. The values will be found in the paper referred to.[1] It will suffice here to reproduce in *Fig.* 4 the forms of the rays for the primary waves and the wave fronts for the primary and secondary waves. The secondary rays are not entered on the figure. They differ so slightly from the primary rays that their presence would have caused unnecessary confusion. The following condensed table gives the more important constants of the several rays for which the calculations were completely carried out. The quantities tabulated are: ϕ, initial and final inclination of ray to the normal or radius; r_0, distance of vertex of ray from the centre of the earth; α, arc between epicentre and outcrop of ray on earth's surface; T, time of transit in minutes. In *Fig.* 4, the isochronic lines are numbered in seconds.

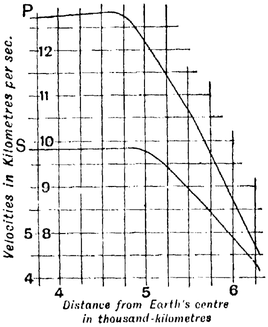


FIG. 3.

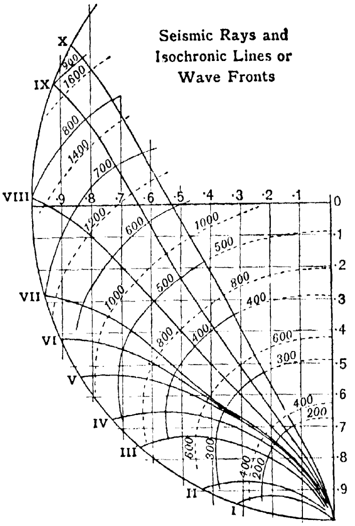


FIG. 4.

Rays of Compressional or Primary Waves.					Rays of Distortional or Secondary Waves.			
ϕ.	r_0.	α.	T.		ϕ.	r_0.	α.	T.
48° 41′	0·935	19° 25′	4·5	I.	..	..	..	..
42 32	·905	26 29	5·9	II.	42° 31′	0·905	25° 37′	10·2
30 18	·815	40 57	8·0	III.	29 40	·815	41 4	14·3
26 39	·77	47 56	8·9	IV.	26 23	·77	54 9	17·2
24 38	·745	57 40	9·9	V.	..	..	..	..
24 18	·735	65 32	10·8	VI.	..	..	..	..
24 3	·725	73 51	11·7	VII.	24 54	·725	76 27	21·7
20 55	·635	91 36	13·4	VIII.	21 39	·635	92 31	24·9
18 1	·545	112 26	15·2	IX.	18 30	·545	110 20	27·6
15	·455	120 8	15·9	X.	15 28	·455	124 44	29·7

The times are worked out by a similar process of quadratures.

Comparison among the corresponding numbers for the two types of wave will show how slight the differences are.

---

[1] *Proc. R.S.E.* xxxix. 199-207.

§ (9) CONCLUSIONS FROM THE THEORY.—As already noted, seismograms obtained at stations distant from the epicentre more than 110° or 120° are difficult of interpretation. The advents of the primary and secondary waves are not clearly defined, and there is great uncertainty in identifying either type. The data of observation can therefore give no information regarding rays which approach the centre of the earth nearer than about half the earth's radius. Down to that depth the earth behaves like an elastic solid through which the two types of elastic waves pass with velocities which steadily increase down to a depth equal to about one-third of the earth's radius. At greater depths the speed of propagation of each wave tends to become nearly constant.

The curious undulatory character of the rays V., VI., VII., VIII., and IX. in the diagram shows that just about this depth of one-third of the earth's radius there is a tendency for a slight decrease in the velocity of propagation at the greater depths. The concavity of the ray towards the centre has this significance according to the mathematical theory on which the whole investigation is based. But, as shown by the almost straight line character of the two deeper rays, this decrease of speed with increase of depth must be very slight.

The very rapid increase of speed of propagation with increase of depth through the first 1200 miles shows that the elastic constants increase more quickly than the density; but that as the critical depth is approached the elastic constants begin to slacken off in their rates of increase with depth and soon change at practically the same rate as the density. This general statement is in full agreement with previous results. It will be observed, however, from the graphs of *Fig.* 3 that this slackening off occurs in the distortional wave at a smaller depth than in the case of the compressional wave. Thus at a depth equal to 0·30 of the earth's radius the ratio n/ρ has become steady or is passing slowly through a maximum, whereas the ratio $(k+4n/3)/\rho$ is still on the increase and continues increasing until the depth 0·36 is reached. Thereafter it becomes steady or passes slowly through a maximum. Now k is just about double the value of n, so that any change in n will produce in $k+4n/3$ a much smaller proportional change. The inference is that the elastic changes which appear at the depths mentioned affect first the rigidity and thereafter the incompressibility. In other words, the changing constitution of the earth's material at these critical depths discloses itself most markedly in that elastic characteristic which belongs to solids as distinguished from non-rigid substances.

What seems to be indicated is that the central core of the earth is unable to transmit the distortional wave. The transition from the elastic solid state to this non-rigid condition will probably take place gradually; and if during this change viscosity comes more and more into play the loss of vibratory energy of the elastic type will become more pronounced. Did the change from elastic shell to non-rigid nucleus take place abruptly across a surface of discontinuity a large part of the energy would be reflected back into the shell and a small fraction only get through to the antipodal side. Such considerations fall in line with the facts established by observation that beyond the arcual distance of 110° the seismograms are meagre and difficult of interpretation.

§ (10) THE SURFACE REFLECTED WAVES.—The fifth Göttingen memoir on Erdbebenwellen is the work of K. Zoeppritz, L. Geiger, and B. Gutenberg, and strikes out on a new line of investigation. This is described by Zoeppritz in an unfinished paper written in 1908 shortly before his death, but not published till 1912, along with the continuation by Geiger and Gutenberg. The weakness of the methods so far given lies, as already indicated, in the uncertainty in the values of dT/da or p as used above. Zoeppritz also pointed out that various assumed laws connecting speed and depth lead to practically the same time graph, and proceeded to inquire if there were no other data of observation which could be used for investigating more accurately the elastic condition of the earth at different depths.

When an elastic wave of either type falls on a surface separating two media of different wave-moduli the incident wave gives rise in general to four distinct waves, a reflected and refracted wave of the same type and a reflected and refracted wave of the other type.[1] Thus a ray of compressional type impinging on a surface of discontinuity within the earth will be changed partly into rays of distortional type. Now the outer surface of the earth, whether bounded by water or by air, is a surface of discontinuity across which only a small fraction of the incident energy within the rocky substance passes. Most of the energy is reflected back into the earth. When, therefore, a primary or P wave radiating out along curved rays from an earthquake source emerges at a particular point of the surface, say at 45° distance from the source, part of it will be reflected as a P wave at the same inclination to the radius and part will be reflected as a distortional wave at a less angle to the radius. The reflected P wave will pass on through another 45° and emerge at 90° from the source at a time a little later than the time taken for the direct wave to pass by its path through this longer distance, and the S reflected wave will emerge at a still further distance. Thus on a well-conditioned seismogram obtained at a station not only will the advents of the P and S waves be distinguishable, but possibly the waves PP and SS once reflected from the middle of the distance may also be seen somewhat later on the seismogram; and even the twice reflected wave PPP, and the alternating reflected waves PS, SP, SPS, Wiechert's Wechselwellen, and so on. The recognition of these reflected waves and their use in helping to determine the distance of the epicentre have increased the accuracy in making this determination.

---

[1] See Knott, "Reflexion and Refraction of Elastic Waves," *Trans. Seismological Society of Japan*, 1886, and *Phil. Mag.*, July 1899; also the greater part of Memoir I. by Wiechert, *Nachrichten d. K. Ges. d. Wiss. Gottingen*, 1907.

Now Zoeppritz's new method for investigating the state of the earth's interior depends on the comparison of the amplitudes of the direct and the reflected P and S waves as shown on one seismogram. We owe to Geiger and Gutenberg the working out of this method in the fifth, sixth, and seventh Göttingen memoirs. For many reasons there are discrepancies and inherent difficulties; but the evidence leads them to the recognition of certain discontinuities within the earth situated at definite ascertainable depths, discontinuities which might easily escape detection by consideration of the time graphs alone. The method has also supplied a corrected table of the times of transit.

On the theory as finally presented the earth is regarded as consisting of a nucleus of radius 3500 kilometres enveloped by a shell whose constitution alters at three surfaces.

The Göttingen seismologists have added greatly to our knowledge of seismology, and their memoirs will always rank high in the literature of the subject, partly on account of the great care taken to utilise the very best data of observation, and partly in virtue of the mathematical skill brought to bear on every branch of the subject.

The significance and interpretation of these reflected waves have also been considered by G. W. Walker in a paper communicated to the Royal Society of London in 1917.[1] All the necessary details of the amplitudes of the various types of reflected wave at different angles of incidence of a wave of each type are worked out afresh in accordance with the recognised theory of Elasticity; and the calculated values of the horizontal and vertical components are tabulated and shown graphically in a form convenient for use. The times of transit of the several types of reflected wave (PP, SS, PS, SP, etc.) and their associated surface effects as given by theory are compared with the corresponding quantities deducible from well-conditioned seismograms supplied by Galitzin instruments. There is fair agreement in some respects but not in others. Certain discrepancies in the times of transit and in the angles of emergence might be removed by suitable correction of the time graph, as previously pointed out by the Göttingen seismologists. The correction suggested is to depress the ordinates of the time graph, that is, shorten the times of transit for distances between 2000 km. and 6000 km. (180° and 54° respectively). In the comparisons made by Walker the condensational reflected waves show best agreement, and the alternating reflected waves worst. Theoretically the PS and SP waves reach a given observing station simultaneously. Otherwise they are quite different; and then interference will produce a resultant effect difficult to analyse. Walker shows, however, that these two simultaneous effects may be separated by means of the vertical component, if the angles of impingences are known. He concludes that, although quantitative analysis of reflection theory on simple lines has removed a number of difficulties that have hitherto attended interpretation of seismograms, there are still difficulties to resolve. The desideratum is a more profound study of the propagation of disturbances through the earth, and (in particular) of the reflection of waves from a variable layer of thickness comparable with that of the earth's crust.

In a later paper Walker[2] shows how the assumption of a deep focus may explain some of the difficulties referred to above.

C. G. K.

Eccentrolinead. See "Draughting Devices," p. 270.

Eddies in the Air. An eddy is a circular motion produced when a fluid flows past an obstacle. At the side of a projecting rock in a stream, or at a ship's side, eddies show as little whirling dimples on the surface of the water. Similar eddies form in the atmosphere, the friction at even comparatively smooth ground being sufficient to account for their formation. Normally the eddies in the atmosphere are invisible, but smoke, dust, or leaves will make them visible. They are seen to consist of rotary motion about an axis, which may be horizontal, or vertical, or may be inclined at any angle to the horizontal. Near an obstacle the flow of air produces a succession of eddies which are formed, detach themselves from the obstacle, move away, and

---

[1] "Surface Reflexion of Earthquake Waves," *Phil. Trans.* Series A, 1919, ccxviii.

[2] "The Problem of Finite Focal Depth revealed by Seismometer," *Phil. Trans.* Series A, 1921, ccxxii.

finally disintegrate. Air is carried from the ground upwards in eddies, checking the flow of the upper current by mixing. According to G. I. Taylor the size of the eddies is determined entirely by the nature of the ground, and is unaffected by the speed of the wind in which they form, but the velocity in the eddies is determined by the velocity of the current in which they are formed.

- Eddy-diffusion, or "Diffusivity." See "Atmosphere, Physics of," §§ (13), (14).
- Eddy-motion in the Atmosphere:
  - Diffusion of heat by. See "Atmosphere, Physics of," § (13).
  - Diffusion of momentum by. See *ibid.* § (14).
  - Equi-partition of energy in. See *ibid.* § (13).
- Effective Diameter of Screw: definition. See "Metrology," VII. § (23) (i.).
- Electric Currents in Atmosphere: continuous currents. See "Atmospheric Electricity," § (21).
- Electric Field of the Atmosphere, measurement of. See "Atmospheric Electricity," § (6).
- Electric Types of Gas Meter. See "Meters for Measurement of Coal Gas and Air," § (5).
- Electrometer, The Pocket: an instrument depending on the quartz fibre. See "Radiomicrometer," etc., § (5).
- Elliptical Trammel. See "Draughting Devices," p. 270.
- Energy:
  - Kinetic:
    - Of the general circulation of the atmosphere. See "Atmosphere, Thermodynamics of the," § (9).
    - Of cyclones and anticyclones. See *ibid.* § (26).
  - Thermal, of condensation. See *ibid.* § (2).
  - For energy equations in thermodynamical processes. See *ibid.* IV., V., and VI. §§ (18), etc.
- Engineers' Scales and Gauges. See "Metrology," VI. § (17).
- Entropy:
  - Calculation of, from pressure and temperature. See "Atmosphere, Thermodynamics of the," § (19). See also Vol. I.
  - Definition of. See *ibid.* § (19).
  - Isentropic equations for dry and saturated air. See *ibid.* §§ (19), (21).
- Entropy, Realised:
  - Calculation of, from pressure and temperature. See "Atmosphere, Thermodynamics of the," § (6).
  - Definition of. See *ibid.* § (6).
  - Distribution of, in the upper air over the globe. See *ibid.* § (6), *Fig.* 9.
  - Distribution of, in high and low pressure. See *ibid.* § (6), *Fig.* 10.
  - Relation of, to convective equilibrium. See *ibid.* § (14).
  - Value of, for saturated air. See *ibid.* §§ (22) and (23), Table VI., *Fig.* 16.
- Entropy-temperature Diagram, construction of, for dry and saturated air. See "Atmosphere, Thermodynamics of the," §§ (22) and (23), and *Figs.* 16 and 17.
- Equation to Scale: expression of length of a line standard. See "Line Standards," § (3).
- Equilibrium Theory of Tides. See "Tides, and Tide Prediction," § (4).
- Error, The Nature of: the difference between an assumed or observed value, and the true value. See "Observations, the Combination of," § (2).
- Errors, absolute, relative, and proportional in metrological work. See "Metrology," II. § (5) (iv.).
  - Symmetrical and asymmetrical, in observational work. See *ibid.* § (5) (v.).
- Errors, The Law of. See "Observations, the Combination of," § (3).
  - Deductions from. See *ibid.* § (4).
- Errors of Screws. See "Metrology," VII. § (24).
- Escapement, Gravity. See "Clocks and Time-keeping," § (9).
- Escapement and Maintenance. See "Clocks and Time-keeping," § (7).
- "Eviction" of Air in Atmospheric Convection: definition and experimental determination of. See "Atmosphere, Thermodynamics of the," § (17).
- Expansion. Explanation of term "thermal coefficient of expansion," and its application in line standard work. See "Line Standards," § (1) (iv.).

— F —

- Fata Morgana. The name given to mirages in which there is considerable distortion and repetition of the images. See "Meteorological Optics," § (10).
- Field Astronomy and Astronomic Atmospheric Refraction. See "Trigonometrical Heights," § (9).
- Fits for Interchangeable Work: definitions: "clearance," "transition," and "interference." See "Metrology," § (29) (iii.).
- Flank of Screw Thread: definition. See "Metrology," § (23) (i.).
- Fog, Translucence of. See "Meteorological Optics," § (16) (iii.) and (v.).

Fog-bow: a white rainbow produced by drops under 0·1 mm. in diameter. See "Meteorological Optics," §§ (14) and (15) (iii.).

Föhn-wind. See "Atmosphere, Thermodynamics of the," § (22).

Forecast. A statement of the weather to be anticipated during a given interval in the near future, based on the study of the synoptic chart and any other available data. A forecast includes a statement of:

(1) The direction and force of the wind at the ground and at moderate altitudes, and the changes anticipated during the period covered by the forecast.

(2) The amount and nature of cloud (whether high or low), precipitation, temperature, and visibility.

(3) Any unusual occurrence, such as sudden frost, fog, or thunderstorms.

The forecaster bases his predictions on the changes in pressure distribution indicated on his synoptic chart, combined with the use of observations of upper winds, temperature, and humidity. Although the general principles of forecasting were laid down in a fairly definite form by Abercromby, a considerable amount of experience is necessary before a forecaster can interpret with any degree of certainty the changes shown on the synoptic chart, and the same can be said of the interpretation of upper air data. The period for which a forecast can be reasonably made varies considerably. When an anticyclone is settled over the country, it is frequently possible to forecast the weather for three or four days ahead, but at times when the Westerly type prevails, secondary depressions come into existence and move so rapidly that it is only possible to forecast the details of the weather a few hours ahead.

Forecasting:
- General principles of. See "Atmosphere, Physics of," § (20).
- Norwegian methods of. See *ibid.* § (21).

Fortin Barometer. See "Barometers and Manometers," § (3) (i.).
- Modifications of: the mountain barometer. See *ibid.* § (3) (ii.).
- Procedure in setting up and reading. See *ibid.* § (3) (i.) (*f*).

Four Metre Comparators. See "Comparators," § (8).

Friction in the Atmosphere. See "Atmosphere, Physics of," § (14).

Friction and Frictionless Motions in Mechanism. See "Metrology," § (34) (iii.).

Friction-rotating: use of friction wheel in mechanism. See "Metrology," § (34) (iii.).

Full Diameter of Screw: definition. See "Metrology," § (23) (i.).

Fused Silica: its coefficient of expansion, suitability for use as material of line standards. See "Line Standards," § (6) (i.).

— G —

Galitzin Seismometer. See "Seismometry," §§ (2) and (4).

Galvanometer, Einthoven, Adaptation of, to the. Measurement of subdivisions of the second. See "Clocks and Time-keeping," § (15).

Gas, Density of a:
- Determined by means of the micro-balance. See "Balances," § (18) (ii.).
- Determined by the methods of Schloesing, Jaquerod and Tourpaian, and Threlfall. See *ibid.* § (18) (iii.).
- Determined by weighing the gas contained in a globe of predetermined capacity. See *ibid.* § (18) (i.).

Gas Burettes. See "Volume, Measurements of," § (23).

Gas Meters. See "Meters for Measurement of Coal Gas and Air," § (1).

Gauge Comparator:
- Hirth "Minimeter." See "Gauges," § (81).
- "Level" type for end-gauges. See *ibid.* § (83).
- "Prestwich" fluid type. See *ibid.* § (80).
- Special end-measuring comparator of high sensitivity. See *ibid.* § (82).

GAUGES

I. Types of Gauges used in Engineering Practice[1]

Broadly speaking, gauges may be divided into two classes:

A. Limit Gauges.
B. Standard Gauges.

The former are used by the mechanic and the inspector to test whether the work being produced is within the specified limits. Standard gauges serve for reference purposes in checking the accuracy of limit gauges, and also for setting up and testing the accuracy of micrometers, measuring machines, and other gauge-measuring instruments.

---

[1] The author wishes to acknowledge his indebtedness to the following firms and individuals for help received in the preparation of this article: Armstrong, Whitworth & Co.; Brown & Sharpe Manufacturing Co.; Alfred Herbert, Ltd.; Adam Hilger, Ltd.; La Société Genevoise d'Instrument de Physique; New Fortuna Machine Co.; Newall Engineering Co.; National Physical Laboratory; Pratt & Whitney Co.; J. A. Prestwich & Co.; Reid Brothers, Ltd.; Dr. P. E. Shaw; Taylor, Taylor & Hobson, Ltd.; Cambridge & Paul Trust Co.; J. Chesterman & Co., Ltd.,; British Engineering Standards Association.

A. LIMIT GAUGES

§ (1) TYPES OF LIMIT GAUGES.—A few of the more common types are described below:

(i.) *Internal Limit Gauges: Double-ended Plug Gauges, Cylindrical and Spherical-ended Bars.*—This type of gauge, in the form of a double-ended plug gauge, is shown in *Fig.* 1. The usual practice is to make the "not go" end shorter than the "go" end. This serves as a ready means of distinction in use, and, moreover, as the "not go" plug seldom enters the hole in the work, it does not require any large amount of surface to resist wear.

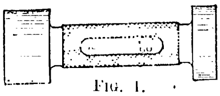


FIG. 1.

The gauge shown is made solid. For larger sizes the gauging portions are often made in the form of hardened steel discs, drilled with lightening holes and pressed on to a tubular mild steel handle. Nevertheless, above about 8 in. diameter, plug gauges are rather heavy and awkward to use, so that for testing the larger sizes of holes bar gauges having cylindrically formed ends are often used. Such a type of limit gauge is shown in *Fig.* 2. The ends are ground on centres, using a mandrel in the central hole.

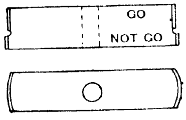


FIG. 2.

Spherical-ended rods are also used for the same purpose. A pair of such rods, mounted in a suitable insulated handle, are shown in *Fig.* 3.

The forms of internal gauges shown in *Figs.* 2 and 3 enable a test to be made on the hole for ellipticity. They give a more critical test of the work at the high end of the limit, whereas the plug form of "go" gauge, in *Fig.* 1, gives the more stringent test at the lower end. The rod form of gauge is the most suitable type for deep holes, as it provides a means of detecting whether such holes are larger at the centre than towards the ends.

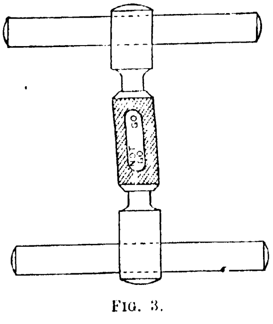


FIG. 3.

(ii.) *External Limit Gauges: Ring Gauges, Snap Gauges.*—*Fig.* 4 illustrates the usual form of external limit gauge for plug work. Such gauges are often made of mild steel plate with hardened steel bushes pressed in. Above a diameter of about 3 in. the gauge is usually made in the form of two separate rings.

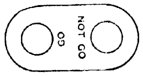


FIG. 4.

Another form of this gauge is shown at A in *Fig.* 5, which consists of a double gap, snap, or horseshoe gauge. For convenience this form of gauge is often made single-ended, as shown at B in the same figure.

Such gap gauges are sometimes made adjustable both for purposes of taking up wear and also for providing a means of varying the tolerance between the "go" and "not go" gaps according to the class of work or "fit" to be dealt with. *Fig.* 6 shows an adjustable snap gauge made by La Société Genevoise; details of the adjusting device are given enlarged. The measuring faces consist of four hardened steel studs E fitted in pairs to each arm of the gauge, the body of which is made of cast iron. The surfaces of the studs are ground accurately flat and parallel and their distances apart are adjusted either to standard block gauges or by the use of a special machine manufactured by the above firm. The method of adjusting the studs will be readily understood from the enlarged detail. The shank of each stud is a good fit in a hole in the gauge and is prevented from turning by a key A. The rear end of the hole is tapped with a fine thread into which a split nut B fits. This nut can be screwed up by means of a special key and serves to feed the measuring face forward. The stud is held back on to this nut by means of a retaining screw C whose conical head fits into a recess

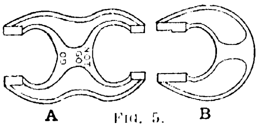


FIG. 5.

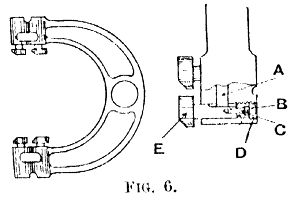


FIG. 6.

in the nut B. The effect of tightening the screw C is to pull the stud well into contact with the nut B, and at the same time the conical head expands the nut and so locks it in the screwed part of the hole. Having made the adjustment, the space behind the nut and screw is filled with a wax seal D which prevents any tampering with the adjustment.

(iii.) *Depth Gauge.*—One form of a limit depth gauge suitable for gauging the depth of a cavity is shown in *Fig.* 7. It consists of a rod A which passes through a hole in a collar B, the under side of which is faced. The upper end of the rod has a line *l* scribed round it, and the position of this line is noted with reference to two fixed lines L and H scribed on the collar, a portion of the latter being cut away for the purpose. The distance between the lines L and H is equal to the tolerance on the depth to be tested.

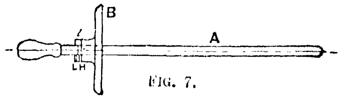


FIG. 7.

(iv.) *Taper Gauges.*—*Fig.* 8 illustrates a pair of limit taper gauges. The end faces of these gauges are usually finished in addition to the conical surface. In testing work the fit of the gauge gives an indication as to the accuracy of the angle of the taper, and this can be checked more definitely by the use of some description of marking material such as prussian-blue paint. The diameters of the work are checked by noting the position of the upper surface of the gauge with reference to a shoulder or some such definite location on the work. The upper surfaces of the gauges are shown stepped, which allows the work to be tested between limits. The depth of the step is equal to the diametral tolerance on the work multiplied by half the cotangent of the semiangle of the taper.

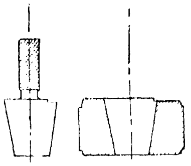


FIG. 8.

§ (2) PROFILE GAUGES.—There are many varieties of this type of gauge. The commonest type is, perhaps, the radius or "fillet" gauge shown in *Fig.* 9. These gauges are made in sets covering convenient ranges of radii, the various plates being pinned together at one end and mounted in a suitable holder. Such sets usually comprise both external and internal gauges for each radius.

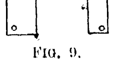


FIG. 9.

A type of profile gauge used for checking the form of the copper driving-band of shells is shown in *Fig.* 10. The example illustrated shows a particularly elaborate form of band: in most cases the outline is devoid of any curves.

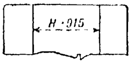


FIG. 10.

Another example of a profile gauge is illustrated in *Fig.* 11. This shows a set of templates for a flat-bottom railway rail. The set comprises three separate pieces; the inner one represents the standard section of the rail, and the two outer pieces are counterparts of the first. Such templates serve the purpose of testing not only the form of the rails, but also that of the rolls for producing them. Similar templates are used for the fish-plates of the rails.

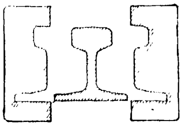


FIG. 11.

Profile gauges are used by placing their edges in contact with the work, the degree of fit being noted either by viewing the junction against a strong light or testing this junction with thin metal feeler pieces. Strictly speaking, they can hardly be classed as limit gauges.

§ (3) POSITION GAUGES.—This type of gauge is used for checking the relative locations of different parts of a piece of work or of an assembly of pieces. A simple example for checking the centring of two holes is shown in *Fig.* 12. It consists of two hardened steel pins *a* fixed parallel to each other in a block *b* at a distance apart equal to the nominal spacing of the holes in the work. In order to allow a certain tolerance on the spacing in the work, the two pins are made slightly smaller than the low diameter of the holes by an amount depending on the tolerance.

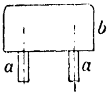


FIG. 12.

A "not go" plug gauge would be used in conjunction with this gauge to check the diameters of the holes on the high limit.

Fig. 13 shows a position gauge for testing the location of the three holes A, B, and C in the fuse body shown in *Fig.* 14. The body of the gauge, which is made of cast iron, is bored out to accommodate the conical surface of the fuse and is provided with a hardened steel axial pin *a*, which enters the hole A of the fuse. Two other pins *b* and *c* slide in hardened steel bushes fixed in the body of the gauge and in the correct positions to correspond with the holes in the fuse. The bushes are ground out to the same size for convenience in manufacture and in testing, the pins *b* and *c* being reduced in diameter at the points to suit the diameters of the holes.

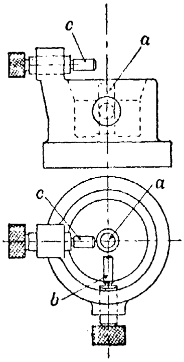


FIG. 13.

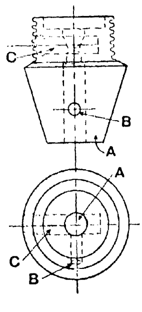


FIG. 14.

The operation of testing the fuses with such a gauge is very simple. The fuse is dropped over the pin *a* and is rotated until the holes B and C come in line with the corresponding pins. For the fuse to be satisfactory it should be possible to insert both pins into the fuse simultaneously.

As in the case of the simple pin gauge described above, the diameters of the gauge pins are made slightly smaller than the diameters of the holes in the fuse in order to allow tolerances on the relative position of the various holes.

§ (4) SCREW GAUGES.—*Fig.* 15 illustrates the usual type of "go" plug and ring screw gauges used for inspection or workshop practice. These are made with threads having the standard profile.

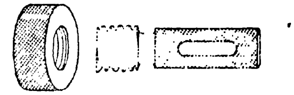


FIG. 15.

The plug gauges are sometimes provided with a plain plug at the reverse end, as shown in *Fig.* 16, having a diameter equal to that of the nominal core diameter of the screw and which serves as a "go" gauge for that element.

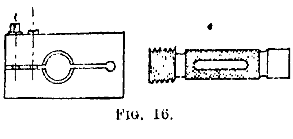


FIG. 16.

The ring gauges are sometimes made split as shown in *Fig.* 16, a pull and a push screw being provided for making the adjustment. This type of gauge is not to be recommended, since the usual results of wear is a malformation of the correct thread form, and this clearly cannot be rectified by any such adjustment.

The "not go" gauge for screwed work usually takes the form of a plain plug gauge for the core diameter of nuts, and a snap or plain ring gauge for the full diameter of the bolts. It is also desirable to ensure that the threads of the work are not unduly thin, and for this purpose "not go" effective diameter screw gauges are used. The form of the thread of such "not go" gauges is as shown in *Fig.* 17, where the crests are truncated and the roots cleared. The standard Whitworth form is shown dotted in this figure. This type of gauge was originated by Mr. W. Taylor.

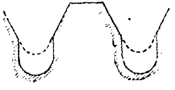


FIG. 17.

Various types of special gauges and instruments have been designed for testing screwed work, a few examples of which are given in § (40).

The screw gauges referred to above are parallel on diameters throughout their lengths. Certain classes of screw threads, however, such as pipe threads and fuse body threads, are made tapered so as to obtain a binding fit, and these threads give rise to taper screw gauges of both plug and ring form. The threads of such gauges are often of approximate Whitworth form; the centre line of the threads can be square either to the axis of the gauge or to its tapering sides (see § (37)).

B. STANDARD GAUGES

As explained in § (14), standard gauges are used as standards for comparison when making accurate measurements on measuring machines. At times they are also utilised in a direct manner when testing limit or working gauges.

§ (5) STANDARD GAUGES.—These are made in a variety of forms, but in every case to a high order of accuracy.

(i.) *Standard Plug and Ring Gauges.*—The first useful type is the cylindrical form, shown in *Fig.* 18. Such gauges are made up to 8 in. in diameter, but they are not commonly used above about 3 in. The gauges are either case-hardened or hardened right through, and their working surfaces are ground and lapped to a high polish.

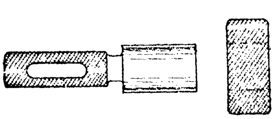


FIG. 18.

The fit of the ring gauge on the plug should be such that the two gauges will assemble without forcing, and without perceptible play, when the working surfaces are clean and free from any trace of grease. The presence of grease or any lubricant will mask any slight play which may exist between the two gauges. Moreover, if the gauges are lubricated it is often possible to insert a plug into a ring of slightly smaller diameter. The explanation is that ring gauges are almost invariably bell-mouthed to some extent and plugs tend to be slightly smaller at the ends, with the result that a very slight wedging effect arises on assembling the gauges, and this can easily cause the ring to stretch without any undue axial force being applied. This stretching of the ring can be definitely proved by using a ring gauge which is ground on the outside. External measurements of the ring when in the free state and when assembled on a plug will indicate the amount of distortion.

When the surfaces of both gauges are highly polished it is possible to obtain a wringing fit between the pair. The gauges can be assembled after lubrication and will remain free so long as one is kept in motion relative to the other. If allowed to rest for only a moment the gauges will seize, and they cannot be separated except by tapping or forcing apart, with the result that the clinging surfaces are more often than not severely damaged in the process.

In using plug gauges as standards for purposes of comparison, it is usual to standardise them locally so as to avoid the effect of even slight variations which may occur in the size of the plug at different parts. A convenient location for the standardisation is half-way along the length of the working surface and across a diameter parallel to a flat on the handle.

Standard plug gauges are usually made correct to size to within ± 0·0001 in. for sizes up to 2 in., the allowable inaccuracy increasing to ± 0·0003 in. for 8 in. gauges.

(ii.) *Standard End Gauges: Bars.* — The second type of standard gauge is the form most frequently termed end gauges. These gauges are made from steel bar and may be hardened throughout their length or only at the ends. *Fig.* 19 shows the Whitworth and Newall types of end gauges. The former are made in ¾ in. square section with the corners rounded off and the ends ground down to ¼ in. diameter. The end faces are flat. The Newall gauges are in the form of cylindrical bars varying in diameter from ⅜ in. to ¾ in. according to length. They are slightly bevelled off at the ends, which are finished flat. The gauges are usually fitted with ebonite sleeves to minimise the thermal effect due to handling. This type of gauge is made up to a length of 72 in. Except in the case of the shorter lengths, these gauges are hardened only at the ends. The end measuring faces of such gauges should be not only flat but also parallel to each other and accurately square to the axis of the rod. The latter condition is essential, as with the longer gauges it is necessary to support them on vee rests during the measurements. It is a fairly easy matter to adjust the height of the rests so as to bring the axis of the gauge parallel to the bed of the measuring machine, but unless the faces are square to the axis, an incorrect measurement of length will be obtained.

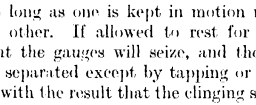


FIG. 19.

End gauges are also made with spherical faces, as shown in *Fig.* 20, the radius of curvature of the faces being equal to half the length of the gauge. Gauges with this type of measuring face lend themselves readily to a process of building two or more together to make a long length. This method usually introduces errors when applied to flat-ended gauges owing to slight lack of parallelism or squareness of the flat faces.

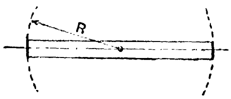


FIG. 20.

When measuring spherical ended gauges under pressure, care should be taken to allow for the elastic compression which takes place at the local contact of each spherical face with the flat faces of the machine (see § (16)).

(iii.) *Standard End Gauges: "Block" Type.* —One of the most useful types of standard gauges both from the point of view of accuracy and adaptability is the type introduced by Messrs. C. E. Johansson of Sweden about ten years ago. These gauges are made in sets, one of which is shown in *Fig.* 21. The particular set illustrated, which was made by Pitters Ventilating and Engineering Co., Ltd., comprises 81 gauges ranging in size from 0·05 in. to 4 in., each gauge being made of hardened steel in the form of a rectangular block. Each block has two opposite faces highly finished, the surfaces of these faces being optically flat and parallel to within 0·00001 in. The maker guarantees all the blocks under 1 in. in length to be accurate to size within ± 0·00001 in. and the 2, 3, and 4 in. blocks within ± 0·00002, ± 0·00003, and ± 0·00004 in. respectively.

The standard set in inch units comprises the following arrangement of 81 sizes:

0·1001 to 0·1009 by steps of 0·0001,
0·101 to 0·149 by steps of 0·001,
0·05 to 1·0 by steps of 0·05,
2, 3, and 4.

The standard set in metric units comprises 103 gauges in millimetre sizes as follows:

1·01 to 1·49 by steps of 0·01,
1·0 to 25·0 by steps of 0·5,
50, 75, and 100.

The most valuable feature of these gauges is the fact that it is possible to "wring" two or more gauges together by first cleaning them and then bringing their faces into contact and applying a combined sliding and twisting motion. When brought together in such a manner it is found that the two gauges adhere strongly to each other and thus form a single gauge whose length is the sum of the component gauges. The adherence is due almost entirely to the presence of a liquid film between the faces. This can be proved by carefully cleaning two gauges with petrol or absolute alcohol, after which it will be found that no appreciable wringing effect can be obtained. On touching one of the faces with the finger so as to deposit a small amount of moisture or grease, the wringing effect will be produced. Pure water appears to be one of the best liquids to give the maximum wringing effect.

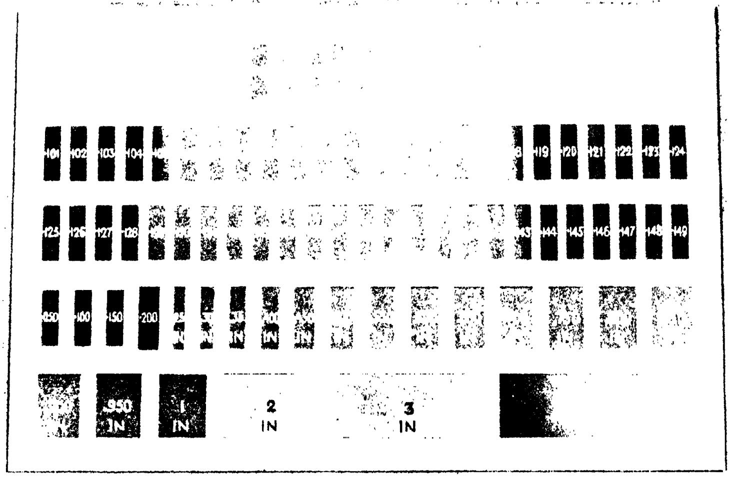


FIG. 21.

Although the presence of a liquid film is essential for the adherence to take place between the two gauges, the thickness of this film is very minute if the gauges are properly wrung. Experiments on the measurement of the thickness of the film are described in vol. xvii. of the *Travaux et Mémoires du Bureau International.* The experimenters, Pérard and Mandet, arrived at the remarkable result that the composite length of two gauges when wrung together is *less* than the sum of their net individual lengths by 0·0000024 in. The thickness of the film must necessarily be positive, but the authors of the paper attribute the shortening to local compression of the gauges at the junction due to the force of adhesion produced by the film, and to the tendency of the faces to fit one into the other. Owing to imperfections in the final polishing the surfaces have an extremely fine matt finish, and the possibility of two such surfaces fitting one into the other to some minute extent can be readily understood.

The force necessary to separate two gauges wrung together with a film of condensed water-vapour has been found to be of an order of 50 lbs. weight per square inch of surface.

Experiments on this subject have been carried out by Mr. H. M. Budgett, the results of which will be found in vol. lxxxvi., 1911, of the *Proc. of the Royal Society.*

If the sizes of the gauges contained in the standard inch set referred to above be examined, it will be seen that by wringing together two or more blocks it is possible to form a gauge whose lengths may involve four figures after the decimal point providing the length is not less than 0·2 in. Up to a length of about 5 in. it is always possible to form a compound gauge without using more than four component gauges. Above this length, however, it is frequently necessary to use a larger number. It is often possible to form a compound gauge in more than one way. The following table illustrates how a length of 2·4787 in. can be arrived at by three different sets of blocks:

·1007	·1002	·1003
·128	·1005	·1004
·25	·130	·137
2	·148	·141
..	·1	·95
..	·9	·85
..	1	·2
2·4787	2·4787	2·4787

By wringing up three such combinations and comparing their sizes in a measuring machine a ready check is obtained on the accuracy of the 18 gauges involved.

Besides their use as standard gauges these block gauges serve many other useful purposes which will be referred to later.

It may be mentioned that this type of block gauge is now being manufactured in America and in England to an accuracy equal to that of the Swedish gauges.

The best known of the American gauges are the "Hoke" type which are being manufactured by Pratt & Whitney Co. These gauges are made uniformly 0·95 in. square in section and of all lengths up to 4 in. Through the centre of each block is a ¼ in. hole for securing caliper-jaws and other attachments. Their accuracy is guaranteed to five-millionths of an inch of marked size up to one half inch and within ten-millionths per inch of length over that size.

A description of the method of manufacturing the "Hoke" gauges will be found in *Machinery* dated June 3, 1920. This article also describes an optical method for testing the flatness of the gauges.

(iv.) *N.P.L. Method of making Block Gauges.*—This method of manufacturing block gauges of the Johansson type was devised at the National Physical Laboratory in 1917.[1]

The material used is a pure carbon steel containing approximately 1 per cent of C. The blanks are carefully hardened after machining and their surfaces are then rough ground on an ordinary magnetic chuck. To ensure freedom from subsequent distortion and change of size, the hardened blanks are subjected to a stabilising heat treatment. This treatment consists of alternately heating and cooling the gauges, the upper limits of the temperature to which they are raised being 210°, 130°, 70°, 40° C. in succession; the lower limit is 10° C. The gauges are then ready for the precision grinding operation. For this purpose the gauges are fixed to a special form of magnetic block chuck shown in *Fig.* 22. This chuck consists of a block of mild steel 1 surrounded by a wood case 2 Each of the large faces of the block has eight raised facets arranged in pairs at each corner as shown. The eight facets on each side are carefully lapped flat and in one plane. The two planes formed by the facets are accurately parallel. The block is wound with coils so that one set of facets can be used as a magnetic chuck. Eight of the gauges to be ground are placed on the magnetic set of facets, and the chuck is placed with the gauges upwards on a surface plate which is bolted to the table of a surface grinder. The upper faces of the gauges are then "spot" ground by sliding the chuck about on the surface plate under the grinding wheel. The gauges are then turned over on the facets, and the remaining faces are ground in a similar manner. The gauges will then be in a state such that they are all of the same thickness to within a few hundred-thousandths of an inch, and they now require to be lapped.

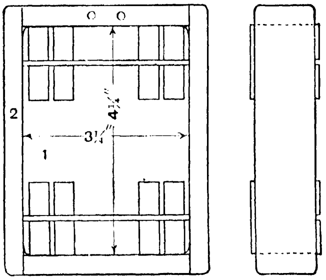


Fig. 22.

The lapping is carried out on cast iron and bronze plates, the upper surfaces of which are charged with very fine abrasive material. Clogging of the plates is prevented by brushing them frequently with a good supply of cold water. The final flatness of the surfaces of the gauges depends upon the flatness of the lapping plates. These are made in threes. They are first "spot" ground on a surface grinder, as described above, and their surfaces are then finished by rubbing the plates together in pairs, using a suitable abrasive between them. If this mutual lapping is done in cyclical order with the three plates, a high degree of flatness is obtained. The plates are about 12 in. diameter, and their upper surfaces are machined with grooves into a chequered pattern for holding the surplus abrasive and lubricant.

---

[1] It is patented under the names of Sir R. T. Glazebrook, J. E. Sears, Jun., and A. J. C. Brookes.

The chuck carrying the eight gauges on one face is placed with the gauges in contact with a lapping plate, and the exposed surfaces of the gauges are lapped down until all grinding marks are eliminated. The gauges are then turned face for face on the chuck and the other faces are lapped in a similar manner. Now, assuming that the face of the lapping plate is accurately flat, the exposed surfaces of the gauges will all lie accurately in one plane, but this plane may not be truly parallel with that formed by the facets on to which the gauges are wrung. In other words, the faces of the gauges may not be truly parallel and their thicknesses may differ somewhat. To counteract this tapering effect, the positions of some of the gauges on the chuck are changed in the manner shown in *Fig.* 23. The upper diagram, A, shows the original distribution of the gauges, and the lower one, B, the relative positions after the gauges have been changed. It should be noted that the upper gauge of each pair is left in its original position, but the other gauges are interchanged in a diagonal direction on the chuck, and they are also turned end for end. A little consideration will show that this rearrangement of the gauges averages out any tapering effect between the two planes referred to above, so that on relapping the gauges they should arrive at a much closer state of parallelism than before. If desired, the gauges can be rearranged once more and lapped for the third time. In practice, however, one interchanging is usually found to be sufficient to render the gauges of equal thickness to within one or two millionths of an inch. The equality of the gauges can be rapidly checked to one millionth of an inch by the use of one of the comparators described in §§ (82) and (83). The type of chuck shown in *Fig.* 22 serves for slip gauges up to about 1 in. in length; for longer gauges up to 4 in. a rather more elaborate chuck is required.

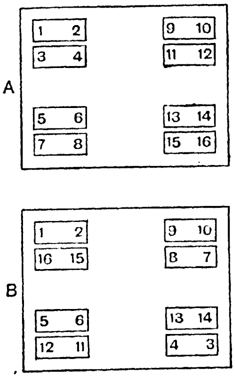


Fig. 23.

It is necessary to know when to cease lapping, so that the thickness of the gauges shall be equal to some desired dimension. The gauges are all removed from the chuck when it is thought that they are approaching size, and, after wringing the eight together, their aggregate length is compared with an appropriate known standard. This comparison is made most readily on the "Level" comparator described in § (83). If the gauges are too thick, they are returned to the chuck and lapped down further. This method of comparing the total length of the eight gauges against a standard, together with the method of obtaining equality of thickness between them, clearly results in a very high accuracy in their final individual lengths. For example, if a 4 in. standard gauge is provided and its absolute length is known to even only ± 0·00001 in., it would be possible to produce eight ½ in. gauges which would be accurate to size to within one-eighth of this uncertainty, *i.e.* about ± 0·000001 in. If it were required to produce some 0·1 in. gauges, eight of them would be lapped up until any five, when wrung together, were the same length as one of the ½ in. gauges. By this method it is possible to generate accurately the sizes for all the gauges comprised in the usual set of 81 slip gauges from a parent 4-in. standard of known length.[1]

(v.) *Secondary Standard Gauges: Balls and Roller Gauges.*—Ordinary commercial steel balls as used in bearings are made to a tolerance of ±0·0001 in., consequently a set of such balls forms an inexpensive set of secondary standards for workshop use. They are made in sizes varying from about $\frac{1}{32}$ to 3 in. diameter.

When measuring steel balls in machines having fairly heavy pressures (such as 5 lbs. weight) between the faces, it is necessary to apply a correction to the measurements on account of the elastic deformation which takes place at the points of contact. A 1-in. ball, under the pressure stated above, will measure small by approximately 0·0001 in., and in the case of an ⅛-in. ball the difference is twice that amount. The general formula for the diminution in diameter due to compression is

$$C = 0{\cdot}0000321 \times P^{\frac{2}{3}}\left(\frac{1}{d}\right),$$

where C is the diminution in diameter in inches due to a pressure of P lbs. weight, the diameter of the ball being *d* in.

---

[1] See *Report of the National Physical Laboratory* for 1921.

Another useful combination of secondary standards is the set of 15 roller gauges supplied by the Hoffmann Manufacturing Co., Chelmsford. These gauges consist of selected commercial rollers as used in roller bearings. They are guaranteed to an accuracy of ±0·0001 in. both on length and diameter, these dimensions being equal on each roller. A set of these gauges is shown in *Fig.* 24.

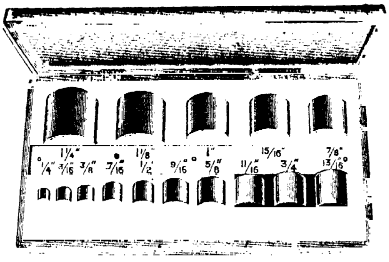


FIG. 24.

II. METHODS OF MEASURING GAUGES

A. LIMIT GAUGES

The choice of instruments and the methods adopted for measuring and testing gauges depend mainly upon the tolerance allowed on the gauges. Generally speaking, limit gauges can be tested without the assistance of so-called "measuring machines," these being used mainly in the verification of standard and reference gauges.

The following paragraphs deal with the general methods of measuring the commoner types of limit gauges.

§ (6) INTERNAL LIMIT GAUGES: PLUG AND BAR TYPES.—The finest tolerance met with on this class of gauge, except where they are used for work of very high-class accuracy, is 0·0003 in. Their measurement is usually carried out by the use of ordinary micrometers or some form of comparator, these instruments being set up on standard gauges. For a description of various types of comparators reference should be made to § (80).

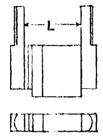


FIG. 25.

Check gauges are not often employed for testing plug gauges, but it is sometimes useful to form a pair of "go" and "not go" snap gauges from block gauges which are built up to represent the high and low limits of the gauge. *Fig.* 25 shows such a snap gauge which is formed by wringing special pieces or "points" on to the outsides of the slips which represent the dimension L of the snap gauge required. The inner surfaces of these points are lapped quite flat. Their outer faces are accurately rounded for about half of their lengths, the thickness of the rounded portions being finished to certain convenient dimensions such as 0·25, 0·2, or 0·1 in. These points will be referred to again later in connection with the measurement of ring gauges.

§ (7) EXTERNAL LIMIT GAUGES: RING AND GAP GAUGES.

(i.) *Ring Gauges.*—Plain ring gauges, such as shown in *Figs.* 4 and 18, can be tested by four methods:

(*a*) "Go" and "not go" check plugs.
(*b*) Block gauges and points.
(*c*) Block gauges and cylinders.
(*d*) The "chord" method.

(*a*) A pair of plain plugs or in the case of the rings larger than about 4 in. diameter a pair of cylindrical-ended bars are made accurately to the high and low limit of the ring gauge. The latter is tested by trying the fit of the "go" and "not go" check gauges in it. The provision of such check gauges becomes expensive unless a fairly large number of ring gauges of the same size have to be tested. Moreover, the difficulty referred to in § (5) in connection with the stretching of ring gauges by the insertion of a plug arises. A further point is that ellipticity in the ring gauge is not readily detected by the use of check plugs.

Generally speaking, except in the case of gauges below 0·5 in. in diameter, the use of check plugs is discarded in favour of the three other methods about to be described.

(*b*) Ring gauges down to about 0·5 in. diameter can be tested by the use of block gauges and points, as shown in *Fig.* 25 above. To facilitate the manipulation of the gauges they are clamped in an adjustable holder as shown in *Fig.* 26. The slip gauges between the points can be varied until the combination just fits the ring gauge, and by this means it is possible to measure the diameter of the latter to an accuracy of about ±0·0001 in. It should be noted that this method provides a means of detecting ellipticity in the gauge by testing the ring across various diameters.

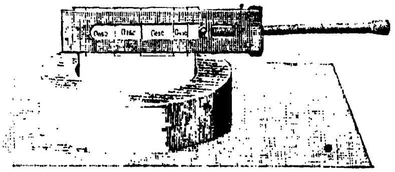


FIG. 26.

It has been found from practice that the curved measuring surfaces of the points wear fairly rapidly, and also the projecting parts of the points are liable to spring slightly in use. The following method of using a pair of plugs instead of the points retains the flexibility of size derived from the use of slip gauges, but avoids the errors mentioned above in connection with the use of the points.

(*c*) The ring gauge is placed on a surface plate and suitable Johannson blocks are wrung together and placed inside the ring, together with two plugs of known diameters, as shown in *Fig.* 27. The slips are varied by 0·0001 in. steps until the plugs are just a nice fit.

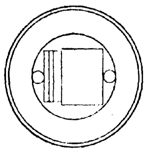


FIG. 27.

This method can be used for rings up to 3 or 4 feet in diameter, in which case the bulk of the space between the plugs is taken up by a long, flat-ended bar gauge, slips being used for the last remaining inch or so.

The Hoffmann roller gauges described in § (5) are very convenient for use as the plugs in the above method.

(*d*) The principle of this method, which is due to Mr. G. A. Tomlinson of the National Physical Laboratory, is illustrated in *Fig.* 28.

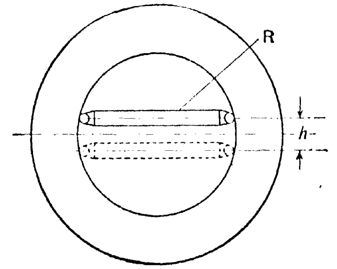


FIG. 28.

The ring is held with its axis horizontal and its diameter is measured with reference to the known length of a ball-ended rod R. This rod is made somewhat shorter than the diameter of the ring, so that when placed horizontally in the latter it is possible to move it vertically through a certain distance h, which depends upon the diameter of the ring, the length of the rod, and the diameter of the balls.

Let D = Diameter of ring.
L = Length of rod *between centres of balls.*
d = Diameter of balls.

Then, $$h^2 = (D-d)^2 - L^2. \quad . \quad . \quad . \quad (1)$$

If the nominal size of the ring is known, then it is possible to calculate the theoretical value of h for a perfect ring of that size when using a suitable rod of known dimensions. An error in the diameter of the ring will be detected if the measured and theoretical values of h do not agree. Moreover, it should be noted that any error in the diameter of the ring becomes magnified in the measurement of h, the degree of magnification depending upon the length of the rod in relation to the diameter of the ring. In practice it has been found convenient to choose the length of the rod for any particular size of ring, so that the diameter errors are magnified five times. That is to say, if the diameter of the ring were 0·001 in. greater than its nominal size, then the rod could be displaced 0·005 in. more than its calculated movement.

The magnification is expressed by

$$\frac{dh}{dD} = \frac{D-d}{h} = 5 \text{ (say)},$$

$$\therefore h = \frac{D-d}{5}, \quad . \quad . \quad . \quad . \quad (2)$$

Then, from equation (1),

$$L^2 = (D-d)^2 - h^2$$
$$= \tfrac{24}{25}(D-d)^2.$$
$$\therefore L = \tfrac{1}{5}\sqrt{24}(D-d). \quad . \quad . \quad . \quad (3)$$

Given D and d, it is possible from (2) and (3) to calculate the required length of the rod and the theoretical value of h. If on measurement, an error δh is found in h, then the error in the diameter of the ring from its nominal size is $\tfrac{1}{5}\delta h$.

The accuracy to which it is possible to determine the error in the diameter of a plain ring depends not only upon the accuracy of the determination of h, but also upon the measurement of the length of the rod L and the mean diameter of the balls d. These three quantities can each be obtained to an accuracy of about ± 0·00002 in., so that the determination of the absolute value of the error of the ring should be known to within ± 0·00005 in. Comparative values of the errors in different parts of the same ring can be obtained true to within ± 0·00001 in.

In addition to the measurement of plain rings this method also lends itself to the determination of the effective diameter of screwed ring gauges. In this case the size of the balls attached to the ends of the rod should be such that they make contact with the threads of the screw about half way between the crests and roots. The calculation of the effective diameter follows the method given above for a plain ring. It has to be remembered, however, that the rod sets itself slightly askew when in contact with the screw thread, and it is necessary to use in the formulae the length of the rod projected in a plane normal to the axis of the screw. This projected length is approximately equal to $\sqrt{L^2 - (p/2)^2}$, where p is the pitch of the screw.

The approximation is very close; in the case of a 3-in. ring having 3½ threads per inch, the error amounts to only 0·00005 in., and for all ordinary purposes the above value for the projected length may be taken as exact.

In the case of a screw the radial distance from the axis to the centre of the ball becomes

$$\tfrac{1}{2}(\mathrm{E}-d+\mathrm{P}),$$

where E is the effective diameter and P is the " P value " corresponding to the diameter of the ball d and the pitch of the thread (see § (63)).

We then have, instead of equation (1),

$$h=\sqrt{(\mathrm{E}-d+\mathrm{P})^2-\left(\mathrm{L}^2-\frac{p^2}{4}\right)}.$$

Equations (2) and (3) become

$$h=\frac{\mathrm{E}-d+\mathrm{P}}{5},$$

$$\sqrt{\mathrm{L}^2-\frac{p^2}{4}}=\tfrac{1}{5}\sqrt{24}\,(\mathrm{E}-d+\mathrm{P}).$$

These equations enable the error in the effective diameter to be determined from the measured value of h in the same manner as for a plain ring.

The method used for measuring the vertical distance h is illustrated in *Fig.* 29. The ball-ended rod A is carried horizontally at one end of the five to one lever B, which is pivoted on two transverse ball feet C spaced 1⅛ in. apart. These feet rest on the upper polished surface of a pile of slip gauges D, the height of which can be readily adjusted by building up on a surface plate below. The balance of the lever is adjusted by the jockey weight W. The height of the gauges D is first chosen so as to bring the lever approximately horizontal when the rod is resting in its lower position under the action of the throw-over weight F. A reading is then made with the fixed micrometer head M by sighting the contact on a thin curved sector E, which is attached to the end of the longer arm of the lever. Under favourable lighting conditions the micrometer can be set to 0·0001 in. The pile of gauges is then increased by the calculated value of h and the rod is brought into gentle contact with the upper part of the ring by changing over the weight F. A second micrometer reading is then taken. The measured error in h for the particular ring is equal to one-fifth of the difference between the calculated value of h and the displacement of the sector E, as measured by the micrometer, with the appropriate sign. Since the error in the diameter is again one-fifth of δh, it will be seen that diametral errors are magnified 25 times at the micrometer.

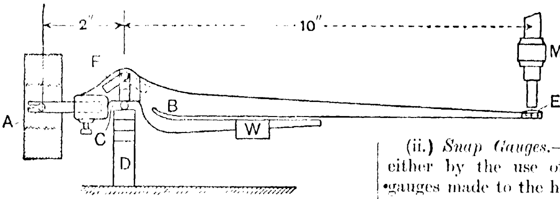


FIG. 29.

When measuring a plain ring the ball-ended rod is set square by eye to the axis of the ring. Any small error in this adjustment produces only a cosine error in the final result and can be neglected. In the case of a screwed ring, the rod must turn through a small angle in the horizontal plane when passing from the lower to the upper position. The method of pivoting the lever allows this movement to occur freely as the balls slide easily over the highly finished face of the supporting gauges.

It is found convenient to use adjustable ball-ended rods made in two parts and fixed in a suitable holder, as shown in *Fig.* 30. The gap between the two parts is set by means of slip gauges to give the required centre distance between the balls. To facilitate this setting, the inner ends of the rods are made plane and spherical respectively. The ball-ends consist of hardened steel balls which have been specially selected and accurately measured. They are fixed to the rod with hard solder.

(ii.) *Snap Gauges.*—Gap gauges are tested either by the use of a pair of special slip gauges made to the high and low limits of the gauge, or by direct use of combinations of block gauges. The gap gauges usually have flat faces and it is necessary to test for any lack of parallelism by trying the fit of the slip gauges between different parts of the face. A note should also be made as to the existence of any springiness of the jaws of the gauge caused by lack of strength in the section of the gauge.

§ (8) DEPTH GAUGES. — Gauges such as shown in *Fig.* 7 are readily tested by the application of one or more piles of slip gauges placed on a surface plate.

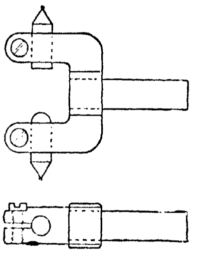


FIG. 30.

§ (9) TAPER PLUG GAUGES. — The most satisfactory method of testing taper plug gauges, both for angle of taper and diameter, is by the use of slip gauges and a pair of roller gauges, as shown in *Fig.* 31. The gauge is stood on a surface plate and, if necessary, is held down by a clamp screw fixed in a suitable bracket fastened to the plate. A pair of roller gauges, as described on p. 303, whose diameters are known accurately, are placed at the base of the gauge, as shown in the diagram. The *distance M_1 over* the outsides of the rollers is carefully measured with a micrometer. This measurement is repeated across different diameters of the gauge, and the two rollers are then raised on equal piles of slip gauges of height H, as shown by the dotted lines, and a further measurement M_2 is made. The height of the piles of slip gauges can be altered to bring the rollers to as many positions as is thought desirable. Measurements are made at each position in turn, and are noted together with the corresponding heights of the two piles of slip gauges.

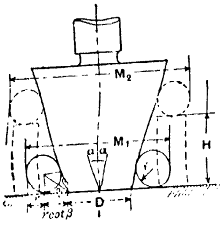


FIG. 31.

The best procedure in working out the results is to take the standard drawing of the gauge and, from the dimensions given, to calculate the theoretical measurements across the rollers at the various heights used. The calculation is of a simple nature. Suppose the diameter of the base of the gauge is given as D, and the semi-angle of taper as α, then, if the radii of the rollers is r, we have

$$M_1 = D + 2r(1 + \cot\beta),$$

where $\beta = \frac{1}{2}(90^\circ - \alpha)$. Similarly

$$M_2 = D + 2r(1 + \cot\beta) + 2H\tan\alpha.$$

As an example, the table below gives a series of measurements made on a taper plug gauge together with the corresponding calculated values of the distance across the rollers as obtained from the above formulae.

Height H.	Distance across Rollers.		Error in Size.
	Measured.	Calculated.	
0″·0	1″·7430	1″·7428	+0″·0002
0·25	1·7680	1·7678	+0·0002
0·5	1·7931	1·7928	+0·0003
0·75	1·8182	1·8178	+0·0004
1·0	1·8434	1·8428	+0·0006
1·25	1·8686	1·8678	+0·0008
1·5	1·8938	1·8928	+0·0010

The last column gives the error in the diameter of the gauge at the various sections at which the rollers made contact with it. It will be noted that these errors are larger towards the upper end of the gauge where the height H is greatest, and moreover, the error towards the centre is less than the mean of the errors at the extremities. This indicates that the general rate of taper is too great, *i.e.* the angle α is too large, and also that the conical surface is slightly hollow towards the centre. The latter error could be readily checked by laying the gauge on a surface plate, when a slight gap of about 0·0001 in. should be noticeable at the middle against a strong light.

§ (10) TAPER RING GAUGES.—A common method of testing taper ring gauges in the *workshop* is by the use of a corresponding taper *plug gauge, of known* accuracy, as a check gauge. The two gauges should assemble and either the shoulder of the plug, or a line scribed round its surface, should come flush with the upper surface of the ring. This method is liable to lead to erroneous results unless special care is used. *Fig.* 32 shows three ring gauges having faulty tapers which could be carelessly passed as "fitting" the same check plug. The errors in the tapers of the rings are, of course, exaggerated. If the actual errors were large they could be detected by the presence of play between the gauges. Such errors could also be detected to within fairly fine limits by the use of some form of marking material applied *very thinly* to the plug gauge before inserting it in the ring and noting to what extent it became removed after rotating the plug. For testing the ring to an accuracy of ± 0·0001 in., however, it would be necessary to resort to direct measurement.

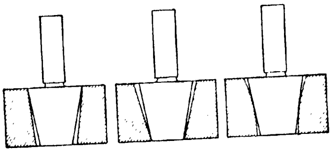


FIG. 32.

The general method of measuring the rate of taper and the diameters of taper ring gauges is by the use of steel balls. When the minimum diameter of the gauges is above about 1½ in. the method shown in *Fig.* 33 can be used. The scheme is analogous to that already described and illustrated in *Fig.* 31 for the measurement of taper plug gauges, the rollers being replaced by a pair of steel

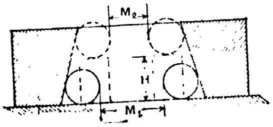


FIG. 33

balls, the distance between which is measured by means of slip gauges. The balls are first placed on the surface plate and are afterwards raised on equal piles of gauges to various heights in succession. The calculation of the results is similar to that explained in the case of the plug gauge above.

For smaller sizes of rings it is not possible to insert the slip gauges, and single different size balls are used in turn, as shown in *Fig.* 34. The balls are allowed to rest in the cone of the ring and the distance from the top of the ball to the upper surface of the gauge is measured with a depth gauge, or, if the ball protrudes, by means of slip gauges. As Hoffmann steel balls are made in $\frac{1}{32}$nd sizes from 1 in. to $\frac{1}{2}$ in., and below that size in $\frac{1}{64}$th-in. steps, it is usually possible to select several sizes of balls which will suit any particular ring.

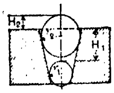


Fig. 34.

It should be noted that by this method of measuring distances along the axis of the cone, the diameters of the ring are obtained to an enhanced accuracy according to the fineness of the taper.

In some cases where the angle of taper is very slight it is not possible to find more than one size of ball which will rest in the cone. For such gauges the method indicated in *Fig.* 35 may be adopted. A ball somewhat smaller than the minimum diameter of ring is chosen and this is inserted together with a small cylinder, D_1, of suitable size, so that the ball rests near the bottom of the ring. After measuring the height, H_1, the cylinder is changed for another of different diameter, D_2, so as to bring the ball higher in the gauge. The standard wires used in the measurement of the effective diameters of screws are very useful for this purpose, as in a complete set they differ consecutively by only a few thousandths of an inch. In special cases it may be necessary to grind and lap up cylinders, but that is a fairly easy matter.

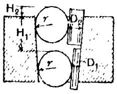


Fig. 35.

§ (11) Taper Plate Gauges.—These are measured by the use of roller gauges and block gauges in practically the same manner as described for taper plug and ring gauges. Instead of standing the gauges on the surface plate, however, they are placed flat downwards; also in the case of the internal tapers, roller gauges are used instead of balls.

If a number of taper plates having the same angle have to be measured, the method shown in *Fig.* 36 is sometimes adopted. Two templates, T, T, fixed together with dowel pins are ground up on all four edges so as to have the correct angle of taper. This angle can be tested by taking measurements over rollers when the square edges of the templates are placed in contact as shown at B. Having obtained the correct angle, the templates are then measured when arranged as at C, thus obtaining the dimension *m*. The two pieces are then placed one on each side of the gauge to be tested as shown at A, and measurements M made at the top and bottom. If the outer edges are found to be parallel the taper of the gauge agrees with that of the template, also the distance across the gauge in the position half-way down is given by (M − *m*), and this can be compared with the corresponding drawing dimension.

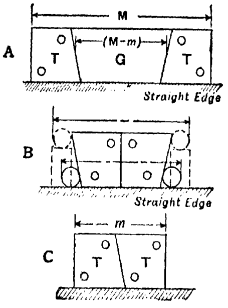


Fig. 36.

Such templates can also be used for checking internal taper plate gauges by measuring the distance between the parallel edges with slip gauges.

§ (12) Profile Gauges.—The smaller sizes of profile gauges such as radius templates and driving-band gauges are most readily checked by one of the following optical methods:

(i.) Optical projection of an image of the profile on a screen to a definite magnification and comparison of this image with an enlarged drawing. A description of the apparatus used for this purpose will be found in § (68).

(ii.) The gauge is mounted on a table whose position is controlled by micrometer screws, and different parts of its profile are viewed in turn through a fixed microscope fitted with cross wires, the movements of the gauge being measured on the micrometers. Descriptions of machines for carrying out such measurements will be found in §§ (34) and (35).

(iii.) *By the Use of a Check Gauge.*—Another method of testing profile gauges is to have a check gauge or counterpart of the gauge, and to try the fit of the two pieces when placed together on a sheet of glass in front of a strong light. Differences between the two edges in contact can be readily detected, especially if the gauges are made of fairly thin plate, but it is not easy to state the amount of these differences. This method is mostly used in the workshop during the manufacture of the gauges, and it is the aim of the mechanic to adjust the profile of the gauge until it is a "light-tight" fit to the check. The check itself must be accurate and should first be verified by optical projection and, if necessary, adjusted until its profile matches the enlarged drawing exactly.

Larger form gauges which very often have symmetrical curved edges can be dealt with by the use of slip gauges and roller gauges. A form of gauge frequently met with on shell work is shown at A in *Fig.* 37, whilst the corresponding check gauge is shown at B in the same figure. If such a check gauge is available the gauge can be readily tested by trying the fit of the two together. In any case it is necessary to verify the accuracy of the male check piece, and in addition to trying the fit of the check it is often desirable to ascertain the actual amount of the errors in the gauge itself. A method of measuring the two pieces is indicated in the figure, and it will be noted that the principle is practically the same as that used in the measurement of taper gauges (see §§ (9) and (10)). In the case of the female gauge, the distances between the rollers, when placed on various equal piles of blocks, are measured with block gauges: for the check piece, the distances over the rollers are measured with a micrometer.

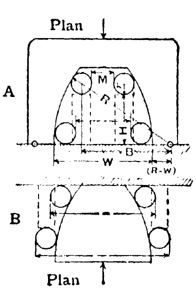


FIG. 37.

The calculation of the theoretical distances from the drawing dimensions of the gauge is simple.

In the case of the female gauge we have

$$B = \sqrt{(R - r)^2 - (H + r)^2},$$

and M, the theoretical distance between the rollers,

$$= 2B - 2R + W - 2r.$$

If both gauges and check are available their symmetry can be checked by trying them together in both positions. The symmetry can also be tested of each piece independently by taking diagonal measurements between or across a pair of rollers, one resting against the straight edge and the opposite one standing on the pile of gauges.

§ (13) POSITION GAUGES.—This type of gauge is most easily tested by means of check gauges. The check gauge is an accurate replica of the piece of work to be tested, but is made in such a form as to be capable of being measured with as little difficulty as possible. The check for the gauge shown in *Fig.* 13 would be of the form shown in *Fig.* 38. The verification of such checks for position gauges usually involves a considerable number of measurements. The various steps in the process for this particular check would be as follows:

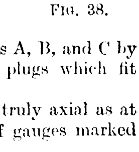


FIG. 38.

(*a*) Measure taper portion of body, using method shown in *Fig.* 31.

Measure parallel part with micrometer. Test concentricity of the two parts.

(*b*) Check diameters of holes A, B, and C by measuring the diameters of plugs which fit them.

(*c*) Test whether hole A is truly axial as at (*a*) in *Fig.* 39. The block of gauges marked J.G. should be equal to the differences between the radii of the parallel part of the body and of the plug in hole A.

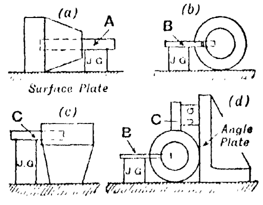


FIG. 39.

(*d*) Test whether axes of holes B and C pass through axis of body as at (*b*) in same figure.

(*e*) Test parallelism of holes B and C with base and their heights as at (*c*).

(*f*) Test right angle between pins B and C in plan as at (*d*). The above case is fairly simple, as the only angle which has to be measured between the axes of the holes happens to be a right angle. A rather more difficult gauge is shown at (*a*) in *Fig.* 40, which also shows the various dimensions to be tested. The method of testing the offset of the pin and the angle of the slot in relation to it is shown at (*b*) in the same figure. The angle of the pin with reference to the flat faces of the gauge is measured by the method shown at (*c*).

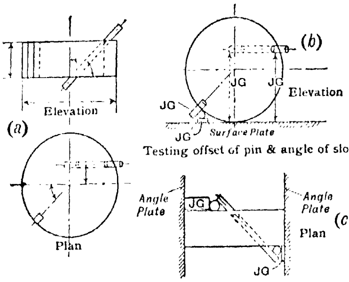


FIG. 40.

B. STANDARD GAUGES

Owing to the accuracy required on this class of gauge it is necessary to use some form of measuring machine, different types of which are described in Part V. § (71), etc. These machines enable the size of the gauge to be determined either directly by reference to a scale, whose calibration is known in terms of the primary standard yard or metre scales, or indirectly by comparison with a reference gauge whose size has been previously determined against the same primary standards. The determination of the difference in size between a gauge and a reference gauge of approximately the same dimension is fairly straightforward. On the other hand, the standardisation of a gauge with direct reference to a line standard is a more difficult matter, and greater precautions in the methods and care in the design of the apparatus have to be taken. The principles of the two systems of measurement are given below without reference to the details of the measuring machines concerned.

§ (14) COMPARISON OF GAUGES. (i.) *Measuring Machine.*—Before dealing with this subject it will be necessary to explain briefly the principles of the measuring machines used. A diagrammatic sketch of the form most frequently used is shown in *Fig.* 41. It consists of a bed A, the upper surface of which is finished and which is provided with a headstock B and a tailstock C. These fittings can be moved to any position of the bed and are provided with fixing clamps. The headstock B carries some form of micrometer screw fitted with a graduated drum, by means of which it is possible to take readings to $\frac{1}{100,000}$th part of an inch. This screw provides a means of measuring movements of one of the contact faces F_1. The other face, F_2, forms part of the tailstock C. These faces are usually flat, parallel between themselves, and perpendicular to the axis of the bed.

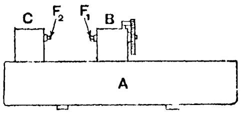


FIG. 41.

In measuring with an ordinary micrometer the screw is adjusted until a certain pressure is set up between the contact faces and the piece being measured, or until the piece feels a nice fit between the faces. Both these effects, however, depend upon the "feel" or "touch" of the particular user of the instrument, with the result that different observers measuring the same piece on the same micrometer often obtain slightly different sizes. (These differences, which are due to the personal element, can be eliminated to a large extent by using the micrometer as a comparator, each observer taking an initial setting on a reference gauge.) When using a measuring machine, however, where the accuracy attainable is much greater, it becomes absolutely necessary to eliminate the personal factor entirely by arranging that the settings shall be made in a mechanical fashion and independent of any consideration of the observer. For this purpose, measuring machines are provided with indicators in some form or other which operate by mechanical, electrical, or optical means. The types in common use are referred to in the detailed description of various makes of machines given in Part V. § (71), etc.

Returning to the method of comparing two gauges on a machine of the type shown in *Fig.* 41, the first operation is to arrange the headstock B so that the distance between the contact faces is approximately equal to the length of the gauges. Having clamped down both head- and tail-stocks, the reference gauge is inserted between the contact faces and the micrometer screw rotated continuously in one direction, until the indicator of the machine registers at its working position; a reading of the micrometer is then taken. The unknown gauge is then substituted for the reference gauge and a similar setting and reading is made. The reading on the standard gauge is now repeated as a check on the constancy of the different parts of the machine. If the settings have been made correctly and if the machine is in good order, the difference between the readings on the two gauges gives the true difference that exists between their sizes.

(ii.) *Conditions of Accuracy.*—Several important factors have to be taken into consideration in order to obtain accurate results. The first is in connection with temperature. Now the normal temperature for standardisation in this country is 62° F. (16·67° C.), and the true size of the reference gauge would be known at this temperature. The size of the unknown gauge is also ascertained at this temperature by making use of the measured difference between the two gauges. In the case of two steel gauges of closely the same dimension, the difference between their sizes will be practically independent of the actual temperature at which the comparison is made, provided the two gauges at the time the measurements are made have a common temperature which does not differ from the normal temperature by more than a few degrees. Variation in the coefficient of expansion will affect the comparison somewhat, especially when dealing with long gauges, and for this reason the temperature of the room in which this class of work is carried on should be kept as nearly normal as possible.

Generally speaking, so long as the temperature of the room remains inside the range 15° C. to 18° C., and does not fluctuate rapidly, no serious trouble will arise due to variation in coefficient of expansion when comparing steel gauges up to about 24 in. in length.

In any case, it is of the utmost importance to ensure that the two gauges shall have the *same* temperature at the moment they are measured. They must be cleaned and left to stand, preferably in contact with the bed of the machine, for at least an hour before any attempt is made to measure them. When inserting the gauges between the faces of the machine they must not be touched with the hands but should be held in a suitable clip having insulated handles. In addition, measurements should be made as rapidly as possible, so as to minimise the thermal effect due to the presence of the observer.

When making a measurement on a fairly long, flat-ended gauge it is not an easy matter to set it correctly between the contact faces so as to avoid any "cross-cornered" effect which gives rise to a large measurement. To avoid this difficulty, such gauges are ground along their length on the cylindrical surface and the faces are finally lapped accurately square to the axis of the gauge. The gauges can then be allowed to rest during the measurement on a pair of suitable vee supports placed on the bed of the machine, care being taken to ensure parallelism between the axis of the gauge and the bed of the machine. Supports are not used for flat-ended gauges below about 6 in. in length, as it is a fairly easy matter to set these shorter gauges between the faces.

To obtain reliable results in the measurements it is essential that the faces of the gauge and of the machine should be free from any traces of dirt or grease. The faces should be cleaned and polished with clean chamois leather and, finally, should be swabbed with a tuft of cotton-wool sprinkled with petrol or benzine. By watching the interference colours on the surfaces as the liquid evaporates it is easily seen whether the last trace of grease has been removed.

Whenever possible, the reference gauge used in a comparison should be of the same type as the gauge to be measured; in other words, flat-ended gauges should be compared with flat-ended reference gauges, plug gauges with plug reference gauges, and so on. By this method the effects due to imperfection in the contact faces of the machine are minimised. In certain special cases, however, it is necessary to compare flat-ended gauges with plug gauges, spherical gauges or balls. Consider the case of comparing a flat-ended gauge with a cylindrical gauge on a machine which has defective contact faces. The most frequent error found in the faces is lack of parallelism, and the effect of this error will be seen from *Fig.* 42. Contact is necessarily made on the flat parallel faces of the end gauge A by the extreme point on each of the measuring faces. On placing the plug gauge B between the faces it might come in contact with the same points if its axis happened to be in a certain direction, but it is more probable that it would miss these highest points and make contact towards the centres of the faces. An error amounting to half the error of parallelism of the faces could thus be introduced into the results. In comparing a ball with a flat-ended gauge the conditions are worse, even apart from any consideration of elastic deformation of the ball due to pressure between the faces, for which a correction is ordinarily applied (§ (5) (v.)).

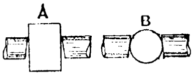


FIG. 42.

Before carrying out any measurements where the highest accuracy is required it is of great importance, therefore, to test the contact faces of the measuring machine for flatness, parallelism, and squareness. The method of carrying out this test is described in § (79).

When making a comparison between two gauges which do not differ by more than a few ten-thousandths of an inch, it is possible on some measuring machines to measure the difference accurately on the indicator when the latter has a certain range of action and is provided with a divided scale which has been calibrated in terms of the divisions on the micrometer drum. If such a procedure is adopted, it is convenient to fit the micrometer with some form of stop so that it can be fed up automatically and definitely to the same position on the two gauges in turn. The difference which exists between the latter is then registered on the scale of the indicator.

In order to test slip gauges of the Johansson type, where the accuracy of the comparison with the reference gauges is required to an order of a millionth of an inch, special comparators have been designed and are used at the National Physical Laboratory. These machines have a range of only 0·0002 or 0·0003 inch. They are devoid of micrometer screws, the difference between the gauges being registered by optical indicators. Descriptions of these comparators are given in §§ (82) and (83).

§ (15) CALIBRATION OF END GAUGES IN SETS.—It is usual to make end gauges in sets so that the lengths of the individual gauges follow a certain sequence. By this means it is possible to make use of the gauges both singly and also when built or wrung together to form a composite gauge. The method thus effects an economy in the number of individual gauges required to cover a certain range of sizes. The Johansson series of block gauges described in § (5) is an example of such a set of gauges. Another familiar series of gauges consists of the following sizes:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 18, 24, 30, and 36 inches.

The measuring faces of such gauges should be either accurately flat, as in the Johansson blocks, so that two or more gauges can be wrung together, or else they should be of true spherical form to allow the gauges to be built together when placed in line on suitable supports. Under these conditions it would be possible to obtain from such a set a gauge of any inch size up to an order of 60 in. or more.

The provision of end gauges in sets is also of considerable service in their initial standardisation. It is possible by suitable means to intercompare the gauges so as to obtain the length of each in terms of that of the longest gauge of the set. The absolute length of the latter can be determined by one of the methods described in § (17), and from this the lengths of the individual gauges are readily obtained.

Taking the series of gauges up to 36 in., referred to above, the first step in the intercomparison would be to determine the 6, 12, 18, 24, and 30 in. gauges in terms of the 36-in. gauge. This would involve the following comparisons:

36 + 24 with 30 + 30,[1]
36 + 18 with 30 + 24,
36 + 12 with 30 + 18 and 24 + 24,[1]
36 + 6 with 30 + 12 and 24 + 18,
36 with 30 + 6 and 24 + 12 and 18 + 18,[1]
30 with 24 + 6 and 18 + 12,
24 with 18 + 6 and 12 + 12,[1]
18 with 12 + 6,
12 with 6 + 6.[1]

The results of these comparisons can be put in the form of fifteen observational equations, such as

$$36 + 24 = 30 + 30 + \alpha_1,$$
$$36 + 18 = 30 + 24 + \alpha_2, \text{ and so on,}$$

where α_1, α_2 are the measured differences.

Now, if

$$30 = \tfrac{5}{6} \times 36 + \alpha_1,$$
$$24 = \tfrac{2}{3} \times 36 + \alpha_2,$$
$$18 = \tfrac{1}{2} \times 36 + \alpha_3,$$
$$12 = \tfrac{1}{3} \times 36 + \alpha_4,$$
$$6 = \tfrac{1}{6} \times 36 + \alpha_5,$$

it is possible to determine the values of α_1, α_2, α_3, α_1, and α_5 by solving the observational equations by the method of least squares. These values give the calibration errors of the five gauges in terms of the 36-in. gauge.

The series of gauges from 1 up to 12 in. can be calibrated in a similar manner in terms of the 12-in. gauge, and, since the value of the latter is known with reference to the 36 in. gauge, it is possible to state the calibration errors of the other eleven to the same basis.

Some interesting work has recently been done at the Bureau International, Sèvres, on the measurements of block gauges. An account of this will be found in vol. xvii. of the *Travaux et Mémoires* of that institution.

§ (16) MEASUREMENTS OF SPHERICAL-ENDED GAUGES AND BALLS UNDER COMPRESSION.—When a steel ball or an end gauge having spherical ends is placed between the parallel flat faces of a measuring machine, and the latter are arranged so as to just touch the spherical surfaces, the contact will take place at two theoretical points. If a compressive force is now applied between the faces

---

[1] For the purposes of these particular comparisons it is helpful to make use of gauges from a second set. The relation between the nominally equal gauges of the two sets should be obtained by preliminary comparison. If a second set is not available, the duplicate gauges may be formed from two others of the same set, *e.g.* the second 30 gauge can be obtained by building together the 24 and 6 or the 18 and 12. The relation of either of these combinations to the 30 is obtained in a later comparison.

—a condition which arises in practically every measuring machine—then the spherical surfaces will become slightly distorted; the points of contact become small circular areas which are capable of resisting the compressive force. This distortion makes an appreciable difference to the measured length over the spherical surfaces, the difference being usually considerably greater than that produced by compression along the length of the gauge. When two or more spherical-ended gauges are placed end to end, the compression takes place, not only at the extreme surfaces, but also at the surfaces of contact between the individual gauges. The actual amount of elastic deformation depends upon the force, the diameters of the spherical surfaces, and the elastic constants of the material.

The elastic diminution in length of a number of spherical-ended steel bars placed end to end and enclosed between a pair of flat parallel faces under an axial pressure of P lbs. weight is given in inches by

$$0{\cdot}000016 \times \mathrm{P}^{\frac{2}{3}}\left[\left(\frac{1}{d_1}\right)^{\frac{1}{3}}+\left(\frac{1}{d_1}+\frac{1}{d_2}\right)^{\frac{1}{3}}\right.$$

$$\left.+\left(\frac{1}{d_2}+\frac{1}{d_3}\right)^{\frac{1}{3}}+\ldots\left(\frac{1}{d_{n-1}}+\frac{1}{d_n}\right)^{\frac{1}{3}}+\left(\frac{1}{d_n}\right)^{\frac{1}{3}}\right],$$

where d_1, d_2, . . . d_n denote the diameters of the spherical ends of the bars in inches.[1]

In the case of a single steel ball of diameter d in. the compression becomes

$$0{\cdot}0000321 \times \mathrm{P}^{\frac{2}{3}} \times \left(\frac{1}{d}\right)^{\frac{1}{3}} \text{ inches.}$$

§ (17) STANDARDISATION OF END GAUGES BY COMPARISON WITH SCALES.—The primary standards of length in this country, the yard and the metre, are defined as "line" measures, and all gauge measurements which are in the nature of "end" measures must be based on these standards. It is not necessary that *every* gauge should be measured with reference to a standard scale, in fact it would be inconvenient to do so; all that need be done is to standardise once and for all in a very accurate manner a single end gauge, which for convenience may be a yard or a metre in length, by comparison with the primary line standard. From this gauge it is possible by methods of subdivision to obtain the individual sizes of a complete series of end gauges,[2] which then serve as reference gauges, and from these, other gauges can be determined by direct comparison.

The standardisation of a yard end gauge from the line standard constitutes one of the most interesting problems in the science of metrology. Some of the methods available are described below.

Method I.—For this determination a gauge A (*Fig.* 43) is used. This gauge is $35\frac{1}{2}$ in. in length and has flat parallel end faces which have a lapped finish and to which are wrung two small rectangular blocks B and C, each half an inch in length. The end faces of these blocks are finished flat and parallel and have wringing surfaces. The upper surface of each is polished and has a fine line scribed on it approximately half-way along the length and in a direction parallel to the end faces. The depth of the short blocks is only half that of the gauge, so that when wrung together with the bottom surfaces of the three in line the upper surfaces of the blocks are on the neutral plane of the gauge.

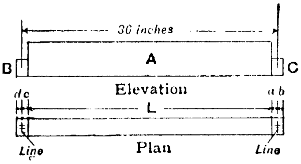


FIG. 43.

The distance between the lines will be approximately 36 in. and it is compared in the usual manner with a standard yard scale, using a line standard comparator of the type referred to under "Line Standards." The inner tank of the apparatus is used simply as an air-bath in this case, as it is not possible to immerse the steel gauges in water. A number of comparisons are made wringing the two blocks on to the ends of the gauge in all possible manners in turn. As arranged in *Fig.* 43, the distance between the lines is (L + a + c). Suppose the measurement of this distance as determined from the comparison with the line standard is M_1, then we have

$$(\mathrm{L}+a+c)=\mathrm{M}_1 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By reversing the blocks in turn we can obtain three other equations,

$$(\mathrm{L}+b+c)=\mathrm{M}_2, \quad . \quad . \quad . \quad (2)$$

$$(\mathrm{L}+b+d)=\mathrm{M}_3, \quad . \quad . \quad . \quad (3)$$

$$(\mathrm{L}+a+d)=\mathrm{M}_4. \quad . \quad . \quad . \quad (4)$$

From (1) and (3) we have

$$\mathrm{L}+\tfrac{1}{2}(a+b+c+d)=\tfrac{1}{2}(\mathrm{M}_1+\mathrm{M}_3),$$

and from (2) and (4) we have

$$\mathrm{L}+\tfrac{1}{2}(a+b+c+d)=\tfrac{1}{2}(\mathrm{M}_2+\mathrm{M}_4).$$

This double determination of the length of the gauge plus the mean length of the two blocks gives incidentally a check on the accuracy of the measurements. Taking a mean of the four measurements we have

$$\mathrm{L}+\tfrac{1}{2}(a+b+c+d)=\tfrac{1}{4}(\mathrm{M}_1+\mathrm{M}_2+\mathrm{M}_3+\mathrm{M}_4).$$

---

[1] *Journal Inst. Mech. Engrs.*, 1920, p. 915; Hertz, *Miscellaneous Papers* (Macmillan), 1896.

[2] See § (15).

The measurements are repeated after turning the gauge A upside down and the results incorporated with those obtained in the original position.

It is not necessary to determine the lengths of the short blocks separately, since for all subsequent purposes for which the gauge will be used it would be possible to take two measurements on it, with each block wrung on in turn. The mean of these two measurements is clearly the length which has been standardised. For the sake of simplicity it could be arranged to make the two blocks equal in length to a millionth of an inch, in which case the standardised length becomes equal to the length of the gauge with either of the blocks wrung on.

Method II.—Gauges of special design, as shown in *Fig.* 44, are used for this determination. They consist of cylindrical bars about 1 in. diameter, having the ends carefully finished to spherical surfaces of a large radius of curvature. A hole about $\frac{1}{8}$ in. diameter is drilled diametrically at the centre of the bar and a taper plug, whose length is equal to the radius of the rod, is fitted into the hole from one end as shown in the diagram. The upper surface of this plug is polished and has a line scribed across it, the direction of the line being at right angles to the axis of the bar. Three such bars are required, each 36 in. in length.

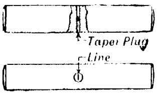


FIG. 44.

Two of the bars, A and B, are placed in an accurately made vee trough so that their inner ends are in contact (*Fig.* 45). The trough containing the gauges is placed on one of the girders of a line standard comparator, of the same type as referred to in *Method I.* The length L between the lines on the plugs is then compared with a standard yard scale, which is placed on the other girder, thus obtaining a measurement M_1. The bars A and B are then reversed in turn, and a measurement is made in each position. Having made the three possible measurements on this pair of bars, the third bar C is exchanged for B and another set of measurements taken between the bars A and C. Finally, a set is made between B and C.

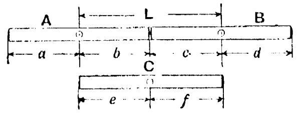


FIG. 45.

Referring to *Fig.* 45, we shall have nine measurements as follows:

Bars A and B.	Bars A and C.	Bars B and C.
$b+c=M_1$	$b+e=M_4$	$f+c=M_7$
$b+d=M_2$	$b+f=M_5$	$f+d=M_8$
$a+d=M_3$	$a+f=M_6$	$e+d=M_9$

where M_1, M_2, etc., represent the actual lengths obtained in the comparisons with the standard yard scale.

The six unknown quantities a, b, c, d, e, and f can be obtained from these nine equations by using the method of least squares, and consequently the total length of each of the three gauges can be readily obtained.

As a check on the results, the lengths of the bars can be intercompared in an end-measuring machine and their differences compared with those obtained from the results of the above method.

Method III.—In the two methods just described it is necessary either to provide the gauge with special attachments or else to have more than one gauge of the same length. It is possible, however, by means of a special type of measuring machine to make a direct comparison between an end gauge and a standard scale. The general arrangement of the machine is shown in *Fig.* 46. One measuring face is attached to the micrometer headstock and the other to a sliding carriage which supports a standard scale, the graduated surface of which is directly in line with the measuring faces. A microscope provided with a pair of parallel wires in the eyepiece is held above the scale in a rigid bracket fixed to the bed of the machine.

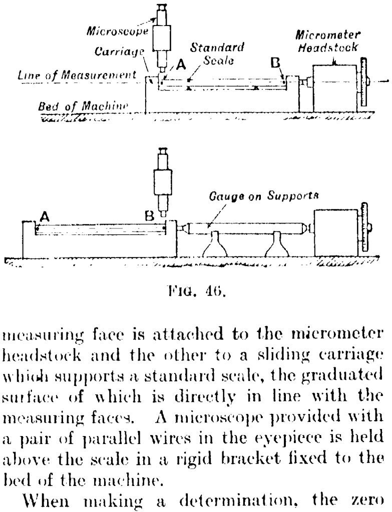


FIG. 46.

When making a determination, the zero line A of the scale is set under the cross wires of the microscope and the carriage is then clamped to the bed. The two measuring faces are now brought into contact by means of the micrometer screw and a reading is taken on the headstock. The carriage is then moved along the bed until the line B on the scale, which corresponds with the length of the gauge, arrives exactly under the cross wires and the carriage is again clamped.

The gauge is now inserted between the faces and a further reading of the micrometer taken.

Assuming the bed of the machine is straight, the distance traversed by the left measuring face will be equal to the length between the two lines A and B on the scale. If this length is denoted by L, the length of the gauge as G, and the two micrometer readings as R_1, R_2 respectively, we have

$$G = L + (R_2 - R_1).$$

The lengths of the scale and the gauge refer to their particular temperatures at the time of the observations, and, in order to deduce the length of the gauge at 62° F., it is necessary to know their coefficients of expansion and to make careful measurements of the temperature of both the gauge and the scale whilst the observations are being taken. It is usual to employ an Invar scale for this determination since, owing to its very low coefficient of expansion, any small uncertainty in the measurement of its temperature will not introduce an appreciable error into the results. With a steel gauge, however, an uncertainty in the temperature as small as 0·1° C. gives rise to an error of approximately one part in a million on the length.

Imperfections in the contact faces of the machine may lead to considerable inaccuracies in the measurements. These errors can be reduced by having spherical-shaped ends to the gauge to be measured, although this would not eliminate the effect due to lack of parallelism of the faces when they are brought together for the zero setting. To avoid this possible inaccuracy the following procedure may be adopted.

Suppose we wish to determine the length of a 36-in. gauge. Take, in addition, two 18-in. gauges and compare on a measuring machine the sum of their lengths with that of the longer gauge, by putting them end to end. Let the small difference be e, as defined by the equation

$$36 = 18(a) + 18(b) + e. \quad . \quad . \quad (1)$$

Now measure the absolute difference between the 36-in. gauge and each of the 18-in. gauges in turn against a standard scale by the method described above, except that it will not be necessary to bring the measuring faces into contact. We shall then have

$$36 = 18(a) + M_1, \quad . \quad . \quad . \quad (2)$$

and $$36 = 18(b) + M_2, \quad . \quad . \quad . \quad (3)$$

where M_1 and M_2 are the absolute lengths measured against the scale.

Subtracting (1) from the sum of (2) and (3) we obtain the equation

$$36 = M_1 + M_2 - e,$$

which gives the absolute length of the 36-in. gauge in terms of known measurements.

The accuracy of the method of comparing the length of an end gauge with that of a scale as illustrated in *Fig.* 46, is governed to a large extent by the accuracy of the movement of the carriage to which the measuring face is fixed. This movement depends upon the straightness of the bed of the machine, which will always be imperfect to some degree. Whatever the errors in the bed may be, the translation of the carriage from one position to another can always be resolved into a linear and a turning or twisted motion. The greater the errors in the bed the larger the turning motion of the carriage will become. Referring to the design of machine illustrated in *Fig.* 46, the effect of this motion on the accuracy of the measurements is illustrated in *Fig.* 47, which shows the scale and the measuring face in two positions corresponding to settings of the microscope on two lines a and b, the actual distance between which is l. The effective translation of the measuring face is represented by the distance d. If the tilt of the scale between the two positions is θ and if the distance of the measuring face from line a is L, then we have

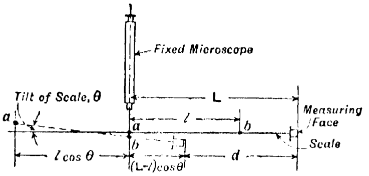


FIG. 47.

$$d = l \cos\theta + L(1 - \cos\theta).$$

The difference between the actual movement of the measuring face and the length as given by the scale, *i.e.* the error introduced into the measurement, is

$$d - l = (L - l)(1 - \cos\theta)$$
$$= \tfrac{1}{2}(L - l)\theta^2$$

approximately, since θ is small. Assuming that the bed of the machine is reasonably straight, the error in the measurement in this case is one of the second order and can be ignored in this particular arrangement, where the scale is directly in line with the gauge being measured.

Consider now the case when the scale is *not* in line with the axis of measurement as given by the line joining the contact faces of the machine. We could still utilise the scheme of fixing the microscope and move the scale, or, what is sometimes done in actual practice, fix the scale to the side of the bed and attach the microscope to either the head or tail stock, according as to which is made to move. The two arrangements are alike from a theoretical point of view. The latter scheme is illustrated in *Fig.* 48, and shows the effect of tilting the carriage holding the microscope through an angle θ.

The difference between the effective movement d of the measuring face and the length l, as given by the distance between the two lines

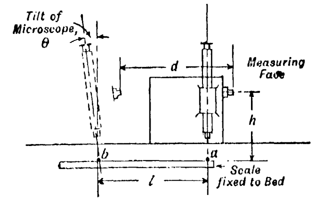


FIG. 48.

a and b on the scale, is equal to $h \times \theta$ if θ is small, where h is the vertical distance between the measuring face and the scale.

A twist of the carriage in the horizontal plane due to lack of straightness of the sides of the bed will produce a similar effect.

This error is of the first order of magnitude, and with this design of machine the surface of the bed needs to be exceptionally straight to limit the error to even 0·0001 inch.

When designing *any* form of measuring machine, careful consideration should be given to the alignment of the axis of the gauge or whatever is to be measured with the axis on which the actual measurements are taken.

III. MEASUREMENTS OF SCREW GAUGES

§ (18) ELEMENTS OF A SCREW.—Screw threads of Whitworth form have seven elements, errors on any one of which may be sufficient to cause a gauge to reject work which ought to pass, or *vice versa*. These elements are :

Full (or major) diameter.
Core (or minor) diameter.
Effective (or pitch) diameter.
Pitch.
Angle.
Radius at crest.
Radius at root.

The definitions of these elements will be found under the theoretical considerations of screw threads.[1]

Screw gauges have to be measured and examined to ascertain whether their dimensions lie between the specified limits. Considering the case of a plug screw gauge, this can be tested on the higher limit by screwing into a check ring gauge whose dimensions conform to those of the high limit. If the gauge refuses to pass through this ring it is clearly too large at some particular part of its profile, and it becomes necessary to measure it in detail to locate this error. Even if the gauge passes the check ring we have still to ascertain whether the gauge is everywhere above the lower limit, and this again involves a detailed examination of its various elements in turn.

The provision of ring check gauges is an expensive matter, and moreover, as it is possible from the individual measurements of the elements—and these measurements, as will be seen later, are necessary to check the minimum dimension—to ascertain whether the gauge is below the upper limit, the use of check rings is practically limited to certain particular sizes of gauges of which large numbers have to be tested.

Ring gauges present rather a different case owing to the fact that it is not such an easy matter to make accurate measurements of the various elements as in the case of the plug gauges. They are usually tested between the limits by a series of plug checks, as will be described later.

§ (19) MECHANICAL MEASUREMENTS OF PLUG SCREWS. (i.) *Full Diameter.*—The measurement of the full diameter over the crests of the threads is usually made with an ordinary micrometer held in the hand, or by placing the screw between the jaws of a measuring machine, which, to avoid compressing the crests of the threads, should have a working pressure of something less than 2 lbs. weight. When making use of an ordinary micrometer, a comparative reading should also be taken on a standard plain plug of closely the same size as the screw. This method eliminates to a large extent the effect of both zero and progressive errors in the screw of the micrometer. The Hoffmann roller gauges described in § (5) (v.) serve admirably as setting plugs for measurements not exceeding $1\frac{1}{4}$ in.

(ii.) *Effective Diameter.*—The effective diameter of a plug screw can be measured mechanically either with a micrometer provided with special contact faces, or by the use of small cylinders or wires of suitable size placed tangentially between the threads. The latter method is usually considered to be the more accurate and is generally adopted in gauge work.

(iii.) *Screw Thread Micrometers.*—The contact faces of micrometers for screw thread measurements take the form shown in *Fig.* 49. The end of the micrometer spindle A is ground conical and is suitably rounded off at the point; the anvil B has an internal vee which fits over the thread of the screw to be measured. The angles of the cone and the vee are made equal to the theoretical angle of the screw thread. The anvil can rotate about its axis so as to allow the vee to accommodate itself to the rake angle of the thread. When the spindle of the micrometer is screwed up to bring its conical point into the vee the reading is then zero; consequently, when the contact points are set so that a screw will just pass between them, as in *Fig.* 49, the reading gives the effective diameter of the screw direct.

---

[1] See "Metrology," Part VII. §§ (23) *et seq.*

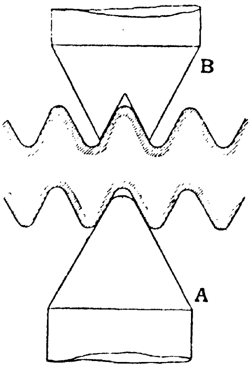


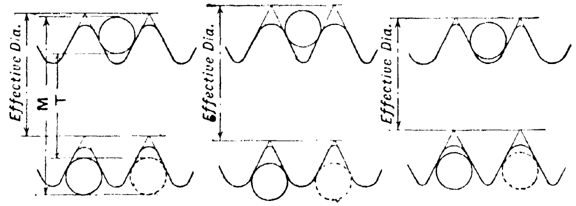
Fig. 49.

Taking a micrometer with a particular pair of contact faces, it is generally possible to use it over a limited range of consecutive pitches of threads, at the same time maintaining contact on the flanks of the threads only. The limit is reached in one direction when the points of the anvil foul the roots, and in the other when the conical point barely enters the space between the threads. For this reason it is necessary to have a number of micrometers, each with different size contact faces, in order to cover a whole range of pitches.

(iv.) *The Wire Method.*—The general principle underlying the second method of measuring the effective diameter is shown in *Fig.* 50, which illustrates three different forms of screw threads having the same full and core diameters, pitch, and angle. The first is a normal, symmetrical thread; the second has the threads thicker than the spaces and consequently has a large effective diameter, whilst the third has thin threads and a small effective diameter. If small cylinders of suitable size are now allowed to rest tangentially between the threads on opposite sides of the screw as shown, it is clear that the diametral measurement M, over the cylinders, will be different in each case. From the definition of effective diameter and the geometry of the figures, it follows that the difference between the measurement over the cylinders and the effective diameter is the same in each case provided the same size cylinders are used throughout. The actual value of this difference, which we may call K, depends upon the diameter of the cylinders and the pitch and angle of the thread. Thus we have effective diameter $= M - K$.[1]

If the actual angle of a screw thread is known, and its particular value is used in calculating the constant K, it is possible to obtain a measurement of the true effective diameter of the screw by using a set of cylinders of *any* size, provided they make contact somewhere on the flanks of the thread. Measurements of the effective diameter of a screw obtained from sets of cylinders of different sizes should agree if the appropriate constant is applied for each set. It is the general practice, however, to select cylinders of such a size that they make contact at approximately half-way down the flanks of the particular pitch of thread. The measurement of the effective diameter from such cylinders is not influenced by errors in the thread angle, and consequently it is not necessary to know the actual angle of the thread.

It is essential that the measurement M (*Fig.* 50) should be made perpendicular to the axis of the screw; consequently, when using a micrometer held freely in the hand it is necessary to place two cylinders on one side of the thread in order to set the micrometer square. This system of measurement gives rise to the term "three-wire measurement of screws." Certain special machines have been designed at the National Physical Laboratory in which the micrometer is set and maintained perpendicular to the axis of the screw but is left free to move in the direction of its own axis. These machines, usually known as "floating micrometer machines," are described in § (23). With these machines, one cylinder only is



Normal Threads Thick Threads Thin Threads

Fig. 50

---

[1] Formulae for calculating the value of the constant K for particular cases are given in § (63).

required on each side of the thread, as shown in full line in *Fig.* 50.

It is essential that the small cylinders or wires (which can be made conveniently by lapping down ordinary sewing needles) should be truly cylindrical over the central portion which comes in contact with the thread. The diameter of this portion must be accurately measured before the corresponding constant can be calculated. It is interesting to note that the value of this constant (which has to be subtracted from the measurement over the cylinders to give the effective diameter) involves a term which is approximately three times the mean diameter of the cylinders. Any error in determining the latter thus becomes trebled in the final result obtained for the effective diameter. To avoid this accumulation of error, it is preferable to measure the distance T, *under* the cylinders (see *Fig.* 50). By this means the error in the measurement of the effective diameter due to this cause is reduced to one of practically the same magnitude as that involved in the determination of the mean size of the cylinders.

The method of obtaining the measurement T, under the cylinders, is shown in *Fig.* 51.

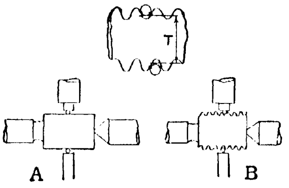


FIG. 51.

A plain plug gauge of approximately the same diameter as the screw is taken, and a micrometer measurement is first made over this plug with the small cylinders included, as shown at A. The cylinders are then transferred to the screw, as at B, and a second measurement is made. The dimension T, for the screw, is then readily deduced from the known size of the plain plug gauge and the difference between the two micrometer readings.

(v.) *Sources of Error.*—There are one or two points to be observed in connection with the measurement of effective diameters by standard wires and by the type of micrometer fitted with special anvils described above. The former method gives what may be called the net effective diameter, when the sizes of the wires used are such that they touch the flanks of the threads at the half depth. Any errors present in the angle affect the measurement only to the second order of accuracy; errors in pitch, either progressive or periodic, also have quite a minor effect on the measurement. If it is desired to ascertain the virtual effective diameter of the screw, *i.e.* the effective diameter of a perfect nut into which the plug will just fit, it is necessary to augment the net effective diameter by amounts corresponding to the errors present in the angle and pitch.

On the other hand, the readings given by a "thread" micrometer are influenced considerably by errors in angle, or periodic errors in pitch. The internal vee on the anvil and the conical-ended spindle may be roughly considered as forming portions of the screw thread of a split nut which are closed round the screw. It should be observed that in screw gauges as ordinarily made, the angular space between the threads has a constant form, since it is the form of the nose of the cutting tool. This space is unaffected by errors either of a progressive or periodic nature which may be present in the pitch. Such errors, however, have their influence upon the thread itself, apart from the space. It is clear that if the pitch between two spaces happens to be too long, then, since the spaces are of constant width, it follows that the intermediate thread must be thicker than the average. Such variation in the thickness of the thread is produced by periodic changes in the pitch.

Now, in the case of standard wire measurements of the effective diameter, the depth to which the wire sinks into the screw depends upon the width of the space between the threads as is shown in *Fig.* 50. Variations in thread thickness due to errors in pitch will not be noticed. When using a thread micrometer, however, where the measurement on one side of the screw is obtained from the contact of an internal vee piece which fits over the *thread*, it is clear that variations in the thread thickness will have their effect upon the measurements. As the other contact face is a truncated conical point which fits into the space between the threads. the radial position of this point will not vary when measurements are made at different threads. Without following the matter further in detail it will be appreciated that in cases where the pitch is variable the readings of a thread micrometer need to be carefully interpreted.

If no errors exist in the angle or pitch, then the two methods referred to should give exactly the same value for the effective diameter. It is essential, of course, that the contact points should have the correct angle and form, as such errors will produce false readings even on a good screw gauge.

(vi.) *Core Diameter.*—The core diameter, which is the diameter at the roots of the threads, is measured with a micrometer, using special vee-shaped pieces (*Fig.* 52) between the measuring faces and the screw. These pieces, which are of hardened steel, have the angle between the sloping sides somewhat less than the thread angle, and the junction of the faces is rounded off to such a radius that the piece can rest on the root of the thread. The back surface is finished accurately flat and parallel to the rounded edge. It is usual to make these vee pieces in about four sizes to accommodate all the usual pitches of threads.

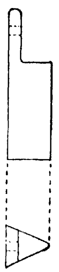


FIG. 52.

The method of making the measurement of core diameter is shown in *Fig.* 53. A micrometer measurement is first made over a plain plug gauge with the vee pieces inserted as shown at A. A second measurement is then taken with the same vee pieces placed in the thread as at B. The core diameter is then obtained by adding to the known diameter of the plug gauge the algebraical difference between the two readings. It should be noted that the thickness of the vee pieces need not be measured. If the measurements are made with a freely held micrometer, it is necessary to have two vee pieces for one of the contact faces to rest on in order to set the instrument square to the axis of the plug or screw. The floating micrometer type of machine described later requires only one vee piece for each measuring face of the micrometer.

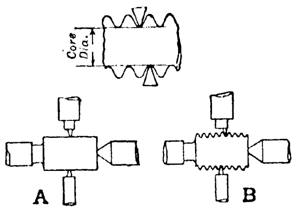


FIG. 53.

(vii.) *Standard Reference Bars for Screw Measurement.*—When making a large number of plug screw gauges of the same size it sometimes simplifies matters to make a reference bar for the effective and core diameters. Such a bar can conveniently take the form shown in *Fig.* 54. It consists of two hardened steel washers with bevelled edges ground and lapped to exactly half the thread angle and mounted on an arbor. If a thread-grinding machine is available the bar can be made solid and the vee groove ground in one end. The vee groove is measured with standard wires as if it were part of a screw thread and its "effective diameter" obtained.

It is useful to grind the groove until its diameter is equal to, say, the nominal effective diameter of the screws to be made, although this adjustment is not absolutely necessary. The cylindrical portion A is finished by grinding and lapping, and its diameter in turn may be made equal to the nominal core diameter of the screw gauge. After standardisation, the bar should be marked with the exact diameter A, and the "effective" diameter" of the groove, together with the pitch for which it is intended.

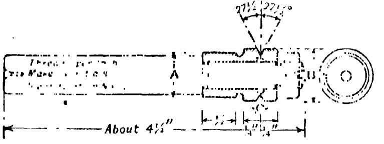


FIG. 54.

To measure the effective diameter of a screw all that is necessary is to mount the bar and the screw in the floating micrometer machine in turn and to obtain a micrometer reading over a pair of suitable size needles when placed in the groove or thread. The exact diameter of the wires need not be known, but it is quite important that they should be accurately cylindrical. The calculation of the effective diameter of the screw is very simple, *e.g.* if the micrometer reading over the wires when in the screw thread is 0·001 in. larger than when in the reference groove, the effective diameter of the screw is that amount larger than that of the bar. The same method applies to the core diameter measurement.

§ (20) TEST ON CONCENTRICITY OF DIAMETERS.—In making a plug screw gauge a common practice is to finish the flanks and roots of the thread simultaneously with one tool, grinding wheel or lap. The full diameter, however, is often finished as a last operation. The result is that although the effective and core diameters are usually accurately concentric, the full diameter sometimes is appreciably in error in this respect with regard to the other two diameters. In examining screw plugs it is necessary that this point should be tested.

The test of concentricity of the diameters is readily made on the floating micrometer type of machine. To test the concentricity of the full and effective diameters, a number of measurements are made over the screw with a standard wire placed between the thread and the micrometer face, the measuring face of the anvil being in direct contact with the full diameter of the screw. If the measurement is constant as the screw is revolved it proves that the concentricity is good. To avoid errors due to bad centring of the screw, a special anvil should be used which has its measuring face reduced in diameter to about 0·05 in., so that for all ordinary pitches it rests on only one crest of the thread.

The concentricity between the full and core diameters is tested in a similar manner, a single vee piece being used instead of the standard wire.

§ (21) Angle of Flanks of Thread.—The angles of the flanks of the thread are most readily measured by the use of microscope and optical projection machines as described in §§ (33), (68), and (69). It is possible, however, to obtain fairly close measurements of the *total* angle by taking measurements of the effective diameter with two or more pairs of standard wires which differ in diameter and which will consequently touch the flanks of the thread at different depths. The theory underlying this point is discussed in § (63). It should be noted that it is not possible by this method to obtain the errors in the angles of the individual flanks. The method of using more than one pair of cylinders gives a ready check not only on the value of the effective diameter, but also of the straightness of the flanks and the total angle of the thread.

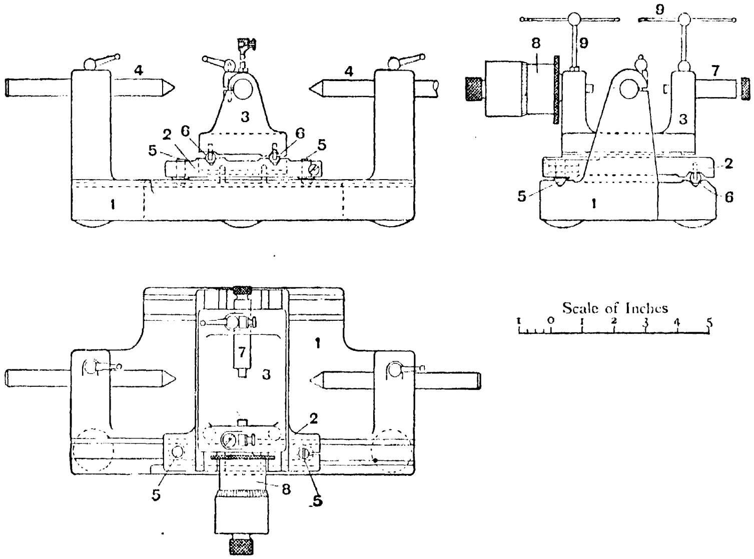


Fig. 55.

§ (22) Roundings at Crests and Roots, and General Form of Thread.—There is no satisfactory mechanical means of examining the general form of the thread of screw gauges. This very important part of the test, however, can be made very readily by optical methods either in the microscope form of apparatus as described in § (33), or on a projection apparatus as described in §§ (68) and (69). The latter method provides a means of making an accurate comparison of the actual form of the thread with that of the standard form.

§ (23) Measuring Machines for the Effective and Core Diameters of Plug Screws. (i.) *Floating Micrometer Measuring Machine.*—This form of machine for measuring the effective and core diameters of plug screw gauges was designed at the National Physical Laboratory early in 1916, and has been made in large numbers by Messrs. Taylor, Taylor & Hobson of Leicester.

A general arrangement of the machine, which will accommodate plug screw gauges up to 2 in. in diameter, is shown in *Fig.* 55. It consists mainly of three parts, the bed 1, the intermediate slide 2, and the micrometer carriage 3.

The bed carries a pair of centres, 4, for supporting the gauge, which is capable of being clamped in position. The intermediate carriage 2 can slide along the bed and is constrained to a straight line motion by virtue of the two longitudinal vee grooves in the bed. One of these grooves forms a guide for the pair of conical-ended studs 5, which are fixed at one side of the carriage; the other side is supported by its flat under-surface resting on a steel ball 6 placed in the second longitudinal groove in the bed.

The upper surface of the carriage 2 is

provided with a pair of parallel vee grooves at right angles to the line of centres. These grooves form a guide for the micrometer carriage 3, which rests on steel balls 6, the under surface of the upper carriage being provided with a suitable vee groove and flat surface. In this manner it is possible to move the micrometer carriage along the bed with a fairly free motion by sliding the lower carriage, and it can also be moved quite freely in a transverse direction by virtue of the rolling contact on the balls.

The two upright parts of the micrometer carriage 3 are bored in line to take an adjustable plunger 7 and a micrometer head 8 respectively. The measuring faces of the plunger and micrometer head are lapped flat and accurately square to the axes of the parts so that they lie accurately parallel in all positions. The plunger is a good sliding fit in its hole and, being provided with a knurled head, its position can be set to a nicety before it is clamped. The shank of the micrometer head is definitely clamped in position. The micrometer head consists of either Brown & Sharpe or Starrett standard make provided with a specially enlarged aluminium barrel and thimble, which was designed at the National Physical Laboratory. The thimble is graduated into 125 parts, each division representing 0·0002 in., and by this means it is possible to take readings to 0·0001 in. with certainty. Two suitable supports 9 are fixed to the micrometer carriage from which the standard wires, or vee pieces, are suspended.

It is essential that the common axis of the micrometer and plunger should be accurately square to the line of centres, and provision is made for making this adjustment. One of the conical-ended studs 5 in the intermediate carriage has its parallel shank eccentric to the cone. A screw-driver slot in the top allows the stud to be rotated, and so alters the direction of the micrometer axis over a small range with respect to the bed. The movable stud can be clamped finally by means of the tightening screw shown.

To test whether the adjustment is correct a well-centred plain plug gauge is mounted between the centres, and a standard wire of any convenient size is suspended on each side. With the plunger clamped in a suitable position a measurement is made over the two wires and the plug when placed as shown at A in *Fig.* 56. The measurement is repeated after changing the wires into the position shown at B in the same figure. If the axis of the micrometer and plunger is accurately square with the line of centres the two measurements will be the same. If a discrepancy occurs a further adjustment of the stud 5 is required.

It should be noted that this machine is not provided with any form of indicator for use when making settings. The micrometer is set by the method of "feel" or "touch," the thimble being rotated until it is felt that contact is made between both measuring faces and the gauge. This method gives quite a definite reading to an order of less than 0·0001 in., provided the micrometer head runs freely and, above all, smoothly. The variation of the "feel" between different observers is eliminated by using the machine as a comparator only, *i.e.* a preliminary measurement is made on a plain plug gauge of known diameter, as described in § (19). Incidentally, this method eliminates the effect of progressive errors in the run of the micrometer head, providing a standard plug is chosen whose size approximates to the diameter of the screw being measured.

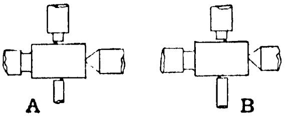


FIG. 56.

When taking measurements on a screw, the setting should be repeated in a number of places both round and along the screw so as to detect any lack of circularity or parallelism.

In the measurements of the effective and core diameters, as described in § (19), it is impossible to arrange the two standard wires or vee pieces to be diametrically opposite; they must necessarily be separated axially by at least half the pitch of the screw being measured. Nevertheless, the measurement made in the machine is a true diametral dimension since the axis of the micrometer is constrained to that direction; the micrometer carriage cannot tilt round so as to measure obliquely over the pair of wires or vees.

(ii.) *Tilting Micrometer Machine.*—Another type of machine which can be used for measuring the effective and core diameters of plug screw gauges is shown in *Fig.* 57. This machine was also designed at the National Physical Laboratory. The gauge is held between centres as before and the measurements are made with a standard type of micrometer 1 attached in a vertical position to a carriage 2, which is capable of tilting about an axis parallel to the line of centres. This carriage has two ball-ended set screws 3 and 4 resting in a vee groove in the bed 5, and which can be adjusted so as to bring the axis of the micrometer square to the line of centres. The carriage can be moved along the vee groove to suit the screw being measured.

The machine is used in the same manner as described in § (19), except that the wires or vee pieces are placed horizontally instead of

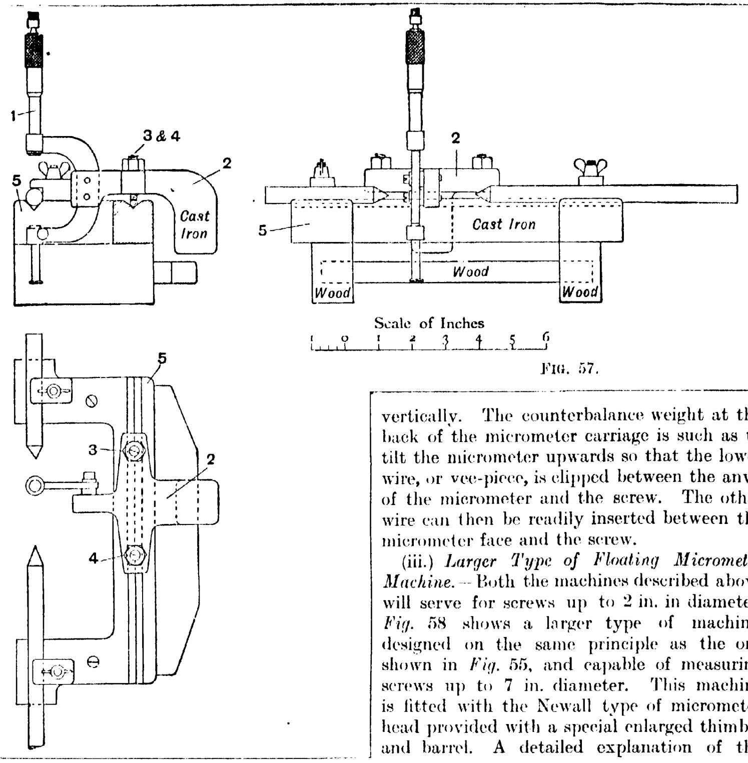


FIG. 57.

vertically. The counterbalance weight at the back of the micrometer carriage is such as to tilt the micrometer upwards so that the lower wire, or vee-piece, is clipped between the anvil of the micrometer and the screw. The other wire can then be readily inserted between the micrometer face and the screw.

(iii.) *Larger Type of Floating Micrometer Machine.*—Both the machines described above will serve for screws up to 2 in. in diameter. *Fig.* 58 shows a larger type of machine, designed on the same principle as the one shown in *Fig.* 55, and capable of measuring screws up to 7 in. diameter. This machine is fitted with the Newall type of micrometer head provided with a special enlarged thimble and barrel. A detailed explanation of the machine is not necessary as the functions of the various parts can be readily followed from the description of the smaller type of machine.

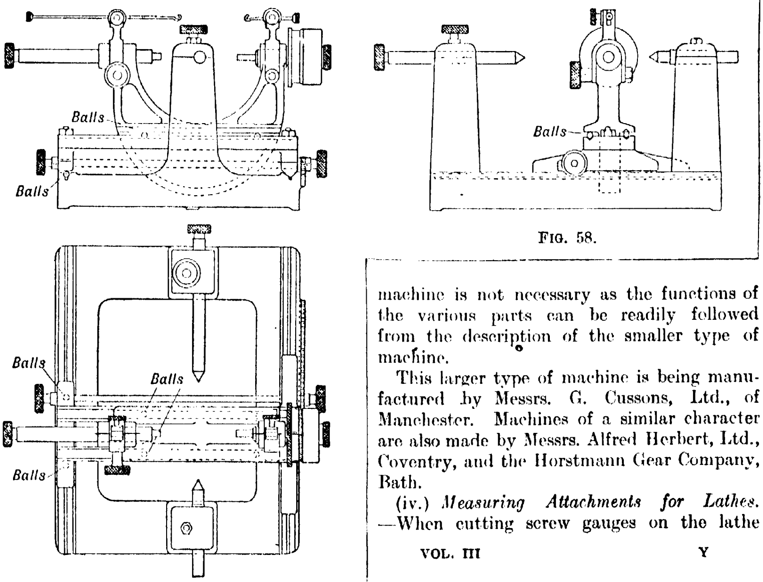


FIG. 58.

This larger type of machine is being manufactured by Messrs. G. Cussons, Ltd., of Manchester. Machines of a similar character are also made by Messrs. Alfred Herbert, Ltd., Coventry, and the Horstmann Gear Company, Bath.

(iv.) *Measuring Attachments for Lathes.*—When cutting screw gauges on the lathe it is desirable to make measurements of the effective and core diameters without removing the gauge from the centres. *Fig.* 59 shows a simple attachment to a lathe for this purpose.

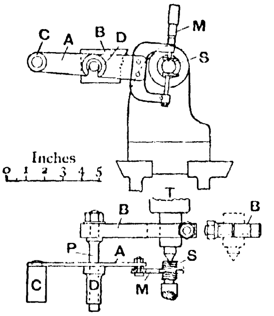


FIG. 59.

A bracket B is clamped on to the spindle of the tail stock of the lathe T, and projects horizontally backwards. Through a hole in B a turned spindle P is bolted. On this a ∩-shaped saddle D slides with an easy fit. On one end of D is fixed a plate A by means of three small screws. This plate carries at the front end a 0 to 2 in. micrometer M, with its spindle in a vertical position. The U-shaped part of the micrometer is fixed to the arm A by two $\frac{3}{16}$ in. bolts spaced $\frac{3}{4}$ in. apart. At the other end of A is a counter weight C, which is sufficiently heavy to slightly overbalance the weight of the micrometer, so that the anvil of the micrometer holds the lower vee-piece or needle gently against the lower side of the screw S, and leaves the workman free to hold the upper one and to operate the micrometer head. *Fig.* 59 shows the micrometer and the vee-pieces in position for making a measurement of core diameter. When a measurement has been made the micrometer and its attachments A, D, and C can be moved away bodily, or preferably slid along the spindle P to the right so as to be clear of the work, with the anvil of the micrometer resting against the lower side of the lathe centre, which should be wrapped with a piece of cloth.

Between the arm A and the micrometer are inserted two thin washers. These serve to reduce the bearing surface between the two, so removing any danger of seriously distorting the micrometer, and also for the purpose of adjusting the micrometer square with the lathe centres. This adjustment can be made by the method indicated in *Fig.* 56 above.

§ (24) THE MEASUREMENT OF THE PITCH OF SCREWS.—Several types of machines have been designed at the National Physical Laboratory for the purpose of measuring the pitch of external and internal screws. Generally speaking, their principles of operation are the same. The screw to be measured is mounted between centres or attached to a face plate, and fixed so that it cannot rotate. A specially shaped steel feeler piece or stylus, as shown later in *Fig.* 63, is traversed along the screw in a direction parallel to its axis by the motion of a micrometer screw, and in such a manner that it rides in and out of the threads. This feeler piece is attached to an appliance which serves to indicate when it rests centrally in the various threads in succession. Readings of the micrometer are taken at the points of indication, and from these the pitch of the screw can be readily obtained.

A. MEASUREMENT OF PLUG SCREWS

The first machine to be described will accommodate plug screws up to 6 in. diameter and an over-all length of 9 in. A photograph of the machine is shown in *Fig.* 60, and a general arrangement drawing in *Fig.* 61.

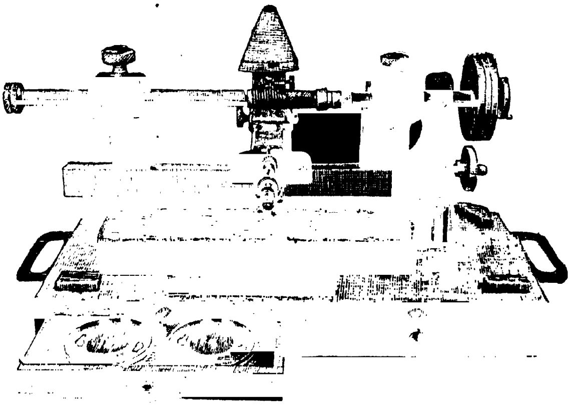


FIG. 60.

(i.) *Structure of the Machine.*—There are three main portions to the machine—*the Bed* A, *the Sliding Bar* B (carrying the micrometer), and *the Indicator and Saddle* K.

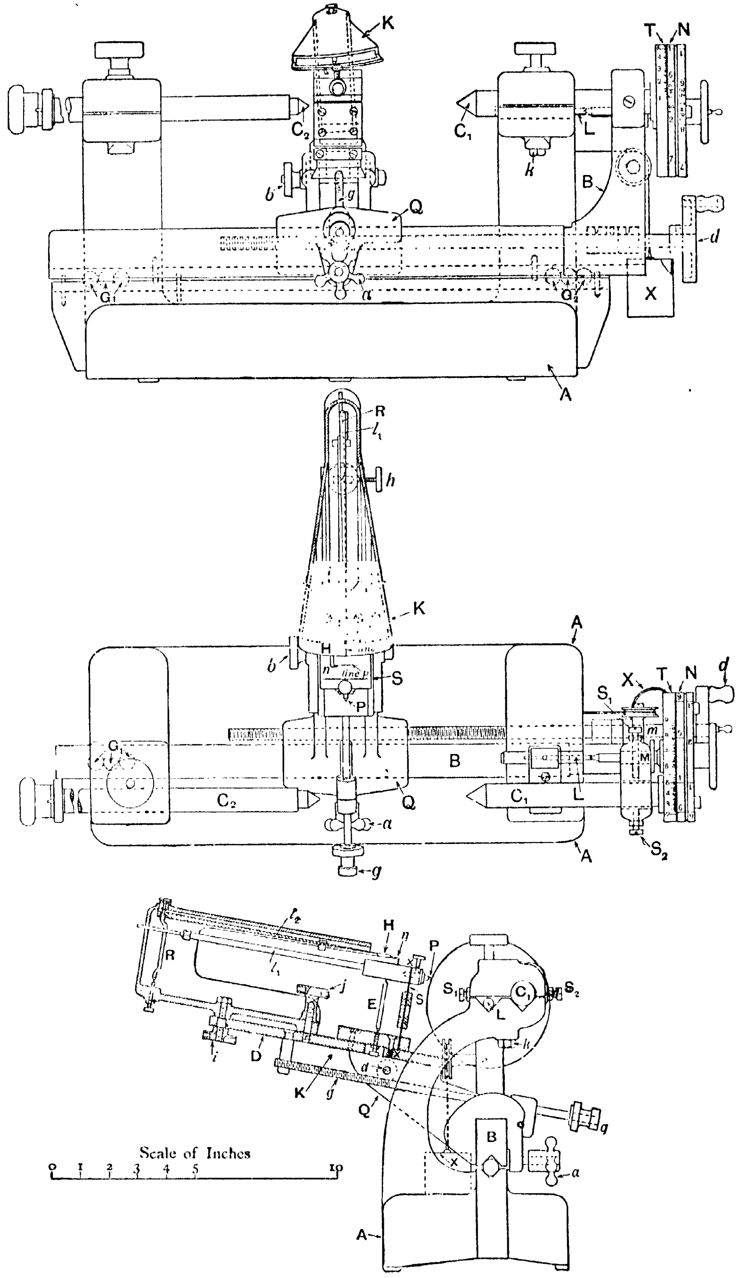

K
T
N
C₂
C₁
L
k
B
b
g
Q
d
G₁
a
G₂
X
A
R
l₁
h
K
A
b
H
n
line
p
S
P
X
T
N
d
S₁
m
M
G₁
B
C₂
Q
C₁
L
a
A
S₂
g
l₂
H
n
P
R
l₁
j
S₁
C₁
S₂
L
E
S
i
D
d
k
K
g
Q
q
B
X
a
A
Scale of Inches
0
1
2
3
4
5
10

FIG. 61.

The bed A carries a pair of centres C_1 and C_2 to hold the screw being measured. The shorter centre C_1 on the right is not moved once it has been adjusted to suit the micrometer travel. The position of the long centre C_2 can be adjusted to suit any length of screw up to 9 in. in length and is fitted with a spring at the point to keep a sufficient pressure on the screw to prevent it rotating during the measurements.

The sliding bar B has a vee groove along its base and runs on two groups of three balls, G_1 and G_2, placed in a similar groove in the bed. The upright portion of the sliding bar has a hole through it which clears the short centre rod C_1 and is guided off the rod by two set screws S_1 and S_2, which are adjusted so as to allow a free endwise motion without too much shake between the points of the two screws and the centre rod. The micrometer head M, which is of the standard Brown and Sharpe pattern of 1 in. range, is fixed in the sliding bar at the top of the upright portion. The thrust of the micrometer spindle is taken by an anvil rod L through an adjustable strut p in a manner detailed in *Fig.* 62. This strut has a centre at the outer end into which fits the conical end of the micrometer spindle. The other end is spherical and bears against the conical end of the hole drilled in the anvil L. By this means any want of truth in the rotation of the conical end of the micrometer spindle is taken up by a swivelling motion of the strut, whilst the actual thrust of the micrometer is not concentrated at a point contact, but is spread somewhat over the surface of the conical centre at the outer end of the strut. A weight X keeps the sliding bar pulled towards the left so that the micrometer remains in contact with the anvil.

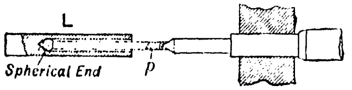


FIG. 62.

The micrometer head is provided with a graduated dial N, which reads against a fixed dial T attached to the end of the short centre rod.

The indicator saddle Q is carried on the sliding bar and can be adjusted to any position along the machine by means of a long screw fitted with a hand-wheel d. When in position, the saddle can be clamped to the sliding casting by the screw a in front.

When testing the pitches of screws longer than 1 in. it is necessary to make the measurements in two or more settings of the saddle on the bar B.

(ii.) *Indicator.*—The indicator is fixed to a cross slide D, which is carried on the saddle Q. This slide allows the whole indicator to be moved towards or away from the line of the centres by an adjusting screw g, to suit the diameter of the screw to be measured.

The hardened steel stylus P which engages with the thread of the screw being measured, is fixed to one end of the first lever l_1, which is supported above the cross-slide by a steel strip S and a strut E. The other end of this lever is forked; the fork embraces a crank R pivoted on a jewelled bearing and has a very light pointer l_2 fixed to it, the crank and pointer thus forming a second lever. The axis of the second lever is tilted 3° sideways so that the weight of the pointer keeps the crank against one side of the fork at the end of the first lever l_1 with a very light pressure. The front end of the pointer moves over a sector-shaped part H of the indicator frame on which a line O is scribed. As the stylus moves in and out of the thread it causes a pure bending of the strip which has no effect on the pointer. If the stylus be subject to a side strain, it causes the strip to twist about the axis XX, which thus becomes the virtual fulcrum of the first lever. Thus, a lateral movement of the stylus relative to the carriage Q is indicated by a movement of the pointer, the magnification of the movement being approximately 750.

The general action of the indicator in a vee thread can be readily seen from the three diagrams A, B, and C in *Fig.* 63. A shows the stylus in contact with the right flank of the thread, and the pressure on it is as shown at U_1; at C the contact is changed over to the left flank and the pressure is U_2. B shows the stylus in contact with both flanks, and it is in this position that the pressure changes from the direction U_1 to U_2, with the result that the stylus twists about the axis XX referred to above. This twisting motion causes the pointer to move from left to right over the section H, and when it is opposite the line O the side pressures on the stylus are equal. The reading is taken on the micrometer when this occurs and the operation is repeated throughout the

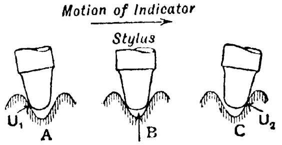


FIG. 63.

length of screw, the stylus riding automatically in and out of the threads during the process. It will be noticed that the steel strip has a dual function. When the stylus is in position B the strip acts as a pivot and the lever turns about XX. In the positions A and C the strip is acting by bending to allow the stylus to ride up and down the flanks.

For measuring the pitch of vee threads the indicator is adjusted by means of the adjusting screw *h* and the two clamping screws *i* and *j*, so that when in its free position, *i.e.* with no side pressure on the stylus, the pointer is opposite the line. For square threaded screws it is necessary to adjust the indicator by means of the screw *h*, so that there is a light side pressure between the stylus P and the flank of the screw chosen for measurement.

(iii.) *Stylus.*—The machine is provided with a set of seven indicator points P, having a graded series of radii at the ends and suitable for measuring the pitches of all Whitworth form threads from 4 to 40 threads per inch. If a thread other than of Whitworth form has to be measured, a suitable point is chosen from among the set by trying them in the thread and selecting the one which touches about half-way down the flanks.

(iv.) *Graduated Dials.*—The thimble of the micrometer head is provided with a sleeve to which a number of interchangeable and specially graduated dials N can be attached. These dials allow rapid measurements to be made of all the commoner pitches without having to resort to any calculations. The dial shown on the machine, which has 9 equal divisions, serves for measuring pitches having 9, 18, or 36 threads per inch.

Consider the case of a screw having 9 threads per inch. The number of revolutions of tne 40-thread per inch micrometer spindle corresponding to a travel of the indicator of $\frac{1}{9}$ of an inch is equal to $\frac{1}{9} \div \frac{1}{40}$, *i.e.* 4 and $\frac{4}{9}$. Consequently, if the zero line on the disc N is opposite a particular mark on the scale of the fixed disc T when the stylus is resting in one thread of the screw, it will be necessary to rotate the disc through 4 and $\frac{4}{9}$ revolutions to bring the stylus into the next thread. Moreover, if the pitch of the thread is quite correct, the fourth line on the disc will arrive exactly opposite the same mark on the fixed disc T. If the pitch is not correct, however, the line will deviate from the mark by an amount proportional to the error in the particular pitch, and this error can be read off directly on the scale of the fixed disc T. If the indicator is now moved into the next thread its error with respect to the zero thread can also be read off in the same manner. Thus, by taking readings on successive threads it is possible to obtain directly the cumulative errors in the pitch of the screw throughout its length. A complement of eight such specially divided dials permits of very rapid measurement of the pitches most frequently met with.

In addition to these special dials, the machine is also provided with another which is fully divided into 250 parts and which is used when measuring any odd size of pitch such as occur on B.A. or S.I. threads. In this case it is necessary to take actual readings of the micrometer disc against the zero line on the fixed disc at each setting of the indicator, and to calculate the errors afterwards.

(v.) *Method of Measurement.*—The method of setting up a plug screw in the machine for measurement is simple. The appropriate dial for the pitch of the screw is put in place on the micrometer, which is unscrewed to the 1 in. end of its run and set so that the zero line on the dial N comes opposite the 5 line on the scale of the fixed dial T. The screw is then set up between the centres, and the indicator brought opposite the first thread at the left-hand end by means of the hand-wheel *d*. The indicator is then fed in towards the screw by means of the adjusting screw *g* until, with the stylus touching both flanks of the thread, the small pointer *n* on the top is about $\frac{1}{20}$ in. away from the line *p*, scribed on the upper surface of the indicator. This ensures a reasonable pressure between the stylus and the screw. The locking screw *b* is then tightened. The indicator is next adjusted by means of the hand-wheel *d* until the pointer comes approximately opposite the line O, and the indicator saddle is then locked on the bar B by means of the screw *a*. To bring the pointer exactly over the line O without disturbing the zero setting of the micrometer, the screw is turned slightly on the centres. The micrometer is now rotated in a right-handed direction until the stylus enters the next thread. The pointer is again brought exactly up to the line O and the reading is taken. The operation is repeated in each of the threads in turn, the pointer automatically climbing up the flanks of the threads and over the crests. Having reached the end of the screw a check reading should be made on the first thread. In this manner a series of readings is obtained such as is given below.

The first set of readings are those obtained from a Whitworth screw having 18 threads per inch, by using the special dial for this pitch. The second set are those of a No. 0 B.A. screw where it was necessary to use the fully divided dial.

SCREW OF NOMINAL PITCH

18 Threads per Inch

No. of Thread.	Reading on Scale of Dial T (Unit = 0·0001 Inch).	Cumulation Error in Pitch (Unit = 0·0001 Inch).
0	5·0	0·0
1	5·6	+0·6
2	6·3	+1·3
3	6·7	+1·7
4	7·5	+2·5
5	7·8	+2·8
6	8·0	+3·0
7	8·2	+3·2
8	8·5	+3·5
9	8·7	+3·7
10	9·0	+4·0
11	8·8	+3·8
12	9·1	+4·1
0	5·1*	..

* Check reading.

The errors in pitch are shown plotted in *Fig.* 64.

This diagram shows that the over-all error in pitch is long by approximately 0·0004 in.

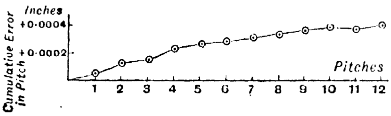


FIG. 64.

and that the bulk of this error occurs over the first half of the screw. The pitch is free from any marked irregularities or periodic error.

No. 0 B.A. SCREW. Nominal Pitch 1 mm.

No. of Thread.	Reading on Fully Divided Dial (Inches).	Difference from Zero Reading (Inches).	Nominal Pitch (Inches).	Cumulative Error in Pitch (Inches).
0	0·14550	0·00000	0·00000	0·00000
1	0·18487	0·03937	0·03937	0·00000
2	0·22394	0·07844	0·07874	− 0·00030
3	0·26369	0·11819	0·11811	+ 0·00008
4	0·30259	0·15709	0·15748	− 0·00039
5	0·34205	0·19655	0·19685	− 0·00030
6	0·38165	0·23615	0·23622	− 0·00007
7	0·42064	0·27514	0·27559	− 0·00045
8	0·46038	0·31488	0·31496	− 0·00008
9	0·49933	0·35383	0·35433	− 0·00050
10	0·53892	0·39342	0·39370	− 0·00028
11	0·57823	0·43273	0·43307	− 0·00034
12	0·61732	0·47182	0·47244	− 0·00062
13	0·65708	0·51158	0·51181	− 0·00023
0	0·14548*	..	..	..

* Check reading.

The diagram in *Fig.* 65 shows the result obtained on plotting the errors in pitch. It will be noted that the mean pitch is short by approximately 0·0003 in. over the length of the screw and, superimposed on this, there is a periodic error of ±0·0002 in. which recurs at intervals of 2½ pitches, *i.e.* 2·5 mm. This indicates that the screw was probably cut on a lathe having a 10 threads per inch lead screw.

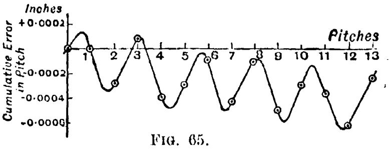


FIG. 65.

It may be noted that by fitting the machine with a metric micrometer head the measurement of screws having millimetre pitches would be greatly facilitated.

The accuracy of the measurements made on the machine depends upon the accuracy of the readings of the micrometer head. As the readings even of a new micrometer head may have an error of as much as 0·0002 in. over the one inch range, it is necessary to make a calibration of the machine to determine the errors. This calibration is readily made by taking a screw gauge of known pitch and obtaining a set of measurements upon it. The difference between the measured and the true errors gives the errors in the readings along the run of the micrometer screw; these errors can then be plotted in the form of a calibration chart for the machine. When measuring the pitches of other screws, reference should be made to this chart and due allowance made for the errors of the machine.

B. MEASUREMENT OF RING SCREWS

§ (25) DETAILS OF MACHINE.—The machine described above can be readily adapted for measuring the pitch of ring screws. The arrangement is shown in *Fig.* 66. The ring to be measured is mounted in a chuck or on a face plate fastened to the right-hand centre. The stylus is fitted to the end of an auxiliary lever which is supported on a steel strip and which engages at the rear end with the main indicator as shown. The measurements are made as described above in the case of plug screws. In order to eliminate possible errors due to the axis of the ring screw not being square to the face on which it is clamped, it is necessary to take two sets of readings along opposite generator lines, the ring being rotated on its axis for this purpose. The mean of two such sets of measurements gives the desired result.

§ (26) PITCH MEASURING MACHINE WITH OPTICAL INDICATOR.—Another design of pitch measuring machine is shown in *Fig.* 67. The general principle is the same as that for the machine just described, but the construction is simpler.

The micrometer head is provided with a graduated brass disc 4½ in. diameter, which is fully divided into 250 parts, each of which represents 0·0001 in. The bar which carries the indicator carriage is supported off the bed on balls and is guided against vertical flat faces on the lower parts of the two brackets which carry the centres. The whole machine is slightly tilted forward in order to keep the bar gently in contact with these guiding faces.

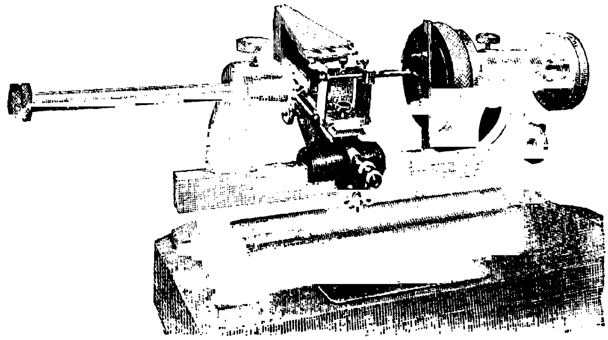


FIG. 66.

The indicator gear is the same as for the machine previously described so far as the first lever is concerned. The first stage of the magnification is obtained mechanically and is about 20 times: the second stage is obtained optically and is about 25 times. The total magnification is thus about 500. The lever *a* carries at its rear end a lens *b*, of about ¾ in. focus. A small 2-volt lamp *c*, with a straight filament, is fixed to a sleeve on a pillar *i* just above the lamp. The image of the lamp filament is formed on a ground-glass screen *g*, placed at the front of the machine, after being reflected from the two mirrors *e* and *f*, which are to be seen in the side view. The mirror *f* is in the form of a long strip which is fixed to the bed of the machine and extends along its length. The mirror *e*, and the lamp *c*, are carried on the post *i*, which is attached to the indicator saddle. The image of the lamp filament can be carefully focussed on the screen by means of the knurled-headed screw *d*, which moves the sleeve relative to the post.

The operation of the stylus is the same as explained in the diagrams A, B, C of *Fig.* 63. Whilst in position B, the image moves across the screen and the micrometer is set so that the image coincides with a line *h* marked on the surface of the glass. This setting is repeated on the various threads in turn and the errors of the pitch obtained as described previously.

An important point to be noted in the designs of the above machines is the position of the micrometer. In each case this is arranged so that its axis is as closely as possible co-linear with the direction of motion of the stylus along the screw. This eliminates to a large extent the effect of errors in the straightness of the bed of the machine.

The two machines shown in *Figs.* 61 and 67 are being manufactured by Messrs. G. Cussons, Ltd., of Manchester.[1]

For descriptions of other methods of measuring the pitches of screws reference should be made to the optical screw-measuring machines described in §§ (33) and (69).

§ (27) MECHANICAL MEASUREMENTS OF RING SCREW GAUGES. (i.) *Tests by Means of Plug Checks.*—Several machines have been designed and various methods devised for taking mechanical measurements of the diameters of ring screw gauges. These are described later, see §§ (30) and (31).

The method most commonly used, however, up to the present, both during the manufacture and for the final inspection, is to make use of a series of "go" and "not go" plug check gauges—a system which was originally devised by Mr. W. Taylor.

The following plug check gauges are necessary for inspection purposes where Whitworth and B.A. screws are concerned:

---

[1] See *Report of the National Physical Laboratory*, 1919 and 1920.

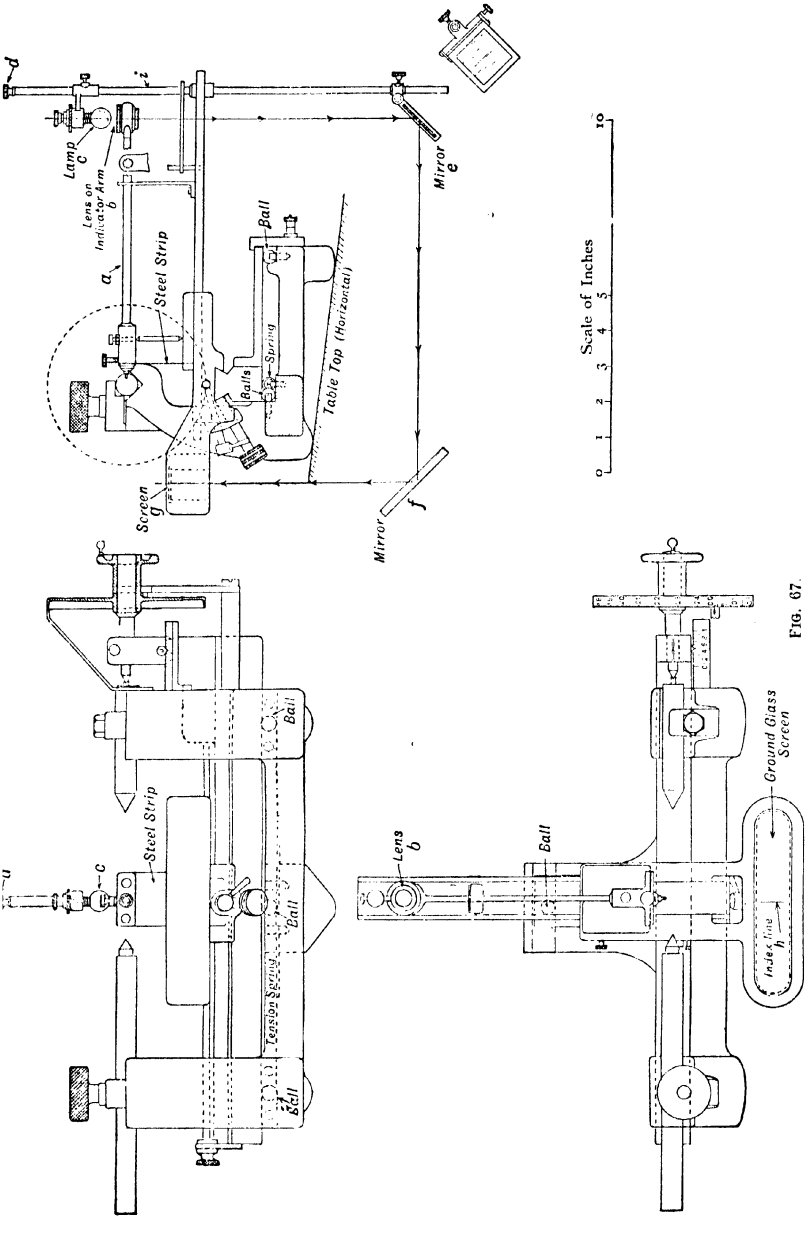

d
i
Lamp c
Lens on Indicator Arm b
a
Steel Strip
Ball
Spring
Balls
Table Top (Horizontal)
Mirror e
Mirror f
Screen g
Scale of Inches
0
1
2
3
4
5
10
Steel Strip
c
Ball
Ball
Tension Spring
Ball
Lens b
Ball
Ground Glass Screen
Index line h

FIG. 67.

(*a*) A "go" screw gauge having the correct form of profile and whose dimensions are all on the lower limit of the ring gauge.

(*b*) A "not go" plain plug gauge whose diameter is equal to the upper limit of the core diameter.

(*c*) A "not go" plug for the effective diameter consisting of a screw plug, whose effective diameter is on the upper limit of the ring, and which is cleared on the full and core diameters.

(*d*) A "not go" plug for the full diameter consisting of a deep and thin thread screw plug, whose full diameter is on the upper limit of the ring.

To make a rigid test on the effective diameter of the ring, the "not go" check (*c*) for that element should be cleared everywhere except for a short length of flank at the half-depth position. It is usual, however, to make the length of the flanks of such checks equal to at least half that of the standard profile. The full form "go" screw plug should have a length of screw at least as great as that of the ring to be tested to enable account to be taken of the whole of any progressive error which may exist in the pitch of the latter. The "not go" screws (*c*) and (*d*) should be only a few threads in length.

For manufacturing purposes it is found desirable to augment the above list of check plugs by three "go" plugs for the individual diameters. If the rings are being screwed on the lathe, or lapped to size after hardening, such "go" checks coupled with the corresponding "not go" checks enable the individual diameters to be opened out separately with minimum risk of making any of them too large. When the state is arrived at where the ring gauge is large enough to take the three "go" checks it should also allow the full form check to pass comfortably, otherwise it would be an indication of probable eccentricity between the diameters.

It should be noted that the system of using plug checks does not take complete account of the rounding of the crests and roots of the threads. The "go" full form check ensures that no extra metal exists on the roundings, but, on the other hand, no means is provided for ascertaining whether any undue clearance has been made. For this reason, and also as a check on the general form of the thread, it is necessary to inspect the profile of the thread in the same manner as for plug gauges. To do this a cast of the thread has first to be made, and this is inspected in an optical projection apparatus such as described in § (69).

(ii.) *Examination of Thread Form by Means of Casts.*—Various materials have been used for making the casts, among which may be mentioned plaster of Paris, dental wax, a mixture of sulphur and graphite containing 93 per cent of the former to 7 per cent of the latter, and various type metals. Whatever the material used, the cast should be taken of only a section of the ring as shown in *Fig.* 68, so that it can be removed from the latter without having to unscrew it, a process which would destroy the accuracy of the cast.

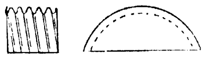


FIG. 68.

Taking the case of plaster, this should be obtained of a good quality and very finely ground—the description used by dentists answers the purpose very well. The gauge should be thoroughly cleaned and its screwed surface afterwards *very lightly* oiled with a brush to prevent the cast adhering to the metal. The ring is then clamped between the jaws of a vice or special clamp, as shown in *Fig.* 69, leaving room to pour in the plaster, which is mixed with water to form a very thin creamy liquid. Just before the plaster has set quite hard, which occurs in about 15 minutes, the cast is very carefully removed from the ring and should be examined immediately on the projection apparatus. The fitting shown in *Fig.* 70, which has the necessary adjustments for setting up the cast, may be used with advantage. It is usual to take two or even three casts from each ring at different positions round the circumference, in order to obtain a check on the concentricity of the diameters.

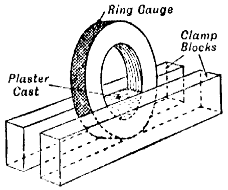


FIG. 69.

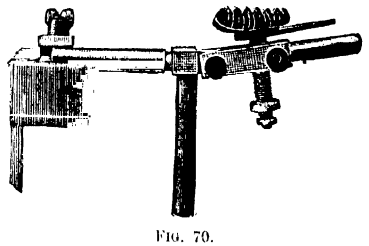


FIG. 70.

Plaster of Paris has been found to give the most satisfactory results for gauges over ½ in. diameter. Under this size dental wax, which softens in hot water, will be found most serviceable, and by careful manipulation it is possible to obtain satisfactory results with this material on very small gauges.

Fig. 71 shows a reproduction from a projection diagram of the thread of a No. 8 B.A. ring gauge from a cast in dental wax. The magnification is 50 to 1.

It is a well-known fact that casts produced from the materials mentioned above are liable to slight warping effects, but so long as the casts are used only for examination of the form of thread, the error due to this effect has been proved to be negligibly small. It would not be safe to attempt to ascertain the pitch of the ring gauges from measurements made on the cast, as a much greater length of cast would be involved and the error due to warping might be serious.

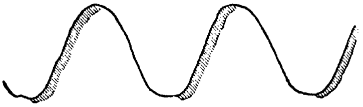


FIG. 71.

§ (28) MECHANICAL MEASUREMENTS OF DIAMETERS BY SLIP GAUGES AND SPECIAL FITTINGS.—Provided the diameter of a screw ring gauge is larger than about ½ in. it is possible to measure the diameters mechanically.

(i.) *Core Diameter.*—This can be measured with a pair of plugs and slip gauges in the same manner as for a plain ring gauge. See § (7) (c).

(ii.) *Effective Diameter.*—This is measured in a similar manner to the measurement of a plug screw with standard wires, except that the wires are replaced by steel balls. Although it is not always possible to obtain a steel ball which will touch the threads exactly at the half-depth, balls can usually be found among the small commercial inch and millimetre sizes which approximate to this condition. It is not possible by this method to deal with pitches much finer than about 24 T.P.I. Whitworth form, as the smallest size of ball made is $\frac{1}{32}$nd of an inch.

The actual measurement with the balls is rather awkward as there are several loose pieces to handle. The method is shown in *Fig.* 72. Two balls, *a*, *a*, are placed in the threads on one side and a third, *b*, is placed diametrically opposite, the balls being conveniently held in position with soft wax. A combination of slip gauges, *c*, is then wrung up and adjusted to size until it will just fit between the balls. The measurement is not at all easy to make, and the result even after careful standardisation of the balls cannot be relied on to an accuracy much better than ± 0·0003 in.

The method can be made somewhat simpler in practice, although at a further sacrifice of accuracy, by omitting the two balls on the one side and by using instead a small cylinder placed parallel to the axis of the gauge and resting on the crests of the threads.

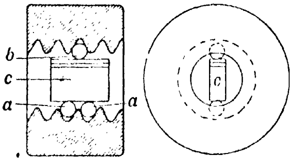


FIG. 72.

(iii.) *Full Diameter.*—The full, or crest, diameter can be measured in a similar manner by making use of a special fitting of the type shown in *Fig.* 73. This consists of a rectangular steel slip, ground and lapped on the under surface, and having on the upper side a truncated conical point, shaped in such a manner that the rounded point will reach to the crest of the thread of the ring. The measurement is made from the core diameter diametrically opposite by using a small cylinder in conjunction with a combination of slip gauges. The full diameter is deduced from a previous measurement of the core diameter together with this measurement of the depth of the thread.

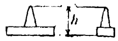
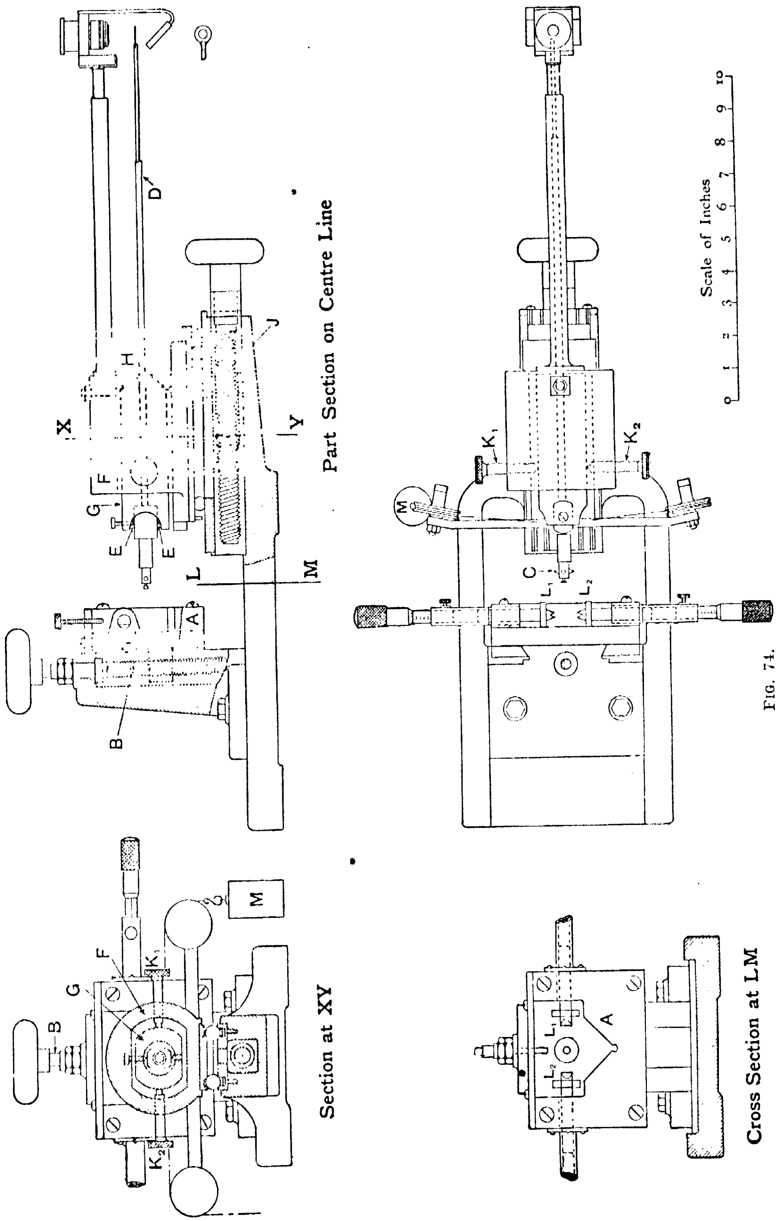


FIG. 73.

§ (29) MEASUREMENT OF EFFECTIVE DIAMETER BY MEANS OF EXPANDING GAUGES.—Several types of expanding plug gauges have been designed for the purpose of measuring the effective diameters of ring screw gauges. Descriptions of such gauges will be found in § (40).

§ (30) EXPERIMENTAL MACHINE FOR THE EFFECTIVE DIAMETERS OF RING SCREW GAUGES.—The experimental machine shown in *Fig.* 74 was designed by Mr. E. M. Eden of the National Physical Laboratory in 1916.

The ring of which the effective diameter is to be measured is clamped in the vee block A, which is capable of vertical adjustment by the screw B. C is a small bar with spherical ends, the diameter of the spheres being the "best" size for the particular thread to be measured. The over-all length of this rod is rather less than the core diameter of the ring to be measured. The bar is rigidly attached at right angles to the end of the pointer D, which is pivoted at the point E, so as to oscillate in a horizontal plane. The carriage F, which is supported on four balls resting in vee grooves, carries the fitting G pivoted at H; this fitting is thus free to swing backwards and forwards in a horizontal plane, the amount of its travel being limited by the



Part Section on Centre Line

Scale of Inches

Section at XY

Cross Section at LM

FIG. 74.

two set-screws K_1 and K_2 which bear against steel plugs let into the fitting G. The carriage can be moved along its bed by means of the screw J, but it has about $\frac{1}{20}$ in. of free movement for any position of the screw J. The end of the pointer carries a fine cross-wire, a magnified image of which is observed on a horizontal scale. The arrangement of the optical system for producing this magnification is indicated in *Fig.* 75. The total magnification should be between 500 and 1000. At L_1 and L_2 are two internal vee pieces which can be moved across the machine (in the plane of the ring which is being measured) by means of the two micrometers. The angles of these vee pieces are each 55°. The weight M serves to keep the fitting G definitely against one or other of the set-screws on the outer carriage.

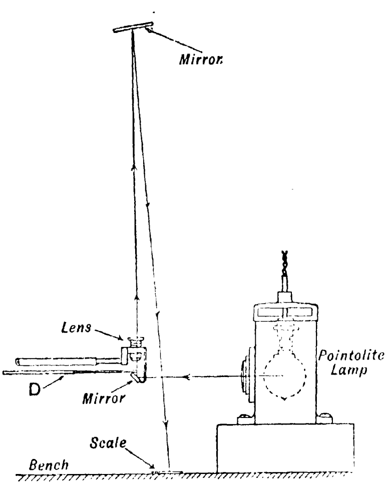


FIG. 75.

The method of using this machine for the measurement of the effective diameter of a screwed ring is as follows:

The ring is fixed into the vee block, and, by means of the screw J, the double-ended rod is fed into the ring, the height of the latter at the same time being adjusted until the bar is approximately along a diameter of the ring. By hanging the weight over one of the pulleys, the fitting G is made to rest against one of the two set-screws, say K_1. This set-screw is then adjusted until the image of the cross-wire on the pointer is near the centre of the scale. The height of the ring to be measured is then carefully adjusted by means of the screw B by observing the position of the maximum deflection of the image of the cross-wire. At the same time, the position of the carriage is so adjusted by means of the screw J that the amount of free movement of the carriage is sufficient to ensure that the spherical end of the bar C is resting freely on both flanks of the screw thread of the ring. When both of these adjustments have been satisfactorily secured the set-screw K_1 is adjusted until the image of the cross-wire on the pointer coincides with a fixed line on the scale. The weight is then hung over the other pulley so that the fitting G is resting on the set-screw K_2 and the other spherical end of the rod C is resting in the opposite side of the screw thread. The position of the carriage is again adjusted by means of the screw J, and the set-screw K_2 is set until the image of the cross-wire is again coincident with the same fixed line on the scale.

The ring is now removed, and one end of the spherical-ended rod is allowed to rest in one of the vee pieces, say L_1. After having adjusted the position of the carriage as above, so that the spherical end rests freely on both flanks of the vee, the micrometer controlling the position of the vee piece L_1 is adjusted until the image of the cross-wire is again coincident with the fixed line on the scale. The same process is then repeated with the spherical end resting in the other vee piece L_2. The settings of the screws K_1 and K_2 are still the same as when the screw gauge was in position. The readings of the micrometers are taken and their sum M_1 noted.

Finally, the spherical-ended rod is removed from the end of the pointer and held between the vee pieces L_1 and L_2, which are adjusted by means of their micrometers until L_1 and L_2 are just in contact with the two spherical ends. The readings of the micrometers are taken and their sum M_2 noted.

Now, if l = overall length of the spherical-ended rod, and p = the "P value"[1] of the spheres for the particular pitch concerned, then the effective diameter E of the ring at the point measured is given by

$$E = (M_1 - M_2) + (l - p).$$

In the case of small rings with coarse pitches the usual correction for rake will have to be made.[2]

§ (31) THE "SHAW" SCREW-MEASURING MACHINE.—This machine was designed by Dr. P. E. Shaw of Nottingham University for the measurement of diameters and pitches of both plug and ring screws.

A detailed description of the first design will be found in *Engineering*, January 24, 1919, and various improvements are mentioned in a further article dated December 19, 1919.

The full diameter of a plug screw has to be measured independently with a micrometer.

---

[1] See § (63).

[2] See *Report of the National Physical Laboratory*, 1921.

The effective and core diameters are then deduced from it by taking measurements of the "effective" depth and total depth of the thread on the machine. In the case of ring gauges, the core diameter is measured independently; the effective and full diameters are then obtained by taking similar measurements of depth on the machine as for plug screws.

The total depth of the thread for either a plug or a ring gauge is ascertained directly by measuring the displacement in a diametral direction A of a conical-ended pin when resting first on the crest of the thread and then at the root, as shown in *Fig.* 76. In the case of the "effective" depth, the measuring pin is furnished at its point with a steel ball which will rest about half-way down the flanks of the thread, as shown in *Fig.* 77. The vertical measurement B is made, and, knowing the diameter of the steel ball and the pitch of the screw, it is possible to calculate from this measurement the difference between the full and effective radii or the core and effective radii of the screw according to whether it is a plug or a ring. From this difference the effective diameter is readily deduced.

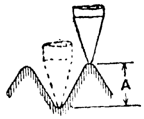


FIG. 76.

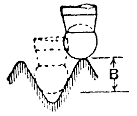


FIG. 77.

It is to be noted that the measurements A and B have to be doubled in deducing the two required diameters from the full or core diameter as the case may be, and consequently these measurements will need to be made to an accuracy of ±0·00005 in. in order to obtain the required diameters to within ±0·0001 in. Moreover, any inaccuracy in the independent measurement of the full diameter of the plug or the core diameter of the ring will recur in the values obtained for the other diameters.

Practical tests on the machine, using screw gauges of known size, have given measurements of the diameters which agreed with the correct values to within ±0·0001 or ±0·0002 in.

The method of making the depth measurements will be readily understood from the diagrammatic general arrangement of the machine, shown in *Fig.* 78, in which a plug screw is shown mounted for measurement. Ring gauges are held in a chuck as shown in the separate view, the shank of the chuck being clamped in the left-hand "centre" bracket. The measuring pin 1, with its axis vertical, is clamped at the end of a tilting lever 2, which has equal arms. This lever rocks on two steel balls bearing in a triangular recess and a slot formed in the upper surface of the carriage 3, which slides freely on the bracket 4, supported from the bed. The outer end of the lever rests in contact with the foot of a dial indicator 5. As the carriage is moved the pin travels along the screw, rising and falling in the thread, and its vertical motion (A or B of *Figs.* 76 and 77) is registered by the dial indicator. The latter can be calibrated over the small range used by means of standard slip gauges.

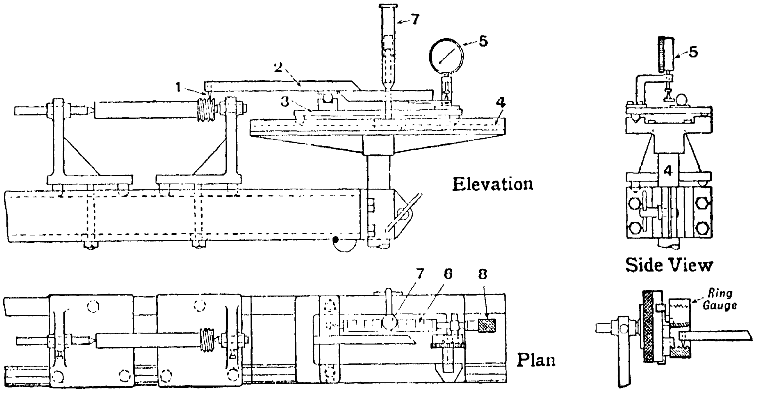


FIG. 78

To measure the pitch of either a plug or ring gauge, the ball-pointed pin is fixed in the end of the lever and the carriage is moved so that the pin registers in the various threads in succession. The translation of the carriage is measured by means of a graduated scale 6 fixed to the slide and viewed through a stationary microscope 7, which is carried off the bracket 4. The scale is divided to suit the pitch of the screw. If the screw being measured has 14 threads per inch, a scale accurately divided into $\frac{1}{14}$ths of inches would be used. Provision is made by means of a micrometer head 8 for adjusting the position of the scale on the slide so as to bring a line under the cross-wire of the microscope when the pin is in the first thread. Any error in the pitches of the subsequent threads will be noted by a lack of coincidence of the cross-wires and the lines on the scale. Such errors can be readily measured on the micrometer head by moving the scale so as to restore the coincidence. The micrometer head should be provided with an enlarged dial to obtain accurate readings.

A mechanical method of measuring the effective diameters of screwed rings, devised by Mr. G. A. Tomlinson of the National Physical Laboratory, will be found in § (7) (i.) (*d*).

§ (32) Optical Measurements of Screw Gauges.—As a general rule optical measurements of screw gauges cannot be made to the same accuracy as mechanical measurements, and, in particular, the results obtained by inexperienced observers are more liable to error. On the other hand, as screw gauges may have errors which cannot be detected by mechanical measurement, the possibility of examining a magnified image of the thread form is an extremely valuable aid both in the production and verification of screw gauges. Further, mechanical measurement of the diameters of a screw gauge can be made conveniently at only a few points on its surface, whereas in an optical apparatus the whole of the screwed surface of a gauge can be examined in detail, the faults seen, and, if necessary, measured. For example, errors in angle, want of straightness of the flanks of the screw, bad form at roots or crests, local thickening of the threads, eccentricity between different diameters, local humps and hollows can be readily detected by optical means.

The present general practice in the examination of plug screw gauges is to measure the diameters and pitch of all sizes larger than about No. 8 B.A. by the mechanical methods described previously. The angle and the general form of thread only are examined by optical means. With the exception of the full diameter it is not possible to make mechanical measurements with any degree of certainty on screws smaller than No. 8 B.A. The examination of very small screws has to be carried out completely on some form of optical instrument.

The optical machines used for this purpose may be conveniently divided into the two classes:

(i.) Microscope Apparatus.
(ii.) Projection Apparatus.

The two types are very similar, in fact the second is a development of the first.

§ (33) Microscope Apparatus for Screw Threads.—The general principle of the microscope apparatus will be gathered from *Fig.* 79. A microscope M held rigidly in a bracket is illuminated from below by a parallel beam of light derived from a suitable lamp and condenser L. A pair of centres C_1, C_2 support the screw gauge S, so that its axis AB is at right angles to that of the microscope. The latter is focussed on the diametral plane DE of the screw and a magnified image of the profile of the screw will be seen on looking through the eyepiece. This image will not be seen clearly unless the profile of the thread be correctly illuminated, and for this to be the case the beam of light from the condenser must be parallel, and its axis must follow the slope of the threads as shown in the front elevation. If the angle of the illumination is not correct, one set of flanks will be less clearly defined than the other. It is essential that the microscope should be accurately focussed on to the plane DE, otherwise the image seen will not be that of a true diametral section of the screw.

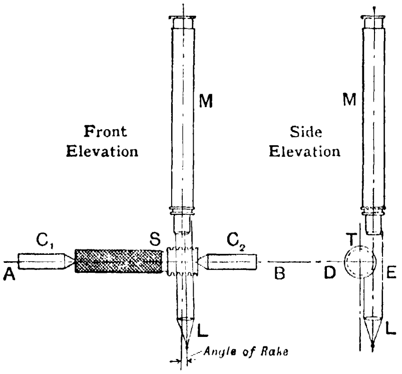


Fig. 79.

The importance of accurate adjustment of the focus when dealing with small screws and using a fairly high magnification cannot be too strongly emphasised. The measurement of the effective diameter is most seriously affected by lack of attention to this point.

From the front elevation view it is evident that where the rake of the screw is large the light which forms the image passes through only one side of the objective, and should the rake be abnormal, no light will reach the eyepiece and no image will be seen. To obtain an image under these conditions, either the aperture of the objective must be increased or special arrangements made for sliding the eyepiece laterally. The relative squareness of the microscope and screw should not be altered to suit any such requirements, either by tilting the microscope or the gauge, although some machines are provided with such false adjustments.

In order to make the required measurements, the eyepiece of the microscope is provided with cross-wires which can be made to coincide with different parts of the image of the profile in turn. These measurements are carried out by means of a pair of micrometer screws which give a relative motion between the microscope and screw in one plane without altering their mutual perpendicularity or the focus of the microscope. In practice, this relative motion is produced by means of a compound slide which gives two motions at right angles to each other. In some machines the microscope is made to move and in others the centres C_1 and C_2 carrying the screw gauge are mounted on the upper carriage of the compound slide, the microscope in this case being fixed. In addition, the microscope is usually mounted so that it can be rotated in its holder about its own axis, the amount of rotation being read on a circular divided scale.

(i.) *Measurement of Full and Core Diameters.*—The microscope is rotated so as to bring the cross-wire parallel to the crests of the threads. The micrometer which controls the diametral movement is then used to bring the cross-wire in coincidence with the images of the crests and roots on each side of the thread in turn, as shown at A, B, C, and D in *Fig.* 80. The full and core diameters are obtained by suitably subtracting the readings in pairs. Account should be taken of any errors in the readings of the micrometer screw, which should be calibrated previously by taking sets of observations on a series of plain plug gauges of known sizes.

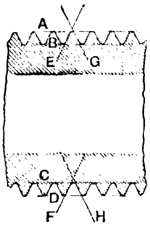


FIG. 80.

(ii.) *Measurements of Effective Diameter.*—The cross-wire is turned so that it becomes parallel to one set of flanks and settings such as E and F in *Fig.* 80 are made by means of the micrometer screw. A similar pair of settings G and H are then made on the other set of flanks. The differences between the readings with the cross-wire turned in each direction are noted.[1]

If the screw has no pitch error and if the motion of the screw has been accurately at right angles to its axis, the difference between the readings referred to above would be the same and would each be equal to the effective diameter. It is usually found, however, that the two values differ slightly, but if the error in pitch and the error in the direction of motion of the screw are both small, the mean of the two differences will give the effective diameter.

If the pitch error is periodic or irregular, or if the error in the mean direction of the traverse of the screw is large, the mean of the differences obtained by the above procedure will not in general be the true effective diameter. Further, if the angle of the thread is out of square with the axis of the screw, or if the flanks are not straight, it is not possible with the ordinary cross-wire method to obtain a reliable measurement of the effective diameter of the screw. Thus in the case of a screw of faulty form, the microscope method gives (by inspection) an indication of the nature of errors, but may fail to measure one principal element of the screw; on the other hand, the mechanical method, which gives this important measurement, may fail to afford any information as to other serious errors. The two methods are thus complementary one of the other; it is only in the case of very good work that they become practically interchangeable.

(iii.) *Measurement of Pitch.*—The cross-wire is adjusted parallel to one set of flanks, as shown at E in *Fig.* 80, and, by means of the second micrometer screw which controls the motion in the axial direction of the screw, settings are made on each flank in turn throughout the length of the screw. A similar set of readings is then made with the cross-wire set parallel to the other set of flanks. The errors in pitch are computed and plotted (in the manner indicated in § (24)) for each of the sets of observations. Generally speaking, the two results obtained will differ if there is any taper in the screw, or if the direction of the motion is not exactly parallel to its axis. Unless the difference is large, however, the mean of the two sets will give the mean errors in pitch along the screw. In order to obtain correct results cognisance must be taken of any errors in the pitch of the micrometer screw itself.

(iv.) *Measurement of Angle.*—The microscope is rotated in its holder so that the cross-wire is brought successively into the positions E, A, and G (*Fig.* 80), on one thread, and the corresponding readings are noted on the divided circle. The difference between the readings at E and G gives the total angle of the thread. The squareness of the thread can be ascertained by comparing the two angles turned through from position E to A and position A to G.

---

[1] As the rake of the thread reverses when opposite sides of the screw are brought into view, it is necessary to alter the direction of the illuminating beam whenever the screw is made to traverse across the field of the microscope.

§ (34) Cambridge Scientific Instrument Company's Microscope Screw-measuring Machine.—This machine, which was one of the first to be designed, is shown in *Fig.* 81, and a complete description will be found in *Engineering*, Nov. 13, 1903.

In this type of machine the gauge is carried on centres or held in a chuck on a compound slide 1, which works on an inclined bed plate 2. The diameter micrometer 3 will be seen at the top and the pitch micrometer 4 at the side. The goniometric microscope 5 is held in a bracket 8 attached to the bed plate. The illumination is obtained from a Nernst lamp 9, shown at the upper right-hand side, and the light after passing through a condenser is reflected in the direction of the microscope by means of a right-angle prism placed beneath the bed plate.

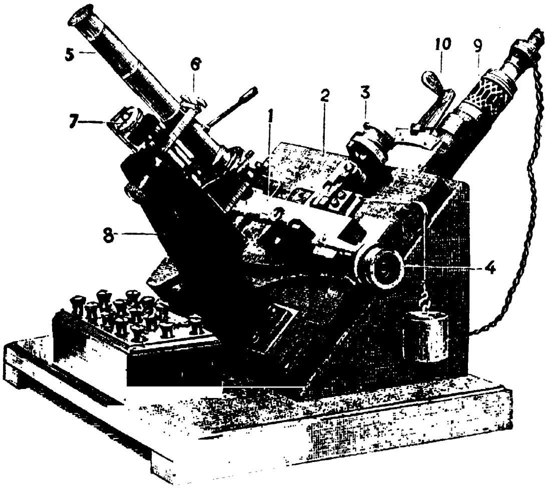


Fig. 81

The beam is directed along the rake of the screw by altering the position of the lever 10, situated in front of the lamp, which tilts the reflecting prism about an axis parallel to that of the microscope. The lever is provided with an index and scale of degrees, so that the rake can be set accurately to any desired angle on either side of the zero. The earlier types of machines had provision made for tilting the microscope to accommodate the rake of the thread. It is essential, however, that the microscope should always be kept square to the axis of the thread, the adjustment for rake being made on the illuminating beam only

The micrometer screws have a pitch of 0·5 mm. and are provided with drums divided into 50 parts, each of which, therefore, represents 0·01 mm. By estimation it is possible to take readings to 0·001 mm.

The microscope can be roughly focussed by sliding it bodily in the tube 6. For fine adjustment the tube and microscope can be moved together relative to the supporting bracket by means of a micrometer screw 7. It is generally sufficiently accurate to set the microscope by focussing on the points of the centres. For small screws, however, it becomes necessary to make a more refined setting, and this is done by focussing on to the crests of the threads at T in *Fig.* 79, and then lowering the microscope by means of the micrometer screw 7 through a distance equal to half the measured full diameter of the screw. This type of machine is now made in two sizes. The smaller size, which is intended primarily for B.A. screws, will accommodate screws up to 12 mm. diameter by 25 mm. length of screw. The larger machine will take screws up to 40 mm. diameter by 60 mm. length of screw.

§ (35) The "Herbert" Optical Measuring Machine.—This machine, which was designed by Messrs. Alfred Herbert, Ltd., Coventry, is shown in *Fig.* 82. The box-shaped bed 1, which is about 18 in. long by 8 in. square section, supports a saddle 2 carrying a pair of centres. This saddle can slide along the bed and is guided in a vee groove in the latter. A bracket 3 is fixed to the rear of bed and supports a compound slide 4 carrying a microscope 5. The position of the latter is controlled by two micrometer screws 6 and 7, as shown, and, in addition, the microscope can be rotated about its own axis as in the case of the machine just described. The illumination is derived from a lamp and mirror attachment 8 fastened to the front of the bed.

The travel of the micrometer screws is 1 in., and by suitably arranging the initial position of the microscope it is possible, by means of these micrometers, to completely measure plug-screw gauges up to 1 in. diameter by 1 in. length of screw in the manner described previously. The pitches of screws up to 12 in. in length can also be dealt with in steps of 1 in. at a time by the use of suitable end gauges 9, inserted between a stop 10 fixed at the left-hand end of the bed, and a similar stop 11 attached to the end of the saddle 2. Gauges differing by 1 in. are used in turn, and the micrometer screw 7 serves for making measurements at intervals in each of the 1-in. steps.

In such a type of machine, where the microscope is made to move, great care has to be taken to prevent any tilt of the microscope axis as it is moved from one position to another. The guiding surfaces must be carefully adjusted for straightness, and there must be no trace of play in the slides. However good the workmanship may be, it is practically impossible to prevent some slight amount of error occurring, due to tilting of the microscope. Provided the error is of a permanent character it can be overcome by calibrating the travel of the two micrometer screws successively against a known scale, the surface of which is placed at the height of the centres, *i.e.* at the normal position of the focal plane of the microscope.

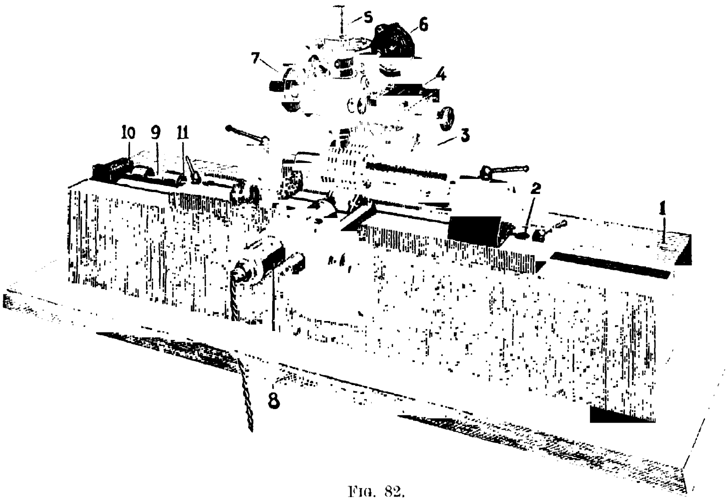


FIG. 82.

In addition to the measurements of plug screw gauges this machine can be used as an ordinary travelling microscope for measuring lengths up to 12 in., the piece to be measured being placed on the movable saddle 2. By the use of the compound micrometer motions and the goniometric microscope, the machine also serves other useful purposes in the workshop, such as the measurement of taper gauges, profile gauges, forms of cutting tools and holes, location of holes in jigs, etc.

Another type of optical screw measuring machine is made by La Société Genevoise of Switzerland. This machine is of the fixed microscope type and comprises a number of special attachments for the standard end-measuring machine made by the same firm. A description of these attachments will be found under the general description of the machine given in § (74).

§ (36) PROJECTION APPARATUS FOR TESTING SCREW GAUGES.—The microscope type of apparatus described above for the measurement and examination of thread form of screw gauges has now been displaced to a great extent by the optical projection type of apparatus. This apparatus enables a shadow-like image of the profile of the screw thread, magnified a definite number of times, to be thrown on to a screen. This image is then compared with a diagram of the standard form drawn to the same magnification, and its variations from the true form can be readily seen and measured with a scale. In addition to the examination of the form of thread on this type of apparatus, it is also possible to carry out complete measurements on a plug screw gauge by mounting it on a carriage whose motions are suitably controlled by micrometer screws.

The projection apparatus has distinct advantages over the microscope apparatus: it is less fatiguing to use, and it allows a direct comparison to be made between the actual form of the thread and the standard form.

The subject of gauge projection apparatus is dealt with more fully in §§ (64) *et seq.*

§ (37) MEASUREMENTS OF TAPER-SCREW GAUGES.—Taper-screw gauges can be divided into two main types :

Type I., in which the thread is square to the axis of the screw (*Fig.* 83).

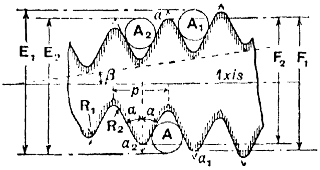


FIG. 83.

Type II., in which the thread is square to the coned surface (*Fig.* 84).

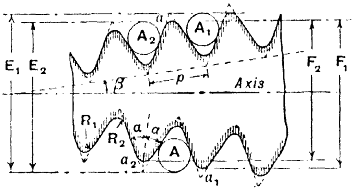


FIG. 84.

§ (38) TAPER-PLUG SCREWS. (i.) *Measurement of Outside Diameter.*—The rate of taper of the outside diameter and the actual diameter at the upper or lower end can be measured by the method described in § (9) for plain taper plugs. A slip gauge should be inserted between each cylinder and the gauge so as to bridge over the threads ; the thickness of the slips should be allowed for in the calculations.

(ii.) *Measurement of Depth and Thickness of Thread.*—The depth and relative thickness of the thread can be readily checked to a reasonable degree of accuracy by the system of optical projection, which allows a comparison to be made between an enlarged image of the thread and corresponding diagram of the theoretical thread form. Measurements can also be made with vee pieces and standard wires, as in the case of parallel screws, but, owing to the difficulty of locating the axial position of such measurements, it is convenient to compare them with measurements of the full diameter made in the same position. The method adopted is as follows :

The gauge is placed between the centres of a floating micrometer machine, such as described in § (23). Referring to *Figs.* 83 and 84, the faces of the machine are brought into contact with any two crests of the gauge at a and a_1 and the diameter F_1 is recorded on the micrometer. The micrometer carriage is then moved along its slide so that a similar measurement F_2 can be made on the crests a and a_2 and the mean value F of the two measurements is calculated. For the thread thickness, two standard wires of suitable size are taken and placed on opposite sides of the gauge at A and A_1, care being taken not to rotate the screw. Having obtained the measurement E_1 over the wires in this position, the wire A_1 is transferred to the position A_2 and a second measurement E_2 is taken. The mean E of the two measurements E_1 and E_2 is then calculated. In the case of the thread depth, vee pieces are used instead of the wires and two similar measurements are taken. Let the mean of these two measurements diminished by the thickness of the vees be denoted by C.

The measurements at the crests and over the wires and vee pieces should be repeated at two or three places along the screw and the difference (E − F) and (F − C) determined at each position.

The difference between the mean measurements E and F is a measure of the relative thickness of the thread, whilst the difference between F and C is a measure of the thread depth. To check the accuracy of the screw from such measurements it is necessary to compare the measured difference (E − F) and (F − C) with those calculated from the theoretical dimensions of the thread. The following formulae give the theoretical values of the two differences for the two types of taper screws :

Let β = Semi-angle of cone,
a = Semi-angle of screw thread,
p = Pitch of screw,
R_1 = Radius at crests of threads,
R_2 = Radius at roots of threads,
r = Mean radius of standard wires used.

Type I.—Thread square to axis of gauge (*Fig.* 83).

$$E - F = 2(r - R_1) + 2(r + R_1) \operatorname{cosec} a - p \cot a,$$

$$F - C = p \cot a - 2(R_1 + R_2)(\operatorname{cosec} a - 1).$$

Type II.—Thread square to coned surface (*Fig.* 84).

$$E - F = 2(r - R_1) + 2(r + R_1) \operatorname{cosec} a \cos\beta - p \cot a \cos\beta,$$

$$F - C = p \cot a \cos\beta - 2(R_1 + R_2)(\operatorname{cosec} a \cos\beta - 1).$$

When comparing the measured differences with those obtained by calculation, due allowances must be made for any errors found in the previous measurement of the full or outside diameter. It may be pointed out that the outside diameter, the measurement of which is described in (i.) above, is not quite the same as that denoted by F in the above formulae. The difference, however, is usually quite small and can be safely ignored.

(iii.) *Measurement of Pitch.*—This element of a taper-plug screw is measured most conveniently on the vertical projection apparatus described in § (69). The method adopted is similar to that described for parallel plug screws in § (69) (viii.), except that in addition to moving the screw axially by the pitch-measuring micrometer screw, it is also necessary to translate it in the diametral direction in order to keep the image of the threads in view.

(iv.) *Measurement of Thread Angle and Examination of General Form of Thread.*—For this purpose the screw should be mounted in either a microscope machine, such as described in § (33), or in a projection machine, as described in §§ (68, 69). The inclination of the flanks to the line joining the crests is measured in the first case by rotation of the cross-wires and in the second by means of a "shadow protractor" (see § (69) (vii.)). The general form of the thread is examined most conveniently in the projection machine by comparison of its image with an enlarged diagram.

§ (39) TAPER-RING SCREWS.—This form of gauge is usually tested by means of a series of taper-plugs, noting the axial positions in which the latter assemble with the ring. A complete set of taper checks would be as follows:

(*a*) A full form screw.

(*b*) A plain taper-plug for the core diameter.

(*c*) A full diameter screw check the threads of which are cleared on the flanks and at the roots.

(*d*) An effective diameter screw check whose threads are cleared everywhere except on the flanks.

Each of these checks should be provided with a stepped shoulder to indicate the upper and lower limits for the ring gauge.

In order to test the accuracy of the taper of the ring it is desirable to make the checks (*b*), (*c*), and (*d*) only a few threads in length and to make them in duplicate, so that one check engages with the ring towards the smaller end and the other towards the opposite end.

It is a difficult matter to make direct measurements of the pitch of a taper-screw ring, but a fairly accurate test can be obtained by having an effective diameter plug check whose length is at least equal to that of the ring and of correct pitch. The behaviour of this check in the ring, as compared with that of the two short effective diameter checks, will give an indication of the accuracy of the pitch of the ring.

The angle and general form of the thread of the ring can be examined by taking a cast of a sector of the gauge as described in § (27) (ii.). The cast is then tested on an optical projection apparatus.

§ (40) SPECIAL GAUGES AND INSTRUMENTS FOR TESTING SCREW THREADS. (i.) *"Go" and "Not Go" Gap Gauge for Screws.*—Designed by Mr. W. Taylor of Taylor, Taylor & Hobson, Ltd., Patent No. 6900.

This gauge is of the form shown in *Fig.* 85, which is taken from *Report No. 38* of the British Engineering Standards Association. The three conical points *a*, *a*, and *b* are accurately ground to an angle of 55° and are truncated so as to clear the root of the thread. The pair *aa*, in the lower jaw of the gauge, are placed at a distance apart equal to exactly twice the number of threads contained in a nut corresponding to the size of bolt to be gauged. The third point *b* is fixed to the other jaw at the midway position. The diametral position of this point is set by means of a standard male screw, correct in pitch and angle and on the *upper* limit of effective diameter. A flat face *c* is fixed in the lower jaw opposite the single point. A standard wire *d*, whose diameter is such that it will touch the thread about half-way down its depth, is chosen, and the face *c* is adjusted so that the wire will just enter the thread of a screw which is correct in pitch and angle and whose effective diameter is on the *lower* limit of acceptance.

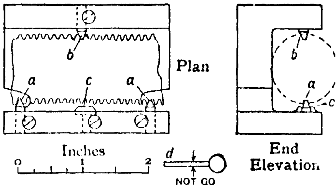


FIG. 85.

In testing bolts or screw, if they enter the gauge and the "not go" cylinder *d* refuses to pass between the thread and the face *c*, then the screw is within the limits on effective diameter, and also the errors present in pitch and angle are not sufficiently large to virtually increase the effective diameter above the upper limit over a length equal to that of a standard nut.

(ii.) *Effective Diameter Gauge for Nuts.*—Designed by Mr. H. S. Rowell.

This gauge is shown in *Fig.* 86. The two limbs *a*, *a*, which work on a pin *p*, are furnished at the ends of their shorter arms with points *b*, *b*, rounded at their ends to such a radius that they sink about half-way down into the thread to be gauged. The other ends have adjustable contacts *c*, *c*, screwed in, and a spring *d* tends to keep these contacts together. The ratio of the arms is 5 to 1.

As originally designed, the instrument was intended as a "not go" effective diameter gauge. It was first inserted in a master ring gauge whose effective diameter was on the upper limit of the work to be tested, and the contacts were adjusted by screwing into the arms until they just met. When testing work the spring forced the gauging points apart, and if the ring was not too large on its effective diameter the contact points *c*, *c*, would fail to meet. If by sighting or by the use of a very thin "feeler" it was found that the points met, then the work was rejected as being too large.

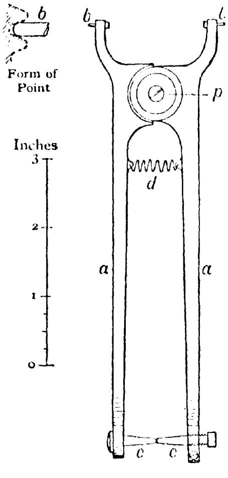


FIG. 86.

This type of gauge can also be used for making comparisons of effective diameter of ring gauges by reversing the contact points *c*, *c*, so that it is possible to micrometer across them. It will also serve for the measurement of the full diameter if the gauging points *b*, *b*, are made sufficiently sharp to reach the crest of the thread.

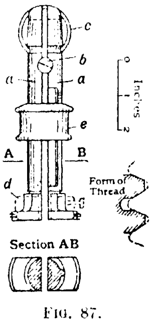


FIG. 87.

(iii.) *Taylor Expanding Effective Diameter Plug Screw Gauge.*—Designed by Mr. W. Taylor. Patent No. 124001.

This gauge takes the form shown in *Fig.* 87 and is made up of two legs *a*, *a*, bearing on a hardened steel pivot *b* at one end, and held together by the hoop-spring *c* at the top. The thread at the other end of the gauge, which is of the usual form for a "not go" effective diameter gauge, is finished to a certain measured size when the two legs are clamped together at the free end by a ring *d* on a suitable size distance piece. Whilst in this state the outer surfaces of the legs are carefully ground cylindrical. A collar *e*, whose bore is somewhat larger than the ground diameter of the arms, is fitted on after the distance piece has been removed. On releasing the screw of the clamp ring slightly, a position can be found when the collar will just lodge on the outside of the legs towards the lower end. The effective diameter across the centre of the screw can then be measured with standard wires, and this diameter will correspond to the position of the collar, which can be suitably marked by a line on the ground surface. In this manner the gauge can be calibrated for a number of positions of the collar along the gauge. The bore of the collar must not be much larger than the ground diameter of the gauge, as the rotational separation of the two halves of the screw about the pivot *b* will cause the threads to become inclined, and if this effect is allowed to go too far a serious error will arise. For the same reason the range of any particular instrument must be somewhat restricted.

When using the gauge to test work it is inserted by closing the two halves together and afterwards allowing them to separate so that the threads bear correctly in the ring. The collar *e* is then allowed to slide down until it takes up its position on the outside of the gauge. Its location on the scale then gives the effective diameter direct.

This form of gauge has a comparatively long life as it is not subject to wear in screwing it into or out of the work. Mr. Taylor has utilised the same principle in the design of split ring gauges, full details of which are given in the Patent Specification.

(iv.) "*Scissor*" *Gauge for B.A. Screws.*—Designed by Mr. W. Taylor.

A diagram of this gauge is shown in *Fig.* 88. It consists of two portions *a*, *a*, approximately rectangular in section, which bear together on a pivot *b*, the necessary force being supplied by the spring C. The two jaws of the gauge can be brought together until contact is made on the faces *d*, *d*. When in this position a number of holes are made half in each jaw. The outer hole is tapped with a full form

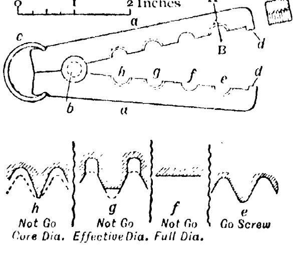


FIG. 88.

screw of the correct nominal size of the screws to be tested. The next hole *f* is not tapped but is finished to a diameter equal to the lower limit of the full diameter of the screw. The screwed hole *g* has the form of thread shown in the enlarged diagram, and is such that it bears only on the flanks of a screw. Its effective diameter is equal to the lower limit. This hole consists of only a few threads, the remainder being cleared away. The last hole *h* is tapped with a steep angle thread and with sharp crests so as to come in contact only with the roots of the screws. Its core diameter is equal to the lower limit of the screws. This hole also comprises only a few threads.

In testing screws they are inserted in the different holes in turn and the two jaws clipped together. When in the hole *e* it should be possible to bring the faces *dd* into contact and yet leave the screw free in the hole. If the screw binds in the hole it is too large. When inserted into any of the other three holes the screw should bind in each case when the jaws are clipped together.

This split type of gauge has the advantage that it is not necessary to screw the thread to be tested in and out of the gauge, so that the gauge has a comparatively long life. The different holes of the gauge are readily checked by a correct nominal size master screw for the hole *e* and by another which has all its diameters on the lower limit for the other three holes.

(v.) *Adjustable Plug Gauges for measuring the Pitch and Effective Diameter of Screwed Rings.*—Designed by Mr. T. Browett, Patent No. 120307, and developed by the National Physical Laboratory.

The first of these two gauges is shown in *Fig.* 89, and is intended for measuring the pitch and effective diameters of screwed rings having 14 threads per inch Whitworth form and full diameters ranging from 1·994 to 2·013 in.

The gauge has an interrupted thread as shown, each portion consisting of two complete turns of thread so as to allow measurements of the effective diameter to be made by placing standard wires in the groove. The form of the thread is shown in diagram A, the crests and roots of the threads being cleared so that the gauge operates only on the flanks of threads to be tested.

The upper of the two portions of the screw is cut on the body of the gauge 1, and the lower portion on the end of the plunger 2, which is an accurate fit inside the body. The screws are cut as parts of a continuous thread when the body and plunger are located together by means of the taper pin 3. On removing this pin it is possible to alter the relative positions of the screws in an axial position by means of the differential micrometer screw 4, the necessary guidance being given by the two parallel pins 5. The micrometer screws on the spindle are threaded 20 and 25 threads per inch R.H., so that one revolution of the spindle causes an axial displacement of (1/20 - 1/25) in., *i.e.* 0·01 in. Backlash is prevented in the micrometer screws by means of the spring shown. The head of the spindle is provided with a thimble 6, divided into ten main parts, each part being subdivided into fifths, so that one small division represents an axial movement of 0·0002 in.

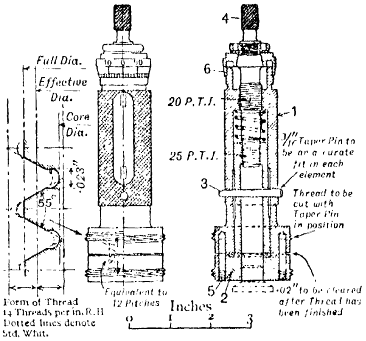

Full Dia.
Effective Dia.
Core Dia.
·023"
55°
Form of Thread 14 Threads per in. R.H. Dotted lines denote Std. Whit.
10 0 50
Equivalent to 12 Pitches
Inches
0 1 2 3
4
6
20 P.T.I.
1
25 P.T.I.
3/16" Taper Pin to be accurate fit in each element
3
Thread to be cut with Taper Pin in position
5
2
·02" to be cleared after Thread has been finished

FIG. 89.

The screw of the gauge is finished to size and the distance between the two portions of the screw is made equal to exactly 12 pitches when the taper pin is in position and when the micrometer reads zero. The effective diameter is such that the gauge will just screw into the smallest size of ring which it is intended to test.

Before screwing the gauge into a ring the taper pin 3 is removed, since, owing to possible errors in pitch of the ring, it may be necessary to adjust the micrometer screw to allow the upper portion of the screw to enter. With the gauge screwed completely in the ring the micrometer spindle is rotated as far as it will go, first in one direction and then in the other, readings R_1 and R_2 being taken on the thimble for each position. The fits of the micrometer screws and the plunger of the gauge are sufficiently free to make the "feel" of the micrometer quite definite.

Diagram A of *Fig.* 90 shows the positions of the two portions of the screw of the gauge when they have been separated as far as possible, and diagram B when they are brought as close as possible. The error in the pitch of the ring over 12 threads is equal to $(a+b)$; in the figure this error is negative, *i.e.* the pitch is short. Now from the two diagrams A and

B, the actual diminution of the distance between the two portions of the gauge from setting A to setting B is given by

$$(P_T + R_1) - (P_T - R_2), \text{ i.e. } R_1 + R_2,$$

and from the diagrams this is equal to $2a + 2(b + R_1)$.

$$\therefore 2a + 2(b + R_1) = R_1 + R_2,$$

$$\therefore a + b = \frac{R_2 - R_1}{2}.$$

In other words, the error in pitch is given by the difference of the mean reading from the zero position of the micrometer. Also, the sense of the error is obtained by noting for which direction of rotation of the micrometer the reading is larger. From the design of the differential screws it is clear that if the reading obtained by screwing clockwise is the larger, then the pitch of the ring is long, and *vice versa*.

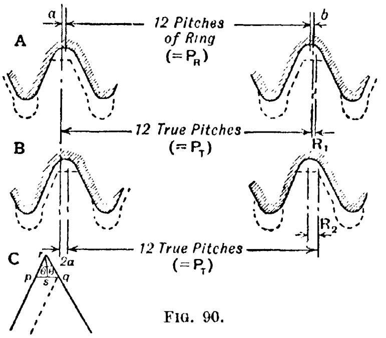


FIG. 90.

From *Fig.* 90 it is obvious that the effective diameter of the ring is appreciably larger than that of the gauge, and the difference between the two is clearly measured in terms of the sum of the readings R_1 and R_2. In diagram C the length pq represents $\frac{1}{2}(R_1 + R_2)$. The difference in the effective radii of the ring and of the gauge is equal to rs, so that the effective diameter of the ring is larger than that of the gauge by an amount $2 \times rs$. This is equal to $pq \cdot \cot \theta$, *i.e.* $\frac{1}{2}(R_1 + R_2) \cot \theta$. In the case of Whitworth thread, $\theta = 27\frac{1}{2}°$ and $\cot \theta$ is equal to 1·92098. Consequently the difference in the effective diameters is equal to $0{\cdot}96 \times (R_1 + R_2)$.

To take a practical example, suppose in a particular ring gauge it is possible to screw up the micrometer spindle in a clockwise direction by an amount registered as 5 divisions from zero, and the amount registered in the opposite sense is 12 divisions. The pitch of the ring is clearly short and by an amount equal to $\frac{1}{2}(12 - 5)$ divisions, *i.e.* 0·0007 in. The effective diameter of the ring is larger than that of the gauge by $0{\cdot}96 \times (5 + 12)$ divisions, *i.e.* 0·0033 in. approximately.

It should be noted that, in addition to standardising the gauge from the dimensions of its screw, it can readily be checked on a ring gauge whose pitch and effective diameter are accurately known.

The gauge described above serves for measuring the pitch over the length taken up by a certain number of threads, and it gives a mean value of the effective diameter of the ring at the two positions corresponding to the two portions of the interrupted thread of the gauge. It is desirable, however, to be able to measure the effective diameter of a ring at several positions along its length, and for this purpose the gauge shown in *Fig.* 91 was designed. This gauge differs from the former, shown in *Fig.* 89, only with regard to the interrupted thread. The end of the plunger is dovetailed into the end of the body and on the enlarged portion two complete turns of thread are cut, two diametral quadrants of the thread being on the body of the gauge and the other two on the plunger. As before, the two portions of the thread can be moved in an axial direction relative to each other by means of the differential micrometer screw. The method of use and the calculation of the difference in effective diameter are the same as for the first type of gauge.

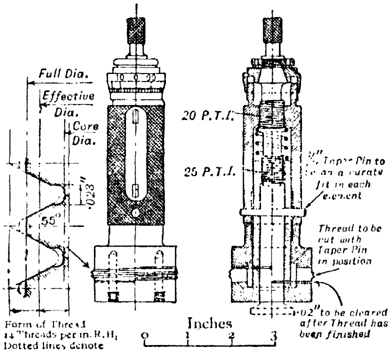

Full Dia.
Effective Dia.
Core Dia.
·023″
55°
10
0
90
20 P.T.I.
25 P.T.I.
⅛″ Taper Pin to be an accurate fit in each element
Thread to be cut with Taper Pin in position
·02″ to be cleared after Thread has been finished
Form of Thread
14 Threads per in. R.H.
Dotted lines denote Std. Whit.
Inches
0
1
2
3

FIG. 91.

This gauge has the advantage over the former type in allowing a number of local measurements of the effective diameter of a ring to be obtained.

(vi.) *Adjustable Ring Gauge for B.A. Screws.*—Designed by Mr. H. I. Brackenbury.

This gauge, which is intended for B.A. screws, is shown in *Fig.* 92. It consists of a circular base 1, which has a recess turned in its upper face. A disc 2 fits in this recess and is held in place by means of the retaining ring 3.

The disc 2 can be turned relative to the base 1 when the ring is screwed home, but there is no axial play between the disc and the recess. An accurate hole is tapped through the two parts 1 and 2 so as to be a continuous thread. This hole forms the gauging parts of the instrument. As originally designed, the hole was threaded with a screw having a full form thread, but better results are obtained if the core and full diameters are cleared, leaving only the flanks of the thread.

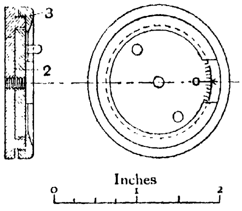


FIG. 92.

The method of using the gauge is to first screw the thread to be tested completely into both parts of the gauge and then to rotate the disc 2 as far as it will go, first in one direction and then in the opposite sense, the amounts of rotation being read on the scale on the disc against a line on the upper surface of the base. From these readings it is possible to estimate the errors in pitch and effective diameter of the screw in a manner similar to that described for the gauges in *Figs.* 89 and 91. The error in the effective diameter is proportional to the amplitude of the angular motion of the disc, and the error in pitch is proportional to the mean of the two readings. The gauge is standardised by taking readings on a plug screw gauge of known effective diameter and pitch.

It should be borne in mind that errors in the angle of the thread will affect the measurements of the effective diameter when using such gauging instruments as shown in *Figs.* 89, 91, and 92. The measurements actually give the virtual effective diameter, *i.e.* the effective diameter at the half depth of the thread augmented in the case of plugs, and diminished in the case of ring screw, by the amount corresponding to the error present in the angle.

Adjustable ring gauges for measuring the pitch and effective diameter of larger sizes of screws can be designed on the principle shown in *Fig.* 92, but an improvement can be made by limiting the thread in each portion to one or two turns as in *Figs.* 89 and 91.

(vii.) *Special Gauge for Plug Screws.*—With the exception of certain of the types of gauges described above it is usually necessary to screw the work and the gauge together. This, in time, causes wear of the gauge with corresponding inaccuracy in the results obtained. The gauge about to be described, which is for plug screws, permits of the work being inserted directly between certain gauging points which not only saves wear on the gauge, but also results in a saving of time for the actual gauging operation.

The gauge which is patented under the names of Sir Henry Fowler, Mr. C. H. Taylor, and Mr. H. J. Alpe (Patent No. 116555) is shown in *Fig.* 93, of which A represents a diagram of the side view with one side cover removed, B a view of the screw and the gauging points, and C a plan of the instrument. The gauge consists essentially of a steel block 2 having a transverse gap 3 for accommodating the screw 1 to be tested. A plate 4, which constitutes the fixed gauging member, is screwed to the block 2 on the one side, and a similar gauging member 5 is pivoted slightly eccentrically to the block and diametrically opposite on the other side of the gap, as shown at B. Each of the gauging members is provided with a pair of projecting teeth, the form of which may be the complete profile of the thread, or they may be cleared at the crests and roots so as to engage only with the flanks of the threads of the screw to be tested, if the gauge is not required to take account of the errors in the full and core diameters. The relative position of the two gauging members is indicated on the scale by the index

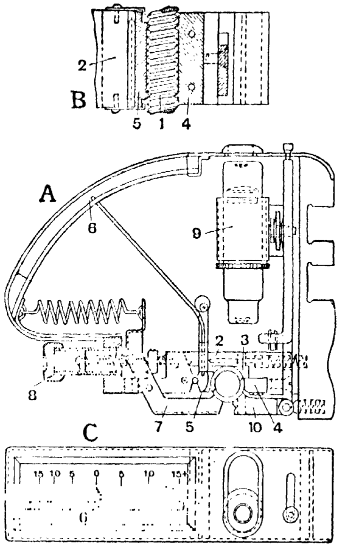


FIG. 93.

point 6, which forms the end of a thin pointer attached to the rotating member 5. A finger-piece 7 is provided for holding the screw firmly in the gap during the measurement. To insert a screw, this finger-piece is raised by means of the spring-controlled plunger 8, which also slightly rotates the gauging member 5 so as to widen the gap between the points. Having placed the screw in the gap, the plunger is released. This first allows the finger-piece to clamp the screw in the gap and in contact with the fixed gauging member, and then permits the movable member to rotate on its pivots until it makes contact with the other side of the screw.

The pointer 6 is set to the zero of the instrument when an accurate screw or gauge has been inserted in the gap. The distance between the gauging points on the fixed member 4 is equal to the length over which it is required to test the screw, and it should be noted that, in the case of the truncated points the reading indicates not only the error in the effective diameter of the screw, but in addition the equivalent of the errors in the pitch and angles of the flanks—in other words, the error of the "virtual" effective diameter. If the points are made with full form profiles the gauge acts as a full form high limit gauge; it then takes account also of the errors in the full and core diameters.

In order to inspect the form of the thread, a suitable microscope 9 is incorporated in the instrument, and provision is made for tilting this microscope so as to view in the direction of the rake of the thread. The tilting adjustment is not strictly correct, as the profile seen is that of an oblique section, normal to the direction of the rake, instead of a true diametral section of the thread. The error involved, however, is not serious for the purpose of the instrument. The field of the micrometer is illuminated through a small hole in the hinged plate. The eye-piece is provided with a graticule as in *Fig.* 94, which allows the image B of the thread of the screw to be compared with the two standard profiles A and B, which are set out to the correct magnification and which are separated diametrically by a distance representing, to the same scale, the radial tolerance on the diameters of the screw.

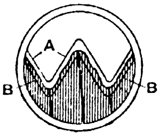


FIG. 94.

The gauging points are hardened, and as the screws do not have to be screwed in and out of the gauge, the points retain their accuracy of form over a relatively long period.

§ (41) METHODS OF GENERATING THREAD FORMS AND PRODUCING SCREW GAUGES.—Accurate single-thread tools designed to complete the whole thread form in one operation are often used in the production of screw gauges on a large scale. These tools frequently take the form of discs, as shown in *Fig.* 95, in which the cutting edge *a* can be ground back as required. All such tools, however, have to be originated from single-point tools; the latter consequently form the basis from which all accurate screwed work is ultimately derived.

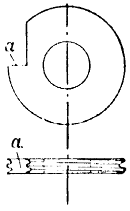


FIG. 95.

One of these single-point tools is shown in *Fig.* 96, and such tools may be used directly in the production of gauges where only small numbers are required. The tools must have the sides of the vee at the correct angle and must also have the correct radius at the point. The sides of the tool can be made to the correct angle, and the angle can be measured without special difficulty, but the exact radiusing of the point is not so easy. If the amount of shortening S is incorrect, the tool will not cut the core and effective diameters of a screw in correct relation to each other. If, for example, the shortening is insufficient, then, when the tool is fed up to the work so as to make the effective diameter correct, the core diameter will be left too small.

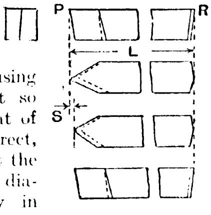


FIG. 96.

To obtain a tool with the correct shortening, a first approximation at any rate may be obtained by first grinding and lapping it perfectly sharp with an angle of exactly 55° and, measuring its length with a micrometer from the sharp point P to a suitable point R at the opposite end. The amount S, by which it should be shortened in forming the radius at the point, is equal to one quarter of the depth of a Whitworth thread of the particular pitch to be cut. If this shortening is carefully done, and the point nicely rounded off, it ought to require very little further modification to adjust the tool finally so as to bring both core and effective diameters down simultaneously within the limits allowed. The exact amount of adjustment required will be ascertained from the results of measurements made on a screw which is being cut by it.

This method is open to the objection that it is difficult to grind or lap the tool in the first place to a perfectly sharp point, and is recommended only as a means of obtaining a first approximation to the form, in the absence of more elaborate apparatus.

A convenient instrument for measuring the amount that the point has been ground back is illustrated in *Fig.* 97. The micrometer head is first set to read on a small cylinder of known size wedged in the vee of the instrument. Preferably, the micrometer head is adjusted in position in its seating until this reading is the exact measure of the truncation corresponding to a cylinder of the size employed. Readings subsequently taken on the point of a screw-cutting tool then give the exact measure of the truncation or shortening of the point of the tool. If C is the diameter of the cylinder used, the truncation S for the Whitworth thread is 0·583 × C.

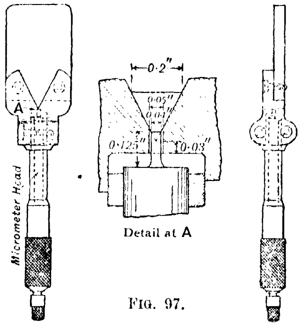


FIG. 97.

The actual cutting part of the tool is, of course, only the extreme tip, and with the smaller sizes this extreme tip will not find a bearing on the sides of the vee in the instrument, owing to the clearance which has to be provided for the micrometer point. The success of this instrument therefore depends on the absolute flatness of the two surfaces forming the sides of the cutting tool. Subject to this condition the correctness of the angle of the tool, and its truncation, can be determined with considerable precision. The tool should have a definite side clearance on each side, as shown in *Fig.* 96, or the test on angle will fail. Usually, however, owing to the rake, the clearance on one side will be very small, as indicated in the same figure.

It is necessary to reduce the diameter of the measuring point of the micrometer head down to about 0·04 in. as shown, until only a small face is left, and the hole left in the vee for the passage of the measuring point should be kept as small as possible, having regard to the sizes of tool to be measured. The dimensions indicated are suitable for measuring tools of any size up to 4 threads per inch, and should not be exceeded unless still larger tools are in contemplation.

It is to be remembered that the 55° is to be the angle of the horizontal cutting surface of the tool, which is to be presented to the work exactly at the height of the lathe centres. No cutting lip should be ground in the top face of the finishing tool, which should be left perfectly flat. Whatever the value of the clearance angle at which the point of the tool is backed away, and whatever side rake is given to the front edge of the tool for cutting coarse pitches, the 55° is to be the angle of the horizontal top face of the tool. If the tool is made to a 55° gauge held square with the backed-off front edge of the tool, the horizontal angle will be too small by an amount depending upon the clearance angle of the tool. It will be seen that the single-point tool used for finishing can only take very light cleaning cuts. The tools used in preparing the threads for the finishing cuts may be given a cutting lip, provided this is not so excessive as to leave a thread of such imperfect form that the final cut with the finishing tool fails to correct it.

The cutting faces of the tool should be finished by lapping. If taken straight from a grinding wheel it is possible for the general angle of the tool to be correct, but for the point, which is the important part in cutting the thread, to be in error owing to elastic deformation during grinding. This error in angle at the point of the tool is fairly frequently found.

To ensure that the tool is satisfactorily radiused it is best to view it under a microscope or to project a magnified image on to a screen. If a projection machine such as is described in §§ (68) or (69) is available, it is easy to compare the tool with a drawing of the standard thread form and to make the tool conform to this. Exceptionally good definition can be obtained in projecting a tool owing to the backing off, and it is possible with a high magnification to check the truncation at the same time as the form at the point.

Whatever method of measurement be adopted in preparing a tool, the final test is to cut a screw, and measure its effective and core diameters, and thread angle, and to examine the rounding at the root. If the shortening of the tool at the point is correct the difference between the effective and core diameters of the screw will be equal to the standard depth of the thread for the particular pitch concerned. If the difference is found to be too small, then the tool has been shortened too much at the point and *vice versa*. The shortening at the point should be adjusted by lapping either the sides or the point of the tool until the correct difference is obtained.

Having produced a tool which will cut the effective and core diameters so that both are within the recognised limits of accuracy, it remains to complete the tops of the threads. The radius of the crest being the same as that at the root, the tool which has been used to cut the screw may be used to cut a suitable groove in the nose of the capping tool, shown in *Fig.* 98. This tool, when in use, is adjusted by eye (or preferably automatically on being placed in the tool-holder), so as to fit properly the flanks of the thread already formed. If, for any reason, the gauge has to be taken off the centres after the operation of forming the flanks and roots, care should be taken to see that the screw runs truly in the lathe before the capping tool is used, otherwise an error in concentricity may be introduced between the full diameter and the remaining elements of the screw.

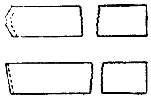


FIG. 98.

From a pair of single-pointed tools as above described, full-form single-thread tools of various types may be generated.

Screw gauges are practically useless unless they are hardened. There are certain methods of hardening which are stated to produce no distortion of the gauge, and when using such processes on gauges which have been accurately cut in the unhardened state, it is only necessary to lightly polish the threads to remove the stain due to the heating effect. It usually happens, however, that the gauge becomes distorted when being hardened, and some correcting process has to be adopted. The most common method is to use suitable laps which for plug screws take the form of cast-iron split rings, screwed internally with the correct form of thread, and which are charged with some fine abrasive powder such as emery or carborundum. Very often several such laps are used, one for the flanks of the thread, one for the crests, and another for the roots.

Another method which is also in use is to grind the thread after hardening. The machines used for this purpose take the general form of a lathe, but the cutting tool is replaced by a thin grinding disc about 4 in. diameter which has its periphery shaped similar to the cutting point of the tool shown in *Fig.* 96. A special appliance which is practically automatic in its action is used for trimming up the cutting edge of the wheel. A second wheel is used for cresting the thread, its periphery being shaped as in *Fig.* 98.

Screwed ring gauges above about $\frac{1}{2}$ in. diameter are usually first screwed on the lathe, using single-point tools or a single-thread chaser. Below this size they are usually threaded with accurately made taps, special jigs being used at times for guiding the tap and to give it the correct lead. If distortion takes place in hardening, the rings are corrected by lapping, the laps taking the form of cast-iron screwed plugs, which are split through the centre and capable of being expanded. Here again it is the common practice to use a number of laps for the different parts of the thread.

§ (42) HARDENING GAUGES.—The working faces of all gauges used for engineering purposes should be of hardened steel. There are various methods of obtaining this hardened surface:

(*a*) The whole of the gauge or, in some cases, its working parts only are made of tool steel and the metal is hardened right through.

(*b*) The gauge or its working parts are made of mild steel which is case-hardened. This process consists of packing the mild steel parts in boxes with a material rich in carbon, the boxes being afterwards heated for several hours to a temperature of about 930° C. This results in an absorption of carbon by the surface of the parts; the depth to which the carbon penetrates depends upon the carbonising material used and the duration of the heating process. If parts which have been carbonised by this process are afterwards subjected to the ordinary hardening process of heating to about 750° C. and quenching in water or oil, it is found that the high carbon steel at the surface becomes hardened and forms a casing round the relatively soft, mild steel core. The thickness of the casing adopted in gauges is of an order of $\frac{1}{32}$ or $\frac{1}{16}$ in.

A modification of this process consists in heating the mild steel parts to a temperature of about 750° C. for about an hour in a bath of molten sodium cyanide, a material which is rich in carbon, and afterwards quenching from a temperature of about 700° C. Parts treated by this process have a hard casing, but its thickness is quite small, being often only a few thousandths of an inch. This thin casing, however, answers the purpose for small light parts or gauges.

Hardening generally is frequently a cause of serious trouble to gauge-makers, especially those engaged on the manufacture of screw gauges. The difficulty arises owing to distortion of the gauge during the hardening process, and, as a consequence, any previous efforts to finish the gauge accurately to size are wasted. For information on this subject reference may be made to a paper read before the Institution of Mechanical Engineers in April 1920 on "The Hardening of Screw Gauges with the Least Distortion in Pitch."

Useful information on the subject will also be found in Reiser's *Hardening and Tempering of Steel* (Scott, Greenwood & Sons).

A further point in connection with hardening gauges, which mainly concerns reference and standard gauges, is the question of stability of the steel. It is found that hardened steel has a tendency to change its size with time. This change occurs most rapidly in the first few weeks after the steel has been hardened, and the metal gradually becomes more stable as time goes on. The change is most noticeable in very hard steels; this is natural, as such a steel must be in a very severe state of internal stress. If the steel is tempered and the strains become somewhat relieved it becomes more stable.

This secular change in hardened steel can be reduced by artificial ageing through a special heat treatment. This method consists in heating the steel up to a temperature of about 200° C. and allowing it to cool down at a uniform rate over a period of several hours. The heating and cooling process is repeated several times, but the maximum temperature attained is gradually reduced on each occasion.

Results on the tests for stability of gauges are given in *American Machinist* for November 27, 1920.

§ (43) STANDARD TYPES OF SCREW THREADS.—There are several well-recognised types of screw threads in common use in Great Britain, America, and on the Continent; some of the best known are the Whitworth Thread, British Association Thread, Square Thread, Acme Thread, Sellers Thread, Metric Thread, and Lowenherz Thread. Particulars of these and other threads are given below.

§ (44) THE WHITWORTH STANDARD THREAD.—The Whitworth Standard screw thread was first proposed by Sir Joseph Whitworth about the middle of the last century, and has been adopted for general engineering purposes in this country. The original tables of diameters and pitches were considered later by the British Engineering Standards Association, which laid down tables for three different systems of threads, known as the British Standard Whitworth (B.S.W.), the British Standard Fine (B.S.F.), and the British Standard Pipe (B.S.P.) threads. The Whitworth form of thread is used in each of these systems, the main difference between them being the correlation of the diameters and the pitches. The standard sizes for the three systems are given in *B.E.S.A. Reports*, Nos. C.L. 7270, 84, and C.L. 6599 respectively.

The form of the Whitworth thread is shown in *Fig.* 99.

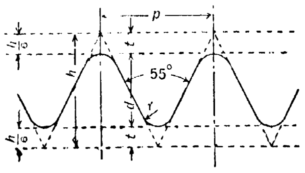


FIG. 99.

Symmetrical thread. Angle = 55°.
Angular depth of thread = $h = 0{\cdot}9605 \times$ pitch (p).
Truncation at crest and root = $t = \frac{1}{6} \times h$.
Finished depth of thread = $d = 0{\cdot}6403 \times p$.
Radius at crest and root = $r = 0{\cdot}137 \times p$.

§ (45) BRITISH ASSOCIATION THREAD.—For screws less than ¼ in. diameter the Standard thread recommended by the B.E.S.A. is the B.A. thread. The standard sizes, which are in millimetre units, are given in the *B.E.S.A. Report*, No. C.L. 7271.

The form of the thread is given in *Fig.* 100.

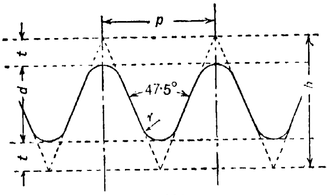


FIG. 100.

Symmetrical thread. Angle = 47°·5.
Angular depth of thread = $h = 1{\cdot}136 \times p$.
Truncation at crest and root = $t = 0{\cdot}236 \times h$.
Finished depth of thread = $d = 0{\cdot}6 \times p$.
Radius at crest and root = $r = \frac{2}{11} \times p$ (approx.).

§ (46) SQUARE THREAD.—This form of thread is used for screws which have to transmit or withstand considerable end thrusts as in screw presses. The form of the thread is shown in *Fig.* 101.

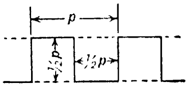


FIG. 101.

This type of thread is difficult to cut with taps and dies and is being superseded to a considerable extent by the Acme thread.

§ (47) ACME THREAD.—This form of thread is used extensively for transmitting thrusts and is the type most frequently used for lead screws of lathes. The form of thread is shown in *Fig.* 102.

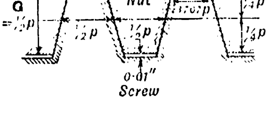


FIG. 102.

A radial clearance of 0·01 in. is allowed at the root of the screw and at the full diameter of the nut. The depth of engagement of the threads is equal to half the pitch.

§ (48) SELLERS THREAD.—The Sellers or United States Standard thread is the type used in that country for general engineering work. The form is shown in *Fig.* 103.

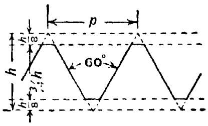


FIG. 103.

Symmetrical thread. Angle = 60°.
Angular depth of thread = $h = 0{\cdot}8660 \times p$.
Truncation at crests and roots = $\frac{1}{8} \times h$.
Finished depth of thread = $d = 0{\cdot}6495 \times p$.
Crests and roots are flat.
Width of flats at crests and roots = $f = \frac{1}{8} \times p$.

§ (49) INTERNATIONAL METRIC THREAD.—The International Standard Screw thread was adopted at the International Congress for the standardisation of screw threads held at Zurich in 1899. It is used extensively on the Continent and also to some extent in this country in the automobile industry. The form of the thread is shown in *Fig.* 104,

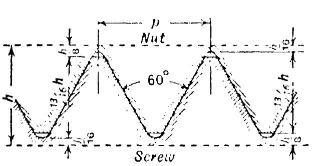


FIG. 104.

from which it will be seen that a definite clearance is made at the roots of the screw and at the full diameter of the nut. This gives the screws a better opportunity of fitting together on the flanks of the thread.

Angle = 60°.
Angular depth of thread = $0{\cdot}8660 \times p$.
Truncation at crest of screw and at core diameter of nut = $\frac{1}{8} \times h$.
Truncation at root of screw and at full diameter of nut = $\frac{1}{16} \times h$.
Radius at root of screw and at full diameter of nut = $\frac{1}{16} \times h$.

§ (50) LOWENHERZ THREAD.—The Lowenherz form of thread is used in Germany for small screws such as are used in instrument work. The form of the thread is given in *Fig.* 105. The angular depth of the thread is made equal to the pitch.

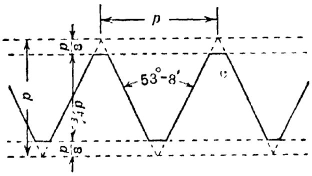


FIG. 105.

Symmetrical thread. Angle = 53° 8′.
Angular depth of thread = p.
Truncation at crests and roots = $\frac{1}{8} \times p$.
Finished depth of thread = $\frac{3}{4} \times p$.

§ (51) BUTTRESS THREAD.—This form of thread is sometimes used for screws which have to withstand an axial force acting in only one direction. The angle between the flanks is 45°, and one flank is normal to the axis of the screw, as shown in *Fig.* 106. The

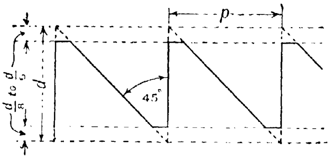


FIG. 106.

truncation at the crests and roots varies in practice from $\frac{1}{6}$ to $\frac{1}{8}$ of the angular depth of the thread. The roots are often rounded for ease in manufacture.

§ (52) CYCLE ENGINEER'S THREAD.—This form of thread was standardised by the Cycle Engineers' Institute in 1902, and is shown in *Fig.* 107.

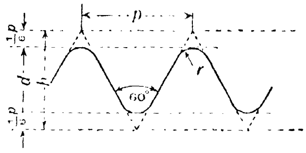


FIG. 107.

Symmetrical thread. Angle = 60°.
Angular depth of thread = $h = 0{\cdot}8660 \times p$.
Truncation at crests and roots = $\frac{1}{6} \times p$.
Finished depth of thread = $d = 0{\cdot}5327 \times p$.
Radius at crests and roots = $\frac{1}{6} \times p$.

§ (53) BRIGGS PIPE THREAD.—This thread is used in the United States, and is now standardised as the National Pipe Thread. The form of thread is shown in *Fig.* 108.

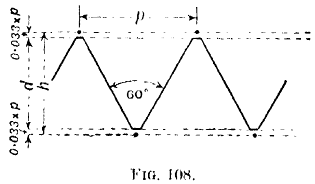


FIG. 108.

Symmetrical thread. Angle = 60°. (Square to axis of pipe.)
Angular depth of thread $h = 0{\cdot}8660 \times p$.

Truncation at crests and roots $= 0{\cdot}0330 \times p$.

Finished depth of thread $= d = 0{\cdot}8000 \times p$.

Crests and roots are flat.

The threads on the pipes and fittings are tapered by $\frac{1}{16}$ in. per inch on diameter.

Particulars of other forms of threads of lesser importance than those given above will be found in *Machinery's Screw Thread Book* (Machinery Publishing Co., Ltd., London).

§ (54) STANDARD SIZES OF SCREW THREADS.—The Tables below give the full diameters and corresponding pitches for the commoner systems of screw threads which have been standardised. References to the official publications are given in each case.

§ (55) BRITISH STANDARD WHITWORTH THREADS (B.S.W.).—Used in general engineering work. Reference: *B.E.S.A. Report*, No. C.L. 7270.

Full Diameter (Inches).	Threads per Inch.	Full Diameter (Inches).	Threads per Inch.
$\frac{1}{4}$	20	$1\frac{3}{8}$ *	6
$\frac{5}{16}$	18	$1\frac{1}{2}$	6
$\frac{3}{8}$	16	$1\frac{5}{8}$ *	5
$\frac{7}{16}$	14	$1\frac{3}{4}$	5
$\frac{1}{2}$	12	$1\frac{7}{8}$ *	4·5
$\frac{9}{16}$	12	2	4·5
$\frac{5}{8}$	11	$2\frac{1}{8}$ *	4·5
$\frac{11}{16}$ *	11	$2\frac{1}{4}$	4
$\frac{3}{4}$	10	$2\frac{3}{8}$ *	4
$\frac{13}{16}$ *	10	$2\frac{1}{2}$	4
$\frac{7}{8}$	9	$2\frac{5}{8}$ *	4
$\frac{15}{16}$ *	9	$2\frac{3}{4}$	3·5
1	8	$2\frac{7}{8}$ *	3·5
$1\frac{1}{8}$	7	3	3·5
$1\frac{1}{4}$	7	..	..

* The *B.E.S.A.* recommends that for general use these sizes be dispensed with.

NOTE.—The table is continued up to 6 in. in the *Report*.

§ (56) BRITISH STANDARD FINE THREADS (B.S.F.).—Used in general engineering work where a somewhat finer pitch is required. Reference: *B.E.S.A. Report*, No. 84.

Full Diameter (Inches).	Threads per Inch.	Full Diameter (Inches).	Threads per Inch.
$\frac{7}{32}$	28	1	10
$\frac{1}{4}$	26	$1\frac{1}{8}$	9
$\frac{9}{32}$	26	$1\frac{1}{4}$	9
$\frac{5}{16}$	22	$1\frac{3}{8}$	8
$\frac{3}{8}$	20	$1\frac{1}{2}$	8
$\frac{7}{16}$	18	$1\frac{5}{8}$	8
$\frac{1}{2}$	16	$1\frac{3}{4}$	7
$\frac{9}{16}$	16	2	7
$\frac{5}{8}$	14	$2\frac{1}{4}$	6
$\frac{11}{16}$	14	$2\frac{1}{2}$	6
$\frac{3}{4}$	12	$2\frac{3}{4}$	6
$\frac{13}{16}$	12	3	5
$\frac{7}{8}$	11	..	..

§ (57) BRITISH ASSOCIATION THREADS (B.A.).—Used in instrument work. Reference: *B.E.S.A. Report*, No. C.L. 7271.

Designating Number.	Full Diameter (Millimetres).	Pitch (Millimetres).
0	6·0	1·0
1	5·3	0·9
2	4·7	0·81
3	4·1	0·73
4	3·6	0·66
5	3·2	0·59
6	2·8	0·53
7	2·5	0·48
8	2·2	0·43
9	1·9	0·39
10	1·7	0·35
11	1·5	0·31
12	1·3	0·28
13	1·2	0·25
14	1·0	0·23
15	0·9	0·21

NOTE.—The table is continued in the *Report* down to the size No. 25.

§ (58) BRITISH STANDARD PIPE THREADS (B.S.P.).—Smaller sizes used in general engineering work. Reference: *B.E.S.A. Report*, No. C.L. 6599.

Nominal Bore of Pipe (Inches).	Full Diameter of Thread (Inches).	Threads per Inch.
$\frac{1}{8}$	0·383	28
$\frac{1}{4}$	0·518	19
$\frac{3}{8}$	0·656	19
$\frac{1}{2}$	0·825	14
$\frac{5}{8}$	0·902	14
$\frac{3}{4}$	1·041	14
$\frac{7}{8}$	1·189	14
1	1·309	11
$1\frac{1}{8}$	1·492	11
$1\frac{1}{4}$	1·650	11
$1\frac{3}{8}$	1·745	11
$1\frac{1}{2}$	1·882	11

§ (59) SPECIAL FRENCH METRIC SCREWS.—This series of screws was standardised by the B.E.S.A. for use in aircraft engine construction. The System International form of thread was adopted. Reference: *B.E.S.A. Report*, No. C.L. 3750.

Nominal Size (Millimetres).	Pitch (Millimetres).	Nominal Size (Millimetres).	Pitch (Millimetres).
3	0·60	8	1·25
4	0·75	9	1·25
5	0·75	10	1·50
6	1·00	11	1·50
7	1·00	12	1·75

For sparking plugs the standardised sizes are:

Pitch . . . . . . 1·5 mm.

	Sparking Plug (Millimetres).		Tapped Hole (Millimetres).	
	Max.	Min.	Max.	Min
Full Diameter .	17·975	17·850	18·312	18·187
Effective Diameter	17·001	16·876	17·176	17·051
Core Diameter .	15·864	15·739	16·201	16·076

Reference: *B.E.S.A. Report*, No. 45.

§ (60) U.S.A. NATIONAL COARSE THREADS.—These screws, which have the Sellers form of thread, are used for general engineering purposes. Reference: *Report of National Screw Thread Commission* (U.S.A.), 1920.

Full Diameter (Inches).	Threads per Inch.	Full Diameter (Inches).	Threads per Inch.
0·073	64	$\frac{9}{16}$	12
0·086	56	$\frac{5}{8}$	11
0·099	48	$\frac{3}{4}$	10
0·112	40	$\frac{7}{8}$	9
0·125	40	1	8
0·138	32	$1\frac{1}{8}$	7
0·164	32	$1\frac{1}{4}$	7
0·190	24	$1\frac{1}{2}$	6
0·216	24	$1\frac{3}{4}$	5
$\frac{1}{4}$	20	2	4·5
$\frac{5}{16}$	18	$2\frac{1}{4}$	4 5
$\frac{3}{8}$	16	$2\frac{1}{2}$	4
$\frac{7}{16}$	14	$2\frac{3}{4}$	4
$\frac{1}{2}$	13	3	4

§ (61) U.S.A. NATIONAL FINE THREADS.—These screws, which have the Sellers form of thread, are used where somewhat finer pitches are required. Reference: *Report of National Screw Thread Commission* (U.S.A.), 1920.

Full Diameter (Inches).	Threads per Inch.	Full Diameter (Inches).	Threads per Inch.
0·060	80	$\frac{9}{16}$	18
0·073	72	$\frac{5}{8}$	18
0·086	64	$\frac{3}{4}$	16
0·099	56	$\frac{7}{8}$	14
0·112	48	1	14
0·125	44	$1\frac{1}{8}$	12
0·138	40	$1\frac{1}{4}$	12
0·164	36	$1\frac{1}{2}$	12
0·190	32	$1\frac{3}{4}$	12
0·216	28	2	12
$\frac{1}{4}$	28	$2\frac{1}{4}$	12
$\frac{5}{16}$	24	$2\frac{1}{2}$	12
$\frac{3}{8}$	24	$2\frac{3}{4}$	12
$\frac{7}{16}$	20	3	10
$\frac{1}{2}$	20		

§ (62) U.S.A. NATIONAL PIPE THREADS.—hese screws have the Briggs form of hread. They are used for connections of ipes and pipe fittings. Reference: *Report of National Screw Thread Commission* (U.S.A.), 1920.

Nominal Bore of Pipe (Inches).	Outside Diameter of Pipe (Inches).	Threads per Inch.
$\frac{1}{8}$	0·405	27
$\frac{1}{4}$	0·540	18
$\frac{3}{8}$	0·675	18
$\frac{1}{2}$	0·840	14
$\frac{3}{4}$	1·050	14
1	1·315	11·5
$1\frac{1}{4}$	1·660	11·5
$1\frac{1}{2}$	1·900	11·5
2	2·375	11·5
$2\frac{1}{2}$	2·875	8
3	3·500	8
$3\frac{1}{2}$	4·000	8
4	4·500	8
$4\frac{1}{2}$	5·000	8
5	5·563	8
6	6·625	8
7	7·625	8
8	8·625	8
9	9·625	8
10	10·750	8
12	12·750	8

NOTE. The full diameter of the thread is somewhat less than the outside diameter of the pipe. The table is continued in the *Report* up to an outside diameter of 30 inches.

§ (63) DATA FOR USE IN THE MEASUREMENT OF EFFECTIVE DIAMETER OF PLUG SCREWS WITH STANDARD WIRES.—The effective diameter of a plug screw thread is equal to the measurement T, under the wires, *Fig.* 50, plus a certain constant which depends upon the mean diameter of the wires d, the pitch of the thread p, the thread angle $2a$, and the rake angle of the thread. This constant is usually denoted by P thus,

Effective diameter $E = T + P$.

The general formula for determining P is

$$P = \tfrac{1}{2} \cot a \times p - (\operatorname{cosec} a - 1) \times d - \left(\frac{\cos a \cot a}{2\pi^2}\right)\frac{p^2}{E^2} \times d \text{ (very nearly).}^1$$

For the Whitworth Thread Form (where $2a = 55°$),

$$P = 0{\cdot}96049 \times p - 1{\cdot}16568 \times d - 0{\cdot}086 \times \frac{p^2}{E^2} \times d.$$

For the United States Standard and International System Thread Forms (where $2a = 60°$),

$$P = 0{\cdot}86602 \times p - d - 0{\cdot}076 \times \frac{p^2}{E^2} \times d.$$

For the British Association Thread Form (where $2a = 47°{\cdot}5$),

$$P = 1{\cdot}13634 \times p - 1{\cdot}48295 \times d - 0{\cdot}105 \times \frac{p^2}{E^2} \times d.$$

---

[1] Jeffcott, "Notes on Screw Threads," *Proc. Inst. Mech. Engrs.*, 1907, ii.

For the Acme Thread Form (where $2a = 29°$),

$$P = 1·93336 \times p - 2·99393 \times d - 0·190 \times \frac{p^2}{E^2} \times d.$$

For the Lowenherz Thread Form (where $2a = 53° 8'$),

$$P = 0·99993 \times p - 1·23594 \times d - 0·091 \times \frac{p^2}{E^2} \times d.$$

The last term in the above formulae arises from the fact that the wires set themselves in directions corresponding to the helix angle of the thread. Generally speaking, the numerical value of this term is small and may be neglected, but it should be included in all cases where the rake angle of the thread is appreciable and the pitch is coarse. For B.S.W. threads the value amounts to nearly 0·0002 in. in some cases.

Assuming that the angle of a screw is correct, its effective diameter can be obtained by using any size of accurate wires provided they make contact somewhere on the straight flanks. The appropriate value for P is calculated by substituting in one of the above formulae the actual mean diameter of the wires used. If, on the other hand, the thread angle is incorrect, the real effective diameter is obtained most readily by using particular size wires which touch the thread at or very close to the half depth. These wires are usually known as the "best" size wires. Other sizes of wires can be used, but it becomes necessary either to substitute the actual measured angle of the particular thread in the above general formula for P, or else to apply a certain correction (formulae given below) to the P values obtained from one of the particular formulae, the constants of which are calculated from the nominal thread angle.

The range of diameter of wire which can be used in a thread is shown in *Fig.* 109. The maximum size rests at the extreme upper end of the straight portions of the flanks, whilst the minimum size, for practical reasons, is such that the top of the wire comes flush with the crests of the threads.

The following table gives formulae for calculating the maximum, minimum, and "best" sizes of wires for various thread forms :

Form of Thread.	Diameter of Wire.		
	Maximum.	" Best."	Minimum.
Whitworth	$0·853 \times p$	$0·564 \times p$	$0·506 \times p$
U.S. Standard and International System	$1·010 \times p$	$0·577 \times p$	$0·505 \times p$
British Association . .	$0·730 \times p$	$0·546 \times p$	$0·498 \times p$
Acme	$0·650 \times p$	$0·516 \times p$	$0·487 \times p$
Lowenherz	$0·978 \times p$	$0·559 \times p$	$0·541 \times p$

When using wires other than the "best" size for measuring a screw having an error of $\delta\theta$ degrees in the thread angle, the corrections to be applied to the P value are given below :

Form of Thread.	Correction to P value.
Whitworth . . .	$(0·036 \times d - 0·020 \times p) \times \delta\theta$
U.S. Standard and International System .	$(0·030 \times d - 0·017 \times p) \times \delta\theta$
British Association	$(0·049 \times d - 0·027 \times p) \times \delta\theta$
Acme	$(0·135 \times d - 0·070 \times p) \times \delta\theta$
Lowenherz . . .	$(0·039 \times d - 0·022 \times p) \times \delta\theta$

If the angle error is positive, and if wires larger than the "best" size are used, then the correction to be applied is also positive. The sign of the correction is given by the formula if the sign of $\delta\theta$ be introduced.

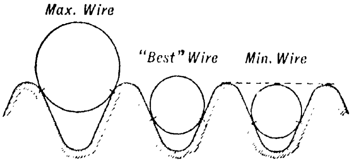


FIG. 109.

Assuming that the flanks of the threads are straight, it is possible to obtain a close estimation of the thread angle by measuring the effective diameter with two sets of wire which approach the maximum and minimum sizes respectively for the particular pitch and form of thread.

If E_1 = Effective diameter as obtained from the larger set of wires (mean diameter, d_1),

E_2 = Effective diameter as obtained from the smaller set of wires (mean diameter, d_2),

then $\delta\theta = (E_2 - E_1)/0·036(d_1 - d_2)$ for Whitworth threads.

Similar formulae can be obtained for the other thread forms by inserting the appropriate factor in the denominator as given in the last table.

This method gives only the total error in the thread angle, and does not differentiate between the individual errors of the two flanks which can only be observed by optical means. For this reason, the method of obtaining the error in the total angle by means of wire measurements, is used merely as a check on the optical measurement.

IV. OPTICAL PROJECTION APPARATUS FOR GAUGE TESTING

Up to about December 1915 the only optical method used in testing screw or profile gauges was the application of the microscope apparatus referred to in § (34). By means of this apparatus it was possible to take complete measurements of the smaller sizes of screws. Although a visual examination of the form of the thread could also be made on this apparatus it was not possible to state directly the amounts in thousandths, or ten-thousandths, of an inch by which the various parts of the profile differed from the standard, or nominal profile. Some assistance is obtained if the eye-piece of the micrometer is provided with a graticule, on the Rheinberg system, of the standard profile made to the appropriate magnification to suit the power of the objective used. By such means it is then possible to fit together the images of the two profiles and to note any departure of the actual image from that of the correct form. A means of measuring the differences, however, would still be lacking. Moreover, the system would make it necessary to change the graticule in the eye-piece for each different pitch of screw to be measured. The use of the microscope for screws having a coarse rake of thread also involves other difficulties as explained in § (33).

The question of the optical examination of screw threads was considered by Mr. E. M. Eden of the National Physical Laboratory in the autumn of 1915, and it soon became clear that some departure from the microscope form of apparatus would have to be made in order to obtain accurate results with semi-skilled observers. The first scheme was to use the optical system of the microscope to produce on a screen a magnified shadow-like image of the profile of the thread. This image was then compared by superposition with a diagram of the standard form drawn to the corresponding magnification. By suitable choice in the powers of the lenses it was possible to obtain a magnification of exactly 50 on the image, this figure being decided upon as being the most convenient for gauge-testing purposes. Knowing the magnification of the image, it became possible to measure its errors with reference to the standard diagram by means of a scale.

§ (64) FIRST PROJECTION APPARATUS FOR SCREWS.—The general scheme of the first projection apparatus is shown in *Fig.* 110, which is self-explanatory. The apparatus was arranged on a horizontal bench, about 6 ft. by 2 ft., and by means of the prism the image was obtained on a horizontal screen placed conveniently at one end. The success of the apparatus depended upon the formation of an image of uniform magnification throughout and free from chromatic defects. It was only after experimenting with several combinations of objectives and eye-pieces that these objects were satisfactorily attained.

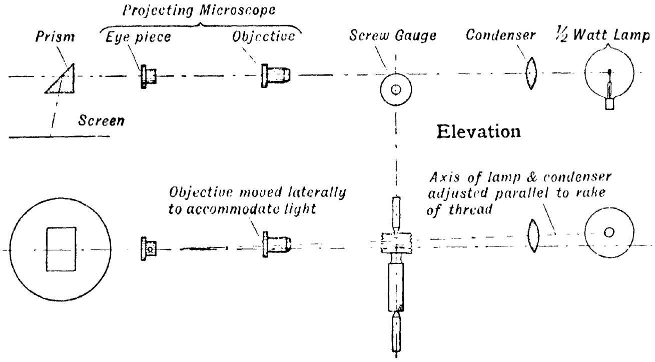

Projecting Microscope
Prism
Eye piece
Objective
Screw Gauge
Condenser
½ Watt Lamp
Screen
Elevation
Objective moved laterally to accommodate light
Axis of lamp & condenser adjusted parallel to rake of thread

FIG. 110.

The objective finally chosen was one marked "3 D," made by R. & J. Beck, Ltd.: the eye-piece was a Kelner's Orthoscopic by Ross. The diameter of the useful field over which no distortion could be detected was about 7 in. This was quite sufficient for ordinary screw gauge work. The two lenses and the screen were suitably disposed so as to give the required magnification. The centres for carrying the screw gauge were then placed at such a distance as to give the best definition of the image on the screen. When placing screws in the machine the longitudinal position of the centres was not changed, but provision was made for raising and lowering them to suit screws of different diameters.

Adjustment for Rake of Screw.—In this type of machine as well as in the microscope apparatus, it is necessary that the illuminating beam should be in the direction of the slope or rake of the threads. It should be carefully noted that, in order to obtain a true image of a diametral section of the screw, the axes of both the lenses have to be kept perpendicular to that of the gauge, and the screen itself must be parallel to the same axis. Any adjustment for the rake of the thread must be made on the illumination. For this purpose, the lamp and condenser were mounted on a frame which could be turned about a vertical axis under the eye piece. This inclination of the beam necessitated a lateral movement of the objective so that its centre might remain on the centre line of the beam, the perpendicularity between the axes of the objective and the gauge being preserved.

§ (65) Single Lens Apparatus for Screws.—The next step in the development of a projection apparatus for screw gauges was the substitution of a single lens for the pair of lenses.

After experimenting with a number of lenses it was found that the "Petzval" series, as used in kinematograph work and made by Dallmeyer, was particularly well suited for the purpose of screw-gauge projection. The No. 1 lens of this type has a 2-in. focus, and the distance from gauge to screen for a magnification of 50 is about 9 feet, whilst the No. 4 lens has a 3-in. focus, the distance in this case being about 13 feet. The definition given by these lenses is good, particularly in the case of the one with the shorter focus. Each lens gives an undistorted image over a sufficiently large field for screw-gauge work.

§ (66) Vertical Projection Apparatus.—In order to make the apparatus convenient to use, and incidentally to reduce the floor space required, Mr. Eden devised the scheme of carrying out the projection in a vertical direction. A horizontal mirror placed above the bench was used to reflect the image on to a horizontal screen which was placed at the level of the screw gauge. The principle of this vertical projection apparatus is shown in *Fig.* 111. A standard type of machine was designed at the National Physical Laboratory in 1917 on this principle. This machine not only allows the form of the thread to be inspected, but, being provided with measuring devices, it also enables plug screws up to 2 in. diameter to be measured completely. This machine is described fully in § (69).

§ (67) Compound Projection Lenses for Large Field of View.—So far we have concerned ourselves with the application of the projection apparatus to the testing of screw gauges. The principle was also developed by Mr. Eden in 1916 in connection with the examination of profile gauges in the form of plates.

The problem presented greater difficulties than the one on screw gauges referred to above. The main object was to be able to

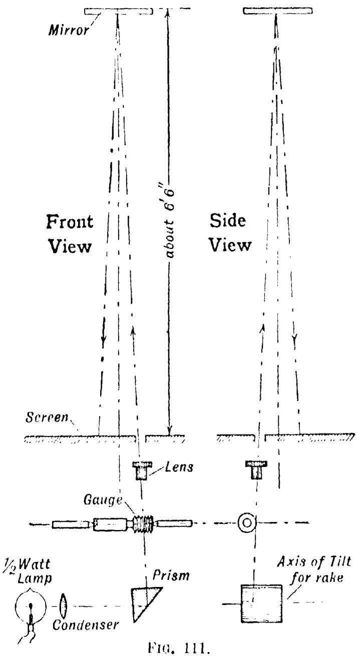


Fig. 111.

project an undistorted image of a plate gauge up to about 1½ in. in size. Such gauges were used in large numbers and varieties during the war on munition work for checking the form of the copper-driving bands of shells. The accuracy required in testing the profile of the gauges was about 0·0005 in.

It was found by experiment that even high-class camera lenses of large aperture when used alone would not give the desired result. The image produced was distorted in every case. This was hardly surprising, as such lenses are designed to give correct results when used for their own particular purpose. Such lenses could be used, however, as projection lenses, provided the whole of the image to be examined fell within an area of not more than about 2 feet in diameter, the distortion over this limited area being usually negligibly small. In order to obtain a sufficiently large undistorted image it was found necessary to use these lenses in conjunction with field lenses having an aperture of about 2½ in. The particular type of lens used for the field is a triple achromatic lens, made by Ross, having a focal length of about 6½ in. Special care has to be used in mounting the two lenses, as it was found that the distortion effects depend upon the distance between the two components. In practice, each pair of lenses should be tested first in an adjustable telescopic mount and their separation varied until the distortion is a minimum. The distance between the lenses is then measured and they are afterwards fitted to a special mount of this length.

The distortion is tested by projecting an image of a plain parallel plug. For the lens to be satisfactory this image should be of equal width from one side of the field to the other, and the definition of the edges should also be constant in different parts.

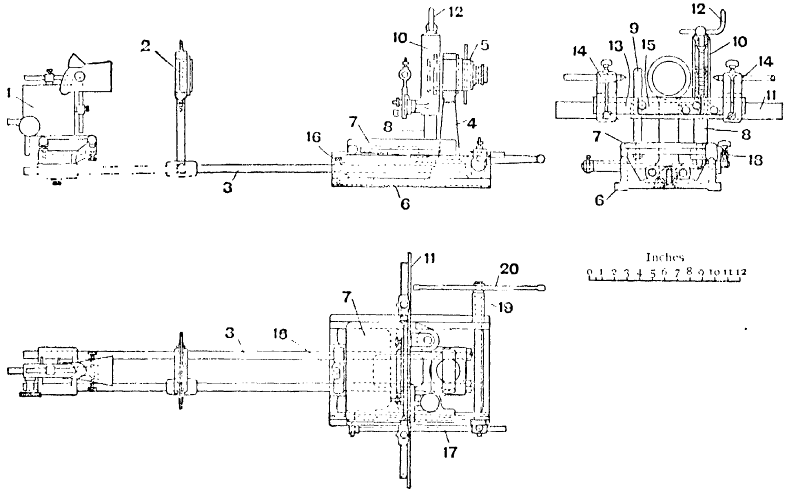


FIG. 112.

The first compound type of lens consisted of a 6-in. Ross Xpres, F 4·5, camera lens combined with a field lens by the same maker. This combination gives an undistorted image over a field of about 80 in. diameter with a magnification of 50. The definition is reasonably good throughout the area of this field. The distance from the gauge to the screen is about 21 ft. 6 in., and the working distance from the back surface of the field lens is 1⅜ in. Other high-class camera lenses are also used in place of the Xpres lens. The following have been tried and an improved definition obtained—Ross Homocentric F 5·6 and F 6·3, Ross "Tessar" and the Dallmeyer "Serrac."

It should be noted that no attempt was made to design a special lens for the purpose owing partly to the difficulty which arose in connection with the supply of special optical glass in the early days of the war. The lenses used were selected from standard types which were stocked by the makers.

§ (68) STANDARD HORIZONTAL PROJECTION APPARATUS.—Having solved the difficulty regarding the lenses, a standard type of horizontal projection apparatus was designed at the National Physical Laboratory in 1916. This apparatus serves not only for plate gauges up to about 1¾ in. in length, but can also be used for inspecting the thread forms of screws.

(i.) *The Apparatus.*—The general principle of the apparatus is extremely simple and is shown in *Fig.* 112. The light is obtained from a small right-angle pattern arc lamp 1, the crater of the arc being placed at the focus of an achromatic condenser 2. Both the lamp and condenser are supported from a pair of rods 3 which, for purposes of rake adjustment, can be rotated through about 10° on each side of the C.L. in a horizontal plane about an axis passing through the centre of the post 4, which carries the compound projection lens 5. When testing plate gauges the lamp can be fixed in the central position by means of a pin 16. The lens post is screwed down to the bed plate 6, which carries a slide 7 on a vee and a flat machined on its upper surface. This slide carries two vertical rods 8 and 9, one of which, 8, is provided with a sleeve 10, to which is fixed a cross bar 11. The sleeve carrying the cross bar can be moved up and down the rod 8 by means of the screw and nut shown, a handle 12 being provided for operating the screw. A small plate having a hole which fits the rod 9 is attached to the cross bar 11, and prevents it from rotating. The cross bar is provided with a slide 13 which carries a pair of centre brackets 14 and a clamp plate 15. When testing screw gauges they are held between the centres. Plate gauges are clamped between the slide 13 and the plate 15. Any special work may be stood upon the upper machined surface of the slide 7. By means of the vertical movement of the cross bar 11 and the sideway motion of the slide 13, it is possible to adjust the gauges with respect to the lens.

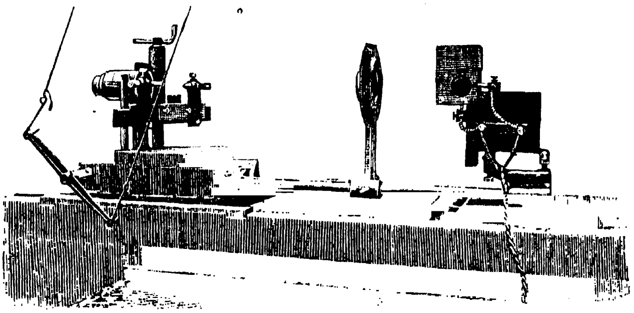


FIG. 113.

In order to focus the image of the gauge the slide 7 is moved on the bed plate so as to vary the distance from the lens. This movement is transmitted through a connecting rod 17, the other end of which is clamped in a block 18 which is operated by an eccentric pin on the spindle 19. This spindle has a cross arm 20, from the ends of which cords are taken over pulleys to the screen at the other end of the room, and by pulling one or other of these cords at the screen the image can be brought into focus.

The accuracy of the results obtained by the use of this apparatus will depend largely upon the alignment and squareness of the various parts of the machine, and the care which is taken in setting up the machine and the screen.

(ii.) *The Screen.*—A general view of the machine is shown in *Fig.* 113 and that of the screen in *Fig.* 114. The latter consists of a vertical board about 8 feet square which is supported in a special manner so as to be movable in its own plane through a limited range. This movement is necessary for the purpose of bringing the standard diagram of the gauge into correct superposition with the projected image. The greater part of the setting can be made by adjusting the gauge on the machine vertically and sideways, but the final adjustment has to be done on the screen itself. The latter can be moved sideways on two horizontal bars fixed to the wall by means of levers. The vertical movement is given by two handwheels.

(iii.) *Diagrams.*—The diagrams of form gauges which serve as the standards with which the projected images are compared, should be carefully drawn to the correct scale on white cardboard having a good smooth surface. It has been found that such diagrams, especially when they reach 6 or 7 feet in length, vary appreciably in their dimensions with the state of humidity of the atmosphere. Before using a diagram which has been drawn out some time before, it is necessary, therefore, to make a few check measurements on the over-all dimensions.

The type of standard diagram for use when examining screw threads is described in § (69) (v.).

FIG. 114.

§ (69) VERTICAL PROJECTION APPARATUS.—The vertical type of projection apparatus designed at the National Physical Laboratory in the latter end of 1916 is intended mainly for screw gauges and is arranged so as to deal with this class of work in a very convenient manner. The machine is compact, requires little floor space as compared with the horizontal machine, and can be operated by one observer. Generally speaking, the apparatus may be considered as being an ordinary pro jector stood vertically on end, but instead of allowing the image to be formed on the ceiling it is reflected down on to the table by a horizontal mirror placed above the machine.

Front and side views of the complete apparatus are shown in *Fig.* 115. There are three main parts. (A) The arc lamp of special right-angle design. (B) The body of the machine which has attachments for holding the projection lens and the gauge. It is fitted with micrometer screws for measuring the gauge and forms a support for the vertical post. (C) The mirror which reflects the image downwards.

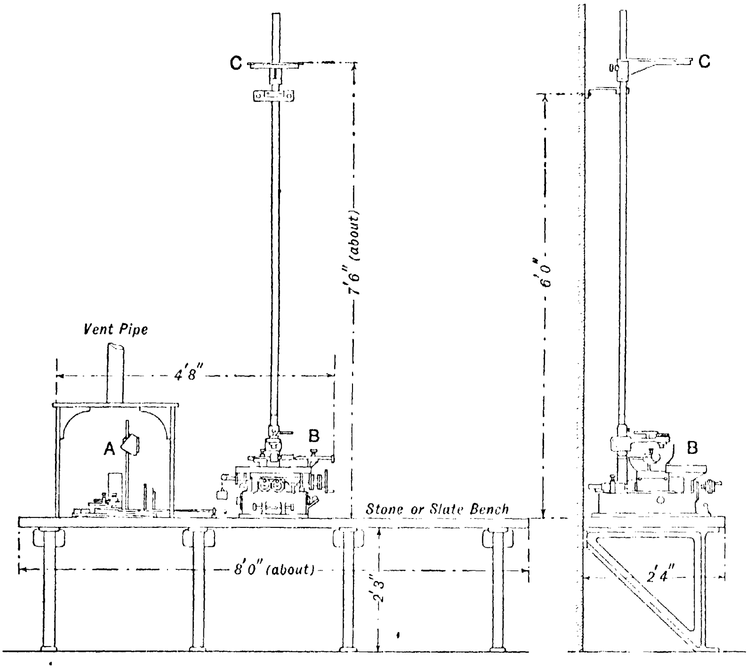


FIG. 115.

(i.) *Optical Scheme of the Machine.*—This is shown diagrammatically in *Fig.* 116. The illuminating beam from the arc lamp A is rendered closely parallel by means of the condenser and passes into the body of the machine through a horizontal tube 1, half way along which it strikes a 45° glass prism 2, which reflects it upwards in the direction of the gauge 3. After passing the gauge, the rays traverse the projection lens 7, and on meeting the mirror C are reflected downwards and come to a focus on the desk 8, which acts as the screen for the reception of the image.

The No. 4 Dallmeyer Kinematograph lens is used in this apparatus. The mirror is optically finished, and in order to avoid ghost effect due to double reflection at the surfaces it is made tapered from front to back. By this means the lesser image produced by reflection at the under glass surface is thrown well clear of the main image.

The direction of the illuminating beam has to be set so as to follow the slope of the threads, and for this purpose it is possible to tilt the prism 2, by means of the lever 5, about a horizontal axis at right angles to the incoming beam. This tilt is produced by moving the prism slightly along the tube 1, which causes the other end of the arm, to which the prism is attached, to move up or down the inclined bar 4. A pair of stops 6 are provided for limiting the travel of the rake lever 5.

(ii.) *Mechanical Arrangement.*—The general arrangement of the body of the machine is shown in *Fig.* 117.

The screw gauge 3 can be moved in a horizontal plane in two directions, one parallel to its axis and the other at right angles. These motions, which are controlled by micrometer screws, allow measurements to be made on the pitch and the diameter.

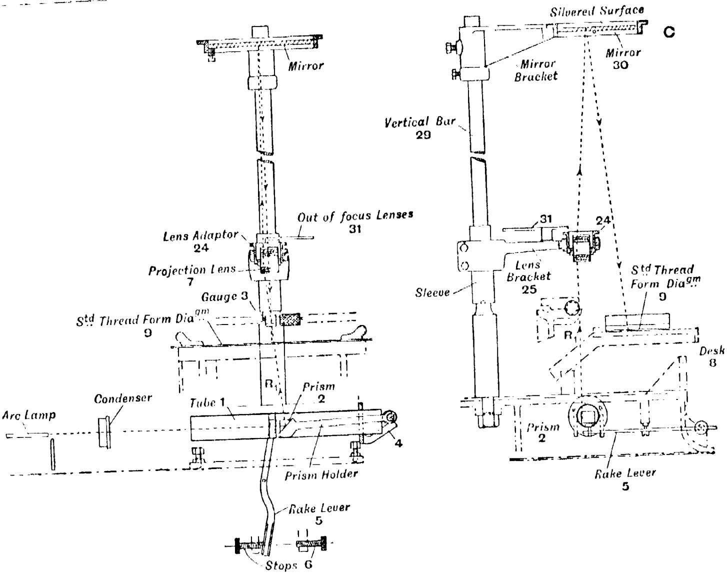
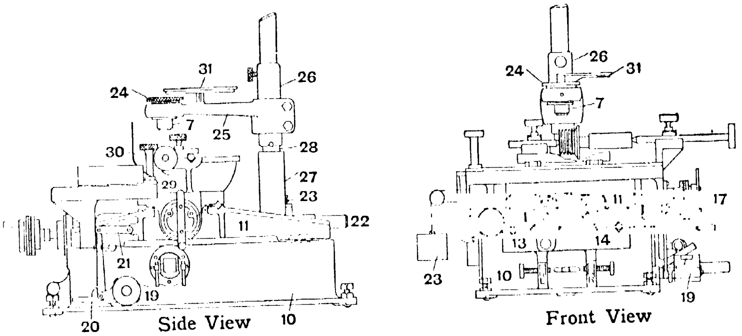
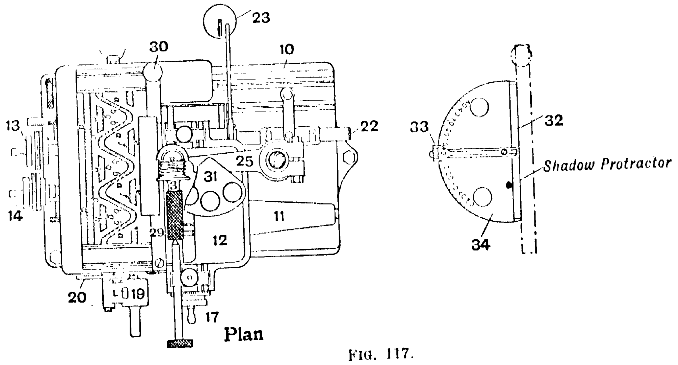


FIG. 116



Side View

Front View



Plan

FIG. 117.

The motions are derived from a carriage 11, which can move along the bed plate 10, and a second carriage 12, which supports the gauge and which can move on the top of the other carriage in a direction parallel to the pair of centres holding the gauge. The motions of the two carriages are guided by means of vee grooves and balls.

The arrangement of the micrometer screws which control the motions of the gauge is shown in *Fig.* 118. The lower carriage 11 is operated by a pair of micrometer screws 13 and 14, the corresponding nuts of which are fixed to the carriage and the bed plate respectively. The corresponding stops 15 and 16 are attached to the bed plate and to the carriage respectively. By this arrangement the carriage can move across freely from one side to the other, the motion being controlled at each end by one or other of the micrometers. This is an advantage over a single micrometer when the traversing motion has to be made frequently.

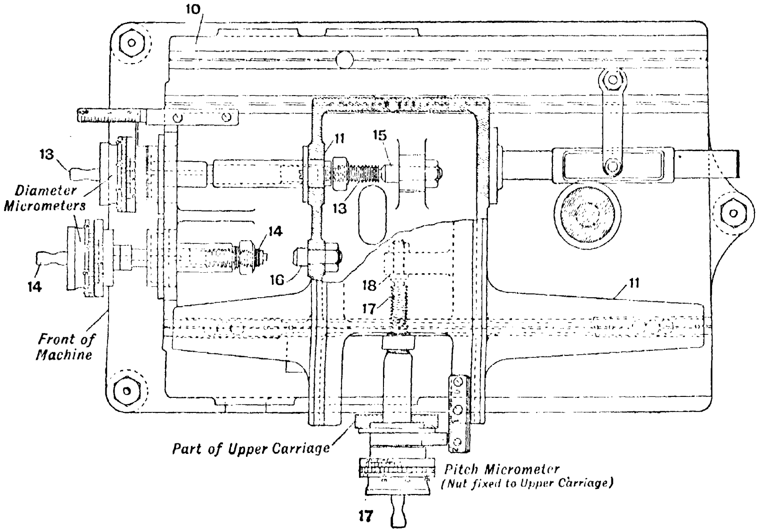


FIG. 118.

A third micrometer 17 is carried by the upper carriage 12, and controls its motions in the pitch-wise direction, the abutment 18 being fixed to the intermediate carriage. A weight 23 (*Fig.* 117) keeps the point of the micrometer in contact with the abutment. The micrometer screws have a pitch of 20 threads per inch and are fitted with dials and verniers graduated to read to 0·0001 in.

The upper carriage has two sets of vee grooves for holding the centres. The front pair is for gauges up to 2 in. diameter, and the rear pair will accommodate gauges up to 6 in. diameter for examination of thread form only.

The front and back motion of the carriage 11 is operated by the "throw-over" gear placed at the right of the bed. This consists of a rocker arm 20, *Fig.* 117, a connecting link 21, and a weight 19, which can be made to rest on one end or the other of the rocker arm according to which direction the carriage is required to move. The motion is steadied by the dashpot 22.

The projection lens is screwed into an adapter 24, in the bracket 25, which is clamped to the sleeve 26. The latter can turn on the post 27, and by this means the lens bracket can be swung aside when inserting gauges in the machine; the pin and vee 28 ensure definite location when the lens is brought back to its working position.

The vertical rod 29, which carries the mirror bracket, is screwed into the post 27, and is steadied at its upper end by a wall bracket.

(iii.) *Setting up the Apparatus.*—This is a simple matter, but requires care in order to obtain correct results from the machine. The body of the machine should be carefully levelled and the post set plumb. The mirror should also be levelled after fixing the mirror bracket at about the right height. The magnification is set by inserting between centres a small plug about 0·08 in. diameter whose actual diameter is accurately known, and focussing its image on the desk. The height of the mirror should then be adjusted until tho width of the image is exactly fifty times the size of the plug.

(iv.) *Setting the Rake of the Light.*—The machine is provided with a set of three auxiliary lenses, ⅛, ¼, and ½ diopters. These are fixed in a mount 31, *Fig.* 117, and can be swung singly into position above the projection lens. If the image of a screw is formed with one of these lenses in position the "out of focus" effect will be clearly indicated by the presence of a dark fringe round the contour of the thread. By moving the rake lever 5, *Fig.* 116, the width of the fringe on the flanks of the image will be found to vary. The light is in the best direction when an equal width of fringe is visible on each flank and the rake lever should be set accordingly.

It is necessary to provide the three "out of focus" lenses, as screws of different diameter or pitches require to be put out of focus by appropriate amounts in order to obtain a suitable fringe pattern in each case.

(v.) *Thread Form Diagrams.*—For the purpose of measuring and inspecting screw threads it is necessary to have diagrams of the standard threads drawn to the appropriate magnification. The first pattern consisted of the outline of the standard thread drawn in thin line on Bristol-board. It was then found to be an improvement to tint the outline on one side of the line a fairly deep grey, a colour which matches that of the actual projected image when seen in a semi-dark room. Using this form of diagram, which was first introduced by the Bentley Engineering Company, the image is fitted into the outline as closely as possible. The presence of excess metal at any point of the thread produces an overlapping of the grey image and grey background of the diagram and is indicated by a black streak or mark at the point concerned. Where metal is missing the two outlines are separated and a white streak is seen. This effect is illustrated in *Fig.* 119, where the image B of a screw having an unsymmetrical thread angle is compared with the standard outline A. Extra metal is indicated over one-half of each flank at the dark places *a*, and metal is missing at the localities marked *b*, *b*.

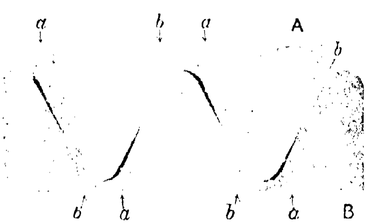


FIG. 119.

For the purpose of taking measurements of the diameters of screw gauges where the image of each side of the screw is viewed in turn, it becomes necessary to modify this simple form of diagram without losing the advantage derived from the grey background. The standard type of diagram adopted for use is shown in *Fig.* 117 in position on the desk of the machine. It consists of two standard outlines *a*, *a*, and *b*, *b*, separated diametrically by a half-inch space which is tinted a dark grey.

The parallel bands *cd*, *ef*, and *gh* are also drawn half an inch wide. The function of the double outline will be explained later. The diagram is printed accurately on zinc or aluminium plate which is stiffened by a steel backing plate. The printing is done from a master plate which has to be drawn out by hand.

(vi.) *Measurements of Plug Screws.*—The form of the thread of plug screws or of casts from ring gauges are examined with reference to standard diagrams as explained above.

(vii.) *Measurement of Thread Angle: Shadow Protractor.*—Errors in the thread angle can be detected against the standard diagram, but, in addition, actual measurements of the angles of the flanks with reference to the axis of the gauge can be made by the use of the "Shadow Protractor" shown in *Fig.* 117. This instrument consists of a semicircular sheet of white celluloid 34 screwed to a metal base, to which is also fixed a straight edge 32 and a movable radial arm of parallel width 33. The outer end of the arm has a line which reads against a half-degree scale engraved round the circumference of the base. When the arm is set square to the straight edge the reading on the scale is zero.

In use, the protractor is placed on the desk of the projector with its straight side in contact with the adjustable straight edge 29, and the image allowed to fall on the celluloid base. The tilt of the adjustable straight edge is then set by means of the eccentric pin 30, so as to bring the straight edge 32 of the protractor parallel to the crests of the image. The radial arm is then rotated and the protractor moved sideways until the shadow of one of the raised edges of the arm becomes parallel to one of the flanks of the image. The angle of that flank is then read on the scale. A similar measurement is made on the opposite flank using the other edge of the radial arm. The pair of measurements is repeated at different positions along the screw.

(viii.) *Measurement of Pitch.*—The screw is arranged between the centres so that when the pitch micrometer screw is towards its zero position the image of the end thread is at about the centre of the desk. The corresponding standard diagram is then placed on the desk and the image of the thread fitted symmetrically into one of the spaces, leaving a thin strip of light showing along each flank. The reading of the pitch micrometer is then taken. The screw is now translated so as to bring the next thread into the space and a further reading is made, the operation being repeated for every thread in succession along the screw. Finally, a check reading should be made on the initial thread. The method of working up the results is similar to that explained in § (24) (v.). Errors in the pitch of the micrometer screw itself must be allowed for in the results.

(ix.) *Measurements of Full and Core Diameters.*—A plain plug of known size P, and of approximately the same diameter as the screw to be measured, is required for setting up the machine. This plug is placed between the centres, and the left diameter micrometer screw is used to bring one edge of its image in coincidence with the edge *dd* of the half-inch band on the diagram, *Fig.* 117. The rock-over gear is now changed over so as to bring the other edge into view, and this is set to the edge *ee* of the band by means of the right diameter screw. The readings of the two micrometers are noted and their sum S obtained.

The screw gauge is now substituted for the plug and similar settings are made on the crests and roots of the thread in turn, thus obtaining two further additions of readings F and C, which correspond to the full and core diameters. The actual diameters can then be calculated on the principle of comparisons from the formulae:

$$\text{Full Diameter} = P + (F - S),$$
$$\text{Core Diameter} = P + (C - S).$$

The concentricity of the diameters can be checked by measuring the thread depth on opposite sides and at three positions round the screw.

(x.) *Measurement of Effective Diameter.*—A preliminary reading is made on the standard plug as before. Readings are then taken on each side of the screw by setting the image with the outline of the thread on the diagram, using the two micrometers in turn. If it is desired to measure the virtual effective diameter, taking account of errors in angle and malformations of the crests and roots, the image should be adjusted diametrically until it first comes into contact with the outline at one point on each flank; a similar setting should also be made on the other side of the thread. If, on the other hand, the net effective diameter is required, *i.e.* the diameter which would be obtained from measurements with best size wires, then the image should be adjusted so that the middle points of the flanks come in contact with the outline on the diagram.

The calculation of the effective diameter is made in a similar manner as for the full and core diameters.

This machine is being manufactured to the National Physical Laboratory design by Messrs. Cussons of Manchester.

§ (70) THE WILSON PROJECTION COMPARATOR.—This apparatus, which was designed and patented by Mr. R. P. Wilson, utilises the general principles of optical projection described previously, and applies them to a scheme for rapidly testing screws up to about 2 in. diameter. In the projection machines described above, the object has been to produce an enlarged image of the profile of the screw on one side only. When it was desired to measure the diameters, the screw was traversed past the projection lens by means of micrometer screws in order to bring the image of each side of the screw on to the screen in turn. When testing screwed work, however, where the accuracy of test is usually not finer than 0·0005 in., it is an advantage to project both sides of the screw simultaneously. This allows the form of the thread to be examined as before and, in addition, it becomes possible by suitable arrangements to obtain a ready check on the diameters of the screw. The essential part of the apparatus consists of two lenses placed side by side, each of which forms an image of one side of the screw. Assuming a standard screw of the maximum allowable size is being projected, the two images can be made to mesh by varying the distance between the lenses. Having adjusted the lenses on the standard, if a screw of larger diameter is now substituted the two images will overlap and the effect will be to produce a black wavy band along the centre of the screen. On the other hand, if the screw is small on diameters the images will become separated and instead of the black band a white space will be seen. The thickness of the band in either case is a measure of the error of the screw.

The optical scheme of the apparatus is shown in plan view in *Fig.* 120. A parallel beam of light from a source A and a condenser B illuminates both sides of the screw C. The optical system consists of two corrected compound lenses D, each of which has a considerable segment removed to allow the centres of the lenses to be brought quite close together if desired. The centres are separated in every case by a distance equal to the effective diameter of the screw being tested. Under these conditions an image of each side of the thread will be produced on the screen F, and their mean distance apart would be equal to the separation of the lenses. Each image,

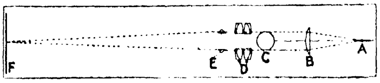


FIG. 120.

however, is brought on to the centre line of the apparatus by means of a small adjustable prism E, one of which is placed just in front of each lens. If the light from one of the lenses is screened off the image produced by the other will, in a dark room, appear black against a white background, as shown at A in *Fig.* 121, which shows the image of a cylinder for simplicity. The effect of allowing both images to be formed simultaneously is to diminish their intensities to a light grey, as shown at B, since each image is illuminated by the free light from the opposite lens. Overlapping of the images immediately produces a black band, as shown at C. It is to be noted that each image is produced by exactly similar parts of the respective lenses, consequently, even though the images may be somewhat distorted, the intermeshing effect in the case of screws will always take place.

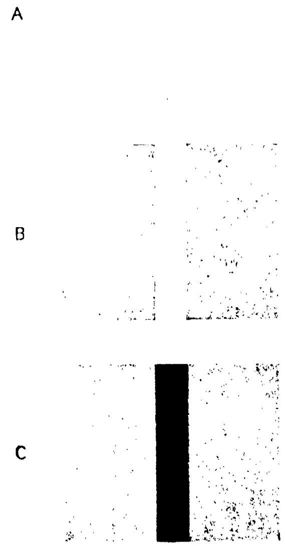


FIG. 121.

A general view of the apparatus is shown in *Fig.* 122. It consists of two stout rods forming a base on which are fixed four brackets. The first of these, A, holds a 500-c.p. "Pointolite" lamp which is placed at the focus of a condenser held in the second bracket B. The screw, or other work to be tested, is held with its axis vertical on a sliding table on the bracket C, which is placed in front of the lens-holder D. A more detailed view of the lens arrangement is shown in *Fig.* 123. Each lens is on a slide, the position of which can be varied sideways by means of one of the micrometer screws I and I′ shown. The micrometers are arranged so that their readings represent the distance between the centres of the lenses. Suitable hoods are provided round the lenses for screening off stray light.

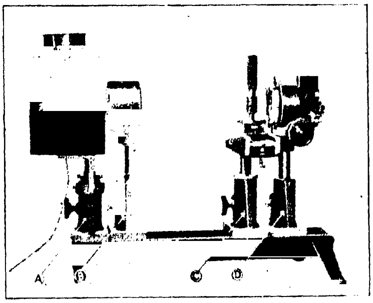


FIG. 122.

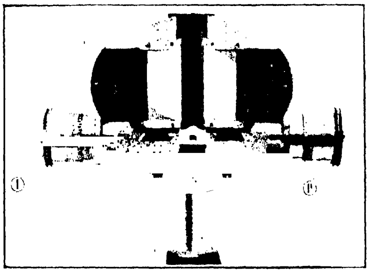


FIG. 123.

The mounting of the reflecting prisms I′ is shown in *Fig.* 124. They are attached to special fittings I provided with adjusting screws for tilting each prism about a vertical

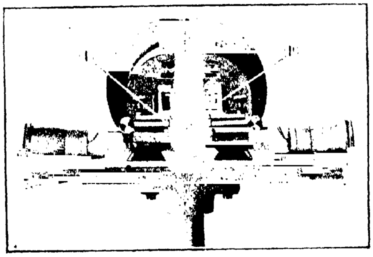


FIG. 124.

and a horizontal axis, and in addition, the fittings can be swung aside from the lenses for purposes of focussing the images.

With regard to the illumination, the beam is parallel, but no adjustments are provided for tilting it in the direction of the rake of the screw which, it should be noted, is in a different direction on the two sides. The resulting images are consequently not equally well defined on the opposite flanks, but provided the rake of the screw is not abnormal and the magnification used is not greater than about 20, the defect in definition is hardly noticeable on the screen. Apart from definition, the threads will appear very slightly too thick, due to this cause.[1]

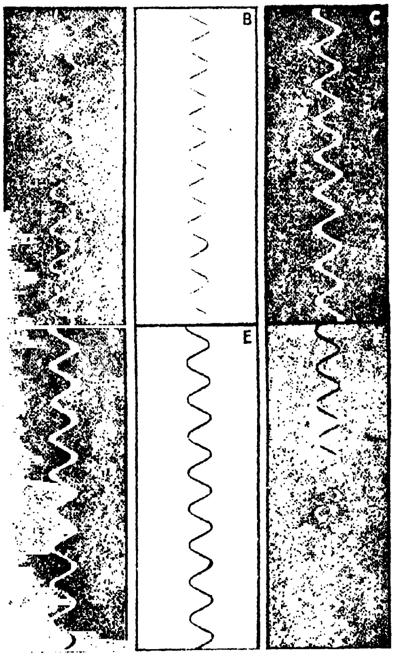


Fig. 125.

It is to be noted that when projecting a screw having *any* symmetrical form of thread the two images can be made to mesh; consequently, no apparent defect would be noticed with regard to the meshing when projecting a screw whose effective diameter was equal to that of the standard screw, but whose angle happened to be seriously in error by the same amounts on each flank. Uniform errors in pitch, whatever their magnitude, would also remain undetected, since each image would be lengthened or shortened to the same extent. To get over this difficulty, it becomes necessary, in addition to the criterion given by the meshing of the images, to refer them by some means to a standard form. This can be done by having a standard outline drawn on the screen to the appropriate magnification or by arranging to project a correct templet of the thread simultaneously with one side of the screw.

Six examples of the images of screw threads having characteristic errors are shown in *Fig.* 125. The errors are readily interpreted from the diagrams.

(A) Effective diameter correct but full and core diameters small.
(B) Full and core diameters correct but effective diameter large.
(C) Small on all diameters.
(D) Tapered thread.
(E) Large on all diameters.
(F) Tapered thread, diameters too large at one end and small at the other.

This apparatus is being made under licence by Messrs. Adam Hilger.

V. Measuring Machines

The machines described in this section are of the type used for the comparison and standardisation of standard gauges referred to in § (5).

There are four well-known forms of measuring machines used in this country, made by Messrs. Armstrong Whitworth, Manchester; The Newall Engineering Company, London; The Pratt and Whitney Company, Hartford, U.S.A.; and La Société Genevoise, Geneva, Switzerland, respectively. Another type of machine has been designed by Dr. P. E. Shaw of Nottingham, but has not been manufactured commercially. Several smaller forms of machines are in existence, and these will be referred to later.

§ (71) The "Armstrong Whitworth" Measuring Machine. (i.) *General Arrangement.*—A general view of this type of machine having a capacity of 12 in. is given in *Fig.* 126, whilst the general arrangement of a 15-in. machine will be seen in *Fig.* 127, the only important difference being in the length of the bed.

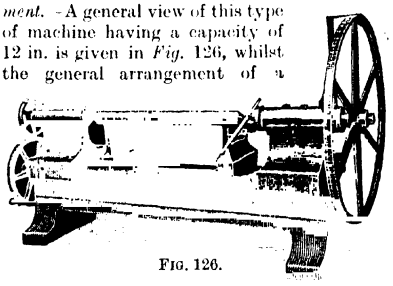


Fig. 126.

---

[1] Since this was written the makers have added an attachment for "raking" the light correctly for each side of the thread. See Patent No. 171764.

DIRECTOR OF INDUSTRIES
Bengal

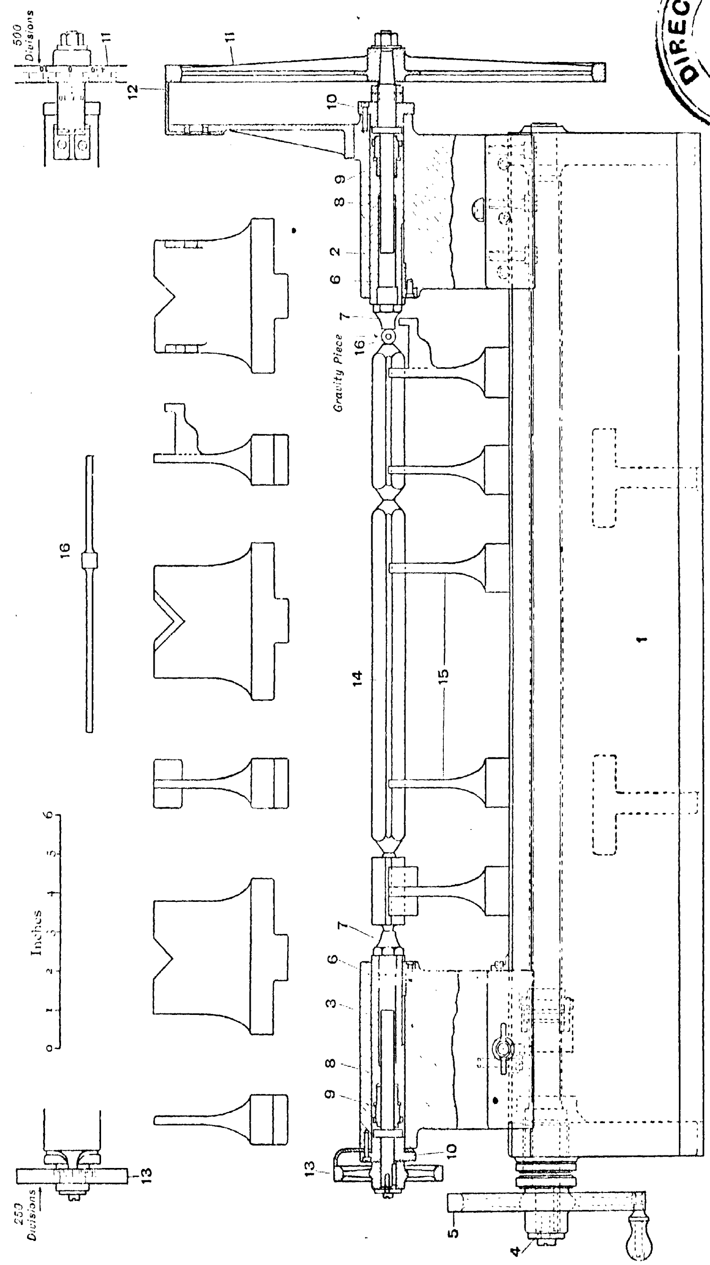


FIG. 127.

The machine consists essentially of a bed 1 which carries a micrometer headstock 2 at the right-hand end, and a tailstock 3 at the opposite end. The bed is supported at the ends, is of hollow section, and has its upper finished surface slotted. The sides are also finished and form a dovetail guide for the tailstock. The micrometer or fast headstock is permanently screwed down to one end of the bed. The tailstock can be moved along the bed to suit the length of gauge to be dealt with, by means of a quick-motion traversing screw 4 held in bearings at each end of the bed and engaging with a nut, fixed to the base of the tailstock. The screw is operated by a handwheel 5 and when in the approximate position the tailstock can be clamped to the bed.

(ii.) *Headstock and Tailstock.*—The headstock and tailstock have practically the same construction. They are bored and fitted with hollow steel plungers 6, ⅞ in. diameter. Hardened steel plugs 7 are screwed into the outer ends of these plungers, and the faces of these plugs, which are lapped flat and parallel, form the measuring faces of the machine. They have a diameter of ⅜ in. The longitudinal positions of the plungers are controlled by steel micrometer screws 8, having square threads, 20 per in., which engage with brass nuts 9 fixed in the rear ends of the plungers. The plungers are prevented from rotating by keys working in slots in the under sides. The thrust is taken between a collar on each screw and the inner surface of two brass bushes 10 let into the end of the ⅞-in. bored holes.

The micrometer screw of the headstock is provided with a 12-in. diameter handwheel 11, which is graduated into 500 divisions, each representing 0·0001 in. As each division actually measures approximately 0·075 in. on the periphery of the wheel, the magnification of the machine is 750. A vernier scale 12 is supported by a suitable bracket and enables readings to be taken to 0·00001 in. The screw of the tailstock, which is intended only for purposes of zero adjustment, is provided with a 3-in. handwheel 13, divided into 250 parts. Each micrometer screw has a range of one inch.

(iii.) *Gravity Piece Feeler.*—The machine has no indicator, all measurements being made by the method of "touch" or "feel." When comparing small gauges under about one inch in size, they are passed in turn between the contact faces, the distance between which is gradually diminished by means of the right-hand micrometer screw until the gauge will just not drop through the gap by its own weight. The method of taking readings on larger gauges is illustrated in both *Figs.* 126 and 127. The gauge, or a number of gauges 14, butting end to end, are placed on suitable supports 15, so that their axes are parallel to that of the machine, and with the left-hand measuring face touching the contact face of the tailstock. The plunger of the headstock is then adjusted by the micrometer screw until a "gravity piece" 16, which consists of a short, accurately finished plug ⅜ in. diameter, just refuses to pass by its own weight through the gap between the right-hand face of the gauge and the contact face of the headstock. With parallel faces this method of taking a setting is sensitive to within 0·00001 in., as a diminution of this order in the size of the gap is quite sufficient to prevent the gravity piece dropping through, where previously it passed quite freely. It will be readily appreciated, however, that the method is rather slow, as the final adjustment of the handwheel must be by hundred-thousandths. It is difficult to judge with any certainty how much reduction in the size of the gap is required to obtain a satisfactory setting, so that it is necessary to proceed very cautiously over the last 0·0001 in. or so.

The thrust collars are held in contact with the abutment faces by suitable lock nuts in the case of the headstock and by the small handwheel in the case of the tailstock. Backlash in the screw is taken up as far as possible by adjusting the brass nuts 9; nevertheless, it is always necessary to turn the handwheel in one direction when making a setting so as to avoid backlash.

The importance of accuracy in the thrust surfaces on the collars and abutments is dealt with in § (79) (iii.), where their effect on periodic error is explained.

These machines are made to suit any reasonable capacity by simply lengthening the bed.

§ (72) THE "NEWALL" MEASURING MACHINE. (i.) *General Arrangement.*—An example of this design of machine having a capacity of 24 in. is shown in *Fig.* 128, from which it will be seen that the machine consists of a hollow boat-shaped bed, carrying on its upper surface a micrometer headstock at the right- and a tailstock at the left-hand end. Neither of these parts is permanently fixed to the bed, but each is provided with a clamp for fixing temporarily at any required position. The machine is also provided with a pair of vee supports for carrying end gauges during measurement. The bed is carried on a pair of feet, the left-hand one of which is attached to the bed by a pivot parallel to the axis. This virtually provides a three-point support, and ensures that the bed is free from any strains due to unevenness of the supporting surface.

(ii.) *Headstock.*—Details of the micrometer headstock are shown in the sectional drawing, *Fig.* 129. The micrometer spindle 1 runs in a thread tapped in steel bush 2, and is supported at each end on its plain portions in hardened steel bushes. The left-hand end of the spindle is hardened and tapered down to a diameter of $\frac{5}{16}$ in. to form the measuring face, which is lapped flat and square to the axis of the spindle. The centre portion is screwed for a length of about 3 in. with a right-hand buttress thread, cut specially deep, the pitch of which is 20 threads per inch. The right-hand end is fitted with a graduated wheel 3, $6\frac{1}{2}$ in. diameter, which is divided into 500 parts each representing 0·0001 in. A vernier 4 enables readings to be taken to 0·00001 in. Complete revolutions of the wheel are noted on the scale 5, on which the vernier slides.

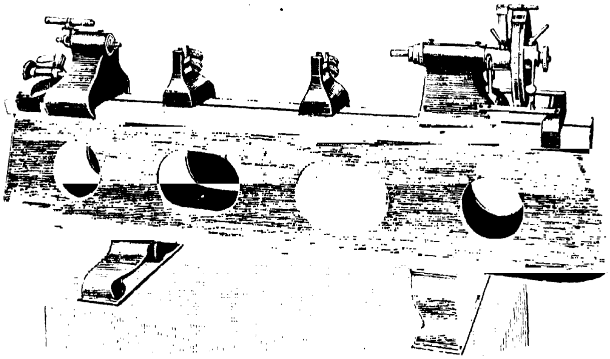


FIG. 128.

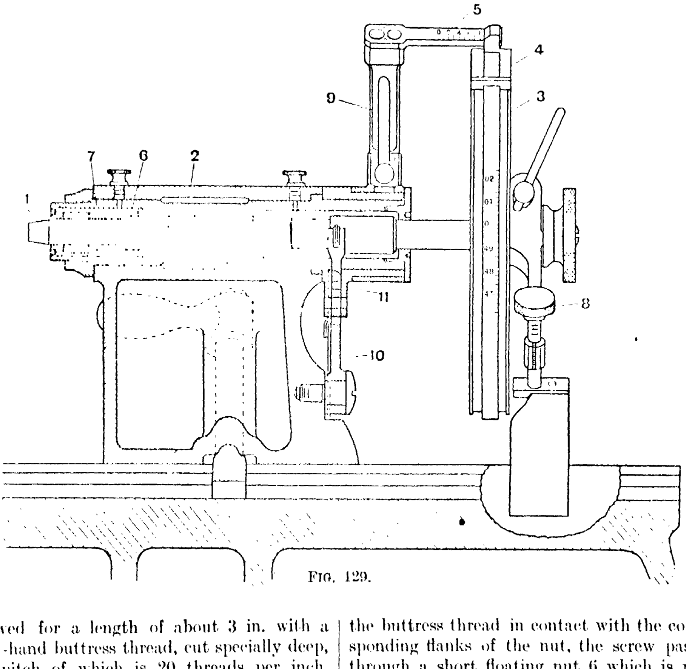


FIG. 129.

In order to keep the rear vertical flanks of the buttress thread in contact with the corresponding flanks of the nut, the screw passes through a short floating nut 6 which is prevented from rotating, and which is pushed to the right by the spring 7. Fine adjustment of the handwheel is made by means of the tangent screw 8 at the end of a radial arm which can be clamped to the spindle at will.

The readings are automatically corrected for progressive errors in the pitch of the micrometer screw by means of a special device which controls the angular position of the arm 9, carrying the vernier. This mechanism comprises a rocker arm 10, pivoted at its lower end and carrying at the other extremity a small roller which bears on an enlarged plain part of the spindle, the diameter of which is varied along its length in accordance with the magnitude of the progressive error in the pitch along the screw. Oscillations of the arm due to the action of the cam are transmitted through a pin joint to the double sleeve 11, which carries the vernier arm. Contact is maintained between the roller and the spindle by means of a spring on the arm 10. It is possible to adjust the position of the vernier for initial settings by unclamping the outer of the two sleeves 11, and rotating the arm 9 about the inner sleeve.

The 0·0001 in. divisions on the handwheel are spaced approximately 0·04 in. apart, so that the magnification of the micrometer is 400. The headstock is also made for metric measurements, in which case the screw has a pitch of 1 mm., and the handwheel has 1000 divisions, each of which consequently represents 0·001 mm. The ranges of the English and metric headstocks are 1 in. and 20 mm. respectively.

(iii.) *Tailstock.*—A sectional drawing of the tailstock is shown in *Fig.* 130. It consists of the main body 12 and a subsidiary block 13 which, by means of the finely threaded screw 14 and a nut in the tailstock, serves for adjusting the position of the tailstock along the bed.

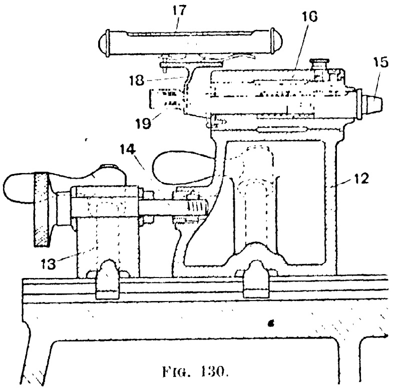


FIG. 130.

The plunger 15, which is hardened, is a good sliding fit in the bush, but is prevented from rotating by a pin working in a slot. Its right-hand end is lapped flat and square to the axis and forms the second contact face of the machine. A pressure of about 5 lbs. weight is exerted between the two contact faces by the spring 16.

(iv.) *Level Indicator.*—In order to facilitate the measurements and to free them from all effects of personal "feel" or "touch," this machine is provided with an indicator fixed to the tailstock. It consists of a fairly sensitive spirit-level tube 17, *Fig.* 130, mounted on a lever 18, which is pivoted at its lower end to a bracket 19 fixed to the rear end of the tailstock. The rear end of the tailstock plunger presses against this lever through the medium of a steel ball, the point of contact being a short distance above the line joining the pivots. The lever is kept in contact with the ball by a small spring-controlled plunger held in the bracket 19. If the plunger 15 is pressed in against the force of the spring 16, it causes the lever supporting the level to turn about its fulcrum and so tilts the level tube. The magnification of the movement of the bubble depends upon the ratio of the arms of the lever and the sensitivity of the level itself: it is usually set to a figure of about 4000. The level tube has a scale engraved on it in tenths of inches, and a movement of the bubble from one line to the next indicates a displacement of the plunger of between 0·00002 and 0·00003 in.

(v.) *Comparison of Gauges.*—When comparing two gauges the headstock and tailstock are clamped to the bed at a suitable distance apart, making use of the adjustment screw at the rear of the tailstock. The reference gauge is then inserted between the contact faces and the micrometer wheel rotated and finally adjusted by the tangent screw until the bubble of the indicator arrives at, say, the central mark on the scale of the level. A reading of the micrometer is then taken. The unknown gauge is now substituted for the reference gauge and another setting made, bringing the bubble up to exactly the same division. This is followed by a second reading of the micrometer. Assuming the micrometer screw is accurate, the difference between the readings gives the difference in the lengths of the two gauges

Instead of using reference gauges for purposes of comparison, this type of machine is fitted at times with a scale and microscope. The scale is made of steel and has the division lines engraved on the polished surface of invar plugs let into the upper surface of the bar. It is rigidly fixed to the side of the bed of the machine at about the level of its upper surface. The microscope is provided with a pair of fixed parallel cross-wires in the eyepiece and is held over the scale in a bracket attached to the tailstock.

The method of taking measurements by the use of a scale and microscope is explained in § (17), and some sources of errors are referred to under the same heading.

The Newall machine is made in four sizes, having capacities up to 12, 24, 48, and 72 in. Machines for metric measurements are also made of corresponding capacities.

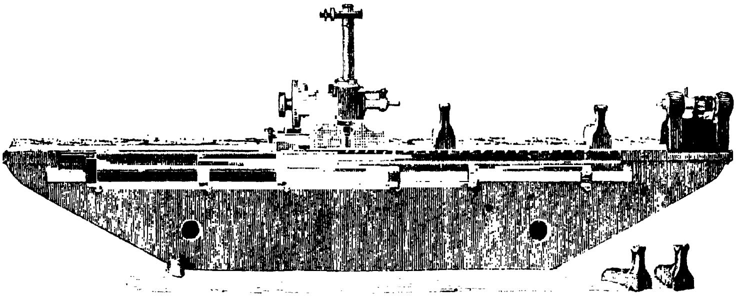


FIG. 131.

§ (73) THE "PRATT AND WHITNEY" MEASURING MACHINE. (i.) *General Arrangement.*—This type of measuring machine, which is of American manufacture, is shown in *Fig.* 131, which is a rear view of a machine of 48 in. capacity. The micrometer headstock is on the left and carries a microscope looking over a scale attached to the rear of the bed. The tailstock is provided with an indicator and is clamped down to the other extremity of the bed.

The bed, which is of massive design, is supported on three feet. Its upper bearing surface consists of a 90°-vee along one edge and a horizontal flat along the other. It has a T-slot along its length for purposes of locking the headstock.

(ii.) *Micrometer Headstock.*—A more detailed view of the headstock is given in *Fig.* 132, which shows the addition of a reading lens over the vernier and a tangent screw adjustment on the micrometer spindle. *Fig.* 133 shows a part sectional line drawing of the headstock in which the internal details can be seen.

The main casting 1 can be accurately set to any position along the bed by clamping the auxiliary block 2 and making use of the fine adjustment screw 3 which connects the block to the headstock. The latter is provided with a fixing bolt for clamping to the bed when the headstock is in the desired position. The headstock is bored and fitted with a hardened steel bush, in which the plunger 4, which is $\frac{7}{8}$ in. diameter, is a good sliding fit. A plug having a taper shank is fitted into the left-hand end of the plunger, and the outer face of this plug, which is lapped accurately flat and square to the axis, forms one of the contact faces of the machine. The diameter of the face is $\frac{3}{8}$ in. The other end of the plunger, which is hollow, carries the nut of the micrometer screw in a conical hole. The brass nut, which is split, is held in position by a retaining ring at the neck, and its fit on the micrometer screw is adjusted by varying its position in the conical hole. The micrometer screw 5, which has a travel of 1 in., has a left-hand vee thread, $1\frac{3}{4}$ in. long, 25 threads per in. The thrust of the screw is taken between a collar on the spindle and the face of a bush fixed in the rear end of the bore of the headstock, which also forms a bearing for the spindle. The latter has a 4-in. graduated drum

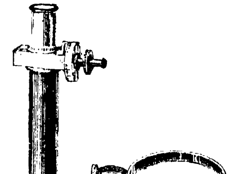


FIG. 132.

divided into 400 parts, each of which represents 0·0001 in. The actual length of each division on the drum is approximately 0·03 in., so that the magnification of the micrometer is closely 300. The readings are made against a single index or vernier 7, which is carried on a sleeve and can be adjusted in a circumferential position by the screw 8. The contact between the thrust collar and the abutment is maintained partly by a light spring next to the handwheel, but mainly by the pressure derived from the tailstock and transmitted through the gauge being measured.

The plunger of the headstock is prevented from rotating by a clip 9 which is attached to its outer end, and which is guided in its longitudinal motion with the plunger by contact with a plate 10. One edge of this plate bears a scale divided into twenty-fifths of inches, and a mark on the clip enables the position of the plunger to be determined so far as complete revolutions of the micrometer screw are concerned.

(iii.) *Compensating Plate.* — The plate 10 also serves as an automatic device for compensating errors in the readings due to inaccuracies in the pitch of the micrometer screw. If the controlling edge of the plate is parallel to the axis of the plunger, the motion of the latter will be purely translatory. Any lack of parallelism, however, will produce a corresponding slight rotational movement of the plunger as it is moved from one end of its range to the other. By virtue of the screw and nut, this rotation will have the effect of slightly advancing or retarding the axial position of the plunger. A calibration of the headstock is made with the plate set accurately in the parallel position, and the errors in the travel of the plunger determined by the method explained in § (79) (iii.). If the error is progressive and is proportional to the travel, it can be corrected by simply tilting the plate 10 about the pivot 11 by means o the adjusting screws 13. After setting the plate it is fixed in position by the screw 1 which passes through a slotted hole in it If the calibration diagram shows a curved line

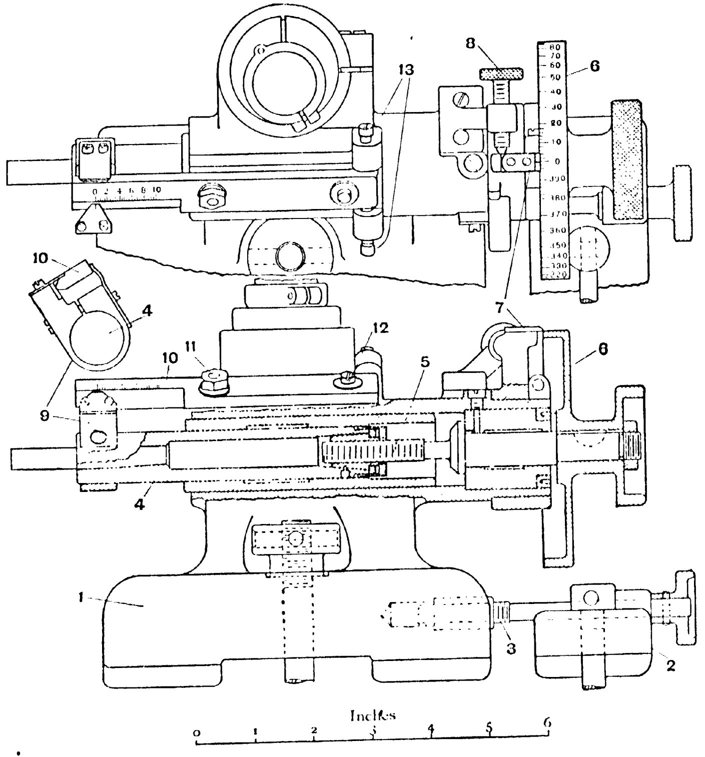


FIG. 133.

it will be necessary to form the edge of the plate to the corresponding shape. By this means of adjustment it is possible to obtain a final calibration which is correct to within 0·00002 or 0·00003 in. throughout the 1 in. range, provided the micrometer screw is reasonably good in itself. This method cannot be applied to the correction of periodic errors in the calibration (see § (79) (iii.)).

(iv.) *Tailstock.* — A sectional drawing is shown in *Fig.* 134. The casting 14, which can be fixed to the bed by a T-bolt, is bored and fitted with hardened steel bushes in which the hollow plunger 15 is a good sliding fit. The plunger is prevented from rotating by a projecting tongue fixed to the base, which runs freely in a longitudinal slot in the lower part of the sleeve 17. The right-hand end of the plunger is fitted with a plug, the outer end of which forms the second contact face of the machine. A spiral spring 16 is placed between the other end of the plunger and a plug in the bush so as to provide a pressure between the measuring faces. This pressure is usually about 5 lbs. weight.

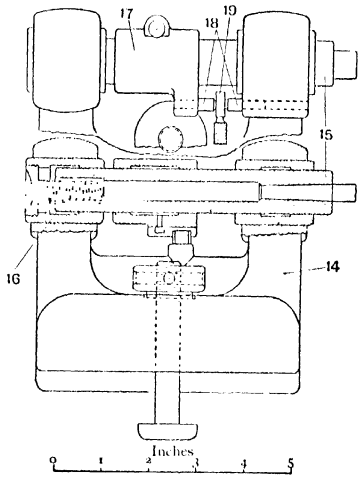


FIG. 134.

(v.) *Indicator.* — The indicator, which is fitted to the tailstock, is of a simple nature. Two auxiliary contact faces 18 are provided, one being fixed to the sleeve 17 which is connected to the plunger, and the opposite one to the base of the tailstock. A small, accurately made plug 19, $\frac{1}{16}$ in. diameter, is clipped between these two faces by the action of the spring at the rear of the plunger. When making a setting on a gauge between the measuring faces, the micrometer hand-wheel is gradually rotated and the pressure on the face of the tailstock plunger increases until it just overcomes the effect of the spring. When this state is reached, the plunger begins to be pressed back and the small plug is released from between the two auxiliary faces and drops. Instead of allowing the plug to actually drop, however, it is arranged initially with its axis horizontal, and the indication is obtained when it swings into the vertical position without falling from between the faces. This criterion is sensitive to an order of 0·00001 in., but has the disadvantage that it gives no preliminary warning as to the nearness of the setting. It is necessary, therefore, to proceed very cautiously over the last few ten-thousandths in order to avoid over-running the setting. The usual procedure is to make a rapid setting so as to obtain an approximate reading to within 0·0001 in. or so, and this is of assistance when making the final setting.

This type of machine can be used for comparing gauges having closely the same lengths by making use of the micrometer headstock. The machines are also provided with scales and microscopes, as shown in *Fig.* 131, for the measurement of gauges by comparison with the line standard. The microscope is mounted in a bracket attached to the headstock and is focussed on to a scale screwed down to a ledge at the rear of the bed. The scale is of steel and has a rectangular section 1 × 1½ in. The lines are engraved on the raised polished surfaces of plugs which are driven into holes in the bar. In some cases, two rows of plugs are fitted to the one bar, those in one row being spaced at intervals of 1 in. and the others at intervals of 2·5 cm. In such cases an additional headstock, fitted with a metric pitch micrometer screw, is usually provided. The scale is fitted with one or more hinged metal shields, which can be swung out of place when readings are to be taken on it.[1]

This make of machine has recently been fitted with a new type of indicator on the tailstock, which consists briefly of a slightly bowed flat spring, one end of which is fixed to the tailstock and the other to the measuring face of the plunger. As the latter is pressed in, the spring is bowed still further and eventually makes an electric contact for a circuit containing an incandescent lamp.

(vi.) *Measurement of Screws on Pratt and Whitney Machine.*— This machine can be used for the measurements of the diameters and the pitch of plug-screw gauges. When measuring the diameters, the screw is supported on centres on a cross-slide, which is placed between the headstock and tailstock, as shown in *Fig.* 135. Standard wires and vee pieces are used as usual when measuring the effective and core diameters.[1] When measuring the pitch, the screw is supported on a pair of centres carried on a bar which is held in brackets from the bed of the machine, as shown in *Fig.* 136. A lever indicator, which has a hardened steel stylus for engaging with the thread, is attached to [illegible] and the plunger of the micrometer headstock is moved along the screw by the motion of the micrometer.

---

[1] The method of taking measurements by the use of the scale and micrometer is described in § (17), *III*. In this design of machines, or in any other where the scale is not in line with the main axis of measurement, the straightness of the bed is a very important factor with regard to the accuracy of the measurements obtained from the scale.

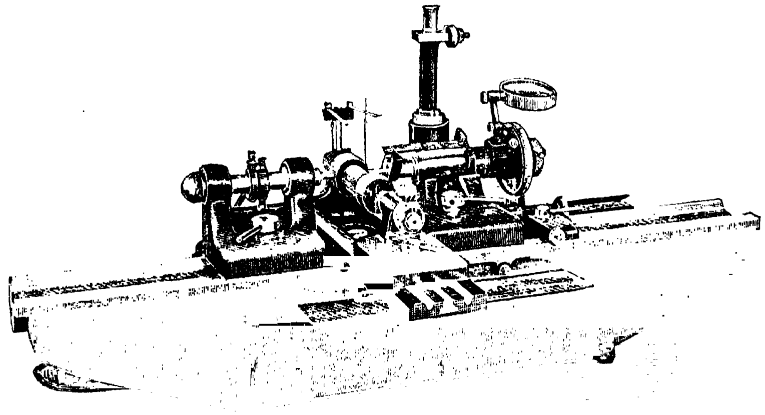


Fig. 135.

The machines shown in *Figs.* 135 and 136 are fitted with the electrical indicator on the tailstock referred to on p. 369.

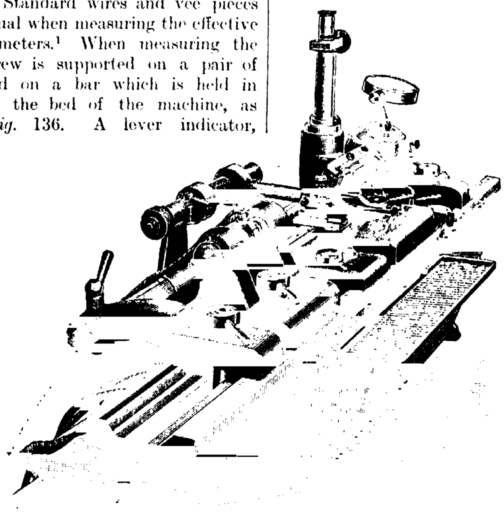


Fig. 136

§ (74) Société Genevoise Measuring Machine. (i.) *General Arrangement.* — A machine of more recent design than either the Newall or the Pratt and Whitney types is made by La Société Genevoise of Switzerland. A general view of this type of machine of 20 in. capacity is shown in *Fig.* 137. The essential parts consist of:

---

[1] For description of method see § (19) (iv.) and (vi.).

(*a*) The bed, at the right-hand end of which is fixed the micrometer headstock furnished with an indicator.

(*b*) The sliding tailstock, which carries a scale in line with the axis of measurement.

(*c*) The fixed micrometer microscope, held in a bracket bolted to the bed and focussed on the surface of the scale.[1]

The machine can be used as a comparator for determining the difference between gauges, provided the measurement does not exceed the 1 in. range of the micrometer screw. It also serves for the determination of the lengths of gauges by direct comparison with the scale; in fact, the machine was designed mainly with that purpose in view.

The construction of the bed, which is of U-shaped section, will be seen in *Fig.* 137. It rests on three feet. The bearing surface for the carriage consists of a vee and a flat.

(ii.) *Tailstock and Scale.*—The tailstock 1 rests on the bed, but the bulk of its dead weight is taken on four rollers 2, which are mounted on spring levers fastened to the underside of the carriage and which roll on auxiliary flat surfaces on the bed. This arrangement serves to reduce the sliding friction and, in addition to giving the carriage a freer motion, helps to preserve the accuracy of the bed. When setting the carriage to any desired point under the microscope, it is first brought into approximate position by noting the reading of an index point on a scale placed along the front of the machine. The final adjustment is then made by closing the clamp 3, by the lever 4, and operating a hand-wheel which, by means of worm-gearing and the traversing screw 5, gives a slow motion to the carriage.

The scale 6 is approximately of rectangular section $\frac{3}{8} \times \frac{15}{16}$ in. It is graduated at every twentieth of 1 in. over a length of 20 in. along the middle portion of the upper surface, which is highly polished. The material of the scale is 58 per cent nickel steel. This alloy is chosen, as it has practically the same coefficient of expansion as that of steel and it remains free from oxidation when exposed to the atmosphere. The scale has two plates screwed to its underside at the "Airy" points. The one on the left is fastened down to the upper surface of the carriage by a screw from below, whilst a shank on the other rests freely in a hole towards the other end of the carriage and is located transversely by a set screw from each side. This arrangement ensures that the scale is not strained, and provides a means of adjusting it parallel to the axis of the bed. A hood is fixed over the scale and removable shutters are provided in short sections, so that only the particular portion which is to be viewed need be uncovered. The graduated surface of the scale is placed

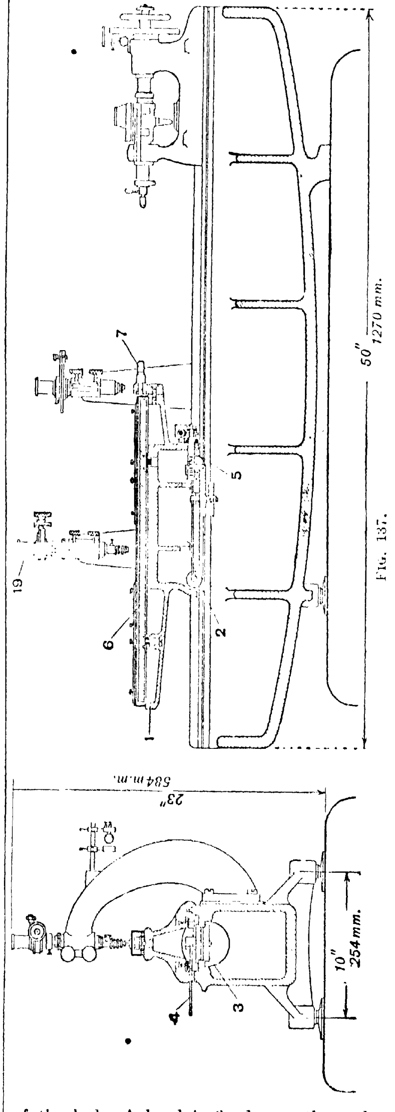


FIG. 137.

---

[1] The right-hand microscope seen in *Fig.* 137 is for screw measuring purposes and will be referred to later.

directly in line with the centre line of the measuring face 7 of the carriage; this constitutes one of the most important features of the design. The advantages of this arrangement are referred to in § (17), *III.*

(iii.) *Micrometer Headstock.* — The micrometer headstock, which is shown in section in *Fig.* 138, is screwed down to the right-hand end of the bed. A hollow steel plunger 8 slides in hardened steel bushes and is fitted at the front end with an internal plunger 9, the face of which constitutes the contact face of the headstock. The rear end of the plunger 9 is fastened to a spring 10, the other end of which is connected to a movable bush 11. By moving this bush axially the spring can be put either in compression or tension, according as to whether external or internal measurements are to be made. This movement is accomplished by rotating the ring 12, which has an internal helical groove into which a stud fastened to the bush fits. The force exerted by the spring is about 2 lbs. weight.

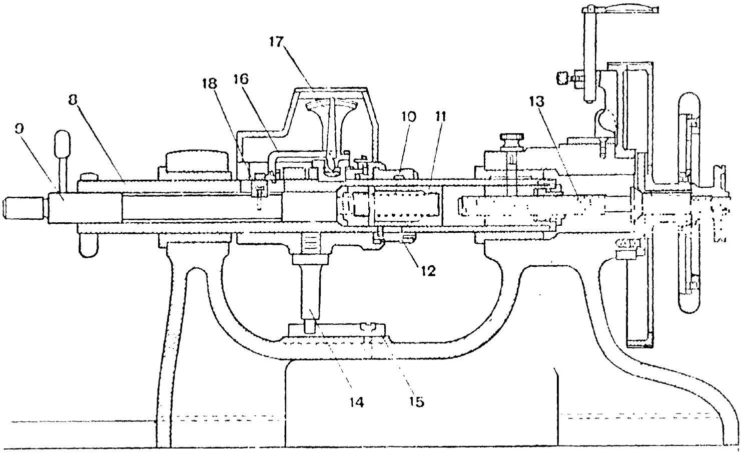


FIG. 138.

The rear end of the main plunger is fitted with the steel micrometer nut. In order to take up backlash a short auxiliary floating nut is provided at the outer end of the main nut, the two being separated by a spring. The micrometer screw 13, which is also of steel, has a specially deep vee thread and is screwed 50 threads per in. right hand. It obtains its thrust from a conical collar which bears against a hardened steel-plate screwed to the rear face of the headstock. The screw is provided with a 5 in. hand-wheel, graduated to read to 0·00005 in. direct, and, by means of a vernier, readings can be taken to 0·000005 in. The magnification of the micrometer screw and wheel is practically 800. The range of the micrometer screw is 1 in.

To prevent rotation of the main plunger, a radial arm 14 is fixed to it by a sleeve. The end of this arm runs between two guide plates 15, screwed to the base of the headstock. As explained in the case of the Pratt and Whitney machine, by suitably forming the edges of these plates, any slight errors in the run of the micrometer screw can be corrected.

(iv.) *Indicator.* — The indicator forms part of the headstock and gives a register of the relative positions of the main plunger 8 and the internal plunger 9. The mechanism of the indicator consists of a pair of multiplying levers 16 and 17 working on hardened steel knife edges. The upper end of the second lever registers over a graduated arc divided into twentieths of inches, each of which represents a movement of the internal plunger of 0·00005 in. This magnification is the same as given by the micrometer screw. The short arm of the first lever is moved by a knife edge 18, which is attached to the inner plunger 9, and the magnification of the indicator can be set by raising or lowering this knife edge and so varying the short arm of the first lever. In order to eliminate backlash the various parts of the indicator are kept in contact by a spring. The whole indicator is totally enclosed in a metal case carried on the main plunger, and a glass top is provided for viewing the scale.

This indicator can be used as a "zero" indicator, or it will serve for measuring differences up to ± 0·0005 in. by means of the scale on the graduated arc.

(v.) *Micrometer Microscope.* — The microscope 19 (*Fig.* 137) is used for reading the position of the scale along the bed. It gives a magnification of 60 and is provided with a pair of parallel wires in the eyepiece, whose position can be varied and read by a micrometer screw and graduated drum. The field of the microscope is illuminated by a small glow lamp placed at the rear of the bracket. The range of the micrometer is $\frac{1}{20}$ in., which is also the distance between the lines on the scale. When making a measurement of a gauge by the scheme outlined in § (17), *III.*, this machine offers two possible methods. The microscope can either be used as a zero indicator, the difference between the length of the gauge and the nearest corresponding line on the scale being measured on the micrometer headstock, or *vice versa.* It should be noted in either case that it is necessary to know the calibration of one or other of the micrometer screws, and, in addition, the length of the scale must be known at every division throughout its length. With regard to the latter, the makers supply a table of errors for each inch division, and they guarantee that the dividing is so uniform that the errors at any of the intermediate divisions can be determined to a sufficient accuracy by interpolation. The most satisfactory method appears to be to use the microscope as a zero indicator, *i.e.* the positions of its cross-wires are not varied throughout a set of readings, and, using only the known inch lines on the scale, to measure the difference on the micrometer headstock, the errors of which should be known throughout its 1 in. travel. The results of tests made by the latter method proved that absolute measurements can be made on this type of machine to an accuracy of within ± 0·00005 in. over a maximum length of 20 in.

(vi.) *Other Uses of the Machine—Internal Measurements.*—In addition to the measurements of end gauges and plain-plug gauges, the machine can be fitted with special adaptors, as shown in *Fig.* 139, for making internal measurements of gap and ring gauges. A U-shaped piece is fitted over each measuring face and is held in position by a clip. The gauge to be measured is held or placed on a suitable support so that the hardened steel internal contact points on the adaptors come in contact with the gauging surface. Having set the spring of the headstock in tension, the plunger is gradually moved to the right by the micrometer screw until the indicator registers at its zero position. The machine has to be standardised on a gap gauge of known size. In making such measurements, care has to be taken to ascertain that no elastic deformation of the gauge occurs owing to the stretching effect of the force applied.

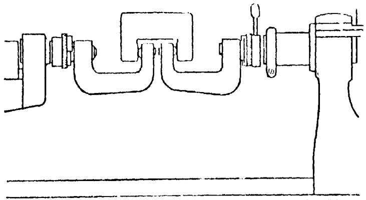
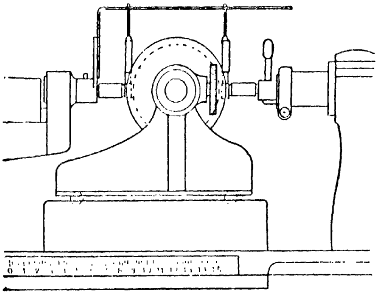


FIG. 139.

(vii.) *Measurements of Plug-screw Gauges.*—Special fittings are provided for the complete measurements of plug-screw gauges.



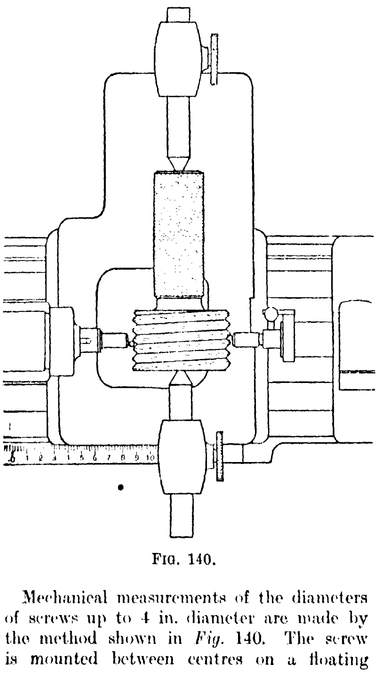


FIG. 140.

Mechanical measurements of the diameters of screws up to 4 in. diameter are made by the method shown in *Fig.* 140. The screw is mounted between centres on a floating

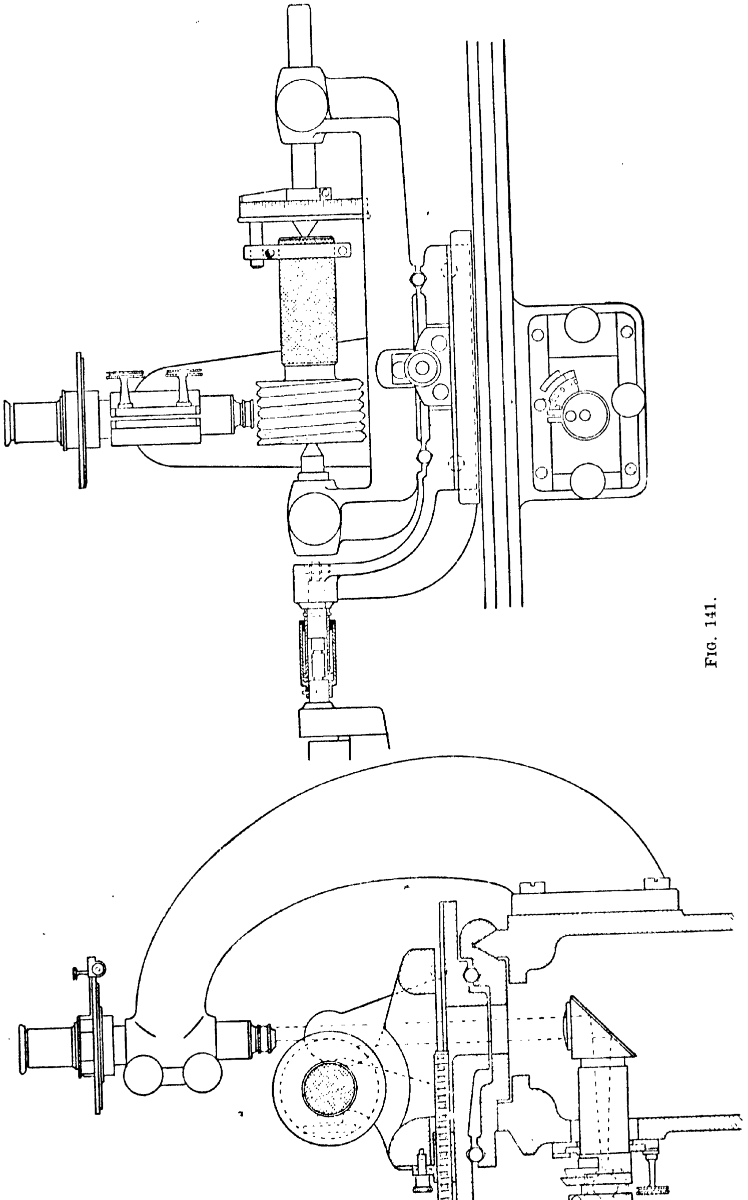


FIG. 141.

carriage, and the measurements are made between the usual contact faces of the machine, standard wires or vee pieces being inserted in the case of effective and core diameters.

The angle and pitch of screws up to 4 in. are measured by the method indicated in *Fig.* 141. An auxiliary microscope is carried in a bracket bolted to the bed. This microscope has a cross-wire which can be rotated, its position being read on a suitable angular scale. The gauge is held between centres with its axis parallel to that of the bed, on a carriage which is connected to the tailstock. This carriage can be moved along the bed with the tailstock, and it can also move transversely on a second slide. The microscope is focussed at the height of the centres and its field is illuminated from below, provision being made for adjusting the direction of the parallel beam of light to suit the rake angle of the thread. The screw is set so that its profile is seen in the microscope and the angles of the flanks are measured with the goniometric eyepiece. The pitch is measured by setting the cross-wire parallel to one set of flanks and moving the screw longitudinally, thread by thread, under the microscope, readings being taken on the scale with the micrometer microscope when the cross-wire coincides with each flank in turn. The observations are then repeated with the cross-wire set parallel to the opposite set of flanks.

The diameters of plug screws up to 1 in. can be measured optically by the scheme illustrated in *Fig.* 142. The screw is again mounted between centres on a carriage which can be moved in the diametral direction by means of a micrometer screw. The crests, roots, or flanks of the thread are set to coincide with the cross-wire of the microscope according as to whether it is desired to measure the full, core, or effective diameter of the screw.

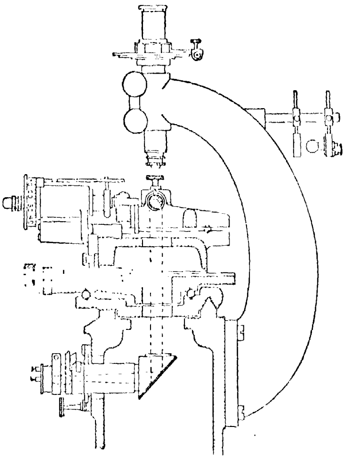


Fig. 142.

The Société Genevoise machine is made in three standard sizes of 20, 40, and 80 in. capacities. Machines for metric measurements are also made in corresponding sizes.

§ (75) "Shaw" Measuring Machine.—A complete description of this type of measuring machine will be found in *Proc. I. Mech. E.*, April 1913. The inventor, Dr. P. E. Shaw, first developed a method of measuring by using micrometer screws furnished with electric contact indicators in 1900. Experience was gained from one or two preliminary designs, and finally the machine shown in *Fig.* 143 was developed in 1912.

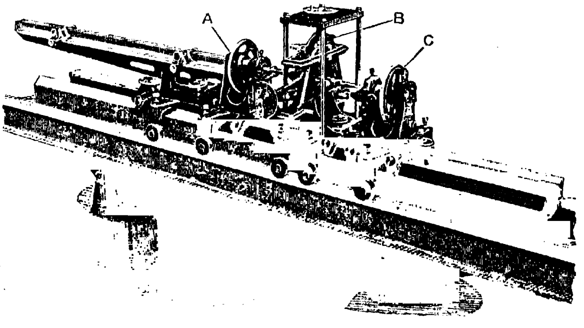


Fig. 143.

(i.) *General Arrangements.*—This machine serves as a gauge comparator, as a comparator between end gauges and scales, and as a means of exploring the parallelism of the faces of a gauge.

The bed supports three carriages, A, B, and C. The central one, B, is the gauge-holder, and consists essentially of a vertical compound slide working on a face which is surfaced to a high degree of accuracy. For purposes of setting up, the gauges are supported on the slide in gimbals, the rotational motions of which are controlled by fine adjustment screws. The other two carriages, A and C, are micrometer headstocks, the former being provided with an extension bracket at the rear for supporting a standard scale in line with the axis of the micrometers. The scale is viewed by means of a micrometer microscope rigidly held in an independent bracket. The bulk of the dead-weight of the carriages is transmitted through adjustable spring feet to a pair of girders placed alongside of the bed.

When comparing two gauges, they are mounted in the gauge-holder and aligned by means of the gimbal support. The first gauge is then brought between the contact points of the micrometers, which are clamped at a suitable distance apart, and a reading is taken on each micrometer. The slide B is then traversed across horizontally so as to bring the second gauge into position and a corresponding pair of readings is taken on it. The difference between the sum of the two micrometer readings on each gauge gives the difference between the lengths of the gauges.

To investigate the parallelism of the faces of an end gauge, a series of measurements are made at various positions on the faces by traversing the slide holding the gauge. It should be noted that such a series of measurements does not give a test on the flatness of either face, since the measurements made on the individual micrometers depend upon the truth of the slide of the gauge-holder. If the faces are shown to be parallel by obtaining a constant sum for the micrometer readings in different parts of the faces, it is quite probable that they are both flat. As a final check the flatness of the individual faces can be readily tested by the application of optical interference methods.

When measuring end gauges with reference to a standard scale, the method adopted is similar to that described in § (17), *III.*

(ii.) *Micrometer Headstocks.*—These are exactly similar in construction so far as the micrometer screw and contact points are concerned. A sectional diagram is shown in *Fig.* 144. The micrometer screw 1, and the nut 2, are of steel and have a pitch of $\frac{1}{2}$ millimetre. The graduated wheel 3, which is attached to the nut, is 150 mm. diameter, and is engraved with 500 divisions. A vernier enables readings to be taken to 0·0001 mm. The magnification of the micrometer and wheel is approximately 940. The nut rotates in a bearing 4, and the thrust is taken by the contact of a conical steel point 5 against a rigid flat face 6, both being hardened. The screw is prevented from rotating by a yoke piece 7, which slides along a guide bar parallel to the screw. Backlash between the screw and nut is minimised by the action of a pair of weights 8, the cords from which pass over pulleys and are connected to the yoke piece on the screw.

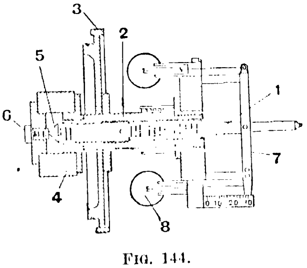


FIG. 144.

(iii.) *The Measuring Ends.*—Each micrometer headstock has a specially designed electric contact measuring end fitted to the micrometer spindle. A section of this fitting is shown in *Fig.* 145. A brass sleeve 9 fits tightly over the end of the micrometer spindle 1 and provides a bearing for the plunger 10, which can slide in and out of the insulated bush shown. The spring 11 normally keeps the plunger away from the point of the micrometer screw. The outer end of this plunger carries an adjustable head 12, which has a flat measuring face at the end. The squareness of the face to the axis of the plunger can be adjusted to a nicety by means of three small set screws, of which only two are shown. For making local measurements a cap 13, which has a small rounded contact point, can be fitted over the flat measuring face.

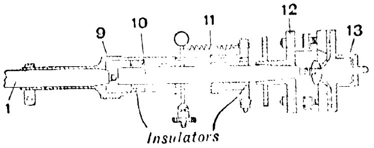


FIG. 145.

When making a measurement, the micrometer spindle 1 is slowly advanced until the measuring point, or face, makes contact with the adjacent end of the gauge. A further motion causes the plunger 10 to slide back in its bearing, and eventually its rear end comes in contact with the tip of the micrometer spindle 1. The two terminals of an electric circuit containing a cell and either a high-resistance telephone receiver or galvanometer are connected to the micrometer spindle and the plunger respectively. When contact is made between these two parts, an indication is immediately given by the telephone, or galvanometer. The contact is made between a bead on the end of the spindle and a plate at the end of the plunger, both being of iridio-platinum. With reasonable care the contacts can be kept free from dust, and it has been found that repeat readings can be made to 0·000002 in. It may be mentioned that this type of indicator gives no warning of the proximity of the setting, and this is a disadvantage if rapid readings are required.

§ (76) HARTMANN AUTOMATIC COMPARATOR.—This machine, which was designed by M. Hartmann of the Section Technique de l'Artillerie of France, is intended for the automatic intercomparison of spherical-ended length gauges; it is not adapted for use with a line standard of reference. The arrangement of the machine is such that a continuous series of intercomparison can be made between a pair of gauges, and the results are permanently recorded on a graduated chart. Thermal effects due to the observer, which present one of the most serious difficulties in using the ordinary type of measuring machine, are entirely eliminated. The apparatus is enclosed in a case provided with glass windows and is operated by a motor placed outside.

(i.) *General Arrangement.*—The bed of the machine carries a micrometer headstock and a tailstock as usual. The position of the latter can be adjusted along the bed to suit any length of gauge up to about one metre. The two gauges to be compared are supported in a carriage which is capable of movement in a direction transverse to the bed and is arranged so that the gauges are deposited between the measuring faces in turn. The headstock has a micrometer screw of 1 mm. pitch left-hand thread, the total run of which is only 2½ mm. The screw is driven by the action of a 40-gr. weight suspended from a cord which passes round a 20-cm. diameter pulley fastened to the spindle. This pulley has ten equally spaced radial arms, the ends of which are furnished with needle-points arranged parallel to the machine axis and on a circle of 2 metres circumference. The graduated chart, past which the needle-points move, is fastened round a drum capable of rotation about a vertical axis.

The action of the machine is simple. The cycle of operations commences by one of the gauges being deposited between the measuring faces. The micrometer screw is then advanced by the action of the weight on the cord, and continues to do so until a definite pressure is set up by its contact with the gauge. The outer end of one or other of the radial arms will be opposite the chart and the needle-point is pressed in. The screw now recedes so as to release the gauge, which is picked up by the traversing carriage, and the second gauge is deposited between the contact faces in its stead. The micrometer then travels forward once more to make contact with this gauge and a corresponding record is made on the chart. Assuming that the gauges do not differ by more than about 0·05 mm., the same needle-point will make the two records, and the circumferential distance between the marks will give the difference between the gauges to a magnification of 2000; *i.e.* a difference of 0·001 mm. will be represented by a space of 2 mm. on the chart.

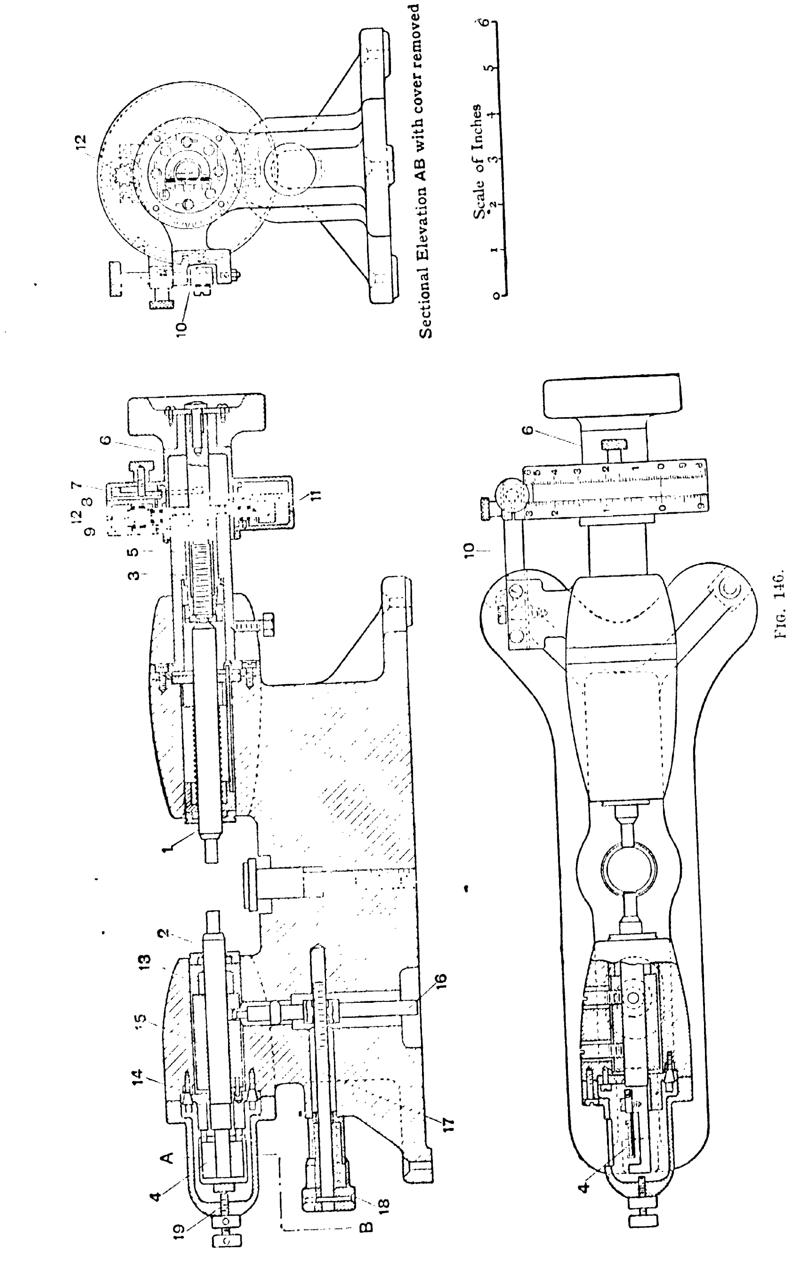
The above cycle of operations is repeated automatically about once every minute so long as the motor is allowed to run, and, as the drum is subject to a small rotation after each record has been made, the result of a series of comparisons is given by two horizontal rows of prick marks in the chart, one corresponding to each gauge. A mean line is drawn through each row and the difference between the gauges is read off on the graduations of the chart. The machine can also be used for the comparison of plug gauges or for investigating the parallelism and circularity of a plug gauge itself.

A full description will be found in *La Nature*, January 8, 1898.

§ (77) MEASURING MACHINE DESIGNED AT N.P.L.—Several types of small special measuring machines have been designed, and are in use at the National Physical Laboratory, Teddington; the one shown in *Fig.* 146 is an example. This machine was designed by Mr. E. M. Eden early in 1918 for measuring gauges up to a size of one inch. By a simple change of one of the measuring anvils the machine can be made to serve for sizes between 1 and 2 in.

There are several novel points in the construction of this machine, among which may be mentioned the optical indicator, the method of varying the working pressure between the contact faces at will, the method of adjusting the parallelism of the contact faces, and the arrangement of the dials of the micrometer to facilitate reading. The general arrangement of the machine is shown in *Fig.* 146. The two hardened steel plungers 1 and 2 are good sliding fits in bushes fitted to the body. The position of the right-hand one is controlled by the micrometer screw 3, and the other is connected to a deflecting mirror 4, which forms part of the indicator gear.

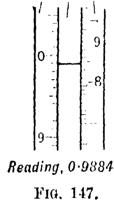
(i.) *Micrometer.*—The steel micrometer screw 3, which has 50 threads per inch, runs in an adjustable brass nut fitted in the sleeve 5, which is screwed to the body. The point of the screw has a hardened steel plug let in,



Sectional Elevation AB with cover removed

FIG. 146.

and this comes in contact with the rear end of the plunger 1, a light pressure being maintained by the spring shown. The plunger is prevented from rotating by a collar which slides along a steel wire. The outer end of the screw spindle carries a thimble 6, which has a 4-in. dial 7, divided into 200 parts, each of which represents 0·0001 in.: the magnification of the reading is roughly 630. Two other dials, 8 and 9, are fitted on the outside of the thimble so as to be free turning fits on it. The central one 7 has a single line engraved across it, and is prevented from rotating by a screw on its periphery, the point of which rests on an adjustable corrector bar 10. Each of the dials, 8 and 9, has an annular toothed disc screwed to the inner face of its boss, the number of teeth on the former being 49 and on the latter 50. A ring 11 is clamped to the thimble between the two bosses, and carries a small pinion wheel 12, which meshes with both the toothed discs. As the micrometer thimble is rotated it carries the ring 11 round with it, and the effect of the toothed discs and pinion is to cause the dial 9 to rotate in the same direction as the thimble, but at one-fiftieth of its speed. Now the fully divided disc 7 is marked with two zero lines diametrically opposite, and the nine intermediate main divisions of each half are numbered 1 to 9 consecutively. If the micrometer screw is at the one-inch end of its run, the dials will read as shown in the lower view of *Fig.* 146. On rotating the thimble in a clockwise direction through half a revolution, the plunger 1 will be made to advance by half the pitch of the screw, *i.e.* 0·01 in., and the opposite zero mark on the disc 7 will be brought to coincide with the single line. The dial 9 will, at the same time, have been driven through $\frac{1}{100}$th of a revolution, and consequently its 99 division will coincide with the single line. The dials now indicate the reading 0·9900. If the thimble is rotated a little further so that it now reads 84·4 small divisions against the line, the latter will be between the 98 and 99 divisions on the dial 9, and the whole reading is seen at a glance to be 0·98844 in. This setting is shown in *Fig.* 147. The gearing of the two dials 7 and 9 at the 50 to 1 ratio thus provides a very convenient means of indicating the complete reading.



Reading, 0·98844

FIG. 147.

(ii.) *Adjustment for Working Pressure.*—The left-hand measuring plunger 2 is supported at each end in the hardened steel bushes 13 and 14. A sleeve 15 is fixed to it by means of two set screws, and the motion of this sleeve is controlled by the vertical lever 16, which rocks about a spherical-shaped boss formed on the rod at about a quarter of its length from the upper end. At about half-way down, the lever is tapped for the tension rod 17, the left-hand end of which is fixed to the cap 18, which fits over a barrel attached to the main casting. This barrel contains a compressed spring, the length, and consequently the strength, of which can be varied at will by screwing the tension rod into or out of the lever. If the measuring plunger 2 is pressed in by the action of the micrometer plunger, the motion is resisted by the spring operating through the tension rod 17 and the lever 16. The pressure can be varied from about 1 lb. to 10 lbs. by altering the strength of the spring, and the amount of pressure is indicated by a scale engraved on the outside of the barrel.

This adjustment for working pressure is useful when dealing with different types of gauges. Small, light, or hollow pieces or balls can be measured under a light pressure of one or two pounds only, whilst for heavier gauges the pressure can be increased to 5 lbs. or more at will.

(iii.) *Adjustment for Parallelism of Measuring Faces.*—The two measuring plungers are ground to the same diameter to within 0·0001 in., and their faces are lapped in the same jig so as to be truly square to their axis. Unless the end bushes for the plungers are all exactly in line, however, the faces will not come parallel when the plungers are assembled. This difficulty could be overcome by lapping the four bushes simultaneously with a long plug lap extending right through the machine, but the adjustment can be made more readily by the following device. The rear bush 14 of the left-hand plunger is a snug fit in a shallow recess bored in the main casting. The actual shape of the bush is shown in *Fig.* 148. It is held in the recess by four cheese-head screws, and in addition, has four conical holes into which fit corresponding taper plugs, the ends of the plugs being screwed into the casting. The flange of the bush is split through the conical holes as shown. If the cheese-head screws are slightly slackened, it is possible by screwing the conical plugs in or out to spring the flange of the bush, and so vary within a certain range the lateral position of the inner diameter of the bush with respect to the recess in the casting. The actual length of the bearing of the bushes and the plunger is quite short, so that the

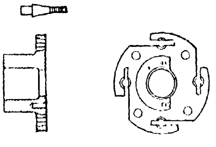


FIG. 148.

direction of the latter answers readily to this fine adjustment without producing a binding fit in the bushes. On completion of the adjustment the cheese-head screws are tightened.

(iv.) *Optical Indicator.*—The relative position of the left measuring plunger and the casting is indicated by a deflecting mirror 4, *Fig.* 146. This mirror is mounted, as shown in *Fig.* 149, on a backing plate 0·02 in. thick, which is turned over at the edges to form a clip. A strip of steel foil, 0·002 in. thick, is soldered down the middle of the plate at the back, and two similar strips are soldered down the two sides on the other face of the plate as shown in the figure. The end of the central strip is fastened to the plunger which is cut away for the purpose, and the outer strips are both clamped to the bush 14. It is clear from *Fig.* 150 that if the plunger is pressed back from its position of rest the strips will be bent, and, assuming that they become arcs of circles, the angular deflection θ of the mirror is equal to the ratio of the movement of the plunger d to the distance between the strips, *i.e.* to the thickness t of the plate on which the mirror is mounted. Thus the magnification can be varied at will by altering the thickness of the plate to which the strips are fastened. To prevent the plunger being pushed back too far and overstraining the strips, an adjustable stop 19 (*Fig.* 146) is screwed into the cap opposite the rear end of the plunger.

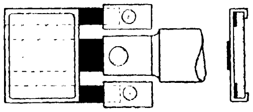


FIG. 149.

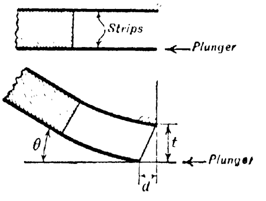


FIG. 150.

The deflection of the mirror is observed by the movement of a spot of light reflected from it by the arrangement shown in plan in *Fig.* 151. The light from a small straight-filament glow lamp A is directed on to the deflecting mirror by a lens B, and the filament is brought to a focus as a bright line on a translucent screen D after reflection from a fixed mirror C. In a particular machine, where the distances from the machine to the fixed mirror C and to the screen D were 27 and 13 in. respectively, a movement of the plunger of 0·0001 in. gave a deflection on the screen of 0·45 in.

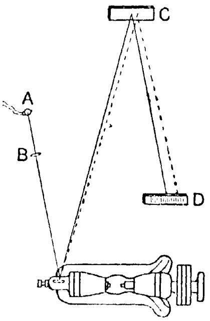


FIG. 151.

The machine will serve as a comparator, using the indicator as a zero-setting device only, the differences being measured on the divided drum of the micrometer. In cases where the difference to be measured does not exceed about 0·0005 in. it can be measured on the scale of the indicator, using a mechanical stop on the micrometer screw. This stop is arranged by clamping together the revolving disc 7 and the central disc 8 which carries the screw stop resting on the corrector bar.

The travel of the micrometer is 1 in. The machine can be arranged to measure sizes up to 1 in., or by substituting a shorter plunger 1, the capacity can be changed to suit sizes from 1 to 2 in.

§ (78) THE "REID" MEASURING MACHINE.—The object of the inventors and makers of this machine was to produce a measuring instrument for use in the workshop, which would be free, as far as possible, from the effects of vibrations and rough handling, and yet capable of giving measurements accurate to within half a ten-thousandth part of an inch.

A general view of the machine is shown in *Fig.* 152. The stiff horizontal arm 1 is bolted down to the base plate 2 and carries at its extremity the vertical measuring head 3 of 1 in. range. The latter is provided with an optical indicator of the deflecting mirror type which gives motion, on a screen 5, to a spot of light produced by the lamp and arrangement of lenses 4.

The capacity of the machine can be readily adjusted to any desired length within reason by inserting distance pieces 6, of suitable size, between the arm and the base.

The measuring head is arranged in such a manner that the spindle does not rotate but is capable of being moved vertically by rotation of the body 3, which carries the nut at its upper end. Referring to *Fig.* 153, the body has an extension at the lower end which forms a guide for the spindle, and which is itself a good running fit in a hardened steel bush fitted to the projecting arm of the machine. The whole micrometer can thus be lifted out of the arm if necessary. When making a setting on a gauge placed on the anvil, the end of the micrometer spindle is allowed to rest on it by the action of the weight of the whole micrometer, and then, by rotation of the micrometer body, the latter can be lowered, with respect to the spindle, until the rim on its under side comes in contact with the steel strip 9 supporting the deflecting mirror. This strip has at the inner end a small ball, which rests in a hollow, being held in place by a light finger spring as shown. This ball forms a pivot about which the mirror can turn. The outer end of the strip is pressed gently upwards by a second finger spring. The strip has two other small balls pressed into holes in it, which come opposite the rim of the micrometer body as shown in the plan view. As soon as the rim makes contact with these two balls, further lowering of the micrometer body will cause the strip supporting the mirror to tilt about the fulcrum formed by the single ball and thus produce a corresponding deflection of the mirror. This deflection is observed in the usual manner by the movement of a spot of light on the scale 5 (*Fig.* 152). The magnification is considerably increased by obtaining a double reflection on the deflecting mirror by the use of a narrow fixed mirror placed just in front of the first (a scheme which was originally due to Mr. E. M. Eden of the National Physical Laboratory). The result of such double reflection is that the rays which eventually leave the mirror are deflected through an angle which is four times as great as the angle through which the mirror itself turns. The magnification obtained on the instrument is 1000 to 1, which is very suitable for workshop measurements. This particular type of indicator is free from lag or backlash, and setting can be repeated to 0·00001 in.

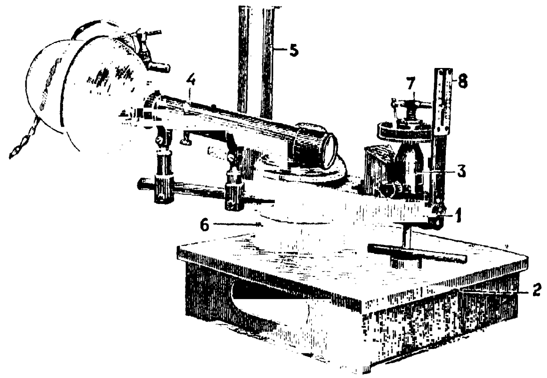


FIG. 152.

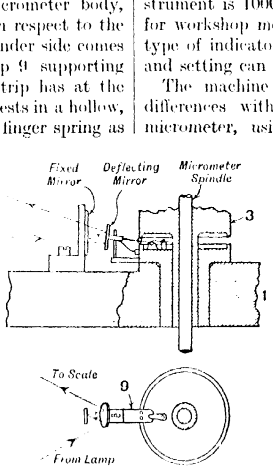


FIG. 153.

The machine can be used for measuring differences within the 1 in. range of the micrometer, using the indicator as a zero setter and bringing the spot of light to the same mark on the scale for each setting. By suitably dividing the scale, differences up to 0·002 in. can be read off direct without making use of the micrometer screw, which would be kept at a constant reading during such comparisons.

The instrument can be very quickly changed over from inch to metric measurements by removing the inch micrometer head and substituting one of metric pitch.

The machine is made by Messrs. Reid Bros., under Patent No. 114,702.

§ (79) TESTS OF MEASURING MACHINES.—The main points which require attention when testing measuring machines are as follows:

(i.) Accuracy of surface of bed.

(ii.) Flatness and parallelism of measuring faces; squareness of faces to axis of machine.

(iii.) Calibration of micrometer screw.
(iv.) Sensitivity of indicator.
(v.) Calibration of scale.

(i.) *Surface of Bed.*—The importance of the straightness of the bed in machines where the scale is not in line with the axis of measurement has already been emphasised (see § (17), *III.*). In such cases sufficient accuracy is hardly obtained by the ordinary straight-edge test, and it becomes necessary to make a more elaborate investigation, using an autocollimating telescope and a mirror. Briefly, this method consists in mounting a mirror on either the headstock or tailstock (whichever moves), so that its face is perpendicular to the axis of the bed, and fixing a telescope opposite the mirror with its axis parallel to that of the bed. When correctly adjusted, an image of the cross wires of the telescope will be seen in the eye-piece. The carriage supporting the mirror is then moved slowly along the bed, and if the latter is not straight the carriage will tilt slightly and give rise to movements of the image of the cross wires in the telescope. The amount of angular motion of the carriage can be readily ascertained if the movement of the image is measured with a micrometer in the eye-piece of the telescope.

(ii.) *The Measuring Faces.*—The flatness of the faces is investigated most readily with an optical proof plane. Want of flatness will be indicated by the presence of "Newton's Rings," and an estimation of the error can be made by noting the number of rings which occupy the area of the face. The faces should, if anything, be slightly convex. By careful lapping it is a fairly easy matter to make them flat to within 0·00001 in. The parallelism is tested by making a number of measurements on a $\frac{1}{4}$ or $\frac{3}{8}$ in. steel ball, when placed in different parts of the faces. For this purpose it is useful to solder the ball on to the end of a short rod as shown in *Fig.* 154. A flat is made on the handle, which is always kept in one position, either vertical or horizontal, in order to confine the measurements to one diameter of the ball.

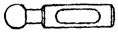


FIG. 154.

Although the faces may be parallel they may both be inclined to the axis of the bed of the machine. Such a defect would produce an error when comparing a flat-ended gauge with one having spherical ends, both gauges being supported. In order to test the squareness of the faces it is necessary to have an end gauge with flat faces which are accurately perpendicular to its axis. The gauge is mounted in the machine on a pair of supports which are adjusted vertically and transversely until the body of the gauge lies accurately parallel to the bed, as tested with a surface gauge and indicator. The end faces will then be square to the bed in both directions. A test of the parallelism of each contact face with the corresponding end face of the gauge is then made as before by taking measurements with a ball at each end of the bar in turn.

(iii.) *The Micrometer.*—The accuracy of the readings of the micrometer screw is tested most readily by taking a series of measurements on a number of standard slip gauges whose sizes are accurately known. The test should first be made using a series of gauges which differ in turn by an amount equal to the nominal pitch of the micrometer screw, or some complete multiple of it. Such readings, when properly corrected for the errors of the gauges themselves, will give a test on the progressive error of the screw. It will usually be found that the total progressive error does not exceed 0·0001 in. over a range of 1 in., and, provided the error is of a fairly uniform nature, it is hardly necessary to apply any correction in the case of comparative measurements where the difference between the standard gauge and the unknown gauge does not exceed 0·1 in.

A more troublesome error, and one which frequently occurs, is that which arises from lack of squareness of the thrust collar of the micrometer screw and the abutment plate with respect to the axis of the screw. Referring to *Fig.* 155, which shows the micrometer spindle in two opposite positions during a revolution, it will be seen that the spindle has moved axially an amount e, equal to the error in the squareness of the thrust face which is the more nearly accurate of the two surfaces. This end movement is reversed during the second half of the revolution. Consequently, throughout its run, the micrometer spindle experiences an oscillatory axial movement which is repeated at every revolution, and which can be represented by the ordinates of a sine curve, the periodicity being equal to the pitch of the screw.

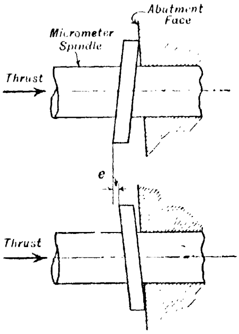


FIG. 155.

In order to test for this periodic form of error, a series of gauges should be chosen so that a number of readings can be obtained within a range of one revolution of the micrometer, *e.g.* if the pitch of the screw is

0·05 in. the gauges could be made to differ by 0·01 in. in turn, so as to obtain 5 readings over a revolution. The test should be repeated over several revolutions in different parts of the screw. The diagrams in *Fig.* 156 represent actual calibration curves obtained for a micrometer screw having a pitch of 0·05 in. The upper curve shows the results of measurements made at every 0·1 in. along the screw, from which it will be noted that the progressive error is within 0·00004 in.

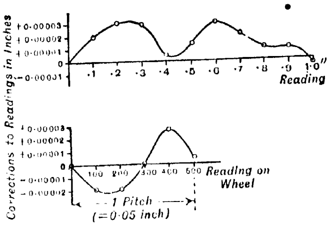


Fig. 156.

The lower curve gives the corrections to be applied to the readings on account of periodic error, which amounts to ±0·000025 in. The curve does not follow exactly a theoretical sine curve probably owing to observational errors.

(iv.) *The Indicator.*—The indicator should be tested initially by taking a number of settings on the same gauge, and noting the accuracy to which the readings agree. The effect of relatively fast and slow settings should also be tried. A test for "stickiness" can be obtained by making a setting and then giving the whole machine a series of jars. If the indicator has considerable friction in its mechanism, the effect will be to make a distinct change in its indication.

The sensitivity of the indicator is tested by noting the change produced in its indication by a known difference in the micrometer reading. The magnification of the indicator should be at least equal to that of the micrometer screw and wheel.

(v.) *The Scale.*—To calibrate the scale it should be removed from the machine, and the distance between its various graduations compared with standard scales by the methods referred to under "Line Measures." The scale should then be replaced on the machine and measurement made on a number of standard end gauges of known length, using the calibrated values of the scale. If the measured lengths of the gauges, after making the various corrections for temperature, etc., do not agree with their accepted lengths, it is probable that either the bed of the machine is in error, or the scale has been strained in fixing it to the machine.

VI. Gauge Comparators

§ (80) Prestwich Fluid Gauge.—Designed and manufactured by Messrs. J. A. Prestwich & Co., Ltd., London.

This form of comparator is shown in *Fig.* 157. It consists of a solid base having an anvil piece E and carrying a vertical bar O at the rear, on which the indicator mechanism is supported. The latter consists of a flat metal fluid chamber A, having a flexible diaphragm B at the under side and connected at the top to a fine-bore glass tube C. The diaphragm carries a contact face D. If this face is pressed upwards the flexure of the diaphragm causes the fluid to rise in the glass tube, the movement of the contact face being magnified approximately in the ratio of the sectional areas of the chamber A and the tube C. In practice this magnification varies in different instruments from 500 to 1200 according to the accuracy desired.

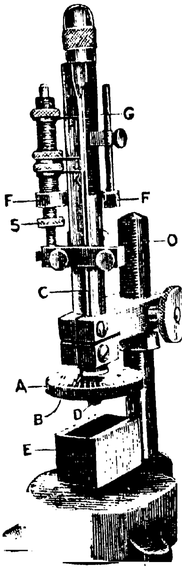


Fig. 157.

The neck of the chamber A has a micrometer thread of short range, and the upper surface is graduated round the periphery to indicate thousandths of inches. The height of the fluid column can be read against a metal scale G, graduated to read to ten-thousandths of an inch, according to the particular instrument. When testing work between limits, the instrument is first adjusted roughly by lowering the indicator by means of the rack and pinion motion on the supporting bar, until the face D comes in contact with a setting gauge or a standard piece of work placed on the anvil. The final adjustment is made by means of the micrometer screw. The two index points H and J are then set to correspond with the prescribed limits. The various pieces of work are then passed through the machine in rapid succession, and it is noted whether the height of the fluid column does, or does not, arrive between the two points in each case.

The instrument is somewhat susceptible to temperature fluctuations as it acts much the same as a thermometer, the effect being to change the zero setting. To overcome this difficulty, a third point K is set initially opposite the normal fluid level when the contact face D is free. The position of this point is then checked periodically, and if it is found that owing to temperature effects the setting is no longer correct, the carrier F, which supports the three index points, is moved bodily by the thumb nut S so as to restore the setting.

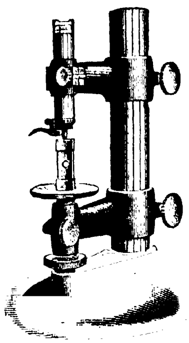


FIG. 158.

The instrument is accurate and rapid in use, and can be made to serve a variety of purposes by providing suitably shaped anvils E.

§ (81) THE HIRTH "MINIMETER."—This instrument, which is made by the New Fortuna Machine Co., Ltd., Bristol, is a form of gauge comparator where the differences are read direct on a graduated scale. *Fig.* 158 shows one type of the complete apparatus for the comparison of end gauges. It consists essentially of a base to which is fixed a stiff vertical post carrying two brackets, the upper one of which supports the measuring head and the lower one the table on which the gauge rests. The brackets can be clamped at any convenient height on the post to accommodate different size gauges. The table has in addition an adjustment in the vertical direction by means of a fine pitch screw. The measuring head is provided with a trigger for raising the contact point when inserting the gauge.

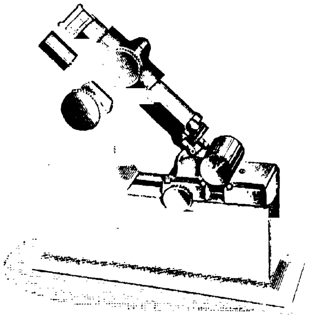


FIG. 159.

A special type of stand for use in comparing plugs is shown in *Fig.* 159. In this case the measuring head is held in an inclined position. The plug to be measured rests on two parallel cylinders fixed to the edges of two blocks, one of which is placed opposite the measuring head, the other being movable. The position of the latter is set against a scale to ensure that the axis of the measuring head always passes through the centre of the plug. The arrangement is shown diagrammatically in *Fig.* 160. Various other types of stands are made for adapting the measuring head to special uses.

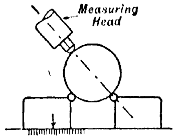


FIG. 160.

A sectional view of the measuring head is shown in *Fig.* 161. The principle of operation will be understood from the diagram to the right of the figure. A block, 2, which has vees

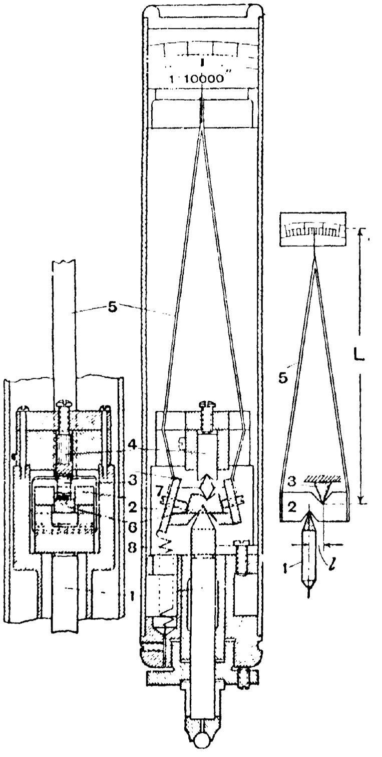


FIG. 161.

cut in its upper and lower faces, is held between two knife-edges, the upper one of which, 3, is fixed, whilst the lower one forms the top of the measuring plunger 1. If the latter is moved upwards, the block 2 will be tilted and the amount of movement of the plunger will be indicated on the scale in the upper part of the instrument, the magnification being equal to the ratio of L to *l*. The magnification can be set to any desired figure by varying the short arm *l* of the lever. This is accomplished by adjusting the lateral position of the small block 6 relative to the main block 2 by means of the two screws 7. The upper knife-edge 3 is in the form of a prismatic-shaped steel piece which obtains its abutment against the stop 4, fixed to the body of the instrument. The knife-edges and vees, which are all glass-hard, are kept in contact by the tension spring 8, which prevents any back-lash in the movement. The whole mechanism is enclosed in a dust-tight steel tube, 1 in. diameter by about 6 in. in length, the upper end being provided with a glass window in front of the index point and scale. The magnification is set in different heads so that the divisions of the scale indicate 0·001, 0·0005, 0·00025, or 0·0001 in. The range of the instrument is about twenty-five divisions.

§ (82) SPECIAL END MEASURING COMPARATOR OF HIGH SENSITIVITY.—Special comparators have been designed at the National Physical Laboratory with the primary object of dealing with slip gauges where the tolerances allowed are of an order of only ±0·00001 in. This figure necessitates being able to make a comparison between an unknown gauge and a standard gauge of the same nominal size to an accuracy of about ± one-millionth of an inch.

The first comparator for this work was designed by Mr. E. M. Eden in October 1918, and was made at the Laboratory. This experimental machine was fully described in *Machinery* of January 8, 1920. The magnification obtained was approximately 18,000, so that a difference of 0·00001 in. between two gauges was recorded by a movement of the image of a cross-wire of nearly 0·2 in. This machine gave satisfactory readings to the millionth of an inch.

One of the measuring faces had a hardened steel ball soldered in the centre and the other had three equal balls also soldered on in mutual contact in the form of a triangle. This type of contact faces simplified the design, but, on the other hand, the machine could only be used for flat gauges.

A more elaborate machine was designed by Mr. J. E. Sears in April 1919 on somewhat the same lines as the earlier machine, but it is more generally useful since it has a pair of flat parallel measuring faces. The capacity of the machine is from 0 to 4 in.

(i.) *General Arrangement.*—In both types of machines one of the measuring faces is held rigidly while the other is brought up into contact with the gauge by the action of a weight which produces an elastic deformation of certain parts of the machine. This deformation is magnified by mechanical and optical means and is recorded by the movement of the image of a cross-wire on a scale.

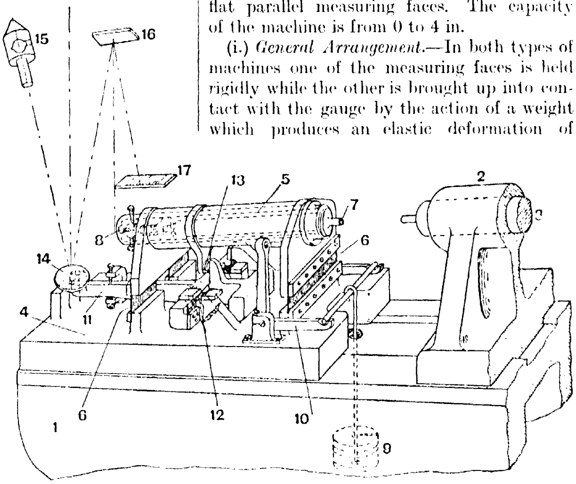


FIG. 162.

A diagrammatic view of the later form of machine is shown in *Fig.* 162.

The bed of the machine carries two headstocks which are securely screwed down in position. The right-hand headstock 2 carries a plunger 3, 1¼ in. diameter, which is lapped to be a good sliding fit in the bore and which can be held in position by two locking screws. The face of this plunger, which is ¼ in. diameter, is lapped in a special jig to ensure it being quite flat and square to the axis of the plunger. The base of the left-hand headstock 4 is screwed to the bed and supports a barrel 5 by means of flexible steel strips 6 at each end. This method of support allows the barrel to move slightly backwards and forwards in a direction parallel to the bed, its axis meanwhile remaining always parallel to itself. The barrel is bored to carry a spindle 7, the right-hand end of which forms the second measuring face of the machine. The spindle has a spherical-shaped collar at the right-hand end which rests on a bevelled facing at the end of the barrel, the contact being maintained by a spring placed inside the barrel. The latter is bored to clear the spindle, the tail end of which is held between the points of two pairs of set-screws 8 placed at right angles and screwed into tongues at the rear end of the barrel. By adjustment of these set-screws the spindle can be tilted on its spherical seating until the two measuring faces are brought accurately parallel. As the faces are required to be parallel to an order of a millionth of an inch, this adjustment needs to be very sensitive, and is provided for by the springing action of the four tongue-pieces in which the set-screws were mounted.

The pressure between the measuring faces is derived from weights 9, suspended by a rod hanging in a recess in the bed. This force is transmitted to the barrel by two bell-crank levers 10, placed one on each side of it. The working pressure of the machine can be readily adjusted to any definite figure by varying the amount of the suspended weights.

To open the measuring faces for inserting a gauge, the barrel is moved slightly to the left on its flexible supports by means of a lever attached to the left headstock. This lever is connected to a spindle carrying an eccentric disc which operates on a stop fixed to a lower part of the barrel. (The lever is not shown in the diagram.) Having inserted a gauge, the lever is turned to disengage the eccentric and the left-hand measuring face is automatically brought in contact with the gauge by the action of the weights, with the result that the gauge is held suspended between the two faces.

(ii.) *Indicator.*—When comparing two gauges of approximately the same size, they are inserted in turn between the measuring faces, as just described, and it is the function of the indicating gear to register the relative displacement between the two settings of the left-hand measuring face or the barrel to which it is fixed.

This indicator gear magnifies the small displacement partly by mechanical and partly by optical means. The mechanical part is performed by a 10 to 1 bell-crank lever 11. This lever is not supported on pivots, which, however well made, would introduce some degree of friction or backlash, but has a virtual axis of rotation formed by the line of intersection of the planes of two pairs of steel strips 12, one pair horizontal and the other vertical, which are fixed to the lever and to lugs on the base of the left headstock. (The perspective view of the machine makes this arrangement clear.) The short arm of the lever, which is vertical, is furnished with a hardened steel ball point which rests against a flat hardened steel stop 13 projecting from the base of the barrel. The face of this stop is arranged to be vertically above the virtual axis of the lever, so that no sliding (with consequent friction) takes place at this contact for small movements of the barrel, the motion being wholly of a rolling nature. It might be mentioned that the correct functioning of the machine depends largely upon the method adopted for supporting this lever and the arrangement of its contact with the barrel.

The longer arm of the lever is horizontal, and the vertical motion of its end is transmitted to a tilting mirror 14. To prevent undue straining of the suspension springs of the lever, its motion is limited by two stops as shown in the diagram. The mirror, which consists of a plano-convex lens of approximately 80 in. focal length with the lower plane surface silvered, is fixed lightly to a thin brass disc, about $\frac{3}{4}$ in. diameter. Three $\frac{1}{16}$-in. steel balls are soldered to the base of this disc, the middle one being approximately 0·1 in. off the centre line of the two outer ones. The middle ball rests between two $\frac{1}{16}$-in. diameter cylinders fixed horizontally to the end of the 10 to 1 lever with their axes parallel to the lever, and the outer ones are supported in a similar manner on brackets fixed to the base of the headstock. The arrangement is shown in *Fig.* 163. The mirror and brass plate are made as light as possible, and the balls and cylinders on which they rested are highly polished so as to reduce the friction to a minimum. When in use the mechanism of the left headstock is protected from dust by an aluminium hood which fits over it.

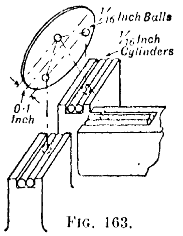


FIG. 163.

A 100-c.p. "Pointolite" lamp in a special holder 15 (*Fig.* 162) is supported from the wall above the mirror. The lamp is provided with a condenser, by means of which the beam of light can be focussed on the small lens-mirror after passing through a small window in the hood. A fine wire is stretched across the front of the condenser and the lens-mirror forms an image of it, which, after reflection from a fixed mirror 16 placed above the machine, is brought to a focus on an opaque scale 17 placed at the rear of the machine. As the height from the lens-mirror to the fixed mirror is $5\frac{1}{4}$ ft., the total length of the optical lever is 132 in. The short arm of the optical lever is equal to the distance between the small balls on the back, which is 0·1 in. The total magnification is thus equal to

$$\frac{10}{1} \times \frac{132}{0 \cdot 1} \times 2,$$

i.e. 26,400.

The last factor 2 is introduced to take account of the fact that the reflected beam of light turns through twice the angle of tilt of the lens-mirror. The result is that a displacement of the left measuring face of one hundred thousandth of an inch gives rise to a movement of the cross-wire on the scale of approximately 0·25 in.

The main divisions of the scale are set out by measuring standard slip gauges differing in size by 0·0001 in., these main divisions being afterwards divided into ten parts to represent 0·00001 in.

It is found that the machine will repeat its readings to within one-tenth of one scale division, *i.e.* to one millionth of an inch, but to obtain such repetition extreme care has to be taken with regard to the cleanliness of the surfaces of the gauges and of the measuring faces, and uniformity of temperature of the gauge and the machine.

§ (83) THE "LEVEL" END-GAUGE COMPARATOR.—This machine was designed by Mr. A. J. C. Brookes of the National Physical Laboratory in 1918, and is the subject of Patent No. 166248. Its function is the accurate comparison of the lengths of flat-ended length gauges. The design of the machine is such that it will deal either with thin slip gauges of the Johansson type or end gauges of bar form up to 6 ft. or more in length. Gauges which do not differ in length by more than one or two ten-thousandths of an inch can be compared to an accuracy of about one-millionth of an inch.

(i.) *General Principle.*—The scheme of this comparator is based on the use of a sensitive spirit-level. The essential parts of the machine are indicated in *Fig.* 164. The cast-iron bed 1 has its upper surface finished accurately flat, and on this surface rests a circular plate 2, the upper and lower faces of which are accurately finished both for flatness and parallelism. A central spigot on the base acts as a register for the plate. By virtue of the extremely high accuracy attained in the parallelism of the two surfaces of the plate, its upper surface remains parallel to its original position to within very fine limits when the plate is revolved. The two flat-ended gauges to be compared, G_1 and G_2, are wrung on to the plate side by side. A sensitive spirit-level 4 has a steel block 5 attached to it as shown. Three steel balls are soldered to the base of the block, two at one end and one at the other, so as to provide a three-point support for the level-tube.

To use the instrument, the level is allowed to rest on the two gauges and the position of the bubble in the tube is noted. The level is lifted off and the plate 2 carrying the gauges is rotated through 180°. The level is again placed on the gauges (without being turned end for end), so that the right-hand ball now rests on G_1 instead of G_2, and the position of the bubble is again noted.

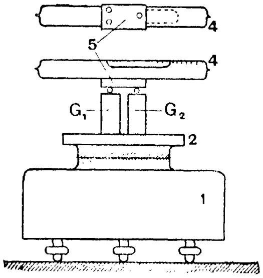


Fig. 164.

Now, since the surface of the plate does not deviate from its original plane during the rotation, the process of turning the plate through 180 is equivalent to interchanging the positions of the two gauges on the plate with the latter kept in its first position. If the two gauges are not of equal length, then the position of the bubble in the level-tube will differ after the plate has been revolved. Moreover, it should be noted that the effect of interchanging the gauges with respect to the level-tube is to tilt the latter through an angle equal to *twice* the difference between the lengths of the gauges divided by the longitudinal spacing of the balls on the base of the level. Assuming then that the calibration of the scale on the level is known with reference to the particular spacing of the balls, the difference between the gauges is obtained by halving the travel of the bubble. The sign of the difference is readily obtained by noting in which direction the bubble moves after the gauges have been interchanged.

It should be appreciated that it is not necessary for the surface of the plate 2 to be level. It is convenient, however, to adjust it by means of the levelling screws on the base, so that if the level-tube is allowed to rest on it, the position of the bubble is fairly central. This adjustment provides that the two positions of the bubble, when measuring, shall be approximately equidistant from the centre of the tube.

(ii.) *Sensitivity.*—The accuracy to which comparisons can be made depends upon two factors—the radius of curvature of the level-tube and the spacing of the balls. With regard to the former, it is possible to obtain ground level-tubes up to a radius of 800 ft. It has not been found necessary, however, to resort to a radius of more than 500 ft. to obtain sufficient sensitivity. Such a tube, with the balls spaced at 0·7-in. centres, gives a magnification of about 17,000 in the reading. The latter figure takes into account the doubling effect produced by rotating the plate carrying the gauge. If the spacing of the balls is reduced to 0·35 in., which is permissible when dealing only with block gauges, then the magnification reaches about 34,000.

It often happens that, even with level tubes which have been ground internally, the radius of curvature is not uniform along the tube. It is necessary, therefore, to test level tubes before making use of them in this apparatus.[1]

The accuracy of the results obtained from the machine can be made independent of any effect arising from slight lack of parallelism of the upper and lower faces of the rotating plate by adopting the following procedure.

The gauges are wrung down to the plate and a pair of readings obtained as indicated in (i.) above. The positions of the gauges on the plate are then interchanged and a second pair of readings obtained. The mean of the distances through which the bubble moves on the two occasions is a measure of the difference between the gauges, quite apart from any error which may exist due to rotation of the plate. This error can be determined by taking readings of the level when it is allowed to rest directly on the upper surface of the plate, which is rotated between the readings. With a plate which has just been trued up the error does not usually amount to more than a millionth of an inch over a 0·7-in. span. Cast-iron plates, however, even after heat treatment and ageing, have been found to warp to some extent, and, to obtain the best results, it is necessary to true up the faces periodically.

The upper surface of the base and the lower surface of the plate are finished by lapping, and, in order that the latter shall revolve freely, the interface is liberally lubricated with paraffin. It has been found that even when a comparatively large amount of paraffin is present between the surfaces, the liquid distributes itself as a film of uniform thickness, and the accuracy of the rotating plate is unimpaired.

(iii.) *Mechanical Arrangements.*—The general arrangement of the machine is shown in *Fig.* 165. The bed plate 1 carrying the rotating plate 2 rests on a stiff bracket 3 bolted to a wall. The measuring head 4, which carries the level-tube, is in the form of an inverted T, the vertical arm of which slides in a tube 8. The latter is supported horizontally by two rods from a crosshead piece 9, which can be clamped at any desired height on the bar 5. The lower end of this bar rests on the bracket 3, whilst its upper end is held by set screws in a bracket 10 bolted to the wall. The level-tube is supported inside the horizontal brass tube forming the lower part of the T-piece 4. This brass tube can be moved longitudinally with relation to the vertical part of the T by means of the traversing spindle 7. The whole of the T-piece and measuring head can be moved in a direction at right angles to the wall by a rack and pinion motion on the rods which pass through the crosshead and which support the tubular piece 8. The milled head for this traverse is shown at 11. Finally, the T-piece can be raised or lowered in the tube 8 by turning the milled head 6. Thus it will be seen that the measuring head can be moved longitudinally, transversely, and vertically. The latter motion allows the level-tube to be lowered into its working position on the gauges when it is desired to take a reading, or to raise it up clear when the plate carrying the gauges is to be revolved. The horizontal motions permit the feet of the level to be brought into contact with the upper surfaces of the gauges in any desired positions.

(iv.) *Setting of Level-tube in Measuring Head.*—The arrangement of the measuring head must be such that when the feet of the level-tube are resting on the gauges the tube must be quite free to take up its natural position, depending only on the relative heights of the upper surfaces of the gauges. The method of supporting the level-tube in the horizontal tube of the measuring head is shown in *Fig.* 166. The tube 1 is first placed inside an aluminium sleeve 2, which has a bush at each end bored so as to be an easy fit on the glass tube. The tube is fixed in the sleeve by packing soft wax round the bushes. The distance between the bushes is such that the level-tube is supported approximately at the "Airy points," so as to minimise distortion. The feet of the level consist of two steel balls soldered into a steel block 3, which is attached to the under side of the sleeve by two finely threaded screws. A piece of steel wire is clamped transversely between the top of the block and the sleeve, and by adjusting the two screws, the angular position of the block 3 can be set with respect to the level-tube so that the bubble of the latter is approximately central when the feet rest on a horizontal surface. The sleeve containing the level-tube is enclosed in a brass tube 4, and is supported, when not in use for measuring, by the two adjustable screws 5. Apertures are left in the upper part of tube 4 for viewing the bubble. The tube is prevented from falling sideways by the pin 6 attached to the sleeve, this pin being guided between two parallel wires off the tube 4.

---

[1] A description of an instrument designed for calibrating level-tubes will be found in the Annual Report for 1920 of the National Physical Laboratory (published by H.M. Stationery Office). For ordinary purposes, where level-tubes are used merely as indicators—the bubble always being brought to the central position—it is not essential that the radius should be uniform. (See also "Spirit-Levels," by E. O. Henrici, *Trans. Opt. Soc.*, Dec. 1918.)

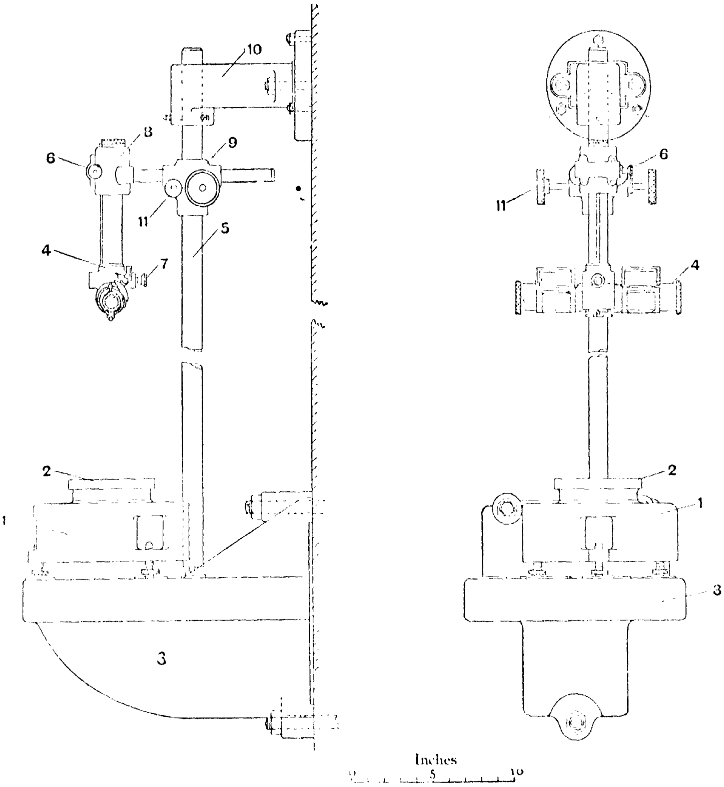


FIG. 165.

The tube 4 forms the horizontal portion of the T-piece referred to in (iii.) above and shown in *Fig.* 165. When it is required to bring the level tube into the measuring position, the measuring head is lowered by means of the knurled head 6 (*Fig.* 165) until the ball feet come in contact with the gauges; the lowering is continued until the level is freed from the supporting screws 5 (*Fig.* 166).

The level-tube now rests freely on the two gauges, the only constraint being in the lateral direction from the pin 6 between its guides. Since the pin is a free fit between the latter, the level is at liberty to set itself in accordance with the relative heights of the upper surfaces of the two gauges. To remove the level from the gauges the measuring head is raised and the level-tube is picked up on the screws 5, and it can be lifted completely clear of the gauges.

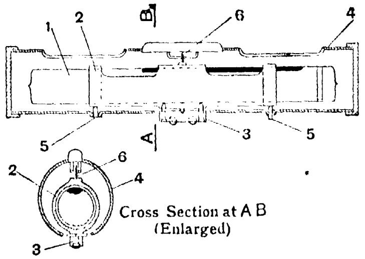


FIG. 166.

It should be noted that it is no longer necessary to have two balls at one end of the block 3 since the pin 6 prevents the level tube from falling over sideways when placed on the gauges.

(v.) *Measuring Scale.*—A special device (due to Mr. E. M. Eden) is used for noting the position of the bubble relative to the tube when taking measurements. It was not convenient to have a scale engraved on the upper surface of the level-tube, since the divisions have to suit both the actual radius of the tube and the distance between the centres of the ball feet. The method adopted to obviate this is shown in *Fig.* 167. A strip of glass, which is silvered longitudinally only over the lower half, is supported at an angle above and slightly to the rear of the tube, so that its surface is parallel to the axis of the latter. A scale is placed behind the glass as shown. The distance of the scale from the glass and the angle of tilt of the latter are adjusted until an image of the bubble is seen superimposed on the lower half of the scale, both being in the same plane, *i.e.* there is no parallax between the image and the scale. The appearance when looking into the glass is shown in the front view of *Fig.* 167. When taking a reading, the position of the extreme end of the bubble is noted against the scale. Only one end of the bubble is read. The glass and scale are held in a suitable fixture which slides over one end of the tube 4 in *Fig.* 166.

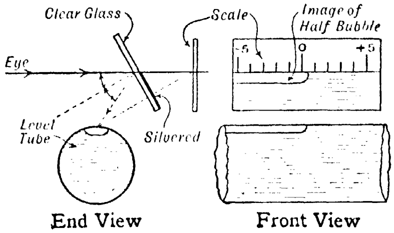


Fig. 167.

(vi.) *Calibration.*—Having chosen a level-tube of uniform radius whose approximate magnitude, in conjunction with the spacing of the ball feet, gives a suitable magnification, it is necessary to determine the spacing of the divisions of the scale so that it will read direct to, say, hundred-thousandths of an inch.

A pair of gauges differing by about 0·0001 in. is wrung on to the rotating plate of the machine. The difference between the gauges must be known beforehand to an accuracy of one-millionth of an inch. Using a preliminary scale divided into tenths of inches, two readings are obtained on the gauges, one before and one after rotating the plate. Suppose that the total distance traversed by the bubble between the two settings, per 0·0001 in. difference between the gauges, is 1·80 in.; a scale would then be drawn out having an over-all length of 1·80 in. and divided into 20 parts. This scale would be substituted for the preliminary scale and its divisions would indicate differences of hundred-thousandths of inches, bearing in mind that the difference becomes doubled when the positions of the gauges are interchanged.

This machine has one great advantage over other comparators of different design. When comparing two gauges, the measurement which gives the difference is made simultaneously on the two gauges, and if during this measurement the gauges are affected thermally by the presence of the observer, the effect on the two gauges should be the same. It is unnecessary to handle the gauges during the comparison even with insulated clips, the procedure being to set them up together on the plate and allow time for the dissipation of the heat due to handling before a measurement is made.

Another very valuable feature of this machine is the facility with which long, flat-ended gauges can be compared. The comparison of a pair of 36-in. gauges can be made with practically the same accuracy as when dealing with a short pair. When measuring fairly long, flat-ended gauges in a horizontal type of measuring machine or comparator, it is most important that the supports used should be adjusted so as to bring the axis of the bar parallel to that of the machine. In the level comparator, supports are no longer required, and the possibility of errors arising from their use is eliminated.

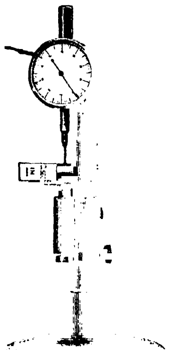


Fig. 168.

§ (84) Dial Indicators.—This type of comparator or indicator is to be found in common use in workshops. It consists of a measuring head which can be used for a variety of purposes by adapting it to suitable fixtures. One form of the complete instrument as made by Messrs. B. C. Ames is shown in *Fig.* 168, where the measuring head is arranged on a bracket over a small horizontal table, on which the pieces to be measured are placed. The height of the table is adjustable within the range of the pillar. The range of the measuring spindle is about ½ in. and the particular instrument shown has 100 graduations round the dial, each representing 0·001 in. Most indicators are graduated to this figure or in metric units to 0·01 mm. They are also made, however, to read direct to 0·0001 in.

The internal mechanism consists of a rack cut on the measuring spindle which operates a train of gears, the last wheel of which is connected direct to the spindle carrying the needle pointer. The spindle is pulled down by a spring, and backlash in the gears is prevented by the action of a hair-spring connected to a separate gear, which is not included in the train, but which meshes with one of the pinions.

The accuracy of these indicators depends upon the precision of the rack and gear wheels. In some makes they cannot be relied on to better than 1 division when used over a range of more than a few divisions.

VII. Workshop Measuring Instruments

§ (85) External Micrometer Calipers.—These instruments usually take the form of a bow frame, one arm of which is fitted with an anvil and the other with a measuring spindle. The axial motion of the latter is controlled by a fine screw-thread working in a nut which is held rigid in the frame. *Fig.* 169 shows a Brown & Sharpe micrometer capable of being used for measurements up to 1 in. The screw has a pitch of 0·025 in. and the thimble is divided into 25 parts, each of which thus corresponds to an axial movement of 0·001 in. of the spindle. A vernier scale is engraved round the barrel to enable readings to be taken to 0·0001 in. For metric measures the pitch of the screw is made 0·5 mm. and the thimble has 50 divisions each representing 0·01 mm. The end of the spindle is hardened and its face finished accurately flat and square to the axis. The opposite measuring face is formed by a hardened steel stud screwed into the bow. Its axial position is adjusted by screwing it forward until the two measuring faces make contact with the instrument reading zero. A locking screw is provided for preserving the adjustment of the stud. A clamp ring is fitted to the spindle to fix it at any desired reading when it is required to use the micrometer as a fixed caliper.

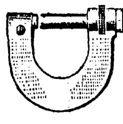


Fig. 169.

In using a micrometer, the thimble attached to the screw is rotated until contact is made between the measuring faces and the object being gauged. The amount of pressure exerted on the object when making contact depends upon the "feel" or "touch" of the manipulator, and for this reason readings obtained by two observers on the same piece may differ by as much as 0·0002 or 0·0003 in. This difference in measurement is avoided if each observer takes a preliminary reading on a reference gauge of known size and so determines the correction to be applied to the reading to suit his own particular touch. This method of using the micrometer as a comparator also enables the errors in the pitch of the screw to be avoided almost entirely, provided the reference gauge selected is of closely the same size as the object to be measured.

When it is necessary to take rapid measurements of a large number of pieces, the micrometer is often fitted with a ratchet or friction stop at the outer end of the spindle. This device ensures a constant pressure of contact.

The larger sizes of micrometers are usually constructed so as to take measurements over a range of several inches. The run of the measuring head is restricted to one inch, and the instrument can be set to suit any particular inch range either by fitting a special anvil or by adjusting the position of a long sliding anvil in a bush at one end of the bow. Reference end gauges are used for checking such adjustment.

When using the larger sizes of micrometers it is usually found that, owing to lack of rigidity of the bow, the delicacy with which the contact can be felt is considerably reduced. This trouble can be overcome to a large degree by using the ratchet stop. Where possible, however, it is preferable to use a bar type of micrometer, such as is shown in *Fig.* 170, for lengths above about 6 in. This type of micrometer, which was designed at the National Physical Laboratory, will be found to be useful for lengths even up to 4 or 5 ft. It consists of a length of ground rod on which two blocks can slide. One of these carries the anvil and the other a micrometer head fitted with an enlarged aluminium barrel and thimble to facilitate accurate reading. The micrometer is used resting on a surface plate, and is satisfactorily rigid even over lengths up to 60 in.

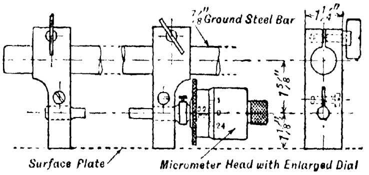


Fig. 170.

§ (86) Internal Micrometer Calipers.—These instruments are used for measuring internal distances between flat surfaces or the diameters of holes. One type is shown in *Fig.* 171. It consists of a holder with a micrometer screw and thimble at one end and a clamp at the other. A number of extension rods of different lengths are provided; these can be held in the clamp and the instrument can be used over a considerable range (8 in. to 36 in.). The rods have a number of grooves spaced 1 in. apart, and in such positions that when the jaws of the clamp engage with one of them, the size over the extreme contact points is an exact number of inches when the micrometer head is set to zero. The contact points are hardened and slightly rounded for measurements of holes.

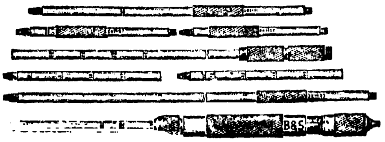


FIG. 171.

Another type of micrometer for the measurement of holes, made by the Newall Engineering Company, is shown in *Fig.* 172. This instrument has three measuring plungers which slide in radial bushes in the body, their inner ends being held against the conical end of the micrometer spindle, which is contained in the handle. Any rotation of the micrometer thimble causes axial motion of the spindle, and this, in turn, gives a radial movement to the three contact points. To check the setting of such three-point micrometers it is necessary to have ring gauges of known size.

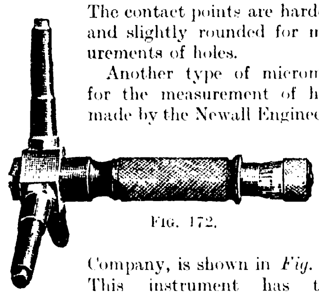


FIG. 172.

The system of three-point measurement is not suitable for detecting ellipticity in holes.

§ (87) DEPTH MICROMETERS.—This form of instrument is intended for the measurement of depths of holes, slots, recesses, etc. It takes the form shown in *Fig.* 173, and consists of a micrometer head to which is fixed a hardened base, flat on the under side. The thimble of the micrometer carries a rod which passes through and projects beyond the base. The rod has a number of grooves spaced 1 in. apart, into which the clamp on the thimble can be fitted as desired. With a set of such grooved bars it is possible to measure any depth up to 24 in.

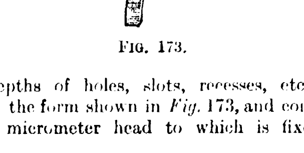


FIG. 173.

§ (88) VERNIER CALIPERS.—These instruments are used for taking both inside and outside measurements. The usual form of the caliper is shown in *Fig.* 174. It consists of a rectangular steel bar with an end projection which forms one of the measuring jaws. The other jaw is attached to a slide which can be moved to any position along the beam, the fine adjustment being made by a thumbscrew attached to an auxiliary slide. The distance between the jaws is given by the reading of the scale engraved on the beam. This scale is usually divided into $\frac{1}{40}$ths of inches, and a vernier on the slide enables readings to be taken to 0·001 in. In some instruments one side of the beam is engraved to read in inch units and the other in millimetres.

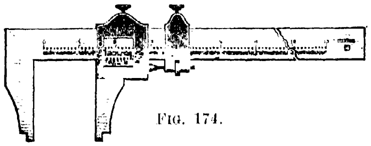


FIG. 174.

The measuring jaws are hardened and their inner faces are finished flat and parallel. The outsides are rounded for a short distance from the ends for purposes of internal measurements. The thickness of the two ends is usually made ¼ or ½ in., and this amount has to be added to the reading when taking internal measurements.

§ (89) VERNIER HEIGHT GAUGE.—This instrument, *Fig.* 175, is used for measuring the heights of different locations on jigs, etc., when stood upon a surface plate. It consists of a base into which a vertical graduated bar such as is used in the vernier caliper is fixed. A slide carrying the measuring piece can be moved up or down

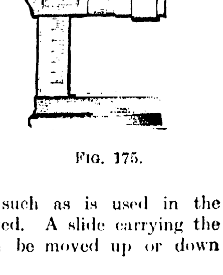


FIG. 175.

the bar, and the reading on the bar gives the height.

§ (90) Surface Gauge.—This instrument, which is shown in *Fig.* 176, can be used in conjunction with a surface plate for testing and marking off heights on jigs, etc. The point of the scriber, which is held by the clamp on the post, is set to a desired dimension by the use of a scale stood on the surface plate. This setting is facilitated by a fine adjustment of the inclination of the post to the base by the arrangement shown. A vee groove is cut in the base to allow the instrument to be used on shafts and spindles.

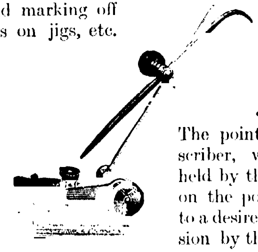


Fig. 176.

§ (91) Dial Surface Gauge.—This type of instrument, which is shown in *Fig.* 177, consists of a dial indicator attached to a base and used for checking heights and various locations. The piece of work to be tested is placed on a surface plate and the height of the dial gauge adjusted to suit the surface to be tested. The variations in the readings of the indicator when the base is moved to different locations give the corresponding differences in height.

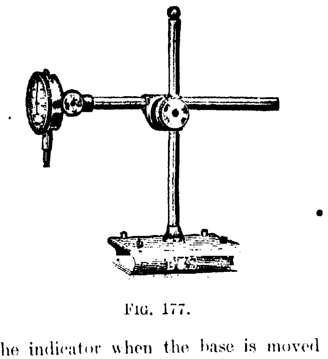


Fig. 177.

§ (92) Thickness Gauge.—This consists of a number of steel blades, usually varying in thickness from 0·0015 to 0·025 in., marked with the corresponding thickness in mils and arranged in a holder as shown in *Fig.* 178. They are used in the workshop mainly by fitters and erectors for determining the widths of the gaps which may exist between adjacent pieces of work. The blades are tried in a gap in turn until one is found to fit. Odd sizes can be made up by putting two or more blades together.

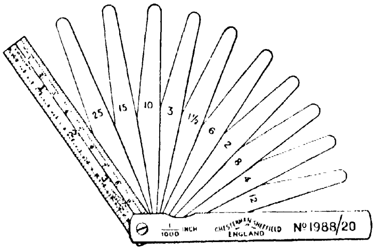


Fig. 178.

§ (93) Screw Pitch Gauge.—This form of gauge, shown in *Fig.* 179, consists of a number of blades, the edges of which are profiled to represent standard pitches and forms of threads such as Whitworth, Sellers, System International. They afford a ready means of identifying the pitches of screws, nuts, bolts, etc.

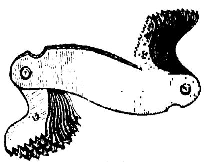


Fig. 179.

§ (94) Wire Gauge.—This gauge is used for identifying the gauge number of wires, etc. A steel plate about $\frac{3}{16}$ in. thick and of either rectangular or circular section, as in *Fig.* 180, has a number of gaps formed in its periphery, the widths of the gaps being equal to the sizes represented by the various gauge numbers. The plates are usually hardened to withstand wear.

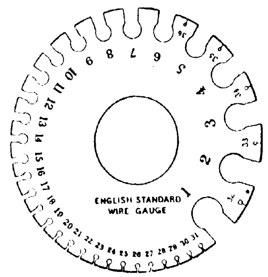


Fig. 180.

§ (95) Surface Plates.—These consist of plates of metal, usually cast-iron, the upper surfaces of which are finished flat. They are made in a large variety of sizes and shapes to suit different purposes. The plates are generally hollow underneath and are suitably ribbed to prevent undue distortion under the weight of objects placed upon them.

The rougher grade of plate has the surface simply machined, and the accuracy depends upon the flatness of the ways of the machine used and the coarseness of the cut. The next grade is produced by taking three plates which have reasonably good surfaces and scraping their surfaces until they match each other in pairs. The quality of the fit between two plates is tested by thoroughly cleaning their surfaces, supplying a very thin coat of some form of marking material, such as Prussian blue paint, to one of them, and then rubbing the two plates together. The points of contact will then be readily indicated on each plate. It is impossible to produce continuous flat surfaces by the process of scraping, and the quality of finish depends upon the proportion of the total area occupied by the high places.

The highest grade of finish is obtained by lapping the plates together by using a special abrasive between them. Here again the plates should be made in batches of three, and the lapping should be distributed between the pairs of plates in rotation. By this means it is possible to produce surfaces which are optically flat, and which will consequently wring together under suitable conditions.

§ (96) STRAIGHT-EDGES.—These consist of narrow bars or plates of cast-iron or steel whose edges are finished plane. They are used for testing the straightness of beds of machine tools, the alignment of the various parts of machines, etc. When made of cast-iron the form usually takes that of an arched beam. This design gives rigidity and freedom from distortion of the true edge due to the weight of the beam. Steel straight-edges, on the other hand, are usually made with the top and bottom edges parallel, and are liable to appreciable flexure due to their own weight. It is a common practice to make a 72 in. straight-edge with a rectangular section of 3 × ⅛ in. which, at first sight, would appear to be reasonably deep and strong. If placed on edge with a support at each end, however, it will be found that the middle sags by 0·0045 in., and if the two supports are brought close together at the middle the ends will be found to droop by 0·002 in. Consequently, if the straight-edge were used to test a surface either concave up to 0·0045 in. or convex up to 0·002 in., the straight-edge would touch the surface throughout its length and the surface might be considered to be quite good by an inexperienced observer. To reduce the flexure to a minimum, the straight-edge should be supported off the surface to be tested on two equal blocks, placed symmetrically with respect to the bar, and at a distance apart equal to 0·55 of its length. The gap between the straight-edge and the surface can then be tested for parallelism by inserting slip gauges in various positions.

Short straight-edges, up to about 6 in. in length, are sometimes made on the "knife edge" principle. The edge of the bar is bevelled off and the sharp edge produced is slightly rounded. Using such a straight-edge in front of a strong light, it is possible to detect errors of an order of a few hundred-thousandths of an inch in the flatness of surfaces.

§ (97). SQUARES.—The usual form of engineer's square consists of a bar, known as the stock, into which a blade is fixed. The edges of the stock and blade are accurately finished and are made as closely as possible at right angles. For tool- and gauge-making, where the highest accuracy is required, the edges of the blade are usually bevelled as shown in *Fig.* 181, and both stock and blade are hardened.

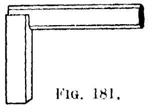


FIG. 181.

Squares can be tested by taking a set of three and comparing them together in pairs with the stocks resting on a good surface plate. Another method is to make a reference square, which consists of a cylinder of cast-iron ground accurately cylindrical and having its base finished truly square with its axis. This is stood upon an accurate surface plate together with the square to be tested. The edge of the blade is brought into contact with the cylindrical surface of the block, using a well-illuminated surface as a background. The square should be tried against the block in several positions round the circumference, and it should be noted whether the error, if present, remains constant. If variations are seen, it indicates that the base of the block is not truly at right angles to the axis of the cylinder, and the block should be corrected before being used further.

§ (98) ANGLE-MEASURING INSTRUMENTS.—There are various types of instruments used for measuring angles in the workshop. One of the commonest, the bevel protractor, is shown in *Fig.* 182. It consists of a plate, the lower edge of which is straight, and which carries a pin on which a second plate can revolve. The latter plate carries a straight-edge blade, and the inclination between this blade and the edge of the lower plate can be varied at will. The angle between the two edges can be read to an accuracy of 5 minutes of arc on a circular scale engraved on the lower plate. A small pinion,

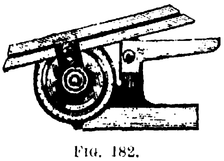


FIG. 182.

which engages with teeth on the upper plate, is provided for making fine adjustments of the angular setting.

Another type of instrument, which was designed at the National Physical Laboratory, is made on the principle of the clinometer and is shown in *Fig.* 183. It resembles the bevel protractor in having two plates, but the upper one, instead of being provided with a straight-edge, has a sensitive spirit-level tube attached to it. In measuring the angle between two edges of a plate the latter is held rigidly in a vice and the straight-edge on the base plate is placed on one of the edges. The second plate is then revolved on the centre pin until the bubble of the level tube is in the middle of its run, the fine adjustment being made by the tangent screw shown. The reading of the angular scale is then noted and the straight-edge is transferred to the second edge of the plate. The level is again adjusted and a second reading is taken. The difference between the readings gives the angle between the two edges. The angular scale is divided on a silver strip and readings can be made to one minute of arc. This instrument proves very useful when setting up jig-work on the milling machine or surface grinder.

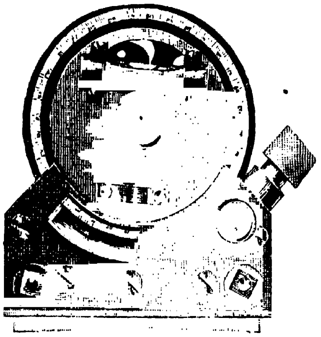


FIG. 183.

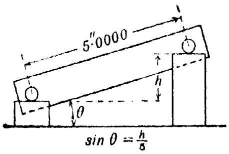


FIG. 184.

The sine-bar, as shown in *Fig.* 184, is a simple device for making accurate angular measurements. It consists of a hardened steel bar with parallel edges and having two plugs, or buttons, of equal diameter inserted in it. The plugs are spaced at exactly 5 or 10 in. centres, according to the length of the bar, and their distances from the edges of the bar are equal. The method of using the bar is indicated in the figure. The plugs are rested on two piles of block gauges, the heights of which are made to differ by an amount h, which is obtained from the formula $h=5$ sine θ, where θ is the angle of inclination of the bar. By making the distance between the plugs exactly 5 or 10 in. the calculation is simplified.

The sine-bar begins to lose its sensitivity for angles above 45°, but this can be remedied by making the bar with a right-angle projection and by setting up the plugs to suit the complement of the angle required if this exceeds 45°.

F. H. R.

GAUGES:
- End: standardisation by comparison with scales. See "Gauges," § (17).
- For depth measurement. See *ibid.* § (1) (iii.).
- For position and location methods of measurement and use. See *ibid.* § (3).
- For profiles. See *ibid.* § (2).
- "Go" and "not-go," theory of. See "Metrology," § (21).
- Limit: theory and use of. See *ibid.* § (17) (ii.).
- Limit: description of various types. See "Gauges," § (1).
- Limit: external types. See *ibid.* § (1) (ii.).
- Limit: internal types. See *ibid.* § (1) (i.).
- Limit: screw. See *ibid.* § (4).
- Methods of measuring external limit gauges of ring and gap type. See *ibid.* § 7.
- Methods of measuring internal limit gauges of plug and bar types. See *ibid.* § (6).
- "Not-go," for screw threads, to control individual elements separately. See "Metrology," § 25 (i.).
- Position type: method of measuring. See "Gauges," § (13).
- Profile type: methods of measuring. See *ibid.* § (12).
- Ring type: Tomlinson's method of measurement. See *ibid.* § (6) (*d*).
- Screw types, measurements of. See *ibid.* § (3).
- Secondary standards: balls and roller gauges. See *ibid.* § (5) (iv.).
- Spherical-end and steel balls: elastic compression during measurement. See *ibid.* § (16).
- Standard: plug and ring types. See *ibid.* § (5) (i.).
- Standard: principle of comparison by measuring-machines. See *ibid.* § (14).
- Standard, reference, check, and master. See "Metrology," VI. § (19).
- Standard block type. See "Gauges," § (5) (iii.).
- Standard block type: method of making at National Physical Laboratory. See *ibid.* § (5) (iv.).
- Standard end bars. See *ibid.* § (5) (ii.).
- Standard end type: their calibration in sets. See *ibid.* § (15).

Taper plate form: method of measuring. See *ibid.* § (11).

Taper plug form: methods of measuring. See *ibid.* § (9).

Taper ring form: methods of measuring. See *ibid.* § (10).

Gauges and Engineers' Scales. See "Metrology," VI. § (17).

Gauging of Casks and Barrels: the determination of the capacity of casks and barrels. See "Volume, Measurements of," § (6).

Gauging of Screw Threads. See "Metrology," § (25). See also "Gauges," III. §§ (18), etc.

Gay Lussac's Alcohol Tables. See "Alcoholometry," § (4).

Geodetic Measures. (i.) *The Nautical Mile.*—According to the definition adopted in England and the United States, the nautical mile is equal to the length of one minute of arc of a great circle on a spherical earth assumed to have the same area as Clarke's ellipsoid (see below).

On the Continent the terms "nautical mile" and "geographical mile" are interchangeable, and both are defined as the mean length of arc of one minute of latitude which varies from 1842·7 m. at the equator to 1861·3 m. at the poles.

Adopting the English definition for nautical mile,

Nautical mile . =1853·152 m. (Admiralty).
=6080 feet.
=1·1515 statute miles.
Geographical mile =1852 m. (Annuaire du Bureau Central des longitudes).
=6076·8 feet.

(ii.) *Clarke's Ellipsoid.*—The surface of the planet as determined by "sea-level" is approximately an ellipsoid, known as Clarke's ellipsoid, with axes as follows:

Semi-polar axis . =6,356,068 m.;
Semi-equatorial axes =6,378,294 m. and 6,376,350 m.;

and according to Clarke's figures:

1 quadrant =10,007,000 m.

The values of the radii have also been given as follows:

	Equatorial Radius.	Polar Radius.
	m.	m.
Clarke, 1880 . .	6,378,249	6,356,515
Helmert, 1906 .	6 378,200	6,356,818
U.S. Survey, 1906.	6,378,388	6,356,909

(iii.) *Geodetic Constants.*—The mean polar quadrant =10,002 kilometres (determined from a mean of Helmert and U.S. Survey).

Value of g: Equator . =978·024 cm./s.^{2}
Lat. 45° . =980·617 cm./s.^{2}
London . =981·19 cm./s.^{2}
Pole . . =983·210 cm./s.^{2}
Mean density of earth . =5·5 g./c.c. approx.
Mean density of surface of earth . . . =2·65 g./c.c.
Volume of earth . . =$1·082 \times 10^{21}$ m.^{3}
Mass of earth . . . =$5·98 \times 10^{27}$ g.
=$5·87 \times 10^{21}$ tons.
Area of land (estimated). =$1·45 \times 10^{18}$ cm.^{2}
Area of ocean (estimated) =$3·67 \times 10^{18}$ cm.^{2}
Mean depth of ocean (Murray) . . . =$3·85 \times 10^5$ cm.
Volume of ocean . . =$1·41 \times 10^{24}$ c.c.
Mass of ocean . . . =$1·45 \times 10^{24}$ g.
Mass of the atmosphere . =$5·33 \times 10^{21}$ g.
Velocity of a point on the equator due to the earth's rotation =$R\omega$=$4·65 \times 10^4$ cm./s.^{2} =1040 miles per hour.

Length of 1° of Longitude in different Latitudes

Latitude.	Metres.	Nautical Miles.	Miles.
0°	111,307	60·064	69·164
10°	109,627	59·157	68·120
20°	104,635	56·463	65·018
30°	96,475	52·060	59·948
40°	85,384	46·075	53·056
50°	71,687	38·684	44·545
60°	55,793	30·107	34·669
70°	38,182	20·604	23·726
80°	19,391	10·464	12·049
90°	0	0	0

See Vol. I. "Measurement, Units of."

Geoid, The. See "Gravity Survey," § (7).

Geometric Design. See "Instruments, Design of Scientific," § (12).

Geostrophic Wind. This is the approximation to the gradient wind obtained when the curvature of the path is neglected. Its magnitude is such that the deviating force due to the earth's rotation is exactly balanced by the gradient of pressure. In medium and high latitudes the geostrophic wind is taken as a reliable measure of the wind at 2000 to 3000 feet. See "Atmosphere, Physics of," § (9).

Computation of. See *ibid.* § (10).

Height of attaining. See *ibid.* § (14).

Variation with height. See *ibid.* § (10).

Gilpin, George, "the founder of alcoholometry." See "Alcoholometry," § (4).

Glaisher: factors for obtaining the dew-point from readings of the dry- and wet-bulb thermometers. See "Humidity," II. § (2) (ii.).

Glaisher's Thermometer Screen. See "Meteorological Instruments," § (5) (iii.).

Glory: a series of coloured rings surrounding the shadow cast by the observer's head on a bank of fog, mist, or cloud. See "Meteorological Optics," § (15) (iii.).

"**Grade, Definition of Term**," as used in connection with screw threads. See "Metrology," § (25) (ii.).

"**Grade of Work**": definition of term. See "Metrology," § (29) (i.) (*b*).

Gradient Wind. The gradient wind is that which will just balance the gradient of pressure, when the deviating force due to the earth's rotation and the centrifugal force due to the curvature of the path are both taken into account. The formulae to be used for the computation is given in article "Atmosphere, Physics of the," § (9), equations (1) and (2). The cyclostrophic component, *i.e.* the term $V^2 \cot r/R$, is only of importance in high latitudes for very strong winds, and it is quite frequently neglected, the gradient wind being assumed to be equal to the geostrophic wind. The calculation of the wind speed is much simplified thereby, since the wind can be immediately computed from the distance apart of the isobars. A scale graduated in accordance with the scale of the map and the pressure interval between consecutive isobars gives a direct reading of the velocity. See "Atmosphere, Physics of," § (9).

Graphical Method: the representation of sets of correlated observations by means of coplanar points, each set of observations representing distances, measured in general from fixed orthogonal axes of reference. See "Observations, The Combination of," § (7).

Gravimetric Determinations of Volume. See "Volume, Measurements of," § (7).

Gravitation, Constant of: the constant G in Newton's equation

Force of attraction $= GMm/d^2$.

See "Earth, Density of the," § (1).

Gravity:

- Determination of intensity of, by Threlfall and Pollock's gravity balance. See "Gravity Survey," § (5) (i.).
- Direction of the force of, the reference spheroid. See *ibid.* § (7).
- Measurement of, by the pendulum. See *ibid.* § (2) (i.).
- Measurement of, at sea, by Duffield's method. See *ibid.* § (5) (ii.) (*b*).
- Measurement of, at sea, by hypsometer apparatus. See *ibid.* § (5) (ii.) (*a*).
- Measurement of, at sea, with a mercury manometer. See "Barometers and Manometers," § (22) (i.).
- Measurements of rates of change of. See "Gravity Survey," § (6).

Gravity, Measure of. The law of universal gravitation states that every particle of matter attracts every other particle with a force which varies directly as the product of the two masses and inversely as the square of the distance between them.

(i.) *The constant of gravitation* is the constant G in the law of attraction set out above, and is defined by the equation

$$\text{Force of attraction} = G\frac{\text{mass} \times \text{mass}}{(\text{dist.})^2},$$

$$G = 6{\cdot}6576 \times 10^{-8}\ \text{cm.}^3/\text{gs.}^2\ \text{(Boys)}.$$

(ii.) *The acceleration of gravity* is the acceleration produced in any body by the force of the earth's attraction; as actually measured the acceleration is that due to the earth's attraction minus the centrifugal force of the earth's rotation.

Owing to the fact that the earth is not perfectly spherical in shape, but is more nearly a spheroid, and also on account of the variation with latitude of the centrifugal force of the earth's rotation, and the irregularities in the density of the earth's surface, the formulae giving the variation of g over the earth's surface are complicated.

A formula of the following form is given by Helmert:[1]

$$g = A(1 + B\sin^2\phi)\left(1 - \frac{2h}{R} + \frac{3h}{2R}\frac{\delta}{\Delta} - \frac{h'(\delta - \theta)}{2R\Delta} + y\right),$$

where ϕ is the latitude, R the mean radius of the earth, h = height above sea-level, h' = thickness of surface strata of low density, Δ = mean density of the earth ($5{\cdot}6 \times$ density of water), δ = mean density of surface strata ($2{\cdot}8 \times$ density of water), θ = actual density of the surface strata in the region, y = orographical correction due to neighbouring mountains. Assuming that $\delta = \theta$ and y is negligible, we obtain

$$g = 978{\cdot}03(1 + {\cdot}005302\sin^2\phi)\left(1 - \frac{5}{4}\frac{h}{R}\right)$$

or

$$g = 980{\cdot}617(1 - {\cdot}00265\cos 2\phi)\left(1 - \frac{5}{4}\frac{h}{R}\right),$$

where $g = 978{\cdot}03$ is the value of gravity at sea-level at the equator and $980{\cdot}617$ in latitude 45°.

Putting $R = 6{\cdot}37 \times 10^6$ metres,

$$g = 980{\cdot}617(1 - {\cdot}00265\cos 2\phi)(1 - 1{\cdot}96 \times 10^{-7}h),$$

where h is in metres.

In British units, putting $R = 2{\cdot}09 \times 10^7$ feet,

$$g = 32{\cdot}172(1 - {\cdot}00265\cos 2\phi)(1 - 5{\cdot}97 \times 10^{-8}h),$$

where h is in feet.

The above formula applies to places on the earth's surface at different heights above sea-level, and takes account of the additional attraction of the high ground; for points at some distance above the earth's surface the factor $R^2/(R + h)^2 = 1/(1 + h/R)^2$, which is approximately equal to $1 - 2h/R$ if h/R is small, replaces $1 - 5h/4R$.

---

[1] Helmert, *Ency. der math. Wissenschaft*, Bd. vi.

(iii.) *Centrifugal Force of the Earth's Rotation.*—On account of the rotation of the earth the acceleration produced in any body is the resultant of the acceleration produced by the gravitational attraction of the earth, and the acceleration produced by the centrifugal force of the rotation. This latter component is equal to $-r\omega^2$, where r is the distance from the axis of rotation, and is equal to $R \cos \phi$; ω is the angular velocity.

At the equator $R = 6{\cdot}37 \times 10^8$ cm., and since $\omega = 7{\cdot}292 \times 10^{-5}$,

$$\therefore R\omega^2 = 3{\cdot}39 \text{ cm./s.}^2,$$

hence for latitude ϕ the value of g is diminished by $R\omega^2 \cos \phi$, *i.e.* by $3{\cdot}39 \cos \phi$ cm./s.^{2}, on account of the rotation of the earth. See Vol. I. "Measurement, Units of."

GRAVITY SURVEY

(The Arabic numbers in the text enclosed in brackets refer to the Bibliography at the end.)

§ (1) INTRODUCTORY.—The primary object of a gravity survey is to obtain values of the force and direction of gravity at various points of the sea-level surface. The secondary object is to make deductions from such results as to the distribution of matter in the earth, thereby throwing light on the structure and internal condition of the earth. This secondary object makes the inquiry of interest not only to geodesists but also to geologists and miners.

Observations in most cases are made at stations above the sea-level surface, and accordingly require to be reduced to this level. The sea-level surface is that surface which would bound the ocean if no tidal action were in force. The ocean may for this purpose be imagined to be continued to any part of the earth by means of deep canals; whereby the term "sea-level" has a meaning all over the earth. This surface is generally designated by the word "geoid."

The form of the geoid has been shown by geodetical measurements to be not very different from a spheroid of revolution about the minor axis, that is an oblate spheroid. So, if a spheroid is selected to approximate as closely as may be to the geoid, the separation of geoid and spheroid at any point is a quantity which is small compared with the axes of the spheroid, the ratio of the two quantities possibly never being so great as 1/50,000. There is, however, no actual proof that the separation does not exceed this amount. At all events the separation has various effects in geodetical problems, and is sufficiently large readily to admit of measurement.

The geoid, being a level surface, is obviously orthogonal to the direction of gravity, and in fact completely defines the direction of gravity at its level. The determination of the form of the geoid is one main division of a gravity survey, the other division being the determination of the force or intensity of gravity.

I. THE INTENSITY OF THE FORCE OF GRAVITY

§ (2) THE MEASUREMENT OF GRAVITY. (i.) *The Pendulum.*—The pendulum has long been recognised as a very precise instrument for finding the force of gravity; and most of the accumulated data of gravity intensity are due to it. If a particle is suspended by an ideal thread of no mass of length l and set swinging through a small arc $2a$, the time[1] of a double oscillation is very approximately $2\pi\sqrt{(l/g)}(1 + (a^2/16))$. This arrangement is the so-called "simple pendulum." When it is replaced *in vacuo* by an actual material pendulum or "compound pendulum" the same formula holds if for l is substituted K/Ma, K being the moment of inertia of the pendulum about the axis of rotation, a the distance of the centre of gravity from the axis of rotation, and M the mass. In other words, l is the radius of gyration about the axis of rotation or the "reduced length" of the compound pendulum. The point on the line through the axis of rotation and the C.G. of the pendulum at a distance l from the former is called the "centre of oscillation." To determine its position for an actual pendulum is a matter of great difficulty when the necessary high precision is sought. Though the pendulum may be made very true to figure, slight variations in density displace the position of the centre of oscillation, and a precise measure—say to 1 in 10^6—of the length l can hardly be obtained directly. In the earlier work of the nineteenth century this led to the introduction of "Kater's reversible pendulum." This was a pendulum fitted with two sets of knife edges, the second being arranged near to the centre of oscillation. A property of the pendulum is that its time of swing is the same whether suspended at the axis of rotation or at the axis of oscillation. In Kater's pendulum, a small adjustable weight made it possible to obtain closely equal times of swing when it was suspended from either knife edges; and when this was achieved the length l was simply the distance between the knife edges, which was susceptible of direct precise measurement.

In modern gravity surveys it is not customary to use a reversible pendulum. The absolute length l of the equivalent simple pendulum is only required for an absolute determination of gravity. Relative values of gravity may be found by swinging a pendulum, for which l is only approximately known, at a standard base station; and then results at any other station become expressible in terms of this standard. The absolute value at the base may be determined by a special research.

---

[1] See "Clocks and Time-keeping," § (4).

(ii.) *Corrections.*—In the practical case the pendulum swings in air, and corrections for the buoyancy of the air, as well as its viscosity, are necessary. Further, the equivalent length *l* varies when the temperature changes, and it is essential to have a good determination of the temperature of the pendulum itself. A further correction, due to the yielding of the stand, called the "flexure correction," was not taken into account in earlier work. Even with a very rigid stand the lateral pull of the pendulum as it swings is sufficient to set up small oscillations in the stand, and this raises the position of the instantaneous centre of rotation and so alters the time of swing. The results of the early observations in India, made by Captains Basevi and Heaviside between 1865 and 1874, in which no measure of the flexure was made, as well as other observations of the same and earlier dates, on this account are burdened with an appreciable error, which cannot now be accurately estimated; so that the precision of these early results is much impaired.

(iii.) *The Indian Survey Apparatus.*—It is impossible to describe in the present article details of all modern pendulum equipments, and for these reference must be made to the publications of the Survey Departments which use them. The modern apparatus, in use in India since 1904, exhibits the main features of modern practice. A long series of observations were made with this in 1903 at Kew and Greenwich. All subsequent observations in India are thus related to Kew as primary base; while, in India, Dehra Dun serves the purpose of a secondary base, observed at both at the beginning and end of each season. Pendulums brought to India by continental observers have strengthened the relation between Indian and European bases. It may be said in general explanation that the usual method is to determine the difference of time of swing of a free pendulum and that of a clock pendulum by the method of coincidences; while the rate of the clock is determined by nightly star observations. The following description of the Indian apparatus is taken from Sir Gerald Lenox Conyngham's account (3, No. 10).

> The apparatus (see *Plate* I.) was made by E. Schneider of Vienna after Colonel von Sterneck's design. The pendulums are four in number, all of precisely similar appearance and very nearly equal times of vibration. Their numbers are 137, 138, 139, and 140. They are made of brass heavily gilded, and have agate edges on which to vibrate; each has a small vertical mirror securely fastened to its head just above the line of these edges.
>
> The stand on which the pendulums hang during the observation is solidly made of brass in the form of a truncated cone with three large apertures in the conical surface. It rests on three footscrews which are capable of being firmly clamped. The stand carries a highly polished agate plane for the reception of the agate edges.
>
> This plane is pierced by an oblong hole through which the head of the pendulum which is to be suspended is passed from underneath; after passage the pendulum is turned through a right angle so that the knife edge bridging the hole rests on the agate surface. In order to avoid accidental injury to the agates, such as might happen if the edges had to be placed on the plane by hand, the edges are divided into two portions, inner and outer, and stirrups are provided on which the operator places the latter in the first instance; then by the action of a slow motion screw the stirrups are gently lowered from under the edges until the inner or true portions rest on the plane, the outer being entirely free.
>
> In the base of the stand a lever is provided for starting the oscillation of the pendulum. It has an adjusting screw so that an oscillation of any desired amplitude can be imparted.
>
> The pendulums swing in air at the natural pressure, but are protected by a cover from draughts.
>
> The flash-box constitutes the other essential part of the equipment. It contains a contrivance whereby a shutter, moving up and down under the control of a break-circuit clock, allows a flash of light to pass through a slit at every beat or every alternate beat. This flash of light is reflected by the mirror on the vibrating pendulum into a small telescope fixed on the top of the flash-box; the times at which the flash passes the horizontal wire in the field of the telescope correspond to the coincidences of the free pendulum with the clock pendulum; the intervals between such passages are therefore the coincidence intervals of the pendulums.
>
> The coincidence interval c of each of the pendulums under discussion with that of a sidereal clock is about 35 seconds. This is connected with the time of vibration s by the equation
>
> $$s = c/(2c-1).$$
>
> If $c = 35$ sec., $s = 0{\cdot}507$ sec. approximately.
>
> On the front of the flash-box there is a porcelain scale graduated into divisions of 3 mm. By observing the reflection of this scale in the pendulum mirror and noting how many divisions pass over the central wire of the telescope as the pendulum vibrates, the amplitude of the vibration is determined, when the distance between the mirror and the scale is known. A convenient distance is about 2 metres and a convenient initial amplitude (semi-arc) of vibration is from 12 minutes to 20 minutes.
>
> Besides the pendulum apparatus the equipment includes a clock with a half-seconds pendulum, specially designed for portability. It has a convenient arrangement whereby the pendulum can be lifted from its bearings and clamped to the back of the case, so that it need not be taken off for a journey. The pendulum, made by Riefler of Munich, is of invar.
>
> The break-circuit arrangement consists of a light lever fixed to one side of the clock case which is lifted by a short arm on the pendulum as it approaches the end of its swing in that direction. The lever is adjustable so that the circuit may be broken for a

longer or shorter fraction of a second at will. The clock was made by Messrs. Strasser and Rohde of Glasshütte.

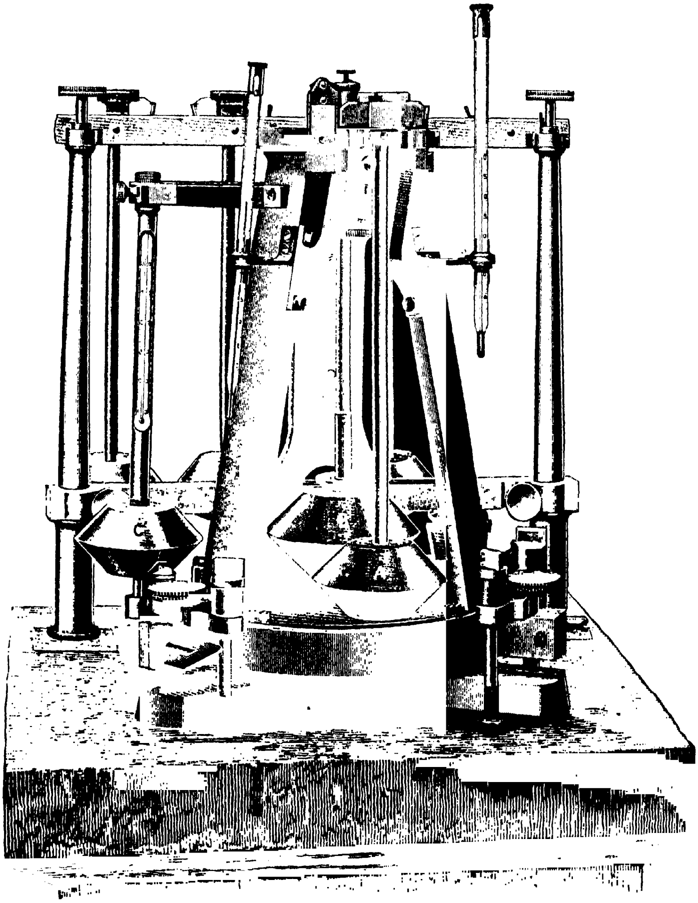


Plate I.

§ (3) Detailed Corrections.—Five corrections to observation results are required, to reduce to (i.) a vacuum, (ii.) temperature 0° C., (iii.) an infinitely small arc, (iv.) sidereal seconds on account of clock rate, (v.) a rigid pillar and stand (flexure). The unit in the correction to the time is in each case in the seventh place of decimals.

(i.) *Vacuum Correction.*—This is proportional to ρ, the air density. If ρ_0 be the density at pressure 760 and temperature zero, T, T_0 the absolute temperatures, p, p_0 the pressures, τ the temperature centigrade, B the barometer reading, γ the coefficient to reduce this to zero temperature, and e the pressure of aqueous vapour, then

$$\rho = \rho_0 \frac{p\,T_0}{p_0\,T} = \frac{\rho_0 B(1+\gamma\tau)(1-3/8 \cdot 3e/8B)}{760(1+\cdot00367\tau)},$$

and the correction becomes

$$\frac{-k' B(1+\gamma\tau)(1-3/8 \cdot 3e/8B)}{760(1+\cdot00367\tau)};$$

and k' is a coefficient depending on the shape, surface, etc., of the pendulum, determined experimentally. The last quantity is found by swinging the pendulum simultaneously with a standard pendulum. Mean k' for the Indian pendulums is 600 with a probable error of 2 or 3 per cent.

(ii.) *Temperature Correction.* — Reduction for the temperature of the pendulum is $-k\tau$, where k depends on the coefficient of expansion of the pendulum and τ is its temperature C. The mean value of k is 49. The temperature of the pendulum itself—not that of the surrounding air—is what is required. On this account Sir Gerald Lenox Conyngham introduced a "dummy pendulum" exactly like the actual pendulums except that its stem was bored to admit a thermometer, which always remained in position. This dummy is kept in the same chamber as the other pendulums, and the temperatures of all are assumed identical. On account of the magnitude of the temperature correction, it has hitherto been found unsatisfactory to observe in a tent, for even when this has double walls large fluctuations of temperature occur. Observations have accordingly always been made in a room; and this restricts the choice of stations.

Conyngham suggested the use of an invar pendulum; but this did not go further at the time, as it was objected that the magnetic properties of invar would render it affected by the earth's magnetic field. Since that date invar pendulums have been used successfully both in America and Germany; the magnetic difficulties have been found to be trifling.

Conyngham has also employed quartz, a material with a minute coefficient of expansion, but found the pendulum too light to be satisfactory.

H. Nagaoka considered the question of invariable pendulums in 1919 (15). He made an elaborate comparison of the suitability of various materials available for the construction of invariable pendulums. For this purpose, on account of its properties, tungsten is strongly recommended. The physical constants of tungsten are given, and it is shown that the pendulum could be entirely constructed of tungsten. The advantage of having every part of the same material is considerable. The points emphasised are the high density, low thermal expansibility, also the great hardness and chemical resistance of tungsten.

(iii.) *Reduction to Small Arc.*—This is simply $-s\alpha^2/16$, where s is the observed time of vibration and α the semi-arc in circular measure. The mean α during the observation may be used with sufficient accuracy.

(iv.) *Time Correction.*—Since the time of vibration is nearly 0·507 second, reduction (iv.) on account of a clock rate of 1 sec. per diem is 0·507/86400 58·7.10^{-7} seconds.

(v.) *Rigidity Correction.*—Reduction to a rigid pillar. When a pendulum swings it gradually sets up an oscillation in the stand supporting it; in other words, the knife edges of the pendulum and the portion of the stand on which they bear have a small lateral movement. This has the effect of raising the geometrical axis of rotation, and increasing the effective length of the pendulum. If an auxiliary pendulum of equal period is suspended on the stand, with its knife edges parallel to those of the first pendulum, an oscillation is set up in it, and by measuring the rate of growth of this oscillation, a measure of the flexure of the stand may be derived. Professor Schumann (see also Abetti (22)) of the Prussian Geodetic Institute investigated the relation, which may be reduced to

$$ds=\left(\frac{\phi_2}{\psi_2}-\frac{\phi_1}{\psi_1}\right)\frac{s^2}{\pi(t_2-t_1)}\,\frac{\mathrm{K}}{\mathrm{K}'},$$

where s is the common time of vibration, and ϕ, ψ are amplitudes of driven and driving pendulum, and K, K′ their moments of inertia. It is usual to observe ϕ, ψ at intervals of two minutes up to sixteen minutes from the start, when ϕ is nearly zero.

§ (4) **Details of a Computation.**—The following computation example of one pendulum, swung at the Indian station Katni, will further illustrate the practical working of the observations and corrections:

Time.	Barometer.		Hygrometer.		Arc.		Pendulum Thermometers.		
							In Dummy.	In Air.	
	H.	T.	Dry.	Wet.	Above.	Below.	No. 516.	Upper. No. 105368.	Lower. No. 105369.
h. *m.*	*mm.*	C.	C.	C.					
1 22	731·2	20·5	21·2	15·0	8·7	8·7	20·20	20·47	20·45
..	..	..	..	..	8·1	8·1	20·20	20·29	20·32
..	..	..	..	..	7·0	6·9	20·21	20·31	20·33
..	731·5	21·1	21·4	15·0	6·7	6·6	20·23	20·60	20·60
Mean =	731·3	20·8	21·3	15·0	7·6	7·6	20·21	20·42	20·43
						Correction		−0·20	−0·24
					Coincidences	Corrected Mean		20·22	20·19

No.	Time.		No.	Time. 20		Difference.		Remarks.
	m.	*s.*		*m.*	*s.*	*m.*	*s.*	
1	29	32·0	62	3	52·2	34	20·2	..
2	30	5·4	63	4	26·0		20·6	..
3		39·8	64	5	0·0		20·2	..
4	31	13·3	65		33·8		20·5	..
5		47·3	66	6	7·6		20·3	.
6	32	21·8	67		41·0		19·2	..
7		55·0	68	7	15·1		20·1	..
8	33	28·6	69		49·0		20·4	..
9	34	2·7	70	8	22·9		20·2	..
10		36·0	71		56·1		20·1	..
11	35	10·0	72	9	30·3		20·3	..
12		43·2	73	10	4·1		20·9	..
				Mean =		34	20·25	s. c = 33·775

Density Correction = −k′D.
k′ = 594.

$$\frac{1}{D}=\frac{760(1+\cdot00367\tau)}{B'(1-\cdot00016\tau)}$$

$$=\frac{760}{B'}(1+\cdot00383\tau)$$

Dry bulb	21·3
Wet bulb	15·0
(1) $-\frac{3}{8}e$ (from Chart I.)	−3·5
(2) Barometer reading	731·3
(3) Index correction	−0·3
B′ = (1) + (2) + (3)	728
Mean of pendulum thermometers in air = τ	20·2
(4) $\frac{760}{B'}$	1·044
(5) $\frac{760}{B'}\times\cdot00383\tau$	0·081
(4) + (5) = $\frac{1}{D}$	1·125
−k′D	−528

Clock rate correction = 58·7*u*.

	Gaining.
Daily rate = *u*[1]	−2·96
u × 58·7	−174

Temperature Correction = −kT.
k = 49·0.

Thermometer in dummy No. 516.

Mean reading	20·21
Correction	−0·08
Temperature = T	20·13
−kT	−986

Arc Correction $=-s\frac{a^2}{16}$
$=-\frac{n^2}{d^2}\times\cdot0178.$

Total arc reading in scale of flash-box (1 div. = 3 mm.) = *n*	15·2
Distance in mm. from scale to pend. mirror = *d*	2205
Correction (from Chart II.)	−9
$\frac{3n}{4d}$ = tan (semi-arc)	·0051701
Semi-arc = *a* (from Table 1)	18′

Observed Time of Vibration $=\frac{c}{2c-1}=s.$

61*c*	2060·25	log.	3·3139200
61 (2*c* − 1)	4059·50	log.	3·6084725
$s=\frac{c}{2c-1}$	0·5075133	log.	$\bar{1}$·7054475

Density Correction	−528	The corrections are all in units of the seventh decimal place.
Temperature Correction	−986	
Arc Correction	−9	
Flexure Correction	−55	
Rate Correction	−174	
Sum of corrections = C	−1752	
Corrected time of Vibration = S = *s* + C	0·5073381	

---

[1] Correction ± as rate is $\frac{\textit{losing}}{\textit{gaining}}$.

The usual programme at a field station is to swing each pendulum twice daily at twelve hours intervals, whereby any diurnal atmospheric variation is compensated. Each complete set of observations on one pendulum occupies about forty minutes; and with the time spent in changing pendulums and allowing temperatures to steady, the observations on four pendulums occupy four hours. Observations usually extend over three days.

§ (5) OTHER METHODS.—Methods of determining the intensity of gravity, other than that of the pendulum, have been proposed.

(i.) *Threlfall and Pollock.* — In Threlfall and Pollock's Gravity Balance (8) a quartz thread is mounted horizontally and is attached, at one end to a spring which takes up variation of tension, and at the other end to an axle which can be rotated, in line with the thread. At the centre of the thread and at right angles to it is attached a light rod or lever bearing a weight at a suitable distance from the thread. The quartz thread is given several complete twists and the weight on the rod is adjusted so that the rod is held in a horizontal position by the two balancing forces due to the torsion of the thread and the weight of the rod. If now the balance is removed to a place where the gravity force is different, the rod will not remain horizontal; but it can be brought back into a horizontal position by twisting the thread, by means of the torsion head. When suitable allowance has been made for temperature change, the pressure having been kept constant, the angular twist of the torsion head gives a measure of the difference of gravity at the two places.

The thread and the framework bearing it were kept in an air-tight enclosure, the torsion head being worked through a stuffing box contrived to be air-tight by means of mercury. The temperature was determined by a platinum wire thermometer placed beside the thread. It was estimated that various errors in deduced value of "*g*" were liable to amount to ·003 dyne; and an error of ·002 was to be expected.

The advantages of such a balance are: (1) time observations are not necessary, so that cloudy skies do not delay the work; (2) the observation can be completed in three hours, of which half is spent in packing and unpacking. Observations have to be made at times of maximum or minimum temperature, as a varying temperature vitiates the results. The complete equipment weighed 226 lbs.

The designers used the instrument in Australia prior to 1900; but nothing later has been published concerning it; and it is understood that the difficulties of accurately allowing for temperature caused the precision of the balance to be considerably less than had been anticipated.

(ii.) *Measurements at Sea.* (*a*) *The Hypsometer Apparatus.*—Attempts have been made to determine gravity at sea. The underlying principles employed were (1) to compare the atmospheric pressure as given by a mercury barometer with that found by a hypsometer or aneroid; (2) to observe the height of a sealed mercury barometer. In both cases a marine pattern barometer or one of special design is used to reduce to the smallest amount the "pumping" of the mercury caused by the ship's motion. In the first case the mercury barometer reading varies with "*g*," while the hypsometer gives the absolute air-pressure. In the second case, the weight of the mercury which varies as "*g*" is balanced against a constant *mass* of air, whose pressure can be computed.

Dr. Hecker of the Prussian Geodetic Institute began observations with the hypsometer apparatus early in this century, and published results for the Atlantic, Indian, and Pacific Oceans, and the Black Sea between 1903 and 1910 (9). Great trouble was experienced as a result of the ship's oscillatory motion, which caused excessive "pumping" in the mercury barometers; and it may be said at once that the results obtained cannot be relied on. The barometers differed from the ordinary land pattern in having a capillary stricture in the centre portion of the main tube. Each side of the capillary portion the bore was reduced to a diameter of 1 mm., and only the upper and lower extremities were of customary bore. In the lower portion an air trap was introduced to prevent the access of air bubbles into the capillary. In this way "pumping" was to some extent overcome.

The hypsometer thermometers gave trouble owing to variability resulting, in Hecker's opinion, from the repeated boiling for considerable periods to which they were subjected. The value of the work lies in the experience gained rather than the values of "*g*" obtained.

(*b*) *Dr. Duffield's Method.* — In 1914 the British Association visited Australia. Dr. Duffield took the opportunity afforded to attempt some determinations of gravity at sea on the voyages out and home. He took with him three apparatuses, in one of which Hecker's hypsometers were replaced by aneroid barometers: while of the other two both used the principle of balancing the weight of a column of mercury against the pressure of a constant mass of air, one being of Duffield's and the other of Hecker's design. Duffield has communicated his experiences with his own apparatus to the Royal Society; and he has written of the other two in the Report of the British Association (10 (*a*), 11).

Further reference here will be restricted to Duffield's own apparatus, from which the most promising results were obtained. Duffield does not consider even these results as entirely satisfactory; but they are undoubtedly an advance on any previous work.

The use of a sealed mercury barometer had been made, but without much success, by Mascart on land in 1882. Duffield's apparatus was in an experimental stage in 1914 when it was taken to Australia and back. In his paper Dr. Duffield says: "A constant volume of air was maintained in the bulb B (see *Fig.* 1) by keeping the mercury always up to the pointer C. The air in the bulb was under reduced pressure in order to keep the apparatus within reasonable dimensions. The barometer tube was bent so that H was vertically over C, the length of the column of mercury HC being approximately 20 cm. The level was kept at C by raising or lowering the level of the mercury in the index tube D, which was of fine bore; an operation which was effected by means of an exhaust pump. It was from the reading of the level of the mercury in this tube, when contact was made with the pointer C, that the value of gravity was calculated. The other side tube E was of wider bore, and was used only for making the initial adjustment at the beginning of the voyage. It was introduced on account of the great difficulty of correctly adjusting the amount of mercury in the apparatus, and to enable it to be used for various ranges of temperature." Contact of mercury with pointer C was revealed by a telephone and trembler in a circuit which was closed thereby. A temperature compensation device, due to Mr. Horace Darwin, in which the size of the reservoir is chosen to make the total volume v of mercury contained such that its rise at H, due to a change of temperature—constant level at C and D being maintained—will just balance the increase of pressure of the air in B, due to the same rise in temperature. In symbols this implies the condition $av=ahT$, where a is the dilatation coefficient of mercury, v is the total volume of mercury, a is area of cross-section of barometer tube at H, and h is the height of the mercury surface at H above that at C. This device makes an absolute compensation at one temperature only, but gives a considerable compensation also for temperatures not widely different.

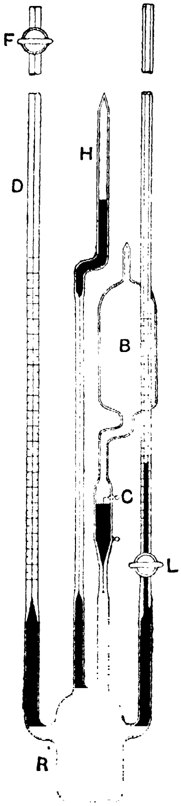


Fig. 1.

It appears that contact at C could be gauged with precision of ·002 mm., corresponding to a precision in "g" of 1 in 10^{5} or 0·01 dyne. In the index tube D a change of level of 1 mm. corresponded to $\Delta g = 0{\cdot}058$ dyne. Capillary tubes, indicated in the diagram, were introduced to prevent "pumping" due to ship's motion. Readings made to evaluate the error due to the viscosity of the mercury in these capillaries suggest that this, combined with errors of contact, vibration, and temperature, would not exceed 0·01 dyne. The whole apparatus was immersed in a tank fitted with suitable windows, etc., and filled with water; and the tank was suspended by cords in the refrigerator rooms of the ships on which the apparatus was tested.

(iii.) *Correction.*—There is a correction to all readings with mercury barometers, whether sealed or open to the air, on account of the ship's motion in longitude. This, due to change in the centrifugal force, is given by the expression $2\omega v \cos\lambda \sin\alpha$, where ω is the earth's angular velocity, v is the speed of the ship, λ is the latitude, and α is the deviation of the ship's course from true north or south. This amounts to about 0·05 millibar per knot at latitude 50°. The reality of this correction has been experimentally verified by Duffield on the destroyer *Plucky* in the English Channel.

In a paper (10 (*b*)) in *Proc. Roy. Soc.*, Professor Sir Arthur Schuster discusses mathematically the effects of oscillation due to ship's rolling and pitching and vertical motion. In this paper certain relations between cross-section and lengths of the various tubes are deduced, which would cause such effects to be a minimum. Schuster estimates the probable error of a determination, and with certain selected dimensions he finds the error in $\Delta g/g$ as great as $1{\cdot}4.10^{-5}$ due to a vertical oscillation of the ship of amplitude 1 metre.

§ (6) MEASUREMENTS OF THE RATES OF CHANGE OF GRAVITY.—An apparatus of type entirely different from any so far described is the gravity balance of Baron Eötvös (see *Fig.* 2 and *Plate* II.). This is a torsion balance in which a horizontal tubular beam is supported by a platinum wire from a torsion head. A mirror is attached to the beam, which reflects a spot of light for reading purposes. At one end of the beam a platinum cylinder is inserted in the tube, while at the other end a second platinum cylinder is either hung by a thread whose length can be varied, or else inserted in the tube. If U is the potential of the gravitational forces, K the moment of inertia of the suspended system, m the mass of the platinum cylinder suspended at the end of a thread of length l, h the distance of the thread from the torsion axis, and a the azimuthal angle of the beam from the x-axis; and further, if θ is the angle of torsion and τ the torsion constant of the platinum wire, the axes of x and y being horizontal, that of z vertical, then (7 (a))

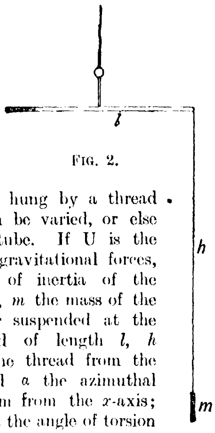


FIG. 2.

$$\theta = \tfrac{1}{2}\frac{K}{\tau}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\sin 2a + \frac{K}{\tau}\frac{\partial^2 U}{\partial x \partial y}\cos 2a$$
$$- \frac{mhl}{\tau}\cdot\frac{\partial^2 U}{\partial x \partial z}\sin a + \frac{mhl}{\tau}\cdot\frac{\partial^2 U}{\partial y \partial z}\cos a.$$

By altering the quantities a and h it is possible to determine the four quantities

$$\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right), \frac{\partial^2 U}{\partial x \partial y}, \frac{\partial^2 U}{\partial x \partial z}, \frac{\partial^2 U}{\partial y \partial z},$$

either by observing the positions of static equilibrium or by observing the times of oscillation about such positions. Instrumental constants to be determined are m, h, l, K/τ, and τ. The first three can be found by direct measurement; K/τ is equal to $(T^2 + T'^2)/2\pi^2$, where T and T′ are times of oscillation of the instrument, with $l = 0$, about two positions for which a and a' differ by 90°. The last quantity τ is found by means of the Cavendish experiment, in which a leaden globe of known size and form is introduced and the resulting deflections are noted.

The four quantities determined by the torsion balance are related to intensity of gravity, "g," and the principal radii of curvature ρ_1 and ρ_2 as follows:

$$\frac{\partial g}{\partial x} = \frac{\partial^2 U}{\partial x \partial z}, \quad \frac{\partial g}{\partial y} = \frac{\partial^2 U}{\partial y \partial z},$$

$$\frac{1}{\rho_1} - \frac{1}{\rho_2} = -\frac{1}{g}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\sec 2\lambda,$$

$$\tan 2\lambda = -\frac{2\dfrac{\partial^2 U}{\partial x \partial y}}{\dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2}},$$

where λ is the angle between the plane of principal curvature ρ_1 and the plane xz.

The sensitivity of this torsion balance is very remarkable. The order of values of the four quantities which are determined is $1 \cdot 10^{-7}$ C.G.S., while the differences obtained in measuring them repeatedly rarely reaches $1 \cdot 10^{-9}$; so that a mean, probably correct to $1 \cdot 10^{-10}$ can usually be obtained.

The instrument is shielded from radiation effects by a triple case, and observations out of doors can only be made satisfactorily during hours of darkness. It will be clear that objects close to the instrument have an appreciable effect on its readings, and it is necessary to select a site free from very near irregularities or else to compute their effect. Baron Eötvös calls this the "terrain effect"; he also considers the topographic effect with reference to a given set of charts showing the topography. Using a given spheroid and a corresponding formula for "g," it is possible to find out how much of the observed values are natural results of the spheroid; and removing these, as well as the terrain and topographic effects, residuals may be formed, which must be attributed to underground irregularities of density.

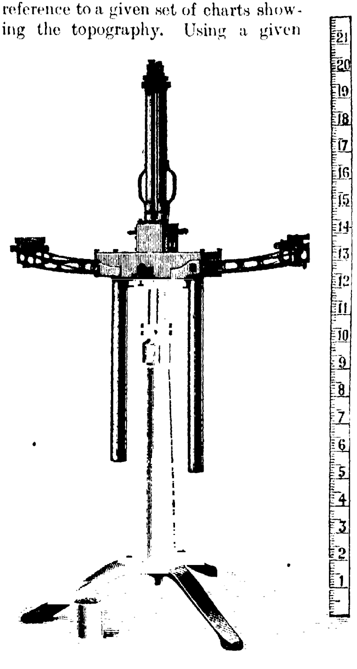


PLATE II.

Without going into that aspect of the results, but stopping with the removal of the terrain effect, it is possible to determine the form of the geoid or level surface above it; but for this supplementary observations have to be made with the pendulum.

The instrument seems to be admirably adapted for detail survey of a district in which rapid variations of gravity occur. As applied to find anomalies of crustal density it should be especially useful. For actual results reference must be made to Baron Eötvös' articles (7 (*a*), 17).

II. THE DIRECTION OF THE FORCE OF GRAVITY

§ (7) **THE REFERENCE SPHEROID.**—As explained in the introduction, gravity at sea-level acts in a direction normal to the level surface which is called the geoid. The geoid has been shown to approximate in form to an oblate spheroid; but it exhibits irregularities with regard to any spheroid selected, which may be revealed by suitable observations. Any point may be chosen as a starting-point or origin for a geodetic survey, and at this point star observations can be made for latitude and azimuth, while the longitude may be decided in relation to any other point on the earth—usually Greenwich—which is selected as the zero for longitude. These values may be called astronomic, and it is possible to assume that the geodetic values of the origin are the same; or, from other considerations which will be more easily understood later on, to assign small differences between the astronomic and geodetic values at the origin. Put otherwise, the reference spheroid for any survey may be placed so as to be parallel to the geoid at the origin, or so as to be slightly inclined to it. In the latter case the inclination of the spheroid to the geoid is defined by the azimuth of the plane containing the two normals and the angle between the normals to the two surfaces, and the components of this angle in meridian and prime vertical are the "plumb-line deflections" in these two directions. These deflections will not ordinarily exceed a few seconds of arc if, as is generally the case, the origin is taken at a place where there is no reason to expect large irregularities in the form of the geoid.

(i.) *Measurement of Position of Point of Observation.*—Now suppose triangulation, emanating from a carefully measured base, is executed in the neighbourhood of the origin, the origin being connected with this triangulation. It is possible to apply this triangulation to the spheroid, if suitable small reductions are applied which depend on the tilt of geoid to spheroid. It is to be remembered that a theodolite, when set up and carefully levelled, has its vertical axis coincident with the normal to the *geoid*. The direction of the normal to the spheroid is a matter of choice, and, strictly speaking, the spheroid is purely a reference figure, introduced for convenience of calculation and expression of results. When observations have been reduced to the spheroid—in horizontal angles the correction is usually very minute and has generally been ignored, though this cannot be fully justified in all cases—calculation may be proceeded with, based on formulae derived from the geometry of the spheroid. In this way the latitudes and longitudes of all points fixed by the triangulation can be determined, and these are generally called the geodetic latitudes and longitudes of the points concerned. If astronomic observations, the same as made at the origin, are made at any of these further points, their astronomic latitudes and longitudes may be found. The astronomic and geodetic values will show a small difference, and this difference will vary from point to point, thus indicating irregularities of the geoid.

(ii.) *Measurement of Height of Point of Observation.*—It remains to refer to the height. Here again the geoidal and spheroidal heights, at the origin, may be regarded as identical or but slightly different. In practice the origin of the survey, being an actual point on the earth, will be above the geoid; and its height above the geoid will have been determined by spirit-levelling operations between some tidal station, where mean sea-level has been determined, and the origin. It has been customary to assume that this height is the same at the origin, reckoned either from spheroid or geoid—which is tantamount to saying that the geoid and spheroid either cut or touch on the vertical through the origin. This is a perfectly proper assumption to make until further considerations show cause for a different assumption; but it is not to be forgotten that thereby the position of the spheroid is restricted, and that after the choice has been made at one point it cannot be made again elsewhere in the same survey. The spheroid so selected as regards height and deflections at the origin of one survey is not identically placed with a spheroid of the same dimensions selected in the same way for a disconnected survey.

As the normals to spheroid and geoid are slightly different, variably so from point to point, it is clear that the two surfaces separate one from the other, the amount of separation sometimes increasing and sometimes diminishing. When in the course of triangulation the angular altitudes of surrounding stations are observed, these altitudes may be put in terms of the spheroid by applying reductions for (*a*) plumb-line deflection; (*b*) atmospheric refraction.[1] The deduction of the height of such points above the spheroid then becomes a matter of simple computation. If also spirit-levelling connects the several points, their height from the geoid becomes known; for spirit-levelling, in virtue of the shortness of the shots and the constant setting up of the instrument levelled in terms of the geoid, clearly yields geoidal heights. The difference of geoidal and spheroidal heights of the same point is a direct measure of the separation of the geoid from the reference spheroid, and thus defines the form of the geoid. When the form of the geoid is known, the direction of gravity, which is normal to the geoid, is also known.

---

[1] See article "Trigonometrical Heights and Terrestrial Atmospheric Refraction."

It will be seen that to investigate fully the direction of gravity the form of the geoid must be found; but the direction of gravity at selected points may be obtained by suitable astronomical observations, combined with properly reduced triangulation connecting the points with the origin of the survey. If all observations were free from error, deflections in prime vertical would be equally well given by observations either for longitude or azimuth. In practice azimuthal error develops in triangulation; and so it is proper to observe at some stations for both longitude and azimuth, whereby the station becomes what is called a "Laplace point," and the longitude result yields a means of correcting the azimuthal error of triangulation.

The necessary work, apart from triangulation itself, accordingly includes astronomic observations for (1) latitude, (2) longitude, (3) azimuth.

§ (8) DETERMINATION OF LATITUDE.—The most precise observations for latitude are:

(i.) Talcott method, with zenith telescope (in meridian).

(ii.) Prismatic Astrolabe method (out of meridian).

Either method is susceptible of great accuracy. The first has been in use for many years, the second is a recent development. Both methods avoid readings of the vertical angles and the attendant errors due to graduation imperfection; further, refraction effects cancel out to a large extent, if the refraction may be regarded as independent of azimuth.

(i.) *Talcott Method.*—This depends on the construction of a star programme in which the stars are selected in pairs, one north and one south of the zenith, and of nearly equal zenith distance at time of meridian transit. The times of transit must be reasonably close together, as both stars of a pair have to be observed before another pair can be dealt with. Their zenith distances must be sufficiently nearly equal that the stars can appear in the field, though not at its centre, when the telescope is set with their mean zenith distance.

The instrument used is called a zenith telescope. It is provided with a vertical circle which is used only for setting. Two sensitive levels are attached to the vertical circle reading verniers. The verniers are set to the mean zenith distance, and the telescope swung round till the bubbles float, and then turned in azimuth into the meridian, either north or south according as the first star of the pair is north or south. When the star appears it is intersected by a wire which is traversed in altitude by a micrometer screw, whose reading is booked as the star crosses the vertical wire; the levels are also read. The telescope is then turned through 180° in azimuth, and the second star observed in the same way. These meridian settings are made by bringing the instrument against one of two stops, which are previously adjusted to agree with the meridian. There is no need to read the vertical circle, which is the same for two stars of a pair. But it is necessary to know the value of the eye-piece micrometer, as well as of the level scales. Both these quantities cancel out to a great extent in the mean of a large number of pairs.

To fix the azimuthal stops in their proper positions a referring mark at a convenient distance is set up prior to the observations. In addition, azimuth is observed nightly by means of timing the transit of a circumpolar star, from which azimuthal deviation may be computed. Time is determined by means of transits of a few stars of small zenith distance, while for collimation one or more stars of zenith distance less than 1° are observed on both faces. The following general rules are followed:

(1) Z.D. of a latitude star should not exceed 40°.

(2) The difference of Z.D.'s of a pair should not exceed 40′.

(3) The interval in R.A. between two stars of a pair should not be less than one minute nor greater than twenty minutes.

(4) The interval between the second star of one pair and the first of the next pair should not be less than one and a half minutes.

(5) No star should be smaller than seventh magnitude; first and second magnitude stars should be avoided.

The general principle of the Talcott method is illustrated by the formulae

$$\phi - \zeta_S = \delta_S,$$

$$\phi - \zeta_N = \delta_N,$$

whence

$$\phi = \frac{\delta_S + \delta_N}{2} + \frac{\zeta_S - \zeta_N}{2},$$

where ϕ is the latitude of the station, ζ_N, ζ_S are the Z.D.'s of a pair of north and south stars, whose declinations are δ_N, δ_S. It will be at once apparent that $\frac{1}{2}(\zeta_S - \zeta_N)$ is immediately deducible from the micrometer eye-piece readings, dislevelment, as indicated by the levels, being duly allowed for.

It is necessary also to determine the inclination of the transit axis to the horizontal, and for this a striding level is used. If W_1, W_2, and E_1, E_2 are the level readings of the western and eastern ends, when the level is placed on the transit axis and reversed, the inclination $i = (W_1 + W_2 - E_1 - E_2)(n/4)$, n being the value in seconds of one division of the bubble. The correction to the time of transit of a star is $i/15 \cos\zeta \sec\delta$, where ζ is the zenith distance and δ the north polar distance. The deviation of the instrument is allowed for in computing the distance from the meridian at which the star was observed; for this must also be known chronometer error, collimation and inclination. Collimation error in azimuth has therefore to be determined.

(ii.) *Latitude by Prismatic Astrolabe.*—A description of this instrument is given, § (9) (iii.). By its means latitude and time are simultaneously determined.

§ (9) DETERMINATION OF LONGITUDE.—Longitude observations all depend on the accurate determination of local time. Time may be found by

(i.) Timing of star meridian transits.

(ii.) Observing time at which east and west stars are at a known altitude, this altitude being observed.

(iii.) Using the Prismatic Astrolabe in method (2), the altitude then being very approximately 60°.

(i.) *Meridian Transits.*—The usual transit instrument is a telescope with bent eye-piece, set up approximately in the meridian. It is equipped with an arrangement by which it may be swung round 180° in azimuth, by lifting the telescope off its Y bearings and replacing it thereon after reversal. In the usual type the eye-piece has three groups of vertical cross-lines, A, B, C, each containing some five lines. The plan of the observation is to observe the time at which a star crosses each of the wires of group A, when the instrument is face west (or east), and, after turning through 180°, observing the time at which the same star crosses the same wires, the instrument now being face east (or west). The mean of these observations is clearly free from collimation error. Timing is usually done by means of a tappet in circuit with a chronograph, and so all the instants of transit are recorded graphically.

An alternative device is to have a micrometer eye-piece which is traversed so that the star remains intersected for some moments, and suitable electric contacts in the micrometer (12 (*e*), p. 10) cause a record on the chronograph at the instant the star reaches certain definite positions. If the same observations are made with instrument reversed the mean of the results is obviously reduced to a centre position free from collimation error.

Errors of adjustment of the instrument, alike as regards horizontality of the transit axis, perpendicularity of this to the line of collimation, and deviation of the line of collimation from the meridian, usually exist. The second of these has an effect which cancels in the means of observations face west and face east. The first may be determined either by striding level or by a method of auto-collimation in which the cross-lines and their image by reflection in a mercury bath (when the instrument is pointed to the nadir) are observed, and the discrepancy of position of a line and its image are measured, in the two azimuthal positions of the instrument. The deviation error is determined by observations to azimuth stars of large zenith distance, whose time of transit is noted. The time stars are chosen of as small zenith distance as possible, when the deviation error has a minimum effect.

So far the determination of local time has been dealt with. To find the longitude it is necessary to find the local time of some occurrence which is observable at both ends of the longitude arc. Formerly this was arranged for by sending a group of signals from one end of the arc to the other by means of electric telegraph. These were duly recorded on the same chronograph as the local time observations, whereby the local time of the signals was recorded. More recently arcs have been measured with the help of wireless telegraphy. This has very much widened the scope of the observation, as it is no longer necessary for a longitude station to be in the vicinity of a telegraph office. In general, stations of principal triangulation and telegraph offices are not closely situated, and connection by triangulation is not easy.

Quite recently, with the introduction of thermionic valves into wireless apparatus and the corresponding increase of the range at which signals can be perceived by a portable field equipment, a still further advance appears feasible. Observations of the daily time signals sent out from one of the large stations such as Eiffel Tower, Nauen, New York, etc., may be made at most parts of the globe, and combined with local time observations should suffice to determine the longitude of the place, relative to the emitting station. This presumes that the time signals are very precise, which is understood to be the case.

(ii.) *East and West Stars.*—The time may be found with considerable precision from observations of the altitudes of a number of east and west stars and the chronometer times corresponding to these altitudes. The stars should be selected in pairs, one east and one west, placed symmetrically with respect to the meridian. This method would not be employed if a transit instrument were available, but can be carried out with a theodolite. The vertical circle has to be read, and any graduation error in it introduces an error into the deduced time.

(iii.) *Prismatic Astrolabe.*—This instrument was invented by MM. Claude and Driencourt and is made by M. Jobin of Paris, in three sizes. The writer has not seen any account of the actual field performance of the two larger size instruments; but even the smallest gives excellent results. The following description is taken from Dr. Ball's handbook (16 (*a*), (*b*)):

> The essential features of the instrument are shown diagrammatically in *Fig.* 3. An equilateral glass prism, E, is placed in front of the object-glass of a horizontal telescope, AA, the back of the prism being normal to the optic axis and the edges of the prism being horizontal. At P is an artificial horizon formed by a horizontal mercury surface. Two parallel rays, SS′, from a star, one incident on the prism and the other on the mercury surface, will form a pair of images in the plane of the principal focus, *f*, of the objective, which images will be coincident at the instant when the star's altitude is exactly 60°, but will be separated as the star's altitude differs from 60°. An observer looking through the telescope at a star which is about to pass the altitude-circle of 60° will see two images which gradually approach and pass each other; and observation of the time at which the images pass each other will give the instant at which the star attains the altitude 60°.
>
> It will be perceived that the reason for making the angles of the prism 60° is that the rays from the star, both direct and reflected, enter the prism-faces normally; they are then totally reflected within the prism, and ultimately leave the back face of the prism, again normally, to enter the telescope. The equilateral form of the prism is the only one by which these conditions can be attained. The prism is the vital feature of the prismatic astrolabe, and the performance of the instrument depends almost entirely on the perfection of its workmanship. The faces must be truly plane, in order to give clear and well-defined images, the edges must be parallel, and the angles must be very nearly 60°. So high is the degree of perfection reached by the maker in the optical workmanship of the prism that he can guarantee the instrument to give altitude constant to within 0″·1. This does not mean, of course, that the prism angles are within 0″·1 of 60°, for it can be shown that, provided the faces are plane, a deviation of 1′ from the 60° angle can be permitted without affecting the constancy of altitude by more than 0″·1. If the working angle of the prism is very slightly greater or less than 60°, the only effect is that the stars will be always observed at an altitude differing slightly from 60°, the altitude being, in fact, equal to $60 \pm 1{\cdot}8a$, omitting refraction where a is the departure of the prism angle from 60°. For it can be shown that the error in altitude is given by $\frac{3n-1}{2}a$, n being the refractive index of the prism If $n = 1{\cdot}52$; the correction is $1{\cdot}8a$.

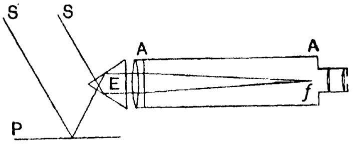


FIG. 3.

In Dr. Ball's method the latitude, time and constant elevation at which stars are observed are regarded as unknowns; and observations to three stars are theoretically sufficient to determine these three quantities. It is desirable, however, to pair stars, so the minimum observation is to four stars, one in each quadrant. For geodetic purposes this number would be considerably increased. For computation, a simple graphic process has been devised. Dr. Ball has also arranged very convenient tables whereby suitable stars are readily selected. Referring to the smallest size instrument, when eight stars are observed, he says, "A very little practice will enable an observer to determine the time within one-tenth of a second and the latitude within about one second, by a couple of hours' observation." This is sufficiently precise for most geodetic purposes even. Moreover, the computations may be nearly completed before the observations are begun, whereby results can be obtained almost immediately after the observations have been completed—in some cases a great advantage.

It will be seen that the observation, combined with wireless signals and triangulation, will give the deflections both in meridian and prime vertical—thus doing the work of zenith telescope and transit instrument.

In a discussion of the prismatic astrolabe at the R.G.S. (16 (*c*)) attention was drawn to an optical difficulty due to the two images being formed, each by only half of the telescope objective. It was stated that this gave an elongated image four times as long as it was wide; and that this would prevent the astrolabe from ever being an instrument of the highest precision. This does not appear to be in accordance with the actual experience of Dr. Ball (16 (*d*)). Another objection mentioned was the disturbance of the mercury surface by wind. Unless this surface can be sufficiently protected observations would only be possible when the air is calm. It would appear that a suitable screen could be arranged to overcome this difficulty. For this Captain H. P. Douglas, R.N., suggests a parallel plate of glass laid on the mercury. This would give a double image of the star by reflection.

§ (10) AZIMUTH BY CIRCUMPOLAR STARS.—

This observation is made by theodolite, generally in the course of geodetic triangulation. Only close circumpolars (north polar distance less than 5°) are used, and these are observed when near to elongation. The criterion generally employed is that the change of the star in azimuth shall not exceed 1″ of arc in ten seconds of time; so that the effect of error in the time shall be negligibly small. It is usual to select two stars, one for eastern and the other western elongation. The time between elongations should be arranged to permit of a sufficiently long set of observations on the first star to be completed before observations on the second are due to commence. The theodolite is set in the meridian given by the triangulation, which is precise enough for finding the stars by setting the computed horizontal and vertical angles on the circles. One station of the triangulation is selected as a referring mark, and the course of the observation is: Referring mark—star, change face, star—referring mark. The chronometer times of the star's crossing of some five vertical wires of the telescope is recorded; and from these it is possible to compute the true azimuth of the star at each wire crossing.

Additional star pairs, near to and north and south of the zenith are observed for time. Instrumental corrections comprise collimation, level, and deviation. The difference between the azimuth deduced from the star observation and that brought up by the triangulation, multiplied by the cotangent of the latitude, is the deflection of the plumb-line in the prime vertical; except in so far as the triangulation is burdened by accumulation of observation error. As remarked above, this accumulation of error can be controlled by the introduction of Laplace stations, at which both longitude and azimuth are observed; the former furnishing a correction to the triangulated azimuth.

III. Discussion of Results and Reductions

The apparatus and observations which have been described serve to determine values of the force and direction of gravity at actual stations, generally situated on the earth's surface. The Eötvös balance gives the means of determining certain differential coefficients of the potential, from which rate of change of "*g*" in any horizontal direction, and the azimuth and difference of principal curvatures of the geoid, can be deduced. It is occasionally possible by observations in mines to find the same quantities at points below the earth's surface. To get the results into comparable terms it is necessary that they should be reduced to one datum level; and the obvious datum is the geoid. For this, the form of the geoid must be found, and this is the first objective of a gravity survey.

§ (11) Deflection of the Plumb-line.—In expressing results of the observations for direction of the force of gravity it is customary to state the value of the "deflection of the plumb-line," *i.e.* the deviation of the vertical—along which gravity acts—from the normal to some spheroid which has been selected as representing in general the form of the geoid. In the same way the force of gravity which would occur if the earth were actually bounded by this spheroid, which is also assumed a level surface of this hypothetical earth, may be represented by a formula; the differences of observed gravity reduced to spheroidal level and the spheroidal formula value may at once be deduced. In this way values of the anomalies of gravity both in magnitude and direction are arrived at. The spheroid to which these anomalies are referred has not necessarily any physical significance, but may without objection be used as a reference solid from which all anomalies may be reckoned, including those of density distribution. Starting with such a spheroid, it is next possible to extend the law of gravity to points situated above it; thus the force of gravity at a height one mile above it may be written down, and also the amount by which the deviation of the vertical (which is a curve) changes in this height. If it is assumed that the *actual* magnitude and direction of gravity change by the same amount, or approximately so, then this procedure will give a means of reducing observed values to geoidal level. This plan has generally been followed in reducing the results of plumb-line deflections, and it has often, but not always, been used for reducing results for the force of gravity. It cannot, however, be completely justified.

§ (12) Cause of Irregularities.—It is clear that at least a portion of the anomaly either of force of gravity or its direction is the result of the irregularities of the earth's surface, more especially those which are local. These irregularities will not have the same effect at the station as at the point vertically below it on the geoid; so that some consideration of irregularities is necessary. There is also a complication due to anomalies of density at points within the geoid. If the form of the geoid be known, the external effects of all internal matter are determinate. As its form is not strictly known, it may be assumed, as a first approximation, to have those deviations from the spheroid which are actually observed at stations above it; and in this way it may be possible to reduce the observed values sufficiently correctly to geoidal level. Meanwhile the method so far adopted is merely to correct the deflection for curvature of the spheroidal vertical—in practice the correction is a very small quantity. As regards reduction of the force of gravity to sea-level, the first plan was merely to introduce the correction $+2gh/R$ which would apply to a point above a spheroid with no intervening mass—commonly called the "free air" reduction. A further correction was introduced by Bouguer, after whom it is named, of amount $-(3\delta h/2\Delta R) \, . \, g$, being the effect of the mass, intervening between the station and the geoid, assumed to be a plateau, where δ and Δ are the earth's crustal and mean densities. With this is usually associated the topographical correction, which takes account of the difference of form of the earth from the Bouguer plateau. These latter corrections, however, are not planned to give the actual values of "g" on the geoid vertically below the station; but the value it would have were the portion of the earth outside the geoid bodily removed. Values of "g" thus arrived at are not *actual* values. They take part in an attempt to *explain* the anomalies of "g," not to state them. Hayford's isostatic reduction, referred to below, has the same object in view.

In some cases the corrections are introduced with opposite sign into the formula for "g." The anomalies are then clearly seen to be those occurring at ground-level.

§ (13) Clairaut's Theorem.—Allusion was made in § (12) to the fact that if the form of the geoid is determined, and the distribution of matter external to it is known, it is possible to calculate the effects of matter interior to the geoid without any further information as to its distribution. It is easy to show that if any function expressing the potential at points exterior to the geoid is found, which satisfies the boundary conditions, this is a unique solution. This function would enable "g" at any point on the geoid to be written down; and by Green's theorem of the equivalent layer the internal portion can be replaced by a hypothetical skin distribution over the geoid of surface density $g/4\pi$.

A particular case of this is Clairaut's equation, generalised by Stokes,

$$\frac{g_{90} - g_0}{g_0} + \epsilon = \tfrac{5}{2}m \quad \text{and} \quad \frac{g_\lambda - g_0}{g_{90} - g_0} = \sin^3 \lambda,$$

where ϵ is the ellipticity of the meridian, λ is the latitude, and m is the ratio of centrifugal force at the equator to g_0. This gives the law of change of "g" at all points on a spheroid, assumed to be the bounding surface of any gravitational system of mass and also a surface of equipotential, but otherwise independent of the internal arrangement of density.

This deduction is made by neglecting squares and higher powers of the ellipticity. Helmert extended the solution to include the squares of the ellipticity, and this leads to a term in $\sin^2 2\lambda$ in the formula for "g." His formula for gravity at sea-level and latitude λ is

$$g = 978{\cdot}030(1 - {\cdot}005302 \sin^2 \lambda - {\cdot}000007 \sin^2 2\lambda).$$

This is for continents; he gives a value greater on average by ·036 for coast districts. For small islands a value greater by as much as 0·3 may be found. The corresponding value of the ellipticity is 1/298·3. These results were found from a consideration of 1600 gravity stations, only 200 of which were occupied prior to 1880.

Under certain assumptions, replacing masses external to a depth of 21 km. below the geoid by a surface distribution at the 21 km. depth level, Stokes showed that the elevation N of the geoid is given by

$$N = R \int_0^\pi \frac{\Delta g_\psi \, d\psi}{Fg},$$

in which R is the radius of the earth, g is the formula value of gravity, and F is a function of ψ, ψ is the spherical distance from the point, and Δg_ψ is the mean anomaly on the circle defined by ψ.

Values of F

ψ	0°	10°	20°	30°	40°
F	1·00	1·22	0·94	0·47	−0·06
ψ	50°	60°	70°	80°	90°
F	−0·54	−0·90	−1·08	−1·08	−0·91
ψ	100°	110°	120°	130°	140°
F	−0·62	−0·27	0·08	0·36	0·53
ψ	150°	160°	170°	180°	..
F	0·56	0·46	0·26	0·00	..

Helmert, assuming the masses external to the geoid, also the inner mass anomalies, condensed on the geoid, where they together are equivalent to a thickness D of matter of density ρ, showed that approximately

$$\Delta g = \tfrac{3}{2} \frac{g}{R} \left(\frac{\delta D}{\rho} - N \right).$$

According to Hayford's theory of isostasy, it appears that $\delta D = 0$ when compensation is perfect; but Helmert in 1910 considered his equation to show the variation in D, regarding N as a very slowly varying quantity. He deduced that the smaller densities which underlie mountains also extend laterally beyond the mountains; and are not confined, as postulated in the present-day theory of isostasy, to the volume directly below the mountains—a very natural arrangement, though not in full accord with Hayford's theory.

§ (14) CONTINENTS AND ISLANDS.—Helmert draws attention to a systematic difference between the actual values of "g" for continents and islands, and says that it is in excess on coasts, small islands, and mountain ranges; and in defect in valleys and at the foot of mountains. Recent results in India uphold this as regards mountain ranges and areas at their feet; but as regards coast stations the following residuals appear, in which γ_c, γ_B are the calculated values on the Hayford and Bouguer hypotheses respectively (3, No. 15, p. 178):

	$g-\gamma_c-\cdot011$.	$g-\gamma_B+\cdot030$.
Colaba (Bombay) .	+0·052	+0·092
Cuttack . . .	−0·005	+0·033
Madras . . .	−0·064	+0·016

The quantities 0·011 and 0·030 are introduced to make the formula values fit India best as a whole.

Helmert also says that the deviation of spheroid from geoid will scarcely exceed ±100 metres; but with the discrediting of Hecker's observations at sea,[1] and considering our lack of knowledge of gravity over the greater part of the earth's surface, it is open to doubt whether this estimate can be accepted.

§ (15) ISOSTASY. (i.) *Hayford's Theory.*—Hayford's theory of isostasy postulates that the amount of matter in any vertical column of the earth, bounded below by the surface of compensation and above by the actual surface of the earth, is the same for all columns of the same cross-section. The compensation surface is a surface parallel to the geoid. He has formulated a computation scheme for calculating the effects at any station; and the calculations have been carried out for the United States by himself and Bowie (7 (*b*) and 12).

From this research 122·2 km. is derived as the most probable depth of the compensation surface. Hayford says (12 (*b*), p. 59): "One may properly characterise the isostatic compensation as departing on an average less than one-tenth from completeness or perfection. The average elevation of the U.S. above mean sea-level being about 2500 feet, this average departure of less than one-tenth part from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet (76 metres) thick on average."

Similar treatment has been applied to numerous stations in India. The first verdict was that compensation was by no means so complete in India as Hayford and Bowie had found it to be in U.S.A. Up to this point no account had been taken in the computations of the known and inferred anomalies of density of the crust. Naturally when matter of low density is found at the surface it is rather a matter of conjecture as to how far this low density will persist. To take the case of alluvium, it is necessary to know to what depth the alluvium extends, and to what extent it becomes compressed in its lower layers. Moreover, the amount of water which it contains is unknown; and this affects the total density. In dealing with the case of the results of the Indo-Gangetic plain, Burrard (3, No. 17) takes anomalous densities into account, and derives depths which will account for the observed facts, on the assumption that compensation is perfect. In this connection also see Bib. 30, 31.

As indicating a view which has been reached on the subject of isostasy in recent years, the following is quoted from Bib. 14 (*a*).

(ii.) *Sir S. G. Burrard.*—"Archdeacon Pratt enunciated this hypothesis (of isostasy) fifty years ago, and although we have been frequently led by unexplained anomalies of gravity to question it, yet the more we investigate the stronger becomes our conviction that Pratt's hypothesis is universally correct. I have often met with gravity anomalies which seemed in opposition to Pratt's view, but after further detailed investigation these anomalies have been actually found to confirm him. Since the days of Pratt the history of isostasy has been to a large extent a record of misconception. It has been stated in text-books that, although large mountain-masses like the Alps are isostatically compensated, small mountains are not. I do not see why such a small body as the Great Pyramid should not be compensated—all we can say is that the means at our disposal are not sufficiently refined to enable us to judge whether it is compensated or not. The alluvial plains of the Mississippi and of the Ganges are always having additional loads of silt deposited upon them, but these loads are compensated as soon as they are laid down by *decreases of density in the crust.* When Pratt enunciated his theory, Airy suggested that the mountains were floating. But isostasy is not flotation. . . ."

(iii.) *Professor Love.* — "The principle of isostasy goes far beyond an empirical assumption designed to co-ordinate the results of geodetic observations. It is an hypothesis concerning the mechanical state of the matter composing the Earth. . . . The distribution of mass in the Earth must be such as not to disturb seriously either the spheroidal form of the geoid or the way in which gravity varies over the geoid. The hypothesis meets these conditions by assuming that the Earth consists of a comparatively thin crust and a core, that the inner boundary of the crust is an equipotential surface at a practically constant distance beneath the geoid, that the matter of the core is so arranged that surfaces of equal density are equipotential surfaces, and that the matter of the crust is so arranged that equal masses of the crust (and ocean) stand on equal areas of the surface of the core. The last is Pratt's principle of compensation. The mechanical state of the core is assumed to be one of hydrostatic equilibrium. . . ."

---

[1] Duffield (14 (*b*), p. 188) says the error is of the order of one-third of a kilometre in the height of the geoid relative to the spheroid of reference.

(iv.) *Sir Joseph Larmor.*—"Stokes recognised that from experiments on the Earth's surface we could draw no certain conclusions as to the nature of what was underground. However, we could make conclusions of more or less probability with regard to the first 60 miles or so below the surface. . . ."

(v.) *Dr. Jeffreys* held that "there is no reason to assume compensation for small areas, for the stresses which would result would be near the surface where the material is strongest."

(vi.) *Further Considerations.*—For the theory, as dealt with by Hayford, to be true, it is necessary to suppose that crustal anomalies of density have no appreciable effect in general. On the other hand, if they have appreciable effect, there is great difficulty in computing these effects, as the extent of the anomaly is not usually known. To derive the extent, as Burrard has done, on the basis of complete compensation is no proof of the existence of such compensation.

There are mechanical reasons, based on the yield of materials to great pressure, which make the idea of isostasy in a general way an inevitable conclusion. Few geodesists would negative the idea of a continent being compensated as a whole. But the number who would go to the other extreme and say that every small earth feature is compensated is very much smaller.

In his observations on the voyages from England to Australia *via* Suez, and back *via* Cape Town, Duffield found a variation of "g" with depth of water (11 (*b*), p. 14). He says: "The results, if confirmed, will very seriously limit the application of the isostatic theory of the earth's equilibrium, since over the Indian Ocean the value of gravity is 0·2 to 0·3 cm./sec.^{2} less than that demanded by the mathematical expression of Pratt's hypothesis—a very appreciable amount in gravitational units. The compensation appears to be less complete than the simple theory had led us to hope. The above suggestions are put forward tentatively and with due regard to the nature of the evidence on which they are based."

§ (16) MAJORANA'S QUENCHING FACTOR.—A number of attempts have been made to see whether matter absorbed gravitation. Erismann (24) found that such effect, if any, was less than 1 in 1000; but Bottlinger stated (25) that the attraction between two masses is reduced when a third mass is introduced between them. In 1919 Majorana (26) found a similar effect and derived a "quenching factor" representing this, which amounted to $6{\cdot}73 \times 10^{-12}$. He made an apparatus whereby a leaden ball could be weighed, and, without any other disturbance, could then be surrounded by a large mass of mercury and weighed again. The ball lost $7{\cdot}7 \times 10^{-10}$ of its weight in the second case. Applying his quenching factor to the case of the sun, he computes that the sun's mean density is actually 4·27, instead of the apparent density 1·41 usually accepted. Further experiments on a larger scale are contemplated.

§ (17) TEMPERATURE EFFECT.—Other physicists have experimented to find whether gravitation is in any way dependent on temperature. Southern (27) found no effect as great as 1 in 10^8 for 1° C. But Shaw (28) found an appreciable effect. This has been criticised by Larmor (29).

§ (18) RELATIVITY.—To Einstein's theory of gravitation, which has occupied so much attention in recent years, especially since confirmed by the solar eclipse observations of 1919, it is not necessary to refer here. Though this fundamentally affects ideas of the *cause* of gravitation, it does not appear to enter the question of a gravity survey over the earth. Publications on this subject are so numerous that they cannot be enumerated here. Those interested should refer to the articles on the subject.

§ (19) SURVEY RESULTS.—For the results of gravity surveys the reader is referred to the publications of the Survey Departments concerned, and of the International Geodetic Conference. All known results for "g" from 1808 to 1909 are collected in one volume (7 (*c*)); while in Bib. 3, No. 16, all deflections and values of "g" in India up to 1920 are tabulated. Bib. 12 (*d*) contains a very complete list of publications on the subject of isostasy.

J. DE G. H.

BIBLIOGRAPHY

Numerous references may be found to gravity survey and theory in the publications of the International Geodetic Association, Prussian Geodetic Institute, Survey of India, United States Coast and Geodetic Survey, and other Survey Departments. Only the more important of these and other publications are referred to below.

1. (*a*) *Geodesy.* By Colonel A. R. Clarke, R.F.S. Oxford, 1880.
   (*b*) *Theorien der höheren Geodäsie.* By Dr. F. R. Helmert. Leipzig, 1884.
2. *Account of the Operations of the Great Trigonometrical Survey of India.*
   (*a*) Astronomical Latitudes.
   Vol. xi. Dehra Dun, 1890.
   Vol. xviii. Dehra Dun, 1906.

(*b*) Telegraphic Longitydes.
Vol. ix. Dehra Dun, 1883.
Vol. x. Dehra Dun, 1887.
Vol. xv. Dehra Dun, 1893.
Vol. xvii. Dehra Dun, 1906.

(*c*) Pendulum Operations.
Vol. v. Dehra Dun, 1879.

3. Professional Papers of the Survey of India.

No. 5. "The Attraction of the Himalaya Mountains upon the Plumb-line in India." By Major S. G. Burrard, R.E., F.R.S. Dehra Dun, 1901.

No. 10. "The Pendulum Operations in India, 1903-1907." By Major G. P. Lenox Conyngham, R.E., F.R.S. Dehra Dun, 1908.

No. 13. "Investigation of the Theory of Isostasy in India." By Major H. L. Crosthwait, R.E. Dehra Dun, 1912.

No. 15. "The Pendulum Operations in India and Burma, 1908–1913." By Captain H. J. Couchman, D.S.O., M.C., R.E. Dehra Dun, 1915.

No. 16. "The Earth's Axes and Triangulation." By J. de Graaff Hunter, M.A., Sc.D. Dehra Dun, 1918.

No. 17. "Investigation of Isostasy in Himalayan and Neighbouring Regions." By Col. Sir S. G. Burrard, K.C.S.I., R.E., F.R.S. Dehra Dun, 1918.

4. *Roy. Soc. Phil. Trans.*, Ser. A, vol. ccv. 1905. "On the Intensity and Direction of the Force of Gravity in India." By Col. S. G. Burrard, R.E., F.R.S.

5. *Encyclopaedia Britannica.* Eleventh Edition. Cambridge, 1911.
Articles: (*a*) "Geodesy."
(*b*) "Earth, Figure of the."
(*c*) "Gravitation."

6. *Veröffentlichung des Preussischen Geodätischen Institutes.*

No. 27. "Bestimmung der absoluten Grösse der Schwerkraft zu Potsdam mit Reversionspendeln." Von Dr. F. Kühnen und Dr. Ph. Furtwängler. Berlin, 1906.

Nos. 32, 49. "Beobachtungen an Horizontalpendeln über die Deformation des Erdkörpers unter dem Einfluss von Sonne und Mond." Von Dr. O. Hecker. Berlin, 1907, 1911.

7. *Comptes rendus des séances de l'Association Géodésique Internationale.*

15me. conference à Budapest, 1906. Vol. i. Leyden, 1908.

(*a*) "Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflachen mit Hulfe der Drehwage." Von Baron Roland Eötvös.

(*b*) Report for the United States, pp. 198-234. By O. H. Tittmann and John F. Hayford.

16me. conference à Cambridge, 1909. Vol. i. Berlin, 1911.

(*c*) "Bericht über die relativen Messungen der Schwerkraft mit Pendelapparaten in der Zeit von 1808 bis 1909 und über ihre Darstellung im Potsdamer Schweresystem." Erstattet von E. Borrass.

8. *Roy. Soc. Proc.*, 1899, vol. xlv.; also *Phil. Trans.* A, vol. cxciii.
"On a Quartz Thread Gravity Balance." By R. Threlfall and J. A. Pollock.

9. *Zentralbureau der internationalen Erdmessung.*

No. 11. "Bestimmung der Schwerkraft auf dem Atlantischen Ozean." 1903. Von Dr. O. Hecker Berlin.

No. 16. "Bestimmung der Schwerkraft auf dem Indischen und Grossen Ozean." 1908. Von Dr. O. Hecker, Berlin.

No. 20. "Bestimmung der Schwerkraft auf dem Schwarzen Meere." 1910. Von Dr. O. Hecker, Berlin.

10. *Roy. Soc. Proc.*, 1916, vol. xcii. Ser. A.

(*a*) "Apparatus for the Determination of Gravity at Sea." By W. G. Duffield, D.Sc.

(*b*) "On the Determination of Gravity at Sea." By Sir Arthur Schuster, F.R.S.

11. *British Association Reports*, Section A.

(*c*) "Determination of Gravity at Sea." Manchester, 1915. By W. G. Duffield, D.Sc.

(*d*) "Determination of Gravity at Sea." Newcastle, 1916. By W. G. Duffield, D.Sc.

(*e*) "Determination of Gravity at Sea." Bournemouth, 1919. By W. G. Duffield, D.Sc.

12. United States Coast and Geodetic Survey.

(*a*) "The Figure of the Earth and Isostasy from Measurements in the U.S." By J. F. Hayford. Washington, 1909.

(*b*) "Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy." By John F. Hayford. Washington, 1910.

(*c*) "The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity." By John F. Hayford and William Bowie. Special publication No. 11. Washington, 1912.

(*d*) "Investigation of Gravity and Isostasy." By William Bowie. Special publication No. 40. Washington, 1917.

(*e*) "Determination of Time, Longitude, Latitude and Azimuth." By William Bowie. Special publication No. 14. Washington, 1913.

13. *Journal of Geology*, University of Chicago Press, Oct., Nov. 1911. "The Theory of Isostasy." By Harmon Lewis.

14. *The Observatory.* London. Taylor and Francis, "Discussions of Geophysical Subjects."

(*a*) "Isostasy." Col. Sir S. G. Burrard, K.C.S.I., R.E., F.R.S., in No. 546 for Dec. 1919.

(*b*) "The Earth's Axes and Figure." J. de Graaff Hunter, M.A., Sc.D., in No. 552 for April 1920.

15. Physical Mathematical Society, Japan. *Proc.*, Nov. 1919, i, 347-351. "The Use of Tungsten for Pendulums." H. Nagaoka.

16. (*a*) *Description et usage de l'astrolabe à prisme.* Par MM. Claude et Driencourt. Paris, 1910.

(*b*) *Handbook of the Prismatic Astrolabe.* By John Ball, D.Sc., and H. Knox-Shaw, M.A. Cairo, 1919.

(*c*) *Geographical Journal*, 1919, liv. 1, p. 29. "The Prismatic Astrolabe." By Ball and Knox-Shaw.

(*d*) *Geographical Journal*, liv. 4, p. 262. "The Prismatic Astrolabe." By Ball and Knox-Shaw.

17. *Zeitschrift für Instrumentenkunde.* Jan. 1914. "The Eötvös Gravity Balance." By Dr. O. Hecker.

18. *American Journal of Science*, Jan. 1911, xxxi. 1-18. "Gravity Determination at Sea." By L. A. Bauer.

19. *Phil. Soc.*, Washington, Dec. 1907, Bulletin 15, pp. 57-74. "The Earth a Failing Structure." By J. F. Hayford.

20. *American Journal of Science*, Aug. 1911, xxxii. 101-113. "Gravity Determination in the U.S.A." By William Bowie.

21. *Washington Academy of Science Journal*, Dec. 1912, pp. 499-501. "Hayford Method of Gravity Reduction." By W. Bowie.

22. *Accad. Lincei: Atti*, 1910, 19, i. 109-114, ii. 272-278. "Bending of the Supports of Pendulums used in the Determination of Relative Gravity." By G. Abetti and C. Cappello.
(The movement of the stand is measured by interferometer.)

23. *Preuss. Akad. Wiss.*, Berlin, 1913, xxviii. 550-560. "Determination of the Geoid in the Harz Mountains." By Dr. F. R Helmert.

24. *Archives des Sciences*, 1911, xxxi. 36-45. "Gravitational Permeability." By T. Erismann.

25. *The Observatory.* Nov. 1912, xxxv. 387-393. "Review of a Pamphlet by K. F. Bottlinger, Munich." By W. de Sitter.

26. *Phil. Mag.*, ccxxxiii. 488. "On Gravitation." By Q. Majorana (Professor, Turin).

27. *Roy. Soc. Proc.* A, 1906, lxxviii. 392-403. "Dependence of Gravity on Temperature." By L. Southern.

28. *Roy. Soc. Phil. Trans.*, 1916, ccxvi. 349-392. "Newtonian Constant of Gravitation affected by Temperature." By P. E. Shaw.

29. *Nature*, 1916, xcvii. 321, 400. Criticism of 28 above by Larmor and Reply by Shaw.

30. (*a*) *Proc. Roy. Soc.* A, 1914, xc. 32-40. "On the Effect of the Gangetic Alluvium on the Plumb-line in Northern India." By R. D. Oldham, F.R.S.

(*b*) *Proc. Roy. Soc.* A, 1915, xci. 220-238. "On the Origin of the Indo-Gangetic Trough commonly called the Himalayan Foredeep." By Col. Sir Sidney Burrard, F.R.S.

31. *Geographical Journal*, 1918, vol. li. "The Problem of the Himalaya and the Gangetic Trough." This is a critical résumé of recent publications, by A. Morley Davies.

Guiding Surfaces, accuracy of, in construction of machines. See "Metrology," IX. § (34) (i.).

— H —

Hail:

Equations for formation of, in adiabatic conditions. See "Atmosphere, Thermodynamics of the," § (21).

The electrification of. See "Atmospheric Electricity," § (24) (ii.).

Hair: variation of length with humidity. See "Humidity," II. § (10).

Halos:

Bouguer's. See "Meteorological Optics," § (14).

Historic examples of. See *ibid.* § (17).

Of 22°. See *ibid.* § (20) (i.).

Of 46°. See *ibid.* § (21) (i.) *et seq.*

Theories of. See *ibid.* § (18).

See also "Parhelion," "Anthelion," "Arcs," etc.

Harmonic Analysis of Tides: a method of tide prediction which represents the tide at any port by a series of simple harmonic terms, and whose period is determined from theoretical considerations, but whose amplitude and phase are found from observation. See "Tides, and Tide Prediction," § (5). See also "Fourier's Series," Vol. I.

Head Rod: an instrument used for measuring diameters of barrels, and also as a computing rule. See "Volume, Measurements of," § (6).

Heat: in the atmosphere. See "Radiation," § (3) (iv.).

Transference of, from the earth to the atmosphere. See *ibid.* § (3) (ii.) and (iv.).

Heat-engine, comparison of atmosphere with. See "Atmosphere, Thermodynamics of the," §§ (1), (19), (22)-(26).

Heights, Computation of, from Pressures, old international meteorological formula for. See "Barometers and Manometers," § (16) (ii.).

Heights, Determination of, by the Barometer. See "Barometers and Manometers," § (16).

Assuming that the temperature decreases uniformly with the height according to the law

$$T = T_0 - \beta H,$$

the height is given by the formula

$$\frac{\beta H}{T_0} = 1 - \left(\frac{p}{p_0}\right)^{\frac{R\beta}{g}}.$$

See *ibid.* § (16).

Assuming that the temperature is constant and equal to T° absolute, the height is given by the formula

$$H = \frac{RT}{g \log_e 10}(\log p_0 - \log p).$$

See *ibid.* § (16).

Hemisphere, Projection suitable for, is Airy's projection by balance of errors, or Clarke's minimum error perspective. See "Map Projections," § (8) (xxxiv.).

Horizon:

Distance of: effect of meteorological conditions on. See "Meteorological Optics," § (8).

Formula for depression of. See *ibid.* § (8).

Horizontal Balance Beam, as used for the determination of the constant of gravitation. See "Earth, Density of the," § (4).

Hudson's Horse-power Computing Rule. See "Draughting Devices," p. 272.

Humidifiers:

Howarth's champion system. See "Humidity," II. § (15) (iii.).

Mather and Platt's vortex system. See *ibid.* II. § (15) (i.).

Smethurst system. See *ibid.* II. § (15) (ii.).

HUMIDITY

I. General

The term Humidity is employed in Physics to denote the presence of invisible water-vapour in a space or diffused through a gas. In mist or fog water exists both as a gas and as a liquid. Invisible water-vapour is dealt with in this section, whilst mists, fogs, and clouds are treated in the sections devoted to meteorology.

The amount of water-vapour may be measured either by its pressure or by its mass in a unit volume. If a be the actual pressure of the water-vapour present, and if b be the maximum pressure that water-vapour in presence of liquid water can exert at the same temperature, then a/b is called the "relative humidity"; and 100 times this quantity is called the "percentage humidity." The same statements are true if the space be occupied by a gas with which the water-vapour does not act chemically, and through which the vapour is uniformly distributed. Such a gas is air, and man is particularly concerned with this case since he lives and works in air.

The maximum vapour pressure b of water-vapour in a vacuum, or, as it is sometimes called, the saturation pressure, depends on the temperature and upon nothing else. This has been regarded as settled since Regnault[1] published his table of the pressure of water-vapour at different temperatures. For the sake of brevity we shall use the letters M.V.P. for maximum vapour pressure.

It was first enunciated by Dalton that the M.V.P. of water in a closed vessel in presence of liquid water at the same temperature is the same, no matter whether there be air present in the vessel or not. Regnault[2] verified this law by experiments *in vacuo*, in air, and in nitrogen. The pressure in air was obtained for thirty-four temperatures lying between the limits 0° C. and 38° C. The results obtained show a pressure in air less than that for vacuum by amounts varying between 0·10 mm. and 0·74 mm., the mean being 0·44 mm. In nitrogen the mean was 0·56 mm. These differences, though very irregular in amount, are considerable and always in the same direction, and might be held to show that Dalton's law is only approximately true, but Regnault himself suggested that they might be due to some constant error. In fact, he attributed the diminished pressure of vapour to the molecular action of the glass vessel producing condensation, the slowness of diffusion preventing the pressure reaching its maximum value by subsequent evaporation. This method of accounting for the discrepancies between calculation and experiment was confirmed by Herwig[3] by experiments upon the compression of vapours. It was found that the pressure of the vapour could be increased beyond the point at which a deposit was first formed on the sides of the vessel, and that the vacuum maximum pressure was this increased pressure. It may be concluded that Dalton's law is strictly true provided that the air is saturated in such a manner as to avoid the molecular action of the sides of the vessel.

We next inquire as to the effect on the M.V.P. when water undergoes the change from the liquid to the solid state. Professor James Thomson[4] has discussed this matter from the theoretical standpoint. The M.V.P. at 0° is only 4·6 mm., consequently if the water be in a vacuum the pressure due to its vapour on it will be nearly one atmosphere less than the ordinary pressure. It can be shown from this that owing to the expansion of water on solidification the temperature of freezing *in vacuo* is +0·0075° C.

If we construct a diagram in which the ordinates represent pressure and the abscissae temperature this point is represented in *Fig.* 1 at T, where PTE is the M.V.P. curve of water in equilibrium with its vapour, NTQ is a line representing the M.V.P. of water-vapour in equilibrium with ice, and MT a line representing the pressure at which water is in equilibrium with ice. James Thomson called the point T where these three lines meet the triple point. Experimental values have been obtained for each of these curves. It is of importance that the ice-vapour and water-vapour curves should be accurately known, as the temperature of the air frequently passes below 0° C. The quantity b, in the opening paragraph, must be obtained from the corresponding curve according to whether we are dealing with a liquid-vapour or a solid-vapour equilibrium. The curve PTE extends upwards to the critical temperature of the liquid, which in the case of water is about 360° C.

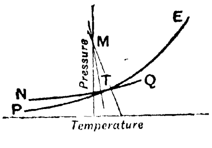


FIG. 1.

As to tables of the M.V.P. for water, that of Regnault-Broch[5] for the range −19·0° to 101° is to be found in Landolt and Börnstein's *Physikalisch-chemische Tabellen*, 1894 edition; the same table, improved by Wiebe and referred to the hydrogen scale of temperature by Thiesen and Scheel, is found in the 1905 edition of the same work, which also contains tables by Scheel for the M.V.P. of water-vapour over ice. The latest table of the Reichsanstalt (Thiesen, Holborn, and others, 1908) is to be found in Kaye and Laby's tables, p. 40 (Longmans, 1911). For ordinary purposes use may be made of any of these tables, as the differences are not large.

II. INSTRUMENTS, CONDENSATION

The methods of finding the amount of water-vapour in air may be arranged in five classes: (1) depending on finding the dew-point, (2) depending on the lowering of temperature of a wet-bulb thermometer due to evaporation, (3) in which the actual mass of water in a given volume of the air is measured, (4) depending on the change of pressure of the air when the water-vapour is removed by absorption, (5) depending on the use of hygroscopic substances. The continuous record of the relative humidity from time to time is made generally by taking advantage of the hygroscopic property of certain substances, hair (De Saussure), thin sheets of gelatine, horn, etc. Many forms of apparatus have been proposed for carrying out these methods, and it would be impossible to attempt the description of them all. Consequently we shall confine ourselves to those which are of more general use, or of special interest.

---

[1] *Relation des expériences, Mémoires de l'Académie*, t. xxi.

[2] *Ann. de Chimie*, 1845, 3rd series, xiv.

[3] *Pogg. Ann.* cxxxvii.

[4] *Proc. R.S.*, 1873.

[5] *Trav. et mém. du Bur. internat. des Poids et Mes.*, 1881. I. A, 33.

The dew-point is that temperature at which the air would be saturated with the water-vapour in it, and in consequence it is the temperature at which condensation of the water to the liquid form should first be observed. From the tables of M.V.P. the pressure corresponding to the dew-point is obtained, and that is the actual V.P. existing in the air. Dalton found the dew-point using a bright metallic cup containing water which was cooled gradually by adding colder water or ice. When the outside of the cup was first observed to become misty from condensation the temperature of the water was taken. Let it be t. Then the cup and its contents were allowed to warm, and the temperature t' at which the mist disappeared was taken. The mean of the two observations was assumed to be the dew-point. The difficulty of the experiment is to make up one's mind when the surface is dimmed first by moisture, since reflections on the bright surface disturb the judgment of the observer.

§ (1) Dew-point Hygrometers.—Daniell designed an apparatus for the purpose. It consists (*Fig.* 2) of a small cryophorus containing ether, one limb of which, *c*, is longer than the other and terminates in a bulb *b*, either of black glass or with a gilt band round it, on which the dew is formed. In the long limb is a thermometer with its bulb immersed in the ether. The other bulb *a* is covered with muslin. When the instrument is to be used all the ether is passed over into the bulb *b* by inversion. On allowing a few drops of ether to fall on the muslin the vapour within *a* is condensed, and fresh vapour rises from the surface of the ether in the blackened bulb; in consequence that bulb is reduced in temperature. This operation is continued until a ring of condensed moisture appears on the outside of *b* at the level of the junction of the ether and vapour. The temperature of the enclosed thermometer is noted, and is again noted when, the whole apparatus being allowed to warm, the condensed moisture disappears. The mean of the two temperatures is taken as the dew-point.

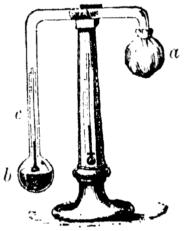


Fig. 2.

Regnault's hygrometer (*Fig.* 3) consists of a glass tube terminating in a very thin and highly polished silver thimble containing ether. The cork which closes the tube has passing through it a long tube going to the bottom of the ether, a short tube, and a thermometer with its bulb in the ether. The short tube is connected to an aspirator. When the aspirator is set running air is drawn through the ether, and soon moisture is formed on the silver thimble. Owing to the high conductivity of silver and the constant stirring caused by the current of air, it is obvious that the thermometer must indicate very nearly the temperature of the external face of the thimble. In order to recognise the first appearance of dew by comparison, a second silver thimble is set near the first, and the two are seen by the observer looking through a telescope some 10 metres away. The telescope must have a sufficient magnifying power to enable the thermometers to be read. Another thermometer, placed in the empty second thimble, is read for the temperature of the air. The observer is sufficiently far off not to affect the hygrometric state of the air. With a little experience the tap of the aspirator can be regulated until the moisture is seen to form, and then the tap is closed. The temperature is read and the apparatus is allowed to heat, when a second reading is made. The mean of the two readings is taken as the dew-point.

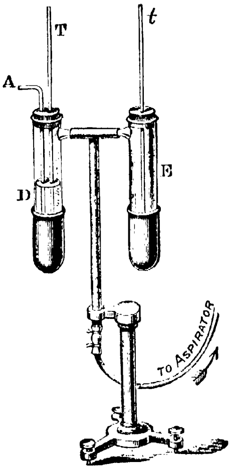


Fig. 3.

The readings of dew-point hygrometers are affected by the presence of the observer; less than 5 grammes of water will saturate a cubic metre of air at 0° C., and the average human being in repose gives off 63 grammes per hour; thus the observer's presence will modify profoundly the hygrometric conditions of the air. The arrangement shown in *Fig.* 4 has been devised to overcome this difficulty. The hygrometer can be enclosed in a wooden box lined with metal, which is cut diagonally as illustrated. By oscillating the lower half of the box—and this can be done from a distance by means of a string—it is possible to obtain an average sample of the air. The box is then closed, the joint being improved by the baize edging. The observer then sets the air bubbling and watches the formation of dew through the double windows in front.

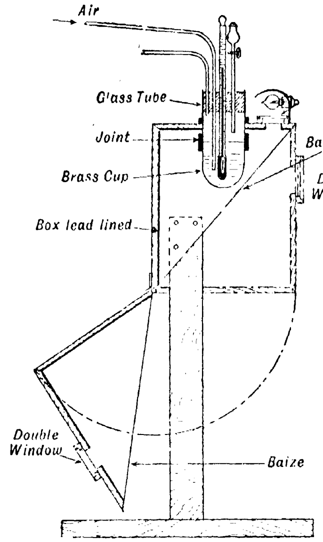


FIG. 4.

In Alluard's[1] modification of Regnault's apparatus the silver tube containing the ether has a flat face. This flat face is surrounded by a flat plate of silver in the same plane as the flat face but not touching it. This plate consequently remains at the temperature of the air, and it is claimed that with this arrangement the first appearance of condensed moisture is more easily recognised.

§ (2) EFFECT OF WIND.—Crova[2] found that Regnault's instrument was not reliable in a high wind with a low humidity. This led him to design a condensation hygrometer in which the formation of the dew occurred on the inside of a vessel. A tube of thin brass, nickel-plated and polished inside, is closed at its far end by a ground glass plate, and at its near end by a lens. Looking in through the lens, the ground glass is seen surrounded by its image reflected in the bright tube. The whole tube is surrounded by a metal box containing carbon disulphide, which can be cooled by drawing air through it. The air to be tested is drawn into the polished tube.

Crova found under these conditions that the point of condensation was higher than that shown by a Regnault's apparatus exposed to the full force of the wind. We may quote as an example from his paper[3] some experiments made at Montpellier in a strong wind from the north-west, which was increasing throughout the experiment.

Time.		Temperature of Air.	Dew-point.	
			Exterior.	Interior.
H.	M.	°	°	°
8	15	19·4	9·5	9·5
8	30	19·9	9·7	10·5
8	45	19·7	8·3	9·8
9	0	19·6	8·1	9·5

His results apparently show that the point of condensation depends on the velocity of the wind.

§ (3) THEORY OF DEW-POINT INSTRUMENTS.—A discussion of the theory of the use of these condensation instruments is necessary. It is assumed that when they are used in the free air they do not affect the total pressure of the atmosphere, and that the fraction of that pressure due to water vapour is unchanged.

We may ask ourselves how these instruments are affected by a wind blowing on them and by radiation. In Regnault's instrument the temperatures of the silver cup and of the layer of air outside are assumed to be the same as that of the ether inside. Let H be the heat withdrawn per second per square centimetre by evaporation of ether; then H = heat received by radiation + heat received by convection + latent heat of the water condensed.

$$H = R(t'-t) + \frac{S_pF(t'-t)}{v} + mL,$$

where R is the radiation constant, t the temperature of the silver surface, t' the surrounding temperature, L the latent heat of evaporation at t, and m the mass of water condensed per second. The expression $S_pF(t'-t)/v$ is the recognised formula for the heat in calories passing from a hot current of gas to a cold surface. S_p is the specific heat of the gas at constant pressure, F the coefficient of skin friction, t' is the temperature of the stream of gas, t that of the surface, and v the mean velocity of the stream. The flow is parallel to the surface, which is not the case when Regnault's apparatus is placed, as it would usually be, at right angles to the wind.

When a dew-point apparatus is used the mass condensed is made as small as possible, and we have reason to believe that the radiation effect will be small compared with the convection effect. Consequently we may write

$$H = \frac{S_pF(t'-t)}{v}.$$

Now it has been shown[4] that approximately F = constant × v^2, therefore H = constant × S_p × $v(t'-t)$. There is nothing in this formula to indicate that it will not be possible by increasing H to compensate for an increase in velocity, and thus to find the same value of the dew-point, provided the humidity is unchanged.

---

[1] *Jour. de Physique*, 1878.

[2] *Ibid.*, 1883.

[3] *Jour. de Physique*, 1883.

[4] *Technical Report of the Advisory Committee for Aeronautics*, 1912-13, p. 40.

We must, therefore, look elsewhere for the effect of a wind, and perhaps we may find an explanation in the note by Dr. Stanton to the paper referred to above. Dr. Stanton pointed out that the flow round a cylinder placed at right angles to a stream would not be stream-line flow, and its character would vary with the velocity. In the case of a pipe of circular section placed with its axis at right angles to the current, the heating effects in the neighbourhood of the points A, B, C (*Fig.* 5) might be widely different, since at A the motion is presumably considerably slower than at B, and C is acted upon by the eddies thrown off by the side. Experiments by Mr. Jakeman with cold air blowing on an arrangement of four electrically heated strips of platinum foil in the positions A, B, C, D on an ebonite rod gave values for the heat dissipated. The diagram given in the paper shows that with the same temperature difference between the strip and the current more heat is dissipated at A than at B; and more at B than at C. Conversely, if the strips were colder than the current, more heat would be transmitted at A than at B, and more at B than at C. Perhaps in the same way in a Regnault's dew-point apparatus in a wind, the cooling would be irregular round the tube of circular section. There is room here for research. Perhaps we might give to the silver tube a stream-line section, and in this way try to overcome the inequality of heating at different parts. But here we must leave the subject to future experimenters.

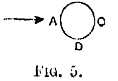


FIG. 5.

§ (4) WET AND DRY BULB HYGROMETER. (i.) *Instruments.*—The lowering of temperature due to evaporation is the principle of the psychrometer or wet and dry bulb thermometers (Leslie). The dry bulb is an ordinary sensitive thermometer, and set by its side is a similar thermometer which has its bulb kept constantly moist by a wrapping of muslin, to which a wick is attached, dipping into a reservoir of pure water. This hygrometer is the most largely used of any of the different types. It is used by meteorologists, gardeners, and manufacturers. The theory of its action has not been very clearly stated, and it is necessary that we should devote more than ordinary attention to it. The difference in the readings of the two thermometers is observed, and from the difference the amount of water vapour present in the air is calculated. *Fig.* 6 represents a common form of the apparatus.

The muslin should be of fairly close texture; if it is too loosely woven it cannot keep the whole surface of the bulb moist. One layer should be sufficient. Experiment has shown that additional layers cause the instrument to be rather slower in reaching a steady temperature. The muslin should be of good cotton, free from oil, and this may be secured by boiling the muslin before it is used in dilute potash solution, taking great care to wash out thoroughly all potash before it is wrapped round the thermometer. It may be tied on securely with cotton thread. The wick may be formed from an extension of the muslin, or by tying in with it some ordinary cotton wick. With the end of the wick in water the thermometer should maintain itself moist by the constant ascent of water by capillarity to take the place of the loss by evaporation. If it shows inclination to become dry, a new wrapping should be substituted. The reading of the wet thermometer will be less than that of the dry, owing to the absorption of heat during evaporation, and the readings will only be the same when the air is saturated with moisture so that there is no evaporation.

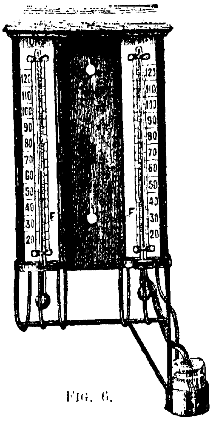


FIG. 6.

Experiment has shown that dry cotton when absorbing moisture causes a slight evolution of heat. This indicates some sort of action, possibly chemical, between cotton and water, but the action is so slight that we may consider the bulb with wet muslin to be virtually a bulb surrounded with a film of pure water.[1]

(ii.) *Formulae for Hygrometry.*—In spite of the general use of the wet and dry bulb thermometers, much vagueness exists as to the formula to be used for the reduction of the readings.[2] The conditions for which the original formulae of August[3] and of Apjohn[4] apply were not sufficiently defined. The consequence is that many modifications of the formula, for which the reader is referred to the *Computer's Handbook*, have been suggested. We give below some of more well-known empirical formulae, but the reader should note that the more scientific method of reducing the observations, and the method that is recommended, is given later in § (8).

The formulae are in general of the type due to Regnault:

$$\epsilon'' = \epsilon' - AB(t - t'),$$

where ϵ'' is the vapour pressure at the dew-point, ϵ' that at the temperature of the wet bulb, t and t' the temperatures of the dry and wet bulbs, B the barometric pressure measured in the same units as those employed for ϵ' and ϵ'', and A a quantity depending on the wind velocity in the neighbourhood of the thermometers.

---

[1] Cobbett, *Proc. Camb. Phil. Soc.* x. 372.

[2] See *The Computer's Handbook*, Section I., published by the Meteorological Office, 1921.

[3] Poggendorff's *Annalen*, II. v. 69.

[4] *Trans. Royal Irish Academy*, 1834.

Pernter has given the values in the following table, if the water on the wet bulb be not frozen:

VAPOUR PRESSURE AT THE DEW-POINT

State of Wind in Screen.	Temperatures in Centigrade Degrees.	Temperatures in Fahrenheit Degrees.
Calm— 0-0·5 metres per second . .	$\epsilon' - \cdot0012B(t-t')\left(1+\frac{t'}{610}\right)$	$\epsilon' - \cdot00067B(t-t')\left(1+\frac{t'-32}{1098}\right)$
Light wind— 1-1·5 metres per second . .	$\epsilon' - \cdot0008B(t-t')\left(1+\frac{t'}{610}\right)$	$\epsilon' - \cdot00044B(t-t')\left(1+\frac{t'-32}{1098}\right)$
Strong wind— Above 2·5 metres per second .	$\epsilon' - \cdot000656B(t-t')\left(1+\frac{t'}{610}\right)$	$\epsilon' - \cdot000364B(t-t')\left(1+\frac{t'-32}{1098}\right)$

Recent experiments in a wind channel at the Royal Aircraft Establishment at Farnborough have shown that for the ordinary range of temperature and humidity the constant A shows no appreciable variation up to speeds of 90 miles an hour (say 40 metres per second) from the values given in the table for a strong wind.

If the wet bulb cover is frozen the coefficients become ·001060, ·000706, and ·000579 respectively, while the 610 in the denominator becomes 689.

The instructions of the Bureau Central Meteorologique prescribe a formula

$$\epsilon'' = \epsilon' - \cdot00079B(t - t'),$$

while in India the formula

$$\epsilon'' = \epsilon' - \cdot00079B(t - t')\left(1 + \frac{t'}{610}\right)$$

is employed. These two practically agree with Pernter's light wind formula. The formulae for a strong wind are suitable for use with Assmann's ventilated and the sling hygrometers. For the latter the following formula—in Fahrenheit degrees—is employed in America:

$$\epsilon'' = \epsilon' - \cdot000367B(t - t')\left(1 + \frac{t-t'}{1571}\right),$$

which, in view of the smallness of the term $(t-t')/1571$, is practically the same as Pernter's strong wind formula.

The tables used by the Meteorological Office[1] are based on Glaisher's factors or multipliers, making use of the equation

$$d = t - A(t - t'),$$

where d is the temperature of the dew-point, t and t' the temperatures of dry and wet bulb respectively, and A a factor determined from the comparison of many thousand simultaneous observations of wet and dry bulb thermometers and of the Daniell Hygrometer taken at the Royal Observatory, Greenwich, from 1841 to 1854, and from observations at high temperatures in India and at low temperatures in Toronto. The values of Glaisher's factor A depend on the dry bulb temperature, and a table is given in the *Computer's Handbook*, from which the following is extracted:

VALUES OF GLAISHER'S FACTORS USED FOR DRY BULB TEMPERATURES FROM 265A TO 314A

Temperature A.	Glaisher's Factor.	Temperature A.	Glaisher's Factor.
265	8·55	290	1·85
266	8·27	291	1·83
267	7·83	292	1·81
268	7·28	293	1·79
269	6·61	294	1·77
270	5·80	295	1·75
271	4·92	296	1·71
272	4·06	297	1·72
273	3·32	298	1·70
274	2·81	299	1·69
275	2·54	300	1·68
276	2·39	301	1·67
277	2·31	302	1·66
278	2·26	303	1·65
279	2·21	304	1·64
280	2·17	305	1·63
281	2·13	306	1·62
282	2·10	307	1·61
283	2·06	308	1·60
284	2·02	309	1·59
285	1·99	310	1·58
286	1·95	311	1·57
287	1·92	312	1·56
288	1·89	313	1·55
289	1·87	314	1·54

The experience of the Egyptian Meteorological Office has confirmed the view that when the air is very dry the linear form of the expression for ϵ'' needs modification. It has occasionally been found to give negative values for the humidity.

---

[1] *Computer's Handbook*, Section I. 3.

§ (5) THE WET BULB THERMOMETER (STILL AIR). (i.) *Theory.*—Clerk Maxwell in his article on "Diffusion" in the ninth edition of the *Encyclopaedia Britannica* gave a theory of the wet bulb thermometer. In calm air we may consider that the steady temperature of the wet bulb is the result of an equilibrium between the heat reaching the thermometer by conduction and radiation, and the heat absorbed in evaporating water from the wet bulb. This water-vapour passes by diffusion outwards through the air. Let Q be the mass of water evaporated, H be the heat gained by conduction, and h the heat gained by radiation. Then, if L be the latent heat of water at the temperature of the wet bulb, $LQ = H + h$.

We shall now express these quantities for a square centimetre of the surface of the wet bulb, and we shall suppose this to be part of a large flat surface. We shall suppose that we are limited to the consideration of the circumstances in a rectangular block of the calm air which has the square centimetre for base. The rectangular block may be supposed to have an end on a surface which remains at a constant temperature θ_0 and at a constant pressure of aqueous vapour p_0, which is the quantity to be determined.

Let the length of the rectangular block be x cm., and let θ_1 be the temperature of the wet bulb, then the heat conducted in time t,

$$H = \frac{tk}{x}(\theta_0 - \theta_1),$$

where k is the conductivity of air in calorimetric units. This according to Stefan is 0·0000558 in C.G.S. units. The heat radiated to the bulb in the time t is

$$h = tR(\theta_0 - \theta_1),$$

where R is the absorption or radiation constant.

The mass of water passing by diffusion in a time t is

$$Q = \frac{tD}{x}\left(\frac{p_1}{P} - \frac{p_0}{P}\right)\sigma\rho,$$

where D is the diffusion constant, P the whole pressure of the air, σ the specific gravity of aqueous vapour, ρ the density of air, and p_1 the M.V.P. at θ_1.

Substituting these values in the equation

$$LQ = H + h$$

we have

$$\frac{LtD}{x}\left(\frac{p_1}{P} - \frac{p_0}{P}\right)\sigma\rho = \frac{tk}{x}(\theta_0 - \theta_1) + tR(\theta_0 - \theta_1),$$

therefore

$$LD\left(\frac{p_1 - p_0}{P}\right)\sigma\rho = k(\theta_0 - \theta_1) + xR(\theta_0 - \theta_1),$$

$$\frac{p_1 - p_0}{P} = \frac{k + xR}{LD\sigma\rho}(\theta_0 - \theta_1).$$

Expressing k, the calorimetric conductivity in units of the thermometric measure of conductivity, $k = S_v\rho K$ where S_v is the specific heat of air at constant volume, we have

$$\frac{p_1 - p_0}{P} = \frac{KS_v\rho + xR}{LD\sigma\rho}(\theta_0 - \theta_1).$$

Since $\gamma S_v = S_p$ where γ is the ratio of the specific heat of air at constant pressure and at constant volume, we have

$$\frac{p_1 - p_0}{P} = \frac{S_p}{L\sigma}\left(\frac{K}{\gamma D} + \frac{xR}{S_p\rho D}\right)(\theta_0 - \theta_1),$$

$$p_0 = p_1 - \frac{PS_p}{L\sigma}\left(\frac{K}{\gamma D} + \frac{xR}{S_p\rho D}\right)(\theta_0 - \theta_1). \quad . \quad (1)$$

Maxwell has shown how the formula (1) may be applied to a thermometer bulb of any shape by using the analogy between the laws of conduction and diffusion and the laws of electrical potential. He shows that if C be the electrical capacity of the bulb and A its area, then

$$p_0 = p_1 - \frac{PS_p}{L\sigma}\left(\frac{K}{\gamma D} + \frac{AR}{4\pi CS_p\rho D}\right)(\theta_0 - \theta_1). \quad (2)$$

If the bulb be spherical and of radius r, $C = r$ and $A = 4\pi r^2$, therefore

$$p_0 = p_1 - \frac{PS_p}{L\sigma}\left(\frac{K}{\gamma D} + \frac{rR}{S_p\rho D}\right)(\theta_0 - \theta_1). \quad . \quad (3)$$

(ii.) *Experimental Verification.*—In an experiment arranged to find the relation between the M.V.P. at the temperature of the wet bulb and the depression of the bulb in order to realise the conditions of calm air, the author arranged wet and dry bulb thermometers inside a porous pot saturated with strong sulphuric acid. The porous pot was of the size used for Leclanché cells, and the wet bulb was placed as low as possible in it. To secure that the sulphuric acid surface should be fresh, acid was poured down the walls of the pot before beginning an experiment. At the surface of the acid the air is dry. The dry bulb thermometer gives the temperature of the enclosure. The only convection possible will be that due to the wet thermometer.

Dry Thermometer, °C.	Depression of Wet Thermometer in °C.	M.V.P. at the Temperature of the Wet Bulb divided by the Depression of Wet Bulb.
16·7	9·0	0·87
16·4	8·7	0·90
7·3	6·0	0·84
6·0	5·5	0·86
5·05	5·2*	0·87
5·1	5·3*	0·85
		Mean 0·865

* Water still liquid.

We notice, if dry air be used, in the formula (3) $p_0 = 0$ and

$$\frac{p_1}{\theta_0 - \theta_1} = \frac{PS_p}{L\sigma}\left(\frac{K}{\gamma D} + \frac{rR}{S_p\rho D}\right), \quad . \quad . \quad . \quad (3^*)$$

and the experiments above show that the value of $p_1/(\theta_0 - \theta_1)$ is 0·865. If we take

$P = 760$ mm. the whole pressure,

$L = 606{\cdot}5 - {\cdot}695\theta_1$ where θ_1 is the temperature of the wet bulb,

$\sigma = 0{\cdot}622$ the specific gravity of water-vapour referred to air.

$K = 0{\cdot}256$ Maxwell's value deduced from Stefan's measurements,[1]

$D = 0{\cdot}198$ Landolt and Börnstein's Tables,

$\gamma = 1{\cdot}41$ the ratio of the specific heat of air at constant pressure to that at constant volume,

$S_p = 0{\cdot}2375$ and substitute in the formula (3*),

we have
$$0{\cdot}865 = {\cdot}478\left({\cdot}92 + \frac{rR}{S_p\rho D}\right),$$
$$\therefore \frac{rR}{S_p\rho D} = {\cdot}89.$$

Thus under the conditions of the above experiments it appears that the two terms, $K/\gamma D$ and $rR/S_p\rho D$, in the bracket have approximately the same value.

The experimental constant 0·865 may be compared with the empirical constants (see *Computer's Handbook*) for still air used by Pernter, 0·93, and by Birkeland, 0·84. It agrees better with the latter. The constant 0·865 is that which should be used where all the bodies in the neighbourhood are at the temperature of the dry thermometer, the condition which is supposed to be the case in a Stevenson's meteorological screen, and when the air is calm.

We may next inquire what change would be produced if dry hydrogen were substituted for dry air. The conductivity of hydrogen is about seven times greater than that of air, and the diffusivity of water-vapour in hydrogen is about $3\frac{1}{2}$ times that in air.

Taking the values of the constants for hydrogen at 760 mm. pressure, we find that
$$\frac{PS_p}{L\sigma} = {\cdot}475,$$
$$\frac{K}{\gamma D} = 1{\cdot}83,$$
$$\frac{rR}{S_p\rho D} = {\cdot}257,$$
from results in the previous section and using the value of D in hydrogen. Therefore, in hydrogen
$$\frac{p_1}{\theta_0 - \theta_1} = {\cdot}99.$$

Some experiments were made to determine this value, using the same apparatus as in the previous section and displacing the air by hydrogen.

	Dry Thermometer.	Wet Thermometer.	Vapour Pressure at Temperature of Wet Bulb divided by the Depression.	
			Experiment.	Theory.
In still air . .	16·7°	7·8°	0·88	0·865
In still hydrogen	16·4	8·75	1·09	0·99

Considering the uncertainty in the actual value of the conductivity of hydrogen and the diffusivity of water-vapour in that gas, there is, therefore, a good agreement with Maxwell's theory.

Another proof of the applicability of Maxwell's theory is to be found in the behaviour of a wet bulb thermometer in a dry atmosphere when the pressure is reduced. Daniell (1834) attached a delicate thermometer with its bulb covered with filter paper to a brass wire sliding through a collar of leather in a ground brass plate. This plate was fixed air-tight on the top of a large glass air-pump receiver, which covered a surface of sulphuric acid of nearly equal dimensions with its base. Upon a tripod of glass, standing in the acid, was placed a vessel containing a little water into which the thermometer could be dipped and withdrawn by means of the sliding wire. At the commencement of the experiment, the pressure gauge stood at 30·2 inches, the temperature of the air being 10° C. On withdrawing the thermometer from the water it began to fall rapidly, and in a few minutes reached a steady temperature. Columns I., II., and III. in the above table contain the results.

Column IV. of the table contains the depression of the wet thermometer, column V. the vapour pressure at the temperature of the wet bulb, column VI. the ratio of the vapour pressure to the depression.

I. Pressure Gauge Inches	II. Dry Thermometer ° C.	III. Wet Thermometer ° C.	IV. Depression.	V. M.V.P. at Temperature of Wet Bulb, mm.	VI. $\frac{\text{M.V.P.}}{\text{Depression}}$	VII. $\sqrt{\frac{\text{New Total Pressure}}{\text{Original Pressure}}}$	VIII. $\frac{\text{Col. VI.}}{1{\cdot}575} \times \frac{1}{\text{Col. VII}}$
30·2	10·0°	5·0°	5·0	6·507	1·301	1	·00083
15·1	9·44	2·77	6·67	5·570	0·835	$\frac{1}{1{\cdot}41}$	·00075
7·5	9·44	1·11	8·33	4·948	0·594	$\frac{1}{2}$	·00075
3·7	9·72	0·277	10·0	4·494	0·4494	$\frac{1}{2{\cdot}86}$	·00082
1·8	9·7	– 2·00	11·7	3·950	0·3376	$\frac{1}{4{\cdot}1}$	·00088

---

[1] *Vide* Maxwell's *Heat*, p. 313, 3rd edition.

We may take formula (3) in the form

$$\frac{p_1}{\theta_0-\theta_1}=\frac{\mathrm{P}}{\mathrm{L}\sigma\rho}\left(\frac{k+r\mathrm{R}}{\mathrm{D}}\right),$$

where k is the calorimetric conductivity, which is independent of pressure. In this formula P/σ occurs and this is constant for all pressures. $\mathrm{P}/\mathrm{L}\sigma\rho$ is equal to 1575. The diffusion constant will vary inversely as the square roots of the total pressures. In column VIII. of the table the value is given of $(k+r\mathrm{R})/\mathrm{D}$ calculated for each pressure. It will be observed that the value is nearly a constant as the theory indicates.

§ (6) THE WET BULB THERMOMETER IN MOVING AIR.—We shall now pass to the consideration of the case in which the air is in motion, that is when the thermometers are exposed to wind. Here we shall avail ourselves of a theory privately communicated by Major G. I. Taylor. The theory takes account of the fact that when wind is blowing over a flat surface (*Fig.* 7) there is against the surface a region free from eddies in which the stream lines are parallel to the surface, and beyond there is a region of turbulence in which eddies effect the transference of heat and water-vapour. The thickness of the eddy-free layer is shown by Major Taylor to be equal to $41/\mathrm{V}_m$ where V_m is the mean velocity. The velocity at the limit of the eddy-free layer is $\cdot 56\mathrm{V}_m$. Major Taylor has shown that in air about one-half of the fall of temperature between the surface and the stream of air occurs in the eddy-free layer.

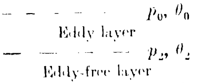


FIG. 7.

The rate at which the interchanges occur in the eddy-layer will be

$$\mathrm{A}\rho\mathrm{S}_p(\theta_0-\theta_2)f(\mathrm{V})=\mathrm{A}(p_2-p_0)f(\mathrm{V})\left(\frac{\sigma\rho}{\mathrm{P}}\right)\mathrm{L}.$$

$$\therefore p_2-p_0=\frac{\mathrm{PS}_p}{\mathrm{L}\sigma}(\theta_0-\theta_2). \qquad . \qquad (4)$$

And in the eddy-free layer

$$\mathrm{D}\left(\frac{p_1-p_2}{\mathrm{P}}\right)\sigma\rho\mathrm{L}=k(\theta_2-\theta_1) \qquad . \qquad (5)$$

or

$$p_1-p_2=\frac{\mathrm{P}}{\mathrm{L}\sigma}\cdot\frac{k}{\mathrm{D}\rho}(\theta_2-\theta_1).$$

If $k/\mathrm{D}\rho=\mathrm{S}_p$, we may combine the effects in the two layers.

$$p_1-p_0=\frac{\mathrm{PS}_p}{\mathrm{L}\sigma}(\theta_0-\theta_1). \qquad . \qquad . \qquad (6)$$

This formula is the same as that deduced by August and Apjohn, who assumed that when the temperature of the wet bulb is stationary in a wind that the heat required to vaporise the water is given out by portions of the surrounding air in cooling to the temperature of the wet bulb, and that every portion thus cooled became saturated with water-vapour. The theory was accepted by Regnault as the basis of his work in his *Études sur l'Hygrométrie.* He found it unsatisfactory for the various conditions under which he worked, conditions which included almost still air as well as air in motion. It will be noticed that the formula (6) is the same as formula (3), based on Maxwell's theory for still air, with the exception that the term involving conduction, diffusion, and radiation is absent.

It appears that the formula (6) is true only when $k/\mathrm{D}\rho=\mathrm{S}_p$. In air the value of $k/\mathrm{D}\rho$ is ·000056/198 × ·001293 or 0·22, which is not far from the value of the specific heat of air at constant pressure, *i.e.* ·2375. In other gases $k/\mathrm{D}\rho$ may have a value which will not justify the simple method used above of combining the effects of the eddy-free and eddy layers.

§ (7) EXPERIMENTAL VERIFICATION.—We shall now test the formula by experiments in air and afterwards examine the behaviour of other gases.

A short test tube 2·3 cm. diameter and 10 cm. long was closed by a cork with three holes in it, through which passed an entrance glass tube leading to the bottom, an exit tube from near the top, and a thermometer graduated in 1/10ths of a degree. The wet bulb was made by wrapping one or two layers of linen gauze about the bulb of the thermometer and tying this in place with cotton thread. The wet thermometer was moistened by immersing it in a beaker of water before each experiment. To prevent heat from coming from the outside to this apparatus it was enclosed in a larger vacuum-jacketed test tube, and the intervening space was plugged with cotton wool. In some experiments this vacuum-jacket was dispensed with, for the complete experiment only lasts about four or five minutes, and not much heat can get in in that short time. The exit tube was connected to the suction nozzle of a Lennox electrical blower so that a rapid draught could be maintained through the apparatus. The entrance tube was connected to a tall tower of pumice saturated with strong sulphuric acid in order to give a supply of dry air.

With dry air the following readings were obtained:

Dry Air. ° C.	Wet Bulb. ° C.	V.P. at the Temperature of Wet Bulb divided by the Depression of Wet Bulb.
15·15	3·1	·477
15·1	2·8	·472
18·2	4·7	·460
16·7	4·0	·474
17·9	4·7	·483
18·2	4·9	·476
		Mean 0·474

The pressure in the apparatus during the passage of the air stream was determined by substituting for the thermometer a pressure gauge containing mercury. This showed that the pressure inside was 25 mm. less than atmospheric when the motor was working at its highest speed. Therefore the theoretical constant is 735 × 0·2375/603 × 0·622, or 0·473, which is in good agreement with the experimental result.

The velocity of the stream of air was ascertained by means of an air meter (Negretti & Zambra) fitted to the side of a box from which the air was drawn. The velocity of the air was found to be 86 feet per minute, and this multiplied by the ratio of the square of the diameter of the aperture of the air meter to the square of the diameter of the tube gave 3·92 metres per second for the velocity of the air passing over the wet bulb. Further experiments were made with hydrogen and with carbon dioxide, to which we have not space to refer, but the results were in agreement with the theory.

Some experiments were made with the same apparatus to measure the amount of aqueous vapour present in air. For this purpose the apparatus for delivering dry air was removed and the air from the room drawn in with the same velocity. At the same time observations were made with Regnault's dew-point apparatus to determine the dew-point and thus obtain the true aqueous pressure in the air.

Dew-point by Regnault's Method.		Air Temperature.	Wet Bulb.	Calculated Aqueous Pressure using 0·47.	Difference.
Temperature.	Pressure.				
	mm.			mm.	
+6·1	7·081	16·1°	11·9°	7·194	+0·113
0·9	4·280	11·4	7·2	4·184	0·096
−1·48	4·097	15·9	8·75	4·294	+0·197
−0·7	4·343	13·3	7·5	4·659	+0·316
				Mean	+0·132

§ (8) VALUES OF THE CONSTANT FOR DIFFERING WIND SPEEDS.—The meteorologist requires to know the constant by which the observed depression of the wet bulb must be multiplied for any degree of ventilation. The experiments which have been described deal with the limiting conditions, namely when there is no wind, and when the wind is sufficiently great to cause the maximum depression. The ventilation may lie between these two limits, and a curve may be given, founded on the results of other experimenters, which must represent very approximately the true relation between the constant and the wind velocity.

Birkeland has given values of the constant for observed wind velocities. When these are plotted with our results for limiting values, such a curve can be drawn, and is given in *Fig.* 8. The value of Pernter's constants are in general agreement with this curve.

Consequently to use this curve the observer must find the velocity of the air blowing over the wet and dry bulbs. The corresponding value of the constant to be used in the theoretical formula $p_0 - p_1 = \text{constant}\ (\theta_0 - \theta_1)$ is then found from the curve; p_1, the M.V.P. corresponding to the temperature θ_1° C. of the wet bulb, is found from the table of M.V.P., and θ_0 is the temperature of the air. It is assumed that the pressure of the air is approximately normal, *i.e.* 760 mm., otherwise a correction should be made.

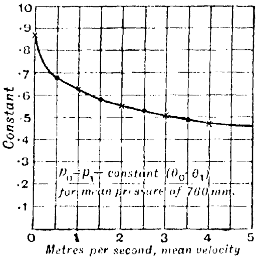


FIG. 8.

§ (9) ASSMANN'S HYGROMETER.—In Assmann's portable psychrometer, shown in *Fig.* 9, use is made of the principle of ventilation. The two thermometers are arranged in metal tubes, through which an artificial current of air is drawn by a clock-work fan. As the fan will give a constant ventilation the appropriate constant may be obtained from the diagram above, when the value of the current is known; or the constant may be obtained by direct comparison with a condensation instrument. It would appear that the fan is usually adjusted to give a velocity between 2 and 3 metres per sec.

The principle is applied in a simpler way in the "sling" psychrometer (*Fig.* 9A). The two thermometers are whirled at the end of an arm in air until the readings of both the wet and dry bulbs are constant. If the number of turns per second and the radius of the whirl are known, we can calculate the relative motion of the air, and then from the curve (*Fig.* 8) the constant may be obtained.

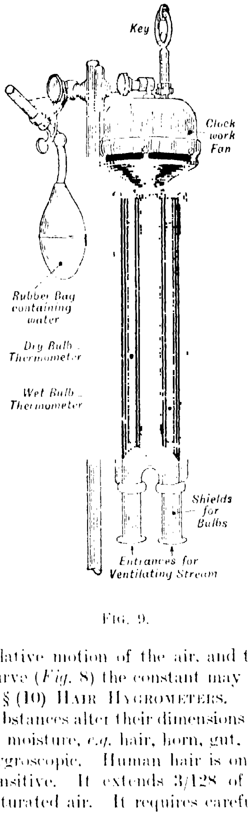


FIG. 9.

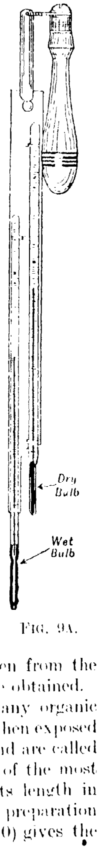


FIG. 9A.

§ (10) HAIR HYGROMETERS. Many organic substances alter their dimensions when exposed to moisture, *e.g.* hair, horn, gut, and are called hygroscopic. Human hair is one of the most sensitive. It extends 3/128 of its length in saturated air. It requires careful preparation to remove oil. The curve (*Fig.* 10) gives the average relation between the percentage humidity and the percentage of the extension for hair. If the hair is wetted with pure water with a brush, and allowed to remain wet for half an hour, the indicator should show 95 per cent humidity. Various methods of magnifying the extension have been used. They usually take the form shown in *Fig.* 11 of a dial with a pointer, which is moved by the contraction of the hair. The dial is graduated to show percentage humidity, or they may be arranged to record the humidity changes on a chart. Recording hygrometers have been constructed with bundles of hair. Such instruments appear to respond quickly to changes of humidity, but they are said to be very uncertain in their readings. Reference should be made to the Physical Society's Discussion[1] on Hygrometry, and specially to Dr. Ezer Griffiths' experiments with the Hair Hygrometer.

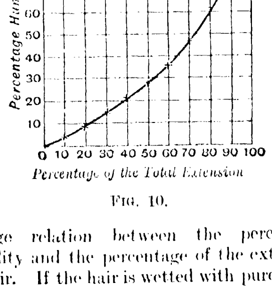


FIG. 10.

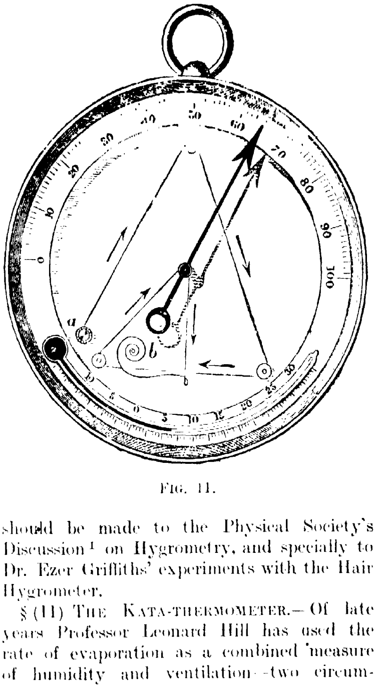


FIG. 11.

§ (11) THE KATA-THERMOMETER.—Of late years Professor Leonard Hill has used the rate of evaporation as a combined measure of humidity and ventilation—two circumstances on which the pleasantness of the air as regards life depend. He calls his instruments the dry and wet Kata-thermometers. They are exposed to the air at a temperature of about 110° F. and the time is taken for a cooling down (κατά) from 100° F. to 95° F. The average 97°·5 F. is near 98°·4 F.—the normal temperature of the human body.

Each Kata-thermometer (*Fig.* 12) has a cylindrical bulb of about 25 sq. cm. surface filled with spirit. The wet bulb is covered with a finger-stall of lisle thread glove. When a reading is to be made the observer immerses the dry Kata in a thermos flask of hot water, wipes it dry, and takes the time of cooling. A similar observation without wiping is then made with the wet Kata. By means of the constant for each instrument marked on it, the heat in millicalories lost per sec. per sq. cm. is calculated. The result with the dry thermometer gives a measure of the loss by radiation and by ventilation (convection), whilst that with the wet Kata gives the same quantity together with the further loss due to evaporation. It is assumed that the two thermometers are exactly similar in all respects other than that one is wet. This is hardly justified, as owing to the rather thick cover used for the wet Kata its area is larger. Also in other respects they are dissimilar; one has a non-conducting coat with a rough surface. Consequently it is not likely that calculations founded on physical constants will agree with experiment.

---

[1] *Proc. Phys. Soc.* xxxiv. pp. v-xcii.

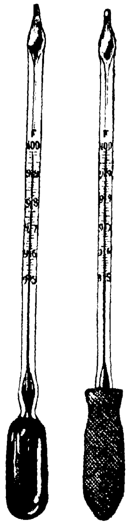


FIG. 12.

Owing to the warmth of the cooling Kata, there is always some convection even when no wind is blowing. A calculation by **Maxwell's** formula for still air shows the heat lost by a dry Kata would only be about half that found by Hill in his experiment in a closed chamber, whilst a wet Kata loses three times the heat the theory requires. When the Katas are used in a current of air using the formulae for heat loss in moving air, we find similar divergences between theory and experiment.

The instrument may, however, be used to obtain useful information about various climates and factory conditions. Professor Hill has written a report on the subject to which the reader interested in hygiene must be referred.[1]

§ (12) THE GRAVIMETRIC METHOD.—The determination of the mass of water-vapour in a given volume of air forms the most accurate method of finding the pressure of vapour in the air (Brunner, 1840), and this method has been used as the standard against which the other methods have been tested. The method was used by W. Napier Shaw[2] to test the fixed points of thermometers between 10° C. and 20° C. assuming Regnault's measures of the M.V.P. of water.

We shall quote a description of the method from Glazebrook and Shaw's *Practical Physics*, § 42, 1885 edition.

The arrangement of the apparatus, the whole of which can be put together in any laboratory, will be understood by *Fig.* 13. As aspirator we may use any large bottle, **A**, having, besides a thermometer, two tubes passing airtight through its cork and down to the bottom of the bottle. One of these tubes is bent as a syphon and allows the water to run out, the flow being regulated by the pinch-cock T; the other tube is for the air to enter the aspirator; its opening being at the bottom of the vessel, the flow of air is maintained constant and independent of the level of the water in the bottle.

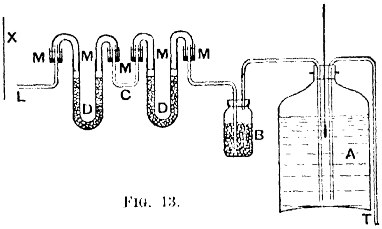


FIG. 13.

The vessel B, filled with fragments of freshly fused chloride of calcium, is provided with two tubes through an airtight cork, one connected with the aspirator passing just through, and the other connected with the drying tube D to the bottom of the vessel. This serves as a valve to prevent any moisture reaching the tubes from the aspirator. The most convenient way of connecting up drying tubes is by means of mercury cups, consisting of short glass tubes with a cork bottom perforated for a narrow tube; over this passes one limb of an inverted U-tube, the other limb of which is secured to one limb of the drying tube either by an india-rubber washer with paraffin or, still better, by being thickened and ground as a stopper. A glance at the figure will show the arrangement. The drying tubes can then be removed and replaced with facility, and a perfectly airtight connection ensured. The space in the little cups M, M, M, M, between the narrow tubes and the limbs of the inverted U's is closed by mercury. Care must be taken to close the ends of the inverted U's with small bungs during weighing, and to see that no globules of mercury are adhering to the glass. The connecting tubes C between the drying tubes should be glass and as short as possible.

Two drying tubes must be used, and weighed separately before and after the experiment; the first will, when in good order, entirely absorb the moisture, but if the air is passed with too great rapidity, or if the acid had become too dilute by continued use, the second tube will make the fact apparent. A thermometer X to determine the temperature of the air passing into the tubes is also necessary.

---

[1] *The Science of Ventilation and Open-air Treatment*, part i., His Majesty's Stationery Office, 1919.

[2] *Camb. Phil. Trans.* xiv. part i.

To take an observation, the tubes are weighed and placed in position, the vessel A filled with water, the syphon tube filled, and the tube at the end of the drying tubes closed by means of a pinch-tap. Then, on opening the tap at T, no water should flow out; if any does, there is some leak in the apparatus which must be made tight before proceeding further. When assured that any air supplied to the aspirator will pass through the drying tubes the observation may be begun. The water is run out slowly (at about the rate of 1 litre in ten minutes) into a litre flask, and when the latter is filled up to the scratch on the neck it is removed and weighed, its place being taken by another flask, which can go on filling during the weighing of the first. This is repeated until the aspirator is empty, when, the weight of the empty flasks being ascertained, the total weight of water thus replaced by air can be found. The height H of the barometer must be determined at the beginning and end of the experiment. During the observation the thermometer X must be read every ten minutes, and the mean of the readings taken as the temperature *t* of the entering air; the thermometer in the aspirator must be read at the end of the experiment; let the reading be *t*. If the aspirator A is but small, it can be refilled and the experiment repeated.

Let *w* be the increase of weight of the tubes D, D, and V the volume of the aspirator at its final temperature T′, B the barometer pressure, *e* the pressure of water-vapour to be found, E the M.V.P. at T′, *ρ* the density of dry air at 0° C. and 760 mm. to the temperature of the moist air, *σ* the specific gravity of steam referred to dry air at the same temperature and pressure. Then it may be shown that

$$\frac{e}{B-e}=\frac{760}{\rho\sigma}\cdot\frac{w\,(1+\alpha T)}{V\quad B-E}.$$

The drawback to this method is that it takes some time, and gives only an average value of *e* during the experiment.

§ (13) THE VOLUMETRIC **METHOD**.—The volume of water-vapour in the air may be ascertained by absorption with a drying agent such as strong sulphuric acid or phosphorus pentoxide. Let the volume absorbed at a pressure P be *v*, and let the whole volume of the moist air at the same pressure be V. Then by Dalton's law the fraction *v*/V of P represents the partial pressure of the water-vapour. Various forms of apparatus, some portable for use in the open air, have been devised, notably that of Schwackhöfer. These are described in the treatises on Meteorology.

(i.) *Norman Shaw's Apparatus.*—We shall describe an improved form of the apparatus, due to W. Norman Shaw,[1] which has a compensating bulb which makes it independent of small variations of temperature and pressure. Its readings were found to be in close agreement with those obtained by the gravimetric absorption method. It consists of four equal bulbs A, B, C, D, joined as in *Fig.* 14. E is a three-way cock, and F and G two taps. C is the measuring bulb with a narrow graduated tube sealed in to its base. H and K are movable mercury reservoirs. In the limb at M is a light non-volatile liquid, high boiling-point paraffin being suitable. This gauge acts as an indicator of the equality of pressure in the bulbs C and D. The apparatus is used thus. The three-way cock E is set so that the air to be tested can be drawn in by lowering the mercury reservoir K to some division on the graduated portion, the taps F and G being open. The taps F and G are now closed. The three-way cock is now set so that the moist air can be passed into the acid chamber A, and this is done by manipulating the reservoirs K and H. After a few minutes, depending on the size of the apparatus, the absorption of the water-vapour by the acid is complete. Then the gas is passed back into the measuring bulb. The acid must not be raised above some mark near the three-way cock, and on no account must be allowed to get into the tube through which the air is drawn in. After the air has been passed into C, the tap F is opened and K is adjusted until the liquid in M is at the same level on both sides of the gauge. The diminution of volume read on the graduated tube gives the volume of water-vapour in the original volume of moist air.

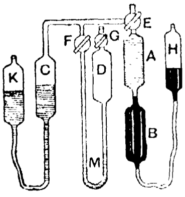


FIG. 14.

It will be obvious how the bulb D acts as a compensator for small pressure and temperature changes. Of course the usual precautions taken in gas analysis should be observed. W. Norman Shaw gives a table to show the satisfactory performance of his apparatus in comparison with other hygrometric methods.

(ii.) *Professor Tyndall's Apparatus.*—Another instrument[2] which depends on the measurement of the change of pressure produced by drying a closed volume of the air under test has been devised lately by Professor Tyndall and the late Mr. Mayo. The air is contained in a brass tube ordinarily open at both ends (*Fig.* 15); the tube is connected to a pressure gauge.

---

[1] *Trans. Roy. Soc. Canada*, 1916.

[2] *Proc. Phys. Soc.* xxxiv. p. lxvii.

A hollow brass plunger with perforated ends fits loosely in the tube and can be made to slide from end to end by tilting it. The plunger is loosely packed with glass-wool which has been previously dipped in powdered phosphorus pentoxide. No P_2O_5 is placed on the glass-wool at the ends of the piston, and its sides are protected by a glass sleeve. To use the instrument the ends of the cylinder are securely closed by close-fitting plugs, and by means of a tap the pressure inside is brought to the same value as that of the outside air; the tap is closed and the cylinder tilted two or three times backwards and forwards. The air is dried by its passage through the glass-wool and the fall in pressure measured on the gauge. From this and a knowledge of the relative volumes of the wet air originally in the cylinder and the dry air in the plunger the humidity can be calculated.

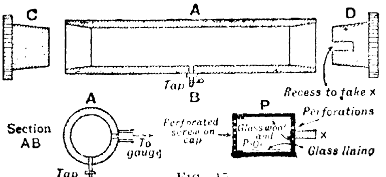


FIG. 15.

§ (14) HUMIDITY AND HYGROSCOPIC SUBSTANCES.—To understand the influence of humidity on manufactures it is necessary to consider in some detail the properties of hygroscopic substances. Let us begin with the cases of wool and cotton. What will be said about these substances will apply in a general way to all vegetable and animal products. The case of wool has been examined carefully by Professor F. T. Trouton.[1] He exposed carefully dried flannel to water-vapour, and measured the amount of water absorbed. Between 4°·8 C. and 18·2 C. he found that the weight of water absorbed depended on the relative humidity only, and not on the temperature. This result is in accordance with theory; and we may regard this law as definitely established. As to the weight absorbed when the flannel was exposed to atmospheres of varying humidity, he found that from 100 per cent humidity down to 47·4 per cent the weight W absorbed at a certain percentage humidity was given by the formula $(W_1 - W)^2 = \text{constant} \times (100 - \text{percentage humidity})$, where W_1 is the weight absorbed in a saturated atmosphere. In other words, the relationship between the humidity and weight of water held was parabolic. It should be noted that the experiments do not extend to low humidities, and that the dry flannel was brought to its equilibrium condition by absorption.

As regards cotton the most recent and careful experiments are due to Orme Masson.[2] Masson's earlier work was concerned with the change of temperature when dry cotton-wool was placed in saturated air. We have already noted that cotton heats itself when absorbing moisture. The later experiments were devoted to ascertaining the weight absorbed by initially dry cotton in air of a given humidity, or the weight which remains in initially saturated cotton when exposed to the same humidity. The equilibrium state in each case is reached slowly; in fact, so slowly that Masson did not wait for what he supposed would be an identical state by either process. He contented himself by finding the average between the results reached in a certain time by absorption and by evaporation. Cellulose in the form of filter paper was also examined. Below (*Fig.* 16) is shown in a diagram the result of plotting separately the results by evaporation and by absorption. Masson's curve as the mean would come between the two curves. It is doubtful whether he was justified in taking the mean, as it is possible that the equilibrium reached by the two methods might be different. There are other instances of different equilibria when dealing with other properties of collodial substances. For the manufacturer, as there is not much difference between the two curves, the mean will probably suffice. Curves similar to the mean curve of Masson had been obtained by Schloesing (1893), Hartshorne (1905), and others for cotton, hemp, flax, jute, and wool.

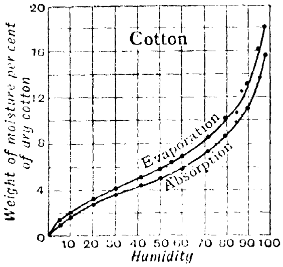


FIG. 16.

The effect of humidity is to prevent evaporation of the moisture in the material, and to prevent electrification by friction during the processes of spinning and weaving. Experiment has shown that cotton and flax are stronger when they have absorbed moisture, and that there is in consequence less chance of breaking the threads; whilst wool apparently becomes slightly weaker. The conductivity for electricity is increased with the amount of absorbed water and any electricity generated can escape readily. Microscopic examination of the threads spun in an atmosphere of high relative humidity shows a more compact structure. For the purpose of raising the relative humidity processes of adding moisture to the air in the workrooms are resorted to, and the air is said to be "conditioned."

---

[1] *Proc. R.S.*, 1906, lxxvii. 292.

[2] *Proc. R.S.* lxxiv. 230, and 1907, lxxviii. 412.

§ (15) HUMIDITY AND VENTILATION.—Water-vapour may be added to the air by two methods: (1) by introducing liquid water either in bulk or in fine drops and allowing it to evaporate, and (2) by sending in jets of steam. The methods in use belong either to one class or are mixed processes belonging to both. Very often systems of ventilation and heating are combined with the operation of humidifying. The climate of the place and the health of the operatives must be taken into consideration. It is not surprising, therefore, that with varying conditions many different processes have been advocated and that the question has been the subject of much legislation.

We shall now give a brief general discussion of the two methods of introducing moisture. In (1) pure cold water is introduced, preferably in small drops. These drops form a cloud which fades away by evaporation. There will be a cooling effect, just as the wet bulb is cooled, and as there is convection the law for the wet bulb $p_1 - p_0 = (\mathrm{PS}_p/\mathrm{L}\sigma)(\theta_0 - \theta_1)$ will determine the exchange of water-vapour and of heat. The effect is analogous to the cooling caused by rain on a sultry day, or to that of a fountain playing in a warm room. The cooling is due to the absorption of the latent heat of evaporation.

In (2) steam is introduced and mixed with the air. If the steam is entering at low pressure it will behave like steam coming from the spout of a kettle. For a short distance from the nozzle it remains gaseous and invisible. Then condensation occurs to a cloud of drops, which may fall as rain or may fade away by evaporation, and provided there is no net condensation we may reach a similar state of humidity with the important difference that we have brought into the chamber the total heat of steam from outside, and have not taken it from the air in the chamber.

Now it would appear at once that in a hot climate the first process has the advantage, whereas in a cold climate the advantage might be with the second. Summer and winter may likewise change the advantage; so too might a change of wind from east to west.

The *Second Report of the Departmental Committee on Humidity and Ventilation in Cotton Weaving Sheds* of the Home Office, 1911 (Cd. 5566), deals with the conditions, etc., in the trade. It contains an Appendix by Sir Henry Cunynghame on Hygrometers. He describes two differential hygroscopes and gives recommendations as to the placing of hygrometers. He draws attention to the influence of draughts, and the position in the weaving shed on the readings of the wet and dry bulb hygrometer.

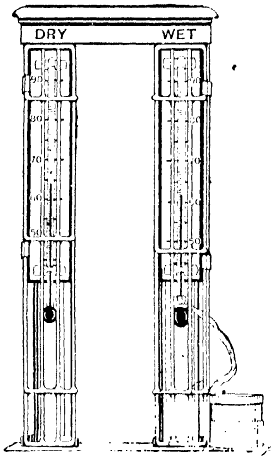


FIG. 17.

A standard form (*Fig.* 17) of the latter to be used by weavers is described in the Report.

§ (16) HUMIDIFIERS.—We shall now describe very briefly a few methods adopted in factories for humidifying the air, commencing with two methods in which cold water is "atomised."

(i.) *Mather and Platt's Vortex System.*—This consists in placing at intervals, and at a convenient height above each floor, a number

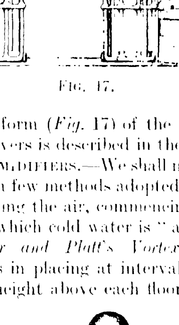


FIG. 18.

of cylinders (*Fig.* 18), and connecting them by piping to a pump, by which they are supplied with water under pressure. The cylinders are the humidifiers proper, and are so constructed and connected that all the water not actually diffused—as exceedingly fine spray—into the atmosphere flows back to a central system of tanks to be filtered and screened before further service.

The tanks, two in number, are fixed at a slightly lower level than the humidifiers, to ensure the return to them of the whole of the surplus water from the machines.

The water from the pump passes, in the first instance, through a large filter on the main delivery pipe, thence along the distributing pipes to the self-cleansing filters, one of which is attached to the side of each humidifier; this final filtration catches any particles of dirt and fibre which may have escaped the main filter. The water now enters the humidifiers, and is expelled from an interior spraying nozzle in a jet at a pressure of about 135 lbs. per sq. in., and impinges immediately on the flat end of an adjustable hardened nickel pin. The result of the impact is that the jet of water splits into an on-rushing cone of fine spray, which, extending to the sides of the cylinder and moving at high velocity, creates a partial vacuum in the upper interior sufficient to induce a strong current of air to pass through the bath of water spray. This cold douche saturates and cools the air, and in addition removes from it much of the suspended dust and fibre; hence the air expelled from the lower part of the machine into the room is cooled and cleansed as well as humidified.

(ii.) *Smethurst System.*—In the Smethurst Air Fountain System (*Fig.* 19) a controlled thread of water under pressure is impinged upon by a jet of compressed air, which expanding breaks up the water, and the surrounding air absorbs it without precipitation at any of the temperatures experienced in the mills. The diagram serves to illustrate the action of the apparatus, which is very like the ordinary spraying bottle used for scent. The jets are distributed about the working room in order that a large bulk of air may be conditioned.

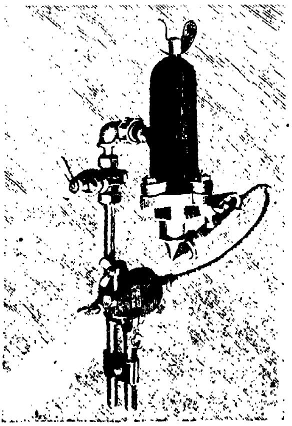
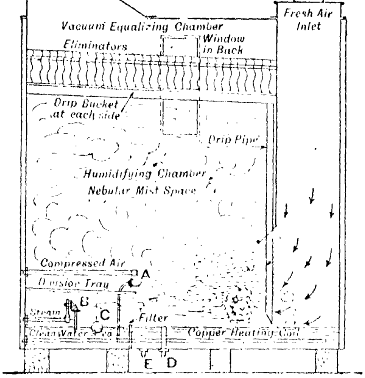


FIG. 19.

(iii.) *Howarth's Champion System.*—In this system (*Fig.* 20) the air, after cleaning and conditioning, is distributed through the workroom by graduated conduits. The air is treated in a special chamber by jets of steam and sprays of water. Any free water is removed by an eliminator. The diagram shows the course of the air and its treatment. The process obviously combines ventilation with conditioning.



A. Patent Air & Water Jets B. Compound Jets, Steam & Water
C. Ball Tap Water Supply D. Overflow E. Flush Out

FIG. 20.

§ (17) WATER-VAPOUR IN THE ATMOSPHERE.—In the lower portion of the atmosphere to a height of some 10 kilometres clouds exist, and various forms of precipitation occur. This portion is called the troposphere,[1] and in it the temperature falls from the surface upwards by the adiabatic law for a rising and expanding gas to a temperature of about 215° A. in January at 11 kilometres. In the upper part of the atmosphere, the stratosphere, up to some 40 kilometres the temperature remains about the same, 221° A. having been observed at 36 kilometres.

In the lower part where precipitation occurs the air may have any value of humidity varying from dry to saturation, but of course the absolute value of the density of the moisture will depend on the temperature, which, as we have seen, is in general falling as we ascend.

---

[1] See "Atmosphere, Physics of."

The following table shows the M.V.P. of ice at temperatures below 0° C.:

Temperature ° C. (Hydrogen Scale).	Temperature A°.	Pressure in mm.*
0°	273°	4·579
-10	263	1·974
-20	253	0·787
-30	243	0·292
-40	233	0·105
-50	223	0·034

* Scheel, *Verh. D. Phys. Ges.*, 1903, v.

S. S.

HYDROMETER:

Constant volume. See "Hydrometers," § (8).

Correction to readings for variations in temperature. See *ibid.* § (7).

Equilibrium of a floating. See *ibid.* § (3).

Metal. See *ibid.* § (16).

Specification for glass. See *ibid.* § (15).

Standardisation of. See *ibid.* § (12).

Testing of. See *ibid.* §§ (12) and (13).

HYDROMETERS

§ (1) GENERAL DISCUSSION. (i.) *Pattern of Hydrometer.*—The type of hydrometer which is in most common use is that shown in *Fig.* 1. It consists simply of a glass bulb A, below which is a smaller bulb B loaded with mercury or lead shot so that the instrument floats with its axis vertical. Above the bulb A is a glass tube C enclosing the scale of the hydrometer, and the whole instrument is hermetically sealed.

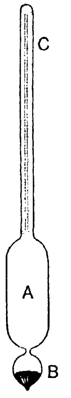


FIG. 1.

The essential function of such an instrument is the determination of the density of liquids. If we neglect for the moment the comparatively small effects due to the surface tension of the liquid in which the hydrometer is immersed, and to the buoyancy effect of the air surrounding the emergent portion of the stem, then the condition of equilibrium of the floating hydrometer is simply that the mass of liquid displaced up to the intersection of the level liquid surface with the stem of the hydrometer, is equal to the mass of the latter. The mass of the hydrometer is a constant. If it is being used at a definite temperature the volume up to any specific graduation mark on the stem is also constant. Thus the indication of the hydrometer is a direct measure of the mass of liquid contained in a certain definite volume, *i.e.* is directly proportional to the density of the liquid at the temperature at which it is being examined.

If the graduations are laid down on the stem of the hydrometer in such a way that the volumes up to consecutive marks increase in harmonical progression, then the distances between the graduations will correspond to equal increments in density.

This is theoretically the simplest, most fundamental, and scientifically the best method of graduating hydrometers. It has a considerable vogue on the Continent and has been officially adopted by the Meteorological Office in Great Britain.

(ii.) *Special Methods of Graduation.*[1]—In industry, however, the property of a liquid which it is desired to measure by means of a hydrometer is usually not the density, but some other more or less directly related property, frequently percentage composition. As a result of this many different methods of graduating hydrometers have been introduced from time to time. For very rough work there may sometimes be an advantage in a hydrometer graduated to indicate directly the particular property of the liquid in which the user is directly concerned, *e.g.* in a hydrometer graduated to indicate percentages by weight of sugar in a sugar solution. Such instruments, however, can only be accurate at one particular temperature, and in most cases the necessity arises sooner or later of using tables of correction in conjunction with the hydrometers. Once this necessity arises, it is equally easy and more satisfactory for all purposes to use a standard type of instrument, graduated to indicate densities.

The indication of the hydrometer is then a definite thing, depending solely on the temperature at which it is used, and, except for the slight effects of surface tension, independent of the influence of variations in the character of the liquid in which it is used.

(iii.) *Hydrometer Tables.*—The whole of the information which concerns the liquid should be incorporated in tables co-relating density at various temperatures with the property of the liquid with which one is concerned. It then becomes possible to prepare either the hydrometer or the tables independently of each other.

Otherwise the hydrometer maker has to assume certain properties of the liquid in which his instrument will be used, and neither he nor the user has any guarantee that the liquids do actually correspond to the assumptions made. But if a hydrometer indicating densities is employed the maker has a definite standard to aim at, and the user has only to assure himself that he has tables suitable to the particular liquids with which he is concerned.

Reliable data for constructing tables co-relating the density of certain liquids, in connection with which hydrometers are extensively used, with their density at various temperatures is already available. For example, there is the work of the Bureau of Standards on alcohol-water mixtures,[1] and that of the Reichsanstalt on sugar solutions.[2]

---

[1] See also "Alcoholometry" and "Saccharometry."

The tables should relate solely to the properties of the liquid concerned, and would then be of general application. Many existing hydrometer tables incorporate allowances for the expansion of the hydrometer and are therefore limited in application to hydrometers made of a particular material. For example, when glass Sikes hydrometers were adopted for revenue purposes in India it was necessary to reconstruct Sikes Tables which can only be true for the particular type of metal instrument for which they were originally constructed.

While hydrometers indicating densities are preferable to those indicating directly the property of the liquid in question, *e.g.* percentage composition, they have even greater advantages over hydrometers with arbitrary scales.

The disadvantages of hydrometers with arbitrary scales are well illustrated by the Baumé hydrometer. The originator of the Baumé scale was a hydrometer manufacturer who adopted the following basis for the scale of his instruments:

For liquids lighter than water the point at which the hydrometer floated in water was taken as 10 and that at which it floated in a 15 per cent salt solution as 0°. The distance between these two points was divided into equal lengths and the scale continued beyond the 10 point by similarly spaced graduation marks.

A second hydrometer for use in liquids heavier than water was constructed on the following basis:

Water was taken as the zero point, and the point at which the hydrometer floated in a 15 per cent salt solution was taken as 15°. The interval was divided equally as with the light hydrometer and the scale continued below the 15° point by equally spaced graduation marks.

Note that one instrument indicates 10° in water and the other 0°. However, Baumé's hydrometers became extensively used, and, as was only to be expected, the need arose for knowing the equivalent density corresponding to degrees on the Baumé scale. Consequently in an attempt to define the scale precisely formulae of the type

$$s=\frac{A}{B \pm n},$$

where s = the density equivalent to the Baumé reading n and A and B are constants, were suggested. The present writer found no less than five different formulae for the heavy hydrometer, and four for the light hydrometer, in a single reference book. The attempt to define the Baumé scale has thus simply resulted in further confusion.

Again, tables have been drawn up co-relating percentage compositions of various liquids, *e.g.* sugar solutions, against Baumé degrees. The tables are of necessity primarily based on the variation in density with composition, and in compiling them some relation between Baumé degrees and density had to be assumed. The particular assumption made is not always stated, and even where it is the user has rarely any guarantee that his particular Baumé[3] hydrometer was constructed on the same assumptions.

The advantage of using hydrometers indicating densities directly, whose basis is unequivocally defined, is obvious.

§ (2) **Specific Gravity Hydrometers.**—A particular class of hydrometer which is largely used and which differs but little in principle from the density hydrometer is the specific gravity hydrometer. In the former the readings give directly the mass per unit volume of the liquid, whereas in the latter they give the ratio of this quantity to the corresponding mass per unit volume of water at the same temperature; or, more simply, the ratio of the masses of equal volumes of the liquid and of water at the same temperature.

In the early days of hydrometers there was, no doubt, considerable advantage in this type of instrument, since the latter ratio is more susceptible of direct measurement than is true density. The density of distilled water at various temperatures has now, however, been most carefully investigated and tabulated,[4] and since the specific gravity of a liquid is merely the ratio of the density of the liquid to that of water at the same temperature, the one quantity is now as easily determined as the other. Specific gravity hydrometers have, therefore, no particular advantage over density hydrometers.

Again, there is a distinct tendency at the present time for the results of investigations concerning the specific gravity of liquids to be given relative to water at 4° C. as basis, instead of relative to water at the same temperature at which the liquid is investigated. Such specific gravities are of course identical with densities expressed in grammes per millilitre (gm. per ml.). It was formerly more customary to express the results relative to water at the same temperature as the liquid under investigation.

A replacement of specific gravity hydrometers by density hydrometers would be in agreement with the above tendency towards uniformity.

---

[1] *Bulletin of the Bureau of Standards*, ix. No. 3; see also *Circular of the Bureau of Standards*, 1916, No. 19.

[2] Plato, *Wiss. Abh. der Kaiserlichen Normal-Eichungs-Kommission*, 1900, ii. 140.

[3] Apropos of Baumé hydrometers the following quotation is not without interest: ". . . in the case of oil, the common expressions 'higher gravity' and 'lower gravity' have directly opposite meanings, depending on whether the specific gravity or Baumé gravity is referred to. The first time this came to the writer's attention was when some 37 years ago he heard two oil manufacturers spend much of an afternoon talking at cross purposes, because when one spoke of certain equipments giving a higher or lower gravity in the product, the speaker had in mind specific gravity, while the other, who was unable to agree with the views expressed, understood Baumé gravity to be meant." *Jour. of Ind. and Eng. Chem.*, June 12, 1920.

[4] P. Chappuis, *Trav. et Mém*, 1907, xiii.

There is a convention established in connection with specific gravity hydrometers which is worthy of notice. A liquid of specific gravity, say 1·035, is spoken of as being 1035° specific gravity. Since the third decimal place is the last which is of significance in many cases, the use of "degrees" with the accompanying suppression of the decimal point is often a convenience.

If densities are expressed in gm. per litre the need for introducing the decimal point similarly disappears.

§ (3) GENERAL EQUATION OF EQUILIBRIUM OF A FLOATING HYDROMETER.—Consider a hydrometer floating stationary in a liquid and having its stem partially submerged.

Let M gm. = mass of the hydrometer *in vacuo*,
V c.c. = volume of the submerged portion of the hydrometer,
v c.c. = volume of the portion of the hydrometer stem not submerged,
ρ gm./c.c. = the density of the liquid in which the hydrometer is floating,
σ gm./c.c. = the density of the air,
T dynes/cm. = the surface tension of the liquid,
α = angle of contact of the liquid surface and the hydrometer stem,
d cm. = diameter of the stem of the hydrometer.

The forces acting vertically downward on the hydrometer are

$$\mathrm{M}g+\pi d\mathrm{T}\cos\alpha,$$

and those acting upward

$$\mathrm{V}\rho g+v\sigma g+\frac{\pi d\mathrm{T}\cos\alpha}{g\rho}\sigma g.$$

Equating these we have

$$\mathrm{M}+\frac{\pi d\mathrm{T}}{g}\cos\alpha=\mathrm{V}\rho+v\sigma+\frac{\pi d\mathrm{T}}{g\rho}\cos\alpha\sigma.$$

When the liquid wets the stem of the hydrometer, and reliable readings cannot be obtained unless this is the case, the angle α vanishes and $\cos\alpha$ is unity; we have then

$$\mathrm{M}+\frac{\pi d\mathrm{T}}{g}=\mathrm{V}\rho+\sigma\left[v+\frac{\pi d\mathrm{T}}{g\rho}\right].$$

For many purposes the smaller terms of the above equation may be neglected and the simple relation

$$\mathrm{M}=\mathrm{V}\rho$$

be used.

§ (4) THE SCALE OF A DENSITY HYDROMETER.—Let D be the density corresponding to the highest graduation mark of a hydrometer, and let the graduation marks represent equal increments of density.

Consider any three adjacent graduation marks corresponding to densities of, say, $\mathrm{D}+nd$, $\mathrm{D}+(n+1)d$, and $\mathrm{D}+(n+2)d$ respectively, d being the increase in density corresponding to each subdivision, and $\mathrm{D}+nd$ being therefore the nth graduation mark from D.

Let V be the volume of the hydrometer submerged when the reading is D, $\mathrm{V}n$ similarly corresponding to $\mathrm{D}+nd$, etc. Then if M is the mass of the hydrometer, we have

$$\mathrm{M}=\mathrm{VD}=\mathrm{V}_n(\mathrm{D}+nd)=\mathrm{V}_{n+1}[\mathrm{D}+(n+1)d]$$
$$=\mathrm{V}_{n+2}[\mathrm{D}+(n+2)d],$$

whence

$$\frac{1}{\mathrm{V}_n}=\frac{\mathrm{D}+nd}{\mathrm{VD}},$$

$$\frac{1}{\mathrm{V}_{n+1}}=\frac{\mathrm{D}+(n+1)d}{\mathrm{VD}},$$

$$\frac{1}{\mathrm{V}_{n+2}}=\frac{\mathrm{D}+(n+2)d}{\mathrm{VD}},$$

and therefore

$$\frac{1}{\mathrm{V}_n}-\frac{1}{\mathrm{V}_{n+1}}=-\frac{d}{\mathrm{V}d}=\frac{1}{\mathrm{V}_{n+1}}-\frac{1}{\mathrm{V}_{n+2}},$$

i.e.

$$\frac{1}{\mathrm{V}_n}-\frac{1}{\mathrm{V}_{n+1}}=\frac{1}{\mathrm{V}_{n+1}}-\frac{1}{\mathrm{V}_{n+2}}.$$

That is, the volumes up to successive graduation marks increase in harmonic progression. The same relation obviously holds for hydrometers graduated to indicate specific gravities.

It follows from the above relation that

$$\mathrm{V}_n-\mathrm{V}_{n+1}=\frac{\mathrm{V}_n}{\mathrm{V}_{n+2}}(\mathrm{V}_{n+1}-\mathrm{V}_{n+2}),$$

i.e.

$$\mathrm{V}_n-\mathrm{V}_{n+1}>\mathrm{V}_{n+1}-\mathrm{V}_{n+2}.$$

That is, the graduation marks corresponding to equal increments in density become progressively more closely spaced towards the lower end of the scale.

The exact spacing of the graduation marks on a density hydrometer may be readily determined, assuming the stem to be of uniform cross-section.

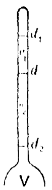


FIG. 2.

Let d_1 = the density corresponding to the highest graduation mark on the stem (*Fig.* 2),
d_2 = the density corresponding to the lowest graduation mark,
d = any intermediate density,
v_1 = volume of the portion of the stem between d and d_1,
v_2 = volume of the portion of the stem between d and d_2,
V = volume of hydrometer below d_2.
M = mass of hydrometer.

Then

$$\mathrm{M}=d_2\mathrm{V}=d(\mathrm{V}+v_2)=d_1(\mathrm{V}+v_2+v_1).$$

Eliminating M and V we obtain

$$\frac{v_1}{v_2+v_1}=\frac{d_2}{d}\cdot\frac{d-d_1}{d_2-d_1}$$

and

$$\frac{v_2}{v_2+v_1}=\frac{d_1}{d}\cdot\frac{d_2-d}{d_2-d_1}.$$

Assuming the stem to be of uniform cross section, we may substitute L for v_1+v_2, l for v_1, and l' for v_2 in the above relations,

where L = distance between the marks d_1 and d_2.
l = distance between the marks d and d_1.
l' = distance between the marks d and d_2.

We thus obtain

$$l=\mathrm{L}\,.\,\frac{d_2}{d}\,.\,\frac{d-d_1}{d_2-d_1},$$

and

$$l'=\mathrm{L}\,.\,\frac{d_1}{d}\,.\,\frac{d_2-d}{d_2-d_1}.$$

Either of the above relations serve to locate the position of the graduation mark corresponding to the density d, the distance apart, L, of d_1 and d_2 being known. It is obvious that, given the densities corresponding to any two points on a hydrometer and their distance apart, similar relations may be deduced to locate any density included in the range of the instrument, whether this density is intermediate between the two known densities or outside them.

By means of the first of the above equations we may calculate the following table for two hydrometer scales ranging respectively from 1·000 to 1·200 and 1·800 to 2·000, assuming the total scale length to be 200 mm. in each case :

Length 1·000 to 1·050 = 57·1 mm.
" 1·000 " 1·100 = 109·1 "
" 1·000 " 1·150 = 156·5 "
" 1·000 " 1·200 = 200·0 "
" 1·800 " 1·850 = 54·1 "
" 1·800 " 1·900 = 105·3 "
" 1·800 " 1·950 = 153·9 "
" 1·800 " 2·000 = 200·0 "

A complete master scale giving the distance of each graduation mark from the highest mark can of course be drawn up in the same manner as above. The above table serves to show to what extent the scales of the two hydrometers, each covering a range of density equal to 0·200, differ from each other.

It might be noted in passing that a scale ranging from, say, 0·900 to 1·000 and divided into intervals corresponding to 0·001 would be identical with one ranging from 1·800 to 2·000 and divided into intervals corresponding to 0·002, provided that the over-all lengths were the same. By the application of this fact the use of liquids of high density, which are often unpleasant to handle, may be minimised in the construction of hydrometers indicating high densities.

An interesting result follows immediately from the relation

$$l=\mathrm{L}\,.\,\frac{d_2}{d}\,.\,\frac{d-d_1}{d_2-d_1}.$$

Suppose that d_1 and d_2 be two chosen densities and that the distance between these points is divided into N equal parts. Let the density d be n divisions from d_1. Then clearly

$$n=\mathrm{N}\,.\,\frac{d_2}{d}\,.\,\frac{d-d_1}{d_2-d_1}$$

or

$$n=\mathrm{N}\,.\,\frac{d_2}{d_2-d_1}\left(1-\frac{d_1}{d}\right).$$

If N, d_1, and d_2 are assigned definite values we may write

$$n=\mathrm{B}-\frac{\mathrm{A}}{d},$$

B and A being constants equal to $\mathrm{N}d_2/(d_2-d_1)$ and $\mathrm{N}d_2d_1/(d_2-d_1)$ respectively.

We may rewrite the above equation thus

$$d=\frac{\mathrm{A}}{\mathrm{B}-n},$$

which is the familiar form of equation used to express the relation between degrees on arbitrary equally spaced hydrometer scales and their corresponding densities.

§ (5) Dimensions of the Stem.—Closely related to the problem of hydrometer scales is the question of choosing suitable stems for bulbs of given displacement and *vice versa*. If M is the mass of a hydrometer, V the volume submerged when it is reading d_1, d_1 being the density corresponding to the lowest graduation mark, and v the volume of the stem between the highest and lowest graduation marks, then we have

$$\mathrm{M}=d_1\mathrm{V}=d_0(\mathrm{V}+v),$$

d_0 being the density corresponding to the highest graduation mark. From the above equations it follows that

$$\mathrm{V}=\frac{d_0}{d_1-d_0}v \quad \text{or } v=\frac{d_1-d_0}{d_0}\mathrm{V}.$$

Consider two hydrometers, one of range 1·000 to 1·050, and the other 1·800 to 1·850. In the first case

$$\mathrm{V}=20v \quad \text{or } v=\tfrac{1}{20}\mathrm{V},$$

and in the second case

$$\mathrm{V}=36v \quad \text{or } v=\tfrac{1}{36}\mathrm{V}.$$

Consequently, if bulbs of equal displacement were used for the two hydrometers of the above ranges, and the stems were equal in diameter, then it follows that the distance between the 1·800 and 1·850 marks would be only $\frac{20}{36}$ths of that between the 1·000 and 1·050 marks. If the same openness of scale were desired in the two instruments the bulb of the 1·800 to 1·850 hydrometer would have to be $\frac{36}{20}$ times as large as the bulb of the 1·000 to 1·050 hydrometer, provided the same diameter of stem were used in each case. On the other hand, if bulbs of equal size were used, then to give an equal over-all length for each scale the diameter of the stem of the 1·800 to 1·850 hydrometer would have to be $\sqrt{\frac{20}{36}}$, *i.e.* 0·745 times the diameter of the stem of the 1·000 to 1·050 hydrometer.

§ (6) Standard Temperature for Hydrometers.—Owing to the changes in volume arising from temperature variations in the material of a hydrometer, the indications of the instrument correspond to varying densities at different temperatures. Hence it is necessary to specify the temperature at which a hydrometer is to be used. This temperature we will term the "standard" temperature of the hydrometer.

A very great variety of standard temperatures are in use. In this country 60° F. is perhaps the commonest; 85° F. is a usual temperature for instruments used in India; 20° C. is commonly used in America; 15° C., 17·5° C., and 20° C. are of frequent occurrence on the continent. Again, for hydrometers intended for special purposes standard temperatures approximating to the normal conditions of use are adopted, *e.g.* in the case of hydrometers used for determining the density of boiler water 200° F., 90° C., and 100° F. are frequently met with.

§ (7) TEMPERATURE CORRECTIONS. (i.) *Density Hydrometers.*—The corrections to be applied to a hydrometer when used at temperatures other than its standard may be readily obtained in the case of density hydrometers as follows:

Suppose a hydrometer to be reading n gm. per ml. in a solution at the temperature $t°$ C., $t°$ C. being the standard temperature of the instrument. Then we have

$$M = nV,$$

M being the mass of the hydrometer and V the volume of liquid displaced when the hydrometer reading is n.

Now suppose the hydrometer to be again reading n, but this time in a liquid whose temperature is $t'°$ C., and let x be the correction which must be added to n in order to give the density of the second liquid in *gm.* per ml. at $t'°$ C. In this case we have

$$M = (n + x)V\{1 + \alpha(t' - t)\},$$

α being the coefficient of cubical expansion of the material of which the hydrometer is constructed. Hence

$$x = n\left[\frac{1}{1 + \alpha(t' - t)} - 1\right],$$

or since α is small we have very approximately

$$x = -n\alpha(t' - t).$$

In the case of glass hydrometers $\alpha = 0·000026$, and therefore a correction of from 2 to 5 (according to the value of n) units in the fifth decimal place must be subtracted from the observed reading for each degree centigrade above the standard temperature, and the same amounts added for temperatures below the standard temperature. Now a variation of about five units in the *fourth* decimal place is negligible in most cases where hydrometers are extensively used. It follows, therefore, that a density hydrometer may in such cases be read at any temperature within about 10° C. of its standard temperature, and its indications will give the density of the liquid in which it floats to a sufficient degree of accuracy without the necessity for applying any temperature correction.

(ii.) *Specific Gravity Hydrometers.*—In the case of specific gravity hydrometers the temperature correction may be similarly obtained. Let s be the reading at $t°$ C. of a hydrometer indicating specific gravities correctly at $t°$ C. relative to water at $t°$ C. as unity. In this case we have

$$M = Vs\rho_t,$$

M being the mass of the hydrometer, V the volume of liquid displaced at $t°$ C. when the reading is s, and ρ_t the density of water at $t°$ C.

Consider the hydrometer to be placed in a liquid whose temperature is $t'°$ C. and such that the hydrometer reading is again s. If x be the correction to be applied to the reading s to give the specific gravity $S_{\frac{t'}{t'}}$ of the liquid we have

$$M = (s + x)V\{1 + \alpha(t' - t)\}\rho_{t'},$$

α being the coefficient of cubical expansion of the material from which the hydrometer is constructed, and $\rho_{t'}$ the density of water at $t'°$ C.

From the two above equations we obtain

$$x = s\left[\frac{\rho_t}{\rho_{t'}\{1 + \alpha(t' - t)\}} - 1\right],$$

from which it is clear that the expansion of water has to be taken into account, as well as the expansion of the hydrometer, in determining the value of x.

The temperature corrections are therefore much larger than in the case of density hydrometers. For instance, a specific gravity hydrometer reading $S_{\frac{60°\text{ F.}}{60°\text{ F.}}} = 1·150$ correctly at 60° F. will require a correction of +0·003 when reading 1·150 at 85° F. in order to give the specific gravity $S_{\frac{85°\text{ F.}}{85°\text{ F.}}}$ of the liquid. Specific gravity hydrometers, therefore, only indicate specific gravities correctly when used at or near their standard temperatures. Compare this with the case of density hydrometers dealt with previously (§ (7) (i.)).

(iii.) *Correction by Variation of Mass.*—The question of temperature correction to specific gravity hydrometers may be looked at from a slightly different point of view. Consider as before a specific gravity hydrometer to be reading s, first in a liquid at $t°$ C. and secondly in one at $t'°$ C. In each case let s be the correct specific gravity, *i.e.* $S_{\frac{t}{t}}$ at $t°$ C., and $S_{\frac{t'}{t'}}$ at $t'°$ C. This can be achieved by varying the mass of the hydrometer suitably. Let M be the mass of the hydrometer when indicating $S_{\frac{t}{t}}$ at $t°$ C., and m the amount by which it must be changed to read $S_{\frac{t'}{t'}}$ correctly at $t'°$ C.

Considering the two cases we have

$$\mathrm{M}=\mathrm{V}s\rho_t,$$

$$\mathrm{M}+m=\mathrm{V}[1+a(t'-t)]s\rho_{t'},$$

ρ_t and $\rho_{t'}$ being the density of water at t and t' respectively. From the above equations we obtain

$$m=\mathrm{M}\left[\frac{\rho_t}{\rho_{t'}[1+a(t'-t)]}-1\right].$$

The term on the right-hand side of the above equation is constant for given values of t and t', and is *independent of s*. Hence, if a hydrometer is furnished with a scale which indicates $\mathrm{S}_{\frac{t}{t}}$ correctly at t° C. the same scale will indicate $\mathrm{S}_{\frac{t'}{t'}}$ correctly at t'° C. provided the mass of the instrument is adjusted by the amount given by the above equation. A similar statement obviously also holds true in the case of density hydrometers.

(iv.) *Hydrometers to indicate Percentage Composition.*—In the case of hydrometers indicating percentage composition directly, then the expansion of the liquid to which the hydrometer relates is an additional factor to be taken into account in determining the temperature corrections to the instrument. For example, take the case of a hydrometer graduated to indicate percentages by weight of sugar in sugar solutions at t° C. The density at t° C. corresponding to each graduation mark is known, being, of course, the same as the density of the corresponding sugar solution. The density at t' corresponding to any graduation mark may be calculated from the known density at t in a similar manner to that adopted in the case of density hydrometers above. The percentage of sugar corresponding to the calculated density at t' may then be determined from the known densities and coefficients of expansion of sugar solutions. The difference between this percentage and that marked on the hydrometer for the graduation mark in question gives the required correction at t'.

Although the readings of a hydrometer can only be strictly accurate at one particular temperature, yet in the manufacture of hydrometers, or in testing them subsequently by comparison with a standard of known accuracy, it is not necessary that the comparisons should be carried out at the standard temperature of the instruments. Provided that two hydrometers are made of the same material, the difference in their readings will be independent of the temperature of the liquid in which they are compared. The readings of each hydrometer will be changed by the same amount by equal changes in temperature, since the instruments being made of the same material their coefficients of expansion will be identical. In comparing hydrometers, therefore, it is only necessary to ensure uniformity of temperature in the liquid in which they are compared, the exact temperature of comparison being a matter of indifference. This of course does not hold true when comparing hydrometers made from different materials, *e.g.* a metal instrument and a glass one. In such cases the difference in the readings will not be independent of the temperature at which they are compared.

§ (8) Constant Volume Hydrometers.—The hydrometers hitherto considered have been of constant mass and variable displacement. It is possible to work with constant displacement and variable mass. Nicholson's hydrometer is based on this principle. The instrument is described in many text-books, but it is not, however, used to any large extent in density determinations.

Buchanan used hydrometers based on the same principle in an extensive series of density investigations. Full details concerning these instruments are given in his paper "Experimental Researches on the Specific Gravity and the Displacement of some Saline Solutions."[1]

§ (9) Hydrometers with Submerged Poises.—A well-known example of this type of hydrometer is the Bates saccharometer. It is a metal instrument with a stem of rectangular cross section above the bulb, and below the bulb a ring is attached by means of a short stem. This ring has a tapered hole drilled in it at A (*Fig.* 3), and a number of poises, each provided with conical pins, may be attached in turn to the hydrometer by inserting the pins in the hole A. The scale of the hydrometer covers a range of 0·03 specific gravity, and with the lightest poise attached readings may be obtained over the range 0·970 to 1·000; with the next poise the instrument reads 1·000 to 1·030, and so on up to 1·120 or 1·150.

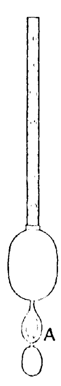


Fig. 3.

The advantage of this method of construction is that an open scale instrument covering a considerable range of density may be made in a very compact form.

We will consider in general terms a hydrometer constructed on the above principle.

Let d_1 be the density corresponding to the highest graduation mark when a particular poise is attached, and d_2 the density corresponding to the lowest graduation mark when the same poise is attached. When the next heavier poise is attached d_2 is the density corresponding to the highest graduation mark, and if d_3 is the density corresponding to the lowest graduation mark the poises are adjusted so that

$$d_2-d_1=d_3-d_2=\delta \text{ say.}$$

Let M be the mass of the hydrometer, including the poise, when the range is from d_1 to d_2, and $M + m$ the mass, again including the poise, when the range is d_2 to d_3. Let V be the displacement of the hydrometer when reading d_2 with the first poise attached, and v the volume of the stem from the highest graduation mark to the lowest graduation mark. Let v' be the difference in volume of the two poises. Then when the first poise is attached we have

---

[1] *Trans. Roy. Soc. Edin.*, 1912 13, xlix. Pt. I. 1-227.

$$M = (V + v)d_1 = Vd_2, \quad . \quad . \quad (1)$$

and when the second poise is attached

$$M + m = (V + v + v')d_2 = (V + v')d_3. \quad . \quad (2)$$

From these it follows readily that $v' = v$, *i.e.* each poise must have a volume greater than that of the preceding poise by an amount equal to the volume of the stem between the highest and lowest graduation marks.

Again, from (1) and (2) we have

$$\frac{M}{M + m} = \frac{(V + v)d_1}{(V + v')d_3},$$

and since $v = v'$,

$$m = M\frac{d_3 - d_1}{d_1} = \frac{2\delta}{d_1},$$

and this relation determines the amount by which the masses of the two poises considered must differ from each other.

One further point is worthy of notice in connection with the above type of hydrometer. The poises may be adjusted so that the highest and lowest graduation marks are correct for each poise, but the intermediate graduations can only be accurate for one particular poise. From § (4), p. 434, we have

$$l = L\frac{d_2}{d}\cdot\frac{d - d_1}{d_2 - d_1}.$$

Let us take two cases as follows:

Case (1) L = 100 mm., $d_1 = 1{\cdot}000$, $d = 1{\cdot}015$, $d_3 = 1{\cdot}030$.
Case (2) L = 100 mm., $d_1 = 1{\cdot}120$, $d = 1{\cdot}135$, $d_3 = 1{\cdot}150$.

The first case gives $l = 50{\cdot}74$ mm. and the second $l = 50{\cdot}66$ mm., *i.e.* the 1·015 graduation mark should be 0·08 mm. further from the 1·000 mark than the 1·135 mark should be from the 1·120 mark. Hence if a hydrometer of the type just considered is graduated so that the graduations are correct over the interval 1·000 to 1·030, then when the poise giving the range 1·120 to 1·150 is used the intermediate graduations will not be quite correctly spaced, the true position of the 1·135 mark being as we have seen, for an over-all length of scale equal to 100 mm., 0·08 mm. above the mark representing 1·015 correctly. This error, however, only corresponds to 0·000024 in terms of density and is therefore negligible for practical purposes.

§ (10) EFFECT OF SURFACE TENSION ON HYDROMETER READINGS.—When a hydrometer is floating in a liquid the liquid surface does not continue to be horizontal up to the point of contact with the hydrometer stem. Owing to the effects of surface tension a small quantity of liquid adheres to the stem of the hydrometer and is lifted above the general level of the liquid as shown in *Fig.* 4. There is in consequence a downward pull on the hydrometer which is equal to the weight of the liquid raised above the general level of the liquid surface. The magnitude of this force is equal to the product of the surface tension of the liquid and the perimeter of the stem. Thus in the case of a circular stem of diameter d cm. the force is πdT dynes, T being the surface tension expressed in dynes per cm. This additional downward force may be regarded as an increase in the mass of the hydrometer, and we may for convenience speak of the term $(M + \pi d T/g)$ as the "effective mass" of the hydrometer.

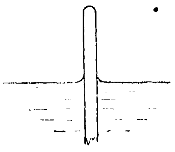


FIG. 4.

The indication of a hydrometer in a liquid depends, therefore, not only on the density of the liquid but also on its surface tension. Suppose a hydrometer to be reading n in a solution at 60° F. whose surface tension is T, and that for this particular liquid n is the correct density of the liquid at 60° F. Then if M is the mass of the hydrometer

$$M + \frac{\pi d T}{g} = nV,$$

V being the volume of liquid displaced when the hydrometer reading is n. Secondly, if the hydrometer is again reading n in a solution also at 60° F., but whose surface tension is T′, let x be the correction to be applied to the reading n in order to obtain the correct density of the second solution at 60° F. In this case

$$M + \frac{\pi d T'}{g} = (n + x)V.$$

From the two above equations,

$$\frac{n + x}{n} = \frac{M + (\pi d T'/g)}{M + (\pi d T/g)}$$

or

$$x = \frac{n\pi d}{g}\left(\frac{T' - T}{M + (\pi d T/g)}\right),$$

or approximately

$$x = \frac{n\pi d}{Mg}(T' - T).$$

The following example will serve to illustrate the magnitude of the surface tension effects. A specific gravity hydrometer whose mass is 37 gm. and range 1·000 to 1·040 is adjusted to indicate $S_{\frac{60^\circ\,F.}{60^\circ\,F.}}$ correctly in dilute sulphuric acid solutions at 60° F. The diameter of the stem being 5 mm. and taking 74 dynes per cm. as the surface tension of a sulphuric acid solution of specific gravity $S_{\frac{60^\circ\,F.}{60^\circ\,F.}} = 1{\cdot}030$, if 50 dynes per cm. is taken as the surface tension of milk, then if the hydrometer is reading 1·030 in a sample of milk at 60° F. the correction required to give the true specific gravity of the milk is

$$\frac{1\cdot030\times3\cdot14\times0\cdot5}{37\times981}(50-74)=-0\cdot0011.$$

Hence a hydrometer of the dimensions and mass given would be 0·001 in error when used in milk if it were correct in sulphuric acid. Such a hydrometer would probably be subdivided in intervals of 0·001 (*i.e.* in 1° intervals), and so the error in milk amounts to one whole subdivision. It is a common practice for hydrometer manufacturers to point lactometers in dilute sulphuric acid solutions, or dilute solutions of salts which have similar surface tensions, and it will be seen from the above example that the errors so introduced are by no means negligible.

Another example is not without interest. Proof spirit is defined as being a mixture of alcohol and water which at 51° F. is $\frac{12}{13}$ths the weight of an equal volume of water. Metal Sikes hydrometers have a mark on the stem indicating the point at which the instrument floats in proof spirit at 51° F. when the 60° poise is used. The instruments are furnished with a metal cap which fits on to the top of the stem and is $\frac{1}{12}$th the combined weight of the hydrometer and the 60° poise. The combined weight of the hydrometer, 60° poise, and cap is hence $\frac{13}{12}$ths of the combined weight of the hydrometer and the 60° poise. If, therefore, there were no surface tension effects, the hydrometer used with the 60° poise but without the cap would float at the same mark in proof spirit at 51° F. as it would in distilled water at 51° F. when used with both the 60° poise and the cap. Owing to the effects of surface tension, however, the ratio of the "effective masses" would not be 13 : 12, and so different readings would be obtained in distilled water and proof spirit.

The problem of the effects of surface tension may be looked at from a slightly different point of view. If there are two liquids of the same density but having different surface tensions, then, as we have seen, a hydrometer will read differently in the two liquids. Let l be the additional length of the stem above the surface of the liquid when the hydrometer is floating in the liquid of lower surface tension T′. The change in the "effective mass" of the hydrometer is compensated for by the decreased displacement, and hence for a stem of diameter d

$$\frac{\pi}{4}d^2ln=\frac{\pi d}{g}(\mathrm{T}-\mathrm{T}')$$

or

$$l=\frac{4(\mathrm{T}-\mathrm{T}')}{gdn}.$$

It is interesting to notice that from the above equation and the equation for the correction x it follows that, other things being equal,

$$x\propto d$$

and

$$l\propto\frac{1}{d}.$$

The significance of this may be illustrated by the following example. Consider two hydrometers of identical range and mass but one having a stem *whose diameter is twice that of the other.* The sensitivity (*i.e.* change in density corresponding to unit length of scale) of the hydrometer with the smaller diameter of stem will be four times that of the other hydrometer, and readings may consequently be taken to a correspondingly higher degree of accuracy. Assume both hydrometers to read correctly in a particular liquid of known surface tension. If they are placed in a second liquid of different surface tension the hydrometers will no longer agree. The densities indicated by each will be in error, but the error in the case of the hydrometer with the stem of smaller diameter will be only half that in the case of the other hydrometer. The sensitivity of the former hydrometer is, however, four times that of the latter, and hence, although the error in the density reading due to surface tension in the first case is only half that in the second, yet compared with the increased degree of accuracy to which readings may be taken on the hydrometer with the stem of smaller diameter, the error is of more serious consequence.

§ (11) Errors due to Surface Tension.—The influence of the surface tension of the liquid in which a hydrometer is floating on the indications of the instrument is a very serious limitation on the use of the hydrometer for determining densities to a high degree of accuracy. Slight contaminations of a liquid surface may alter its surface tension very considerably,[1] and in consequence the indications of a hydrometer in a liquid may vary appreciably even though the density of the liquid remains constant. It is therefore essential to observe scrupulous cleanliness of the hydrometer itself, of the liquid in which it is read, and of the vessel in which the readings are taken if consistent results are to be obtained. Dilute aqueous solutions are particularly liable to give false readings, and in distilled water itself it is perhaps most difficult of all to obtain reliable hydrometer readings. This fact is important, because statements such as "observe the indication of the instrument in distilled water" so frequently occur in descriptions of methods of graduating hydrometers. Mineral oils, alcohol solutions (except when very dilute), strong acid solutions, sugar solutions, and sodium carbonate solutions are some of the more suitable liquids for use in connection with hydrometers.

The following simple criterion as to the cleanliness of the stem of a hydrometer and the condition of the liquid surface is useful. If a hydrometer is submerged a little beyond its position of equilibrium and then released, it will oscillate up and down for a while. If the stem is clean and the liquid wets it completely, the stem of the hydrometer will move through the liquid surface during the above oscillations without in any way *disturbing or deforming* the meniscus surrounding the stem. If, on the other hand, the stem is dirty or not completely wetted, then as it passes through the surface the stem will drag the meniscus out of shape. This effect is generally most noticeable when the hydrometer commences to descend after reaching its highest position during an oscillation. If any such deformation of the meniscus takes place it is useless to take a reading on the hydrometer. An idea of the errors which may arise in such cases may be gained from the fact that by manipulating a metal Sikes hydrometer in a dilute alcohol solution (say 95° Sikes) so as to obtain an unfavourable meniscus, it is possible to make the instrument read in error by as much as 0°·4.

---

[1] See F. Nansen, *Scientific Results of the Norwegian North Polar Expedition 1893-96*, London, 1903, vol. iii. chap. x., "On Hydrometers and the Surface Tension of Liquids," for an extensive series of investigations on the variation in the surface tension of liquids.

In order to minimise the errors arising from contamination of the liquid surface, it is recommended by some authorities that the liquid should be allowed to overflow immediately before taking a hydrometer reading. By this means a newly formed surface is obtained which is more likely to be free from contamination than the surface obtained without overflowing.

§ (12) Standardisation of Hydrometers.—With the exception of the few cases in which a particular hydrometer is legalised as the standard instrument, the standardisation of hydrometers primarily depends upon density determinations. Arbitrary scales may be expressed in terms of density, scales indicating percentage composition are based on the densities of the liquids to which they relate, and so on. Generally speaking, the standardisation of a hydrometer resolves itself into determining the density of a liquid and noting the indication of the hydrometer in the same liquid and at the same temperature, the temperature chosen being the standard temperature of the hydrometer.

The arrangement of apparatus illustrated diagrammatically in *Fig.* 5 affords a convenient and accurate method of standardising hydrometers. The liquid in which the hydrometer is read is contained in a glass vessel A, the front and back faces of which are parallel, ground plane, and polished on the outside, so that the scale of the hydrometer does not appear distorted when viewed through the front of the vessel. The vessel A stands underneath a balance B. A thin rod C, which passes through a hole in the base of the balance case and also through a hole in the table supporting the balance, is attached to one scale pan of the balance at its upper end, and carries a hook at its lower end. From this hook a plummet D is suspended inside the vessel A by means of a thin platinum wire. The hydrometer to be standardised floats alongside the plummet, and a thermometer is suspended with its bulb situated near both the hydrometer and the plummet. The vessel A should be sufficiently large to allow the contained liquid to be stirred vigorously without removing the hydrometer or plummet, the beam of the balance being, of course, arrested whilst stirring is in progress.

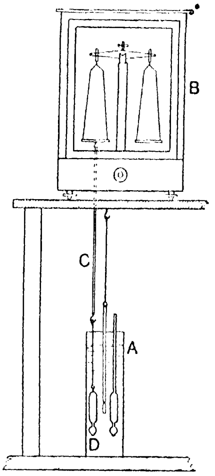


Fig. 5.

A convenient mode of procedure when the standard temperature is not far removed from room temperature is as follows. The liquid to be used is adjusted so that the hydrometer, when floating in the liquid at its standard temperature, gives a reading close to the point at which it is desired to determine the error of the instrument. The temperature of the liquid is then adjusted, so that it is a few degrees centigrade above or below the standard temperature of the hydrometer. The temperature of the liquid will gradually approach room temperature, and the initial temperature is adjusted so that the temperature of the liquid passes through the standard temperature of the hydrometer as it rises or falls to room temperature. A suitable initial temperature having been obtained, the vessel A is placed in position under the balance, and the plummet, hydrometer, thermometer, and stirrer are inserted in the liquid. The level of the liquid is then adjusted to a previously fixed position, indicated by a mark on the vessel, so that a constant length of the wire supporting the plummet is immersed. After the plummet and hydrometer have had sufficient time to attain the temperature of the liquid, the latter is thoroughly stirred to obtain uniformity of temperature. One observer then determines the apparent weight of the plummet. The balance case is closed during the final adjustment of the rider on the beam, and the vessel A, being entirely outside the balance case, introduces no disturbing effect on the balance.

Whilst the plummet is being weighed, a second observer sees that the hydrometer is clear of the plummet and in a suitable position for him to take its reading. When the first observer has adjusted the weights and rider so that the balance is in equilibrium, the second observer notes the hydrometer reading. Temperature observations are taken before the balance is adjusted and immediately after taking the hydrometer reading. When the above observations are complete, the liquid is once more thoroughly stirred, without removing either the hydrometer, plummet, or thermometer, the stirrer itself having remained throughout in the liquid. A second set of observations is then taken, and so on, until the temperature of the liquid is finally as much below (or above, as the case might be) the standard temperature of the hydrometer as it was initially above (or below) this temperature. The apparent weights of the sinker are then plotted against the corresponding temperatures, and a second curve is also obtained by plotting hydrometer readings against temperature. The distribution of the points about the two curves affords a valuable indication of the consistency of the observations. The apparent weight of the plummet and the reading of the hydrometer, which correspond to the standard temperature of the hydrometer, are obtained from the two graphs. A previous determination of the weight of the plummet in air and in distilled water respectively provides the remaining data required for calculating the density of the liquid, and hence the correction to the hydrometer.

The procedure outlined gives results sufficiently accurate for most hydrometers. For instance, if a plummet whose volume is 50 c.c. is used, and the weighings are accurate to within 5 milligrammes, the densities will, so far as the weighings are concerned, be correct to one unit in the fourth decimal place. Actually, of course, the weighings can be carried out to a closer accuracy than 5 milligrammes with an ordinarily good balance. An error of 0°·1 C. in temperature observations will, generally speaking, correspond approximately to an error of one unit in the fourth decimal place.

For more accurate determinations, the temperature of the liquid in the vessel A may be controlled by surrounding it with a water-bath provided with a thermostat.

Another improvement is to use two plummets of equal mass but of different volumes, one suspended from each arm of the balance. The difference in the apparent weights of the two plummets when immersed in a liquid (*i.e.* the weights required on the side of the balance from which the plummet of larger volume is suspended, in order to produce equilibrium) is then the weight of a volume of liquid equal to the difference in volume of the two plummets. This difference in volume is determined by weighings in distilled water. The plummets can be readily interchanged on the balance and the advantages of "double weighing" obtained. Further, if the plummets are suspended by means of platinum wires which are equal in diameter, the downward pull on each wire due to the effects of surface tension will be equal and hence need not be taken into consideration.

The outstanding advantage of the above method of standardising hydrometers is that the density of the liquid is determined simultaneously with the hydrometer reading, in the same liquid and under identical conditions.

If a pyknometer is used to determine the density, then the sample of liquid in the pyknometer may not be identical with the bulk of the liquid in which the hydrometer is read, neither as regards composition nor temperature.

§ (13) COMPARISON OF HYDROMETERS.—The method of standardising hydrometers described in the preceding paragraph is, of course, impracticable where large numbers of hydrometers have to be dealt with because of the amount of time which it would involve. In dealing with numbers of hydrometers either in the course of manufacture or testing, it is customary to make use of a standard hydrometer whose corrections have been carefully determined.

The comparison of hydrometers with the standard instrument is best carried out in a vessel which is large enough to allow two hydrometers to float side by side without danger of fouling either each other or the sides of the vessel. A convenient vessel for the purpose is one of rectangular cross section, and if its internal dimensions are 14½ in. by 2½ in. by 4½ in., it will be big enough to accommodate all but exceptionally large hydrometers. The front and back faces, *i.e.* the two broad faces, of the vessel should be ground plane and well polished, and the glass should be quite clear and free from striae and similar defects. By this means distortion of the hydrometer scale when viewed through the front face of the vessel is avoided.

A screen placed behind the vessel, as shown in *Fig.* 6, facilitates taking readings on the hydrometers. The top half of the screen is painted a dead black, and the bottom half is white. The screen is inclined at an angle

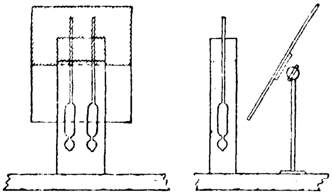

Front Elevation **Side Elevation**

FIG. 6.

behind the vessel, so that the bottom half reflects light through the liquid. The junction of the black and white portions of the screen is arranged to be horizontal and slightly below the level of the surface of the liquid in the vessel. In order to obtain accurate hydrometer readings, it is necessary to place the eye exactly on the level of the liquid surface. The screen helps to secure this condition. If the eye is placed much below the level of the liquid surface, the latter appears as a white rectangle. On raising the head, the rectangle becomes narrower, and changes from a white to a dark rectangle, whose front edge is bounded by a very sharply defined white line formed by the bottom edge of a narrow white band, which crosses the front of the vessel at the base of the meniscus formed against the inside of the front face. At the same time a white ellipse appears around the stem of the hydrometer. On raising the head still further the ellipse becomes thinner, and the rectangular surface becoming more foreshortened, the ellipse at the same time appears closer to the white line defining the front edge of the rectangle. Finally, when the eye is exactly on the level of the liquid surface, the ellipse, which has now become practically a thin straight line, merges into the white line forming the front edge of the liquid surface. On dropping the head slightly the ellipse immediately becomes visible once more in the dark surface of the liquid. A little practice soon enables one to bring the eye to the correct position with much more certainty than when viewing the surface without the screen, and simply using the foreshortening of the liquid surface into a straight line as a criterion of the correct position.

The solutions to be used for the comparisons should be stored in the room in which they are to be used. It has been shown previously that the difference between the readings of two hydrometers is independent of the temperature at which they are compared. The temperature should, however, remain as nearly as *possible constant throughout the comparisons*. The solution should be *thoroughly* stirred after being transferred to the vessel in which the hydrometers are to be read.

The hydrometers, particularly the stems, should be thoroughly cleaned before use. Once cleaned, the hydrometers should, as far as possible, only be handled by taking hold of the extreme top of the stem above the highest graduation mark, and on no account should the portion of the stem occupied by the graduation marks be fingered.

The hydrometers having been cleaned and the liquid made ready for the comparisons, the following is a convenient method of carrying out the comparisons. First, the standard is placed in the liquid, care being taken only to release the instrument when it is near to its position of equilibrium, and so to avoid wetting the stem for any appreciable distance beyond the point at which it intersects the liquid surface when the hydrometer is floating freely. Whilst placing the second hydrometer in the liquid, the standard is lifted slightly, and held so that the introduction of the second hydrometer does not cause the stem of the standard to be wetted beyond its reading. Both hydrometers are released only when they have been adjusted approximately to their position of equilibrium. The reading of each is then noted approximately. The top of each hydrometer is then gripped by the thumb and first finger of the left and right hand respectively, and both instruments are immersed, so that their stems are wetted for an equal distance (1 cm. to 2 cm. is a convenient distance) beyond their reading. When in this position, the grip on the hydrometers is entirely relaxed, and the hydrometers are kept in position simply by the top of the stems pressing against the V formed by the finger and thumb (see *Fig.* 7). The hands may then be withdrawn without disturbing the hydrometers, and the latter then rise and finally settle down after a few oscillations into their position of equilibrium, and when both are quite stationary the reading on each is noted.

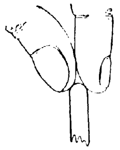


FIG. 7.

The standard is then held with one hand, and the other hydrometer is removed from the vessel. The next hydrometer to be compared with the standard is then introduced, with the same precautions as before, and the interval during which this and the standard are settling down to their final reading may be occupied in drying the hydrometer first compared, as it is inadvisable to leave the instrument lying about wet if a second comparison is to be made against the standard at another point on the stem.

It is sometimes recommended that hydrometers may be compared with a standard using an ordinary cylindrical hydrometer trial jar. The standard hydrometer is first read in the liquid, and then a small number, say six, of the hydrometers to be checked are read successively in the same liquid. The standard is then read once more in the liquid, a second set of six are then read successively, and so on, readings being taken on the standard between each set of readings on the hydrometers to be checked. This method is not nearly so satisfactory as having the standard hydrometer and the one to be compared with it floating side by side in the same vessel.

§ (14) ERRORS DUE TO SURFACE CONTAMINATION.—Errors in hydrometer readings due to contamination of the liquid surface may be minimised by overflowing the liquid just before taking a reading, and so obtaining a freshly formed liquid surface. An apparatus designed to permit renewal of the liquid surface by overflowing is shown in *Fig.* 8.

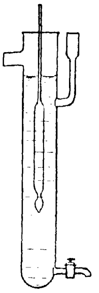


FIG. 8.

"The manipulation[1] is as follows: The cylinder is filled nearly to the spout by the liquid whose density is desired, or in which hydrometers are to be compared. The hydrometer is then immersed in the liquid, and permitted to float freely until it has assumed the temperature of the liquid. The hydrometer is raised to permit thorough stirring of the liquid. The temperature is observed if desired. From a beaker of the same liquid enough is poured into the funnel to cause the liquid to overflow and run out the spout, where it is caught in a convenient vessel.

"The hydrometer is then read. The completeness of the cleansing of the surface of the liquid may be tested by repeating the operation as the readings will approach a constant value as the surface becomes normal.

"The necessity for such special manipulation is confined to the reading of hydrometers in liquids which are subject to surface contamination. Such, in general, are aqueous solutions or mixtures of acids, alkalies, salts, sugar, and weak alcoholic mixtures. Oils, alcoholic mixtures above 40 per cent by volume, and other liquids of relatively low surface tension are not, in general, liable to surface contamination sufficient to cause appreciable changes in hydrometer readings."

One other point in connection with the comparison of hydrometers should be noted. If the hydrometers compared have very nearly the same range and dimensions, in particular, stems of approximately the same diameter, then it is immaterial in what liquid they are compared. If, on the other hand, the standard differs appreciably from the hydrometer to be checked, then the comparisons should be carried out in the liquid in which the hydrometer is to be used, or else an allowance made for the difference in surface tension between the liquid used for the comparisons and the liquid for which the hydrometer is required.

§ (15) GENERAL SPECIFICATION FOR GLASS HYDROMETERS.—Hydrometers should be made from glass free from striae and similar defects, and in particular the surface of the stem should be quite smooth. The glass should be of a kind which sufficiently resists the action of chemicals, and possesses properties such as would render it suitable for use for thermometers. Hydrometers should be thoroughly annealed before they are graduated. A hydrometer should be everywhere symmetrical about its vertical axis.

When mercury is used for loading, it should be contained in a bulb at the base of the hydrometer, which is sealed off from the rest of the instrument. When lead shot or other loading material is used, it should be fixed in position by means of a suitable cement, which will not soften at the highest temperature at which the hydrometer is likely to be used. No loading material should be left loose inside the hydrometer. The hydrometer must be loaded so that the instrument floats with its stem vertical.

The scale should be fixed in position, so as to prevent all possibility of it slipping, and paper of high quality should be used.

The graduations should be without evident irregularities. The graduation marks should be made by fine straight lines, which lie in planes perpendicular to the axis of the hydrometer, so that they are horizontal when the stem is vertical. The shortest graduation marks should extend at least one-quarter the way round the stem. Sufficient lines should be numbered to enable the exact reading at any point to be readily noted. Generally speaking, at least every tenth line should be numbered. The use of abbreviated numbers should be confined to the central portion of the scale, and the end graduation marks should be numbered in full. The numbers should not encroach on the space occupied by the shortest graduation marks. The numbered marks should be carried at least half-way round the stem, and the scale should be straight and without twist. The graduation marks should in general be not less than 1 mm. nor more than 2 mm. apart.

The stem should extend from 2 cm. to 3 cm. beyond the highest graduation mark, and the lowest graduation mark should be at least 5 mm. from the junction of the stem and the bulb.

A fundamentally important point is that each hydrometer should bear an explicit inscription giving the basis on which the scale is constructed. The inscription can, of course, be abbreviated. For example, the inscription "$S_{\frac{60^\circ\text{ F.}}{60^\circ\text{ F.}}}$ at 60° F." is quite sufficient to indicate that the hydrometer is intended to give specific gravities at 60° F. relative to water at 60° F. as basis, the readings being taken at 60° F.

If the hydrometer has an arbitrary scale, *e.g.* Baumé, the exact definition of the scale should be given on the instrument.

The inscriptions on hydrometers are often inadequate, and at times misleading. As an example of the former, a hydrometer marked

---

[1] *Circular of the Bureau of Standards*, No. 16, 1916, p. 12.

"Alcoholometer 15° C." is quite inadequately marked. The inscription might be equally true if the hydrometer indicated percentages by weight, percentages by volume, proof strength or degrees Sikes, and is not sufficient to define the scale precisely. A misleading inscription met with by the writer was "specific gravity, 85° F." on a hydrometer whose readings in sugar solutions at 85° F. were intended to give the specific gravity $S_{\frac{60^\circ \text{F.}}{60^\circ \text{F.}}}$, which the solutions would have had if cooled down to 60° F.

§ (16) Metal Hydrometers.—Hydrometers constructed of metal are open to very serious criticism, and they have many disadvantages as compared with glass hydrometers.

In the first place, metal hydrometers are very liable to undergo changes in weight, due partly to corrosion and partly to wear. In order to lessen the changes due to the former, metal hydrometers are plated sometimes with nickel, more frequently with gold. The Sikes hydrometer, legalised in this country for determining the strengths of spirits, is a gold-plated instrument. Whilst lessening the liability to corrosion, gold plating increases the liability to change in weight due to wear. Gold is a soft metal, and also a heavy one. A gold-plated Sikes hydrometer has an outer layer whose density is about twenty times the bulk density of the instrument. The plating inevitably wears away in the course of time, and so the hydrometer becomes progressively lighter. The seriousness of this is clearly indicated by the fact that hydrometer makers regularly enter into contracts with users of these instruments to adjust and regild them at intervals.

Another disadvantage of metal hydrometers is that they are liable to develop leaks at the joints.

The bulbs of metal hydrometers are necessarily made of thin sheet metal, and are liable to be dented, and thus to have their volume changed.

A metal surface is much less readily wetted than a glass one. In weak alcohol solutions, for example, it is extremely difficult to obtain a well-formed meniscus around a metal stem.

The graduation marks and numbers are engraved on the stems of metal hydrometers. Air bubbles are apt to cling to the indentations thus formed on the surface of the stem.

The above disadvantages do not occur with glass hydrometers. There is, however, one undeniable advantage which metal hydrometers possess over glass ones, viz. they can be made much more compact and portable. To obtain the same range and openness of scale as an ordinary metal Sikes hydrometer provided with nine poises, it would be necessary to have at least five glass hydrometers, in order to keep the size of the glass instruments within reasonable dimensions.

Glass hydrometers are also more likely to be broken than metal ones. There is reason, however, for preferring the glass hydrometer even on this score. So long as it remains intact, a glass hydrometer can be relied upon, and when broken the fact is apparent. A metal hydrometer, on the other hand, may become seriously out of adjustment without the defect being realised. It is also quite probable that the cost of replacing breakages in the case of glass hydrometers would not exceed the cost of readjustments in the case of metal ones.

§ (17) Tolerances.—A reasonable tolerance to allow for the error at any point on a hydrometer scale is plus or minus the scale equivalent of 1 mm. to 1·5 mm. The difference in the errors at any two points on the stem should not exceed the maximum error allowed at a point. This represents a degree of accuracy which should be attained by manufacturers without entailing excessive cost of production.

§ (18) Different Types of Glass Hydrometers.—The form of hydrometer which is perhaps of most frequent occurrence is that already shown in *Fig.* 1, p. 431. This is a very satisfactory and serviceable form of instrument. The hydrometer being, however, a lamp-blown article, variations in form can be readily introduced. Consequently, the cylindrical bulb is frequently replaced by pear-shaped, spherical, and a variety of other shapes of bulbs. No particular advantage attaches to any of the types, except in very special cases, which would make them preferable to the simple cylindrical form for general use.

One particular case where a bulb other than cylindrical in form is advantageous is that of accumulator hydrometers used for determining the density of the acid *in situ* in accumulator cells. In this case a hydrometer with a flattened bulb is preferable, as it can be inserted between the plates of the accumulator with less risk of touching the plates and so giving a false reading.

Most hydrometers are adjusted to be read at the intersection of the level liquid surface with the stem. In some cases, however, they are adjusted to be read at a definite distance above the liquid surface. This mode of adjustment is convenient in the case of accumulator hydrometers, as readings can then be taken above the level of the plates. A hydrometer adjusted on this principle is shown in *Fig.* 9. The hydrometer scale is read opposite to the top edge of the float shown in the diagram, and the float also serves as a fender to keep the hydrometer from coming into contact with the plates of the accumulator.[1]

Hydrometers are frequently made with thermometers enclosed inside them. Quite as reliable results may be obtained, however, with an ordinary hydrometer, using a separate thermometer for reading the temperature.

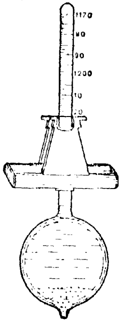


FIG. 9.

The scales adopted for hydrometers are almost infinite in variety. Each industry in which hydrometers are used appears to have its own selection of scales. The following quotation from *Dairy Chemistry* by H. D. Richmond illustrates this:

> "Soxhlet's lactometer contains a scale from 25° (1·025) to 35° (1·035) divided up into suitable divisions ($\frac{1}{2}$ or $\frac{1}{10}$).
>
> "Vieths' lactometer has a globular body: it requires a smaller bulk and depth of milk than Soxhlet's, and is suitable for taking the specific gravity in a half-pint can. The scale reads from 25° to 35°.
>
> "Quevennes' lactometer has a scale from 15° to 40° and is marked to show proportions of water added to milk and skim milk respectively. The auxiliary scale is useless.
>
> "Another form of lactometer, the name of whose inventor is deservedly lost in oblivion, has a scale from 0 to 100, 0 being equal to a specific gravity of 1·000 (water), and 100 being equal to a specific gravity of 1·029. It is of no practical use in milk-testing.
>
> "Still another form is marked M at 1·029 and W at 1·000, the intermediate space being divided into quarters; this form is a mere toy."

§ (19) ALCOHOLOMETRY AND SACCHAROMETRY.—Particulars of the more important instruments used for the determination of mixtures of alcohol and water or the strength of sugar solution are given in the articles on Alcoholometry and Saccharometry, to which reference should be made. A comprehensive account of a large number of types will be found in the *Handbuch der Aräometrie*, by Dr. J. Domke and Dr. E. Reimerdes.

V. S.

HYETOGRAPH: a self-recording rain-gauge. See "Meteorological Instruments," § (14) (ii.) (*b*).

HYGROMETER:
- I. Assmann's. See "Humidity," II. § (9).
  - Formulae for reduction of readings of. See *ibid.* II. § (4) (ii.).
- II. Dew-point:
  - Types of:
    - (i.) Crova's instrument. See *ibid.* II. § (2).
    - (ii.) Daniell's apparatus. See *ibid.* II. § (1).
    - (iii.) Regnault's apparatus. See *ibid.* II. § (1) *et seq.*
  - Effect of wind on. See *ibid.* II. § (2).
  - Theory of. See *ibid.* II. § (3).
- III. Hair. See *ibid.* II. § (10).
- IV. Wet- and dry-bulb:
  - Description of. See *ibid.* II. § (4) (i.).
  - Formulae for reduction of readings of. See *ibid.* II. § (4) (ii.).
  - See also "Thermometers, Wet-bulb."

HYGROSCOPIC SUBSTANCES:
- Effect of humidity on. See "Humidity," II. § (14).
- Properties of. See *ibid.* II. § (14).

HYPSOMETER APPARATUS FOR MEASUREMENT OF GRAVITY AT SEA. See "Gravity Survey," § (5) (ii.) (*a*).

— I —

ICE, range of, between summer and winter. See "Atmosphere, Thermodynamics of the," *Figs.* 1-4.

ICE-CRYSTALS IN THE ATMOSPHERE. See "Meteorological Optics," §§ (17), (18).

INDIAN GEODETIC SURVEY FOUR-METRE COMPARATOR: description. See "Comparators," § (9).

INDIAN SURVEY APPARATUS FOR MEASUREMENT OF GRAVITY. See "Gravity Survey," § (2) (iii.).

INDICATING DEVICES USED IN METROLOGICAL MEASUREMENTS: conversion of linear to angular movement. See "Metrology," § (36) (i.).
- Fizeau dilatometer. See *ibid.* § (36) (v.) (*e*).
- Magnification of linear motions by mechanical levers. See *ibid.* § (36) (ii.).
- Michelson interferometer. See *ibid.* § (36) (v.) (*e*).
- Proportionality of scales. See *ibid.* § (36) (iv.).
- Pure mechanical lever magnification (Dr. P. E. Shaw's machine). See *ibid.* § (36) (ii.).
- The gravity piece. See *ibid.* § (36) (v.) (*a*).
- The liquid indicator. See *ibid.* § (36) (v.) (*c*).
- The optical lever. See *ibid.* § (36) (iii.).
- The telephone contact (Dr. P. E. Shaw's machines). See *ibid.* § (36) (v.) (*d*).
- The tilting level. See *ibid.* § (36) (v.) (*b*).

---

[1] See "Secondary Cell Maintenance," *Post Office Electrical Engineers' Journal*, January 1914, for information regarding the use of hydrometers in connection with accumulators.

Tutton wave-length comparator. See *ibid.* § (36) (v.) (*e*).
Ultra-micrometer (Whiddington and Dowling). See *ibid.* § (36) (v.) (*f*).

Indicator, used to read the top of the mercury column in terms of the barometer scale: the vernier is the usual device used. See "Barometers and Manometers," § (3) (i.) (*d*).

Indicator, Calibration of, for use in metrological measurements. See "Metrology," IX. § (32) (ii.).

Indicator-diagrams for Saturated Air. See "Atmosphere, Thermodynamics of the," §§ (22), (23).

Indicators and Measurers, Functions of, in Metrological Observations. See "Metrology," IX. § (32) (i.).

INSTRUMENTS, THE DESIGN OF SCIENTIFIC

§ (1) Introductory.—The advancement of knowledge in many branches of science is the result of making accurate measurements of natural phenomena or of the physical properties of matter. These measurements are made with scientific instruments, or philosophical instruments, as they were formerly called. Also successful teaching of most sciences can only be given with the aid of scientific instruments. This is especially true in the case of Physics. It is now realised that efficiency in many industrial processes can only be secured from the data obtained by accurate measurement, and scientific instruments are largely used as manufacturing tools. Instruments may be used for producing and exhibiting some phenomena giving only qualitative results. This is especially the case in the early stages of the development of a branch of pure science. Generally, however, little progress is made until measurements can be taken, though the qualitative instruments are useful in teaching. The essential qualities of a good design are different in instruments used for each of these classes—research, teaching, or industry. Before detailing these qualities we will first consider a few fundamental points in the method of taking measurements and the use of instruments.

§ (2) Methods of Measurement.—Measurements by an instrument are made by means of some phenomenon capable of giving signals to the observer's brain. This is also the case when photography is used or autographic records are made. The most usual, most convenient, and generally the most accurate method of measurements is by observing the coincidence of two objects in space by the eye; however, when the coincidence is not perfect the distance between the objects should be estimated. But the fact that the observer has two eyes or two ears allows other methods to be used occasionally. The head is itself an instrument, which can be used in an approximate way as a range-finder, or as a direction indicator for sounds without the use of coincidences. Apart from these slight exceptions, every scientific measurement ultimately depends on observing coincidences. The physical property being observed is tested against a similar physical property capable of variation, and when the balance is correct the fact is signalled to the observer in one of several ways. This may be called the null method. The signal is usually given by the eye, occasionally by the ear, and may be given by touch. When the eye is used the most common method is to observe the position of some indicator in space. In weighing a mass on a balance the observer obtains the signal by eye when the pointer is not deflected from its equilibrium position. With an instrument for use in a laboratory in which the forces available for giving indications are small, the indicator is often a spot of light. If the forces are large enough, but still small, and especially if the instrument is required for workshop use, the indicator may be a pointer moving near, but not in contact with the scale. Lastly, if the forces available are large, the indicator may be a vernier in contact with the scale. Each of these designs has inaccuracies peculiar to itself which can generally be reduced by good design. The eye can also be used to observe the brilliancy and the colour of light. The equality of two colours can be used to indicate the equality of the observed physical properties, or the brilliancy of the illumination of two optical fields can be used as the signal; photometers, and some optical pyrometers, are examples. If the signal is a sound it may alter in magnitude, in pitch, or in the phase of the sound waves. These methods are not often used, but examples can be found of each of them. For instance, the magnitude of the sound on the telephone is used with a Wheatstone bridge for indicating the balance-point for alternating current. The difference of pitch between two sounds is used for adjusting tuning-forks, and the difference of phase between two sounds, combined with bi-aural hearing, has been used for locating submarines and aeroplanes.

In many instruments the null method is not used. In order to save time and to make the reading simple, scales or units of material, such as weights or electric resistances, are often used. But every measurement taken by such means depends on the calibration of a scale in which the null method was used.

Consider the measurements of length, mass, and time.

The unit of length is given by a certain piece of material which is supposed to remain constant in shape, and is the legal standard of length. Scales equal in length to the standard are made by the null method; scales of other lengths are made by a process of adding and subtracting.

The standard of mass is a certain piece of matter which is assumed to remain constant. Then by a null method of weighing this mass against an equal mass in a balance a series of equal masses is obtained, and by a process of adding and subtraction a series of masses of different amounts is made.

In the case of time the rotation of the earth is the unit, and the scale of time is obtained by dividing the mean solar day into equal parts. These subdivisions of time are compared by observing the motion of bodies which are oscillating or moving at uniform rates.

In designing a definite instrument we have first to find a chain of physical phenomena, the last of which can be measured in some easy and accurate way. For example, suppose the measurement of a high temperature is required, and we adopt as the first link in the chain of phenomena the thermoelectric effect between two dissimilar metals. The electromotive force obtained is a known function of the temperature. If a workshop instrument is required the use of complicated potentiometer methods must not be used, and the next step is to convert the electromotive force into a current, which is measured by converting it into a force by a galvanometer. This force is measured by a spring, and the temperature is read by the movement of a pointer over a scale.

In this example the five steps in the chain are—

(i.) Conversion of temperature into electromotive force.

(ii.) Conversion of electromotive force into current.

(iii.) Conversion of current into electromagnetic force.

(iv.) Balancing this electromagnetic force against the force of a deflected spring.

(v.) Measuring the force causing the deflection of the spring by a pointer moving over a scale.

§ (3) Requirements of Good Design.—The following chief requirements of a good design in instruments will be considered in detail: *accuracy, sensitivity, robustness, convenience and rapidity in use, simplicity and cheapness, bad design and good workmanship, durability, damping*. The important question of geometric design will then be considered and the method of procedure in preparing a new design.

§ (4) Accuracy.—An instrument should have the following qualities:

(i.) It should have the required accuracy. The highest accuracy is not always necessary, or even advisable.

(ii.) It should have constant accuracy, not dependent on position in which it is used or the magnitude of the measurement taken.

(iii.) Every important source of inaccuracy or error should be known.

(iv.) If possible each important error should be capable of elimination by taking two or more readings or by adjustment of the instrument without other special apparatus.

(v.) When an error cannot be eliminated it should not vary with circumstances.

(vi.) When an error cannot be eliminated it should, if possible, be capable of measurement by the instrument itself, without other special apparatus, so that the results may be corrected accordingly.

The accuracy which can be obtained is largely dependent on the costliness of the instrument, and there has usually to be a compromise between a simple and cheap design and accuracy. In many instruments there are parts which require good and accurate workmanship, such as a divided circle or micrometer screw; but the accuracy of the results should, as far as possible, be independent of the perfection of the construction in the other parts. The accuracy of any instrument depends upon a great number of errors introduced in many ways. These errors vary greatly in amount; some are very small and some comparatively large. The larger errors are the most important to reduce, and it may be an improvement in the instrument to reduce these errors by making the instrument less simple and more costly. On the other hand, to reduce errors that are already small may actually reduce the usefulness of the instrument by increasing its complication and cost, as in this case the resulting improvement in accuracy may not be perceptible. It may even happen that the inaccuracy of the final reading is increased by reducing one of the errors which is already small, as by so doing another error which is not small may be greatly increased.

Let us consider the accuracy of the pyrometer described above. It depends on the errors introduced at each step. The electromotive force is due to a difference in the temperature of the two junctions of two dissimilar metals or alloys. The pureness of the alloy will cause errors, and an error in knowing the temperature of the cold junction will cause an error in the final readings. Also the hot junction may not be at the temperature to be measured. In the next step the electromotive force produces a current in a resistance which will vary with change of temperature and from other causes, and these errors may be considerable. The magnetic field of the galvanometer may alter and cause errors. The spring will have errors due to fatigue, permanent set, corrosion, and temperature. Friction of the pivot will cause errors in the position of the pointer, and there may be errors in the scale from which readings are taken, and personal errors in reading the position of the pointer on the scale.

§ (5) Sensitivity.—The final accuracy of an instrument partly depends on its sensitivity. In a sensitive instrument a small change in the property to be measured or observed gives a large change in the observed phenomenon. To obtain a certain amount of accuracy the sensitivity must not be too small; but increasing the sensitivity may not increase the accuracy, and a large increase may often diminish it. An instrument with a high degree of sensitivity is harmful, as it gives a false impression of accuracy; and this may seriously mislead the observer.

In order to obtain the required sensitivity in an instrument, optical magnification is often used. This may consist of a high- or low-power microscope or a simple lens. Or mechanical magnification may be used.

Magnification by resonance is sometimes used, for instance, in the measurement of alternating electric currents, and largely in wireless telegraphy.

The phenomenon of resonance, besides being useful to obtain magnification, may be a serious source of trouble and error. The pointer of an instrument may vibrate so much that its position cannot be read.

Another method of obtaining magnification is by using relays or amplifiers, particularly in electrical work. The distinction between these magnifiers is important. In the null method a relay may be useful to magnify the signal which shows out of balance and make it appreciable to the observer. When the result is obtained by reading the amount of a movement an amplifier is useful, but not a relay.

§ (6) Robustness.—An instrument should be robust, and it should not be easily damaged by rough handling; if possible, all parts should be strong enough to withstand accidents and the usual stresses due to transit. It need not be clumsy or unnecessarily massive, but delicacy is essential in some parts of certain instruments. An instrument is sometimes praised because it is delicate, whereas the design should be condemned if it is unnecessarily delicate.

Great sensitivity may be the highest praise that can be given to an instrument, as it can be used to measure very small quantities. A robust instrument can be very sensitive and a delicate instrument far from sensitive; and we may define delicacy as the opposite of robustness. Some instruments must be delicate in order to fulfil their duty, and require great skill in their manufacture. In such instruments it is the skill rather than the design to which the praise is due.

§ (7) Convenience in Use and Need for Dexterity.—An instrument should not require skill or dexterity in its use, and the results should be obtained quickly; but this is less important in a laboratory than in a workshop instrument for commercial use. If possible, an instrument should be fit to bear rough handling. Safety devices may be introduced which make improper manipulation impossible, or protect the instrument from their bad effect. For instance, the coils of galvanometers may be automatically clamped by the shutting of the covers, or by the act of lifting the instrument from the table, preventing the suspensions from being broken, or the pivots being damaged.

Parts which require manipulation should be easy to handle, and movements should be without backlash, so that no special care is required to avoid errors from this source. Any parts that require lubrication should be accessible. Scales should be easy to read, and parts which may be damaged by dust or dirt should be covered, or they should be easy to clean. The design of the instrument should differ according to the skill of the observer who will use it and the place where it will be used. The method of taking observations should be easy and the process pleasant to perform, not trying to the eyes, and as little fatiguing as possible. Fatigue is bad in itself; and a tired observer will increase errors in the results.

§ (8) Simplicity and Cheapness.—In order to reduce the cost it must be known who is to use the instrument, where it is to be used, and for what particular purpose it is required. An experimenter may require apparatus for a special piece of new work, and often good results are obtained with roughly made apparatus when in skilful hands. Its design requires great knowledge and skill, and the greater the ability of the experimenter the simpler the apparatus will be. Probably most instruments were first made for use in this way.

The manufacturer of scientific instruments has a different problem. The design should differ according to the number required. The method of manufacture, and consequently the design, should depend on whether the number required is very large, considerable, or very small. The designer should be a mechanical engineer with much scientific knowledge. He should be well acquainted with the methods of manufacture available, and, in order to avoid unnecessary cost, the instrument should not require great skill to make. He should be familiar with the properties of many materials, and he should know the accuracy of the machine tools available and the skill of the workmen employed.

It is often most difficult to compare the relative merits of the several methods of making one part of an instrument. It perhaps might be of cast iron or gun-metal, or a forging, or built up of plates, rods, or tubes. The advantages of the cast iron in hardness, strength, stiffness, and cheapness must be compared with ease of working and absence of corrosion in the case of gun-metal. Iron often has the advantage, and is more used than was formerly the case. Then, again, a considerable increase of cost, or a slight increase of sensitivity or accuracy, must be compared. Castings must be suitable for the foundry; holes must be easy to drill, and worked surfaces be in positions to suit the machine tools, and it must be easy to fix accurately the various parts together. Every detail should be examined from many widely different points of view, and the result is generally a compromise, often not easy to arrive at. An expensive instrument, if much used, may lead to real economy, as it may enable results to be obtained more rapidly than a cheaper and slower instrument, and much time will be saved.

§ (9) Bad Design and Good Workmanship.—Some of the evil effects of bad design in an instrument may be reduced by excessive care and costliness in the workmanship. But when this is the case the evil effects will return when the instrument becomes worn or slightly damaged. In a well-designed instrument accurate results can be obtained even if its parts become worn or damaged, and the most skilful workmanship is not required.

§ (10) Durability.—An instrument may be required for one piece of research, after which it may not be used again, but in most cases an instrument is for continued use, and should have a long life. Corrosion and chemical changes should not cause damage. Suitable materials should be employed and the surfaces well protected. Simple readjustment should enable the wear of the rubbing surfaces to be compensated. The wear should also be small, and the instrument should not be easily broken or damaged.

§ (11) Damping.—Many instruments require damping, and the most suitable amount requires consideration. Accurate instantaneous readings of a changing phenomenon may be required. The phenomenon may be the fluctuating electric current in a galvanometer, which has some moving part with inertia. Therefore it cannot indicate the correct instantaneous value of the current. It is only by proper damping that approximately accurate readings can be obtained. If an instrument is used ballistically the damping should be small, and its amount should be known in order that the results may be corrected.

We may wish to measure the mean value of a regularly alternating phenomenon. The phenomenon may be always positive and a true arithmetic mean be wanted, or the phenomenon may be alternating between equal positive and negative values, and the root mean square be wanted. The instrument must be well damped if a steady mean value is to be read, and also the damping must be of correct type to give the true mean value.

The designer must also remember that, apart from considerations of damping, if an instrument is to give approximately correct indications of a fluctuating phenomenon the free period of the instrument undamped must be considerably shorter than the period of the changing phenomenon, or else the lag of the instrument will be so great that the readings are valueless.

§ (12) Geometric Design. This form of design can often be adopted with great advantage in scientific instruments.

A rigid body has 6 degrees of freedom, and if it is desired to have a certain relative movement between two parts corresponding to one degree of freedom it is necessary that there should be a constraint between the two pieces at five points. For example, a rectilinear movement can be given to a moving piece by five points on it in contact with a fixed piece, and each sliding along one of five parallel straight lines on the surface of a fixed piece. The correct shape can be given to the sliding piece very easily, as no accurate machining or fitting of large surfaces is required, but the fixed piece must have an accurate form. This form, however, can usually be cheaply and accurately produced by a machine tool. For instance, four of the straight lines on the fixed piece may be on the surface of a cylinder, a form easy to make in a lathe or grinding machine with great accuracy. The fifth straight line, however, cannot be on the surface of the same cylinder. The actual diameter of the cylinder is unimportant, and no accurate fit is required. Geometric design in an instrument gives movement of great truth with slight and uniform friction and a reduction of cost.

If a solid piece of an instrument is constrained in more than six ways it will be subject to internal stress, and will become distorted, as it is not perfectly rigid. It may be impossible, however, to detect the distortion without the most exact micrometrical measurements. Geometric design reduces these internal strains which cause the bending of parts, thus giving rise to serious errors in badly designed instruments.

In instruments which are exposed to rough usage it may sometimes be advisable to secure a piece from becoming loose, even at the risk of straining and jamming it; but in apparatus for accurate work it is essential that the bearings of every piece should be properly defined, both in number and in position.

Two pieces must remain in contact at the requisite number of points. The forces required can be given by gravity, but in instruments springs are often used as the forces required are small. Much care and thought should be given to the design of springs. Satisfactory action of springs is most important, and it is a common experience to find them the most troublesome part of an instrument to design. Sometimes a single spring will give all the desired constraint, and this is satisfactory when the parts are stiff, but usually it is better to increase the number of springs and so diminish the stresses in the parts. It is usually advisable to have the spring under a fairly constant tension, and often the extension should be considerable and not vary greatly.

We may give a definition of a geometrical design as follows: Two bodies designed to work together with certain relative motion are said to fit geometrically when the number of relative degrees of freedom added to the number of relative points of constraint is exactly six.

Many types of design may be called semi-geometric, in which the bearing surfaces are small surfaces instead of a near approach to points, or are lines of considerable width. A line bearing will support a much greater pressure than a point bearing, although much less than a surface bearing; it is not so simple as a point bearing, but is often simpler to adjust than a surface bearing. For example, theodolites and astronomical instruments are universally supported by geometrical bearings consisting of cylindrical trunnions resting in two parallel Y supports. If the instrument is very light these Y supports may be rounded so as to give four-point contacts with the two trunnions. This is a true geometric design. In heavier instruments each Y-piece will be formed of two planes. In order to secure proper line contact the plane surfaces in the supports must be in a definite position with regard to each other. The design then may be termed semi-geometric.

Not only does geometric design in general secure cheap construction and accurate movement, but backlash is easily avoided between the different parts. The trunnions in the theodolite are held down in the Y supports by springs. There can never be shake in the movement even after much wear of the trunnions and supports.

Clerk Maxwell, in the *Handbook of the Special Loan Collection of Scientific Apparatus* (1876), writes:

"When an instrument is intended to stand in a definite position on a fixed base it must have six bearings, so arranged that if one of the bearings were removed the direction in which the corresponding point of the instrument would be left free to move by the other bearings must be as nearly as possible normal to the tangent plane at the bearing."

This is an important principle, and applies also to moving pieces. The pressure at each bearing is then a minimum, and thus the friction is also at a minimum, giving smooth movement, and risk of jamming is greatly diminished: also springs required to avoid shake and backlash can be smaller and weaker than in other cases. Also any alteration of the distances between the original surfaces due to wear, elastic compression, denting, or dirt between the surfaces, will cause the minimum displacement of the part.

It is usually easy to arrange that a point on a piece of apparatus be adjustable in some direction. For instance, the point may be the end of a projecting screw. If this adjustable point is one of the contact points in a geometric design, adjusting the screw will slightly change the direction of a relative linear movement, or change the axis of a relative rotational movement, or the relative position of a piece which cannot move. This allows cheap construction with a high order of accuracy. This adjustment can also be used to compensate for wear at the rubbing surfaces, securing long life in the instrument without inaccuracies. Good geometric design therefore gives a good method of reducing the fault it possesses of rapid wear due to small bearing surfaces.

The use of the geometric principle also is most helpful in the design of rough apparatus when made by the experimenter himself or in the laboratory workshop.

The advantages of adopting a geometric design are many and very great, but there are cases when it is best to disregard the principle entirely. Although ball bearings are not scientific instruments they sometimes form part of them. They are most useful and successful, and as their design is far from being geometric the consideration of their design is most instructive. Their success depends on good workmanship and excellent material. The balls must be as nearly spherical as possible and of the correct diameter, and the ball races must also be true and of the correct size. The balls and races must be very hard, but not so brittle as to be liable to break. These conditions can only be fulfilled by manufacture on a large scale with good machinery. The interest in this case is that, although the design is not geometric, the bearings are a most valuable invention, and they are now used in large numbers and undoubtedly their use will increase. They require little or no attention, and greatly reduce friction.

§ (13) CONTRAST IN DESIGN OF INSTRUMENTS AND MACHINES.—In instruments the magnification and transposition of displacements is usually required, in machines more often the magnification and transposition of force.

Strength and efficiency are most important in the design of machines, and slight flexibility in its parts is often an advantage, as it relieves strains; but in instruments questions of rigidity may be of the greatest importance. Parts of the instrument often have to be so stiff that they do not bend or deflect a perceptible amount under the small forces to which they are subjected. In machines large bearing surfaces where movement takes place should be provided, both to reduce wear and to avoid too great local pressures between the parts. In instruments, on the other hand, the pressure between moving parts may be so slight that contacts may take place almost at points.

§ (14) ALTERNATIVE METHODS.—When the exact requirements of the instrument are thoroughly realised many chains of physical phenomena should be considered, and the most suitable selected. Alternatives in the mechanical design of details should be similarly considered. A skilful designer will realise that his first design is extremely unlikely to be the best he is capable of making. He should therefore devise alternative designs, consider which is best, and discard the other. The process should be repeated again and again, and the more often it is repeated the better the ultimate result will be. The first design may seem satisfactory, but it is essential that its worth should be tested by comparison with alternatives. Patience is required, and the designer should adopt a frame of mind which will enable him to compare two of his designs as if they were not made by himself. The process of considering alternatives should be adopted down to every detail, and thus the number of alternatives compared becomes very great. In essential parts an alternative can often be made by fixing a part which moved and allowing a part which was fixed to become movable. For instance, a galvanometer can have a moving magnet or a suspended coil; or a clock-driven drum can have the clock itself rotating with the drum or the clock itself can be at rest. The temptation to consider the first invented design the best that can be found is great, and too much stress cannot be laid on the importance of comparing many alternatives. Usually the first conception of the design is complicated, and the development which takes place during the process of comparing alternatives is towards simplicity without loss of accuracy or efficiency.

H. D.
C. C. M.

INTEGRATING MECHANISM (KELVIN) FOR WATER METER. See "Meters for Measurement of Liquids," § (3).

INTEGRATION, MECHANICAL METHODS OF

§ (1) INTRODUCTORY.—Practically all physical investigators rely ultimately on some form of graphical representation as a convenient and suggestive mode of expression for their experimental results. Analysis of the curves then provides the interpretation of the data found. Not the least frequent step in such forms of analysis is the determination of the area included in a closed curve, or, what amounts to the same thing, the area between the curve and some given datum line. No less than the physicist, the engineer is concerned with the measurement of area in a multitude of ways, as in the evaluation of the work done during expansions and contractions in steam cylinders, for example, by integration from indicator pressure diagrams, in the determination of centres of gravity, of buoyancy, and of pressure, and moments of inertia of various figures that arise in the treatment of strength of materials or the pressure of winds or water on immersed surfaces. Mathematically the question arises in the graphical analysis of differential equations which cannot be solved by purely analytic processes. Here usually the quantity required at any step could be written in the form $\int_a^x \phi(x)dx$, implying that what is desired is the area enclosed between the curve $y=\phi(x)$, the axis of x, an ordinate at $x=a$ and an ordinate at all other values of x. The result will, of course, be itself a function of x, and will consequently be represented as a curve. For most experimental and engineering purposes, however, the upper limit of this integral is a definite fixed quantity.

§ (2) ARITHMETICAL METHODS. — Various arithmetical methods have been devised for estimating the area enclosed between the x-axis and a given curve. These are, in general, based on the assumption that for the purpose in question the curve is represented with sufficient accuracy by another passing through a definite number of fixed points on the original curve, usually but not always equally spaced x-wise, and consisting of a series of arcs of simple curves drawn through consecutive groups of such points. Perhaps the best known, and in some respects the simplest, arithmetical expression for the area is that provided by Simpson's Rule, where the arcs are simple parabolas: If the given area be divided into an even number of strips of equal breadth, h, parallel to the axis of y, and if the ordinates of the edges of these strips be consecutively $y_1, y_2, \ldots y_n$, there being, of course, an odd number of such ordinates, then the area is very approximately given by

$$\frac{h}{3}\{y_1+y_n+2(y_3+y_5+\ldots)+4(y_2+y_4+\ldots)\}.$$

A less well known but more accurate formula is due to Gauss, in which the ordinates are measured at places not equally spaced but at positions definitely selected in the range irrespective of the actual form of the curve. As compared with Simpson's Rule for the same number of ordinates, the accuracy of the estimate in area is nearly doubled by the use of this formula. For a curve of fairly continuous shape, speaking generally, a system of seven ordinates, for example, spaced in the special manner to be explained, gives results which differ from a careful planimeter reading by less than ·4 per cent. If the total interval over which the area is required is from $x=0$ to $x=p$, the ordinates are taken at the positions $x_0, x_1, \ldots x_1'$, etc., where

$$\begin{array}{lll} x_0/p=\cdot 5 & x_1/p=\cdot 9745 & x_1'/p=\cdot 0255 \\ & x_2/p=\cdot 8707 & x_2'/p=\cdot 1293 \\ & x_3/p=\cdot 7209 & x_3'/p=\cdot 2791. \end{array}$$

Let the actual corresponding ordinates be y_0, y_1, y_1', y_2, y_2', y_3, y_3'. These ordinates, be it noted, are symmetrically spaced about the central one y_0. The area A is then given by

$$\frac{A}{p}=\cdot 2090y_0+\cdot 0647(y_1+y_1')+\cdot 1399(y_2+y_2')$$
$$+\cdot 1909(y_3+y_3'),$$

an estimate as accurate as that obtained by taking thirteen ordinates and using Simpson's Rule.[1]

§ (3) PLANIMETERS. *Theory.*—To avoid the not infrequent heavy computation involved in such arithmetical methods of evaluating areas, many mechanical contrivances have been designed with a view to the direct estimation of the area by sweeping it out or tracing the enclosing curve. Such instruments are known generally as planimeters. Although historically most planimeters have been designed to follow principles presumed special to the particular instrument, the general theory of the various types of planimeter is included in the following theorem concerning the area swept out by a moving line.

Consider the two curves C and C′ (*Fig.* 1), enclosing the two areas S and S′, and let a line PP′ of fixed length l move round the curves so that P and P′ trace out the whole contours C and C′ respectively. PP′ will then sweep out a total area S − S′ in making the complete circuit, whether C′ is entirely interior or exterior to C. If PP′ and QQ′ be any two consecutive positions of the line, the inclination between them being the small angle $\delta\phi=\angle RQ'Q$, where RQ′ is parallel to PP′, then the area δA of the small quadrilateral PP′Q′Q swept out by the line is equal to the sum of the area of the small rectangle PP′Q′R and the triangle RQ′Q.

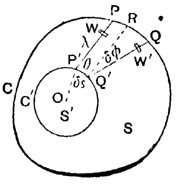


FIG. 1.

Let $P'Q'=\delta s$, and $\theta=$ angle between P′P and the tangent at P′, that is between PP′ and P′Q′ ultimately, then area of $PP'Q'R=l\delta s\sin\theta$, and of $RQ'Q=\frac{1}{2}l^2\delta\phi$,

$$\therefore\ \delta A=l\delta s\sin\theta+\tfrac{1}{2}l^2\delta\phi.$$

Integrating this expression over the complete contour,

$$S-S'=l\int ds\sin\theta+\tfrac{1}{2}l^2\int d\phi. \quad . \quad (1)$$

Two cases must be considered. When the curve C completely encloses C′, PP′ moves always in the same direction round C′ and ϕ increases from zero to 2π as the complete circuit is made. In this case, therefore,

$$S-S'=l\int ds\sin\theta+\pi l^2. \quad . \quad . \quad (2)$$

When C and C′ are quite external to one another PP′ returns back upon itself as the circuit is completed, so that $\int d\phi=0$; in this case

$$S-S'=l\int ds\sin\theta. \quad . \quad . \quad (3)$$

§ (4). The apparently diverse principles upon which the various types of integrating mechanisms are based are all particular cases of the foregoing general theorem. Let C′ be a given curve of known area S′, and let C be the curve whose area S is required, then, from the above formula, S is definitely calculable provided the quantity $\int ds\sin\theta$ can be evaluated. As will be seen presently, a simple mechanical method can easily be devised for evaluating this quantity directly, while a rod of definite fixed length is guided round with its extremities on C and C′. In the main, planimeters differ simply in the choice which is made of the basic curve C′ and the consequent differences in mechanical detail.

§ (5) PLANIMETERS. *The Registering Mechanism.*—The simple standard mechanical device whereby the quantity $\int ds\sin\theta$ is measured is as follows. Let a graduated wheel W (*Fig.* 1) be fixed with PP′ as axis, or on an axis parallel and rigidly attached to PP′, so that the wheel rests lightly and can roll on the chart. Suppose the rolling edge of the wheel is at a distance λ from P′, then for any small displacement of PP′ to QQ′ the roller will slip along the component of its path in the direction PP′ and roll for the component at right angles to PP′. Now the distance travelled by the roller W at right angles to PP′ to its position W′ is $\delta s \sin\theta + \lambda\delta\phi$, which is therefore the distance δR rolled by the edge of the wheel. Thus

---

[1] "On Gauss's Theorem for Quadrature and the Approximate Evaluation of Definite Integrals with Finite Limits," by A. R. Forsyth, F.R.S., *Brit. Assoc. Reports on the State of Science*, 1919.

$$\delta R = \delta s \sin\theta + \lambda\delta\phi.$$

For the complete circuit of the curves the total distance R registered on the graduated wheel is

$$R = \int ds \sin\theta + \lambda\int d\phi.$$

As before, the last term is either zero or $2\pi\lambda$, according as C′ is wholly external or internal to C. From this it follows that the rolling wheel, if appropriately graduated, will automatically register the actual integral required. If, therefore, P′ is constrained by the mechanism to move round the curve C′ which encloses a known area, while the other extremity P of the rod traces out the curve whose area is required, this required area will be immediately deducible from the reading registered on the integrating wheel.

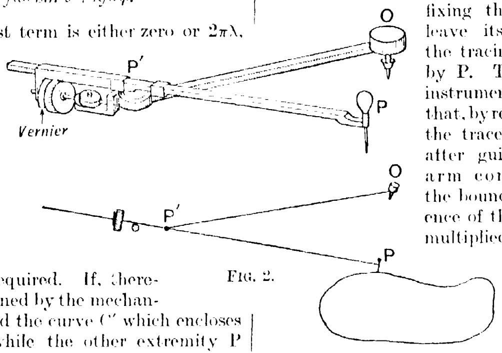


FIG. 2.

§ (6). The two main types of planimeter in common use depend in principle on two particular cases of the general theorem given in § (3). The general class of *Polar Planimeter*, of which perhaps the best known are the Amsler types, corresponds to the case in which C′, the guiding curve, is a circle, while in the *Linear Planimeter* type the guiding curve is a straight line, or rather a closed curve consisting of a straight line returning on itself.

§ (7) POLAR PLANIMETER.—If two rods OP′ and P′P (*Figs.* 1 and 2) of fixed lengths be jointed at P′ while O is fixed in position so that the whole mechanism can turn freely about O as centre, then when the tracing point P is guided round the curve C whose area is required, the point P′ will trace out the circle C′ with O, the pole, as centre. If in addition a recording wheel, of the type already referred to, be fitted to P′P with axis parallel to P′P, the fundamental elements of the Amsler planimeter are provided; any other elements in the instrument proper are mere minor additions to be dealt with later. In this case, if the length of OP′ be r, and as above R be the distance by the tracing wheel rolled through, then the formula for S, the area of a curve external to S′, is $S = lR$, since the point P′ will trace out an arc of C′ and return on itself, so that in effect $S' = 0$, while, when S encloses S′, C′ is completely traced out, the value of $S' = \pi r^2$ while $\int ds \sin\theta = R - 2\pi\lambda$, and

$$S = \pi r^2 + lR - 2\pi\lambda l + \pi l^2.$$

Usually the pole O is selected so that C′ is external to C.

An illustration of the Amsler planimeter is shown in *Fig.* 2, with a diagram indicating the manner in which it is used. A small weight is usually provided, and rests on the rod at O to ensure that a sharp point pressing into the paper at O, and fixing the pole, does not leave its position during the tracing of the contour by P. The details of the instrument are so adjusted that, by reading the roller on the tracer arm before and after guiding the tracing arm completely round the boundary C, the difference of the readings, when multiplied by a constant factor, directly provides a measure of the area described.

§ (8) LINEAR PLANIMETERS.

General Principles.—The second class of planimeter, the linear type, is constructed to have the curve C′ a straight line, so that in tracing out the curve C with one end of a rod of fixed length the other end is constrained to move along C′, only a finite portion of the straight line being traced out, since the constrained end returns on itself to the starting-point. The area S′ is consequently zero, and the general formula (1) takes the simple form $S = l\int ds \sin\theta$.

The same device as before, in the form of a recording wheel, will evaluate this integral. Since many modifications of the planimeter designed to determine moments of inertia, etc., are based on this form of instrument, it is advisable to develop the comparatively simple theory of this type without recourse to the general formula.

Let OP represent a rod of fixed length l (*Fig.* 3), the point O being constrained to slide along the guide bar XOX. Consider what effect is produced on the recording wheel W whose axis is OP, as the tracing point P describes the rectangle PQQ′P′. As the tracing point moves from P to Q a certain reading will be recorded on the wheel W.

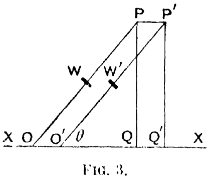


FIG. 3.

Along QQ' the direction of motion of the tracing point is along the direction of the axis of the wheel, so that the latter does not roll and no additional reading will be recorded. From Q' to P' the recording wheel will exactly reverse the reading registered during the passage from P to Q, since all the previous conditions are duplicated, except that O will have travelled to O'. The description of the three sides PQQ'P' will therefore leave the reading at P' identical with that at P. From P' to P, however, a definite reading will be *recorded*, and this it is necessary to estimate exactly. Since the rolling of W is merely the component motion of P perpendicular to OP, the distance rolled by W is $OO'\sin\theta = PP'\sin\theta = PP' . PQ/OP$. Accordingly the wheel W during the motion of the point P directly from P' to P, or along PQQ'P'P, registers a distance equal to PP' . PQ/OP, that is, equal to the area of the rectangle PQQ'P' divided by l. It is to be noticed that the reading is thus independent of the position of the recording wheel along the turning arm OP. Such a simple planimeter, therefore, will give a measure of the area of any rectangle PP'QQ' by the difference in reading in passing from P to P'. If this rectangle be regarded as one element of the area between any curve AB (*Fig.* 4) and the guiding axis OX, the whole area between the curve and that axis will be found by running the planimeter tracing point from C to A, then along the series of elements of arc such as PP', that is, along the curve AB, and finally down from B to D. It is clear that for the present purpose it is immaterial whether the tracing point be taken from C to A and finally from B to D, or from C' to A and finally from B to D', so long as C' and D' are equidistant from OX, since the reading along CC' neutralises that along D'D. Accordingly, if A and B be two points on the closed curve APBR equidistant from OX, on guiding the tracing point along ARB the area CARB will be registered, while on returning along BPA the area DBAC will be registered, in the opposite direction, however, so that the net effect of tracing out the contour curve ARBPA will be to provide a reading on the wheel equivalent to the area enclosed.

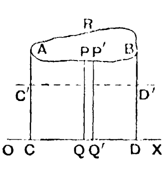


FIG. 4.

§ (9) LINEAR PLANIMETERS.—These instruments as a class are based essentially on the foregoing principles, and differ among themselves only in the details that render particular forms more suitable to special kinds of diagrams than others. The Coffin planimeter, for example, is perhaps the simplest form of the linear type, and is much used for indicator diagrams, for which it is specially adapted. It consists of little more than the guide rail, tracer arm, and recording wheel as described above. A more elaborate form of linear planimeter is shown in *Fig.* 5, a type, as will shortly be seen, that lends itself easily to modifications that enable moments of inertia, etc., also to be measured.

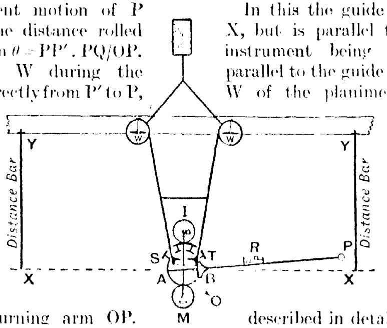


FIG. 5.

In this the guide bar is not the axis of X, but is parallel to it, the pole of the instrument being constrained to move parallel to the guide bar by the two wheels W of the planimeter which run along grooves in the bar. By this means the region in the neighbourhood of the pole is free of the guide bar and enables modifications for moments, etc., to be easily added. This planimeter will be described in detail later.

§ (10) ACCURACY, ERRORS, AND SOURCES OF ERROR.—In planimeters of the types under discussion there are two main sources of error; they are that due to any lack of parallelism between the axis of the recording wheel and the tracing arm, and the slipping instead of rolling of this wheel due to frictional causes or unevenness in the chart paper.

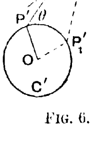


FIG. 6.

As far as first order errors are concerned, due to the former causes, certain modifications in the Amsler Planimeter, giving what are called compensating planimeters, enable these to be eliminated with comparative ease. If the angle between the axis of the wheel and P'P, the tracing arm (*Fig.* 6), be a small quantity α, then the reading recorded by the wheel will be $ds\sin(\theta-\alpha)$ instead of $ds\sin\theta$ for a small displacement PQ. For the position on the circle C' symmetrical about the position OP the reading due to an equal small displacement will be $ds \sin(\pi - \theta + \alpha)$ instead of $ds \sin(\pi - \theta)$. In tracing the curve C, therefore, starting with the first position the total reading will be $\int ds \sin(\theta - \alpha)$, while with the second position it is $\int ds \sin(\pi - \theta + \alpha) = \int ds \sin(\theta + \alpha)$. If the arithmetic mean of these two readings be taken, the result will therefore be

$$\tfrac{1}{2}\int ds\,[\sin(\theta - \alpha) + \sin(\theta + \alpha)] = \int ds \cos\alpha \sin\theta$$

$$= \int ds \sin\theta - \frac{\alpha^2}{2}\int ds \sin\theta + \ldots ,$$

indicating that the mean of the two readings gives a result which differs from the true readings $ds \sin\theta$ only by a quantity of the second order in α. To conduct this double operation necessarily involves taking two readings, one with the pole to the left and one with the pole to the right of the tracer arm. This is not possible with the ordinary simple Amsler type, since the tracer arm is mounted above the pole arm so that the range of the tracer arm is restricted (cf. *Fig.* 2), but simple mechanical changes have been introduced to facilitate this, producing what are called compensating planimeters.

As regards the second source of error, that occasioned by the slipping or jumping of the integrating wheel over unavoidable unevenness or crinkling on the paper, a special form of instrument designed to minimise this effect, if not actually to eliminate it, has been devised. In this modified form the recording wheel, instead of rolling directly on the paper, is mounted to roll in contact with a smooth horizontal disc which is geared to a larger wheel rolling on the diagram. The use of this type is rather limited, and for normal work the simpler form generally suffices.

Quite apart from the fineness of adjustment of the various parts of a planimeter and the smoothness or otherwise of the paper, the accuracy of the instrument depends very materially on the possible accuracy in reading the wheel. This is increased by two methods: in the first place a fine vernier (*Fig.* 2) is usually fitted alongside the recording wheel to allow fractions of a small division to be accurately read; in the second place, since the distance traversed by the wheel, and therefore the reading on the wheel, will increase with the length of the tracing arm, the accuracy of measurement of small areas may be considerably enhanced by a lengthening of this arm. This is accomplished by sliding this piece through a socket at P′ (*Fig.* 2), an arrangement moreover that enables the length to be so adjusted as to provide the reading directly in various alternative units, *e.g.* square inches or square centimetres. A verification of the setting is normally made by means of a simple accessory provided in the form of a flat metal strip at one end of which, and at right angles to the flat face of which, is a needle point. This point, when pressed into the paper, furnishes a centre about which the strip may rotate. If the tracing point of the planimeter is inserted into one of a series of holes drilled through the strip at various graduated marks along its length, the planimeter can be guided to trace very accurately a circle of known area, and the reading on the roller of the instrument may be compared with this value.

While lack of careful adjustment of the various parts of a planimeter and friction at the junction may seriously militate against the accuracy and reliability of a planimeter, nevertheless in general a very high standard of accuracy is easily attained, higher certainly, as far as most experimental applications are concerned, than the diagrams themselves warrant. Extensive and careful tests by numerous observers have brought out the limits of accuracy of the instrument. In the case of the polar type, Amsler found that the maximum error occurs when the direction of motion is inclined at an angle of 45° to the axis of the integrating wheel, and of magnitude ·1 per cent at most. With a disc type of planimeter he found that on making a series of successive readings equivalent to 130 revolutions of the disc, the ratio of disc and roller readings did not vary in the worst case by more than 0·0003, and generally by less than 0·000001.

It is obvious from these figures that the planimeter is an instrument of an exceptionally high degree of accuracy, and for most practical purposes of an engineering or physical nature may be presumed to introduce errors of much smaller magnitude than those involved in the plotting and sketching of a graph.

§ (11) THE RADIAL AVERAGING INSTRUMENT.—Mention may here be made of an exceedingly simple instrument known as the *Radial Averaging Instrument*. Strictly regarded, it is perhaps not a planimeter in the ordinary sense of an instrument for measuring areas; its more direct application is to determine the mean value of a series of quantities such as are plotted on a polar diagram. It is, however, usually contained as a component part of the *Universal Planimeter*, designed for the computation of areas and mean ordinates of diagrams of self-recording instruments, drawn either on strips or on circular charts. The averaging instrument consists essentially of an ordinary tracer arm grooved on the lower surface and resting on the ball-shaped head of a centre pin fixed at the centre of the radial chart. By this means, when the tracing point passes over the curve, making one complete circuit, the arm lengthens or shortens by sliding on the head in the groove. The normal planimeter recording wheel on the tracer arm with its axis parallel to the latter gives the reading in the ordinary way. If the registration is less or greater than one round of the chart, the multiplying constant must be accordingly altered. The theory of the instrument is simple. If $r_1, r_2 \ldots r_n$ be the n values of r corresponding to n readings spaced equally at intervals $\delta\theta$ round the whole circuit, then $n\delta\theta = 2\pi$. The mean value of r is

$$\frac{(r_1 + r_2 + \ldots r_n)}{n} = \frac{\Sigma r_n\delta\theta}{2\pi} \rightarrow \frac{1}{2\pi}\int_0^{2\pi} r d\theta.$$

Since the axis of the recording wheel is parallel to the radius vector, the former will register only the components perpendicular to r of all motions of the tracing point derived by running round the curve. For an elementary displacement of angle $\delta\theta$, when the radius vector varies from r to $r + \delta r$, this component is $r\delta\theta$; hence the total reading recorded on the roller is $\int r d\theta$, from which the required mean value is at once obtained.

§ (12) INTEGROMETERS.—The expression integrometer is a general term for any integrating instrument, and as such embraces planimeters designed purely for the measurement of area. In general usage, however, it is restricted to apply to what are also termed moment planimeters, that is to say, instruments for evaluating the moment and the moment of inertia about a given line of the area enclosed by a given curve. These quantities occur, of course, very frequently in most branches of engineering design, and whenever centres of gravity, of pressure, etc. are required.

If the ordinate of the curve in question above the given datum line XX, chosen for the purpose to coincide with the axis about which the moments, etc. are required, is represented by y, then the area, moment of area, and moment of inertia may be represented symbolically as $\int y dx$, $\frac{1}{2}\int y^2 dx$, and $\frac{1}{3}\int y^3 dx$ respectively, the integral in all cases being taken right round the closed curve. The evaluation of the first integral has already been accomplished by means of the linear planimeter, where one end of the tracing arm is constrained to move along XX by means of a guide bar, an integrating wheel being attached at some intermediate point on the tracing arm. The remaining two integrals can be evaluated quite easily by a comparatively simple development of this instrument. The requirements and nature of this additional device will be obvious from the following elementary theory.

§ (13) MOMENT OF AREA.—If OP be a rod (*Figs.* 3 and 4) of constant length l, constrained at O to travel along XX, while P traces out the curve C, then for any position $y = l \sin\theta$. The integral corresponding to the moment of the area consequently requires the evaluation of $\int y^2 dx = l^2\int \sin^2\theta dx = \frac{1}{2}l^2\int dx(1 - \cos 2\theta)$, the integral being taken right round the curve. But on making such a complete circuit $\int dx = 0$,

$$\therefore \text{Moment of area} = -\tfrac{1}{4}l^2\int dx \cos 2\theta$$

$$= -\tfrac{1}{4}l^2\int dx \sin\left(\frac{\pi}{2} - 2\theta\right).$$

Now it has already been seen that when an integrating wheel is attached to OP and rolls on the diagram so that its axis makes an angle θ with XX, its reading will record $\int ds \sin\theta$. In the same way, if a recording wheel could be attached to OP in such a way that its axis made an angle of $\pi/2 - 2\theta$ with XX the value of the integral would be registered directly on the wheel. Such are the mechanical requirements of an instrument for measurement of moments of area.

§ (14) MOMENT OF INERTIA.—For the estimation of moments of inertia, the quantity to be evaluated is

$$\tfrac{1}{3}\int y^3 dx = \frac{l^3}{3}\int \sin^3\theta dx = \frac{l^3}{4}\int dx \sin\theta - \frac{l^3}{12}\int dx \sin 3\theta.$$

Now the ordinary integrating wheel for measurement of area will record $\int dx \sin\theta$, while if in addition a wheel could be attached so that its axis made an angle 3θ with XX when OP made an angle θ, the reading on such a wheel would provide the last integral.

The readings thus obtained would provide the requisite data for estimating the moment of inertia of the area. In the Amsler integrometers all three devices—for the measurement of area, of moment of area, and moment of inertia—are incorporated in the one instrument, although there is a smaller form of the same instrument where only the first two are embodied. The integrating wheel giving the area rolls on the paper, the remaining wheels rolling on discs in a manner similar to the disc planimeter. *Fig.* 5 gives a diagram of the Amsler integrator. YY is a guide rail so adjusted as to ensure that O, the centre of rotation of the tracing arm OP, lies on the axis of moments XX, the adjustment being effected by means of the two distance bars which at one end run in a slot in the guide rail. A simple integrating wheel R on OP running on the chart provides $\int ds \sin\theta$ in the usual manner, and consequently measures the area of the curve traced out. Rigidly attached to the rod OP is a frame consisting of a circle AB of radius $2a$, say, and a portion ST of a circle of radius $3a$, both with their centres at O.

An inclination θ to OP causes the circular frame to rotate about O through an angle θ. Geared to the arc of radius $3a$ is a circular disc I of radius a, and to the arc of radius $2a$

another circular disc M, also of radius a. When OP, consequently, turns through an angle θ, M will turn through an angle 2θ, and I through 3θ. In M and I simple recording wheels are set with their axes in the plane of the discs, and such that when OP lies along XX they are respectively perpendicular to and parallel to OP. It follows that when P traces out the curve, R will record $\int ds \sin\theta$, the roller on M will record $\int ds \sin(\pi/2 - 2\theta)$, *i.e.* $\int dx \cos 2\theta$, and the roller on I will record $\int dx \sin 3\theta$, the three integrals which, as has been seen, are required for the measurement of moment, and of moment of inertia.

§ (15) INTEGRAPHS.—While the instruments for the mechanical evaluation of areas and moments in common use in engineering and physical laboratories consist usually of planimeters and integrometers, there is a class of instrument known as an integraph whose function is to graph the integral curve directly on the chart. The earliest form of integraph is apparently due to Abdank-Abakanowicz (1878), and through the researches of Professor Pascal of Naples a number of integraphs have been invented for special purposes. It is proposed here merely to explain the principles upon which two of these depend.

§ (16) ABDANK-ABAKANOWICZ INTEGRAPH.—This instrument is designed to trace the curve $y = \int dx f(x)$ from the known curve $y = f(x)$. PQRS is a frame whose base (*Fig.* 7) RS is constrained to travel along the guide XX, while the variable point C on QS traces out the given curve $y = f(x)$. At any position therefore $SC = f(x)$. O is a fixed point in RS at unit distance from S. A variable point C′ in RP fitted with an integrating wheel is constrained by a suitable mechanism always to move parallel to the rod OC. The tangent to the curve traced out by the integrating wheel at C′ will therefore be parallel to OC; that is to say, for the curve at C′

$$\frac{dy}{dx} = \frac{SC}{OS} = f(x),$$

$$\therefore y = \int dx f(x);$$

consequently C′ traces out a curve, the ordinates of which give the required integral $\int dx f(x)$.

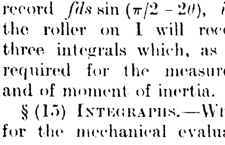


FIG. 7.

§ (17) PASCAL INTEGRAPH.—A more general form of integraph, invented by Professor Pascal, enables the solution of the differential equation $\lambda dy/dx + y = f(x)$ to be traced as a curve directly from the graph of $f(x)$. Once more PQRS (*Fig.* 7) is a frame capable of being adjusted to a breadth λ and constrained as before to slide along XX. The graph of $y = f(x)$ meets SQ in C. The rod CC_1 is slotted at C to run easily in any position through SQ, while at C_1 there is an integrating wheel whose axis is at right angles to CC_1. It follows that as the frame slides along XX and C traces out the curve $y = f(x)$, the tangent to the curve traced out by C_1 will always be C_1C. The path of C_1 is the integral curve, for if (x, y) be the co-ordinates of C_1 on the curve traced out

$$\frac{dy}{dx} = \text{slope of } C_1C = \frac{SC - RC_1}{RS} = \frac{f(x) - y}{\lambda}.$$

Hence the co-ordinates of C_1 satisfy the equation

$$\lambda\frac{dy}{dx} + y = f(x).$$

The particular curve of the series of integrals required in any particular case is, of course, determined by fixing the position of C_1 or C′, as the case may be, to satisfy the initial conditions.

Neither of the instruments whose principles have been described in this and the preceding paragraph have come into general use to any great extent.

§ (18) POLAR INTEGRAPH.—Another instrument, the *Polar Integraph*, has also been invented by Professor Pascal, which aims at constructing a curve in polar co-ordinates whose radius vector at any point is the value of the integral $\int r d\theta$, where $r = f(\theta)$ is a given polar curve.

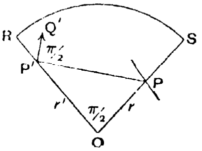


FIG. 8.

The frame ORS (*Fig.* 8) in the shape of a circular sector—for illustration, the particular case of a quadrant is taken—is capable of rotation about the fixed pole O, and runs on a heavy wheel at S. A variable point P on OS traces out the curve $r = f(\theta)$ as the sector rotates. At P′, the point of intersection of the rod PP′ with OR, is a tracing wheel whose axis makes a constant angle—taken here for illustration as zero—with PP′. Here PP′ coincides with the axis of the wheel and therefore will always be normal to the curve traced out by the wheel at P′. The tangent at P′ is P′Q′. Then if $OP' = r'$ and $OP = r$,

$$\tan OP'Q' = r'\frac{d(\theta + \pi/2)}{dr'} = r'\frac{d\theta}{dr'},$$

from the ordinary expression for the angle between the radius vector and the tangent. But $\tan OP'Q' = \cot OP'P = r'/r$,

$$\therefore \frac{r'}{r} = r'\frac{d\theta}{dr'},$$

$$\therefore \frac{1}{r} = \frac{d\theta}{dr'},$$

$$\therefore r' = \int r\,d\theta = \int f(\theta)\,d\theta.$$

It follows that the radius vector of the curve traced out by the wheel at P′ is the integral with respect to θ of the polar curve $r = f(\theta)$.

REFERENCES TO PUBLICATIONS ON PLANIMETERS

(1) "Mechanical Integrators," by Hele Shaw, *Min. Proc. Inst. C.E.*, 1885, p. 75.

(2) "The Theory of Continuous Calculating Machines and of a Mechanism of this Class on a New Principle," by Hele Shaw, *Phil. Trans. R.S.*, 1885, clxxvi. ii. 367.

(3) *Amsler Planimeter Report*, *Brit. Assoc.*, 1892, p. 40.

(4) "Report on Planimeters," Henrici, *Brit. Assoc. Report*, 1894, p. 496.

(5) O. Lang, *Zeitschrift fur Vermessungswesen*, 1894, vol. xii.

(6) F. W. Hill, *Phil. Mag.*, 1894, xxxviii. 265.

(7) "Über das Roll Planimeter von Coradi," Franz Lorber, *Zeitschrift des Osterreichen Ingenieur- und Architecten-Vereins*, xxxvi. 135.

(8) *Les Appareils d'intégration*, by Morin, 1913.

(9) A. O. Allan, *Phil. Mag.*, April 1914, p. 643.

(10) *Handbook of the Napier Tercentenary Exhibition*, Math. Lab. Instruments, p. 181 *et seq.*

HY. L.

INTERCHANGEABILITY OF STANDARDS, partial and universal. See "Metrology," § (20).

INTERFERENCE FITS: definition of term. See "Metrology," § (29) (ii.) (*b*).

INTERNATIONAL METRIC STANDARD THREAD. See "Gauges," § (49).

INVAR: a material with low coefficient of expansion. See "Metrology," § (4).

And other nickel steel alloys: general discussion of properties. See "Line Standards," § (4).

Composition, coefficient of expansion: its anomalies of expansion, its instability, method of ageing, etc. See *ibid.* § (5).

See also "Invar and Elinvar," Vol. V.

INVERSIONS IN THE ATMOSPHERE. Regions in which temperature increases with height, occurrence of, in the atmosphere. See "Atmosphere, Thermodynamics of the," §§ (5), (7). See also "Atmosphere, Physics of," §§ (5), (13).

In meteorology the word "inversion" usually refers to an inversion of the lapse-rate of temperature, the temperature increasing instead of decreasing with height. Inversions near the ground are frequent in winter. They occur on clear calm nights, and are the result of radiation from the ground into outer space continued through the long nights. Similar inversions occur at the top of stratus clouds, and occasionally inversions occur in the troposphere not in association with clouds. The latter do not extend over a great range of height. At the base of the stratosphere there is frequently an increase of temperature with height, and for this reason the stratosphere is sometimes called the "upper inversion."

IONISATION IN THE ATMOSPHERE. See "Atmospheric Electricity," § (10).

IONS:

Number and mobility of, in atmosphere. See "Atmospheric Electricity," § (12).

Source of, in atmosphere. See *ibid.* § (13).

IRIDESCENT CLOUDS. See "Meteorological Optics," § (15) (ii.).

IRREVERSIBLE CHANGES OF NICKEL STEEL ALLOYS. See "Line Standards," § (4) (ii.) and (iii.). See also "Invar and Elinvar," Vol. V.

ISALLOBARS: lines showing equal change of pressure. See "Atmosphere, Physics of," § (20).

ISENTROPIC, without change of entropy. See "Entropy" and "Adiabatic."

ISOBARS: lines of equal pressure.

Isobars are lines on a chart showing equal barometric pressure at mean sea-level. It is important to note that the pressures read from the barometer are corrected for the height above mean sea-level and for variation of gravity with latitude before they are plotted on the synoptic chart. The drawing of isobars is strictly analogous to drawing contour lines on a map. For a discussion of types of isobars see article "Atmosphere, Physics of the," § (17).

Calculation of flow of air over, for different distributions of velocity. See "Atmosphere, Thermodynamics of the," § (16). See also "Pressure."

Straight. See "Atmosphere, Physics of," § (18) (vii.).

Types of. See *ibid.* § (18).

ISOGRAPH. See "Draughting Devices," p. 272.

ISOTASY: Sir S. G. Burrard. See "Gravity Survey," § (15) (ii.).

Hayford's theory. See *ibid.* § (15) (i.).

Professor Love See *ibid.* § (15) (iii.).

— J —

Jordan Sunshine-recorder. See "Meteorological Instruments," § (25). See also "Radiant Heat and its Spectrum Distribution," § (2).

— K —

Kata Thermometer. See "Humidity," II. § (11).

Kater's Pendulum. See "Gravity Survey," § (2) (i.).

Kennedy Meter. See "Meters for Measurement of Liquids," § (1).

Kilogramme: the metric unit of mass, originally defined in terms of a cubic decimetre of water. See "Volume, Measurements of," § (1).

International prototype, material of: platinum-iridium alloy (10 per cent iridium). See "Balances," § (8).

Kinetic Energy:

Of the general circulation of the atmosphere. See "Atmosphere, Thermodynamics of the," § (9).

Of cyclones. See *ibid.* § (26).

Kites, as used for the Investigation of the Upper Air. See "Air, Investigation of Upper," § (2).

Knife-edges of Equi-arm Balance. See "Balances," § (1) (iii.).

— L —

Lapse: a word used to denote the variation of any element of the atmosphere with height. It corresponds with "gradient," which in meteorology is used to denote variation in a horizontal direction. See "Pressure," "Temperature," etc. See also "Air, Investigation of the Upper," § (11).

Lapse-rate of Temperature. The rate of change of temperature per unit of vertical height is called the lapse-rate of temperature, the term gradient of temperature being retained for variations in the horizontal plane.

Latent Heat of Water and Water-vapour. See "Atmosphere, Thermodynamics of the," § (2). For measurement of. See "Latent Heat," Vol. I.

Laths for Curve Drawing. See "Draughting Devices," p. 272.

Latitude, Determination of, by meridian altitudes. See "Latitude, Longitude, and Azimuth, by Observation in the Field," § (6).

By observation in the field, the time being known. See *ibid.* § (4).

Prismatic astrolabe method. See "Gravity Survey," § (8) (ii.).

Talcott method. See *ibid.* § (8) (i.).

LATITUDE, LONGITUDE, AND AZIMUTH BY OBSERVATION IN THE FIELD

§ (1) **Instruments and Methods.**—In order that any survey may be placed upon the earth in its true position and orientation we require the latitude and longitude of one point and the azimuth of one line. These must be determined by astronomical observation in the field. The instruments used are—

(*a*) The sextant, necessary at sea, but never to be used on land if a theodolite is available, unless concealment must be practised. The sextant used with artificial horizon cannot measure altitudes greater than 65°, and is therefore useless for noon observation of the sun in low latitudes; and it cannot measure azimuths without complication and inconvenience.

(*b*) The theodolite, the normal instrument for survey, which for astronomical work should be fitted with dark glasses for sun, a sensitive level (1 div. = 5″) on the microscope or vernier arm, and diagonal eyepiece for altitudes higher than 45°. The five-inch micrometer theodolite is the most suitable for general use in the field, though in hot still weather convection currents by unequal heating disturb the microscope adjustments, and verniers may give as good results, except for the difficulty of illuminating them at night.

(*c*) The zenith telescope, or the theodolite fitted with an eyepiece micrometer, for measuring small differences of zenith distance, is used in precise latitude work, in first order surveys.

(*d*) The prismatic astrolabe is much used by the French, and lately by the Survey of Egypt, for latitude and time, but cannot determine azimuth.

(*e*) Half-chronometer watches are carried in the field in preference to box chronometers.

(*f*) Field sets for the receipt of wireless time signals have immensely simplified and improved the accuracy of field determinations of longitude.

(*g*) Logarithm tables of the trigonometrical functions to single second of arc are essential to rapid work: those of Bagay or of Peters are the best.

The tendency of modern practice has been to break away from the text-book methods of determining position in the field, and it is difficult to specify those that should now be considered the standard methods of observation. All methods must be arranged to eliminate the errors of adjustment of the instruments, and the text-book method of measuring an altitude with the theodolite requires that the instrument shall be reversed after one observation and a second made with face of the vertical circle on the opposite side: the mean of the two, which is free from errors of collimation and zero, being reckoned as one complete observation, and one half useless without the other. There will, however, still remain in the corrected altitude any errors due to flexure (or more probably shake in the telescope of the theodolite), to personality of bisection, and to systematic errors in the calculated refraction; to eliminate which it is necessary to observe a second star on the opposite side of the zenith, so that these errors shall enter into the result with opposite sign, and be eliminated also in the mean of the two. But if this is necessary, then the necessity for changing face really disappears, because the errors which it eliminates are equally eliminated by combination of the pair of stars, north and south for latitude, or east and west for time and azimuth. On the other hand, if the weather be uncertain, there may be time to get a complete observation of one star on both faces, but not to get the incomplete observation of two opposite stars on one face, which together will make a complete observation. Hence in doubtful weather the former method is surer than the latter, and it must be left to the judgement of the observer how far he can go in abandoning the canonical method of changing face on each star. We will describe first the strict text-book methods, and indicate later the modifications that an experienced observer may make with advantage.

§ (2) Time and Azimuth, the Latitude being known approximately.—All stars which cross the prime vertical are moving most quickly in altitude and least in azimuth when they are upon that circle: they are then best suited for determination of time and azimuth. For convenience of explanation we may treat the two together as derived from one observation, though the best results are obtained by concentrating attention on one or the other. Suppose, then, that in latitude ϕ (approx.) a star near the prime vertical east, of north polar distance p, is observed in altitude h at time by chronometer T, and that the reading of the horizontal circle on the star is A_1, while that on a terrestrial reference object (R.O.) is A_0; then if $s = \frac{1}{2}(\phi + p + h)$ we have, by a slight adaptation of the ordinary formulae of spherical trigonometry,

$$\tan \tfrac{1}{2}t = [\cos s \sin (s - h) \operatorname{cosec} (s - \phi) \sec (s - p)]^{\frac{1}{2}},$$

$$\tan \tfrac{1}{2}\mathrm{A} = [\sec s \sin (s - h) \sin (s - \phi) \sec (s - p)]^{\frac{1}{2}},$$

from which the hour angle t or the azimuth A may be calculated. From the hour angle of a star of known right ascension we can calculate immediately the local sidereal time, and thence the local mean time, for comparison with the recorded chronometer time and the derivation of the chronometer error. The azimuth A given by the above expression is reckoned from the elevated pole to the celestial object, east or west as the case may be: it is transformed into azimuth measured clockwise from south in geodetic work.

From the calculated azimuth of the star, and the observed difference between the azimuths of the star and the R.O., we have the azimuth of the R.O. and thence of any other terrestrial mark that may be observed with it at the same station.

In practice the observation on the star must be repeated several times, to average out the accidental errors of observation, and in these repetitions the theodolite must be reversed, according to the programme: face L, R, R, L, so that the errors of zero and collimation shall be eliminated from the mean of the results, an equal number of observations being made on each face; and each altitude reading must be corrected for the readings of the bubble on the microscope arm, to allow for the small dislevelments of the instrument during the course of the observations, or slight eccentricity of the microscope arm on the horizontal axis. Finally, an equal number of stars should be observed east and west, so that the mean result may be free from any error constant in amount and systematic in character, such as personality in setting, or shake of the telescope objective or diaphragm or inner tube.

Such in their main lines are the standard observations for time and azimuth, by east and west stars. The many important details of good practice may be studied in a text-book such as that of Close and Cox, where the forms of computation are well set out.

§ (3) Azimuth by Stars at greatest Elongation East or West.—Any star that crosses the prime vertical, that is, whose declination is less than the latitude of the place, is moving slowest in azimuth when it is upon the prime vertical, and should be observed about that time when it is observed for azimuth by the ordinary method of I. But stars farther from the equator, which pass between the pole and the zenith, will, instead of continuously increasing their azimuths, reach a maximum azimuth east, decline to azimuth zero again as they cross the meridian, and pass to a maximum azimuth west. At their "greatest elongations" they may be observed very accurately for azimuth, provided that the altitude of the star at elongation is not too great. In latitude ϕ and for star of N.P.D. p the azimuth at maximum is given by $\sin A = \sec \phi \sin p$; the hour angle by $\cos t = \tan \phi \tan p$; and the altitude by $\sin h = \sin \phi \sec p$. From these simple expressions it is easy to prepare a programme for observation.

For stars within a few degrees of the pole, and when great accuracy is not required, it is sufficient to calculate the hour angle and thence the chronometer time of the maximum elongation, and take a series of pointings within say four minutes on each side of the maximum: calculating the azimuth at maximum from the expression above, and assuming that it is sensibly constant during the observation. But for stars farther from the pole, and when accuracy is required, this is not sufficient. The azimuth at the moment of each pointing must be computed from the hour angle by the expression

$$\tan A = \tan t \cos M \operatorname{cosec} (M - \phi),$$

where $\tan M = \cot p \sec t$.

Since the azimuth is passing through a maximum it is not sufficient to calculate it for the mean of the hour angles, but it must be calculated separately for each value of t. And the hour angles must be calculated with precise local time, an observation for error of chronometer having been made immediately before or after the observation for azimuth.

§ (4) Latitude, the Time being known.—The normal determination of latitude[1] is made by observation of the altitudes of stars in or near the meridian. So long as it is necessary to make repeated observations, with change of face, it is impossible to make the observations exactly on the meridian; but with an approximate knowledge of the local time of transit and of the error of the chronometer on that time, each observation in the neighbourhood of the meridian may be corrected to the meridian altitude by the expression

$$2 \sin^2 \tfrac{1}{2}\Delta t \cos \phi \sin p \sec h \operatorname{cosec} 1'',$$

varying with the square of the hour angle Δt, and requiring the addition of a second term if the hour angle is greater than about ten minutes, which should never be necessary. For the same reasons as above the stars should be observed in pairs north and south of the zenith, at nearly the same altitudes to avoid errors of refraction, and with the usual systematic reversal of the theodolite and readings of the microscope levels.

Observations of the sun, with pointing alternately at the upper and lower, east and west limbs, may be made on the same principles for latitude, time, and azimuth, but give accuracy much less than that from stars, because the observations cannot be taken in balanced pairs, cannot usually be made at a good altitude on the prime vertical, and are inherently less exact than observations on the minute images of the stars.

The above standard methods of finding time, azimuth, and latitude have been found by long experience to be the best for general use on boundary and reconnaissance surveys, by observers of perhaps no great experience, in uncertain conditions of weather. The experienced observer in favourable conditions may with advantage depart from the above rigorous methods by omitting the constant change of face if he is certain of getting well-balanced pairs of stars on one face or the other, or by adopting one of the methods which follow; but he should not do this until he is thoroughly skilled, and an accomplished computer.

Since latitude is required in determination of time, and time in determination of latitude, the process is clearly one of successive approximation. The latitude will usually be known by account within a minute or two, which is sufficient for finding time, especially since if the star is close to the prime vertical the calculation is nearly independent of errors in the latitude; the local time being then known, and the right ascension of the star, it is easy to calculate the small hour angles required to reduce the circum-meridian to the meridian altitudes. In practice it is usually sufficient to take out the values of $2 \sin^2 \tfrac{1}{2}\Delta t \operatorname{cosec} 1''$ (from tables) for each observed altitude and multiply the mean by the value of $\cos \phi \sin p \sec h$ for the mean altitude.

The method of finding time from altitudes of stars, on or about the prime vertical, is used in the field because it is easy to control the corrections to an observed altitude; the fixed observatory method of transit over the meridian cannot be employed in the field because the theodolite is not stable enough for adjustment in the meridian, nor would it be possible to obtain readily the corrections for collimation, level, and azimuth.

§ (5) Time and Latitude by equal Altitudes of three or more Stars.—This method has been much employed of late years; it is fundamental in the use of the prismatic astrolabe (*q.v.*) but has many advantages in theodolite work, since the absolute altitude is not required: only that the three observations be made at the same altitude; no circle readings are wanted, but the theodolite must have a sensitive level on the vernier or microscope arm of the vertical circle, to control slight dislevelment during the observations. The rate of the watch must be known.

---

[1] See also "Gravity Survey," § (8).

With assumed (approximate) values of latitude ϕ and time t of the observation of a star of declination δ, calculate the altitude h and the azimuth A from the formulae

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t,$$

$$\sin \text{A} = \cos \delta \sin t \sec h.$$

Then if h_0 is the unknown constant altitude at which the stars have been observed, subject only to the small correction n for the readings of the level; and if $d\phi$ and $d\text{T}$ are the corrections to the assumed latitude and watch error respectively, each star gives an equation

$$h + \cos \text{A} d\phi + 15 \cos \phi \sin \text{A} d\text{T} - h_0 - n = 0,$$

and with three stars giving three such equations we may solve for $d\phi$, $d\text{T}$, and h_0.

If latitude is the principal consideration, the best result is obtained by choosing one star close to the meridian north, and the two others about an hour on either side of the meridian south. If time is the principal object, one chooses two stars near the prime vertical east and west, and one near the meridian north or south.

The method is not confined to three stars; any number may be observed and the solution made by least squares, or preferably by the graphical construction described below: an enlargement of the well-known method of the "new navigation." This method by equal altitudes requires time in the preparation of a programme (which may be much shortened by the use of the tables when using the prismatic astrolabe) and skilled computing; but those who have used it extensively in the field are convinced that it is more accurate than the ordinary methods.

(*i.*) *Graphical Methods of Reduction.*—The usual method of graphical reduction is based on the following process: assume an approximate latitude and longitude for the place of observation (the position "by account"), and with the assumed longitude and the Greenwich mean times of the observations (from the chronometer, corrected for error and rate) calculate the corresponding altitudes of the stars observed. Calculate also from the same data their azimuths, or in rough work take them out from azimuth tables. Now compare with the observed altitudes corrected for refraction. The point of the earth having the star in its zenith at the instant of observation is in the direction of the calculated azimuth, and all points on the earth having the star at a given altitude lie on a small circle round that point, cutting the azimuth line at right angles. If the observed altitude is greater than the calculated by a small quantity dh, this position circle cuts the azimuth line at the distance dh towards the star; and the small arc of this circle required is practically identical with a line—the "position line"—drawn at right angles to the azimuth line at distance dh from the assumed position, paying attention to the sign of dh. A position line is drawn thus for each star, and the concluded position is the centre of the circle which is most nearly tangent to all the position lines.

The process thus described, which is equivalent to the method of the "new navigation," has the disadvantage that the calculation cannot be begun until the observations have been made; and if the results are required at once, the calculation must be made late at night, with likelihood of error due to fatigue. Messrs. Ball and Knox-Shaw have pointed out (*Geog. Journ.* liv. 37) that in good climates, when observations can be secured with certainty, it is often convenient to compute the observations of an equal-altitude series by an alternative process, as follows: with an assumed latitude, and as close an approximation as possible to the constant altitude (corrected for refraction), calculate the corresponding local times, and compare with the recorded G.M.T.'s. Calculate the azimuths from the formula

$$\sin \text{A} = \cos \delta \sin t \sec h,$$

as before. Plot the differences, local time minus G.M.T. (or longitude), along an axis as abscissae, and through these points draw position lines at right angles to the calculated azimuth lines. Find the centre of the circle which most nearly touches the four position lines. The abscissa of the centre will give the correction of the G.M.T. to local mean time, that is, the longitude of the place; its ordinate, multiplied by the cosine of the latitude, will give the correction to the latitude. The radius of the circle is the correction to the assumed constant altitude.

A similar method can be used for graphical reduction of any set of altitudes of stars, not necessarily at the same altitude. From the measured altitudes calculate the local times; compare with the G.M.T.; calculate the azimuths, and plot the position lines as above. No change of face is required, as any constant errors due to collimation, index error, etc., will affect only the radius of the circle, and not the position of its centre.

For a good result by this graphical method four stars at least are required, in azimuths differing by 90° as nearly as is convenient.

And since it is difficult to find stars crossing a given altitude circle near the meridian, it is best to choose stars reaching the desired altitude about the middle of the quadrants, *i.e.* S.E., S.W., N.W., and N.E. Theoretically, three stars are sufficient; but there is then no check against errors in observation and calculation. With four stars any serious error is immediately apparent, though it may not be possible to say which of two stars in opposite azimuths is wrong. With more than four stars the observation in error is obvious, and may be rejected or weighted down according to circumstances.

Method I.—Rectangular axes through O, the position by account, meridian and parallel through O. Scale: say 1 mm. = 1″. OA, B, C, D are the azimuths of four stars observed; OM = obs. − calc. altitude of star A; M*aa* is the position line at right angles to OA. Similarly constructed position lines are *bb*, *cc*, *dd*. The true position is Z, centre of circle most nearly tangent to four position lines. Correction to position by account is, in latitude ZN, in longitude ON sec ϕ, where ZN is perpendicular to the parallel through O. As the figure is drawn the displacements of the pairs of position lines in opposite azimuths are not due principally to a constant error in observed altitudes.

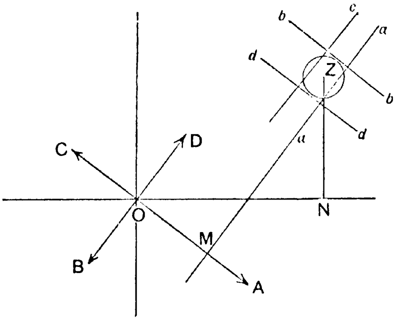


FIG. 1.

Method II.—Horizontal axis is the parallel of the atitude by account. Scale: say 10 mm. = 1s. The points *a*, *b*, *c*, *d* are the plotted longitudes deduced from the four observed altitudes of stars in deduced azimuths *a*A, *b*B, etc. The position lines *aa*, *bb*, etc. are drawn perpendicular to these azimuths. The true position is Z, centre of circle most nearly tangent to four position lines. Its longitude is represented by N; the correction to the latitude by account is +ZN cos ϕ. As the figure is drawn the displacement of the position lines is due principally to a constant error in the observed altitudes of amount radius of circle × cos ϕ, such as might be caused by error in the angle of prism of the prismatic astrolabe.

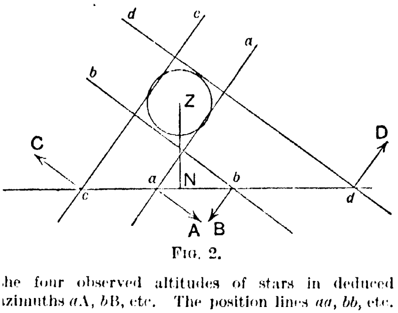


FIG. 2.

§ (6) LATITUDE BY MERIDIAN ALTITUDES.—This is an admirable method for a skilled observer. A preliminary observation for time and azimuth enables the observer to prepare a list of chronometer times of transit of his stars, which should be equally balanced north and south of the zenith, and to set his theodolite so that the telescope describes the meridian—not with sufficient accuracy to allow of transit observations for time, but quite well enough to ensure that the star is sensibly at its maximum altitude when crossing the centre wire. The reduction is very simple, since the latitude is the complement of the altitude of the equator, *i.e.* of the altitude of the star (corrected for level readings and refraction), + the star's declination. If the observations are equally balanced north and south it is not necessary to change face, and the process is rapid.

An accurate modification of the method—known as Talcott's—is possible when the theodolite is provided with an eyepiece micrometer for measuring small differences of altitude. With a preliminary approximate latitude, pairs of stars are chosen which transit north and south respectively at small differences of zenith distance within the range of the micrometer, which measures this difference directly, and no circle readings are made except for setting; but the level must be sensitive and accurate, to correct for slight displacements of the vertical axis during the observations. To find sufficient pairs of stars it is necessary to go below the magnitude conveniently observable in all but the largest theodolites, and the Talcott method is perhaps more adapted to the zenith telescope; but this instrument is outside the scope of the ordinary astronomical work in the field.

§ (7) LATITUDE BY ALTITUDE OF THE POLE STAR AT ANY TIME.—The pole star describes so small a circle about the celestial pole that its motion in altitude is very slow, and its altitude relative to the pole may be calculated if the time is known approximately. The formula for computation of the latitude from an observed altitude h (corrected for refraction) is

$$\phi = h - p\cos t + \tfrac{1}{2}p^2 \sin^2 t \sin 1'' \tan h,$$

when p is the north polar distance of the star in seconds of arc, taken from the daily ephemeris in the *Nautical Almanac*, and t is the hour angle of the star. For rough determinations it is sufficient to use the table calculated in the *Nautical Almanac* which gives an approximation to the above expression. Such observations of Polaris are often useful to balance observations south of the zenith. But a better way of using the pole-star is to include it in an equal altitude series, and work it up with the others, without any special form of computation as above.

§ (8) Latitude and Time with the Prismatic Astrolabe.—This comparatively new instrument, the invention of MM. Claude and Driencourt, has thoroughly established itself in use by the French Colonial surveyors, and more recently has been used on boundary survey and similar work by British officers. The instrument itself is described elsewhere.[1]

Its merits are in some respects still matter for controversy, but generally those who use it become enthusiasts for it. Its undoubted merits are (1) ease of manipulation: the adjustment is simple, and small errors of adjustment have no effect on the observed times; (2) freedom from any instrumental readings, either of vertical circle or level: the observation is of time of passage over the horizontal wire; (3) comfort of position: the telescope is always horizontal and at a convenient height for observation from a chair; (4) superior accuracy.

Its alleged demerits are: (5) images elongated, each being formed by only half an objective. Some observers find serious difficulty in this: others find none; (6) disturbance by wind. It is agreed that observation is impossible in a moderate breeze, which disturbs the reflecting mercury surface; (7) trouble in preparing programme of stars to be observed: this is now obviated by the tables in the Handbook by Dr. Ball and Mr. Knox-Shaw; (8) restricted usefulness of instrument: it is incapable of determining azimuth, and cannot deal with any altitude but 60°.

The last is the most serious: as the instrument cannot determine azimuth, a theodolite must always be taken with it, and will do the astrolabe's proper work nearly though not quite so well. A theodolite gives good results also in a wind that puts the astrolabe out of action. Hence the latter cannot be carried unless there is transport for both. Nevertheless the instrument has undoubted value, and will probably come into more general use as it becomes better known.

The observations are reduced by the second graphical method given above, which was devised for the purpose, and is markedly superior to the method described by the original inventors.

§ (9) Longitude in the Field.[2]—The longitude is the difference between the local time and the time of the standard meridian of Greenwich, which for the comparison must be (1) determined by independent methods, or (2) carried by chronometers, or (3) sent by electric telegraph or wireless signal.

The methods of class 1, by Lunar Distances, Moon-culminating Stars, Occultations of Stars by the Moon, Observation of Eclipses of the Sun or of Jupiter's Satellites, can never give much accuracy, and may now be considered obsolete, or nearly so. For the method of Occultations—the last survivor—see Close and Cox, or *Hints to Travellers*. An average of only about five occultations per month can be observed with field instruments at any place, even if none are lost by bad weather; the average error of a longitude from a single occultation will be at least one second of time, or say 500 yards on the equator. The recent divergence of the moon by some 12″ of arc from her tabular place makes it necessary to apply to Greenwich for corrected places of the moon if for any reason it has been necessary to use one of these lunar methods.

The method of class 2, by transport of chronometers or watches, is also fast becoming obsolete. The care of a large number of watches is onerous, and they have nearly always a different rate travelling and standing still. The system on which they are compared daily and their rates determined is too long to be described here: see Close and Cox, p. 257, or for methods of the highest elaboration see *Documents scientifiques de la mission Tilho* (1906-1909), vol. i. part 2, chap. i.

The methods of class 3 have been revolutionised by the rapid extension of wireless time signals and the immense improvements in the receiving apparatus following on the invention of the amplifying valve. Exchange of time signals by electric telegraph is necessarily limited to countries of some development, is often hampered by the difficulty of securing uninterrupted use of the line or its bad condition, and is affected also by obscure sources of error that produce discordances of about two-tenths of a second of time even in elaborate determinations made between well-equipped observatories, with automatic transmission of signals. Under field conditions the error of a telegraphic longitude may very well average several tenths, and the systematic effect of personality in observing and in transmitting signals makes it hard to improve on this by repetition of observations, unless the observers can be interchanged. Hence longitude by electric telegraph is an obsolescent method, and the future is with wireless.

The time signals from the Eiffel Tower are controlled by the Paris Observatory, working in co-operation with other French and with British observatories: for current arrangements see the *Yearbook of Wireless*

---

[1] See "Gravity Survey," § (8) (ii.) and § (9) (ii.).

[2] See also "Gravity Survey," § (9).

Telegraphy. The signals comprise three series of "ordinary" time signals, usually accurate within 0·1 second, and one series of "scientific" signals, the *vernier acoustique*, of 300 beats in 295 seconds, every sixtieth beat being suppressed for identification. These signals may be compared with clock or chronometer by the method of coincidences to an accuracy of about 0·02 sec., which is well within the absolute accuracy of the time signal, and much within the limits of time-determination in the field. A few minutes after these signals the precise time of the first and last, as observed at the Paris Observatory, is sent out by code signals. The reason for this procedure is that the transmitting clock cannot be brought so precisely to the correct time at the desired moment that its error is not appreciable when it is compared with the standard clocks of the very elaborate installation of the international time service.

The ordinary signals of the Eiffel Tower have been used for some years already in the determination of longitudes by the officers of the Mission Tilho in the French Sudan, and the signals from Arlington near Washington are easily received on the Amazon. As a preliminary to the improvement and extension of the system of time signals throughout the world there is much activity in re-determining the longitudes of the principal observatories; but at the time of writing there is little information as to the average and extreme errors of the various services.

§ (10) Deflection of the Vertical.—The astronomical determination of position on the spheroid involves the assumption that the direction of gravity, which controls the bubble of the level or the plane of the mercury surface, is normal to the spheroid at the point, which is only approximately true, owing to the irregular distribution of density in the crust of the earth, as well as to the existence of visible disturbing masses: the mountains in excess, and the deep oceans in defect of density. The average value of this deviation of the vertical is several seconds of arc, even in country where the visible disturbing masses are slight. On the other hand, in very mountainous country, where the deviations calculated from the visible masses are large, the deviations are on the whole not nearly so great as they would be if those masses exerted their whole effect; from which it is concluded that the visible excesses and defects of density are to a large extent compensated in the crust below. Nevertheless, the discrepancies between positions observed astronomically and the relative positions of the same points determined by triangulation are so considerable that their effect may become visible even on relatively small scales such as 1/250,000. Hence even if the observation in the field could be made errorless, a framework of observed latitudes and longitudes is an unsatisfactory basis for a map, as compared with a framework of triangulation: the points thus determined astronomically might be visibly discordant when plotted on the plane-table. On the other hand, there is something to be said for mapping by astronomical positions rather than geodetic in country such as the Amazon basin, where it is impossible to see fixed points at any distance, and isolated positions must be determined astronomically, with resultant discordance from the map based on triangulation.

References

Text-Book of Topographical Surveying, by Colonel C. F. Close, C.M.G., R.E., revised by Captain E. W. Cox, R.E. Second edition. H.M. Stationery Office. 1913. 7s. 6d.

Hints to Travellers. Published by the Royal Geographical Society. Tenth edition. 1921. 2 vols. 21s. net.

A Handbook of the Prismatic Astrolabe, by John Ball, O.B.E., and H. Knox-Shaw, M.A. Published by the Egyptian Government. Cairo. 1919. 10s. 6d.

"Modern Methods of finding the Latitude with a Theodolite," by John Ball. *Geographical Journal*, xlix. 6, June 1917. (Abstract. The complete paper to be published in the Technical Series, R.G.S.)

A. R. H.

Law of Errors, deductions from. See "Observations, the Combination of," § (4).

Least Squares: the principle that in measurements of unequal weight the most probable values of observed quantities are those that render the sum of the weighted squares of the residual errors a minimum. See "Observations, the Combination of," § (10).

Length, Units of.[1] (i.) *The Metre.*—The *metre* is defined as the distance, at the melting-point of ice, between the centres of two lines engraved upon the polished neutral web surface of a platinum-iridium bar of nearly X-shaped section called the *International prototype metre.* This is a copy of the original Borda platinum standard—the *mètre des archives*—which was intended to be equal to 10^{-7} or one ten-millionth of the length of the meridian through Paris from Pole to Equator. According to Clarke's figures the correct relation is a quadrant $= 1{\cdot}0007 \times 10^7$ metres, the mean of the values obtained by Helmert and the U.S. Survey for the mean polar quadrant is $1{\cdot}00021 \times 10^7$ metres. The length of the bar as constructed is now taken as an arbitrary standard.

(ii.) *The Yard.*—The *yard* was defined by the Weights and Measures Act, 1878, as the distance at temperature 62° F. between the central transverse lines in two gold plugs in the bronze bar called the *Imperial standard yard*, when supported on bronze rollers so as best to avoid flexure of the bar. The bar is of 1 inch square section, and is 38 inches long; the defining lines are at the bottom of two holes so as to be in the median plane of the bar.

---

[1] Most of the information has been taken by permission from the *Computer's Handbook* of the Meteorological Office, to which the reader is referred for further details.

(iii.) *Equivalents.*—
(*a*) *Metric Units.*

Metre . . . 1 m. = 39·370113 in.
= 3·280843 ft.
= 1·093614 yd.
Kilometre . . 1 km. = 0·6213717 mi.

(*b*) *British Units.*

Mile . . . 1 mi. = 1609·343 m.
Yard . . . 1 yd. = 0·9143992 m.
Foot . . . 1 ft. = 0·3047997 m.
Inch . . . 1 in. = 2·5399978 cm.
Nautical mile (English)[1] = 1853·182 m. (Adm.)
= 6080 ft.
= 1·1515 statute mi.

(*c*) *Astronomical Units.*—For astronomical work it is convenient to use larger units than those defined above.

The astronomical unit is equal to the semi-major axis of the earth's orbit.

1 astronomical unit = 1·495 × 10^8 km.
= 9·289 × 10^7 mi.
Parsec . . . = distance at which the astr. unit subtends 1 second (1″).
= 206,000 astr. units approx.
= 3·083 × 10^{13} km.
= 1·9158 × 10^{13} mi.
Light-year . . = the distance travelled by light in 1 year (velocity of light = 2·9986 × 10^{10} cm./sec. = 186,326 mi./sec.)
= 0·31 parsec.

(iv.) *Small Units.*—For measurements of the wave-length of light and X-rays the unit is one ten-thousand millionth metre, and is known as a "tenth-metre" or the Ångström unit.

Ångström unit . . . 1 A.U. = 10^{-10} m.
Micron . . . . . 1 μ = 10^{-6} m.
Millimicron . . . . 1 $\mu\mu$ = 10^{-9} m.
Mil . . . . . . 1 mil. = 10^{-3} in.

(v.) *Ancient French Units.*

1 toise = 6 ft. = 1·9490366 m. = 2·1314918 yd.
1 foot = 12 in. = 0·3248394 m. = 1·0657461 ft.
1 inch = 12 Paris lines . . = 27·069953 mm. = 1·0657461 in.
1 line . . = 2·255829 mm. = 0·0882165 in.

(vi.) *Russian Measures.*

1 verst = 1·06678 km. = 0·663 mi.

UNITS OF AREA.—Measures of area are based on the standard of length.

(i.) *Equivalents.*—
(*a*) *Metric Units.*

Square centimetre: 1 cm.^{2} = 0·1550 in.^{2}
= 0·001076 ft.^{2}
= 0·0001196 yd.^{2}
100 m.^{2} = 1 are.
= 0·0988 rood.
10,000 m.^{2} = 1 hectare.
= 2·4711 acre.

(*b*) *British Units.*

1 in.^{2} . . . . = 6·4516 cm.^{2}
1 ft.^{2} . . . . = 929·03 cm.^{2}
1 yd.^{2} . . . . = 8361·3 cm.^{2}
1 acre . . . . = 4840 sq. yds.
= 0·4047 hectr.
1 square mile . . = 259·00 hectr.
= 2·59 sq. km.

See Vol. I., "Measurement, Units of."

LENGTH STANDARDS (LINE), various, multiples and submultiples of yard and metre. Form and material of same. See "Line Standards," § (3).

LIGHTNING. See "Atmospheric Electricity," § (20).

LIMIT GAUGES. See "Metrology," VI. § (17) (ii.).

LINE, DATE: the line at which the date changes when crossed from east to west or west to east. See "Clocks and Timekeeping," § (1).

LINE STANDARDS OF LENGTH

§ (1) DESCRIPTIVE.—A Line Standard of Length has been fully defined elsewhere, *e.g.* the yard[2] and the metre, and the essential desiderata to be borne in mind in designing and using the actual material bar carrying the defining lines[3] have also been considered. The paramount quality aimed at in such a bar, and at the same time the most difficult of realisation, is the *invariability* of the defined length, and this end is always kept in view, whether it be in the selection of a suitable material of which to construct a bar, the choice of its shape or section, the manner of supporting it, the quality of the lines, or considerations of expansibility and temperature.

(i.) *Section.*—Line standards are almost invariably of uniform section. Early types of bars were of rectangular or square section (*Fig.* 1, *a* and *b*), with the defining lines ruled on the upper surface. Later bars of similar section were made with the defining lines lying in the neutral plane,[4] and this marked an important step in the progress towards greater refinement and accuracy. *Fig.* 1, *c* and *d*, show clearly how this is done, either by sinking the lines to the bottom of cylindrical holes (as in the case of the Imperial Standard Yard,[1] or by ruling them on the surface of a step at either end, the surfaces which bear the lines being, in both cases, in the neutral plane. From this limited use of the neutral plane it was but a step to the X or Tresca section type (adopted for the International Prototype Metre), and the H section type (*Fig.* 1, *e* and *f*). In both these similar types the defining lines are engraved on the upper surface of the horizontal web, which is the neutral plane of the bar. Both too are of much lighter construction than those already referred to, and yet owing to their design lose nothing in rigidity. The large area of surface which they present to surrounding media is a great advantage, as it facilitates the rapid equalisation of temperature throughout the bar, while the whole length of the neutral plane is rendered available for subdivision if required. All modern bars of first quality are of the H or X section type, with the defining lines and other graduation lines in the neutral plane.

---

[1] See "Geodetic Measures."

[2] See "Metrology," § (1).

[3] *Ibid.* § (1).

[4] *Ibid.* § (7).

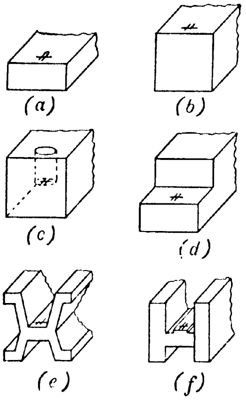


FIG. 1.

(ii.) *Material.*—No material has yet been discovered which, at one and the same time, possesses all the qualities essential to a satisfactory bar, such qualities as chemical simplicity and purity and hence homogeneity, molecular stability under all conditions of use, hardness, rigidity, susceptibility to a high degree of polish, capability of being ruled with fine permanent lines, a low or even zero expansibility, absence of thermal hysteresis, etc. Certain metals and alloys are the best materials which conform in varying degrees to these requirements, and for this reason most standards are constructed of such material. Allusion has been made elsewhere[2] to the properties of secular growth and thermal hysteresis possessed by some of the alloys, particularly some nickel-steel alloys, and the limitation of their use (with discretion) to working standards.

A particularly valuable alloy is platinum-iridium, the material of the International Prototype Metre; this, when subjected to suitable treatment during manufacture and a subsequent artificial ageing process, shows no perceptible secular change.[3] Its high cost strictly limits its use to primary and secondary standards. For everyday standards pure nickel proves an admirable and certainly cheaper substitute,[4] but it is difficult to forge, and satisfactory lengths exceeding one metre have not so far been produced. The value of invar[5] has already been noted, but its behaviour is very complex and it must be employed with a full knowledge of its properties, which are discussed in more detail in a later section (§ (4)). The pure precious metals, such as platinum and gold, can reasonably be expected owing to their chemical simplicity to possess considerable stability, but apart from the deterrent matter of cost they are lacking in certain mechanical qualities. Bronze and brass have served their purpose well in the early days of standards, but their instability, coupled with other defects, has led to the discontinuance of their use in present times.

Of non-metallic substances the only one that has proved of any value is fused silica,[6] already mentioned, and discussed in more detail in § (6).

(iii.) *Method of Support.*—The elastic distortion of a bar due to its own weight may prove a source of error unless properly controlled, and this is done first by placing all graduation marks in the neutral plane, and secondly by a definite method of supporting it when under observation. With regard to the latter, it is obvious that a variation in the method of support will as a rule bring about a change in the amount of bending, and therefore an alteration in the defined length. It is vitally important, therefore, to ensure that the amount of distortion shall be constant, and for this reason it is the practice to use the bar supported at a limited number of definite "points," the latter being really lines of contact made with small rollers placed under the bar; the points, being once decided upon, are strictly adhered to. It may be argued that support by a flat surface would be preferable to this, since it would guard altogether against bending. But, first of all, the really flat surface is not practicable, and secondly, the method resolves itself into support at an indefinite number of irregularly disposed points, a condition that is evidently not capable of exact reproduction. Further, the system of employing a limited number of points of support has the advantage that all surfaces of the bar are equally exposed to the surrounding medium, thus facilitating equality of temperature distribution throughout the bar.

---

[1] See "Metrology," § (1).

[2] *Ibid.* § (1).

[3] See "Metrology," § (4).

[4] *Ibid.* § (4).

[5] *Ibid.* § (4); also "Invar and Elinvar," Vol. V.

[6] See "Metrology," § (4).

The "best" points to choose for supporting the bar have been discussed elsewhere[1] and are given by Airy's formula

$$b = \frac{a}{\sqrt{n^2 - 1}},$$

where a is the total length of the bar considered as having uniform section, n the number of symmetrically and equally spaced points, and b the distance between any two consecutive points. When $n = 2$, the usual case,

$$b = \frac{a}{\sqrt{3}} = 0{\cdot}577a, \quad \text{or} \quad \frac{b}{a} = {\cdot}577.$$

Broch, however (*loc. cit.*), gives

$$\frac{b}{a} = {\cdot}559.$$

There is little to choose between these two values, though Airy's value is most commonly employed; and if b/a is varied between the two values, no appreciable error is introduced in the distance between the defining lines; and further, this distance differs from that existing when the bar is unaffected by gravity, by a negligibly small amount.

The important point, however, to be borne in mind is that for a particular bar a definite value of b/a should be adopted and subsequently strictly adhered to, even if this value is afterwards found to be outside the limits quoted above. The danger, however, of a value outside the limits is that a small accidental error in setting the points may introduce a serious error in the observed length of the bar, whereas when the value lies within the limits, a small accidental setting has no appreciable effect in altering the definite length.

(iv.) *Expansion.*—The important part played by temperature is fully discussed[2] elsewhere, and it is only proposed here to consider the term "thermal coefficient of expansion," and how it is interpreted. In line standard work it is usual to express the expansion of a bar by the formula

$$L_\theta = L_0(1 + a\theta + \beta\theta^2 + \gamma\theta^3 + \ldots), \quad . \quad (1)$$

where L_0 is the length of the bar at zero temperature, L_θ that at temperature θ and a, β, etc., the expansion constants. The terms beyond the quadratic term are generally negligible, and the formula reduces to

$$L_\theta = L_0(1 + a\theta + \beta\theta^2). \quad . \quad . \quad (2)$$

If L_0 be known, then L_θ can be readily calculated.

If in equation (2) L be differentiated with respect to θ we get

$$\frac{dL}{d\theta} = L_0(a + 2\beta\theta),$$

or

$$a + 2\beta\theta = \frac{1}{L_0} \cdot \frac{dL}{d\theta}, \quad . \quad . \quad (3)$$

and this gives the rate of change of length of the bar at temperature θ, $a + 2\beta\theta$ being the *true* coefficient of expansion at temperature θ.

Sometimes the *mean* coefficient of expansion ${}_1a_2$ between two temperatures θ_1 and θ_2, L_1 and L_2 being the corresponding lengths, is used, and is expressed thus:

$${}_1a_2 = \frac{1}{L_0} \cdot \frac{L_2 - L_1}{\theta_2 - \theta_1}, \quad . \quad . \quad (4)$$

If $\theta_2 - \theta_1$ is large, this is not sufficiently accurate for line standard work. When, however, $\theta_2 - \theta_1$ is small, equation (4) approximates to equation (3), and in the limit when $\theta_2 = \theta_1$, they become identical.

Let θ be the mean value between the temperatures θ_1 and θ_2, that is, let $\theta = \frac{1}{2}(\theta_1 + \theta_2)$. If β be small, and $\theta_2 - \theta_1$ also small (say two or three degrees), then it can be shown that the mean coefficient between θ_1 and θ_2 is equal to the true coefficient at θ, without giving any appreciable error, *i.e.*

$${}_1a_2 = a + 2\beta\theta. \quad . \quad . \quad . \quad (5)$$

Fig. 2 shows this graphically.

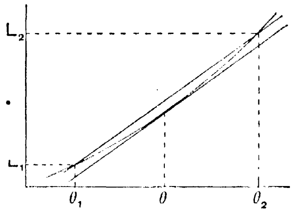


FIG. 2.

Since in actual practice β for most line standards is small compared with a, equation (5) is extremely useful for correcting results from one temperature to another, where the range is small.

The expansion of a bar may also be expressed as a curve, which is equivalent to (1) or (2), but as the greatest possible accuracy is always aimed at in fundamental line standard work, the formula (2) is almost invariably used.

(v.) *Defining Lines.*—The defining lines are naturally not the least important feature of a bar, and depend in the first instance on the quality of the material of which the bar is made and the nature of the surface. They are usually ruled with the help of a good dividing engine (see § (2)), the tool being a fine diamond chip; the surface carrying the lines is previously very carefully polished and therefore rendered highly reflecting. Each line is really a very shallow and narrow groove, and when illuminated and viewed through a microscope presents itself, in contrast to the bright surface, as a narrow dark band. The appearance of this line or band depends on the nature of the groove and on the way in which it is illuminated. With regard to the groove, the best result is achieved when it is cut with uniform width and depth, and in such a manner that its cross-section is symmetrical about a line at right angles to the surface, the section approximating to a sharp V shape (*Fig.* 3). The most satisfactory illumination for such a line is obtained by directing diffused light so that it falls vertically on the surface and is reflected back along the incident path through the microscope.[1] A line thus ruled and illuminated will have the appearance best suited for a line standard, since it will have perfectly straight and parallel edges, which will be very sharply defined. The blackness of the lines, with vertical illumination, depends of course on the fact that the sloping sides of the cut (*Fig.* 3) do not reflect the light vertically, but there may be a slight scattering of light which would affect the quality of the blackness.

---

[1] See "Metrology," § (7).

[2] *Ibid.* § (4).

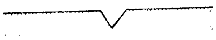


FIG. 3.

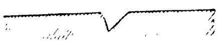


FIG. 4.

Variations in the nature of the groove and in the illumination usually result in an undesirable appearance in the line. For instance, with oblique illumination of a satisfactorily ruled line, the light falls unevenly on the side of the cut, there is a different scattering effect, and consequently one edge of the line will appear considerably sharper than the other. A groove of unsymmetrical section (*Fig.* 4), illuminated vertically, will give a similar result. Also, in the case of a groove with section of shape as in *Fig.* 5, with vertical illumination, the flat bottom of the cut will reflect the light vertically and the line will appear double. The bright central portion will, however, be usually very uneven, as shown in *Fig.* 6, a state brought about by the chattering of the blunt tool during the ruling process.

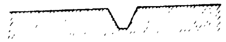


FIG. 5.

Across the two defining lines are ruled two longitudinal lines at right angles to them (*Fig.* 7). These assist in the location of the same portions of the lines each time they are observed. In fact, the distance between the middle points of the portions of the defining lines included between the two parallel longitudinal lines is the actual defined length of the bar.

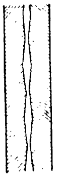


FIG. 6.

(vi.) *Wave-length Rulings.*—As already noted, if a defining line is well ruled, it will have quite straight edges and be of unvarying width. The central axis of the line is really the defining limit of the standard, and must be estimated when the cross wires of a micrometer microscope are set on it. The accuracy of the operation determines the accuracy with which the bar may be used for comparison purposes. It is found that the lines on some standards are far from complying with the conditions necessary for a perfect line, their edges appearing quite irregular even under low-power microscopes; with higher-power instruments, of course, the defect is only more apparent, and adds to the doubt of the observer as to the position of the central axis. Some lines which appear quite regular under low-power objectives are found to have an irregular appearance under greater magnification. Thus increased magnification is not always an advantage, and may be the reverse. If, however, the defining line can be made fine enough and at the same time quite regular—and this becomes increasingly difficult as greater refinement is aimed at—advantage certainly accrues from increased magnifying power.

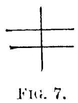


FIG. 7.

The necessity for greater refinement in ruling lines was brought home to Tutton in connection with his wave-length comparator (*q.v.*).[2] This instrument makes use of the wave-length of monochromatic light for the measurement of the small difference in the lengths of the two standards, the number of the wave-lengths in the interval being determined by an interference method. The exact location of the beginning and end of the interval measured is determined by the accuracy with which the central axes of two lines can be located by means of a micrometer microscope, and it is not possible to do this without introducing uncertainties, amounting in some cases to several wave-lengths.

Tutton had been attracted by an account of the very fine rulings, ranging from 10,000 to 120,000 lines to the inch, by H. J. Grayson,[3] of Melbourne, on a machine of his own construction, and in the hope that they might assist in giving the refinement he required, ordered several experimental rulings which were to be made on various materials: glass, silvered glass, speculum metal, gold, silver, platino-iridium, Bailey's metal and invar, and with lines 40,000, 50,000, and 60,000 lines to the inch. Each ruling consisted of a group of five lines (*Fig.* 8) together with two thicker "finder" lines placed symmetrically one on either side of the group at a suitable interval, the latter assisting in locating the group under a low-power objective. Two transverse lines also were ruled. The rulings proved more than equal to expectation, the exceedingly fine lines being wonderfully sharp when viewed under a high-power microscope with a $\frac{1}{15}$-in. dry objective. Experiment led to the selection, as the best ruling, of the one on speculum metal, on the scale of 40,000 lines to the inch.

---

[1] See "Comparators," § (1) (*h*).

[2] Tutton, *Phil. Trans. R.S.* A. 210, i.; also "Comparators," § (4).

[3] H. J. Grayson, "A New Dividing Engine for Ruling Diffraction Gratings," *Roy. Soc.* (*Victoria*), xxx. Part i., 44.

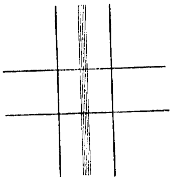


FIG. 8.—A Location Signal.

The group of lines, together with the "finders," was called by Tutton a "location signal," and it was his idea to use "location signals" for a line standard, the central one of the group being the defining line. They are eminently suitable for that purpose, but owing to the limitation of Grayson's machine, it was not till later that an experimental bar could be ruled.

The distance between the lines on such a ruling is of the order of one wave-length, a cadmium red ray having a wave-length equal to [illegible] in., and hence the rulings are frequently referred to as wave-length rulings.

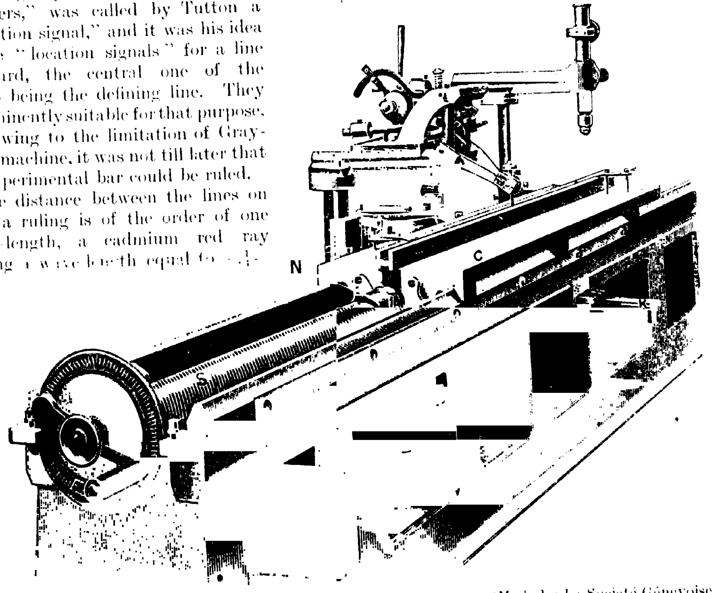


FIG. 9.—The Dividing Engine at the National Physical Laboratory. (Made by La Société Génevoise.)

When a location signal is viewed under a less powerful objective, say, a $\frac{2}{3}$-in., the five lines are not individually resolved, but appear as one line, with of course the edges of the line exceedingly well defined. It thus makes an ideal line under these conditions, which can readily be compared by ordinary comparator methods with the somewhat similar lines on existing fundamental standards. It must be realised, however, that although if all standards had "wave-length" rulings as location signals a higher accuracy of comparison might be obtained, the accuracy of the link with existing standards is limited to that attainable in the ordinary comparisons for which alone the existing standards are suitable.

§ (2) DIVIDING ENGINE.—As already indicated, a good dividing engine is a necessary adjunct to line standard work, and a typical example of such a machine is that in use at the N.P.L. (*Fig.* 9). This apparatus, designed and made by the Société Génevoise, is capable of producing the highest and most accurate class of work called for in connection with line standards or scales.

It consists mainly of two portions, corresponding to its two main functions. One part comprises the dividing head, which carries, and controls the movement of, the tool; and the other part, controlling the spacing of the lines on the standard or scale, consists of a carriage for supporting the work, and an accurate lead screw for moving the carriage along under the tool. The whole is supported on a rigid cast-iron bed, 2 metres long by 20 centimetres wide, which rests on two solid piers of concrete.

The top front edge of the bed is accurately planed in the form of an inverted V, while the back edge is finished flat, the two thus serving as supporting rails for the flat carriage C, on which the work rests. (In *Fig.* 9 an H section standard is clamped in position ready for graduation.) The greater part of the weight of this carriage is borne by six small wheels which rest on flat ways just below the V and plane edges. They are attached to the ends of cantilever springs, which bear on the under-side of the carriage, the pressure due to the springs being adjustable. This arrangement, by removing most of the dead sliding weight from the V and plane, lessens the amount of work entailed in driving the nut, and therefore saves wear on the screw.

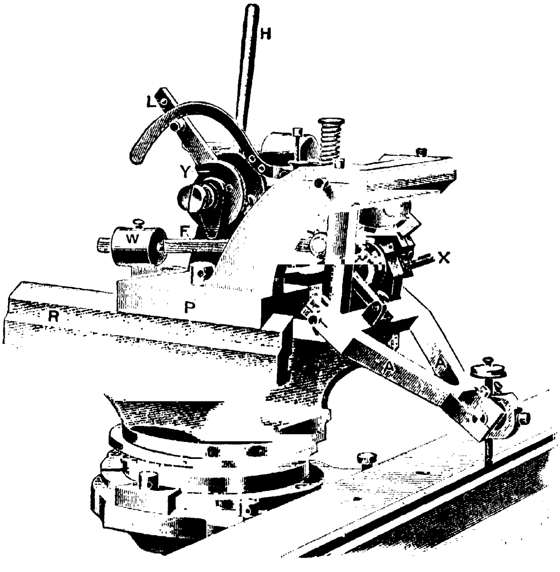


FIG. 10. The Dividing Head of the National Physical Laboratory Dividing Engine.

The screw S is of one mm. pitch and is very accurately cut. It works in a brass split nut N which bears against one end of the carriage C. The two halves of the nut are secured together by a hinge on one side and a spring on the other, and when the latter is released the nut can be disengaged and bodily removed to another part of the screw. This saves not only time, but wear and tear on the screw. A tail-piece on the nut fits into a slot in a steel plate under the screw. This slot, usually not parallel to the axis of the screw, gives the nut a slight rotation as it advances, and thus corrects for any small progressive error that may be in the screw. Also, by means of a lever K, the direction of this slot, and hence the rotation of the nut, can be altered so as to correct for changes in the length of the screw, due to changes in temperature. The screw is operated by a divided head, which with the help of a vernier reads to microns.

Attached to the rear of the bed is a rail, along which may slide, and to which may be securely fixed, the base of the dividing head and two microscope supports.

The dividing head shown in enlarged view in *Fig.* 10 is a complicated piece of mechanism whose features can only be briefly indicated. The various moving parts are secured to a steel plate P, which slides on two V rails R, thus giving the tool a coarse adjustment in a direction at right angles to the movement of the carriage. The tool may consist of a diamond chip mounted on a brass rod (as in *Fig.* 10), or may be made of steel. It is fixed at the end of a pivoted arm A, the weight of which, with the tool, is balanced by a counterpoise W sliding on a steel rod F. The required pressure on the tool is exerted by placing weights on the flat head of the tool. The mechanism is set in motion by means of the handle H, operated by hand. At the commencement of the ruling of a line, with the tool resting lightly on the work, the handle is in its rearmost position. Fixed relatively to this handle is a lever L carrying a small roller, and when the handle is pulled forward the lever follows it, bringing the roller into contact with a curved brass piece, eccentric with the centre of motion of the lever. As H, and therefore L, are pulled forward, the brass piece is slowly lifted, the arm A is pulled backward, and the tool rules the line. At the same time the length of stroke is controlled by two sets of pins (one set X can be seen in *Fig.* 10) which engage with a special rotating cam, the various recesses in the latter regulating the lengths of successive lines. As soon as the stroke is finished a cam Y comes into operation and depresses F, thereby raising A and the tool from the work. This completes the forward movement of H. The handle is now moved about two-thirds of its motion backwards, the brass curve moves in the opposite direction, and the tool is brought forward again ready for the next stroke. Before lowering the tool once more, the work is advanced the required amount by means of the screw S. The handle is then pushed completely backwards, lowering the tool on to the work ready for the next line to be ruled.

§ (3) VARIOUS LENGTH STANDARDS.—It has been noted elsewhere[1] that for everyday purposes there exist numerous (ternary and working) standards, which are copies of the primary standards (either the metre or the yard), and for reasons that should now be quite clear, it is necessary from time to time to compare them one with another in order to check possible secular changes in length (for method, see "Comparators"), and at least one of them, the most stable, must at less frequent intervals be referred, either directly or indirectly, to the primary standard.

It should be noted, however, that it is not possible to make an *exact* copy of a primary standard, but one can be made so that its length differs from that of the original by only a small and readily determinable amount. For example, a copy of the metre may be found to be 10 μ short at 0° C., that is, its length may be stated as equal to 1 metre − 10 μ at 0° C., an expression which is usually referred to as its "Equation to Scale."

Reference, so far, has been largely made to metre and yard standards, but it is obviously necessary to have, for practical purposes, both longer and shorter standards. The latter are most conveniently obtained by a process of subdivision of a standard length, and the best standards for this purpose are of course the working standards. Exact subdivision is however impossible, and consequently it becomes necessary to find out by how much each interval or subdivision, before it can be put to any practical use, is in error; in other words, the scale must be calibrated. The method by which this is done is described under "Comparators," § (9), Longitudinal Comparator.

Longer standards, usually complete multiples of the yard or metre, may be made in the form of bars, tapes, or wires. Owing to the limitations imposed chiefly by bulk and portability, such bars, when used as accurate standards, rarely exceed 4 metres in length. For standards longer than this, tapes and wires are used. These are very convenient in connection with surveying work, and owing to the smallness of cross-section are very flexible and can be wound on drums. They are thus very light and very portable (see "Tapes and Wires"). Before such bars or tapes can be used they must, however, be standardised by ultimate reference to the yard or metre. This can be done by dividing them into yard or metre sections, and then finding the length of each of these sections by comparison with a standard yard or metre. It is usual, however, with tapes to introduce for this purpose an intermediate standard, a common example of which is a bar with its defining lines 4 metres apart. Such a *four-metre bar* must first of all be compared metre by metre with a standard metre, and its length thus accurately determined. It can then be used for standardising a tape. For example, the length of a 24-metre tape can be readily determined by dividing it into six sections, each 4 metres in length, and comparing each section with the four-metre bar. It is easily seen that the introduction of an intermediate standard is a great convenience in that it results in a saving of time and a gain in accuracy.

The four-metre bars in use at the National Physical Laboratory and at the International Bureau are made of invar and are of the H section type. They are necessarily of stout construction, the area of cross-section being about twice that of a corresponding metre bar.

§ (4) INVAR AND THE NICKEL STEEL ALLOYS.[2] (i.) *Temperature Difficulties.*—The importance of an exact knowledge of the temperature and of the coefficient of thermal expansion of a standard has already been alluded to.[3] The determination of the latter is a purely laboratory task, and can be carried out, as a rule, to any degree of accuracy; but in certain circumstances it is difficult to measure the temperature with certainty. Such circumstances arise when it is not possible to control the temperature, and especially where the latter is apt to vary rapidly over short intervals of time. Difficulties of this kind are inseparable from surveying work, where long tapes and wires are used. The work is necessarily done in the open air under all sorts of conditions, and consequently the temperature can only be approximately determined. Such a disturbance as a sudden burst of sunshine after a cloudy interval, and just when an observation is being, or is about to be taken, may give rise to great uncertainty and consequent error. Further, a series of observations taken under such varying conditions must subsequently be adjusted for temperature, that is, all the readings must be corrected so that they appear as if taken at a particular standard temperature, and this involves both the coefficient of expansion of the tape and the exact difference between the temperature of observation and the standard temperature. Hence, an error in the observed temperature gives rise to an error in the correction, and therefore in the final result. In addition, the value of the expansion coefficient has an effect on the result in such a case, since the greater the coefficient of expansion the greater still must be the error of correction.

---

[1] See "Metrology," § (6).

[2] See also article "Invar and Elinvar," Vol. V.

[3] See "Metrology," § (4).

As already mentioned,[1] these difficulties have been met in recent years by the employment of the nickel steel alloy "invar," which has such a low coefficient of expansion, and which in consequence is now extensively used for the manufacture of line standards, particularly the longer ones in the form of tapes and wires. The remarkable properties of invar have been briefly indicated,[2] but it is necessary at this stage to enter into more details concerning it and the family of alloys to which it belongs. Such knowledge is supremely essential to a proper use of invar in line standard work.

M. Guillaume, the discoverer of invar, directed his early researches to the discovery of some material which would possess properties rendering it perfect for use in the manufacture of standards, and which would be considerably less expensive than platino-iridium, hitherto the most satisfactory. He soon recognised the eminent suitability of pure nickel, but the hope that he would be able to use it for making a four-metre standard was not realised, for it was found impossible to produce sound pieces of nickel having lengths greater than about one metre.

(ii.) *Nickel Steel Alloys.*—By a series of incidents he was led to study the alloys of nickel and steel, the remarkable properties of which (contradicting all preconceived notions respecting mixtures of metals) are of such great importance to metrology.

His investigations led him to classify these alloys into two groups, those containing more than 25 per cent nickel, and those containing less. The former possess properties that are reversible with temperature under ordinary conditions, while the latter are correspondingly irreversible. Further research by different workers has shown that this classification is approximate only, and that under certain conditions the two groups may overlap to some extent.

The various physical properties of these alloys, magnetic, electrical, mechanical, etc., are exceedingly interesting, but it is necessary to confine attention here to expansion phenomena only.

A nickel steel alloy is composed mainly of pure nickel and pure iron, together with small quantities, amounting to not more than one per cent in all, of one or more other substances, viz., manganese, carbon, silicon, chromium. Manganese has been proved to be a necessary constituent in all the alloys, since without it it is not possible to forge the metal. Slight variations in the proportions of any of these added constituents have a marked effect on the various properties of the alloys. For the sake of uniformity, Guillaume adopted as a typical alloy one containing 0·4 per cent Mn and 0·1 per cent C.

(iii.) *Irreversible Changes.*—The thermal expansion of an irreversible alloy is illustrated by *Fig.* 11. When cooled from a high temperature it contracts uniformly as indicated by the straight line ABC. At a certain temperature the contraction slows down, and on further cooling the alloy expands as indicated by CD. After this, normal contraction follows as DE. On re-heating, it expands uniformly as EDF, contraction then takes place along FB, after which the line BA is followed. If cooling is arrested at C′ and the alloy reheated subsequent changes follow the line C′B′BA. The temperatures at which the various changes take place depend upon the composition of the alloy. Such alloys as this are, of course, unsuitable for metrological purposes.

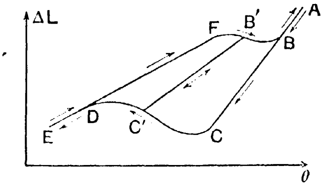


FIG. 11.—Expansion of a Nickel-steel in the Irreversible State (Guillaume).

(iv.) *Reversible Changes.*—The behaviour of a reversible alloy under ordinary conditions is quite different, the changes due to rise and fall of temperature following the same line. *Fig.* 12 shows generally the changes that take place. On cooling from a high temperature, there is a normal contraction as FE, then a rapid change as ED, followed by a slow contraction DC. A further change as CB brings the contraction to a normal one as BA. The temperatures at which these changes take place again depends on the composition of the alloy. For example, at ordinary temperatures (say about 20° C.) FE corresponds to an alloy containing about 25 per cent Ni, ED one between 27 per cent and 32 per cent Ni, CD one between 32 per cent and 36 per cent

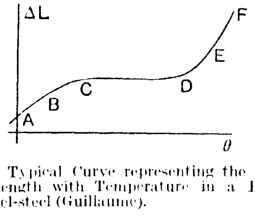


FIG. 12.—Typical Curve representing the Variation of Length with Temperature in a Reversible Nickel-steel (Guillaume).

---

[1] See "Metrology," § (4).

[2] *Ibid.* § (4).

Ni, etc. At higher temperatures a higher proportion of nickel corresponds to each portion of the curve.

(v.) *Fig.* 13 shows how the expansion of a reversible alloy varies with the percentage of nickel content. The curve represents a series of alloys having expansibilities ranging from about 1 to about 15 millionths. The series, moreover, is continuous, and it is thus possible to obtain alloys having any desired expansibility within the limits named above, a hitherto unattainable state of affairs with such a discontinuous series as iridium, tantalum, and tungsten. In contrast to these latter, also, is the cheapness of the alloys.

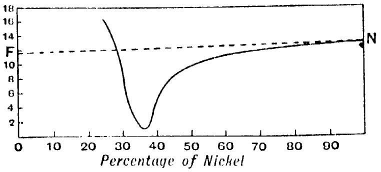


FIG. 13. True Expansibilities at 20° C. (in millionths) of the Typical Alloys of Iron and Nickel (containing 0·4 Mn and 0·1 C per cent) (Guillaume).

In the same *Fig.* 13 the dotted line FN represents the expansibilities anticipated by applying the usual law of mixtures.

The curve of *Fig.* 13 is, however, only applicable at normal temperatures. At higher temperatures it is modified in form, as clearly indicated by *Fig.* 14, the trough gradually becoming shallower with increasing temperatures, accompanied by a continuous movement of the minimum point to the right.

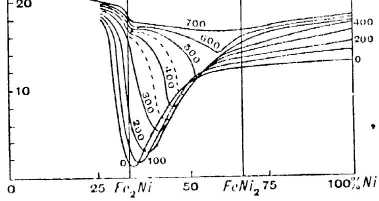


FIG. 14.—Variations in the True Coefficient of Linear Expansion of Ni Steels at various Temperatures (Chevenard).

§ (5) INVAR.—The minimum point of the curve of *Fig.* 13 represents an alloy containing 35·6 per cent Ni, with an expansibility of about 1·2 in a million per 1° C., and the name Invar (diminutive of invariable) is now applied to all the alloys having expansibilities in the neighbourhood of this point. The lowest coefficient of expansion hitherto associated with a pure metal is that of tungsten, 4 in a million per 1° C.

The value of the coefficient of expansion at the minimum point is not invariable; it is influenced by the amount of manganese and carbon present, and may be varied accordingly. But, as already noted, a certain quantity of manganese is essential to assist forging. Also all treatment, whether mechanical or heat, to which invar may be subjected, modifies the value. Heating followed by slow cooling increases the value, but heating followed by rapid cooling reduces it. Cold-rolling or drawing further assists in reduction. The important fact follows that it is thus possible by suitable treatment to reduce the coefficient of expansion so much that it becomes negative, Guillaume producing by such a means invar with a coefficient equal to $(-0{\cdot}552+0{\cdot}00377\theta)10^{-6}$ at ordinary temperatures. An adjustment of the conditions renders it possible to produce invar of almost zero expansibility, and as early as 1903 several kilometres of wire were produced having a coefficient equal to $(0{\cdot}028-0{\cdot}00232\theta)10^{-6}$. The value of this in accurate surveying work is too obvious to require labouring.

Unfortunately invar possesses a slight instability which manifests itself in various ways, but which it is necessary to take into account where the highest accuracy is required.

A forged bar of invar shows small transitory changes of length after change of temperature. Consider it as observed at a certain temperature θ. If it has been previously exposed at a higher temperature it will, after the main contraction is effected, continue to elongate slowly. If, however, it has been brought from a lower temperature, there is a corresponding slow contraction. The time during which such a change occurs is greater when θ is reached by cooling than when it is reached by heating. Also the higher the temperature the greater the rate of these changes. The amount of these anomalous changes for a rapid change of temperature, between the limit 0° and 100° C., may be represented by the following empirical formula (due to Guillaume), θ being the temperature,

$$\frac{\Delta L}{L} = -0{\cdot}00325\,10^{-6}\theta^2.$$

A forged bar of invar also possesses a slow progressive and permanent change extending over several years. This secular change is independent of the transitory changes just referred to and is continuous. A freshly forged bar, cooled to and kept at normal temperature, expands slightly, at first quickly and then more slowly, approaching a definite limiting length. The extent of the change can, however, be reduced by employing a suitable ageing process, called by Guillaume *étuvage* (stoving or stewing), to distinguish it from a similar but distinct process, annealing. It consists of maintaining the material at a steady temperature of 100° C. for 100 days, and then very slowly cooling it over a period of two or three months. *Fig.* 15 will give an idea of the extent of this secular change

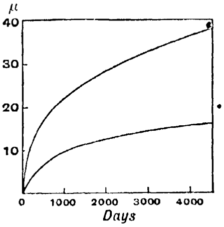


FIG. 15.—Elongations in Microns per Metre of Invar with Time (Guillaume). Upper curve, forged bar not treated; lower curve, forged bar cooled in 50 days from 150° to 40° C.

for a bar not treated and for a bar subjected to an ageing or stoving process.

An aged invar bar (No. 27) has been under observation at the N.P.L. for 19 years, and the total elongation during that time amounts to 21 μ (*Fig.* 16).

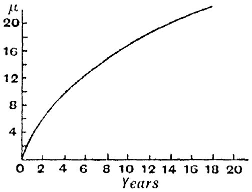


FIG. 16.—Curve showing Secular Growth of N.P.L. Invar Standard Metre No. 27 during 19 years.

A drawn bar of invar behaves in a manner similar to that of a forged bar, but in addition exhibits a change not possessed by the latter. If it rests for some time at normal temperature, and is then brought to 100° C., it shortens, then elongates, and afterwards again shortens.

The facts outlined in the preceding paragraphs serve to show how complicated is the thermal expansion of invar, and emphasises the necessity for care in using the metal for standards. It is obviously not suitable for use in making absolute or reference standards, but it is of considerable value provided it is used with precaution, and provided it is checked at intervals, at least once a year, against more stable standards.

A more stable alloy is the 42 per cent or 43 per cent nickel-steel, which shows only a very small secular growth after it has been suitably aged. On this account it is often preferred to invar, although its expansion is about $+8 \times 10^{-6}$ per 1° C.

Quite recent researches (1920) by Guillaume point to the presence of carbon as the cause of instability, and that therefore there is every prospect of producing eventually an invar which shall be absolutely stable.[1]

§ (6). (i.) *Fused Silica* has also been mentioned[2] as having a low expansibility ($+0{\cdot}4 \times 10^{-6}$ per 1° C.), which gave rise to the idea of utilising it for a line standard. Its lack of thermal hysteresis also recommended it in this direction, and its secular stability has since been proved by observations extending over a number of years. It is simple in composition, durable, chemically stable, and is certainly not costly. Its chief drawbacks are its fragility and the difficulty of rendering visible through water lines ruled on its surface. The latter has been overcome in a simple manner as described below, while the former resolves itself into a matter of careful handling, such as it would naturally receive in the hands of a person habitually using important standards.

(ii.) *Silica Metre.*—The idea of utilising silica as the material for standards of length was first conceived at the N.P.L., where, after preliminary investigations, was designed a silica metre standard, which was completed in 1911.[3] It consists of a tube of transparent silica about ¾ inch in diameter, terminating at each end in a horizontal silica slab, which is about ⅛ inch thick and semicircular in shape. To prevent rolling of the bar when under observation, silica trunnions are fused into the rod at one of the correct positions of support, the other point being indicated by an etched line round the tube. The trunnions are wedge-shaped, with the edges underneath. Special supports are provided, one with two flats for the trunnions, the other being an ordinary roller for the other points. As the bar is observed in water, it is provided with a hole at each end so that the inside of the tube may be filled with water.

The slabs and the edges of the trunnions are very accurately finished. The surface of each slab is polished optically flat and parallel. The edges of the trunnions are parallel to the surfaces of the slabs to within 1 in 500. The upper surfaces of the slabs are parallel to within 1 in 15,000, and are coplanar to within 1 mm., and the two slabs are the same thickness within 0·02 mm.

The *under* sides of the slabs are platinised, the films being very thin and adherent. The defining lines are ruled completely through these films, and are viewed *through* the slabs from above. It is for this reason that the slabs are so accurately finished to prevent optical displacement of the images. The films are protected by cover slips cemented over them.

---

[1] Guillaume, "Aciers au Nickel"; *Proc. Phys. Soc.*, xxxii. 374; B. of S. Circ. 58, "Invar and Related Nickel Alloys"; article "Invar," Vol. V.

[2] See "Metrology," § (4).

[3] G. W. C. Kaye, D.Sc., "A Silica Standard of Length," *Proc. Roy. Soc.* A. 85, 1911.

If the points of support of the bar are correctly placed in accordance with Airy's formula, the end slabs will be horizontal, and any slight error in fixing the points will cause points in the neutral plane near the ends of the bar to be displaced in a vertical direction only, that is, there will be no horizontal displacement. For this reason, the end slabs are so arranged that the image of the lines shall always lie in the neutral plane. As the images are formed slightly above the platinised surfaces, it follows that the latter must be placed just as much below the neutral plane. Thus the slabs are not symmetrically placed with respect to a horizontal axial plane of the bar, the neutral plane being in fact 0·3 mm. above the platinised surfaces.

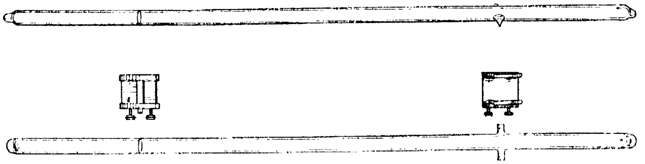


FIG. 17.—The N.P.L. Silica Metre and Special Supports.

The bar was made by the Silica Syndicate, the polishing of the surfaces was carried out by Messrs. Hilger, and the platinising and ruling of the bar done by the staff of the N.P.L., where the bar has since been used as a ternary standard. Since its completion, it has been under continuous observation, except during the war, and there is no evidence so far of any sensible change in length, any slight variations in results obtained being attributable to errors of observation.

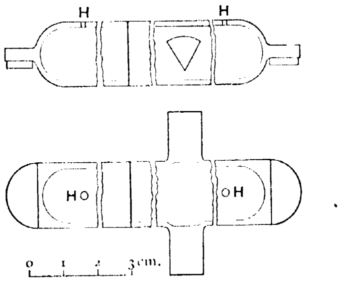


FIG. 18.—The N.P.L. Silica Metre (scale drawing).

Figs. 17 and 18 give an idea of the appearance and dimensions of the bar.

MICHELSON'S EXPERIMENT

§ (7). (i.) No account of length standards is complete without a description of the experiment whereby Professor Michelson measured the metre in terms of the wave-length of monochromatic light. This attempt to establish a "natural" standard of length is best understood by reference to his own explanatory diagram, *Fig.* 19, the apparatus which he used being based on his well-known interferometer.[1]

Two pencils of light from a source *s* (*Fig.* 19) are collimated and fall on the back surface (as viewed from *o*) of the plate *a*; part of the light is reflected towards the mirror *d*, and back on its path through *a* to an observer at *o*, while the other part passes on to mirror *b*, through the compensating plate *c*, and is reflected back along the same path, one pencil from the mirror *m*, and the other from the mirror *n*, and finally by reflection at *a* reaches *o*. Two sets of interference fringes can be seen at *o*, corresponding to the two pencils of light. If *m* or *n* be moved in a direction at right angles to its reflecting surface, the corresponding set of fringes will move across the field of view, and the number passing any particular fiduciary mark can be counted.

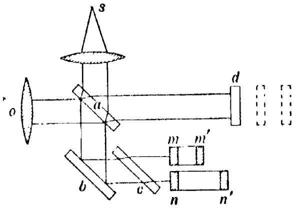


FIG. 19.—Michelson's Experiment. The optical arrangement.

Owing to the limited distance at which interference fringes can be observed, Michelson found it necessary to adopt as an intermediate standard a length shorter than the metre. He proceeded to measure the number of wave-lengths in this intermediate standard, and then used the latter for measuring the whole metre in much the same way as a yardstick is used for measuring a length of cloth. This intermediate standard had to be as long as possible to reduce errors of addition when it was applied to the metre, and yet to be short enough to give distinct fringes. The length of one decimetre was decided upon as best fulfilling these conditions.

---

[1] *Trav. et Mém. de B.I.P.M.* xi. 1; A. A. Michelson, *Light Waves and their Uses*. See also "Interferometers," Vol. IV.

Michelson found it convenient to construct his decimetre or intermediate standard in a form differing from that of the ordinary line standard, and *Fig.* 20 shows how this was done.

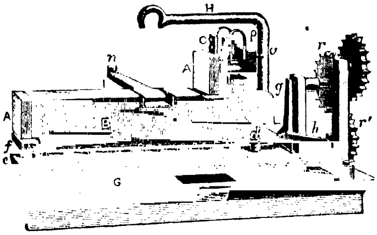


FIG. 20.—An Intermediate Standard.

On a brass casting B are mounted two mirrors A and A′, which are adjustable by means of three small pins to their rear, so that their front faces are truly parallel to a high accuracy, the distance between the front faces constituting the length of the intermediate standard.

The number of wave-lengths concerned in one decimetre is of the order of 300,000, the difference of path of the waves being of course two decimetres. The counting of such a large number with absolute certainty as to the result would have proved an enormous task had it been attempted. Michelson, however, decided on another process of counting. He constructed a number of still smaller standards of the same type as the intermediate standard described above, their lengths being such that, together with the decimetre standard, they formed a geometrical progression with a common ratio of $\frac{1}{2}$.

Altogether nine such standards were used, varying in length from about $\frac{1}{2}$ mm. for the smallest to one decimetre for the largest. The counting with certainty of the number of wave-lengths, a matter of about 1200 in the shortest of these lengths, was a comparatively simple matter. The numbers of wave-lengths in the other standards were obtained by a series of comparisons with this smallest standard, no further counting being necessary. How this was done will be clear from a description of the method of carrying out the observations.

The source of light for the apparatus is a cadmium vapour lamp which gives out three simple radiations—red, green, and blue—and the particular radiation required is selected by means of a dispersion prism.

Returning to *Fig.* 19, *mm′*, *nn′* represent the mirrors of the two smallest standards. *d* will be referred to in what follows as the reference plane. *d*, *mm′*, and *nn′* are arranged so that they can be moved in the direction of the rays of light. The two front mirrors *m* and *n* are adjusted in position to give fringes in white light with the reference plane *d*. The central fringes being black and the others coloured, it is easy to distinguish them, and when the central fringes occur in the same relative position on *m* and *n*, then the latter are in exactly the same plane. In this position the surface of the reference plane may be said to be coincident with *m* and *n*. The reference plane is next moved slowly backwards the length of *mm′*, the shorter standard, that is, until the reference plane is coincident with *m′*, the fringes due to the selected radiations being carefully counted as they pass across the field of view. The coincidence of *m* and *d* is detected by the fringes due to white light. This may be repeated once or twice as desired in order to verify the number counted. The fraction of fringe at the beginning and at the end of the interval can be estimated to about $\frac{1}{20}$th of a fringe, corresponding to about $\frac{1}{40}$ μ. The next step consists in moving the standard *mm′* backwards through its length, that is, until *m* is again coincident with the reference plane, which is then once more moved through the distance *mm′* and becomes coincident with *m′* again. At this point, the appearance of the white light fringes will indicate when *nn′* is exactly twice the length of *mm′*, and if it is not so, it is possible by a simple process to measure the difference in fringes with the same degree of accuracy as noted above. Thus the length of *nn′* is found by direct comparison with *mm′*.

mm′ is now removed and replaced by the standard which is twice *nn′* and the whole process repeated. By a succession of such steps the decimetre is at last reached, and its length in wave-lengths is determined. *Fig.* 21 gives a general idea of the apparatus used, the source of light, dispersion prism, etc., being omitted.

It now remains to compare the decimetre with the metre, and this is done in ten steps by exactly the method described. Resort is made to the microscope in the final comparison with the line standard, and although the accuracy obtained is not nearly so good as with the wave measurements, the total error due to the only two observations necessary is about equal to that due to the probable sum total of error due to the wave measurements.

The number of wave-lengths in one metre was thus found to be as follows:

1,553,163·5 for the red radiation,
1,966,249·7 for the green radiation,
2,083,372·1 for the blue radiation,

all being observed in air at 15° C. under normal atmospheric pressure.

In the design and construction of the apparatus, various mechanical difficulties arose, such as the production of accurately lapped ways for the carriages supporting the reference plane and standards, but these were all overcome with much prolonged and patient labour.

(ii.) A re-determination of the metre in terms of wave-lengths was made in 1906 by MM. Benoit, Fabry, and Perot.[1] The method employed and the apparatus used differed considerably from Michelson's, the manner of counting the wave-lengths depending on the coincidence of fringes of different wave-lengths, though recourse had still to be made to a succession of subsidiary standards of a special type. The number of wave-lengths of the cadmium red radiation was found to be 1,553,164·13, thus confirming very closely Michelson's figure.

W. H. J.

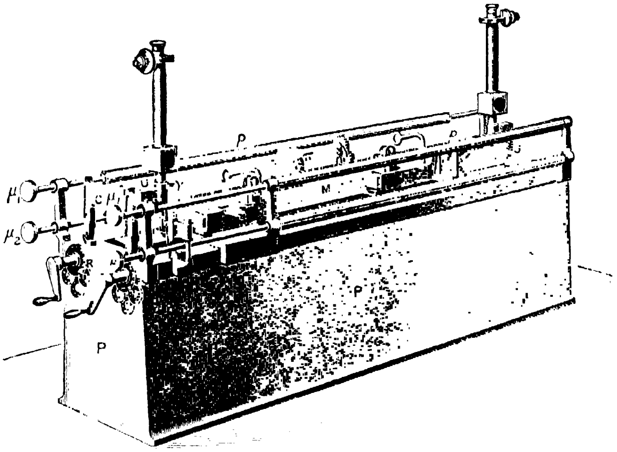


FIG. 21.—Michelson's Experiment.
The apparatus with cover removed (lighting arrangements not shown).

Liquid, Density of, determined by means of total immersion floats. See "Balances," § (15) (v.).

Determination of, by means of a pyknometer. See *ibid.* § (15) (i.).

Determined by means of sinker weighings. See *ibid.* § (15) (ii.).

Liquid Depth Gauge, distant reading type. See "Liquid Level Indicators," § (3).

Litre: the metric unit of volume. See "Volume, Measurements of," §§ (1) and (2).

Location Signal: term applied to a group of wave-length rulings. See "Line Standards," § (1) (vi.).

Log-log Rules. See "Draughting Devices," p. 273.

Longitude, Determination of. See article "Gravity Survey," § (9).

By prismatic astrolabe. See *ibid.* § (9) (iii.).

Longitudinal Comparator at the Bureau International, Sèvres: description. See "Comparators," § (11).

Longitudinal or Subdividing Comparators: general description, scope. See "Comparators," § (10) (i.).

Looming. See "Meteorological Optics," § (8).

Lowenherz Standard Thread. See article "Gauges," § (50).

— M —

Majorana's Quenching Factor. See article "Gravity Survey," § (16).

Manometer:

High precision, designed by Lord Rayleigh for use with a constant head of liquid, to determine the densities of gases: a steel rod, containing two fiducial points, determines, and also measures, the distance between the upper and lower mercury surfaces. See "Barometers and Manometers," § (20) (*a*).

Metal Diaphragm, used as a sphygmomanometer to measure blood-pressures. See *ibid.* § (19) (vi.).

---

[1] *Trav. et Mém. de B.I.P.M.* xv. 1.

Optical lever, for measuring small-pressure differences. See *ibid.* § (20) (*c*).

Simple U-tube (siphon). See *ibid.* § (19) (i.).

Manostat: an instrument, used chiefly in chemical experiments, to control the pressure within an apparatus. See "Barometers and Manometers," § (21).

MAP PROJECTIONS

The word Projection is used here in a sense much wider than it bears in geometry. A map is the representation on a flat sheet of the relative positions of features which lie on a curved surface, the surface of the Earth. A perfect representation being impossible, since the surface of a spheroid is not developable on a plane, we have to find some construction on the plane corresponding to the meridians and the parallels of latitude on that portion of the spheroid to be represented on the map: this construction is the Map Projection. A geometrical projection, such as the projection from the centre of the spheroid on a tangent plane, has not necessarily any particular advantage, and the projections in actual use are mostly not of this kind. Any orderly method of constructing meridians and parallels may be considered a map projection or "graticule," whose value will depend on fulfilment of conditions appropriate to the map under construction.

The criteria of a Map Projection are: (1) uniformity of scale; (2) accurate representation of area; (3) true representation of shape; (4) ease of construction; and (5) adaptability to methods of survey and computation. The first three are criteria of principle: they cannot all be satisfied at once over any large area of the Earth's surface, though for an area a few degrees square the representation may be practically perfect. The last two have little theoretical interest but great practical importance, which is sometimes overlooked in treating the subject.

§ (1) Four Classes of Maps.—The choice of a projection is governed first by the class of map required: (*a*) the World map, covering at least a hemisphere; (*b*) the Atlas map, covering a continent, or at least a country, on scales smaller than 1/Million, with generalised topography; (*c*) the smaller scale Topographical map, between 1/Million and perhaps 1/100,000, either compiled and reduced from larger scale surveys, or the result of rapid and approximate work in the field, and covering at most a few degrees square; and finally (*d*) the large scale Topographical map, each sheet covering only a few hundred square miles on not less than 1/50,000, representing the detail surveys of the field and the calculations of the computing office. In maps of class (*a*) the defects of projection are obvious, and the choice is a choice of evils; in class (*b*) a judicious choice can generally avoid the grosser errors, though the map is sensibly imperfect in its representation of distance or area or shape; in class (*c*) the representation on a single sheet is to the eye practically perfect, though the errors are susceptible of measurement; and in (*d*) the errors of a well-chosen projection may be smaller than can be drawn, but are still readily appreciable in the numerical work that is the basis of the map. In the first two the errors that result from neglecting the ellipticity of the Earth's figure are so much smaller than the defects of the projection that it is hardly worth while to trouble about them. In the two last it is essential to take into account the figure of the Earth; not because it produces any visible difference in a single sheet of the map, but because it is involved in the survey calculations.

§ (2) The Properties of Projections.—The criteria by which a projection is judged are defined above as:

(i.) *Representation of Scale:* A small distance on the ground should bear a constant proportion to the corresponding distance on the map. This cannot be secured completely, or the map would be a perfect representation, which is impossible. But the scale along the meridians may be correct, or along one or more parallels, or along the radii from the centre of the projection.

(ii.) *Representation of Area:* Although the scale cannot be correct in all directions all over the map, it may at every point of the map be such that the scale in one direction is inversely proportional to the scale at right angles to the first; each element of area has then the same proportion to the corresponding element on the ground, and the map is "equal area."

(iii.) *Representation of Shape:* Although all shapes cannot be represented truly, the scale in one direction may be at every point proportional to the scale at right angles, so that the shape of *elementary* areas is preserved, and the projection is called orthomorphic or conformal.

(iv.) *Ease of Construction:* which depends more on the technical equipment of the cartographer than on the complexity of the projection. In most cases the projection of a map is plotted in rectangular co-ordinates, and smooth curves drawn through the plotted points. Unless the curvature is very sharp, the class of curve makes little difference, at any rate in a large establishment, where the calculation and drawing are done by different people.

(v.) *Adaptability* to methods of survey and computation is much more important in the work of a large survey, where it is desirable that the numerical co-ordinates of all trigonometrical points shall be calculated once for all in a single system, ready for immediate use on any sheet of the series. This is especially important in war, for successful co-operation with the artillery.

§ (3) Relative Importance of these Properties.—This depends on the class of map. For a World map orthomorphism is of small value, since the shape of large areas cannot be preserved correct; equivalence of area is desirable, at least approximately; accuracy of scale sufficient to allow of distances being measured in any direction can hardly be obtained; but ease of construction is useful. Most important is the choice of centre and boundaries, so that the area to be represented is well arranged, and the relative situation of its parts preserved so far as possible. But, speaking generally, no measures can be made on a World map without precautions and subsequent reduction: the misrepresentation on a plane is too great.

In an Atlas map, as defined above, the errors are not so unmanageable. On a well-chosen equal area projection the areas are preserved and distances may be measured roughly, with precaution. But true bearings or azimuths are generally unmeasurable to a greater or less degree according to the size of the area, and orthomorphism is not of much account except in the special case of sea-charts on Mercator's Projection.

In the smaller scale Topographical maps it is easy to have scale and areas sensibly correct, though not exact, and orthomorphism is important as facilitating the compilation of material reduced from varying but larger scales. In the larger scale Topographical maps and plans the errors of the projection may be made insensible to measurement, and the projection is chosen to facilitate the orderly and convenient arrangement of the triangulation and reductions.

§ (4) Nomenclature of Projections.—Practice varies very much; but a convenient way is to divide the projections into genera according to the method of construction: conical, cylindrical, zenithal (including perspective as a sub-genus), and conventional: the last a wide class, which may at discretion include or exclude the modifications in the former, such as Bonne's or the Polyconic. The genera are then divided into species according to their properties of correct representation of length, area, or shape of an elementary area: thus we have the conical equal area, the zenithal orthomorphic, and so on. This principle cannot be used to the full without pedantry, as there are some projections always called after their inventors, as Mercator's, or after their principal advocate, as Bonne's.

§ (5) Classes of Projections.—Perspective projections are those which are projections in the geometrical sense, produced by the intersection of a plane with a bundle of rays drawn through points on the spheroid from a common vertex of projection. Such are the Gnomonic, Stereographic, Orthographic, and Clarke's Minimum Error Perspective Projections.

(i.) *Conical Projections.*—These are defined most generally as those in which a set of straight lines radiating from a common vertex are cut at right angles by a set of concentric circular arcs described about that vertex. In practice, though not necessarily, the first set are equally spaced, and the different properties required are obtained by modifying the spacing of the circular arcs. The figure thus bounded by the extreme radii and the circular arc of radius r, furthest from the vertex, is a fan-shaped figure with an angle at the vertex equal to $2\pi n$, where n lies between 0 and 1. This fan-shaped figure may be rolled into a cone of which the solid angle depends on the magnitude of n, which is called the constant of the cone. The radiating straight lines have become generators of the cone, and the circular arcs have become its circular sections.

Let one of these circular sections be the same size as a chosen parallel of latitude of the spheroid, and be made to coincide with it. The vertex of the cone will then lie in the polar axis of the spheroid produced. The circumference of the circular section is $2\pi rn$, and of a parallel of the spheroid is $2\pi\nu \cos \phi$, where ϕ is the geographical latitude and ν the radius of curvature at right angles to the meridian in that latitude. Hence the constant of the cone is $\nu \cos \phi / r$.

The two coincident circles, the section of the cone and the parallel of latitude, being divided similarly, the generators of the cone will correspond to meridians of the spheroid, and will make angles with one another equal to n times the corresponding angles between the meridians. The position of the vertex on the polar axis produced, and consequently the length of r, are still at disposal, and there is evidently an infinite series of cones based on a given standard parallel, but with different radii and different values of n, the constant of the cone. If the vertex be so chosen that the cone is tangent to the spheroid along the parallel of latitude, then $r = \nu \cot \phi$ for that parallel, and the constant of the cone equals $\sin \phi$. If, further, we have the radii of the concentric set of arcs so spaced that their distances apart are the same as the distances of the other parallels of latitude of the spheroid, then we have on the cone a construction of radiating lines and orthogonal concentric arcs which, when the cone is developed on to a plane, gives the simple conical projection.

By keeping the meridians the same, but modifying the spacing of the parallels, one may make the projection equal area or orthomorphic; and one might do the same on any one of the cones of the infinite series mentioned above. But in fact this great variety is never used; if one parallel is to be made the right length, the cone is always supposed to be tangent at that parallel for the simple conic with true meridians, and for the simple conical orthomorphic, though not for the conical equal area with one standard parallel.

In all such conical projections with only one parallel its true length, the scale along the other parallels rapidly becomes seriously untrue, which can be to a great extent remedied by making two parallels standard, or true to scale. But it is important to note that the cone with these two parallels standard is not the cone which cuts the spheroid in these two parallels, and to speak of these projections as "secant conical" projections is misleading.

(ii.) *Oblique Conical Projections.*—Up to the present we have supposed the radiating lines to correspond to meridians of the spheroid, and the concentric circular arcs to its parallels of latitude: this is the normal case, and the vertex of the cone may then be pictured as in the polar axis produced, though by this time we may very well dispense with picturing the cone at all. But if the conical projection is not normal, the radiating lines from its vertex will correspond to radiating great circles from a point on the sphere—to speak of the spheroid makes difficulties—and the concentric arcs correspond to concentric small circles of the sphere about the point. Such oblique conical projections have no value, except in the particular case when the constant of the cone becomes unity, and the cone may be pictured as degenerating into the tangent plane at the point. This gives the valuable series of Zenithal or Azimuthal Projections. Zenithal projections may of course be centred on the pole (Polar Zenithal), or on the equator, but more generally are centred on some point between the two, and called Horizontal. They are usually true to scale at the centre, though there is no reason why some other small circle instead of that at the centre should not be made true; and the radii are divided to give true distances from the centre, equal areas, or orthomorphism, on the same principles as the conical projections from which they may be considered derived. The meridians and parallels of the sphere are represented by complex curves, which are, however, constructed without much difficulty by calculating the distances and bearings from the centre of their principal intersections, plotting the bearings true, and the distances along the radials according to the law of the projection. One zenithal projection may therefore be readily converted into another, of different properties, by simply modifying the radial distances, preserving the bearings from the centre. Zenithal projections are much used for Atlas maps, but never for maps on a large scale, and it is rarely if ever necessary to take account of the ellipticity of the Earth further than to use for the radius the geometric mean of the radii of curvature at right angles to and in the meridian through the centre of the map.

(iii.) *Cylindrical Projections.*—If the constant of the cone becomes zero, or the vertex is supposed to be at infinity, the cone becomes a cylinder, which may be thought of as touching the spheroid along the equator (Normal Cylindrical), or along a meridian (Transverse Cylindrical), or along some other great circle (Oblique Cylindrical), the last being of little importance. The distances from the vertex of the cone, the radii, all become infinite, and our formulae have to be adapted to give differences of radii. Three of the cylindrical projections—Mercator's, Cassini's, and Lambert's first or the Gauss Conformal—are among the most important of all projections.

§ (6) UNIFORM NOTATION FOR FORMULAE OF PROJECTIONS. (i.) *Conical:* r_0 is the radius of a single standard parallel of latitude ϕ_0 or co-latitude χ_0.

r_1, r_2 are the radii of two standard parallels of latitudes ϕ_1, ϕ_2, or co-latitudes χ_1, χ_2.

r is the radius of any other parallel of latitude ϕ or co-latitude χ.

R is the radius of the Earth reduced to the scale of the projection: or if the ellipticity is taken into account, ρ is the radius of curvature in the meridian, and ν at right angles to the meridian.

n is the constant of the cone;

$\Delta\lambda$ the difference of longitude from the central meridian;

θ the angle a meridian of the projection makes with the central meridian; whence $\theta = n . \Delta\lambda$.

(ii.) *Zenithal:* r is the radial distance from the centre of the projection corresponding to an angular distance ζ on the sphere.

β is the angular radius of the boundary of the map.

§ (7) SCALE VALUE OF A PROJECTION AT DIFFERENT POINTS. (i.) *Conical:* The scale along a meridian is $dr/\rho d\phi$. The scale along a parallel is

$$rd\theta/\nu \cos\phi \, d\lambda = nr/\nu \cos\phi,$$

and then scale values can be found very easily for any point from the formulae given for the radii in the various conical projections, it being practically sufficient to put both ρ and $\nu = \text{R}$ in the calculation.

(ii.) *Zenithal:* The scale along a radial is $dr/\text{R}d\zeta$ and along a parallel small circle is $r/\text{R} \sin\zeta$.

Considerations of symmetry show that the above found scale values are maximum and minimum values of the scale values at the point. Call them a and b $(a > b)$. Then an elementary small circle about the point on the Earth is transformed by the projection into an ellipse with semi-axes a and b. This ellipse is called the Indicatrix. It is easily shown that the maximum deformation of angle in the immediate neighbourhood of the point is 2ω, where $\sin\omega = (a-b)/(a+b)$. The values of a, b, and 2ω are often tabulated for intervals over the projection, as a criterion of its deformation; and for all orthomorphic projections $2\omega = 0$. But this gives a false impression of accuracy. A better test of the freedom of a projection from serious distortion is to calculate from the rectangular co-ordinates the distances and bearings between selected points far apart on the projection, and compare with their true distances and bearings on the sphere. The result of such comparison is by no means in favour of the orthomorphic projections for maps of great extent.

§ (8) PRINCIPAL PROJECTIONS.—The principal projections for maps are described briefly in the following paragraphs, arranged alphabetically, not systematically. When a projection is of value only for small Atlas maps, the Earth is generally taken as spherical; but when necessary the radii of curvature ρ and ν are used instead of R in the formulae which follow:

(i.) *Airy's Projection by Balance of Errors.*—Invented by Sir George Airy when Astronomer Royal (*Phil. Mag.*, Dec. 1861; corrected by Capt. Clarke, *Phil. Mag.*, April 1862). A zenithal projection on a complicated formula constructed to make the "total misrepresentation" (expressed by the integral over the surface to be represented of the sum of squares of errors of scale in two orthogonal directions) a minimum. The law of the radii depends on the spherical radius to be represented. The original investigation by Airy was erroneous, and although the error was quickly pointed out by Clarke, the numerical results of Airy are often quoted in text-books. A small example, with centre in lat. 23½° N. and spherical radius 113½°, is frequent in atlases: but its only serious use is for map of United Kingdom by Ordnance Survey on 10 miles to inch, covering area much too small to exhibit the merits of the projection, which is suitable for hemispheres.

Formula:

$$r = 2\mathrm{RM}^{-1}\{\cot\tfrac{1}{2}\zeta \log\sec\tfrac{1}{2}\zeta \\ + \tan\tfrac{1}{2}\zeta\cot^2\tfrac{1}{2}\beta\log\sec\tfrac{1}{2}\beta\},$$

where logs are common, M their modulus, $\mathrm{M}^{-1} = 2{\cdot}30259$, and β the spherical radius.

(ii.) *Bonne's Projection, or Projection du Dépôt de la Guerre.*—First used in a rough form by Ptolemy for his second projection for World-map; by Bonne (1752); and adopted by the Dépôt de la Guerre (1803) for the map of France on the scale 1/80,000. Much used throughout last century in continental surveys, for the O.S. maps of Scotland and Ireland, and in atlases. A modification of the simple conic, in which all the parallels are divided truly and meridians are curves passed through these dividing points. The projection is equal area; and scale along and perpendicular to parallels (but not along meridians) is correct. Its great defect is the obliquity of meridians to parallels, increasing with distance from centre. This proved so disadvantageous for artillery work that at the end of the Great War the tactical maps of the Allies were all being converted to Lambert's Conical Orthomorphic Projection (*q.v.*).

(iii.) *Breusing's Projection.*—A mean between the Zenithal Equal Area and Zenithal Orthomorphic (Stereographic). Its inventor employs the geometric mean, with radii given by

$$r = 2\mathrm{R}\sqrt{\tan\tfrac{1}{2}\zeta\sin\tfrac{1}{2}\zeta}.$$

But it has been shown recently by Young that the harmonic mean gives a better result, with radii

$$r = 2\mathrm{R}\tan\tfrac{1}{4}\zeta.$$

(iv.) *Cassini's Projection* (Transverse Simple Cylindrical).—Used by César François Cassini for the Carte de France, 1745–1793, and by the Ordnance Survey for the maps of England (Scotland and Ireland are on Bonne's Projection). Taking a central meridian and fixed point on it as origin (for England the meridian through the point of the principal triangulation at Delamere, Cheshire), the length of the spheroidal perpendicular from any point to the central meridian, and the length of the arc of meridian from the origin to the foot of this perpendicular, are plotted as rectangular co-ordinates. The scale at right angles to the central meridian is correct; the scale parallel to it is too great by a quantity varying as the square of the distance from the meridian. Hence the projection should not be used for a map covering a great extent of longitude.

For spherical Earth the co-ordinates x and y are given by

$$\sin x = \sin\Delta\lambda\cos\phi,$$

$$\cot(\phi_0 + y) = \cos\Delta\lambda\cot\phi,$$

ϕ_0 being latitude of origin. For spheroidal Earth the expressions for x and y are complicated, and calculation proceeds by successive approximations, or by the expansions in series (see *The Mathematical Basis of the Ordnance Maps of the U.K.*, Major A. J. Wolff, D.S.O., R.E., Southampton, 1919).

(v.) *Clarke's Minimum Error Perspective Projection.*—About 1860 Colonel A. R. Clarke applied Airy's principle of making total may make the projection equal area or orthomorphic; and one might do the same on any one of the cones of the infinite series mentioned above. But in fact this great variety is never used; if one parallel is to be made the right length, the cone is always supposed to be tangent at that parallel for the simple conic with true meridians, and for the simple conical orthomorphic, though not for the conical equal area with one standard parallel.

In all such conical projections with only one parallel its true length, the scale along the other parallels rapidly becomes seriously untrue, which can be to a great extent remedied by making two parallels standard, or true to scale. But it is important to note that the cone with these two parallels standard is not the cone which cuts the spheroid in these two parallels, and to speak of these projections as "secant conical" projections is misleading.

(ii.) *Oblique Conical Projections.*—Up to the present we have supposed the radiating lines to correspond to meridians of the spheroid, and the concentric circular arcs to its parallels of latitude: this is the normal case, and the vertex of the cone may then be pictured as in the polar axis produced, though by this time we may very well dispense with picturing the cone at all. But if the conical projection is not normal, the radiating lines from its vertex will correspond to radiating great circles from a point on the sphere—to speak of the spheroid makes difficulties—and the concentric arcs correspond to concentric small circles of the sphere about the point. Such oblique conical projections have no value, except in the particular case when the constant of the cone becomes unity, and the cone may be pictured as degenerating into the tangent plane at the point. This gives the valuable series of Zenithal or Azimuthal Projections. Zenithal projections may of course be centred on the pole (Polar Zenithal), or on the equator, but more generally are centred on some point between the two, and called Horizontal. They are usually true to scale at the centre, though there is no reason why some other small circle instead of that at the centre should not be made true; and the radii are divided to give true distances from the centre, equal areas, or orthomorphism, on the same principles as the conical projections from which they may be considered derived. The meridians and parallels of the sphere are represented by complex curves, which are, however, constructed without much difficulty by calculating the distances and bearings from the centre of their principal intersections, plotting the bearings true, and the distances along the radials according to the law of the projection. One zenithal projection may therefore be readily converted into another, of different properties, by simply modifying the radial distances, preserving the bearings from the centre. Zenithal projections are much used for Atlas maps, but never for maps on a large scale, and it is rarely if ever necessary to take account of the ellipticity of the Earth further than to use for the radius the geometric mean of the radii of curvature at right angles to and in the meridian through the centre of the map.

(iii.) *Cylindrical Projections.*—If the constant of the cone becomes zero, or the vertex is supposed to be at infinity, the cone becomes a cylinder, which may be thought of as touching the spheroid along the equator (Normal Cylindrical), or along a meridian (Transverse Cylindrical), or along some other great circle (Oblique Cylindrical), the last being of little importance. The distances from the vertex of the cone, the radii, all become infinite, and our formulae have to be adapted to give differences of radii. Three of the cylindrical projections —Mercator's, Cassini's, and Lambert's first or the Gauss Conformal—are among the most important of all projections.

§ (6) UNIFORM NOTATION FOR FORMULAE OF PROJECTIONS. (i.) *Conical:* r_0 is the radius of a single standard parallel of latitude ϕ_0 or co-latitude χ_0.

r_1, r_2 are the radii of two standard parallels of latitudes ϕ_1, ϕ_2, or co-latitudes χ_1, χ_2.

r is the radius of any other parallel of latitude ϕ or co-latitude χ.

R is the radius of the Earth reduced to the scale of the projection: or if the ellipticity is taken into account, ρ is the radius of curvature in the meridian, and ν at right angles to the meridian.

n is the constant of the cone;

$\Delta\lambda$ the difference of longitude from the central meridian;

θ the angle a meridian of the projection makes with the central meridian; whence $\theta = n \,.\, \Delta\lambda$.

(ii.) *Zenithal:* r is the radial distance from the centre of the projection corresponding to an angular distance ζ on the sphere.

β is the angular radius of the boundary of the map.

§ (7) SCALE VALUE OF A PROJECTION AT DIFFERENT POINTS. (i.) *Conical:* The scale along a meridian is $dr/\rho d\phi$. The scale along a parallel is

$$rd\theta/\nu \cos\phi \; d\lambda = nr/\nu \cos\phi,$$

and then scale values can be found very easily for any point from the formulae given for the radii in the various conical projections, it being practically sufficient to put both ρ and $\nu = \text{R}$ in the calculation.

(ii.) *Zenithal:* The scale along a radial is $dr/\text{R}d\zeta$ and along a parallel small circle is $r/\text{R} \sin\zeta$.

Considerations of symmetry show that the above found scale values are maximum and minimum values of the scale values at the point. Call them a and b ($a > b$). Then an elementary small circle about the point on the Earth is transformed by the projection into an ellipse with semi-axes a and b. This ellipse is called the Indicatrix. It is easily shown that the maximum deformation of angle in the immediate neighbourhood of the point is 2ω, where $\sin \omega = (a-b)/(a+b)$. The values of a, b, and 2ω are often tabulated for intervals over the projection, as a criterion of its deformation; and for all orthomorphic projections $2\omega = 0$. But this gives a false impression of accuracy. A better test of the freedom of a projection from serious distortion is to calculate from the rectangular co-ordinates the distances and bearings between selected points far apart on the projection, and compare with their true distances and bearings on the sphere. The result of such comparison is by no means in favour of the orthomorphic projections for maps of great extent.

§ (8) PRINCIPAL PROJECTIONS.—The principal projections for maps are described briefly in the following paragraphs, arranged alphabetically, not systematically. When a projection is of value only for small Atlas maps, the Earth is generally taken as spherical; but when necessary the radii of curvature ρ and ν are used instead of R in the formulae which follow:

(i.) *Airy's Projection by Balance of Errors.*—Invented by Sir George Airy when Astronomer Royal (*Phil. Mag.*, Dec. 1861; corrected by Capt. Clarke, *Phil. Mag.*, April 1862). A zenithal projection on a complicated formula constructed to make the "total misrepresentation" (expressed by the integral over the surface to be represented of the sum of squares of errors of scale in two orthogonal directions) a minimum. The law of the radii depends on the spherical radius to be represented. The original investigation by Airy was erroneous, and although the error was quickly pointed out by Clarke, the numerical results of Airy are often quoted in text-books. A small example, with centre in lat. 23½° N. and spherical radius 113½°, is frequent in atlases: but its only serious use is for map of United Kingdom by Ordnance Survey on 10 miles to inch, covering area much too small to exhibit the merits of the projection, which is suitable for hemispheres.

Formula:

$$r = 2RM^{-1}\{\cot \tfrac{1}{2}\zeta \log \sec \tfrac{1}{2}\zeta + \tan \tfrac{1}{2}\zeta \cot^2 \tfrac{1}{2}\beta \log \sec \tfrac{1}{2}\beta\},$$

where logs are common, M their modulus, $M^{-1} = 2{\cdot}30259$, and β the spherical radius.

(ii.) *Bonne's Projection, or Projection du Dépôt de la Guerre.*—First used in a rough form by Ptolemy for his second projection for World-map; by Bonne (1752); and adopted by the Dépôt de la Guerre (1803) for the map of France on the scale 1/80,000. Much used throughout last century in continental surveys, for the O.S. maps of Scotland and Ireland, and in atlases. A modification of the simple conic, in which all the parallels are divided truly and meridians are curves passed through these dividing points. The projection is equal area; and scale along and perpendicular to parallels (but not along meridians) is correct. Its great defect is the obliquity of meridians to parallels, increasing with distance from centre. This proved so disadvantageous for artillery work that at the end of the Great War the tactical maps of the Allies were all being converted to Lambert's Conical Orthomorphic Projection (*q.v.*).

(iii.) *Breusing's Projection.*—A mean between the Zenithal Equal Area and Zenithal Orthomorphic (Stereographic). Its inventor employs the geometric mean, with radii given by

$$r = 2R\sqrt{\tan \tfrac{1}{2}\zeta \sin \tfrac{1}{2}\zeta}.$$

But it has been shown recently by Young that the harmonic mean gives a better result, with radii

$$r = 2R \tan \tfrac{1}{4}\zeta.$$

(iv.) *Cassini's Projection* (Transverse Simple Cylindrical).—Used by César François Cassini for the Carte de France, 1745–1793, and by the Ordnance Survey for the maps of England (Scotland and Ireland are on Bonne's Projection). Taking a central meridian and fixed point on it as origin (for England the meridian through the point of the principal triangulation at Delamere, Cheshire), the length of the spheroidal perpendicular from any point to the central meridian, and the length of the arc of meridian from the origin to the foot of this perpendicular, are plotted as rectangular co-ordinates. The scale at right angles to the central meridian is correct; the scale parallel to it is too great by a quantity varying as the square of the distance from the meridian. Hence the projection should not be used for a map covering a great extent of longitude.

For spherical Earth the co-ordinates x and y are given by

$$\sin x = \sin \Delta\lambda \cos \phi,$$

$$\cot (\phi_0 + y) = \cos \Delta\lambda \cot \phi,$$

ϕ_0 being latitude of origin. For spheroidal Earth the expressions for x and y are complicated, and calculation proceeds by successive approximations, or by the expansions in series (see *The Mathematical Basis of the Ordnance Maps of the U.K.*, Major A. J. Wolff, D.S.O., R.E., Southampton, 1919).

(v.) *Clarke's Minimum Error Perspective Projection.*—About 1860 Colonel A. R. Clarke applied Airy's principle of making total misrepresentation a minimum to the perspective projections. If the projection is made on the tangent plane from an external point distant hR from the centre of the sphere, the radii are given by

$$r = \mathrm{R}(1+h)\frac{\sin\zeta}{h+\cos\zeta}$$

and h depends upon the spherical radius β of the map for which the total misrepresentation is to be a minimum. There is no simple relation between h and β, but h has been found for various values of β, and others can be obtained by interpolation. If the plane of projection is moved parallel to itself to a distance kR from the centre, the scale at the centre is sacrificed, but with advantage to the outer zones; k is substituted for unity in the expression for r; and k also has to be adapted to suit β. The projection is interesting, but scarcely used (see *Phil. Mag.*, 1860).

(vi.) *Conical Projection, Simple, with One Standard Parallel.*—First described by Ptolemy; developed on cone touching the sphere along the standard parallel. The radius of the standard parallel is R cot ϕ_1 (or ν cot ϕ if Earth's ellipticity is regarded). Other parallels are concentric circles, their true distances apart. The standard parallel is divided truly, and points of division joined by straight lines to the vertex to make the meridians. Constant of the cone $n = \sin\phi$. Scale along meridians and along standard parallel correct; along other parallels too great. Much used in atlases, but greatly inferior to the little used

(vii.) *Conical Projection with Two Standard Parallels.*—Sometimes called by the name of De l'Isle, but used two centuries earlier by Mercator for map of Europe (1554). Radius of standard parallel of latitude ϕ_1 (the other ϕ_2) given by

$$r_1 = \nu(\phi_2 - \phi_1)\cos\phi_1/(\cos\phi_1 - \cos\phi_2).$$

This parallel is constructed and divided truly by straight meridians drawn from the vertex. Constant of the cone is

$$n = (\cos\phi_1 - \cos\phi_2)/(\phi_2 - \phi_1).$$

By proper choice of standard parallels various conditions may be satisfied; practically, parallels one-seventh whole extent of latitude from bounding parallels give a good result. Scale along meridians and two standard parallels correct; along other parallels too small within and too large without the standard parallels. Often improperly called the Secant Conic projection.

(viii.) *Conical Equal Area, with One Standard Parallel.*—Lambert's fifth Projection, 1772. The cone on which projection may be considered as developed is no longer tangent along the standard parallel. Radius of standard parallel $r_0 = 2\mathrm{R}\tan\frac{1}{2}\chi_0$, and of any parallel $2\mathrm{R}\sec\frac{1}{2}\chi_0\sin\frac{1}{2}\chi$; $n = \cos^2\frac{1}{2}\chi_0$. Scale along the meridian too great on the polar side of the standard parallel, and too small on the opposite side; scale along the parallels inversely as the scale along meridians. The vertex of the projection represents the pole, but the whole polar area is included within the angle $2\pi\cos^2\frac{1}{2}\chi_0$. The projection is little used, being much inferior to the

(ix.) *Conical Equal Area with Two Standard Parallels.*—Albers, 1805. The constant of the cone $n = \frac{1}{2}(\sin\phi_1 + \sin\phi_2)$; the radii of the standard parallels are $r_1 = \mathrm{R}\cos\phi_1/n$ and $r_2 = \mathrm{R}\cos\phi_2/n$; the radius of any other parallel is given by

$$\begin{aligned} r^2 &= 2\mathrm{R}^2/n\,.\,(\sin\phi_1 - \sin\phi) + {r_1}^2 \\ &= 2\mathrm{R}^2/n\,.\,(\sin\phi_2 - \sin\phi) + {r_2}^2. \end{aligned}$$

Outside the standard parallels the scale along the parallels is too large and along the meridians too small; between the standard parallels the opposite. The projection was used for the new edition of the Austrian Staff Map on scale 1/750,000, but very rarely in atlases. To take account of ellipticity of the Earth, use the product $\rho\nu$ of radii of curvature in the two directions at centre of map, instead of R^2: this gives an approximate result sufficient for Atlas maps. For closer approximation use the geocentric instead of the geographical co-latitudes.

(x.) *Conical Orthomorphic.* — Lambert's second (1772), frequently known by the name of Gauss (1825). If the radii of a conical projection are given by $r = m(\tan\frac{1}{2}\chi)^n$, when n is the constant of the cone, and m a scale constant, the projection is orthomorphic. If n is chosen so that the cone is tangent along the standard parallel, $n = \cos\chi_0$; and if the scale constant is chosen to make the standard parallel its true length, $m = \mathrm{R}\tan\chi_0/(\tan\frac{1}{2}\chi_0)^{\cos\chi_0}$. Hence the general expression for the radii is

$$r = \mathrm{R}\frac{\tan\chi_0}{(\tan\frac{1}{2}\chi_0)^{\cos\chi_0}}\,.\,(\tan\tfrac{1}{2}\chi)^{\cos\chi_0}.$$

The scale away from the standard parallel is now everywhere too large, and there are pairs of parallels north and south of the standard parallel, along which it is too large by the same amount. Any such pair of parallels is connected by the relation

$$n = \frac{\log\sin\chi_1 - \log\sin\chi_2}{\log\tan\frac{1}{2}\chi_1 - \log\tan\frac{1}{2}\chi_2},$$

but this does not allow us to find conveniently the parallels which will be made standard by a given reduction in the scale value. If n is maintained in its original value, the new standard parallels will not be quite symmetrical with respect to the original standard. But if we are content to modify n slightly we choose the two parallels to be made standard, calculate n from the above expression, and then the scale value

$$m = \frac{R \sin \chi_1}{n(\tan \frac{1}{2}\chi_1)^n} = \frac{R \sin \chi_2}{n(\tan \frac{1}{2}\chi_2)^n}.$$

To take account of the ellipticity of the Earth it is very nearly sufficient to use, instead of χ_1, χ_2, the astronomical co-latitudes, the corresponding geocentric co-latitudes, and to replace R by ν. But more accurately we have

$$n = \frac{\log \nu_2 \sin \chi_2 - \log \nu_1 \sin \chi_1}{\log\left[\tan \frac{1}{2}\chi_2\left(\frac{1 + e\cos \chi_2}{1 - e\cos \chi_2}\right)^{\frac{e}{2}}\right] - \log\left[\tan \frac{1}{2}\chi_1\left(\frac{1 + e\cos \chi_1}{1 - e\cos \chi_1}\right)^{\frac{e}{2}}\right]},$$

$$m = \frac{{}_1\nu \sin \chi_1}{n(\tan \frac{1}{2}\chi_1)^n\left(\frac{1 + e\cos \chi_1}{1 - e\cos \chi_1}\right)^{\frac{1}{2}ne}} = \frac{\nu_2 \sin x_2}{n(\tan \frac{1}{2}\chi_2)^n\left(\frac{1 + e\cos \chi_2}{1 - e\cos \chi_2}\right)^{\frac{1}{2}ne}},$$

and $$r - m(\tan \tfrac{1}{2}\chi)^n\left(\frac{1 + e\cos \chi}{1 - e\cos \chi}\right)^{\frac{ne}{2}}.$$

(See Germain, *Traité des projections*, p. 56.)

This projection, until lately little used, became prominent by its adoption for the tactical maps of the Allied armies towards the end of the war. Extensive tables were calculated by the French and the Americans (for the latter see Special Publ. U.S. Coast and Geodetic Survey, Nos. 47, 49, 53). To secure homogeneity in such tables it is necessary to work from the rigid theory, with consistent values, to eight or nine places of decimals, for the fundamental constants in the figure of the Earth: both tables are open to slight criticism in this respect. The especial advantage of the projection is its adaptability to co-operation between the Survey battalions and the artillery, for calculation of battery zero-lines, bearing pickets, grid-references of batteries and targets, etc.

(xi.) *Conventional Projections.*—A name perhaps originally used for all projections not perspective, and consequently not geometrically projections at all; but now used loosely for miscellaneous varieties not included in the principal classes nor having any geometrical properties of interest, *e.g.* the Globular. The polyconic and Cassini's should not be called conventional.

(xii.) *Cylindrical Projection, Simple.*—The most conventional of all projections: meridians and parallels equidistant lines cutting orthogonally, forming squares if the equator is the standard parallel, and rectangles if some other parallel is standard. Used by early map-makers, as Marinus of Tyre, but now of no importance except in its transverse form, which is Cassini's Projection (*q.v.*).

(xiii.) *Cylindrical Equal-area Projection.*—A true geometrical projection of sphere on circumscribing cylinder, by perpendiculars to the axis of the cylinder. Distance of any point from the equator = R sin ϕ (for cylinder touching equator). A projection of little importance.

(xiv.) *Cylindrical Orthomorphic Projection* = Mercator's (*q.v.*), if the cylinder touches the equator; or = Gauss Conformal (*q.v.*), if cylinder is transverse, touching the meridian.

(xv.) *Gauss Conformal Projection* (Transverse Cylindrical Orthomorphic).—Really due to Lambert: first employed for the Survey of Hanover, and lately for the Survey of Egypt. Described by Mr. J. I. Craig (*Survey Department Papers*, No. 13, Cairo, 1910). Resembles the Cassini Projection, slightly modified to make it orthomorphic. Complicated in theory and not markedly superior to simpler projections, except for certain computing advantages, arising from its orthomorphism, which are valuable in precise cadastral survey. Suitable only for country of little extent in longitude, as the valley of the Nile.

(xvi.) *Globular Projection.*—Conventional, used only for map of the world in two hemispheres; first by Nicolosi, 1671; sometimes called by name of Arrowsmith, who published large map of two hemispheres in 1794. The equator, the central meridian, and the circumference of the bounding circle are divided into equal parts; the meridians and parallels are arcs of circles passing through the points thus determined. The projection has no merit but simplicity of construction; the zenithal equidistant or zenithal equal area make better projections for a hemisphere.

(xvii.) *Gnomonic Projection* (so called from its use in graduating sun-dials) is the perspective projection from centre of the sphere on plane tangent at point chosen for centre of map.[1] Radial distance from centre equals tangent of angular distance: hence distortion increases quickly away from the centre. Great circles of the sphere project into straight lines; hence charts on the gnomonic projection have been constructed to facilitate great-circle sailing, but are not much used, owing to enormous distortion of chart covering large area. Problems in great-circle sailing are solved more readily by calculation. The projection is also used for plotting nautical plans on scale greater than 1/50,000 (see *Report International Hydrographic Conference*, London, 1919), and during the war was used on charts for direction-finding by wireless. A gnomonic projection of the world on a circumscribed cube has interesting properties, facilitating study of great-circle routes (see Hinks,

---

[1] Invention ascribed to Thales, *c.* 548 B.C.

Map Projections, p. 41, and *Geog. Journ.*, June 1921). Celestial photographs are on gnomonic projection if distortion of objective is negligible.

Formula: $r = R \tan \zeta$.

For geometrical properties *v.* H. H. Turner, *Monthly Notices R.A.S.* lxx. 204.

(xviii.) *International Map Projection.* For the *Carte Internationale du monde au millionième* the Conference which met in London (1909) adopted a modification of the polyconic proposed by M. Charles Lallemand. Each sheet covers 6° of longitude by 4° of latitude. The central meridian is made its true length less a very small quantity, whose effect will be explained below. The top and bottom parallels are then constructed as in the polyconic projection, and divided truly. Corresponding points on these parallels are joined by straight lines to make the meridians, and the meridians are divided each into four equal parts by points which define the intermediate parallels. The meridians on each side of the central meridian are longer than the central, and the shortening of the latter, mentioned above, is so chosen that the meridians 2° on each side are of their true length. The projection differs, then, from the ordinary polyconic in the following respects: the meridians are straight, instead of being curves; their lengths are slightly diminished; and they are divided equally. Practically the only sensible difference is the first, which allows a perfect instead of a rolling fit with adjoining sheets east and west. The fit with sheets north and south is perfect in either case. But the angles at the corners are slightly less than right angles, and a block of four sheets, two in each row, does not make a perfect fit. For tables see *Resolutions of the International Map Committee*, London, 1909; or Hinks, *Map Projections*, p. 114. Tables on the same projection for maps on the scale 1/Two Million are in *Geographical Journal* (Oct. 1918).

(xix.) *Mercator's Projection* (Cylindrical Orthomorphic).—Constructed empirically by Gerard Mercator, and used first in his World map of 1569. The theory first investigated and tables published by Edward Wright (*Certaine Errors in Navigation corrected*, 1599). The logarithmic formula below, first given by Henry Bond, 1645. Meridians are parallel straight lines cutting the equator orthogonally at distance apart true to scale; parallels are straight lines parallel to equator at continually increasing separation; scale value at any point proportional to secant of latitude; hence the poles are at infinity. Unsuitable for general map of world owing to great exaggeration of scale north and south. But generally used in navigation because, being orthomorphic with meridians and parallels orthogonal straight lines, the line of equal bearing, constant compass course, or loxodrome between any two points is a straight line, and the course is found conveniently by use of parallel ruler and the compass rose engraved on the chart.

Distance of parallel lat. ϕ from the equator is, in minutes of arc:

$$7915'.705 \log \tan (45° + \tfrac{1}{2}\phi)$$
$$- 3437'.7(e^2 \sin \phi + \tfrac{1}{3}e^4 \sin^3 \phi),$$

where e is the eccentricity of the Earth's figure.

Numerous extensive tables are published; *e.g.* Germain, p. 290.

(xx.) *Mollweide's Projection* (sometimes called Babinet's Homalographic) is a representation of the whole sphere in an elliptical figure with major axis twice the minor. Described by the author in Zach's *Monatliche Korrespondenz*, August 1805. Projection is equal area, and much used of late for distribution diagrams of the whole world; but though areas are preserved, the distortion of shape is great. To get areas true to desired scale take major axis $2\sqrt{2}\,.\,R$ and divide equally for equator. Meridians are ellipses through these points and the extremities of minor axis of figure, length $\sqrt{2}\,.\,R$. Parallel of latitude ϕ is at distance from equator $\sqrt{2}\,.\,R \sin \theta$, where θ given by

$$\pi \sin \phi = 2\theta + 2 \sin \theta.$$

An interesting transverse Mollweide, constructed by Col. Sir Charles Close, is given as frontispiece in Hinks's *Map Projections*; useful for representation of areas and distributions in British Empire.

(xxi.) *Orthographic Projection* (ill-named, as by no means a correct representation), the perspective projection on tangent plane by lines parallel to diameter through point of contact; or centre of projection at infinity. Invention ascribed to Hipparchus. Useful in astronomy, especially for maps of moon and planets, but little used in geography.

Formula: $r = R \sin \zeta$.

(xxii.) *Perspective Projections.* The relatively small class of strictly geometrical projections on a tangent plane touching the sphere at centre of map, by rays from a vertex in the diameter through this point of contact.

Vertex at centre of sphere—Gnomonic.

Vertex opposite end of diameter—Stereographic.

Vertex distant from centre 1·367R—Sir Henry James's Projection (as corrected by Clarke).

Vertex distant from centre 1·71R—La Hire's Projection.

Vertex at infinity—Orthographic.

Corresponding to various spherical radii of surface to be represented are positions of vertex giving minimum error of representation. The series comprised in general name Clarke's Minimum Error Perspective Projection, in which, however, the minimum is obtained by bringing the plane of projection in from tangency to a parallel position nearer the centre. The projections of James and La Hire are special cases of Clarke's (*q.v.*).

(xxiii.) *Polyconic Projection.*—Each parallel is plotted independently, as if it were the standard parallel in a simple conic projection, and the parallels cut the central meridian orthogonally at their true distances apart. Hence the circular parallels have their centres on a straight line, but are not concentric; they diverge from one another on each side of the central meridian. The meridians are smooth curves passed through the points dividing the parallels truly, and are curved, as in the Bonne Projection. The polyconic projection is neither equal area nor orthomorphic, and it is quite unsuitable for maps covering large areas. But for a single topographical sheet it is indistinguishable from other projections with better theoretical properties; and it has the great advantage that as each parallel is constructed and divided independently of the chosen centre, tables may be calculated to serve for the projection of any sheet, without further calculation. (See *Tables for a Polyconic Projection of Maps based upon Clarke's Reference Spheroid of 1866*, U.S. Coast and Geodetic Survey, Washington, 1884, and subsequent editions.) Modifications in use are the Rectangular Polyconic and the Projection of the International Map (*q.v.*).

(xxiv.) *Rectangular Polyconic.*—Introduced about 1858 by Colonel A. R. Clarke of the Ordnance Survey. The parallels are constructed as in the ordinary polyconic. One is divided truly; and meridians are formed by curves passing through these points and cutting all the parallels orthogonally. May be constructed from tables calculated by Major Leonard Darwin, R.E., published by the War Office, 1890. Recently re-examined by Capt. G. T McCaw, General Staff, who proposes a modification, making the projection practically orthomorphic, and is publishing new tables.

(xxv.) *Polyhedric Projection*.—Much used in continental surveys for projection singly of topographical sheets, by supposing perpendiculars from points of the spheroidal surface on a plane passing through the points marking the corners of the sheet. Of no scientific interest, and without the convenience attaching to a single system of co-ordinates for all sheets of a survey. In any single sheet indistinguishable from Cassini's or the polyconic.

(xxvi.) *Retro-azimuthal Projection.*—A new type devised by Mr. J. I. Craig of the Survey Department, Egypt, in which the azimuth of the centre is correct at any point of the map (see Technical Lecture, No. 3, 1908-1909, Egyptian Survey Department). Used in map constructed to show the true bearing of Mecca at any point. Two classes distinguished, the first with parallel straight meridians, so that azimuth measurable on compass rose; the second with curved meridians, from which azimuth at any point not so easily measured. The stereographic is retro-azimuthal in the second class.

(xxvii.) *Sanson's Projection*, or the Sanson-Flamsteed, used by Sanson (1650) in his atlas, and by Flamsteed (1729) for his star maps. The particular case of Bonne's Projection in which the equator is the standard parallel, and the other parallels straight lines parallel to it at their true distances apart. Much used in atlases for countries near the equator. Equal area: but suffers from obliquity of meridians to parallels. Constructed very easily by spacing parallels truly and dividing them truly, proportional to cos ϕ.

(xxviii.) *Stereographic Projection* (Zenithal Orthomorphic) is the perspective projection on a tangent plane from the point on the sphere diametrically opposite the point of contact. Invention ascribed to Hipparchus, 150 B.C. Much employed in geography during sixteenth and seventeenth centuries, but now little used except for solving problems in crystallography. Both meridians and parallels project into circles, so that projection may be constructed geometrically, and has been used as a type from which to construct other zenithal projections by transformation. Practically it is more convenient, and especially more accurate, to construct from plotted rectangular co-ordinates. Although orthomorphic, the projection gives bad distortion of shape towards the margins of a hemisphere. Hence generally abandoned for maps, though important and interesting historically, particularly as foundation of graphical methods of calculation: *e.g.* the astrolabe.

Formula: $r = 2R \tan \frac{1}{2}\zeta$.

(xxix.) *Tissot's Projections.*—In his *Mémoire sur la représentation des surfaces*, 1881, M. A. Tissot developed from the most general analytical considerations a system of projection in which the distortion in angle is of the third order only, and the distortion in length of the second. In the general form the method does not seem to have been used, but it has been shown recently that Tissot's projection is to the third order the same as the stereographic, while the projection for a zone is to the same order equivalent to Lambert's Conical Orthomorphic, and the projection for a "lune" between two meridians is equivalent to the Gauss Conformal. Tissot's work should be taken, therefore, as leading to these projections as the best, rather than as proposing alternatives.

(xxx.) *Werner's Projection.*—The particular case of Bonne's in which the standard parallel is the pole; the parallels concentric circles about the pole, divided truly. Described by Werner in a treatise on Ptolemy, 1514: of antiquarian interest only.

(xxxi.) *Zenithal Equidistant Projection.*—Employed in its simple polar form by Glareanus (*c.* 1510), and in general form first studied by Lambert (1772). Distances from the centre their true length, and azimuths from the centre true. May be considered as special case of simple conic, generally oblique, with constant of cone unity, or cone degenerated into the tangent plane. Best constructed from tables of distances and azimuths for given differences of latitude and longitude; *e.g.* in Hammer, *Die geographischen wichtigsten Kartenprojektionen*, Stuttgart, 1889. When plotted to true scale at centre the scale at right angles to the radial becomes rather rapidly too large. But it is possible to apply a general scale correction, depending on the spherical radius of area to be represented, that reduces the error of the outer zones, though at the expense of the inner, and renders this projection as good as any of the zenithal (see A. E. Young, *Map Projections*, R.G.S. Technical Series, No. 1).

(xxxii.) *Zenithal Equal-area Projection.*—Lambert's sixth, wrongly called Lorgna's. The radii, instead of being their true length, are calculated from $r = 2R \sin \frac{1}{2}\zeta$, which gives an equal-area projection. Easily calculated from the tabular distances used in the zenithal equidistant. One of the best projections for a large continent, as Asia. A particular case of the conical equal area with one standard parallel. If it is necessary to take account of the Earth's ellipticity one may replace R by $\sqrt{\rho\nu}$, and calculate the radii from the spheroidal distances and azimuths. But it will rarely be worth while to do so, as the projection is used only for Atlas maps.

§ (9) ZENITHAL ORTHOMORPHIC PROJECTION is treated under its particular name of Stereographic (*q.v.*). Interesting as belonging to the general group of conical projections, the particular class of perspective projections, and the group included under general name of Lagrange's Circular Orthomorphic.

Projections in Use.—Of the above principal projections, many are scarcely used. In atlases we find chiefly simple conic and Bonne, zenithal equidistant or equal area, and Sanson. In the larger scale maps of topographical surveys Bonne is most widely used, the polyconic is the most convenient, while Cassini, the "Gauss Conformal," and Lambert's Conical Orthomorphic have each important advantages where area to be covered is not too large to allow of a single projection for the whole. Mercator's Projection should be confined to marine charts, or a narrow belt along the equator.

If a choice of projection has to be made, the following will serve:

For a hemisphere: Airy's Projection by Balance of Errors, or Clarke's Minimum Error Perspective.

For a continent N. or S. of equator: Zenithal Equidistant or Zenithal Equal-area.

For a continent cut by the equator: Sanson is good and easy to draw.

For Atlas maps of smaller areas: Conic with two standard parallels.

For extensive topographical map on small scale: Polyconic, Rectangular Polyconic, or Projection of the International Map.

For a topographical series on large scale: Polyconic if plotted independently; Lambert's Conical Orthomorphic, or Gauss Conformal, for series on a single projection, the former if extent E. and W., the latter if extent N. and S.

TABLES FOR PROJECTIONS

The most general are Tables for a Polyconic Projector of Maps, United States Coast and Geodetic Survey, Special Publication, No. 5. Figure: Clarke's Spheroid of 1866.

Tables for Maps on scales 1/125,000 and 1/250,000 between latitudes 0° and 60°, and lengths in feet of second of arc of meridian and parallel are given in Close, *Text-book of Topographical Surveying*, H.M. Stationery Office, 1913. Figure: Clarke's Spheroid of 1858.

Similar Tables from equator to 40° N. in Auxiliary Tables, Survey of India, Dehra Dun. Figure: Everest, 1830.

Tables for Lambert's Conical Orthomorphic Projection: For the Western Front, Paris, *Service géographique de l'armée* (lithographed and not published)—Figure, Plessis, modified by Puissant; or U.S. Coast and Geodetic Survery, Special Publ. 47—Figure, Clarke, 1866.

For the United States: Special Publication, 52.

PRINCIPAL WORKS ON PROJECTIONS

Germain, A.: *Traité des projections des cartes géographiques*, Paris, Bertrand (*c.* 1866). Out of print.

Lambert, J. H.: *Anmerkungen und Zusätze zur Entwerfung der Land- und Himmelscharten*, 1772. Reprinted in Ostwald's *Klassiker der exacten Wissenschaften*, No. 54.

Fiorini, M.: *Le projezioni delle carte geografiche*, Bologna, 1881.

Tissot, A.: *Mémoire sur la représentation des surfaces et les projections des cartes géographiques*, Paris, 1881.

Craig, J. I.: *The Theory of Map Projections*, Survey Department, Paper No. 13, Cairo, 1910.

The above are severely mathematical: an elementary book is

Hinks, A. R.: *Map Projections*, Cambridge University Press, 1912. Second edition in the press.

A valuable work, with many new and interesting developments, is

Young, A. E.: *Map Projections*, R.G.S. Technical Series, No. 1, 1920.

A. R. H.

MARVIN SUNSHINE-RECORDER. See "Radiant Heat and its Spectrum Distribution," § (4).

Mass, Measure of. (i.) *Metric.*—The *International prototype kilogramme* is the mass of a cylinder of platinum-iridium, which is a copy of the original Borda kilogramme—the *kilogramme des archives*; this was intended to be equal to the mass of a cubic decimetre of pure water at its maximum density.

(ii.) *British.*—The *Imperial standard pound* is the weight *in vacuo* of a platinum cylinder.

(iii.) *Equivalents.*—

(*a*) *Metric Units.*

Kilogramme . .	1 kg.	= 2·2046223 lbs.
Gramme . . .	1 g.	= 15·432356 gr.
Metric tonne . .	1 t.	= 1000 kg.
		= 2204·622 lbs.
		= 0·9842 ton.

(*b*) *British Units.*

Pound . . .	1 lb.	= 453·59243 g.
Ounce (avoir.) .	1 oz.	= 28·3495 g.
Ounce (troy and apothecary) .	1 oz.	= 31·10348 g.
Grain . . .	1 gr.	= 0·06479892 g.
Ton . . .	1 ton	= $1·016047 \times 10^6$ g.

See "Measurement, Units of," Vol. I.

Master Gauge: definition of. See "Metrology," § (19).

Maximum Thermometer. See "Meteorological Instruments," § (7) (ii.).

Measurement:

Methods of, applied to instruments. See "Instruments, Design of Scientific," § (2).

Units of, in Meteorology. See "Atmosphere, Thermodynamics of the," § (2).

Measurement of Volumes of Gases. See "Volume, Measurements of," § (22).

Measurers and Indicators, functions of, as used for metrological observations. See "Metrology," § (32) (i.).

Measuring Flasks:

Construction and tolerances. See "Volume, Measurements of," § (12).

Testing of. See *ibid.* § (13).

Measuring Glasses, Graduated. See "Volume, Measurements of," § (20).

Measuring Instruments used in Workshop: external micrometer calipers. See "Gauges," § (85).

Measuring Machines:

"Armstrong Whitworth" type. See "Gauges," § (71).

Cambridge Scientific Instrument Company's microscope type for screw gauges. See *ibid.* § (34).

For comparison of standard gauges, general principles. See *ibid.* § (14).

For effective and core diameters of plug screws. See *ibid.* § (23).

For pitch of ring screw gauges. See *ibid.* § (25).

Hartmann automatic comparator. See *ibid.* § (76).

"Herbert" microscope machine. See *ibid.* § (35).

National Physical Laboratory type. See *ibid.* § (77).

"Newall" type. See *ibid.* § (72).

"Pratt and Whitney" type. See *ibid.* § (73).

"Reid" type. See *ibid.* § (78).

"Shaw" type. See *ibid.* § (75).

"Shaw" type for screw gauges. See *ibid.* § (31).

"Société Genevoise" type. See *ibid.* § (74).

Tests on. See *ibid.* § (79).

Types for end-gauges. See *ibid.* Section V.

Types for measuring pitch of screws. See *ibid.* § (24) (A).

Mendeléeff's Work on Alcohol. See "Alcoholometry," § (4).

Mercedes-Euklid Machine. See "Calculating Machines," § (10) (i.).

Mercury, Density of, in grm. per c.c., tabulated. See "Balances," Table III.

Meteorograph: a self-recording instrument giving a record of two or more of the ordinary meteorological elements. For kite balloon see "Meteorological Instruments," § (36).

Meteorographs, as used for the investigation of the upper air. See "Air, Investigations of Upper," § (8), etc.

METEOROLOGICAL INSTRUMENTS

Instruments for measuring the pressure of the air are described under the article "Barometers," to which reference should be made.

The following article includes instruments for the measurement of

II. Temperature,[1]
III. Precipitation,
IV. Wind Velocity and Direction,
V. Sunshine,[2]
VI. Solar Radiation,[2]
VII. Clouds, and
VIII. Instruments for use in Aircraft.[3]

I. General Remarks

§ (1) **Desirable Characteristics.**—The fundamental characteristics of good meteorological instruments in general are the following, arranged in order of importance:

(1) Accuracy.
(2) Reliability, *i.e.* little tendency to change in accuracy.

---

[1] See also "Thermometry," Vol. I.

[2] See also "Radiation"; "Radiation, The Measurement of Solar, etc."; "Radiant Heat and its Spectrum Distribution," Vol. IV.

[3] "Instruments used in Aircraft," Vol. V.

(3) Simplicity of design.
(4) Ease of reading and of manipulation.
(5) Strength of construction.
(6) Durability.
(7) Low cost of maintenance.
(8) Low initial cost.

The desirability of these qualities needs only to be stated to be appreciated, but it is necessary to add some remarks on the need for simplicity of design.

§ (2) Simplicity of Design.—It has to be borne in mind that most meteorological instruments are to be maintained in continuous operation, and that the majority of them are either wholly or partially exposed to the weather. Further, amateur observers, to whose work the science is much indebted, must always form the great majority of contributors to the whole stock of observational data, and most of them are not trained physicists or skilled mechanics. Consequently excessively fragile or non-durable instruments and highly complicated instruments are suitable only for use at observatories or other places where the necessary skilled attention can be continuously devoted to them. Scores of designs for meteorological instruments, which have functioned satisfactorily in the laboratory, have come to nothing because they are unsuitable for the conditions under which they must be used. Especially does this apply to self-recording instruments, in the design of which simplicity should be constantly aimed at. Every recording meteorological instrument, except the Campbell-Stokes or Jordan sunshine recorder, necessarily contains a clock, and the object of the designer should be to reduce all other moving parts to a minimum.

§ (3) Self-recording Instruments.—There are two further necessary characteristics of a *self-recording* instrument, viz.:

(9) Minimum of friction between moving parts.
(10) Provision of adequate control.

(i.) *Friction.*—In condition (9) is included the friction between the writing-pen, if any, and the paper, as well as that in the bearings of the instrument; indeed, the former is often much the more important of the two. It can only be reduced by adjusting the pressure of the pen on the paper to the lowest possible amount, taking care at the same time that the pen is quite clean and in good condition.

The fact is that provision must be made for adjusting the pressure of pen on paper; but it must be confessed that, occasionally, the adjustment is misused, for it is far easier to apply more pressure than to clean, adjust, or change a pen which refuses to write, and it is not always realised by the operator that the resulting record may be practically valueless. Designers have often felt this difficulty and have sought to avoid it by providing some system of recording other than that of a simple pen, carrying ink. The most elegant of these methods is the photographic method, which will be exemplified later on. In this case the "pen" is really a narrow pencil of light which is accurately focussed upon bromide paper which receives the record. Pen friction is entirely absent, but there are three disadvantages:

(*a*) The record is not visible until the sheet has been removed and developed.

(*b*) The recording portion of the instrument must be kept in a dark room.

(*c*) The initial cost, and (more serious) the cost of maintenance is high.

Another system of recording which may be mentioned is that in which the indicating arm of the instrument is only brought into contact with the paper at definite and regular intervals (say one minute). A typewriter-ribbon is usually interposed between the arm and the paper, and the armature of an electro-magnet forces the arm upon the ribbon at instants which are separated by the chosen interval. The resulting record consists of a chain of dots.

(ii.) *Control.*—Condition (10) is often exemplified in large aneroidographs in which the scale value of the pressure scale is two, three, or more times that of the scale of a mercury barometer. The "control" is provided by a pile of aneroid boxes, and sometimes the magnification of the motion of the moving end of the pile, by the levers of the instrument, is so great that the "control" at the pen (which is less than the control at the moving end of the pile divided by the magnification) is insufficient to overcome friction of pen on paper, and a "steppy" record results.

II. INSTRUMENTS FOR MEASURING THE TEMPERATURE[1] OF THE AIR

§ (4) Preliminary.—The accurate measurement of the temperature of the air in the open is one of the most difficult of all meteorological measurements, for it is so readily affected by effects of radiation. Radiation from the sun, the clouds, the sky, the ground, and surrounding objects passes in straight lines through the air without appreciably affecting its temperature, for air is very transparent to radiant heat, especially if it is dry. But the instrument that is used to measure the temperature of the air is some kind of thermometer, and is made of material which intercepts radiant heat to an appreciable extent. In consequence the reading of the instrument may differ from that corresponding with true air temperature by any amount up to 50° F., or even more. Such differences depend partly upon the nature of the thermometer, partly upon the amounts of the different kinds of radiation experienced, and partly upon the wind velocity and other extraneous factors. The reading of a thermometer freely exposed in the open may thus bear no determinable relation to the temperature of the particles of air in which it is placed.

---

[1] See also "Thermometry," Vol. I.

§ (5) THERMOMETER SCREENS.—It is therefore usual to provide some form of thermometer shelter, or "screen," which will serve to support the thermometers and to protect them from the weather and accidental damage, and at the same time shield them from radiation, without impairing the free passage of air over the bulbs of the thermometers. Many types of screen have been devised, and several types are in actual use in different countries. It will, therefore, be convenient to consider first what are the properties of an ideal thermometer screen, and subsequently to trace how far these properties are exemplified in screens which are in use.

(i.) *An Ideal Screen.*—Ideal thermometer screens could probably be defined in more than one way, but it seems that the following characteristics, if realisable, would satisfy the necessary conditions:

(1) The screen should be a "uniform temperature enclosure."

(2) The temperature of the inner walls of the "enclosure" should be the same as that of the external air.

(3) The "enclosure" should completely surround the thermometers.

(4) The "enclosure" should be impervious to radiant heat.

These conditions are sufficient to ensure that the temperature of the thermometers is the same as that of the inner walls of the screen, and therefore, by hypothesis, the same as that of the external air.

Of these conditions the third and fourth are easy of attainment, but the first and second are difficult. It is usual to provide an approximation to conditions (1) and (2) by (*a*) constructing the screen of non-conducting material, such as wood or straw; (*b*) providing the screen with double walls, with ample air circulation about them; and (*c*) painting the screen white so as to reflect as much radiation as possible. The outer wall of the screen may become heated to a considerably higher temperature than the air, on a day of strong sunshine, but in consequence of the non-conducting layer of air between the outer and inner walls, the temperature of the inner wall will not differ greatly from that of the air. The difference will be reduced if there is appreciable wind, when this intervening layer of air is constantly being changed, and has no time to acquire by conduction and convection the temperature of the outer wall and pass it on by the same agencies to the inner wall.

The four conditions given above do not include provision for free circulation of air inside the screen, but this is always provided in practice, partly to assist in realising conditions (1) and (2), as just explained, and partly in order that rapid changes of air temperature may be immediately communicated by convection to the thermometers. The process of radiation exchange between the thermometers and the inner walls of the "enclosure" requires some time for the adjustment of temperature as between walls and thermometers, following on a change of temperature of the walls, so that, in the absence of wind circulation inside the "enclosure," an appreciable time-lag would be introduced between corresponding temperature changes in the air and in the thermometers. On days of strong solar radiation, however, it is evident that the free circulation, inside the screen, of air which has passed over the heated outer walls of the screen will cause the thermometers to read too high. The omission of provision for free circulation would, however, in that case give much more erroneous results. For the temperature of the inner walls would rise to that of the outer walls (supposed thin) of the screen, and the readings of the contained thermometers would therefore approximate closely to that of the outer walls. Should, however, the outer walls be made of thick non-conducting material, the temperature of the inner walls, and therefore of the contained thermometers, would, in the absence of circulation, tend to remain constant, and would fail entirely to respond to the air temperature outside.

By way of illustration of these remarks it is a matter of common experience that the air inside an ordinary stone or brick house, of which the doors and windows are kept closed, remains at an equable and comparatively low temperature throughout the hottest day, the walls serving to prevent passage of heat from the outside to the inside. The opening of a window, however, is sufficient to raise the temperature quickly, and if the window is on the sunny side of the house it will probably admit air which has been heated by contact with the outer surface of the wall. On the other hand, the temperature inside a hut with thin walls would, on the same occasion, be raised to a high value, far above that of the external air, if the hut were kept closed. In this case, relief would be obtained by opening doors and windows, thus introducing cooler air from outside.

We thus arrive at the conclusion that, on occasions of strong sunshine, the combination of a non-conducting shelter, and a free circulation, inside the shelter, of air which has not been heated by conduction from the walls of the shelter is likely to give, inside the shelter, a good approximation to the true air temperature. If the shelter were also made to protect the thermometers inside from being wetted by precipitation it would be satisfactory on all other occasions, but the non-conducting material would not then be an urgent necessity.

We proceed to trace the application of these ideas to types of thermometer screens which are in use.

(ii.) *Wall Screen.*—One of the earliest types of screen used in this country is the "wall screen"—a wooden thermometer shelter which is arranged for suspension on a wall facing north. It consists merely of a wooden back with a narrow roof projecting forwards and slanting downwards from the upper edge of the back. The thermometers are suspended upon the wooden back, and the roof protects them from most of the precipitation which occurs.

Such a screen is unsatisfactory for the following reasons:

(*a*) The thermometers are not adequately protected from precipitation which falls with northerly winds.

(*b*) In summer the sun shines directly upon the thermometers in the early morning and late evening.

(*c*) Solar radiation reflected from surrounding objects raises unduly the temperature of the thermometers during the day time.

(*d*) The thermometers are influenced to some extent by their close proximity to the wall.

(iii.) *Glaisher Screen.*—This is a wooden structure with a roof which slopes towards the south, and with single-louvred walls on the east, south, and west sides of the screen. The bottom and the north side are open, and the thermometers are suspended within the screen in such a way that direct solar radiation never falls upon them. The screen is supported upon a strong wooden stand about 6 feet above the ground, and it is exposed on an open site as far as possible from buildings or trees. This screen gives temperatures which are too high on sunny days, on account of the indirect radiation which reaches the thermometers through the open bottom and north side. On the other hand, the minimum temperatures which are registered are too low on clear nights for a similar reason.

(iv.) *Marine Screen.*—In the screens described above it is usual to expose four thermometers, viz. dry-bulb, wet-bulb, maximum thermometer, and minimum thermometer; these being the usual equipment of a climatological station for observations of air temperature and humidity.

At sea, self-registering thermometers (maximum and minimum) cannot be used because of the motion of the ship, so that the dry and wet bulbs are the only thermometers which require to be sheltered. The wooden screen which is used is therefore a small one, 16 inches high, 5 inches deep, and 7 inches wide, with panels of single louvres in the sides, bottom, and door (in front), and solid roof and back. It is fixed on deck against a bulkhead or other suitable support, in a position which is protected from spray as far as possible, and where hot air from funnels, etc., does not spoil the exposure.

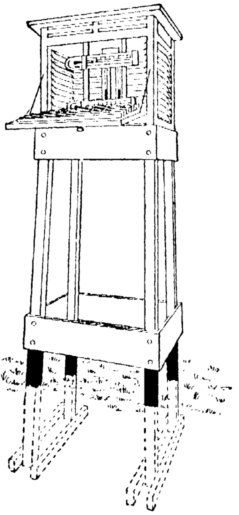


FIG. 1.

(v.) *Stevenson Screen.*—This is the standard screen in use in the British Isles, and is illustrated in *Fig.* 1. It was designed by Thomas Stevenson,[1] C.E., of Edinburgh, in 1866, and has since been slightly modified in detail. It is a wooden box, with double roof, and doubly louvred sides, supported on a wooden stand so that the thermometers are about 4 feet above the ground. One of the louvred sides is mounted on hinges along its lower edge, and acts as a door to give access to the thermometers, which are mounted on vertical wooden supports in the middle of the screen. The outer roof slopes down from front to back. The inner roof is separated from the outer by an inch or two; it is horizontal, and is drilled with a number of ventilating holes. The clear internal dimensions of the screen are 18 inches long, 11 inches wide, 15 inches high. This size accommodates comfortably the dry- and wet-bulb thermometers, and the maximum and minimum thermometers. A screen twice as long as the standard screen is in use at several official stations, to accommodate a thermograph and hair hygrograph in addition to the thermometers. The screen is set up so that the door faces due north, or slightly east of north, so as to ensure that direct solar radiation does not fall on the thermometers when the door is opened at the hours of observation.

---

[1] *Journ. Scott. Met. Soc.*, 1866, i. 122.

Numerous experiments have been made in different countries to determine how closely the readings of the thermometers approximate to the true temperature of the external air, from which it appears that during calm, hot, sunny days the temperature inside the screen may be 2° or 3° F. above the true value, and during calm clear nights the temperature inside the screen may be as much as a degree below the true value. These results arise from radiation effects upon the screen itself, which are partially communicated to the thermometers inside by the processes which have already been explained.

(vi.) *Tropical Screens.*—Owing to the intense solar radiation experienced in the tropics, it has been held for a long time that the ordinary Stevenson screen is unsuitable for use in low latitudes. The approved arrangement consists of a hut with open sides and a thatched roof sloping upwards from low eaves to a point in the middle of the roof. In the middle of the hut, and fixed at a height of about 4 feet above the ground, is a wire cage containing the thermometers. The cage acts merely as a support and as a guard against accidental damage to the thermometers; the whole of the screening is performed by the thatch.

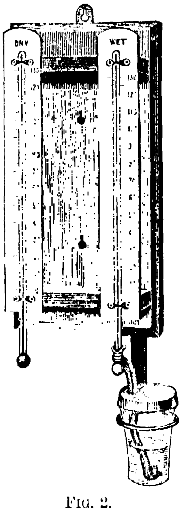


FIG. 2.

It seems reasonable to expect, however, that the thermometers inside the cage will be affected by indirect radiation from the hot ground outside the hut; and, indeed, it is understood that recent experiments in India have suggested that the use of a Stevenson screen for observations of air temperature in India gives values which are more reliable than those from the orthodox thatched hut.

There are several other types of screen in use in different countries, but they do not differ so markedly from the types described above as to require detailed particulars.

§ (6) THE ASSMANN PSYCHROMETER.—The standard method of measuring air temperature consists in surrounding the bulb of the thermometer by a coaxial thin-metal tube heavily nickel-plated, which is protected by a larger coaxial tube similarly nickel-plated. A current of air is drawn by a fan through the inner tube and past the thermometer bulb, the velocity of the current being not less than 2 metres per second. Such an arrangement can be used in strong sunshine without being affected by radiation, but, of course, it is desirable to avoid unnecessary exposure to direct sunshine. This arrangement has been standardised to give an instrument which will indicate true temperatures of both dry and wet bulbs, which is known as the Assmann psychrometer (see *Fig.* 2A). Further particulars about this instrument, and of the method of use, will be found in the article on "Humidity."[1]

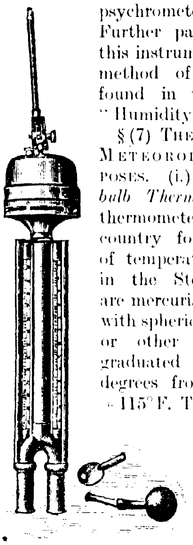


FIG. 2A.

§ (7) THERMOMETERS FOR METEOROLOGICAL PURPOSES. (i.) *Dry- and Wet-bulb Thermometers.*[2]—The thermometers used in this country for eye readings of temperature (dry bulb) in the Stevenson screen are mercurial thermometers with spherical bulbs of Jena or other suitable glass, graduated in Fahrenheit degrees from −15° F. to +115° F. They are mounted on porcelain mounts upon which every whole ten degrees is plainly figured. The scale is not less contracted than 18° F. to the inch. The thermometer must pass the Class B test of the National Physical Laboratory, for which the errors must not in general exceed 0·2° F.

The wet bulb is an exactly similar thermometer, over the bulb of which is stretched tightly a thin covering of nainsook, which is kept moist by a few strands of cotton-wick attached to the bulb, and dipping into a small vessel containing pure water (see *Fig.* 2).

(ii.) *Maximum Thermometer.*—The standard maximum thermometer is a mercurial thermometer very similar to that just described, except

(*a*) The range is 0° F. to 130° F.

(*b*) The porcelain mount is supported upon a mahogany frame, with a metal guard at one end to protect the bulb, which projects from the mount, from injury.

---

[1] See "Humidity," §§ (7), (9).

[2] See "Humidity," § (4).

(*c*) There is a fine constriction or other equivalent arrangement at a point in the bore of the stem, near the bulb, which offers considerably more resistance to the flow of mercury than the remainder of the bore. When temperature is rising, and the column of mercury in the thermometer is continuous, the mercury is forced past the constriction and the end of the column registers correctly the prevailing temperature; but when temperature falls, the mercury column breaks at the constriction as soon as the mercury in the bulb contracts, but the further end of the column remains at a point corresponding with the highest temperature attained, for the contraction of the detached column itself due to the fall of temperature is quite inappreciable for meteorological purposes. The thermometer therefore always indicates the highest temperature to which it has been subjected since it was last "set." Setting is performed by holding the instrument by the end of the wooden mount remote from the bulb, and vigorously swinging it at arm's length so as to drive the mercury by centrifugal force past the constriction and into the bulb, until the whole volume of mercury is continuous again. A well-made maximum thermometer is a very satisfactory instrument and seldom gets out of order. It is usual to support the tube of the thermometer in a nearly horizontal position in the screen, with a *slight* slope downwards towards the bulb.

(iii.) *Minimum Thermometer.*—The standard minimum thermometer is very similar in form to the maximum thermometer, but the liquid used in the thermometer is uncoloured alcohol, and the range of graduation is −20° F. to 110° F. There is no constriction in the bore of the tube, but a black or dark blue glass index in the form of a long dumb-bell is inserted in the bore, and is normally completely immersed in the alcohol. The tube being supported horizontally, the flow of alcohol produced by a rise of temperature passes the index without moving it. Likewise the return of the alcohol towards the bulb due to a fall of temperature has no effect upon the index until the end of the column reaches the end of the index, when, in consequence of the surface tension at the free surface of the alcohol, any further decrease of temperature results in the withdrawal of the index towards the bulb, the further end being always at the end of the alcohol column so long as temperature continues to fall.

The thermometer is read by noting the graduation on the scale which corresponds with the end of the index further from the bulb; this reading will be the lowest temperature attained since the instrument was last set. The necessity of placing a glass index in the bore of the tube requires the use of a considerably wider capillary for the stem of the thermometer than is needed for the maximum or the dry- and wet-bulb thermometers. The openness of the scale is, however, the same, for, on the other hand, the coefficient of expansion of alcohol is nearly six times that of mercury. The increase in bore is therefore compensated to a considerable extent by the increased coefficient of expansion, the result being that the bulb of the minimum thermometer is not markedly different in size from that of a maximum thermometer; generally the former is slightly the larger. Minimum thermometers are usually less sensitive to change of temperature than mercurial thermometers.

The chief defect of the minimum thermometer consists in the great tendency of the alcohol to evaporate from the end of the column and to condense at the further end of the tube, where it may form a bubble of liquid equivalent in length to 3° or 4° F. of temperature on the thermometer scale. It is evident that this will cause the thermometer to read too low by an equal amount, resulting in the registration of correspondingly erroneous minimum temperatures. This tendency appears to be most marked on days of considerable range of temperature. Even if a bubble is not actually produced, a very thin film of alcohol is liable to be formed upon the internal walls of that part of the tube which is not occupied by the column of spirit, and this may introduce an error of nearly 1° F. in the reading. Mercury, being a non-volatile liquid which does not "wet" glass, is much to be preferred on that account for use in thermometers.

It is the practice among thermometer makers to seal alcohol minimum thermometers while their bulbs are immersed in a freezing mixture. In this way as much air as possible is included in the thermometer, for it is found that this air appreciably diminishes the diffusion of alcohol vapour along the tube, and therefore tends to reduce the defect. A mercurial thermometer is, on the other hand, usually sealed when the bore of the tube is filled with mercury, that is, when the thermometer has attained the maximum temperature which, when sealed, it can withstand with safety. At lower temperatures therefore there is a vacuum in the tube above the mercury column. Thus, at moderate temperatures the pressure inside a mercurial thermometer is less than atmospheric, while that inside an alcohol thermometer is greater than atmospheric. F. J. W. Whipple has suggested that the excess of pressure inside a spirit thermometer may in course of time slightly distend the bulb of the thermometer, causing it to read too low. This would explain the circumstance that these thermometers are inclined to develop errors which require the application of positive corrections, even when care is taken that all the spirit in the thermometer is continuous in the column, and not condensed upon the walls of the tube above the column.

The minimum thermometer is placed in the screen with its stem horizontal, and it is "set" daily by removing it and holding it bulb upwards, when the index descends by gravity until its further end coincides with the free end of the alcohol column.

§ (8) Arrangement of the Thermometers inside the Screen.—As explained above, it is usual to place four thermometers inside a Stevenson screen, viz. dry-bulb, wet-bulb, maximum, and minimum thermometers. The first two thermometers are read with the stems vertical, while the last two are read with the stems horizontal. It is necessary to remove the first two from the screen only occasionally, whereas the maximum and minimum thermometers have to be removed daily for setting. Consequently the usual arrangement of the thermometers in the screen is to fix the dry and wet bulbs side by side towards the back of the screen, and to hang the maximum and minimum thermometers upon two vertical wooden battens fixed in the screen, forward and also to the side of the dry and wet bulbs, the maximum thermometer being near the top and the minimum near the bottom of the screen. In this way the scales of the dry and wet bulbs, between about 30° F. and 80° F., which is the portion most frequently required, fall between the mounts of the maximum and minimum thermometers, and are readily visible on opening the door of the screen. No difficulty in reading these thermometers, between these limits, without introducing errors of parallax, is experienced.

§ (9) Thermographs, or Self-recording Thermometers.—A thermograph is an instrument which automatically records temperature as a function of time. The sheet of paper upon which the record is obtained is wrapped around a metal cylinder, with axis vertical, which is caused to rotate uniformly by means of suitable clockwork. Some arrangement is provided whereby either a pen or a spot of light, which moves vertically up and down in response to changes of temperature, is caused to touch the paper and leave upon it a permanent impression. In this way a continuous record of temperature is obtained. Considerable variation is found in different instruments as regards the method of indicating temperature change: it is this variation which gives rise to different types of thermographs, which may be classified thus, according to the type of thermometer used:

(i.) Mercury thermometer:
  (1) Photographic registration.
  (2) Mechanical registration.
(ii.) Bimetallic thermometer }
(iii.) Bourdon tube } Mechanical registration.
(iv.) Resistance thermometer }

(i.) *Mercurial thermographs*, with photographic registration, have been in continuous operation at Meteorological Office observatories since 1867, and are now in use at Kew, Eskdalemuir, Aberdeen, and Valencia observatories. Each thermograph comprises two thermometers, one acting as a dry bulb, the other as a wet bulb. The glass tubes of the thermometers are bent twice at right angles to enable the bulbs to be exposed outside the building in a louvred thermometer screen fixed to a north wall of the observatory. Small air specks are left in the mercurial columns at convenient positions in the parts of the tubes which are inside the building. Light from two lamps is thrown upon the tubes by passing it through two condensers and reflecting it at two plane mirrors, the lamps and the tubes being at conjugate foci of the condensers. The whole of this light is intercepted by suitable screens except that which finds its way through the light specks, which passes on and is brought to a focus upon a piece of bromide paper fastened around the clock cylinder by means of suitable lenses placed in the paths of the beams. Matters are so arranged that the image of the speck in the wet-bulb thermometer falls on the paper vertically below that of the speck of the dry-bulb, so that corresponding portions of the two records are in vertical line. The lights are completely cut off automatically for four minutes once every two hours, so as to provide convenient "time-marks" whereby the records are easily tabulated. Each sheet lasts for forty-eight hours, when it is taken off and replaced by a new one. It is essential that the apparatus be installed in a dark room, and the record cannot be inspected and tabulated until it has been developed, fixed, washed, and dried. Although the photographic method is expensive and troublesome, the apparatus is well made and robust, and the method has the great advantage of being entirely frictionless—there is not even a pen pressing against the paper. Perhaps the most serious objection that can be brought against the method when used at an observatory is that it is impossible for an observer to watch the record being made: he cannot, therefore, correlate, on the spot and at the time, corresponding changes of temperature and other meteorological phenomena. A minor objection is that the thermometers one end to protect the bulb, which projects from the mount, from injury.

(*c*) There is a fine constriction or other equivalent arrangement at a point in the bore of the stem, near the bulb, which offers considerably more resistance to the flow of mercury than the remainder of the bore. When temperature is rising, and the column of mercury in the thermometer is continuous, the mercury is forced past the constriction and the end of the column registers correctly the prevailing temperature; but when temperature falls, the mercury column breaks at the constriction as soon as the mercury in the bulb contracts, but the further end of the column remains at a point corresponding with the highest temperature attained, for the contraction of the detached column itself due to the fall of temperature is quite inappreciable for meteorological purposes. The thermometer therefore always indicates the highest temperature to which it has been subjected since it was last "set." Setting is performed by holding the instrument by the end of the wooden mount remote from the bulb, and vigorously swinging it at arm's length so as to drive the mercury by centrifugal force past the constriction and into the bulb, until the whole volume of mercury is continuous again. A well-made maximum thermometer is a very satisfactory instrument and seldom gets out of order. It is usual to support the tube of the thermometer in a nearly horizontal position in the screen, with a *slight* slope downwards towards the bulb.

(iii.) *Minimum Thermometer.*—The standard minimum thermometer is very similar in form to the maximum thermometer, but the liquid used in the thermometer is uncoloured alcohol, and the range of graduation is 20° F. to 110° F. There is no constriction in the bore of the tube, but a black or dark blue glass index in the form of a long dumb-bell is inserted in the bore, and is normally completely immersed in the alcohol. The tube being supported horizontally, the flow of alcohol produced by a rise of temperature passes the index without moving it. Likewise the return of the alcohol towards the bulb due to a fall of temperature has no effect upon the index until the end of the column reaches the end of the index, when, in consequence of the surface tension at the free surface of the alcohol, any further decrease of temperature results in the withdrawal of the index towards the bulb, the further end being always at the end of the alcohol column so long as temperature continues to fall.

The thermometer is read by noting the graduation on the scale which corresponds with the end of the index further from the bulb; this reading will be the lowest temperature attained since the instrument was last set.

The necessity of placing a glass index in the bore of the tube requires the use of a considerably wider capillary for the stem of the thermometer than is needed for the maximum or the dry- and wet-bulb thermometers. The openness of the scale is, however, the same, for, on the other hand, the coefficient of expansion of alcohol is nearly six times that of mercury. The increase in bore is therefore compensated to a considerable extent by the increased coefficient of expansion, the result being that the bulb of the minimum thermometer is not markedly different in size from that of a maximum thermometer; generally the former is slightly the larger. Minimum thermometers are usually less sensitive to change of temperature than mercurial thermometers.

The chief defect of the minimum thermometer consists in the great tendency of the alcohol to evaporate from the end of the column and to condense at the further end of the tube, where it may form a bubble of liquid equivalent in length to 3° or 4° F. of temperature on the thermometer scale. It is evident that this will cause the thermometer to read too low by an equal amount, resulting in the registration of correspondingly erroneous minimum temperatures. This tendency appears to be most marked on days of considerable range of temperature. Even if a bubble is not actually produced, a very thin film of alcohol is liable to be formed upon the internal walls of that part of the tube which is not occupied by the column of spirit, and this may introduce an error of nearly 1° F. in the reading. Mercury, being a non-volatile liquid which does not "wet" glass, is much to be preferred on that account for use in thermometers.

It is the practice among thermometer makers to seal alcohol minimum thermometers while their bulbs are immersed in a freezing mixture. In this way as much air as possible is included in the thermometer, for it is found that this air appreciably diminishes the diffusion of alcohol vapour along the tube, and therefore tends to reduce the defect. A mercurial thermometer is, on the other hand, usually sealed when the bore of the tube is filled with mercury, that is, when the thermometer has attained the maximum temperature which, when sealed, it can withstand with safety. At lower temperatures therefore there is a vacuum in the tube above the mercury column. Thus, at moderate temperatures the pressure inside a mercurial thermometer is less than atmospheric, while that inside an alcohol thermometer is greater than atmospheric. F. J. W. Whipple has suggested that the excess of pressure inside a spirit thermometer may in course of time slightly distend the bulb of the thermometer, causing it to read too low. This would explain the circumstance that these thermometers are inclined to develop errors which require the application of positive corrections, even when care is taken that all the spirit in the thermometer is continuous in the column, and not condensed upon the walls of the tube above the column.

The minimum thermometer is placed in the screen with its stem horizontal, and it is "set" daily by removing it and holding it bulb upwards, when the index descends by gravity until its further end coincides with the free end of the alcohol column.

§ (8) ARRANGEMENT OF THE THERMOMETERS INSIDE THE SCREEN.—As explained above, it is usual to place four thermometers inside a Stevenson screen, viz. dry-bulb, wet-bulb, maximum, and minimum thermometers. The first two thermometers are read with the stems vertical, while the last two are read with the stems horizontal. It is necessary to remove the first two from the screen only occasionally, whereas the maximum and minimum thermometers have to be removed daily for setting. Consequently the usual arrangement of the thermometers in the screen is to fix the dry and wet bulbs side by side towards the back of the screen, and to hang the maximum and minimum thermometers upon two vertical wooden battens fixed in the screen, forward and also to the side of the dry and wet bulbs, the maximum thermometer being near the top and the minimum near the bottom of the screen. In this way the scales of the dry and wet bulbs, between about 30° F. and 80° F., which is the portion most frequently required, fall between the mounts of the maximum and minimum thermometers, and are readily visible on opening the door of the screen. No difficulty in reading these thermometers, between these limits, without introducing errors of parallax, is experienced.

§ (9) THERMOGRAPHS, OR SELF-RECORDING THERMOMETERS.—A thermograph is an instrument which automatically records temperature as a function of time. The sheet of paper upon which the record is obtained is wrapped around a metal cylinder, with axis vertical, which is caused to rotate uniformly by means of suitable clockwork. Some arrangement is provided whereby either a pen or a spot of light, which moves vertically up and down in response to changes of temperature, is caused to touch the paper and leave upon it a permanent impression. In this way a continuous record of temperature is obtained. Considerable variation is found in different instruments as regards the method of indicating temperature change: it is this variation which gives rise to different types of thermographs, which may be classified thus, according to the type of thermometer used:

(i.) Mercury thermometer:
    (1) Photographic registration.
    (2) Mechanical registration.
(ii.) Bimetallic thermometer }
(iii.) Bourdon tube } Mechanical registration.
(iv.) Resistance thermometer }

(i.) *Mercurial thermographs*, with photographic registration, have been in continuous operation at Meteorological Office observatories since 1867, and are now in use at Kew, Eskdalemuir, Aberdeen, and Valencia observatories. Each thermograph comprises two thermometers, one acting as a dry bulb, the other as a wet bulb. The glass tubes of the thermometers are bent twice at right angles to enable the bulbs to be exposed outside the building in a louvred thermometer screen fixed to a north wall of the observatory. Small air specks are left in the mercurial columns at convenient positions in the parts of the tubes which are inside the building. Light from two lamps is thrown upon the tubes by passing it through two condensers and reflecting it at two plane mirrors, the lamps and the tubes being at conjugate foci of the condensers. The whole of this light is intercepted by suitable screens except that which finds its way through the light specks, which passes on and is brought to a focus upon a piece of bromide paper fastened around the clock cylinder by means of suitable lenses placed in the paths of the beams. Matters are so arranged that the image of the speck in the wet-bulb thermometer falls on the paper vertically below that of the speck of the dry-bulb, so that corresponding portions of the two records are in vertical line. The lights are completely cut off automatically for four minutes once every two hours, so as to provide convenient "time-marks" whereby the records are easily tabulated. Each sheet lasts for forty-eight hours, when it is taken off and replaced by a new one. It is essential that the apparatus be installed in a dark room, and the record cannot be inspected and tabulated until it has been developed, fixed, washed, and dried. Although the photographic method is expensive and troublesome, the apparatus is well made and robust, and the method has the great advantage of being entirely frictionless—there is not even a pen pressing against the paper. Perhaps the most serious objection that can be brought against the method when used at an observatory is that it is impossible for an observer to watch the record being made; he cannot, therefore, correlate, on the spot and at the time, corresponding changes of temperature and other meteorological phenomena. A minor objection is that the thermometers have large bulbs, thus rendering them more sluggish than the thermometers in general use for meteorological purposes. The instrument is described in the Report of the Meteorological Committee for 1867.

A thermograph containing a mercurial thermometer arranged for mechanical registration is on the market. The mercury is contained in a steel tube, which is connected by a flexible steel pipe to a steel box with a corrugated steel diaphragm. Increase of temperature is accompanied by an expansion of the mercury relatively to the steel, so that the diaphragm is pressed outwards, while decrease of temperature produces the opposite result. The motion of the diaphragm is magnified by a system of levers terminated either by an index hand moving over a circular dial, which is graduated to show temperature on the desired scale, or by a pen arm carrying a pen which writes upon a chart in the usual way.

(ii.) The *bimetallic thermograph* is now in use at a fair number of stations in this country. It writes a visible record upon a chart which can be printed beforehand to indicate the time and temperature scales to which the instrument is adjusted. The thermometer is a long strip consisting of two pieces of brass and invar electrically welded together and coiled in the form of a spiral, the invar being on the outside and the brass on the inside of the coil. The coil is mounted upon a horizontal axis at the end of and outside the case of the instrument. One end of the coil is fixed to the case, through a fine adjustment screw, by which the pen can be set to agree with a standard thermometer; the other end is connected through the horizontal axis to the arm which carries the pen (*Fig.* 3). When temperature rises the end of the coil carrying the pen arm causes the latter to move upwards, and when temperature falls the arm moves downwards. Corresponding movements are therefore transmitted to the pen itself, which writes upon the sheet mounted upon the clock drum.

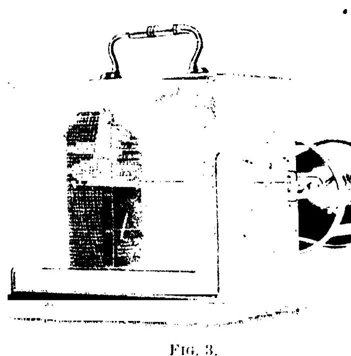


FIG. 3.

A clamp for fixing the pen arm to the horizontal axis is provided at the pivot of the pen arm, by means of which the position of the pen upon the paper can be considerably varied. This is used when the seasonal change of temperature is more than can be accommodated upon the sheet. The instrument is small and portable, and in careful hands gives excellent results. It has to be standardised by comparison with a standard thermometer, and when used to measure air temperature it should be exposed in a Stevenson screen.

(iii.) The *Bourdon tube thermograph* is similar in general arrangement to the last type, but in this case the thermometer is a tube of thin metal, called a Bourdon tube, of which the transverse section is in the form of a very flat ellipse, and the longitudinal section is in the form of a quadrant of a circle. The tube is completely filled with alcohol. When temperature rises the alcohol expands and the volume of the tube increases. This increase produces a decrease in the curvature of the longitudinal section of the tube, so that if one end of the tube is rigidly attached to the case of the instrument, the other end moves relatively to the case, and this motion is magnified by a system of levers terminating with the pen arm which carries the pen. This instrument possesses the advantage over the bimetallic thermograph, that the temperature scale value can be regulated to a nicety to suit the chart, whereas that adjustment is not provided on the bimetallic instrument. On the other hand, the Bourdon tube thermometer is much less sensitive to change of temperature than the bimetallic thermometer, and the absence of multiplying levers in the latter instrument is an advantage, because that tends to the elimination of friction.

(iv.) *Electrically recording thermographs* are exemplified by the resistance thermometer used in conjunction with a Callendar electric recorder. The thermometer consists of a fine platinum wire wound round a mica frame which is protected by a glass or metal sheath. The electrical resistance of such a wire is connected with its temperature by a relation which is very nearly linear, so that the problem reduces itself to the recording of the resistance of the platinum wire. This is done in a very elegant and interesting manner by the Callendar Electrical Recorder,[1] which may be briefly described as a self-balancing and recording Wheatstone bridge, in which two of the resistance arms are equal to one another.

In the figure, R and R are the equal resistances in the arms AB and AE of the Wheatstone bridge, which is shown diagrammatically in the usual manner. T represents the resistance thermometer, and S is a compensating resistance. CD is a resistance wire with a sliding galvanometer contact at X. If r be the resistance of CD per unit length, then CX . r and XD . r are the resistances of CX and XD, and when the bridge is balanced

---

[1] See "Resistance Thermometers," § (20), Vol. I.

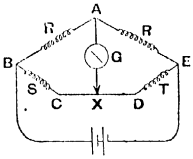


FIG. 4.

$$R(S+CX\,.\,r)=R(T+XD\,.\,r).$$

Thus
$$\begin{aligned}T&=S+(CX-XD)r\\&=S+(CD-2XD)r\\&=\text{const.}-2r\,.\,XD.\end{aligned}$$

In other words, the distance of X from either end of the wire CD is a measure of the resistance T, and therefore of the temperature. In the Callendar recorder the sliding contact at X is adjusted automatically to secure a balance, and the recording pen, being attached to X, produces a visible record of the temperature.

III. INSTRUMENTS FOR MEASURING PRECIPITATION

Precipitation may take the form of rain, snow, hail, sleet, dew, mist, or fog, but the first-named form, rain, is by far the most important in all except arctic and northern or southern continental climates, where snow becomes the main constituent at some, if not all, seasons of the year.

§ (10) THE RAIN-GAUGE.—The instrument used to measure rainfall is the rain-gauge, with its graduated glass measure. The gauge consists of (a) a cylindrical metal vessel, (b) a funnel with a long delivery tube, the funnel being soldered or brazed inside another cylinder, open at both ends, which fits over the opening of the vessel, and (c) a glass or metal receiver placed inside the vessel to collect the rain which falls into the funnel and passes down the delivery pipe. If a glass receiver is used it is advisable to place it inside a metal receiver, which in turn rests on the bottom of the vessel. The cylinder into which the funnel is fixed is terminated at its upper end by a strong brass rim with a bevelled upper edge, so that the collecting area is bounded by what amounts practically to a circular knife edge, of which the radius must be made accurately to a standard size. Except for this rim, rain-gauges are usually made of strong sheet copper, which is very durable. Galvanised iron is, however, cheaper, and can be recommended, but it will not wear so well as copper. In this country the diameter of the brass rim of most gauges is 5 inches, but there is a considerable number of gauges in use of which the diameter is 8 inches. An important feature of a good gauge is the deep side of the movable cylinder, between the lip of the gauge and the upper edge of the funnel, amounting to some 4 or 5 inches. This prevents splashing out of heavy rain, and also assists materially in retaining snow. The gauge is set up so that its rim is 1 foot above the ground, the cylindrical vessel being buried sufficiently in the ground to secure that.

The "Meteorological Office" pattern gauge is provided with a splayed base, as shown in the illustration (*Fig.* 5). The cylindrical vessel is in this case replaced by a vessel in the shape of a frustum of a cone surmounted by a short open cylinder. The advantage of this type is that, when fixed in the ground, the weight of part of the earth surrounding the frustum is carried upon the sloping side, and thus the gauge is firmly secured. It is found that cylindrical gauges invariably work loose in the ground as a result of the daily removal and replacement of the funnel in connection with the measurement of the collected rain.

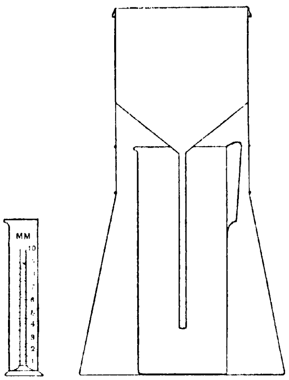


FIG. 5.

§ (11) THE RAIN MEASURE.—Rainfall is measured in terms of the *depth* of water which would be collected upon a level area of any size, supposing the rain to fall uniformly over the area at the rate at which it falls into the gauge. In a 5-inch diameter gauge, therefore, the water collected, if poured into a cylinder 5 inches in diameter, would fill it to a depth which is taken as the measure of the fall. In order to secure accuracy of measurement to the hundredth of an inch or tenth of a millimetre, and also for the sake of convenience, a measure 5 inches in diameter is never used, but the collected rain is poured into a measure from 1½ to 2 inches in diameter, which is graduated in such a way that the rainfall is read off directly in the desired units.

If R is the radius of the gauge, and P is the rain fall, while r is the internal radius of the glass measure, all expressed in inches or in millimetres, then the distance p from the bottom of the measure, of the graduation corresponding with rainfall P, expressed in the same units as before, is given by

$$\pi R^2 P = \pi r^2 p,$$

i.e.

$$p = P\frac{R^2}{r^2}.$$

If $R = 4$ inches, and $r = 1$ inch, then $p = 16$ P, so that half an inch of rain from an 8-inch rain-gauge when measured in a glass 2 inches in internal diameter will fill it to a depth of 8 inches. It is therefore quite practicable to divide such a measure to show tenths and hundredths of an inch of rain up to half an inch, or in whole millimetres and tenths up to 10 millimetres. It is obvious that a measure graduated to suit a particular size of gauge is unsuitable for another size, and it is therefore necessary to engrave on the measure the diameter of the gauge with which it should be used. For example, 2 millimetres of rain in an 8-inch gauge will be shown as $2 \times (8^2/5^2)$ or 5·1 mm. in a measure graduated to suit a 5-inch gauge.

§ (12) OBSOLETE RAIN-GAUGES.—The rain-gauge described above is that usually known as the Snowdon pattern. There are many other patterns which are officially regarded as obsolete, although examples of them are still to be found in use. They are not to be recommended, either because of some fault in design (*e.g.* lack of the deep rim, causing the catch to be too low on account of the out-splashing), or because of some weakness of construction which unduly shortens the life of the gauge.

§ (13) VARIOUS PRECAUTIONS, EXPOSURE, ETC. (i.) *Evaporation.* The Snowdon rain-gauge described above is so constructed that evaporation of the collected rain is negligible. The volume of the outer vessel below the funnel, when the latter is in position, is practically shut off from the external air, the only connections being (1) along the narrow and long delivery pipe, and (2) round the close-fitting sliding joint between the lower vessel and the cylinder which supports the funnel.

(ii.) *Height of Rim above Ground.*—In this country rain-gauges are exposed with their rims 12 inches above the ground-level. It is undesirable to set the rim flush with the surface of the ground since this would result in the splashing into the gauge of rain which had fallen and rebounded from the ground near the rim. On the other hand, increasing the height of the gauge above ground-level, especially if this is done by placing the gauge on the roof of a building, causes the catch to be appreciably reduced. It has been shown that this is due to deflection of the air currents by the gauge itself, and by the building, if any, so that the current, as it passes over the gauge, is resolved into a succession of eddies; and, in consequence, rain which should properly be retained by the gauge is carried completely over it. This matter will be discussed more fully when considering the measurement of snowfall.

(iii.) *Rim to be Horizontal.*—The rim of the gauge must be kept horizontal. When rain falls vertically the catch in a gauge which is not level will be that falling within the cylinder with vertical axis, of which the rim of the gauge is an oblique section. The normal section of this cylinder is an ellipse of which the major axis is equal to the diameter of the gauge; consequently the area of the normal cross-section is less than that of the rim of the gauge, and the catch is reduced in proportion. If we neglect wind eddies round the gauge and assume that when rain and wind occur together the drops fall at a uniform oblique angle into the gauge, then it is easy to see that the amount collected represents the true fall at a point more or less distant on the windward side. Thus the rain collected at the gauge G (*Fig.* 6) would, under the assumption made, have originated from the cloud sheet at C. Had there been no wind, this same rain would have been exactly collected in a similar gauge at G′, since the cross-section of the cylinder shown dotted at G′ is equal to the cross-section of the same cylinder at C, which again is equal to the oblique section at G of the elliptical cylinder CG. But, in fact, rain does not descend in oblique straight lines when it is windy, but eddy currents pass round and over the gauge with the result that some rain which belongs to the section at C is not collected in the gauge at all, and the catch is too low. This difficulty is especially marked in mountain and moorland sites, and in such cases it is necessary to select a more or less sheltered place for the gauge, such as a natural depression in otherwise generally level ground. Failing that, it is usual either to construct an earthwork around the gauge at a distance of about a yard, of which the top is level with the rim of the gauge; or to provide a metal shield around the gauge, such as the Nipher shield (see below, § (15)).

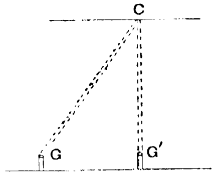


FIG. 6.

(iv.) *Site of Gauge.*—At ordinary stations it is necessary for accurate results that the gauge should be exposed in an open site so as to avoid the errors due to wind-eddies round buildings, trees, etc.; and to secure this, it is sufficient that the distance of every object from the gauge should be not less than twice the height of the object.

§ (14) SELF-RECORDING RAIN-GAUGES.—A large amount of ingenuity has been expended upon designs of instruments which will automatically record both the amount of rainfall and the time at which the fall occurs. A considerable number of different kinds are in actual use, but it is doubtful if finality in the matter has been reached. Self-recording rain-gauges are usually divided into three types:

(1) Tilting-bucket gauges.
(2) Float-gauges.
(3) Balance gauges.

(i.) *Tilting-bucket Gauge.*—In the tilting-bucket gauge the rain, as delivered from the funnel, is collected in one or other of two small elongated "buckets," one on each side of the arm of a balance. When one bucket is nearly filled (having collected a definite quantity of rain, say ·01 inch) the side of the arm which carries it is automatically depressed by the weight of the collected rain, and the bucket is emptied. At the same time the other bucket comes into use as collector, and so the process goes on, the arm rocking like a see-saw for every ·01 inch of rain that falls. The rocking is communicated to a pen lever and pen recording on a drum rotated by clockwork through a reversed escapement motion and a "snail" mounted on the axis of the escapement wheel. The pen, on reaching the top of the chart, falls automatically to the bottom again, since these two positions correspond with the points of the maximum and minimum radii of the snail, these radii being superimposed over one another.

(ii.) *Float-gauges.*—In float-gauges the water is collected in a cylindrical vessel containing a light metal float, and the vertical motion of the float is communicated through a float-rod to the pen which records upon a clock drum in the usual way. It is obvious that the vertical motion of the pen is proportional to the rainfall. A continuous record is thus obtained instead of the step-by-step record of the tilting-bucket gauge. When the pen reaches the top of the chart, it is usually arranged that a siphon comes automatically into operation, which empties the vessel to such an extent that the pen falls to the base line of the record in a few seconds. The siphon then ceases to act and the record starts again from the base line.

The automatic siphon is usually the weak feature of these instruments, and is frequently a continual source of trouble and failure. A plain siphon is sometimes used, but care in the selection of proper proportions, and of the proper diameter of the siphon, is necessary, otherwise the siphon will "dribble," *i.e.* fail to start fully at a definite time, and merely carry over the excess of water as it is collected. Dribbling may also occur at the conclusion of the siphoning process, if rain is falling sufficiently rapidly just to prevent air from entering at the bottom of the short leg of the siphon in sufficient quantity to stop the flow. Another defect of the automatic siphon is its liability to start for slightly different quantities of water in the collecting vessel. In any gauge of this type, rain which falls during the time the siphon is in action is not recorded, but unless the siphon empties the gauge at an abnormally slow rate, the resulting loss is negligible.

(*a*) Messrs. C. F. Casella & Co., Ltd., and Messrs. Negretti & Zambra have, quite recently, and almost simultaneously, brought out new recording rain-gauges with "plain" siphons in the sense that there is no moving subsidiary device for fully starting the siphon at the right time. The two siphons present several interesting features. They differ from one another in some respects, but agree in that they are arranged with the long and short legs coaxial, the long delivery tube being inside the short leg, which is connected to the collecting vessel. The advantage of this arrangement is evident from the figure, which represents a longitudinal central section of the knee or upper end of the siphon where the direction of flow of the water is reversed from an upward to a downward direction, and the water enters the long leg of the siphon.

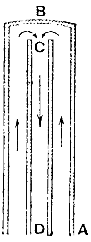


FIG. 7.

The outer tube AB (*Fig.* 7) represents the upper end of the short leg, and the inner tube CD represents the upper end of the long leg. It is clear that if a is the radius of the tube CD, and d is the vertical clearance between the ends of the two tubes at the top, then d need not be greater than the value which is determined by the condition that fluid motion down CD is not throttled at the bend. In other words, d need not exceed the value given by

$$2\pi ad = \pi a^2,$$

or

$$d = \frac{a}{2}.$$

That is, the necessary clearance is only one-quarter of the diameter of the inner tube. If that tube is $\frac{3}{8}$ inch in diameter, the clearance required is only $\frac{3}{32}$ inch, and is practically of capillary dimensions. This fact ensures that the starting of the siphon is a definite occurrence, for as water passes over the bend capillary action drives all air in front of it, and a full flow commences forthwith. Similarly, after siphoning, the first small quantities of air which are passed up the short limb immediately occupy the capillary space at the top, and effectively check the flow. The width of the annular space in the short limb inside the outer tube and outside the inner tube may clearly be still less than *d*, in so far as the condition of freedom from throttling of the flow is concerned, but it is much better not to reduce this distance to capillary dimensions, for this would mean that the level of water in the short limb would be higher than that in the collecting vessel by an amount which would be subject to variation according to the state of the tube as affecting surface tension.

Other methods of overcoming the difficulties connected with plain automatic siphons have been brought forward from time to time. These are embodied in different instruments, of which it will be sufficient to refer to the hyetograph, the Dines gauge, and the Fernley gauge.

(*b*) In the hyetograph the automatic siphon is entirely dispensed with, and the float rises steadily until the collecting vessel is filled, the amount collected being equivalent to over 4 inches (about 100 mm.) of rain. The pen, instead of being mounted directly upon the float-rod, is supported on an arm pivoted to a fixed axis on the instrument. Projections on the side of the float-rod come successively into action by bearing against a cam mounted upon the pen arm. When the pen reaches the top of the chart, which occurs after ½ an inch (or 10 mm.) of rain has been collected, the projection then in action automatically overruns the end of the cam, the pen arm falls until its motion is arrested, when the pen has returned to the base line of the chart, by the cam coming into contact with the next lower projection on the float-rod, which now takes up the control of the pen arm and pen. The hyetograph is a simple instrument, and it is reliable in action if care is taken to avoid friction between cam and projections.

(*c*) The Dines recording rain-gauge is described by Mr. W. H. Dines, F.R.S., in the *Meteorological Magazine* for 1920, vol. lv. p. 112. In this instrument the collecting vessel containing the float is mounted upon one arm of the lever of a balance, and an adjustable counter-weight is placed on the other arm. When the required amount of rain (10 mm.) has been collected in the vessel, the pen has reached the top of the chart, the weight of the vessel and of the collected rain just overbalances the counter-weight, and the vessel descends a short distance, which is sufficient to depress the "knee" of the siphon attached to the vessel to such an extent that it is completely and suddenly filled with water. The siphon is therefore started without dribbling or hesitation when the required quantity of rain has been collected. As the vessel is emptied, the counter-weight raises it to its initial position again.

(*d*) The Fernley recording rain-gauge was designed by J. Baxendell of the Southport Meteorological Observatory. It has an exceptionally open rainfall scale, and was intended primarily for observatory use. The siphon is started by being "primed" by the sudden introduction of water near the bottom of the long limb at the required moment. The water used is a part of the last catch of rain in the collecting vessel, arrangements being made for it to be collected in a tipping-bucket, which is overturned through the agency of a cam attached to the float-rod and an intermediate trigger action. The satisfactory action of this instrument depends upon the proper performance of a series of motions, each one of which is consequential upon the previous one.

(iii.) *Balance Gauges.*—In balance gauges the receiving vessel is suspended on one arm of a curved lever balance, or is floated in a mercury trough. As rain is collected the vessel is depressed in proportion to the quantity received, and the amount of depression is communicated to a pen writing on a chart in the usual manner. When the requisite amount of water has been collected, corresponding with the rain scale of the chart, the vessel is emptied either by being automatically inverted or by the aid of a siphon. The large Casella gauge is perhaps the best-known form of the curved-lever balance gauge, and the Beckley gauge is representative of floating receiver gauges.

In the Beckley gauge the neck of the receiver is much constricted, and a siphon is fixed to the receiver with its knee at the level of the narrowest part of the neck. This arrangement is effective in ensuring certain action of the siphon, for even in very light rain the rise of the water-level in the neck of the receiver, relative to the siphon, is appreciable, so that "dribbling" of the siphon is unlikely to occur. This method of starting a siphon is clearly inapplicable to float-gauges.

§ (15) THE MEASUREMENT OF SNOWFALL.—There are numerous difficulties connected with the measurement of snowfall, which render necessary the adoption of special precautions. A large proportion of snow falls when the wind is blowing strongly, so that the snow in falling is carried by wind eddies around and over obstructions, and is also piled up in drifts. These two effects, if not guarded against, vitiate the two usual methods of measurement, viz.: (*a*) collection of the snow in a rain-gauge, and subsequent melting and measurement as rain; and (*b*) measurement of the depth of the fallen snow. It may, however, be noted that the measurement of the depth of the snow, even if satisfactorily performed, does not meet full requirements, for the density of recently fallen snow varies considerably according to the air temperature which prevails at the time, so that the factor of conversion to equivalent rainfall is not constant. The factor is, however, usually taken to be one-tenth or one-twelfth; hence the convenient rule to meteorological observers —" measure the depth of snow in centimetres (or feet), and convert to equivalent rainfall by calling centimetres millimetres, or feet inches, and retaining the numeric." It is usual to reserve a level patch of ground for this purpose, and after each measurement to clear away the snow so that further falls can be measured without ambiguity.

There are also difficulties connected with the collection and subsequent melting of snow in a rain-gauge. If the wind is strong it eddies over the mouth of the gauge, so that most of the snow which would otherwise be collected is carried over the gauge, and the actual catch is an indeterminate fraction of the real fall. The wind may also remove from the funnel some snow which had been previously collected there.

Different expedients have been suggested for overcoming this difficulty. Nipher's shield, which is perhaps the best known of these, will be briefly described, and a few remarks added as to the properties of the ideal shield.

(i.) *Nipher's Shield.* — Nipher's shield consists of a frustum of a cone fixed around the gauge with its wide end uppermost, and with its axis coincident with that of the gauge. There is a clear space between the shield and the outside of the gauge, to allow the snow or rain which falls upon the inside of the shield to drop through to the ground. The shield is sufficiently far away from the rain-gauge to prevent rain which falls upon it from splashing into the gauge. The upper edge of the shield is fixed level with the rim of the gauge. The object of this arrangement is to prevent the formation of the usual wind eddy over the gauge and to replace it by an eddy which is formed at the upper edge of the external surface of the shield, and deflected downwards by the shield. The following diagrams (*Figs.* 8 and 9) will explain this point.

(ii.) *The Ideal Shield.*—The problem appears to be the following. It is required to provide such a shield as will ensure that the stream-lines of air above the gauge are as nearly as possible horizontal and free from eddies. For in such a current, snow, or any other form of precipitation, will fall with a uniform vertical component of velocity, which will be the same as that in still air, and the only difference will be that the precipitation will possess also a horizontal component of velocity corresponding with that of the air. The quantity collected by the gauge will be correct, but it will have originated from a cloud somewhat to windward of the gauge.

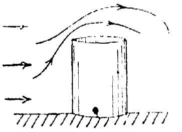


FIG. 8.

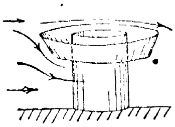


FIG. 9.

IV. INSTRUMENTS FOR MEASURING WIND VELOCITY AND DIRECTION

(A) *Wind Velocity*

§ (16) INTRODUCTORY.—The instruments used for the measurement of wind velocity are of many kinds, and vary considerably in size and price. They may all be divided, however, into three main types.

(*a*) *Cup and Fan Anemometers.*—Broadly speaking, these instruments measure the quantity of air passing them in an interval, expressed in miles or feet run of wind.

(*b*) *Pressure-tube Anemometers.*—These instruments depend upon the pressure effects produced by an air current in pipes which are presented to the current in different ways.

(*c*) *Pressure-plate Anemometers.*—In these instruments a plate of known dimensions is placed in the current normal to its direction. Either the force exercised by the wind upon the plate is measured, or the plate is hinged about its upper edge and swings freely from that edge, and the angle is measured by which the plate, being in equilibrium, is deflected out of the vertical by the wind pressure.

In instruments of the first type the indications scarcely depend at all upon the density of the air, but in instruments of the second and third types the density of the air occurs as a factor in the expression for the pressure exerted by the wind upon the plate. The relation between wind pressure, density, and velocity has been proved experimentally to be satisfied by the equation

$$p = k\rho v^2,$$

where k is a constant depending upon the instrument, p is wind pressure, and ρ is air density; whence it follows that the wind velocity varies as the square root of the quotient of pressure and density. This fact needs to be borne in mind in considering anemometers for use at high elevations where the air density is appreciably less than at sea-level. It is even more important in dealing with instruments, which are generally of the second type, for measuring the velocity of an aeroplane relative to the air. Since variations of air density depend mainly upon height above sea-level, the interpretation of the readings of such instruments depends upon the height at which the machine happens to be flying.

§ (17) THE ROBINSON CUP ANEMOMETER. (i.) *Description.*—The cup anemometer consists of four hemispherical cups of thin metal attached in such a way to the ends of two horizontal arms, crossed at right angles to one another at their middle points, that the open cups face horizontally, and are in turn exposed to the wind as they rotate under the action of the wind about a vertical axis through the point of intersection of the arms. If two cups are mounted at the ends of a single horizontal arm which is free to rotate about a vertical axis at its middle point, the resultant pressure of the wind upon the cups is in general such as to cause rotation about the axis in the sense corresponding with a recession, from the direction from which the wind is blowing, of a cup which is open to the wind. Thus in *Fig.* 10, if the long arrow represents the wind direction, then the wind pressure on cup B, which in the position shown is open to the wind, is greater than that on cup A, which is presented to the wind as a convex surface, and the arm will rotate in an anti-clockwise direction. If now the second arm, with its cups C and D, is added, we have similarly, for the position shown in *Fig.* 11, that the pressure on C is greater than that on D, which again produces an anti-clockwise rotation. Hence the whole system will rotate in the same direction. Two cups on a single arm are not sufficient to give steady motion, because the torque or turning moment fluctuates considerably in the course of a rotation, being a maximum when the arm is at right angles to the wind direction and zero when the arm lies along the wind direction. Three cups are also inappropriate for similar reasons, but four cups are suitable for an anemometer, both from the point of view of nearly uniform torque during a revolution, and also because such an anemometer is well adapted to manufacture in a simple and strong design.

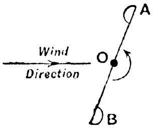


FIG. 10.

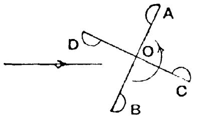


FIG. 11.

Considerable attention has been devoted to the mathematical theory of the cup anemometer exposed in a steady wind-current, but the problem is too difficult for a complete solution to be obtained, and the most important results of these attempts have been to suggest different *forms* of the mathematical relation which exists between the velocity of the wind and the corresponding speed of the cups, leaving the constants of the relation to be determined subsequently by experiment. The actual case is complicated still further by the fact that the natural wind is not a steady current, but one which is constantly fluctuating both in velocity and direction. The effect of fluctuation in velocity is generally to exaggerate the true average velocity of the wind, because of the inertia of the cup system, which is affected by the higher more than by the lower velocities.

An account, with a discussion, of some of the theories of the Robinson anemometer, by F. J. W. Whipple, has recently been published by H.M. Stationery Office as *Reports and Memoranda, No. 669, of the Advisory Committee for Aeronautics*, and the following remarks are mainly based upon this paper.

(ii.) *The "Factor" of the Cup Anemometer.*—Dr. Robinson's original experiments at Armagh showed that the speed of the wind was approximately three times the speed of the centres of the cups in their circular path, and this result was, until comparatively recently, assumed to be true for all sizes of cup anemometer, although experiments had been made by several investigators which did not support it. It has now been shown, by experiments in a wind channel at the National Physical Laboratory, and by comparisons between a cup anemometer and a pressure-tube anemometer exposed side by side, that the ratio between the speed of the wind and the speed of the cups, usually known as the "factor" of the anemometer, is not constant but varies with the wind speed, being greater at low than at high speeds. This variation is independent of frictional effects in the instrument itself, but the fact that it operates in the same direction as those effects led the early investigators to suppose that when due allowance had been made for friction the factor was constant at all speeds. It has also been shown that anemometers of different sizes, as specified by the diameter of the cups and length of the arms, have different factors for the same wind speed, but it appears from the theory of dimensions that the factors of anemometers of similar dimensions (both as regards cups and arms) are proportional to the product wind speed × linear dimensions. A further result of experiment is that the variation of factor with speed is smaller according as the ratio of length of arm to diameter of cups is reduced; and as a nearly constant factor is obviously a convenience, it seems desirable to construct anemometers with short arms and large cups. Dr. Robinson's original experimental anemometer had 3-inch cups on 5·36-inch arms, and his observatory anemometer had 12-inch cups and 23-inch arms. Dr. Robinson concluded that the same factor 3 was applicable to both.

(iii.) *The "Factor" of the Observatory Cup Anemometer.*—The standard recording cup anemometers at British observatories, as arranged by Robert Beckley, have 9-inch cups and 24-inch arms; for these the factor 3 was assumed in official publications to be true until 1905, when the factor was changed to 2·2, on the recommendation of the Wind Force Committee of the Royal Meteorological Society, which was based on experiments by Mr. W. H. Dines. The accompanying table sets out the relation between wind speed in metres per second as tabulated on the basis of constant factors 2·2 and 3·0 and the probable true speed, together with the percentage errors thereby introduced. The probable true wind speeds of the table are deduced, as regards the higher speeds, from comparisons between Robinson and pressure-tube anemometers; and as regards the lower speeds, by an application of the theory of dimensions, as referred to above, to experiments at the National Physical Laboratory on an instrument of about one-third the linear dimensions of the Beckley instrument.

Observatory Cup Anemometer (9-inch Cups, 24-inch Arms)

A	Nominal wind speed (factor 2·2) (m/s) .	1	2	4	6	8	10	12	14
B	Nominal wind speed (factor 3·0) (m/s) .	1·4	2·7	5·5	8·2	10·9	13·6	16·4	19·1
C	Probable true wind speed (m/s) . .	1·5	2·7	4·7	6·5	8·2	9·9	11·6	13·4
D	Error of A from C, expressed as a percentage of C	−33	−26	−15	−8	−2	+1	+3	+4
E	Error of B from C, expressed as a percentage of C	−0·7	0	+17	+26	+33	+37	+41	+43

The nominal wind for factor 2·2 is therefore correct at about 10 m/s or 22 mi/hr. For stronger winds the factor exaggerates the real speed; for lighter winds it would appear to understate the true speeds. At low speeds, however, the comparison between the two types of instrument is not very reliable. The constant factor 2·2 gives therefore fairly good general agreement with fact.

On the other hand, factor 3·0 gives results which are in general considerably in excess of the true wind speed, especially at higher speeds where the percentage error is as much as 43 per cent.

(iv.) *The "Factor" of the Portable Cup Anemometer.*—The anemometer referred to above as being of about one-third the linear dimensions of the Beckley anemometer is known in the Meteorological Office as the portable cup anemometer. It has 3-inch cups and 7⅜-inch arms. The factor of the instrument is taken to be 2·73, each mile run of wind corresponding with 500 revolutions of the cups. A test at the National Physical Laboratory of one of these instruments gave the following result:

Portable Cup Anemometer (3-inch Cups, 7⅜-inch Arms)

Nominal wind speed (factor 2·73) (m/s)	1	2	4	6	8	10	12	14	16	18
True wind speed (m/s) . .	1·2	2·3	4·5	6·4	8·3	9·9	11·5	13·0	14·4	15·8
Error expressed as percentage of true wind speed	−17	−13	−11	−6	−4	+1	+4	+8	+11	+14

The portable cup anemometer is arranged to indicate the run of wind which has passed in any desired interval, by a simple system of gears, terminating in a counting mechanism similar to that used as a mileage indicator on the speedometer of a motor car, or (less satisfactorily) by a system of dials as on a gas-meter. By suitably arranging the gears and incorporating the value of the factor in them, the figures on the indicator will represent miles—or kilometres—run of wind, so that average values of wind speed in miles per hour or kilometres per hour over a period can readily be obtained, by taking the difference between the readings of the indicator at the beginning and end of the period and dividing that difference by the period expressed in hours.

(v.) *The "Factor" of the Electrical Cup Anemometer.*—The electrical cup anemometer—Meteorological Office pattern—has 3-inch cups and 4-inch arms, and the factor of the instrument varies less than that of the portable cup anemometer, as the following table shows :

Electric Cup Anemometer (3-in. Cups, 4-in. Arms)

Wind speed appropriate to the constant factor 2·65 (m/s)	2	4	6	8	10	12	14	16	18	20	25
True wind speed (m/s) . . .	2·5	4·5	6·4	8·2	10·0	11·9	13·6	15·3	17·1	18·9	23·4
Error of speed as determined by the constant factor, expressed as a percentage of the true wind speed	−20	−11	−6	−2	0	+2	+3	+5	+5	+6	+7

The abnormally high percentage error at 2 m/s is to be ascribed to the effect of friction.

The cups turn an electrical contact-maker, which completes a circuit once in every twenty-five revolutions of the cups. When setting up the instrument for use the contact-maker is connected in series with a bell, or buzzer, a bell-push, a switch, and a 4-volt battery. When taking a reading the switch is closed, and the interval from one contact to the next, or for ten contacts, is timed, and the corresponding wind speed is taken from a table which takes account of the variability of the factor at different speeds.

(vi.) *Recording Form of Cup Anemometer.*—The cup anemometer is readily adapted to write on a sheet a permanent record, from which the run of the wind during any interval can be measured. In the Robinson-Beckley instrument this is effected by gearing the cups to a cylinder upon which a complete turn of a projecting metal helix is mounted. The axis of this cylinder is parallel to that of another cylinder, which is rotated once in twenty-four hours by a clock. Upon the latter cylinder is mounted a piece of "metallic" paper which can be changed every day, and the helix is lowered until it touches the paper. On the assumption that the factor of the anemometer is 3 the helix makes a complete revolution for every fifty miles run of wind. If the clock is not running such a revolution is indicated on the paper by a straight line parallel to the axes of the cylinders, and equal in length to the distance, measured parallel to the axes, between the ends of the helix. If, now, we suppose the clock to be started, it is plain that the record will consist of a line inclined to the line just described at an angle which varies with the speed of the wind, being smaller for strong winds and larger for light winds, a calm being shown by a line at right angles to that above described, *i.e.* at right angles to the axes of the cylinders. If v is wind speed, and θ is the angle referred to, then $v = k \cot \theta$, where k is a constant depending on the units in which v is measured and also upon the scale values of the chart. The wind speed at any instant may therefore be determined approximately by measuring the angle between the base line of the chart and the tangent to the record at the point corresponding with the instant. The records, are, however, usually tabulated to show the run, expressed in miles, of wind which has passed the anemometer in periods centred at exact hours of Greenwich mean time. These figures are equivalent to the corresponding mean wind speeds expressed in miles per hour. As already explained, the instruments were made to suit factor 3, and the present practice in the Meteorological Office is to use a table to convert these figures to metres per second, and at the same time to reduce them in the ratio 11 : 15, so as to change the factor from 3·0 to 2·2. A specimen record is reproduced in *Fig.* 20.

§ (18) The Fan Anemometer.—Similar in principle to the cup anemometer is the fan anemometer, or air-meter, which is like an ordinary wind-mill in construction. There is a light wheel comprising a number of spokes upon each of which is mounted a light vane set obliquely to the plane of the wheel. When the wheel faces the wind it rotates at a rate which is very nearly proportional to the speed of the wind. The wheel is connected by a worm-gear to an indicator dial, arranged like a gas-meter, whereby the "run" of wind which has passed the instrument can be read, expressed in feet or other suitable unit of length. The mean velocity over any interval, say a minute, is obtained by reading the dials at the beginning and end of the interval and subtracting the readings, so as to obtain the "run" of wind during the interval. The readings are facilitated by the provision of a simple device for throwing the counting mechanism in or out of gear with the wheel.

The indications of this instrument, like those of the Robinson cup anemometer, are nearly independent of air density, so that no corrections are needed to the readings of instruments exposed on kite-balloons or other aircraft, or at mountain observatories.

§ (19) Pressure-plate Anemometers.—In instruments of this type a plate is exposed normally (or nearly so) to the wind and the force exerted by the wind upon it is measured, either by means of springs controlling the motion of the plate, or by allowing the plate to be deflected from the vertical plane until it is in equilibrium under the action of the wind force and the weight of the plate. The latter principle is exemplified in the swinging-plate anemometer.

(i.) *The Swinging-plate Anemometer.*—Although the design of this instrument is based on that of one of the first anemometers ever constructed the instrument in its modern form, as brought forward during the war, is a very useful and simple portable anemometer. It is illustrated in *Fig.* 12. It consists merely of a metal plate 5¾ inches long and 4⅞ inches wide, suspended on knife-edges from one end to a light rectangular framework. The instrument is held facing the wind, the framework being vertical, and the plate assumes an equilibrium position at an angle from the vertical which depends upon the wind speed. A scale of miles per hour is engraved upon a curved piece under the lower edge of the plate, and indicates the wind speed, that reading being taken which is opposite the edge of the plate. In the actual wind, which is made up of gusts and lulls in a continuous series, the plate is always oscillating, but in practice an observation extends over half-a-minute or so, and it is easy to note either the mean position or the extreme positions of the plate during that interval, according to the character of the information required.

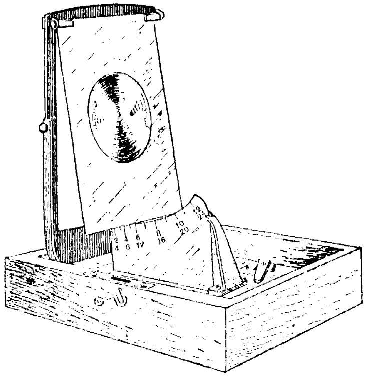


FIG. 12.

The instrument is calibrated in a wind channel where the speed of the current is known, for although the wind force upon a plate can be computed under certain assumptions as to the character of the flow, yet in practice these assumptions are liable to be departed from somewhat seriously, owing to sudden changes in the form of the stream lines at critical velocities. When the deflection of the plate exceeds about 30° the indications become unreliable. For the higher speeds it is found better to load the plate with " bosses " consisting of a pair of flat weights, one on each side of the plate, screwed together through a hole in the centre. If the bosses together weigh three times as much as the plate the total weight will be four times that of the unloaded plate, and as the centre of gravity is unchanged, the pressure required to produce a given deflection of the loaded plate will be four times that required for the same deflection of the unloaded plate. Since wind speed varies as the square root of wind pressure, the corresponding velocity will be twice that indicated by the same deflection of the unloaded plate.

Referring to the illustration, the vertical framework upon which the swinging plate is mounted, and the curved plate upon which the scales are engraved, both fold into the box, and there is a lid, not shown, which closes the box when the instrument is carried. There is a spirit-level upon the box to indicate when it is level, and, therefore, when the framework is vertical.

(ii.) *Osler Pressure-plate Anemometer.*—This is another anemometer which depends upon the pressure of wind upon a plate, and it was for many years regarded as the standard instrument for determining wind pressures. A vertical plate, 190 square inches in area, is held normal to the wind by mounting it upon a wind vane. The plate is forced forward by the wind against a series of springs which are arranged on its lee side, and the motion of the plate against the springs is communicated to a pencil which writes upon a travelling chart. The pressure scale of the chart is obtained by direct calibration of the springs.

It is found, however, that this instrument gives wind pressures in excess of the correct amounts. A strong gust of wind produces a rapid motion of the pressure plate against the springs, and the inertia of the plate, which is, of necessity, considerable, carries it beyond the point appropriate to the true maximum wind pressure.

§ (20) THE DINES TUBE ANEMOMETER.[1] (i.) *Introductory.*—The pressure-tube anemometer depends upon the principle that when an open tube is placed in a current of air there is produced inside the tube a pressure which differs from that outside, supposing the current stilled (commonly known as the " static " pressure) by an amount which is generally proportional to the square of the velocity of the current, and to the density of the air, *i.e.*

$$p - p_s = k\rho v^2,$$

where p is the pressure inside the tube, p_s is the static pressure outside the tube, v is the velocity of the current, ρ is the density of the air, and k is a constant depending upon the shape of the tube and the manner of presentation of its opening to the direction of motion of the current. In certain cases k will also depend upon the velocity of the current, in the sense that at certain critical velocities a rapid change in the value of k may take place, due to a change in the shape of the stream lines of the air in the neighbourhood of the opening.

---

[1] See also " Friction," § (11), Vol. I., dealing with the Pitot Tube.

The shape of opening which corresponds with the maximum value of k is that of an open tube lying in the direction of the current, the open end directly facing the current.

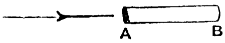


FIG. 13.

Thus if AB (*Fig.* 13) is a tube open at A and closed at B, and the current is blowing directly into the aperture, as shown by the arrow, then it has been shown experimentally that

$$p - p_s = \tfrac{1}{2}\rho v^2,$$

k in this case being $+\frac{1}{2}$. Since $\frac{1}{2}\rho v^2$ represents the kinetic energy of the current, this equation means that such a tube completely converts the kinetic energy of a horizontal pencil of current, of which the cross-section is equal to that of the tube, into potential energy represented by a correspondingly increased pressure inside the tube.

It has been found experimentally that there is a certain shape of opening which produces a value of k equal to zero. In this case $p = p_s$, and hence this shape enables us to make direct measurement of the static pressure of the current.

The shape referred to is shown in *Fig.* 14. AB is a tube closed at both ends, but open at C through a series of small holes. The current blows directly against the end A of the tube. The pressure inside the tube is then equal to the static pressure outside, and the arrangement is usually known as a "static head."

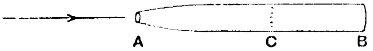


FIG. 14.

On the other hand, there are numerous shapes for which the value of k is negative. Common examples are the numerous forms of chimney cowls, ranging from a plain vertical pipe open at the upper end to the various complicated shapes associated with smoky chimneys where down draughts occur as the result of unfavourable eddy-motion over the chimney, due to obstructions in the neighbourhood. The particular shape of this kind which will be considered is that consisting of an annular space between two co-axial vertical tubes, which is connected to the outside air by rows of holes drilled horizontally round the circumference of the outer tube, as shown in *Fig.* 15. The current is supposed to blow horizontally past the tube. Provided the holes are drilled symmetrically round the tube, it is clear that the particular direction from which the wind blows is immaterial. Many experiments have been made at the National Physical Laboratory upon shapes of this type, and it has been shown that the value of k depends partly upon the number and size of holes in relation to the size of the outer tube and the width of the annular space. It also varies according to the presence or absence of a rain-shield, which is sometimes placed behind the holes to prevent rain water which has been driven through the holes from being carried down the annular space. For the standard shape of this kind the value of k is $-\frac{1}{4}$.

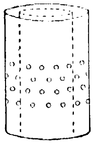


FIG. 15.

(ii.) *Description.*—The tube anemometer consists of three portions:

(1) The head and vane.
(2) The connecting pipes.
(3) The recorder.

The head and vane contain two of the shapes described above, arranged for correct presentation to the wind. One of the shapes is mounted on the vane, the other is in the "head" which also serves to support the vane. The two shapes used are (1) the horizontal tube of the vane, open at the end which is facing the wind, and (2) the shape last described, consisting of a vertical annular tube connected to the outside air by a series of small circular holes drilled into the outer wall of the tube. The whole head and vane is arranged as shown in *Fig.* 16. Since the vane keeps the former shape properly facing the wind, it is clear that this arrangement

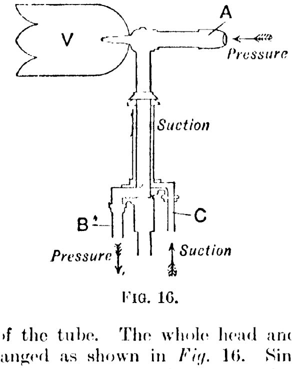


FIG. 16.

gives two pressures, each of which differs from the static pressure by small amounts which are proportional to the square of the velocity of the wind; the two values of the constant k being $+\frac{1}{2}$ and $-\frac{1}{4}$ respectively. The two openings are usually known as the pressure and suction openings, because the pressures produced by them are respectively greater and less than the static pressure.

The two connecting pipes are usually compo tubes which convey these air pressures from the head and vane to the recorder. They can be of considerable lengths, up to 100 feet or more if desired, and can follow any convenient route from the vane to the recorder. These properties of the connecting pipes constitute important advantages of the tube anemometer, for it becomes possible to mount the head and vane in a very exposed position, away from buildings or other local obstructions, by placing them at the top of a mast or light girder steel tower, while the recorder can be in a building or hut at the foot of the tower.

The recorder consists essentially of (1) a cylindrical tank containing liquid; (2) an inverted bell jar or "float" of sheet copper, which is placed in the liquid, and is capable of motion up and down under the influence of the "pressure" and "suction" conveyed by the compo pipes from the vane and head respectively to the inside and outside of the float; a vertical rod fixed to the top of the float passes through a moderately close-fitting collar in the cover of the tank, and carries an arm bearing a crow-quill pen; (3) a rotating clock drum, upon which the vertical movements of the float, as communicated to the pen by the vertical rod, are recorded. It is clear that the combined effect of the excess air pressure inside the float and the diminished pressure outside it, as communicated by the compo pipes, both tend to raise the float by an amount which will depend upon the difference between these pressures, and therefore upon the speed of the wind.

There are two kinds of recorder in common use, viz. (*a*) the original Dines pattern, made by R. W. Munro, Ltd., and (*b*) Halliwell's modification of that pattern, as supplied in Negretti and Zambra's "anemobiagraph." Sections of these two recorders are shown in *Figs.* 17 and 18. The most important consideration in the design of these recorders is the fact that, whereas a record of wind *speed* is required, the pressure difference between the inside and outside of the float is proportional to the *square of that speed*. Arrangements must therefore be made for the lift of the float above the position of equilibrium to be proportional to the square root of the difference of pressure in the two connecting tubes. This is secured in the original Dines model in a very elegant manner by suitably fashioning the shape of the internal surface of the float, as shown in *Fig.* 17. In this model the float is subjected to no external mechanical forces whatever. It can be shown that if x and y are the co-ordinates of any point upon a median section of the internal surface of the float, measured from an origin at the centre of the uppermost cross section of the float, the axis of x being drawn vertically downwards, and the axis of y being horizontal, then the equation to the median section is

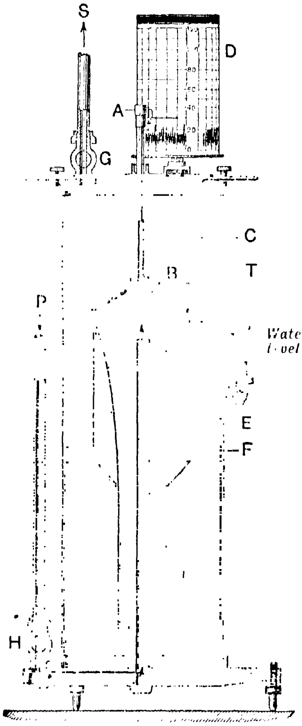


FIG. 17.

A, float rod, carrying pen; BF, float; E, inner surface of float; D, clock drum; P, pipe connecting to pressure; S, pipe connecting to suction.

$$y^4(\kappa + h - x) = \kappa c^4,$$

where κ is a constant, h is the constant depth of water in the tank outside the float, and c is the internal radius of the float at the cross section through the origin. The surface is, therefore, a quartic hyperboloid of revolution. It is assumed that when floating in equilibrium with the pipes disconnected the water level passes through the y axis, and that is arranged for in practice.

In the Dines recorder the liquid used in the tank is pure water, which has the advantage of being easy to replenish, and the disadvantage of being subject to freezing in cold weather, with disastrous results to the float.

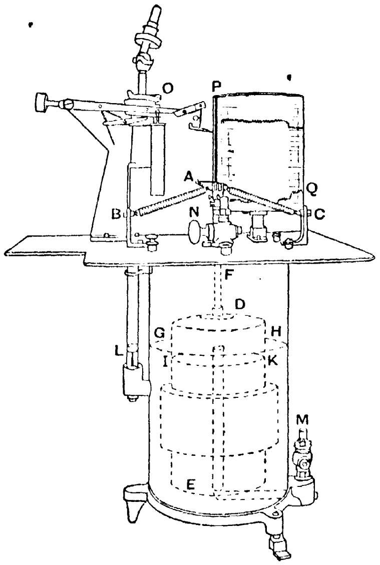


FIG. 18.

A, pen; AB, AC, springs controlling vertical motion of pen; DE, float; F, float-rod, carrying pen; GH, level of liquid outside float; IK, level of liquid inside float; L, level of liquid for zero velocity; M, connection for pressure pipe; N, connection for suction pipe; O, direction cam; PQ, clock drum.

In Halliwell's modification of the Dines recorder, an approximation to the same result is obtained by using a plain cylindrical float with a cylindrical protuberance round it to secure the necessary buoyancy, and constraining its motion in a vertical direction by two coiled springs mounted upon the cover of the tank. Each spring is attached to the cover at one end and to the vertical spindle at the other (*Fig.* 18). In the zero position of the float the springs are horizontal and under no tension, so that their effect is nil. At increasing velocities the float is raised, the springs are extended, and affect the motion of the float in two separate ways, (*a*) by the increase in tension due to extension of the springs, (*b*) by decrease of the angle between the direction of the springs and the float spindle. Each of these changes increases the vertical component of the force due to the springs, and it is clearly this component which is effective so far as the motion of the float is concerned. In this recorder the liquid used is a mixture of glycerine and water of a definite specific gravity. This mixture possesses the advantage of not freezing except at a very low temperature; it has the disadvantage of not being easily replaced when necessary. It is to be noted, however, that the velocity scale value of the recorder does not depend at all upon the particular liquid used—the function of the special liquid is to balance the standard float at the zero position when the level of liquid is the same inside as out.

It may, further, be noticed that in the Dines recorder the water level outside the float remains constant when the instrument is in operation, because the float is mechanically unconstrained in the vertical direction. On the other hand, the level of the liquid outside the float of the Halliwell recorder rises as the wind velocity increases, because of the action of the springs, and the scale value of the record is correspondingly contracted. In both instruments the level of the liquid inside the float falls as the velocity increases—it is evident that the difference in levels inside and outside the float is that corresponding with the pressure difference in the air spaces above the free surfaces of the liquid.

(iii.) *Calibration.*—The recorders must be calibrated to suit (*a*) the differences of pressures at different velocities between the two connecting tubes, due to the openings at the upper ends of these tubes at the vane and head of the anemometer, (*b*) the condition already mentioned, that the vertical motion of the float for any velocity should be proportional to that velocity, and not to the square of the velocity, to which the pressure difference is proportional, and (*c*) the velocity scale value of the chart. The relation between pressure difference and wind speed, given by the standard head and vane, is

$$W = \cdot 000731 V^2,$$

where W is pressure difference measured in inches of water, and V is the speed in miles per hour.

(iv.) *Comparison of Records from Cup and Tube Anemometers.*—Records of wind from tube anemometers are graphs showing wind velocity as ordinate and time as abscissa. The "structure" of the wind, *i.e.* the irregular succession of gusts and lulls, is also shown, and the record appears as a more or less broad band running across the sheet, indicating at a glance the velocity limits between which the wind has oscillated. A reproduction of a record from a Dines anemometer is given in *Fig.* 19.

If R = run of wind as measured by the Robinson anemometer, expressed, for example, in miles, and t = time expressed in hours, the record of the Robinson - Beckley instrument is a relation between R and t.

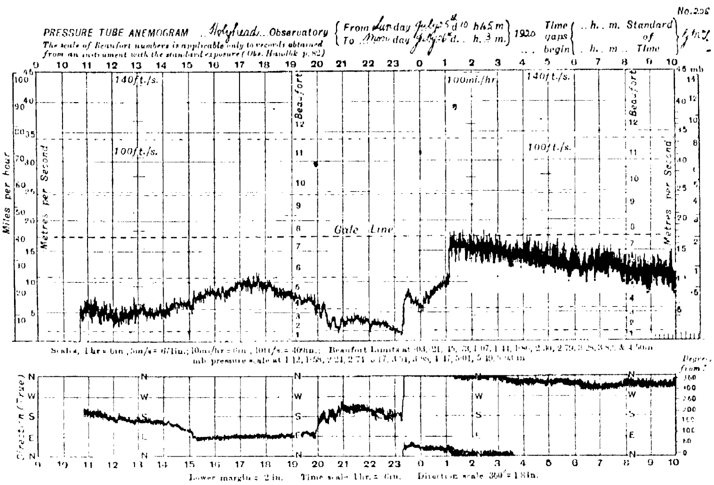


FIG. 19.

Now if v is wind velocity, it is clear that

$$v = \frac{dR}{dt} = k \cot \theta$$

in the notation used above, § (17) (vi.). Hence, theoretically, the record from the tube anemometer is a graph of the first differential coefficient with respect to time of the Robinson record. Actually the median line of the broad band of the tube anemometer record is a close approximation to this theoretical result. The Robinson-Beckley instrument is ineffective for the study of wind structure.

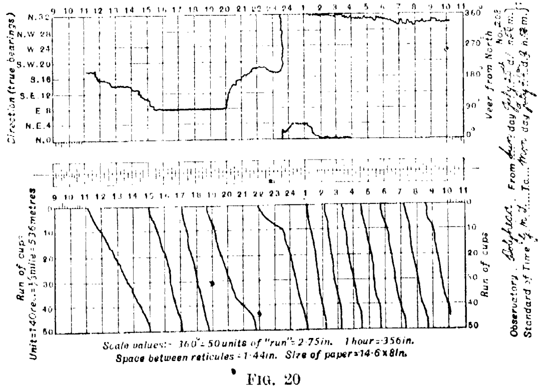


FIG. 20

Figs. 19 and 20 are reproductions of simultaneous records obtained at Holyhead from the pressure-tube anemometer and the cup anemometer there, which bear out these remarks. Wind-direction is also recorded by each instrument.

(B) *Wind Direction*

§ (21) WIND-VANES.—A wind-vane is an extremely simple instrument, but if it is to be used for scientific purposes, there are a few points to which careful attention must be paid. Thus

(*a*) The vane must turn upon its pivot with as little friction as possible.

(*b*) It must be properly balanced about its pivot, especially in the fore-and-aft direction.

(*c*) If it has to control an instrument for recording wind direction it should be designed so as to produce the maximum torque, in relation to its weight, for a given change of wind direction.

Condition (*a*) is secured by supporting the vane upon a hardened steel cup and a point or ball-bearing, arranged at the highest point of the fixed vertical support carrying the vane. The vertical sleeve attached to the vane, which serves to prevent it from falling from position on its support, should be a loose fit over that support.

Condition (*b*) is most important, for if a proper balance is not secured the vane acquires a permanent bias towards one direction of the compass. For in practice it is impossible to maintain the support in an absolutely vertical position, and it slopes slightly forward, say, for example, towards the north. Then the vane will always tend to point towards the north or south according as the arrow end of the vane is too heavy or too light in comparison with the fin end of the vane, for the weight of the vane has a definite moment about the sloping axis of rotation in every other position.

Condition (*c*) has received attention at various times. One of the methods of securing greater torque and steadiness which is in fairly common use is to use a splayed vane; that is, the vane consists of two blades inclined at an angle of about 30° to each other, and arranged symmetrically about the direction of the arrow-head of the vane. While such an arrangement increases the torque, it is doubtful if greater steadiness is secured, for it is known that the conditions of flow behind such a vane will be unsteady because there will be a considerable wake behind the pocket of the splay which will consist of two series of eddies rotating in opposite directions (*Fig.* 21).

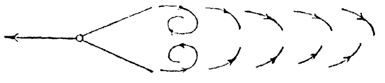


FIG. 21.—Eddies in Wake of Splayed Vane.

A new vane has recently been designed in the Royal Aircraft Establishment, South Farnborough, which represents a considerable advance upon earlier models. The fin is entirely redesigned, its longest dimension being now vertical instead of horizontal, and it is situated at a considerable distance from the axis of rotation. In *Fig.* 22 BC is the fin, A is a balancing weight, so that the centre of gravity passes through D where the hardened steel bearing is placed. EF is the loose sleeve, and G is the support upon which the vane turns. H is a cross-section of the fin BC; it is of aerofoil shape to secure steadiness in the wind. According to tests at Farnborough the torque or turning moment of this vane is about eight times as great as that of an ordinary vane of equal weight, and recent experience of the vane, as adapted to wind direction recorders, shows that it is extremely sensitive to changes of direction even when the wind is light and when the vane has to control the recorder some 40 feet below it.

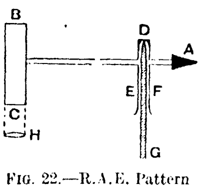


FIG. 22.—R.A.E. Pattern Wind-vane.

§ (22) WIND-DIRECTION RECORDERS. (i.) *Introductory.*—A large number of instruments have been designed to record the direction of the wind, and at first sight the problem appears to be quite a simple one. It is an easy matter to construct a wind-vane which will always indicate wind direction, and all that remains is to devise means of causing the vane to record its motion upon a suitable chart which is moved uniformly by clockwork. Difficulties, however, occur in the design of the recording apparatus, owing to the fact that the most convenient form of wind-direction chart shows all the points of the compass arranged in sequence from north through east, south, and west to north again. There are thus two north lines, one at the top and the other at the bottom of the chart, and provision is required for a discontinuity in the record at the north point as the vane passes continuously through north. Thus in *Fig.* 23 the diagram of rectangles represents the usual form of wind-direction chart, the cardinal points being shown by horizontal lines and the hours of the day by vertical lines. A hypothetical record of wind direction starts at 9 h. (*Fig.* 23), with the wind between S. and W. The wind veers until, at about 10 h. 20 m., the direction is north. The veer continues and the record reappears at the top of the chart, until at 12 h. the direction is south. The figure represents matters as being much simpler than they are in reality, for in fact every wind-vane is continually oscillating, except when there is a calm, so that an actual record consists of a ragged band representing oscillations about the mean direction. When the mean direction is north the record consists of two parts, one at the top, the other at the bottom of the sheet, corresponding with the two parts of the oscillation on either side of north. Reproductions of actual records are shown in *Figs.* 19, 20.

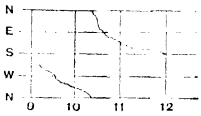


FIG. 23.

It will be sufficient to mention some of the methods adopted for producing this result, as exemplified by the Baxendell recorder, the Beckley recorder, the Casella recorder, the Munro-Rooker recorder, the Dines recorder, and the Halliwell recorder.

(ii.) *The Baxendell Recorder.*—This instrument, designed by Mr. Joseph Baxendell, meteorologist to the Southport Corporation, is described in the *Quarterly Journal of the Royal Meteorological Society*, 1899, vol. xxv. p. 326. The drum upon which the chart is fixed has its axis vertical and is rotated by a spindle which turns with the vane. The chart is placed on the drum with the hour lines horizontal and the direction lines vertical, the two north lines on the chart falling one exactly above the other at the overlap. The recording pen is moved by clockwork at a uniform rate along a fixed vertical line.

This is a very simple and effective method of meeting the difficulty, but it is impossible to combine *on the same chart as the direction record* a record of wind velocity.

(iii.) *The Beckley Recorder.*—In this instrument, which has stood the test of time at the British observatories since 1867, the difficulty is surmounted in a simple and ingenious way. The drum carrying the chart is horizontal and is rotated by a clock, so that the hour lines are parallel to the axis of the drum, and the lines showing cardinal points become circles surrounding the drum. Above the drum is mounted a smaller solid brass cylinder of which the axis is parallel to that of the drum. The cylinder is connected with the wind-vane in such a way that it makes a complete revolution for one complete revolution of the vane. Upon the cylinder is coiled one complete turn of a thin metal helix, of such a pitch that the distance from one turn to the next, measured parallel to the axis of the cylinder, is equal to the distance between the two north lines of the chart. The chart is printed on specially prepared "metallic" paper, and is marked by pressure of the helix due to the weight of the cylinder upon which it is mounted. When adjusted so that the ends of the helix are vertically above the two north lines it is clear that this arrangement will give a record of wind direction of the required type. The chief objections to this instrument are—(1) the record has a somewhat blurred appearance, and (2) considerable friction must necessarily occur between the helix and the paper—otherwise the record would be invisible—and to overcome this the vane has to be of the wind-mill type. Such a vane is very heavy and sluggish and subject to backlash, and is therefore not adapted to meet modern demands. This instrument has, however, given results of the greatest value, and owing to its substantial construction it is probable that modern instruments will be unable to compete with it on the score of durability.

The instrument, with which is combined a velocity recorder of the cup type, so that the two records appear on the same sheet, is fully described in the *Report of the Meteorological Committee for 1867.*

A reproduction of a specimen record is shown (*Fig.* 20).

(iv.) *The Casella Recorder.*—In this instrument an endless cord passes tightly round two equal pulleys in the same plane. One of the pulleys is geared to the vertical spindle of the vane, so that corresponding angles turned through by vane and pulley are equal. The length of the cord is exactly three times the circumference of each pulley, and three pencils are mounted upon the cord at equal distances from one another. The width of the chart from north to north being equal to the circumference of each pulley, it is seen that a true record of wind direction is obtained by mounting the pulleys with cord and pencils in front of the chart, so that the axes of the pulleys are parallel to the two north lines in the two planes passing through those lines and perpendicular to the plane of the chart. The pencils are adjusted so that they touch the paper when passing between the pulleys on the side next the paper. It will be seen that if P_1, P_2, and P_3 are the pencils (*Fig.* 24), P_1 will leave the chart at the top of the sheet as P_2 comes on at the bottom, and conversely.

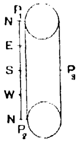


FIG. 24.

This design is interesting and simple, but the adjustment of the pencils is not very easy, and it is impossible to use pens which will give an ink record.

The newer recorders are based upon the principle of the Beckley recorder, in that helices of different kinds are used to convert angular motion into linear motion parallel to the axis of the angular motion.

In order to overcome the blurring of the record as produced by the Beckley recorder, due to contact of the helix itself with the chart, the helix on the newer instruments acts as a cam to move a pen up and down the chart. By mounting the pen at the end of a pivoted arm which is controlled by the cam at a point between the pen and the pivot, a magnification of the motion of the helix is produced. In these instruments the difficulty referred to at the beginning of this section is much felt, and different devices for overcoming it are adopted, as now to be described.

(v.) *The Munro-Rooker Recorder.*—In this instrument the helix does not form quite a complete turn, about 3° of the angular motion round about the north points of the record being replaced by a steep cam connecting the upper and lower ends of the helix. The helix and the connecting cam are cut in the form of a channel into the surface of a cylinder, and a roller mounted upon a short stud projecting from the side of the pivoted arm referred to above runs in the channel. At the north point the arm, which is balanced about its pivot, is rapidly forced by the connecting cam from top to bottom of the chart, or conversely. A simple arrangement is provided for lifting the pen from the chart when the roller is travelling up or down the connecting cam.

(vi.) *The Dines Recorder.*—Two pens, two helices, and two pivoted arms are used. Both pens are slightly out of balance, one so that it falls to the lower north line from the upper north line of the chart, when unconstrained by the helix, the other so that it is carried up from the lower to the upper north line in similar circumstances. One of the two pens is always at rest in its unconstrained position, and makes a "zero" line on either the lower or upper north line. The use of two pens is a disadvantage from the operational point of view, but an advantage from the point of view of the finished record, because of the provision of a "zero" line which is independent of the ruling of the chart (see *Fig.* 19).

(vii.) *The Halliwell Recorder.*—The chart of this instrument shows a complete revolution and a half of wind direction; that is, starting from north at the top of the chart, we pass successively through east, south, west, north, east to south at the bottom. The helix, which, as in the Munro-Rooker instrument, is in the form of a channel, consists of a complete turn and a half to match the chart, and is provided with two vertical channels on opposite sides of the cylinder, connecting respectively the two points on the helix corresponding with north, and the two points corresponding with south. Arrangements are provided for the roller on the pen arm to drop down the former channel when it reaches the top of the helix, and to be forced up the latter when it reaches the bottom. In the new position there is ample space on the chart for any oscillations about north or south, without fear of the pen reaching the edge of the chart again. This instrument has the advantage of using only one pen.

The last three types of instrument are easily adapted to work in conjunction with a tube anemometer so that the combined instruments may give records of wind speed and of wind direction on the same chart with a common time-scale.

§ (23) EXPOSURE OF ANEMOMETERS.—The question of exposure is extremely important in anemometry. An anemometer can only measure the wind speed or wind direction which it actually experiences, consequently it is necessary to ensure that it is subject to a fair sample of the "wind conditions" of the immediate locality. Anemometers fixed on the ground are intended to measure the wind at the surface, but it is impossible to take this expression too literally, for at the ground itself the speed must be zero and it increases rapidly with height. Apart from that difficulty, however, the measurement of wind near the ground is usually subject to effects of neighbouring objects which cause local eddies. In anemometry the local eddy is especially to be avoided, consequently it is desirable that the instrument be exposed from 30 to 40 feet above the top of the highest neighbouring obstruction. If possible, a flat stretch of country should be selected as site for an anemometer, and the instrument should be exposed at the top of an open-work tower or of a pole 30 to 40 feet high. If an anemometer is set up in a place surrounded by trees it is found that an exposure 30 feet above the tree-tops is satisfactory, but it is to be observed that the eddies caused by tree-tops are likely to be much greater than those due to a flat ground surface, and are therefore likely to extend to a greater height above the tree-tops in the former case than above the ground in the latter.

V. INSTRUMENTS FOR RECORDING THE DURATION OF SUNSHINE

There are two types of sunshine recorder which have been in general use in this country, viz. the Campbell-Stokes sunshine recorder and the Jordan sunshine recorder. In neither instrument is there a clock, for advantage is taken of the apparent motion of the sun to indicate time of day upon the records.

§ (24) CAMPBELL-STOKES SUNSHINE RECORDER. (i.) *Description.*—In the Campbell-Stokes sunshine recorder (*Fig.* 25) a homogeneous glass sphere, 3 inches in diameter is set concentrically within a brass bowl which forms part of a spherical shell. The bowl supports the card upon which the record is obtained in such a position that the sun's rays are always focussed upon it. A charred black mark is produced where the sun's image meets the card when the intensity of solar radiation is sufficiently strong. When the recorder is in correct adjustment the image of the sun passes along the card during the course of the day, parallel to its edges. Owing

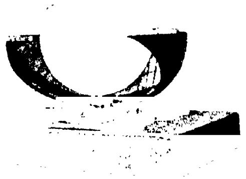


FIG. 25.

to the apparent motion of the sun in declination during the year, the image of the sun is higher upon the bowl in winter than in summer, while at the equinoxes when the sun is in the celestial equator its image takes up a position midway between the extreme positions at the solstices. The planes through these extreme positions and the centre of the bowl make angles of 23½° with the equinoctial plane or celestial equator.

It is therefore found convenient to provide three overlapping slots to take three different kinds of card which are used at different seasons of the year, as follows:

Description of Card.	Position in Bowl.	Period of Use.
Long curved or summer	Bottom slot	13th April to 31st Aug.
Short curved or winter	Top slot	13th Oct. to end of Feb.
Medium, straight, or equinoctial	Middle slot	1st March to 12th April and 1st Sept. to 12th Oct.

(ii.) *The Adjustments of the Sunshine Recorder.* Consideration of the matter will confirm the following conditions to which a recorder which is in accurate adjustment must conform:

(*a*) The centres of the glass sphere and of the bowl must be coincident.

(*b*) The plane containing the central longitudinal line of the equinoctial card, when in position in the recorder, must pass through the centre of the glass sphere, or of the bowl, and must coincide with the celestial equator.

(*c*) The vertical plane through the centre of the sphere which passes symmetrically through the bowl must coincide with the geographical meridian.

(*d*) The principal focal length of the glass sphere for heat rays must be equal to the radius of the bowl measured to the surface of the card when in position.

(*e*) When a card is in position the hour lines printed transversely across it must lie in meridians of the celestial sphere corresponding with hour angles of 15°, 30°, 45°, 60°, etc., measured from the geographical meridian. The linear distance between consecutive hour lines on the equinoctial card is therefore

$$\frac{2\pi}{24} \cdot a = \frac{\pi a}{12},$$

where a is the radius of the bowl measured to the middle line of the equinoctial card. In particular, it follows that the line joining the points of intersection with the central longitudinal line of the card of the six-hour and eighteen-hour lines on the equinoctial card passes through the centre of the bowl, and is bisected at that point.

Full information regarding methods of setting sunshine recorders is given in the *Observer's Handbook* of the Meteorological Office. They depend necessarily upon the adjustment of the recorder and fixing it to comply with the conditions (*a*), (*b*) and (*c*) above. Conditions (*d*) and (*e*) are to be observed by the maker of the instrument and of the record cards.

(iii.) *Effects of Errors of Adjustment or Manufacture.*—It is important to determine in what manner and to what extent errors of adjustment or construction of the instrument affect the records obtained, and in what follows each of the possible errors will be considered in turn.

(*a*) *Error of Concentricity.*—There are two cases for consideration: (α) Centre of sphere situated in the plane through the celestial equator, but displaced from its correct position in that plane.

(β) Centre of sphere in the line joining the centre of bowl to the celestial pole, but displaced from its correct position in that line. Any other case can be treated as a combination of these two.

In case (α) the error in recorded duration varies in different parts of the scale. The general formula for the proportionate error at hour angle h, measured from south, is

$$\frac{x}{a} \cos \delta \cos (\theta - h),$$

where x is the (small) displacement of the centre in the equatorial plane, θ is the angle between the direction of the displacement in that plane and the line drawn to the south, δ is the sun's declination, and a is the radius of the bowl.

The error over an interval from hour angle h_1 to hour angle h_2 will be

$$\frac{24x}{\pi a} \cos \delta \, . \int_{h_1}^{h_2} \cos (\theta - h) \, . \, dh \text{ hours},$$

$$\text{or} \quad \frac{48x}{\pi a} \cos \delta \cos \left(\theta - \frac{h_1 + h_2}{2}\right) \sin \frac{h_2 - h_1}{2} \text{ hours}.$$

The burn will be in correct position at the equinoxes, but it will be slightly displaced and not parallel to the central white line of the card at other times.

It follows from the above formula that the error is numerically greatest in those parts of the scale where the line joining the displaced centre to the true centre meets the card, and zero in the direction at right angles thereto. The recorded duration will be exaggerated on the whole if the displaced centre is south of the true centre, and it will be reduced if the displaced centre is north of the true centre. Thus a displacement of 0·1 inch of the centre towards south will increase a twelve-hour sunshine record at the equinoxes by ·53 hour, since a is 2·86 inches in the standard instrument.

In case (β), where the displacement of the centre is along the polar axis, the proportionate error is nil upon the equinoctial card, and it will be $+\{x \sin \alpha / a \cos (\delta - \alpha)\}$ on the summer card and $-\{x \sin \alpha / a \cos (\delta - \alpha)\}$ on the winter card, where x is the amount of the displacement measured upwards, and α is the semi-vertical angle of the cones of which the summer and winter cards, when in position, are frusta. For the standard instrument, $\alpha = 16^\circ$, so that the effect of a displacement of 0·1 inch of the centre along the polar axis introduces an error per hour of summer or winter record of

$$\frac{0 \cdot 1}{2 \cdot 86} \frac{\sin \alpha}{\cos (\delta - \alpha)},$$

which is numerically about ⅔ minute for all possible values of δ. In case (β) the burn remains parallel to the edges of the card, but it is displaced parallel to itself by a distance x on the equinoctial card, and a distance $x \cos \delta \sec (\alpha - \delta)$ on the summer and winter cards; so that there is a risk of records being lost owing to burns falling on the frame.

(*b*) *Latitude and Level Errors.*—If the plane passing through the longitudinal median line (central white line) of the equinoctial card and the centre of the bowl does not coincide with the celestial equator, the apparent displacement of the pole of the plane can be resolved into two parts: one, $d\lambda$, along the geographical meridian; the other, di, at right angles to the meridian. It can be shown that the corresponding errors, dh_1 and dh_2, in the times indicated by the recorder, expressed in angular measure at the rate of 1° to 4 minutes of time, are given by

$$dh_1 = d\lambda \,.\, \tan \delta \sin h,$$

$$dh_2 = di \,(\cos \lambda + \sin \lambda \tan \delta \cos h),$$

where h is hour angle measured from noon, δ is sun's declination, λ is latitude, and $d\lambda$ and di are so small that their squares and higher powers can be neglected. dh_1 vanishes at noon and also at the equinoxes. It has numerical maxima at sunrise or sunset and at the solstices. Thus if $d\lambda = 2^\circ$, $\delta = 23\frac{1}{2}^\circ$, $h = 90^\circ$ (*i.e.* 6 A.M. or 6 P.M. local time), dh_1 is nearly equivalent to 4 minutes of time, which is the error in the recorded duration from noon to 6 P.M.

For a given latitude, dh_2 has numerical minima at the equinoxes and at 6 A.M. and 6 P.M., and maxima at noon and at the solstices. If $di = 2^\circ$, $\lambda = 55^\circ$, $\delta = 23\frac{1}{2}^\circ$, and $h = 0^\circ$ (*i.e.* at noon), dh_2 is equivalent to 7¼ minutes of time. The recorded duration of continuous sunshine from 6 A.M. to 6 P.M. will not be affected by this error.

The error $d\lambda$ affects the curvature of the record, the burns appearing slightly curved upon the equinoctial card and not parallel to the edges of the summer and winter cards.

The error di does not affect appreciably the curvature of the records, but it causes a burn to cross its true position at an angle equal to $di \sin \lambda \sec \delta$. When $di = 2^\circ$, $\lambda = 55^\circ$, and $\delta = 0^\circ$, this angle is 1·6°.

In both cases there is risk of the burns passing off the edges of the card, resulting in loss of record.

(*c*) *Meridian Error.* If the vertical plane through the centre of the sphere which passes symmetrically through the bowl does not coincide with the geographical meridian, but makes a small angle $d\phi$ with it, then the recorded duration is correct, but the burns are not parallel to the edges of the card. The angle between the actual and true course of a burn is $d\phi \cos \lambda \sec \delta$. If $d\phi = 2^\circ$, $\lambda = 55^\circ$, and $\delta = 23\frac{1}{2}^\circ$, this angle is 1¼°. The indicated time is incorrect by an amount equal to $(12 \,.\, d\phi)/\pi$ hours. When $d\phi = 2^\circ$, this is 8 minutes of time. As before, there is a risk of loss of record, especially near the times when a new type of card is taken into use.

(*d*) *Error in Focal Length.*—If the principal focal length of the glass sphere for heat rays is not equal to the radius of the bowl, the image of the sun will be imperfectly focussed upon the card, and the resulting burn will be thicker than is necessary. Further, the focussed heat will be distributed over the enlarged image, and when radiation is feeble, as near sunrise and sunset, some record will, in consequence, be lost.

(*e*) *Errors in Record Cards.*—Errors of printing the cards, so that the scale value is incorrectly adjusted to suit the diameter of the bowl, as required by condition (*e*), are obviously important. A contracted scale value will lead to excessive records of duration of sunshine. Similarly, a bowl of which the diameter is greater than the standard, if used with standard cards, will indicate too much sunshine, and *vice versa*. The recorded duration in these cases bears to the true duration the same ratio as the actual diameter of the bowl bears to the diameter of that bowl which is appropriate to the card, as defined under paragraph (*e*) above.

(iv.) *The Record Cards.*—The cards upon which the records are obtained should be made of a standard substance, in order that different records may be comparable. The surface of the card must be printed in a colour which absorbs heat radiation; a white surface is ineffective, because it reflects most of the heat. It appears from experiments made that there is little, if any, difference between records obtained from cards of different colours. Black would be the best colour for the purpose, but it would be inconvenient because the burns are necessarily black. Prussian blue has accordingly been adopted for use in the British Isles, and, generally, in the British dominions and colonies. This colour gives a good contrast with black, and it absorbs freely the red and infra-red rays which are principally concerned. A grey tint is used in the Netherlands.

It will be seen from the above discussion that the sunshine recorder is an instrument which is liable to a large number of errors, some of which may be due to the instrument itself, or the cards used with it, while the remainder are due to faulty adjustment.

(v.) *The measurement of the records* produced by the Campbell-Stokes sunshine recorder is not free from difficulty. The image of the sun which is produced upon the card by the glass sphere is not "sharp," but has a certain area with an indefinite edge, and the charring which takes place when the sun shines occurs throughout this area and also spreads a little around it, especially when radiation is intense. Thus the length of the burn, as indicated by the time scale of the card, will be slightly in excess of the actual duration of sunshine, especially when the record is made up of a number of short bursts of strong sunshine separated by intervals of heavy cloud. Some allowance is therefore made for the "spreading of the burn" when measuring the records. On the other hand, at sunrise and sunset the radiation is always feeble because of the absorption in the long path of atmosphere through which it has then to pass, and the area of the burn is reduced to unusually small dimensions in the centre of the image. In this case it is usual to measure to the extreme limit of the burn, "as far as it can fairly be seen." The *Observer's Handbook* should be consulted for further particulars.

§ (25) The Jordan Sunshine Recorder.—This instrument consists essentially of a metal camera with a slit through which the sun's rays are admitted. The wall of the camera is cylindrical, and the record is obtained on a piece of light-sensitive paper of the ferro-prussiate type, which is arranged around the inner cylindrical wall, in such a position that the sun's rays pass along it throughout the day. In a later form there are two cameras and two slits, one for the morning, the other for the afternoon record.

This instrument is not recommended officially, but it is interesting as depending upon the actinic rays of the spectrum (violet and ultra-violet), whereas the Campbell-Stokes recorder uses the heat rays in the red and infra-red portion of the spectrum.

§ (26) The Night-sky Recorder.[1]—The sunshine recorder, by giving information as to the duration of sunshine, is useful also as indicating the amount of cloud during the day hours. The corresponding indication during the night hours is afforded by the use of a camera which is fixed with the axis of the lens pointing to the celestial pole. The shutter being opened at night when the sun is 10° below the horizon, and closed in the morning when the sun is again 10° below the horizon, an impression is left on the plate of the images of Polaris and δ Ursae Minoris and other neighbouring stars for such time as they are not obscured by cloud. The images are, of course, in the form of circles centred at the image of the celestial pole. This instrument is now in continuous operation at Greenwich Observatory, and the measurements of the time during which the two stars mentioned are recorded as shining are published each day in the *Daily Weather Report.*

VI. Instruments for measuring Solar Radiation [2]

A measurement of the duration of sunshine is obtained by the Campbell-Stokes sunshine recorder, but this instrument gives practically no indication of the *intensity* of the solar radiation received throughout the day or from one day to another. The intensity of solar radiation is a matter of prime concern, not only to meteorologists, but to the whole human race, yet there are very few instruments which can claim to measure this quantity with any accuracy. Such instruments are usually somewhat easily deranged and also expensive.

§ (27) Black-bulb *in Vacuo.*—The commonest instrument used in this connection is the black-bulb thermometer *in vacuo.* This consists of a mercurial maximum thermometer with its bulb coated with lampblack, mounted inside a glass protecting sheath from which the air has been exhausted. The instrument is exposed in the open to the sun's rays, which are readily absorbed by the lampblack, and converted into heat, which is indicated by a high reading of the thermometer. The purpose of the sheath is twofold: first, to protect the lampblack from the weather; and secondly, to help provide an insulating vacuum which reduces the conduction of heat from the thermometer by wind or rain. It is usual to assume that the difference between the day's maxima of temperature of the black-bulbed thermometer and of a thermometer in the screen is a measure of the maximum solar radiation for the day, and this general statement is probably an approximation to the truth, provided that all black-bulb temperatures are obtained from a single thermometer. The difficulty in the use of this method is that readings of different black-bulb thermometers, when exposed side by side, may differ considerably from one another, so that the measure of radiation obtained in this way is expressed in different units according to the particular black-bulb thermometer in use. The variation appears to be due to slight differences in the dimensions of the thermometers, in the quality of the vacuum, and in the character and thickness of the black coating. Considerable numbers of these thermometers are in use.

§ (28) Ångström Compensating Pyrheliometer.—The instrument for the measurement of solar radiation, which has been adopted as standard by the International Meteorological Congress (Innsbruck, 1905), and by the International Union for Co-operation in Solar Research (Oxford, 1905), is the Ångström pyrheliometer. In this instrument solar radiation is received alternately on two thin strips of metal, coated with black. To the back of each strip is attached a sensitive thermo-junction, and the two junctions are connected together through a sensitive galvanometer. The junctions are electrically insulated from the strips, but are placed so close to them that they take up their temperatures. An electric current can be passed through either strip as desired. To make an observation, one of the strips is exposed to solar radiation, and a current is passed through the other (which is then shielded from radiation), the current being adjusted so that the temperatures of the strips are equal, as indicated by absence of deflection of the galvanometer. The current is varied by means of a rheostat and measured by a milliampèremeter. Assuming that the solar energy received by the exposed strip is equal to the electrical energy communicated to the other, it is easy to determine the former in terms of the measured strength of the current and the constants of the strips (viz. dimensions, absorbing power of blackened surface, and resistance per unit length).

---

[1] *Q.J.R. Met. Soc.* xlvi. 243.

[2] See also "Radiant Heat and its Spectrum Distribution," "Radiation," "Radiation, Measurement of Solar, etc."

§ (29) Other Absolute Pyrheliometers.[1]—Abbott and Fowle have also constructed absolute pyrheliometers of different designs, which have been employed in a number of important investigations.

§ (30) Silver-disc and Michelson Pyrheliometers.—Accurate, but not absolute instruments for determining the intensity of solar radiation are the silver-disc pyrheliometer and the Michelson pyrheliometer. In both of these instruments solar radiation is absorbed by a receiving surface, and the increase in temperature of that surface is suitably measured. These instruments need to be standardised by comparison with absolute instruments before the readings can be expressed in absolute units.

§ (31) Callendar Radiation Recorder.—As usually constructed this instrument gives a continuous record, with the assistance of a Callendar electric recorder, of the vertical component of radiation received from sun and sky. The receiver consists of two resistance wires coiled on a horizontal mica frame placed inside a small glass bulb, one wire being bright and the other black. The recorder measures and records simply the difference of temperature between the coils. Each receiver is calibrated before issue, so that the chart upon which the record is obtained can be ruled to show absolute values in watts per square centimetre.

For information regarding other instruments, past and present, for measuring solar radiation, a paper by R. S. Whipple, in the *Transactions of the Optical Society*, London, 1915, should be consulted.

§ (32) Ether Differential Radiometer.—This is an instrument recently designed by W. H. Dines, F.R.S.[2] The purpose of the instrument is to determine the radiation from the sky by finding the temperature of a full radiator which produces the same radiation effect upon the instrument as the actual radiation from the sky. The instrument consists of an ether differential thermometer, one bulb of which is exposed to the sky, and the other to a source of heat or cold, which is radiating fully, of which the temperature is measured. When a balance is secured the temperature of the radiating source is the "equivalent radiative temperature" of the sky. At night the mean radiative temperature is stated to vary in England from about − 20° F. in winter to about + 15° F. in summer on clear nights. In order to overcome the practical difficulty of providing radiators at these low temperatures there is an arrangement whereby the bulb, which is normally exposed to the sky, is only partially so exposed, the remainder of the bulb receiving radiation from a warm source which is provided. The combination of cold and warm radiations upon this bulb can be balanced by a radiation upon the other bulb which is derived from a source of intermediate temperature. The apparatus is arranged so that the calculation of the equivalent radiative temperature of the sky can be performed without difficulty with the aid of tables.

VII. Instruments for determining the Motion of Clouds

§ (33) Nephoscopes.—Information regarding the direction of motion and speed of clouds is regularly obtained at meteorological observatories, and to secure this some form of nephoscope is used. Nephoscopes are of two kinds—(*a*) direct vision, and (*b*) reflection instruments. In the former the observer watches directly a definite cloud, choosing the cloud so that he can conveniently interpose the indicating part of the nephoscope between his eye and the cloud; in the latter a mirror is ruled and divided to provide the indicating part of the instrument, and the observer watches the *image* of a cloud pass across the mirror.

(i.) *Besson and Fineman Nephoscopes.*—In this country two types of nephoscope are in general use, one of each of the above kinds. They are the Besson comb nephoscope (direct-vision instrument) and the Fineman nephoscope (reflection instrument). For details *The Observers' Handbook*, published by H.M. Stationery Office, should be consulted.

---

[1] See also "Radiant Heat and its Spectrum Distribution."

[2] *Q.J.R. Met. Soc.* xlvi. 399.

Nephoscopes enable an observer to determine the direction of motion of the cloud, but no nephoscope can of itself permit an observer to determine the absolute speed of the cloud; all that is possible in the absence of further information is a determination of the ratio between the speed and the height of the cloud, *i.e.* the angular velocity of the cloud about a point on the ground vertically below it.

(ii.) *Principle of Nephoscopes.*—The principle of the nephoscopes is the same, and may be briefly stated thus. Five points in space are considered (*Fig.* 26); two (A and B) represent the two positions at the same height of the cloud under observation at the beginning and end of a convenient interval of time, a third (C) is a fixed point, while the fourth and fifth (D and E) are points in a horizontal plane near C, and at a known vertical distance from C, lying in the two lines CA and CB. Since AB is also horizontal it is clear that DE is parallel to AB. Further, if h is the height of the cloud above C, and h_0 is the vertical distance of the horizontal plane containing D and E from C, then

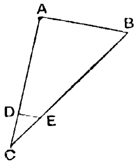


Fig. 26.

$$\frac{AB}{h} = \frac{DE}{h_0}.$$

Thus DE is the direction of motion of the cloud, and since the length AB, in conjunction with the time taken by the cloud to pass from A to B, measures the speed of the cloud, the above equation gives the speed-height-ratio in terms of the speed along DE and the known quantity h_0. DE is in the indicating part of the instrument, and the direction of DE, and the speed along DE, are both measureable with the help of the instrument and a stop-watch. It will be seen that if h, the height of the cloud, is otherwise known, the absolute value of the speed of the cloud can be determined.

§ (34) Camera Obscura.—A simple and effective instrument for observing clouds in the zenith is provided by a "camera obscura," with a revolving ground glass or screen placed in the focal plane of the lens. The ground glass should be ruled with a number of equidistant parallel lines. An ordinary camera serves quite well for the purpose, if fixed with lens axis vertical, the lens being above the ground glass. The camera is focussed for "infinity" and the ground glass is rotated until the image of the cloud appears to move parallel to the lines ruled on the glass. The direction of the lines gives at once the direction of the cloud motion. The speed-height-ratio can be determined by noting the time required by the image to traverse a distance on the ground glass equal to a convenient fraction (say $1/n$) of the focal length of the lens. If that time is t seconds, then the speed-height-ratio is $1/nt$ sec.$^{-1}$, *i.e.* $1000/nt$ milliradians per second.

§ (35) Darwin-Hill Mirror.—Another instrument of this type is the Darwin-Hill mirror,[1] which is a sheet of plate-glass, 50 cm. square, silvered on the back, which is fixed in a horizontal position and ruled in centimetre squares. A movable eyepiece is provided 10 cm. above the surface of the glass, through which the reflection of a cloud can be viewed. The eyepiece being set vertically above a definite intersection of the lines on the mirror, the time is noted, and by means of a stylographic pen the course of the cloud, as defined by the intersection at the mirror-surface of the line of sight from the eye to the cloud-image, is marked upon the mirror. Equal intervals of time are shown by cross-marks upon the course as drawn on the mirror, and in this way the co-ordinates x and y of the apparent position of the cloud on the mirror, referred to the horizontal axes on the mirror passing through the point vertically below the eyepiece, can be written down in centimetres. If H km. is the height of the cloud, and X and Y km. are its actual horizontal distances from the vertical at the point of observation, measured parallel to the axes defined above, then it is clear that

$$H : X : Y = 10 : x : y.$$

Hence the direction of the marked course on the glass is parallel to that of the cloud, and the rate of progress upon the marked course is to the speed of the cloud as 10 is to H. Two such mirrors can be used at the ends of a measured base line to observe the same cloud, and a comparison of the co-ordinates obtained on the mirrors for simultaneous observations of the cloud will enable a determination of the height of the cloud to be made. For, using suffixes 1 and 2 to denote corresponding quantities measured from the two mirrors, we have

$$H : X_1 = 10 : x_1,$$

and $$H : X_2 = 10 : x_2.$$

Thus $$H : X_1 - X_2 = 10 : x_1 - x_2.$$

If the axis of x be chosen to be in the direction of the line joining the mirrors, then $X_1 - X_2 = D$, the length of the measured base line in kilometres, thus

$$H : D = 10 : x_1 - x_2,$$

which is an equation for finding H. It is, however, difficult to be quite certain that both observers are dealing with the same portion of cloud, even if they are in telephonic communication. The method is effective for rapidly finding upper-air currents on a clear day, provided means is available for projecting vertically a shell which is timed to burst and produce a visible puff of smoke at about the desired height.

---

[1] See "Position-Finder, The Mirror," Vol. IV.

VIII. Various Instruments for Use on Aircraft[1]

§ (36) Meteorograph for Kite Balloons.—A meteorograph for kite balloons has been devised at the Royal Aircraft Establishment, and is used with success. It consists of a number of separate instruments (altimeter, thermometer, aeroplane compass, fan anemometer) mounted upon an aerofoil. The indicating dials or scales of the instruments are brought together upon a vertical partition arranged athwart the aerofoil, and a small camera with a kinematograph film is fixed in the aerofoil in such a way that a composite photograph of these indicators can be secured. By a clockwork mechanism exposures are made at regular intervals throughout an ascent. The aerofoil is suitably mounted upon the cable of the kite balloon, sufficiently far from the balloon itself to be free from the large disturbing influence on the wind which is produced by the balloon, and its shape ensures that it is always correctly orientated to agree with the prevailing wind direction. Consequently the aeroplane compass correctly indicates the wind direction at the position of the aerofoil. From the series of photographs on the film it is easy to draw up a table showing corresponding values of temperature, wind direction, and wind speed at the different heights attained, since each photograph shows clearly simultaneous readings of each of the instruments.

§ (37) The Aeroplane Psychrometer is a combination of dry- and wet-bulb thermometers which is used for obtaining observations of temperature and humidity in the upper air by the pilot of an aeroplane. The cockpit of an aeroplane is an unsuitable place for measuring air temperature, since conditions there are affected by various sources of error, of which the heat from the engine is the chief. It is necessary to expose a thermometer for obtaining air temperature on a strut upon the wing of the machine in order to secure a good exposure, and a thermometer so placed may be 4 feet or more distant from the observer. Some means of magnifying the stem and scale is therefore required to enable readings to be made. The earlier aeroplane thermometers were spirit thermometers, the liquid being coloured deep red and the bulb being large in relation to the bore of the tube, so that the scale was very open, about 5° F. to the inch. The stem was made of lens-fronted glass, which magnified the bore laterally but not longitudinally, and the mount of the thermometer was a large block of wood, of which the back was hollowed to fit the strut, while the front was painted white with black graduations and figuring for the thermometer scale. The bulb was surrounded by a metal "honey-comb" bounded by a sheet of bright metal so arranged that when fixed to the machine air passed freely over the bulb, but direct solar radiation could not reach it. This thermometer was easy to read from the pilot's seat, but it suffered from the disadvantage that it was sluggish in action, and was therefore unsuitable for observations of temperature when the machine was climbing or descending rapidly.

The more modern instrument consists of an ordinary mercurial thermometer with a small bulb fixed upon a porcelain mount. The graduations are on the stem of the thermometer. The scale value is not particularly open, but is about 18° F. to the inch. Such a thermometer is much more sensitive to changes of temperature than the large-bulbed spirit thermometer just described. To render it readable at a distance of 4 feet a 2-inch diameter lens, of focal length about 4 inches, is mounted on a slide in front of it, the axis of the slide being parallel to the stem of the thermometer, and a two-string or a Bowden-wire control is provided so that the lens can be moved up or down by the observer until the end of the mercury column and the scale in the immediate vicinity of it are seen in the field of view, when reading is easy. A second thermometer is mounted alongside the first to act as wet bulb, and a similar reading lens is provided for it, the two lenses being arranged in the same sliding mount and moved together. The bulbs of the thermometers are not protected specially from radia-

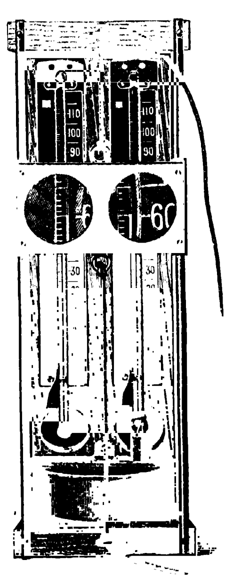


Fig. 27.

---

[1] See also Vol. V., "Instruments for Use in Aircraft."

tion since the ventilation experienced is in general sufficient to counteract radiation effects. In any doubtful case it is not difficult to manipulate the machine so that the thermometers are in the shade of the wings while observations are made. *Fig.* 27 shows the arrangement.

Special tables for obtaining relative humidity, vapour pressure, etc., from the dry- and wet-bulb readings are used, to suit the special conditions of ventilation to which the thermometers are subjected.

§ (38) DOBSON BAROTHERMOGRAPH.—A useful instrument for use on aeroplanes is the Dobson barothermograph, which records temperature as a function of pressure as the Dines balloon meteorograph.[1] The scale, however, is not microscopic, but readable without special appliances other than a transparent celluloid scale, which is ruled to show isobars and isotherms in two series of curves intersecting approximately at right angles. Pressure is measured by two parallel sets of aneroid boxes, of which the terminals move in opposite directions and are connected to a cross-lever which is pivoted at a point midway between the terminals. The sets of boxes therefore assist each other so far as rotation of the cross-lever due to change of pressure is concerned, but any inertia effects due to vibration of the engine of the aeroplane are cancelled, because they are felt as equal and opposite forces on the two sets of aneroid boxes. The temperature element is a bimetallic coil of steel and brass of the usual type placed in a cubical cell which is provided with numerous ventilation holes on three of its faces. The cross-lever of the aneroid system and the arm of the thermograph are connected together by a system of jointed rods with a writing-point at the middle joint of the system. The result is that the isotherms are very nearly circles described about a series of definite positions of the end of the thermograph arm, and the isobars are nearly circles described about a series of definite positions of the end of the aneroid cross-lever arm. The joints are jewelled to eliminate friction and all the moving parts are very carefully balanced to eliminate blurred records due to engine vibration. The records are made by a sharp metal point writing on a smoked card, and are "fixed" before tabulation by flowing a quick-drying lacquer over the surface. The case of the instrument is stream-lined and arranged for attachment to a leading edge of a plane.

The scale values are approximately as follows: 10 mb. = 10° F. = 4 mm. From a tabulation of pressure and temperature which is made direct from the record it is easy to draw up a corresponding tabulation of temperature at definite heights, using the barometric formula or its equivalent in table or diagram form.

Figs. 28 and 29 are a view of the instrument and a reproduction of an actual record. The latter shows separate curves for the ascent and descent.

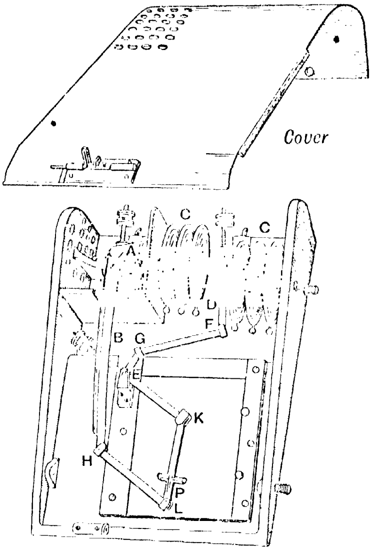


FIG. 28.

A, bimetallic thermometer, controlling lever B; CC, aneroid boxes of barograph, controlling lever D; E, fixed pivot; GEK, bell-crank lever; GF, KL, HI, jointed rods; P, pen.

It will be noticed that this instrument is well adapted for obtaining air densities.

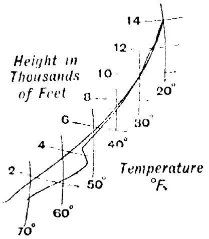


FIG. 29.

Density depends upon pressure and temperature only, consequently lines of equal density could also be drawn upon the celluloid scale, with the aid of which a tabulation of pressure or temperature or height against density could be made. A knowledge of the density at different levels is important in gunnery.

R. C.

---

[1] See "Air, Investigation of Upper," § (7).

METEOROLOGICAL OPTICS

§ (1) INTRODUCTION.—The study of meteorological optics is concerned with those properties of light which depend on the atmosphere and on the occurrence of dust, water, or ice suspended in it.

More or less complete accounts of various phenomena coming within the province of meteorological optics are to be found in treatises on light, notably in the *Traité d'optique* of M. E. Mascart (Gauthier Villars, 1893). The most exhaustive account of the subject is in *Meteorologische Optik*, a work which was mostly written by J. M. Pernter and was completed by his pupil F. M. Exner (Braumüller, Wien und Leipzig, 1910).[1] It should be noted, however, that some of Pernter and Exner's work requires revision in the light of modern knowledge, and, moreover, that there are still a certain number of phenomena which have not yet received satisfactory explanation. Amongst short summaries of the subject that of Humphreys (*Physics of the Air*, Philadelphia, 1920) must be mentioned

Notes for the use of observers will be found in the *Observers' Handbook* of the Meteorological Office. These are based on instructions prepared by J. M. Pernter for the International Meteorological Committee.

§ (2) THE APPARENT FORM OF THE SKY.—The actual boundary of the atmosphere being indefinite and the heavenly bodies seen in the sky the sun by day and the stars by night being practically at an infinite distance, it might be supposed that the sky would seem to be at the same distance from an observer in all directions. This is not the case, however. The sky seems nearer overhead than close to the horizon; constellations, as well as sun and moon, appear to be larger when low down. The apparent form of the sky is to most people a segment of a sphere. The ratio of the dimensions of the segment can be estimated by judging the position of the middle points of arcs on the sky and determining instrumentally the angles the arcs subtend to the eye. For example, the elevation of a point which to the eye appears as the middle point of the arc from horizon to zenith is found to be about 22°, whereas if the sky were a hemisphere the middle point of such an arc would have had an elevation of 45° The observations are consistent with the assumption that the apparent distance of the horizon is about four times that of the zenith.

The explanation given by Humphreys for this curious phenomenon is that when looking at clouds we realise that the clouds overhead are actually nearer to us than those at less elevation. Even when the sky is blue one accepts it as a slightly curved surface: the heavenly bodies seem to be embedded in this surface rather than far beyond it. Some theorists have put forward a more elaborate explanation according to which the scale to which the mind refers the size of distant objects depends on the position of the eye in its socket. A simple illustrative experiment is to gaze for a few seconds at the sun when it is not too bright. On turning away from the sun complementary images appear on the tired retina. Such images are said to look large when near the horizon and diminish as the eye is turned upwards, but in the experience of the present writer this is not the case, so that, for his eyes at any rate, the explanation breaks down.

This physiological or psychical phenomenon has to be remembered when various observations are reviewed. It explains the great apparent height of mountains, which appear to shut off half the sky when their elevation is only 22°; it makes the ends of a rainbow appear wider than the middle, and makes a halo, when seen round a low sun, elliptical rather than circular. In the routine of practical meteorology it probably leads to exaggerated estimates of the proportion of the sky that is cloud-covered.

§ (3) REFRACTION IN THE ATMOSPHERE.—The atmosphere being a medium whose density is not uniform, light is propagated through it not in straight lines but along curved rays.[2] How this happens may be pictured most readily by reference to the wave-theory of light. Suppose a wave to be advancing from a distant source of light at nearly the same level as the observer, the upper and lower parts of the wave will be in air differing slightly in density, and the upper part will generally move the faster. Accordingly the wave front will rotate, and the light reaching the observer's eye will seem to come from a point above the actual source. A useful analogy is that of sea waves which wheel round when one end, being in shallower water, moves more slowly than the other.

The radius of curvature of a ray may be calculated from the formula

$$\frac{1}{r}=-\frac{dv/dn}{v}=+\frac{d\mu/dn}{\mu},$$

in which r is the required radius, v is the speed of light, μ is the index of refraction, and dn is measured at right angles to the ray from the lighter to the denser medium.

It follows that if, as is usual, the distribution of density is symmetrical and the ray is therefore in one plane throughout its course, the total rotation of the wave front is given by the equations

$$\theta=\int\frac{\tan\zeta}{\mu}\frac{d\mu}{ds}ds=\int\tan\zeta\,.\frac{d\mu}{\mu},$$

in which ζ denotes the angle between the ray and the normal to a surface of equal density.

---

[1] A second edition has just appeared.

[2] See "Trigonometrical Heights and Terrestrial and Astronomic Refraction."

§ (4) ASTRONOMICAL REFRACTION.—The displacement of the stars by refraction is of great importance in observational astronomy, and much attention has been given to the estimation of its magnitude.[1]

To evaluate the displacement theoretically we may use the integral obtained in the last paragraph.

To a first approximation

$$\theta = (\mu_0 - 1) \tan \zeta,$$

where ζ is the zenith distance of a star and μ_0 is the index of refraction of the air at ground level.

When a second approximation is required the curvature of the atmosphere must be allowed for, and the following formula is obtained

$$\theta = C \tan (\zeta - A\theta),$$

where $$C = \tfrac{1}{2}(\mu_0 - 1)(3 - \mu_0),$$

and $$A = \frac{H}{(\mu_0 - 1)R} - \frac{1}{2},$$

R being the radius of the earth and H the height of the homogeneous atmosphere.

A formula of this type was given by Bradley; the theoretical evaluation of the coefficients appears to be due to Rayleigh.

The value of the constant A determined by Biot and Arago from astronomical observations is 3·25. Rayleigh's theory would give a somewhat higher value. The constant C is 60·6 seconds or nearly ·0003 radian.

The following table (after Bessel) gives the correction for refraction at 760 mm. and 8·5° C.:

Zenith Distance.	Correction.	Zenith Distance	Correction.
10°	10″	60°	100″
20	21	70	157
30	33	80	316
40	48	85	586
50	69	90	2094

§ (5) RISING AND SETTING OF THE HEAVENLY BODIES.—It will be seen that according to the preceding table when the zenith distance of a heavenly body is apparently 90° its actual distance exceeds 90° by 2094″ or 35′. This correction is greater than the apparent diameter of sun or moon, so that when either of these luminaries is seen on the horizon its geometrical position is completely below that level.

In estimating the time of sunrise the practice of meteorologists[2] is to allow for refraction and go by the time at which the centre of the sun is on the horizon; the true distance of the centre of the sun from the zenith is assumed to be 90° 34′.

It should be remarked, however, that any unusual distribution of temperature may modify the course of the sun's rays and occasion a distortion of the image, and the time of sunrise or sunset may be affected by many seconds.

§ (6) THE GREEN RAY.—When the sun sets under favourable conditions the last glimpse of it is coloured a brilliant green. The phenomenon and the corresponding one at sunrise are explained by the unequal refraction of light of different colours.

§ (7) REFRACTION IN SURVEY WORK.—An important part of the accurate survey of a country lies in the determination of level.[3] To ascertain the difference of level between two spots the observer has two graduated staves set up and views them from an intermediate position. The instrument he uses is provided with a telescope which can rotate about a vertical axis, the optical axis remaining horizontal, and he examines the two staves in turn, ascertaining the graduations which appear to be at the same level. The most satisfactory time of day for surveying is when the sun is low, as the air is least turbulent, but at such times the layers near the ground are considerably cooler than those above and the rays of light are not straight but curved with the convexity upwards. In the circumstances the staff-graduations which appear to be at the same level are really at different heights. The error introduced in this way is quite appreciable, but it can be eliminated by arranging the routine of observation so that positive and negative errors cancel one another.

The error in a single observation may be computed by the formula

$$\epsilon = \frac{1}{2}\frac{s^2}{\mu}\frac{d\mu}{dh} = -\frac{1}{2}\frac{s^2}{10^6}\frac{dT}{dh},$$

where s is the distance from telescope to staff,
μ is the index of refraction of the air,
T is the temperature on the centigrade scale,
h is the height above ground, and
ϵ is the error in the observed height in so far as it is due to the cause under discussion.

Any, the same, unit may be used for ϵ, s, and h.

§ (8) THE DISTANCE OF THE HORIZON.—The distance of the horizon which bounds the field of view of an observer looking over the sea or a level plain depends to a certain extent on the meteorological conditions.

The horizon is determined by the cone of rays which reach the observer's eye having originated as tangents to the globe. If the air were of uniform density these rays would be straight lines, and in that case the distance of the horizon could be found from the simple formula

---

[1] See "Trigonometrical Heights," etc.

[2] *International Meteorological Tables*, p. B. 15.

[3] See "Surveying and Surveying Instruments," §§ (31), (32), Vol. IV., also "Trigonometrical Heights, etc."

$$s_0 = \sqrt{2hR},$$

in which h is the height of the observer and R the radius of the globe, whilst the depression of the horizon, *i.e.* the inclination of the rays as received by the observer, would be δ_0, given by the relation

$$\delta_0 = \sqrt{\frac{2h}{R}}.$$

When the air is not of uniform density the horizon is still determined by rays which start as tangents to the globe, but these rays are curved instead of being straight. If the air is warmer below than above, the rays curve upwards, and the distance of the horizon corresponding with a given height of the observer's eye is decreased. At the same time the inclination of the rays received by the eye is increased. On the other hand, when the lapse-rate[1] of air-density is high, as, for example, when there is warm air over cold water, the horizon may be very considerably raised.

A classical instance of the latter phenomenon occurred on July 26, 1797, when the whole French coast from Calais to St. Valery was seen from the shore at Hastings. A good example of the former effect is the observation of Cook, who, in 1773, saw a large iceberg disappear below the horizon as the result of the lowering of air temperature on the passage of a snow squall.

The depression of the horizon is given by the formula

$$\delta^2 = \frac{2h}{R} + \frac{2}{10^6}(T - T_0),$$

where h is the height of the observer's eye, R the radius of the earth ($6{\cdot}4 \times 10^6 m$), T is the temperature (centigrade) at the ground level on the actual horizon, T_0 is the temperature at the observer's eye, and δ is the depression in radians.

In the theoretically simple case of a uniform lapse-rate of temperature

$$\delta = \sqrt{\frac{2h}{R}}\left[1 - 3{\cdot}2\frac{dT}{dz}\right],$$

and in this case the distance of the actual horizon is found from the relation

$$s = 3580\sqrt{h}\left[1 + 3{\cdot}2\frac{dT}{dz}\right],$$

the unit of length in these formulae being the metre.

Looming.—The word looming is used for an illusion which makes objects appear bigger than they are. When conditions are favourable for increasing the distance of the horizon the vertical scale of distant objects is also increased and they "loom," but this is not the only possible cause of looming.

§ (9) INFERIOR MIRAGE.—When the air very close to the ground is much heated it may happen that rays of light can pass into this lowest layer and curve upwards again. In such circumstances an observer will receive the same impression as if the rays had been reflected from the surface of water; when he can thus see an object and its inverted image he is said to observe a mirage; that term is used somewhat loosely, however, for other kindred phenomena. The illusion of the presence of water is frequent, but it does not always occur.

In the diagram (Fig. 1), which is drawn with the vertical scale enormously exaggerated, O represents the eye of an observer, S the actual position of a conspicuous object. The observer sees the object directly at P (he is unconscious of the fact that P is slightly below S) and an inverted image at Q. The condition for the mirage to be possible is that light can pass from S to O by two different paths. The illusion of the presence of water is assisted by the reflection of the sky B above S. This is seen in direction OL. It may not be produced if the effective background behind S consists of sand-hills instead of sky.

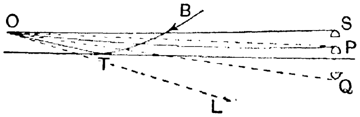


FIG. 1.—Production of Inferior Mirage.

When the sheet of water is apparently present its near edge will be determined by rays which almost graze the ground. The illusion of reeds growing by the water-side is produced by the vertical extension of the images of pebbles which happen to be suitably placed (as at T in the diagram).

It has been mentioned that a rapid increase of air density[2] from below upwards leads to a narrowing of the horizon; the horizon in this sense is really identical with the near edge of the sheet of "water," but it may happen that it does look like the ordinary horizon, and in such circumstances an object beyond the "horizon" and its reflection or mirage may be merged and seem to float in the air.

Though typical of tropical deserts, mirage is not infrequently to be observed in England. Excellent examples have been seen on level wood pavement in the London suburbs, wide roads appearing to be completely flooded.

---

[1] See "Atmosphere, Physics of the," § (5).

[2] When air is heated by contact with a hot surface the pressure, which is determined by the weight of the superincumbent atmosphere, is not altered, but the density is reduced. In spite of the convection currents the average density of the air very close to the ground is lower than that of the air above it.

The same type of mirage can occur at sea; much more frequently it happens that when a distant coast is looked at a bright whitish strip is seen along the sea-horizon in front of the coast. To the casual observer the strip seems to be the actual shore. In this case the mirage or reflection of the land is of little depth and is not noticeable, the bright strip is the reflection of the sky. The phenomenon is an indication that the sea is warmer than the air above it.

Shimmering.—An invariable accompaniment of the inferior mirage seen over a hot land surface is shimmering, a tremulous movement evidently due to refraction through the irregular boundaries of the convection currents. It is not uncommon, especially among riflemen, for mere shimmering to be spoken of as mirage when no true mirage is to be observed. It will be noticed, however, that a shot directed along the line of sight, OP of *Fig.* 1, would pass below the target S, so that in such circumstances it is necessary to aim high.

§ (10) SUPERIOR MIRAGE.—Another type of mirage is that in which the reflecting layer is above the observer. The necessary condition is an exaggerated falling off of air density at a certain height, implying a sharp "inversion" or increase of temperature with height. In this case the object is seen inverted, and frequently there is an erect image above the inverted one, indicating that three alternative routes are available for the light from object to observer. The most striking examples of the phenomenon have been reported from the Arctic regions; on one occasion a ship twenty-eight miles away, and therefore far below the horizon, was seen reversed in the air and actually recognised.

In the diagram (*Fig.* 2) BC is the layer in which the lapse rate is high and the rays are curved—the curve is a parabola. The ray which reaches the observer O as if coming from P appertains to an inverted image, that from Q to an erect one.

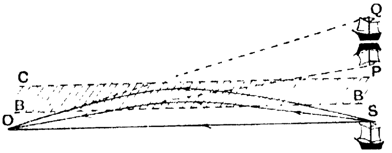


FIG. 2.—Production of Superior Mirage.

Fata Morgana.—*Fata Morgana* is the Italian name for Morgan the Fairy, the legendary half-sister of King Arthur. The mirage seen across the Straits of Messina suggested to the poetical palaces in the fairyland where Fata Morgana reigned, and her name is now given to mirages seen in like situations where there is much distortion and repetition of images. In such cases superior and inferior mirages may be produced simultaneously by interlacing currents of air.

§ (11) SCINTILLATION OR TWINKLING OF STARS.—Every one is familiar with the twinkling of stars, a rapid and irregular fluctuation of the colour as well as of the intensity of the light. The nearer the horizon the more vigorous the twinkling. Except when very near the horizon the planets do not twinkle appreciably, but their images, like those of the sun and moon, when seen in the telescope exhibit an analogous phenomenon, a rippling of the contour. The spectrum of a twinkling star shows light and dark bands which move sometimes in succession from the violet to the red or *vice versa* and sometimes oscillate.

The explanation of scintillation is found in the irregularities in the distribution of temperature and humidity in the air; the surfaces of equal index of refraction are not everywhere flat and horizontal but twisted into all sorts of irregularities, so that the atmosphere may be regarded as containing flaws, and the fronts of the waves of light on their way from a star to the eye do not remain plane. If such irregularities did not occur, the pencil which just fills the pupil of the observer's eye would have throughout its course a cross-section equal in area to the pupil. In the actual atmosphere the cross-section of the pencil as it enters the atmosphere must vary, and, therefore, the amount of energy entering the eye fluctuates. Moreover, the path of the red light which reaches the eye is not quite so much curved as that of the violet light (the paths may be separated by a metre or more), and the fluctuations in the brightness of different colours will be more or less independent. The theory explains not only the ordinary direct observations of twinkling but the passage of the bright and dark patches in the spectrum. The regular movement of these patches shows that the "flaws" in the atmosphere move mostly in one direction, which can be shown to be from west to east. How it is that the movement is slow enough to be visible does not seem to have been adequately explained.

Shadow Bands.—Another phenomenon which has been explained by the movement of these "flaws" in the atmosphere is observed just before the totality of a solar eclipse, when a white wall, for example, may look as if light reflected from a rippled water surface were falling on it.

§ (12) DIFFRACTION. THE COLOURS OF THE SKY.—Lord Rayleigh's theory[1] provides a general explanation of the blue of the midday sky. To explain the appearance of red and other shades at sunset the absorption, or rather the extinction, of the light either before or after the diffraction to which the colour is mainly due must be allowed for. The short waves which are produced most freely when the sunlight is incident on the atmosphere are also scattered most readily on their passage through the lower strata.

---

[1] See "Scattering of Light," Vol. IV.

To estimate the brightness of the light of any colour reaching the eye from a particular elementary portion of the atmosphere, the intensity of the same colour in the original beam from the sun must be multiplied by three factors. The first represents the proportion of the light which gets as far as the diffracting element, the second the fraction which is diffracted there, the third the proportion of this light which reaches the eye without being diverted by secondary diffraction. The first and third factors decrease as the length of the path of the light through the atmosphere is increased; these factors are smaller for the smaller wave-lengths. On the other hand the second factor is greater for small wave-lengths. When the total length of the path is moderate this second factor is of most importance and blue light reaches the eye most readily, but when this total length is greater, *i.e.* for the sky near the horizon, with a low sun it is the red light which prevails, the path being long enough to provide an adequate amount of red diffracted light, not long enough to extinguish it. When the sun is low it appears a golden yellow, whilst the sky near the horizon is red, shading off through pale yellow to pale blue and the deeper blue overhead.

Pioneer work in the way of numerical computation of the intensity of different colours of the spectrum in skylight when the sun is on or below the horizon has been published by P. Gruner.[1] His calculations, though not carried far enough for detailed comparison with observation, establish the fact that diffraction by air molecules is sufficient to account for the red colour of the sunset sky.

The experiments of Professor R. W. Wood [2] show that if the light of the sky were due entirely to diffraction by the air there would be no glare or excess of brightness in the region near the sun. The actual excess (the light one diameter from the sun is about five times as bright as more remote parts of the sky) [3] is to be attributed to diffraction by foreign matter, *i.e.* dust, and possibly water-drops and ice-crystals in the air. The most brilliant sky colours, notably those which followed the Krakatoa eruption, are to be attributed to fine dust.

The colours of the sky are affected by the presence of clouds. If the line of sight passes for long distances in shadow the sky frequently takes a greenish hue, the blue light being reduced in strength in comparison with that of greater wave-length and not being reinforced by sunlight diffracted in the lower levels.

The phenomena of sunset and sunrise, as observed with cloudless skies and a clear atmosphere, must be set out in further detail.

Two series of phenomena are recognised, which may be denoted primary and secondary, the primary occurring before the secondary at sunset, after at sunrise. For convenience of description, only the sunset sequence will be detailed.

Sunset and Sunrise Colours. Primary Series.—(*a*) The ruddy *counterglow* in the east appears when the sun is still above the horizon. Its upper limit remains steady after the sun sets, but from that time it is encroached on by the *earth shadow* or *dark segment* which gradually eclipses it.

(*b*) The *oversun glare* (Dämmerungsschein) is a white luminous area which is seen well above the sun as it nears the horizon.

(*c*) After sunset there is a *bright segment* of red or gold, the upper boundary of which is the *twilight arch* (Dämmerungsbogen).

(*d*) The oversun glare develops at sunset into the *purple-light*. According to observations made by Miethe and Lehmann [4] in Assouan this development proceeds from the outer edge inwards. The purple-light is said to be at its brightest when the sun is about 4° below the horizon. The purple-light is initially almost a circle with radius between 25° and 45°. It disappears rather quickly, subsiding on to the twilight arch when the

---

[1] *Beiträge z. Physik d. Freien Atmosphäre*, 1919, viii. 120.

[2] *Phil. Mag.*, 1920, Ser. 6, xxxix. 430.

[3] This refers to visible light; the contrast in the case of thermal energy is more striking. The following table refers to observations at Mount Whitney (*Annals of the Astrophysical Observatory of the Smithsonian Institution*, iii. 145). The unit of brightness is $1/10^8$ of the brightness of the centre of the sun. The brightness of any part of the sky is regarded as the sum of two parts—A, a function of the elevation of the point, and B, a function of the distance from the centre of the sun.

A

Solar Distance.	Brightness.	Solar Distance.	Brightness.
2°	5566	10°	208
3	2586	15	142
4	1366	30	62

B

Elevation.	Brightness.	Elevation.	Brightness.
0-10°	320	40-55°	89
10-20	174	55-75	70
20-30	137	75-90	59
30-40	112		

[4] *Met. Zs.*, 1909, xxvi. 101.

sun is some 6° below the horizon. The beautiful Alpine *afterglow* is seen on mountains illuminated by the purple-light.

Secondary Series.—The secondary series includes the secondary counterglow with a secondary dark segment below it, a secondary oversun glare, a secondary twilight arch, and finally a secondary purple-light. Observations of the complete secondary series are rare. The secondary twilight arch has been identified with the boundary of the primary dark segment after its passage through the zenith.

The phenomena fall into two classes, those due to diffraction by the gaseous molecules and those due to diffraction by dust in suspension in the upper regions of the atmosphere.

In the latter class are included the oversun glare and the purple-light. In this connection it is important to notice that the purple-light was seen at its best at the time of the Krakatoa eruption, and that the distance from the sun of the brightest region agreed with that of Bishop's ring. Spectral analysis indicates that the purples may be regarded legitimately as the sum of red and sky-blue.

The secondary series is attributed by Pernter to light which has undergone diffraction twice, but the theory presents difficulties.

§ (13) POLARISATION OF LIGHT. (i.) *From the Sky.*—The polarisation of the light from the sky was discovered by Arago. The maximum polarisation occurs at 90° from the sun. As was pointed out by Stokes, if the skylight were entirely due to diffraction of direct sunlight by very small particles the polarisation at this angle would be complete, and the fact that the polarisation is in the plane through the sun was of importance in establishing the theory that the displacement of the ether in the light wave was perpendicular to the plane of polarisation.[1] Actually the polarisation is not complete. Some of the light reaching the eye has been diffracted more than once, and the existence of this secondary diffraction, arising from the small particles of the air or less regularly from grosser matter in suspension, explains the observed facts.

The primary diffracted light from the sky in the immediate neighbourhood of the sun should not be polarised. Observation shows that there is polarisation in this region. This polarisation probably is a consequence of the secondary diffraction. The primary diffracted light which reaches any point of the atmosphere comes from all directions, but most of it must come from the directions in which the air extends furthest, *i.e.* from near the horizon. Such light is polarised in such a way that the "displacement" in the light vector is nearly vertical, and the statement is also true after the light is diffracted again. On the vertical great circle through the sun there are neutral points where the primary and secondary diffraction just balance and the light is unpolarised.

These points are:

Arago's neutral point about 160° from the sun.
Babinet's ,, ,, ,, 20° above the sun.
Brewster's ,, ,, ,, 20° below the sun.

The distances from the sun vary through ranges of 5° or more.

The hypothesis of secondary diffraction is due to Soret. It should be mentioned, however, that it is doubtful[2] whether secondary diffraction by air molecules would be adequate to produce the observed effects, and Exner has recently[3] suggested that it is caused by reflection of the primary diffracted light by the grosser particles in the air.

(ii.) *From a Landscape.*—With a clear sky the light received from solid objects is polarised in the same way as the sky would be in the same directions. With a cloudy sky, on the other hand, the polarisation is in a vertical plane. Exner explains these facts[4] by consideration of the minute facets from which the light must be reflected, pointing out that in the case of the cloudy sky the most brilliant illumination is from the zenith. The light from the clouds is not itself polarised.

§ (14) RAINBOWS.—Rainbows are seen when the sun shines upon falling rain. Sometimes only one bow or part of it is seen, sometimes two. Both bows have their centres at the point opposite to the sun below the horizon of the observer. The inner or primary bow, which is the brighter, has an angular radius of about 41°. The outer or secondary bow has a radius of about 52°. The primary bow is normally coloured red on the outside, and shows colours in the order of the spectrum with violet inside, whilst in the secondary bow the colours are in the reverse order. The space between the bows is somewhat darker than the rest of the sky. In favourable circumstances, with a low sun and a heavy cloud for background, the contrast is striking. The colours of the bows are not always developed to the same extent, the widths and brightness of the successive bands being variable. Moreover, some of the colours, notably the violet or pink, may be repeated, sometimes with a colourless interval, so that supernumerary bows are seen. These supernumerary bows are inside the primary.

---

[1] According to the electromagnetic theory of light it is the electric force which is at right angles to the plane of polarisation.

[2] Marie A. Schirmann, *Meteorologische Zs.*, 1920, xxxvii. 12.

[3] *Meteorologische Zs.*, 1920, xxxvii. 115.

[4] *L.c.* p. 114.

The general explanation of the rainbow appears to have been first given by Theodorich[1] about 1311, and Antonius de Dominus, at the end of the sixteenth century, gave an experimental demonstration with globes filled with water. The theory was placed on a numerical basis by Descartes in 1637, and the explanation of the colours was given by Newton in his *Opticks* (1704). According to the geometrical theory of Newton, rainbows should always have the same distribution of colour. To account for the variations, and especially for the supernumerary bows, the wave theory is required. This was pointed out by Young in 1804. The way in which the problem could be treated mathematically was shown by Potter in 1835, and his work was completed by Airy in 1836 in the same volume of the *Transactions of the Cambridge Philosophical Society*. It is a curious fact that the general recognition of Airy's work was delayed by the unsuggestive title of his paper, "On the Intensity of Light in the Neighbourhood of a Caustic." The detailed specification of the colours of rainbows caused by drops of various sizes was made possible by Mascart, who pointed out the law according to which the intensity of light of various wave-lengths should vary, and was completed by Pernter.[2]

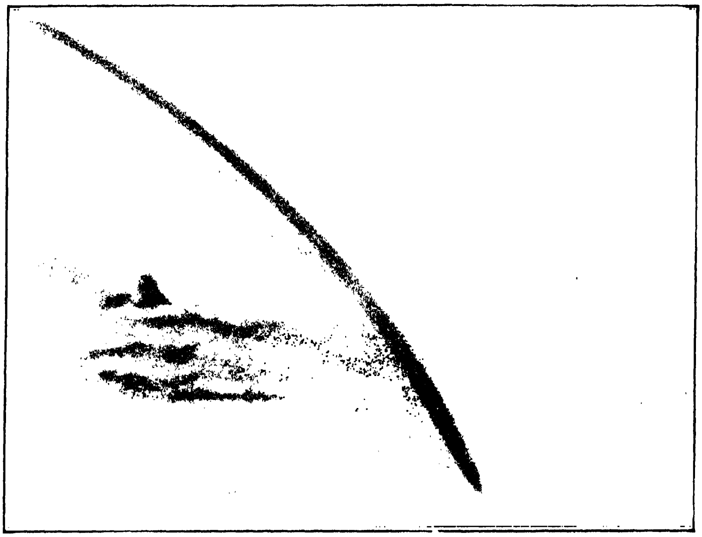


FIG. 3.—Rainbows: Primary and Secondary. From a Photograph by G. A. Clarke.

The Newtonian theory is given in elementary text-books of optics and need not be discussed here.[3] We shall endeavour to show to what extent the actual colours of the rainbow depend on the undulatory nature of light.

The primary rainbow is due to light which has been reflected once internally in the raindrops. To simplify the consideration of how the bow originates we may confine attention at first to the course of monochromatic rays which come from a particular point of the luminary and enter a spherical drop in a specified diametral plane. After the first refraction the rays touch a certain caustic, after the internal reflection they touch another caustic, and on emergence they will touch a third. The form of this third caustic

---

[1] Preston, *Theory of Light*, § 317.

[2] *Wiener Akad. Sitz.* Bd. 106, § 153.

[3] For convenience of reference the essential formulae for the rays with minimum deviation may be written down. They are:

Primary bow—

$$\sin i = \mu \sin r, \quad 2\cos i = \mu \cos r,$$

$$\sin i = \sqrt{\frac{4-\mu^2}{3}}, \quad \cos i = \sqrt{\frac{\mu^2-1}{3}},$$

$$\sin\frac{\pi - D}{2} = \frac{1}{\mu^2}\left(\frac{4-\mu^2}{3}\right)^{\frac{3}{2}}.$$

Secondary bow—

$$\sin i = \mu \sin r, \quad 3\cos i = \mu \cos r,$$

$$\sin i = \sqrt{\frac{9-\mu^2}{8}}, \quad \cos i = \sqrt{\frac{\mu^2-1}{8}},$$

$$\sin\frac{2\pi - D}{2} = \frac{1}{8\mu^3}(\mu^2-1)^{\frac{1}{2}}(9-\mu^2)^{\frac{3}{2}}.$$

In these formulae i is the angle of incidence, and r the angle of refraction into the drop, both for the rays with minimum deviation. D is this deviation, and μ the index of refraction.

is indicated in *Figs.* 4 and 5. It has two asymptotes which are themselves rays. For these rays the deviation from the direction of the incident light is a minimum. This is easily verified by placing a straight-edge as a tangent to the caustic and gradually turning it whilst maintaining the tangency. The deviation is a maximum, 180°, for the ray which is directly reflected at the back of the drop.

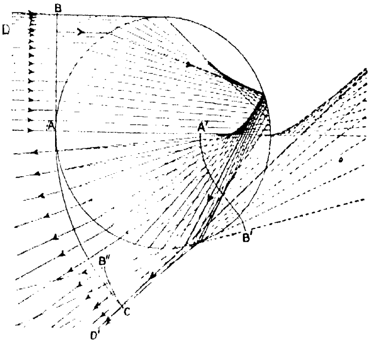


FIG. 1.—The Passage of Light through a Raindrop. From Humphreys' *Physics of the Air.* AB, A'B', and ACB'' represent wave-fronts.

A good approximation to the form of the wave-fronts can be determined by thinking of the *loci* swept out by the ends of strings unwrapped from the two branches of the caustic. In the neighbourhood of the limiting or Descartes ray these approximate wave-fronts are quartic curves with cusps. They indicate an infinite disturbance at the cusps. The leading part of the wave corresponds to light coming from the central part of the drop, the spur with light from the further part, the cusp with light along the Descartes ray. Propagation of such waves past an undisturbed region or shadow is not possible. Some of the energy must pass into the shadow, and the form of the waves must be modified. It is of interest, however, to trace the interference pattern corresponding with the approximate wave-fronts.

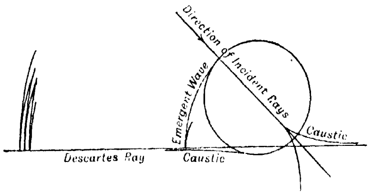


FIG. 5.—The Interference of Light-waves emerging from a Raindrop.

As the waves follow in succession there is reinforcement on the lines where crest intersects crest and interference on intervening lines.

It can be shown that the separation between the two branches of the approximate wave-front at an angular distance θ from the Descartes ray is $c\,.\,\theta^{\frac{3}{2}}$, where

$$c = \frac{8}{9}a\frac{(\mu^2-1)^{\frac{1}{2}}}{(4-\mu^2)^{\frac{3}{2}}},$$

a being the radius of the drop, and μ the index of refraction.

Interference occurs where the interval is equal to an odd number of half wave-lengths, *i.e.* where

$$\theta^{\frac{3}{2}} = \frac{2n+1}{2}\,.\,\frac{\lambda}{c},$$

n being any integer, and λ the wave-length.

This equation determines the points where the intensity of the light vanishes. The maximum intensity occurs where

$$\theta^{\frac{3}{2}} = n\,.\,\frac{\lambda}{c}.$$

In the more accurate theory of Airy the intensity of the light is found by the evaluation of a certain integral, and calculation shows that the maxima and minima are determined to a high order of approximation by the conditions

$$\theta^{\frac{3}{2}} = (n+\tfrac{1}{4})\frac{\lambda}{c} \text{ and } \theta^{\frac{3}{2}} = (n+\tfrac{3}{4})\frac{\lambda}{c}$$

respectively.

It will be noticed that the first maximum is no longer on the Descartes ray but at an angular distance from it, which is given by

$$\theta^{\frac{3}{2}} = \tfrac{1}{4}\frac{\lambda}{c}$$

(more precisely by $\theta^{\frac{3}{2}} = 0{\cdot}22(\lambda/c)$).

Now consider the observer at a point which may be called P. He receives from the particular drop A light of the given wave-length of maximum intensity, provided that the angle between the lines of vision AP and the direction of the light coming from the luminary is D + θ, D being the deviation of the Descartes ray, and θ being given by the formula last stated.

He will also receive light of like character from all the other drops of the same size which are at the same angular distance from the sun. It follows that in the circumstances postulated, a point source of monochromatic light and equal raindrops, the observer would see a large number of bows decreasing in brightness, the outermost bow being the brightest.

In practice we have to deal with composite light from a source of considerable magnitude and with drops which are not of uniform size. Pernter has worked out the details of the bows which would be due to drops of various magnitudes. He finds that with drops of diameter exceeding 1 mm. the maxima of illumination due to a point source of light would be fairly close together. When allowance is made for the angular diameter of the sun it is found that distinct colours must appear only as a single spectrum. Thus large drops give approximately the rainbow of Descartes' theory.

On the other hand, with very small drops, with a diameter of 0·1 mm. or less, the maxima for monochromatic light are spread out. The first maxima for all colours nearly coincide, so that the rainbow is almost a pure white. Such a rainbow is only seen under favourable circumstances when the observer is very near a cloud; it is sometimes known as a fog-bow or as Ulloa's Ring. Three concentric white bows have been observed on occasion.

As with coronas, so white rainbows may be occasioned by droplets far below the freezing-point. Pernter (*Meteorologische Optik*, p. 393) does not contemplate the existence of such droplets, and provides a far-fetched explanation of what he calls Bouguer's halo by three internal reflections in ice-crystals of a special form.

The shape of the white rainbow is determined by the intersection of a cone with the surface of the cloud. In the case of a horizontal cloud or fog seen from above, the curve of intersection is a hyperbola, and the "bow" may appear as two long white streaks.[1]

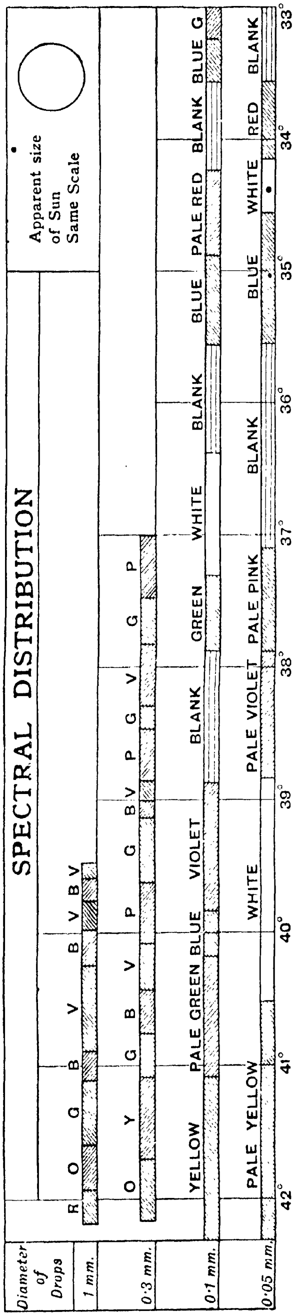


FIG. 6.—The Distribution of Colour in Rainbows produced by Drops of Various Sizes.
The radii of the arcs of the various colours are given by the horizontal scale.

In *Fig.* 6 (based on Pernter's results) the distribution of the colours in primary bows due to drops of different sizes is shown. It will be noticed that the presence of supernumerary bows, which are usually recognised as pink, indicates the predominance of drops with diameter about ¼ mm.

The composition of the secondary or outer bow formed by light which has suffered two internal reflections has not been worked out in the same detail. The secondary bow is generally fainter than the primary, and apparently no observations of the supernumerary arcs which might theoretically be expected outside the secondary bow have been reported.

Rainbows of higher orders than the second can be produced under laboratory conditions. They are not to be seen in nature, though there seems to be no good reason why careful observations should not in fortunate circumstances detect the tertiary bow at 50° from the sun.

Rainbows due to light from the sun's image in a sheet of water are observed occasionally; such bows have their centres above the horizon. The course of the light is from sun to water, to drop, to observer. On the other hand, when a bow is seen reflected in the water, the course of the light is from sun to drop, to water, to observer.

---

[1] Baldit, *Annuaire Soc. Météorologique de France*, 1907, lv. 61.

Lunar rainbows are reported occasionally. The colours are difficult to detect.

§ (15) DIFFRACTION PHENOMENA. (i.) *Coronas.*—Frequently, when the moon is seen through clouds, it is surrounded by coloured rings. Sometimes the colour is confined to a brownish patch in which the moon seems to be embedded. More frequently the most conspicuous part of the phenomenon is the scorched appearance of the neighbouring clouds. Under favourable circumstances, however, there are well-defined rings, blue being innermost, and the spectral colours are repeated more than once. The repetition of any particular shade is seen best when coloured glass is used. The brownish inner ring with the bluish-white inner field between ring and luminary form the *aureole*, the complete phenomenon is the *corona*. Coronas round the sun can seldom be seen with the naked eye owing to the general brilliance, but they are to be observed with smoked glasses.

The condition necessary for the formation of a well-developed corona is the presence in the cloud which covers the luminary of drops of approximately uniform size. The light is diffracted by these drops. When a wave of light strikes such an obstacle as a drop of water secondary wavelets travel away from back and front of the drop. If the waves in the incident light come at regular intervals there will be the same interval between the wavelets. In some directions the crests of these wavelets coincide, in other directions the crests of one set coincide with the hollows of the other set. Accordingly, when observed from some points of view, the drop will be an effective source of light, but not in others. Equally effective drops will appear to lie at equal angular distances from the source of light, which will therefore appear to be surrounded by luminous rings. The angular radii of the rings are determined by the ratio of the wave-length of the light to the size of the obstacle, the larger the ratio the larger the rings. Hence it follows that with natural light the blue rings are smaller than the red, and also that the largest coronas are caused by the smallest drops.

The theory of coronas, as usually stated,[1] assumes that the drops are equivalent in their action to thin discs of the same diameter. On this hypothesis the angular radii of the rings of maximum brightness for any particular wave-length are given by the formula

$$\sin\theta = k\cdot\frac{\lambda}{a},$$

where θ is the radius, λ is the wave-length, $2a$ the diameter of a drop, and k has the values 1·64, 2·69, 3·72, 4·72, . . . for successive maxima.

The intensity of the successive maxima falls off rapidly, the third being only one-tenth as bright as the first.

Measurements of coronas have served to determine the size of the drops which produce them. These average about ·02 mm. in diameter, the range being from ·01 mm. to ·06 mm.

(ii.) *Iridescent Clouds.*—Patches of colour like mother-of-pearl are occasionally seen on high clouds such as cirro-stratus. These patches are probably portions of coronas of very large radius. The classical explanation attributes them to diffraction by ice-needles. The complete theory of diffraction by a cloud of needles with fortuitous orientation does not seem to have been worked out, but it is improbable that sufficiently brilliant colours could be produced by such a cloud. Simpson[2] prefers to attribute the phenomenon to minute water-drops, supporting this view by his observation that a corona and a halo are never seen at the same time on the same cloud. There is abundant evidence for the existence of drops at temperatures far below the freezing-point.

(iii.) *Glories.*—On mountains an observer standing with his back to the sun will sometimes see coloured rings round the shadow cast upon mist by his own head. The whole phenomenon is known as the *Brocken Spectre*, the coloured rings being called a *glory*. Similar observations have been made from balloons. A striking feature of the Brocken Spectre is that, even if the shadows of the observer's companions are seen, they are without glories. The explanation is that the glory is no personal attribute of the observer himself. If he used a periscope for his observations he would see the glory round the shadow of the top of the periscope; in fact, Franz Mierdel,[3] who has been successful in viewing glories in an artificial cloud in the laboratory, adopted such a device.

Glories and coronas are twin phenomena. It has been noticed how, when light waves fall on a drop of water, secondary waves are produced. The secondary waves, whose general direction is forward, produce the corona; those whose general direction is opposed to the incident light will play their part in the formation of a glory. Owing to the symmetry of these secondary waves the dimensions of corona and glory will be equal.

---

[1] The complete mathematical solution of the problem of the scattering of plane electric waves by spheres has been given by T. J. I'A., Bromwich (*Phil. Trans. Roy. Soc.* A, 1920, ccxx. 175), but the application to the case in which the radius of the sphere is comparable with a wave-length has not been worked out numerically.

[2] *Quarterly Journal R. Met. Soc.*, 1912, xxxviii. 291.

[3] *Beiträge zur Physik der freien Atmosphäre* (München), 1919, viii. 95.

A white rainbow, Ulloa's ring, is often seen outside a glory (see § (14)). Roughly speaking, the rainbow is produced by light reflected inside the drops, the glory by light scattered without penetrating the drops.

(iv.) *Bishop's Ring.*—Though not due to the presence of water-drops, Bishop's ring may appropriately be mentioned here. This ring was seen after the eruption of Krakatoa in 1883. Careful observation has led to its detection in subsequent years. The ring is a faint reddish corona of large radius, the inner edge about 12° from the sun, the outer at about 22°. These dimensions indicate that the diffracting particles which produce the corona must have diameters about ·002 mm. Their small size accounts for the slowness with which the particles settle down to earth, the phenomenon lasting for many months.

§ (16) GENERAL EFFECTS OF WATER-DROPS. (i.) *The Opacity of Clouds.*—When the sun is seen through cloud the intensity of the light is much reduced. Light which is refracted through the drops or diffracted by them increases the general luminosity of the cloud, and the contrast between the sun and the cloud depends only on the rays which get through the cloud without meeting a drop.

Consider a pencil of light of cross-section A coming from a point of the sun. Let S be the cross-section of one of the drops which happen to be in the pencil. The chance of a particular ray avoiding the drop is (A − S)/A or (1 − S/A). Hence the probability of the ray avoiding all the drops may be likened as $(1-S_1/A)(1-S_2/A)\ .\ .\ .\ (1-S_n/A)$, where $S_1, S_2\ .\ .\ .\ S_n$ are the areas of the cross-sections of all the drops, n in number, in the pencil. This expression is very nearly equal to $e^{-B/A}$, where B is the sum of the cross-sections. It follows that if I be the intensity of the sunlight before entering cloud, and J the observed intensity of contrast on emergence,

$$\log\left(\frac{J}{I}\right) = -\frac{B}{A}.$$

Accordingly if the drops are of uniform size with radius a, and the number in unit volume is q, whilst the thickness of the cloud measured along the line of sight is l, then

$$\log\left(\frac{J}{I}\right) = -\pi a^2 ql,$$

The volume of a drop is $\frac{4}{3}\pi a^3$. Hence the volume of water per unit volume of cloud is $\frac{4}{3}\pi a^3 q$; the vertical thickness of the cloud is $l\cos\zeta$, where ζ is the sun's zenith distance. Thus the volume of water in a vertical column of unit area is $\frac{4}{3}\pi a^3 ql\cos\zeta$; the diameter of a drop is $2a$. Thus the equivalent rainfall measured in drop-diameters is $\frac{2}{3}\pi a^2 ql\cos\zeta$ or $\frac{2}{3}\cos\zeta\log(I/J)$.

Richardson's observations[1] show that when the sun is just visible through stratus cloud the equivalent rainfall is equal to about four drop-diameters, the proportion of sunlight passing through the cloud without obstruction being about one part in 10,000.

The brightness of the sun when seen through fog or haze is affected in the same way. If the drops or dust particles are small enough diffraction affects the question. The light of longer wave-length is not so much obstructed as that of shorter wave-length, and the sun appears red.

(ii.) *The Opacity of Rain.*—The same line of reasoning may be applied to the question to what extent falling rain obscures a landscape. Although large drops cut off more light than small ones, they get out of the way quicker and so are not so efficient in producing opacity. Since the rate of fall of a drop is nearly as the square root of the diameter, it follows that for a given rate of precipitation the obscuring power, the $\pi a^2 q$ of the last formula, varies inversely with the square root of the volume of a drop.

(iii.) *Translucence of Cloud or Fog.*—The proportion of sunlight transmitted through a sheet of cloud has been discussed[2] by L. F. Richardson. By consideration of the distribution of the reflected and refracted rays he has shown that approximately 6 per cent of the energy of the light falling on a spherical drop is scattered backwards, *i.e.* in directions deviating by more than 90° from the incident light, the remaining 94 per cent passing onwards through the drop.

As the light penetrates deeper into the cloud it falls off in intensity, but so does the intensity of the reflected light from below. The difference between the amounts of energy flowing up and down is the same all through the cloud, and therefore equal to the difference between the amount emerging at the bottom of the cloud and the amount reaching that level from the ground.

Richardson deduces the approximate formula

$$\frac{I\cos Z}{E} = 2\left[1+\cdot06(1-B)\int_0^c \pi a^2 q\,dh\right],$$

in which

I is the intensity of the incident sunlight,
Z is the zenith distance of the sun,
E is the intensity of the emergent light,

---

[1] *Proc Roy. Soc.* A, 1919, xcvi. 23.
[2] *Loc. cit.* pp. 23-31.

B is the reflectivity of the earth's surface,
a is the radius of a drop,
q is the number of drops in unit volume,
and the integration is with respect to h the height above the base of the cloud.

In a particular case in which a strato-nimbus was under observation, the ratio on the left of this equation was found to be 4·0, and the reflectivity B being 0·15, it was deduced that the equivalent rainfall in the cloud was 24 drop-diameters.

(iv.) *Albedo of Cloud.*—It will be noticed that the thicker the cloud the more light is reflected by it. Moreover, the brilliance of the clouds depends on the angle between the rays from sun to cloud and from cloud to observer. The larger this angle the brighter the cloud. According to observations[1] of valley fog seen below the level of the observatory on Mount Wilson, California, when this angle is 165° the cloud is six times as bright as when the angle is 10°. According to the Mount Wilson observers the mean reflecting power of the clouds may be taken as 65 per cent.

(v.) *Visibility.* The visibility of a distant object depends mostly on the contrast with its surroundings. The condition of the atmosphere affects visibility in two ways. The suspended dust or mist cuts off the direct rays of light, and, on the other hand, by reflecting the diffuse sunlight it adds to the total light received by the eye and so reduces the contrast. In a fog by day the latter effect is the more important; the lights on a passing train do not remain in sight much longer than the train itself; and white objects are only seen a very little farther than darker ones. Even in the most favourable conditions, in the middle of the day, the light diffracted by the air itself veils the distance in blue and reduces contrasts so that the sharpest detail is noticed about sunset, when the air between observer and object is not directly illuminated.

In the neighbourhood of large cities haze is mostly due to coal smoke; it is most prevalent when the air is comparatively stagnant, and especially when an inversion of temperature at a few hundred feet confines the smoke to the lower layers. In country districts in England visibility generally improves in the middle of the day, when a large proportion of the dust which had settled down towards the ground is carried up again as the stratification of the air becomes less stable. Visibility is better with winds from the north and west, presumably because there is no pollution over the North Atlantic.

§ (17) ICE CRYSTALS IN THE ATMOSPHERE.—The phenomena to be considered in the following sections include various rings, arcs, and patches of light which are explained by the presence of ice-crystals in the atmosphere. These phenomena are seen in most favourable circumstances in the polar regions, where small ice-crystals frequently occur at low levels, but they are by no means rare in temperate latitudes. The rings round the luminary are called haloes, the patches of light mock-suns or parhelia; the group of phenomena observed on a particular occasion is known as a halo complex. *Fig.* 7 is a simple example—the "halo of 22°" with the "upper tangent arc," the "mock-sun ring" and "mock-suns," sketched at Aberdeen by G. A. Clarke on March 5, 1908—and is reproduced as giving the impression received by an observer;

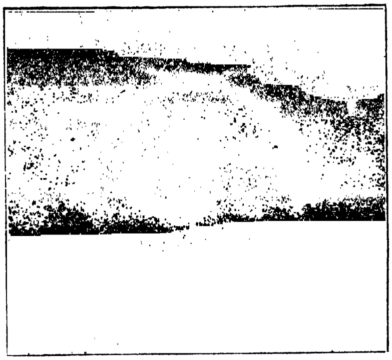


FIG. 7.—Solar Halo of 22° Radius with Upper Arc of Contact, Mock-sun Ring and Mock-suns. From a sketch by G. A. Clarke.

whilst in *Fig.* 8 we have one of the most elaborate complexes, the historic Petersburg halo, recorded by Lowitz on July 18, 1794.

The classical work on the subject is the *Mémoire sur les Halos* of Bravais, Paris, 1847. The valuable monograph by M. Louis Besson, "Sur la Théorie des Halos," Paris, *Annales de l'Observatoire de Montsouris*, ix. x., 1908, 1909, should also be mentioned.

§ (18) THE FORMS OF SNOW CRYSTALS.—Dobrowolski, meteorologist of the Belgian Polar Expedition, classifies snow crystals in three types according to the ratio of the length of the principal axis to that of the secondary axes; two extremes, laminae and needles, and an intermediate type, prisms. Transitional forms, thick lamina, and thick needles are very rare.

(a) Ice-needles have usually been found stuck together in little bundles, but no doubt they are formed separately. Their length, averaging about 2 mm., far exceeds their thickness, which is of the order 0·1 mm. Although earlier writers always describe the needles as hexagonal prisms, Nordenskjöld and

---

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, 1908, ii. 141.

Dobrowolski were not able to observe any definite crystalline form. Moreover, these ice-needles are believed to be derived from relatively low and warm clouds, and are therefore not available for halo formation.

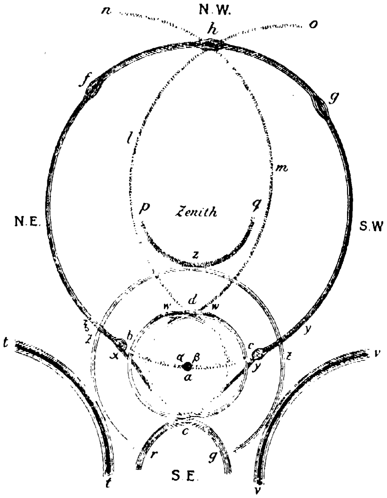


FIG. 8.—A Halo Complex, St. Petersburg, July 18, 1794, 10 A.M.

The sun is shown at *a*. The observer recorded two circles with centres *α* and *β*, but theorists regard these as the circular halo of 22° with the upper and lower tangent arcs completed to make a circumscribing ellipse. Parhelia are shown at *x*, *y*, the anthelion at *h* and paranthelia at *f*, *g*, all on a horizontal circle, the parhelic circle.

The parhelia *x*, *y* were coloured, as were the arcs from them to the inner halo, the " Arcs of Lowitz," so named after this observation.

The faint arcs *nhmd*, *ohld* are the oblique arcs through the anthelion.

The brightly coloured arc *rcg* is perhaps the Lower Arc analogous to Parry's Upper Arc.

The halo of 46° *zzz* is touched by the circumzenithal arc *pzq*, and by two infra-lateral arcs.

(*b*) The fundamental prismatic form is the " hemimorphic prism," which is a hexagonal prism surmounted by a pyramid, the principal section of the prism being a regular hexagon. The complete prisms which are found appear to be formed by the fusion of the pyramids of two hemimorphic prisms. There is usually a cavity inside each hemimorphic prism. As the cavity is near the base Dobrowolski thinks that there is probably a tendency for the prism to fall point downwards.

(*c*) The flat crystals, which have been so frequently photographed, are very thin, the thickness being about one-tenth of the diameter. The simplest form is hexagonal, but this form is developed by a process of growth from the crystals with symmetrical branches (see *Fig.* 9). Probably the branches are formed when conditions are favourable for rapid growth, and the crystal is consolidated subsequently. Sometimes the processes alternate three or four times, as is shown by the complex pattern. Even when the crystal takes the simple hexagonal form it is not suitable for halo formation. The greater part of the light must impinge on the large hexagonal faces and pass straight through. It is only rays which happen to be exactly parallel to these faces which can pass through the crystal edgeways, and owing to the mode of growth such light is likely to be diverted by flaws. Moreover, there appears to be no evidence as to whether the other faces are at right angles to the hexagons.

(*d*) Types (*b*) and (*c*) are frequently combined, giving a hemimorphic prism surmounted by a hexagonal lamina or a complete prism with a lamina at each end. The former arrangement is of importance, as the lamina will keep the prism vertical as it falls. According to Dobrowolski this is because the lamina will act as a parachute. The experiments of Besson, who observed the behaviour of ebony models in water,[1] show that such an object will fall point downwards provided that the width of the cap is greater than the height of the prism. Otherwise it will fall point upwards.

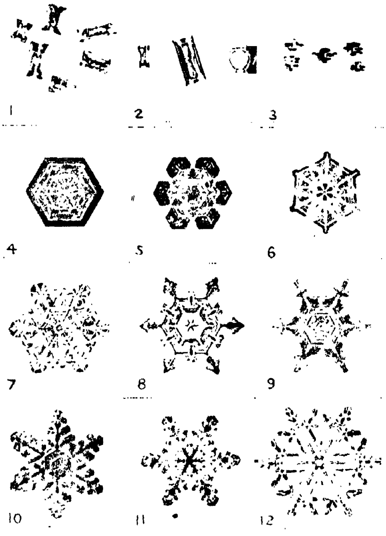


FIG. 9.—Snow Crystals. From Humphreys' *Physics of the Air.* Microphotographs by W. A. Bentley.

---

[1] *Ann. Soc. Météorologique de France*, 1907, lv 46, and *Ann. de l'Observatoire de Montsouris*, 1910.

In the classical works on the subject it was assumed without question that a falling body would set in such a way that it would meet with least resistance; the ice-needles, which were invoked to explain some of the phenomena of the halo complex, were supposed to tend to set themselves vertically whilst the laminar crystals were expected to fall edgewise. Everyday experience of falling bodies challenges the validity of these assumptions, and it is curious that they were accepted for so long. Some doubts on the subject were expressed by Bravais in a note published later than his famous memoir on halos, but it seems that it was not until 1901 that the opposite principle, that bodies fall in such a way that the air offers the maximum resistance, was enunciated by C. S. Hastings in this connection. Experiments by Besson indicate that this principle is valid in so far as flat discs fall flat and long prisms with their axes horizontal. It is of interest to notice, however, than an hexagonal prism falls with an edge downwards, not a face.[1] This observation appears to invalidate the halo theory of Hastings,[2] which requires the presence of a large number of crystals with one of the diagonals of the prism horizontal.

The general result of Dobrowolski's discussion is that the crystals effective in the production of the halo complex are likely to belong to classes (*b*) and (*d*) — the hemimorphic prisms with or without the cap having vertical edges and the complete prisms with or without caps having horizontal edges. He considers, however, that the evidence that these complete prisms are sufficiently numerous is insufficient.

In the classification of the elements of the halo complex we shall use the following notation:

X	Prisms with random orientation.
Y	Prisms with edges vertical.
Z	Prisms with edges horizontal.
YX	Prisms with edges nearly vertical.

In earlier works on the subject the Y group are regarded as ice-needles, the Z group as flat hexagons; following Dobrowolski we suppose the Y group consists of the hemihedral crystals with or without caps, the Z group of complete prisms. Hastings[3] makes the Y group contain the flat hexagons, the Z group elongated prisms.

§ (19) THE DEGREES OF FREEDOM OF ICE-CRYSTALS.—In the examination of theories put forward to explain the various phenomena, it is useful to count the degrees of freedom of the crystals which satisfy the conditions postulated. The distance from the observer being regarded as irrelevant, there are two degrees of freedom of position in the sky, two degrees of freedom for the orientation of the axis, one for rotation about this axis. The condition that the rays which pass through the crystal reach the observer deprives the crystal of two degrees of freedom, leaving three for the special circumstances. If two of these are used up the phenomenon produced is an arc, if all three are used up the light must appear to come from one point of the sky, and the phenomenon is a mock-sun or parhelion.

If, for example, the crystal must have the axis horizontal and the rays pass with minimum deviation two degrees of freedom are used up and the phenomenon is an arc. Making the axis vertical takes away two degrees of freedom, and if minimum deviation takes a third the phenomenon is a parhelion.

§ (20) REFRACTION THROUGH TWO FACES INCLINED AT 60°. (i.) *Halo of 22°.* X. *Double Minimum Deviation.*—Alternate faces of a hexagonal prism are inclined at 60°. The index of refraction of ice being known, the minimum deviation of rays which are refracted at two such faces can be computed.

The minimum has the smallest value when the axis of the crystal is perpendicular to the plane of incidence. It is then approximately 22°. No sunlight passing through X crystals can reach the observer in any direction inclined at less than 22° to the direct beam. The angle is rather smaller for the red end of the spectrum. The halo of 22° which is red on the edge nearest the sun and falls off gradually in luminosity outwards is accordingly explained. The rays which have deviation more than the minimum account for a certain amount of luminosity outside the halo.

(ii.) *Parhelion of 22°.* Y.—When the prisms are all vertical a ray from sun to observer cannot pass through a prism in a principal plane. The minimum deviation is accordingly greater than for X crystals. Since there are three special restrictions on the degrees of freedom (vertical crystal 2, min. deviation 1) the image of a point source of monochromatic light would be a point. The parhelia explained in this way are highly coloured patches red on the side nearest the sun; horizontal white streaks extending from the parhelia away from the sun are explained by rays which have not had minimum deviation. When the sun is low the parhelia are close to the halo of 22°, but for greater altitudes they are well outside it.

For example, with the sun 55° above the horizon the inner edge of the parhelion is distant 36½° from the sun.

(iii.) *Extended Parhelia.* YX.—Oscillation of the crystals about the vertical in all possible planes would give blurred and extended parhelia; such have been photographed by Schultz.[1]

---

[1] *Ciel et Terre*, 1907, xxviii. 310.

[2] *Monthly Weather Review*, 1920, xlviii. 322.

[3] *Ibid.*, June 1920.

(iv.) *Arcs of Lowitz.* YX.—Tilting of the prisms is also the accepted explanation of these very rare arcs which are seen below the parhelia connecting them obliquely with the halo of 22° (see *Fig.* 8). To produce arcs there must be some restriction on the freedom of oscillation. Possibly capped prisms oscillate most readily in such a way that a diagonal of the hexagonal cap remains horizontal.[2]

(v.) *Upper and Lower Tangent Arcs.* Z.—Minimum deviation through prisms with their axes horizontal accounts for these arcs which touch the 22° halo at its highest and lowest points. The shapes of the arcs depends on the elevation of the sun. When the elevation is very low the upper arc is a cupid's bow, when the elevation is 35° or more the two arcs combine to form a curve approximating to a circumscribing ellipse.

(vi.) *Parry's Upper Arc.* ZZ.—The arc to which this name is given by Hastings is above the upper tangent arc. It is attributed by this author to the Z crystals which float not only with the axis horizontal, but also with a diagonal of the hexagonal cross-section horizontal. The deviation is not a minimum. Besson's observation mentioned above indicates that isolated crystals would not be stable in the position postulated by Hastings. M. Besson has pointed out, however, that complex crystals built up of several prisms fitted together like so many spokes would float with the spokes horizontal, and it may be that in such a case the cross-section of a spoke would satisfy the condition laid down by Hastings.

(vii.) *Parhelic Sun Pillar.*—A pillar of light such as appears over the sun in the photograph by Schultz, mentioned above, may be regarded as the combined halos of the two parhelia, *i.e.* it is formed by light passing through pairs of crystals giving equal and opposite deviation.

§ (21) REFRACTION THROUGH TWO FACES INCLINED AT 90°. (i.) *Halo of* 46°. X. *Double Minimum Deviation.*—The ends of a hexagonal prism, whether simple or capped, are at right angles to the other faces. Light can enter at the end and emerge at another face or *vice versa*, so that the effective refracting angle is 90°. The corresponding minimum deviation is 46°. The halo of 46° about the luminary is explained at once.

(ii.) *Circumzenithal Arc.* Y.—These arcs are analogous to the Parry arcs. They are due to hexagonal prisms floating with the bases horizontal. They are frequently called horizontal tangent arcs of the 46° halo, but, as Bravais shows, they are not true tangents; they can appear when the 46° halo is missing, and they are especially brilliant when the only other phenomena are the parhelia of 22° (also due to Y crystals). The arc is part of a true circle about the zenith.[3] The circumhorizontal arc at the bottom of the 46° halo has only been recorded on few occasions. (Bravais quotes two examples—one solar, one lunar. Recent observations will be found in the *Monthly Weather Review*, 1916, 1917, 1918.)

(iii.) *Kern's Arc.* Y.—This arc is at the same height as the circumzenithal arc on the opposite side of the zenith. Ekama[4] explains it as due to light entering the horizontal cap, reflected from one vertical face and emerging through the opposite face.

(iv.) *Lateral Tangent Arcs of the* 46° *Halo.* Z.—The infralateral tangent arcs of the 46° halo are convex to the sun. The height of the point of contact depends on that of the sun. Theorists agree that these arcs are due to such rays as enter a horizontal prism at one end and emerge through one of the long faces. In the theory of Bravais the prisms giving minimum deviation are selected. Hastings[5] postulates crystals with one face horizontal, and supposes that the light enters through the vertical end and emerges through a face inclined at 60° to the horizontal. Symmetry suggests that supralateral tangent arcs should exist. If they do they have been confounded by observers with a brightening of the shoulders of the 46° halo.

(v.) *Parhelia of* 46°.—Parhelia at the intersection of the parhelic circle and the 46° halo are observed, occasionally even in the absence of the halo. Unlike the parhelia of 22°, these parhelia are precisely on the 46° circle at all elevations of the sun. A satisfactory explanation is wanting.

§ (22) REFLECTION PHENOMENA. *The Parhelic Circle.* Y.—A conspicuous feature of many halo complexes is a horizontal circle through the sun; this is known as the mock-sun ring or parhelic circle. It is explained most simply by the external reflection of light from vertical faces of Y crystals, but it may be reinforced by light passing through such crystals and reflected internally and perhaps by the light reflected from the ends of Z crystals.

---

[1] Reproduced by Hastings, *Monthly Weather Review*, 1920, xlviii. 322.

[2] Cf. Hastings, *l.c.* p. 329. Hastings postulates hexagonal laminae spinning with a diagonal horizontal. Fujiwhara and Oti (*Bulletin of the Central Meteorological Observatory of Japan*, 1919, iii. 33) discuss the theories of Galle and Pernter and consider that no restriction on the mode of oscillation is necessary for the true arcs of Lowitz; their theory of what they call the Lowitz arcs of the second kind appears to involve such an assumption as is made above. The work of these authors is open to criticism, however.

[3] Pernter, following Galle, believes that the phenomenon actually observed is not a true circumzenithal arc but a tangential arc formed by refraction at minimum deviation through prisms with the refracting edge horizontal. Pernter gives no mechanical reason for the edge remaining horizontal.

[4] *Onweders in Nederland*, 1895, p. 66.

[5] *L.c.* p. 326. Cf. Bravais, *l.c.* p. 189.

The Anthelion. (i.) ZZ.—Observations of the anthelion, a luminous patch at the same height as the sun and opposite to it, are not common. In many cases this point is merely noticed as the intersection of certain oblique arcs, but Bravais considers its independent existence established.[1] The suggestions put forward for its explanation depend on reflection in succession from two faces at right angles with a common vertical edge. Bravais proposes internal reflection, the light entering and emerging by the same face, whilst Humphreys puts forward the possibility of external reflection from one face and from the under side of the cap of a hexagonal prism. In either case the prism must be floating with a diagonal of the hexagonal section vertical.

(ii.) *Oblique Arcs through the Anthelion.*—These two arcs cut the parhelic circle at the anthelion. They are usually confined to this neighbourhood, but occasionally they form a loop surrounding the zenith. Bravais's theory of the arcs depends on the striation on the surface of tabular crystals and does not seem tenable. Hastings regards the arcs as produced by light passing through Z crystals entering by a horizontal surface, reflected by the vertical end and emerging through a surface inclined at 60° to the horizon, but the details of this theory have not been published.

(iii.) *Paranthelia.* Y.—Bright spots on the half of the parhelic circle nearer to the anthelion are known as paranthelia. The best authenticated are 60° in azimuth from the anthelion, or 120° in azimuth from the sun. These are due in part to light which has been reflected internally at two vertical faces of a Y crystal. One way in which this may happen was shown by Soret; numbering the vertical faces consecutively from 1 to 6 we may suppose the light to enter by 1, to be reflected by 3 and 5, and to emerge through 2. Light reflected externally from re-entrant angles of star-shaped laminar crystals would also reinforce these paranthelia.

(iv.) *Paranthelic Arcs.*—A short arc is occasionally observed passing obliquely through the paranthelia of 120°. The explanation is still in doubt.

(v.) *Sun Pillars.* Y.—Sun pillars are vertical white streaks passing through the sun. They are best seen when the sun is below the horizon. They are accounted for by the external reflection of light by flat laminar crystals. Such crystals will not remain quite horizontal as they flutter downwards. The drawn-out image of the sun is analogous to the reflection in the rippled surface of a lake.

(vi.) *Pseudhelia.* Y.—Analogous to the sun-pillars are pseudhelia, reflections of the sun seen when the observer is above the cloud.

(vii.) The *Halo of 90°* and the *Paranthelia of 90°* are white and should perhaps be classed as reflection phenomena. They are rare, however.

§ (23) REFRACTION THROUGH TWO FACES INCLINED AT ANGLES OTHER THAN 60° OR 90°.—*Halos with abnormal radii* have been reported occasionally. The evidence has been summarised by Besson, who groups the observations as follows:[2]

Name of First Observer.	Number of Observations.	Radius observed.
Van Buijsen .	8	7° 30′ to 10°
Rankin . .	6	17°, 18°
Burney . .	6	18° 30′, 20°
Dutheil . .	1	21°
Scheiner . .	5	28°
Feuillée . .	4	32° to 35°

In some of the observations three of the abnormal halos were seen simultaneously and in others they were seen together with the 22° halo. Besson comes to the conclusion[3] that all with the exception of Scheiner's can be explained by a single hypothesis. This hypothesis is that each of the operative ice-crystals is in the form of a hexagonal prism with edges cut off by facets inclined at 25° 14′ to the principal axis. Rays of light may intersect either two such facets or one facet and the base or a side, so that sufficient variety in the refracting angle is provided for. It should be mentioned, however, that Besson is not able to quote direct evidence for the existence of crystals with facets at the inclination he postulates.

F. J. W. W.

METERS[4]

I. TACHOMETERS, SPEEDOMETERS, AND MISCELLANEOUS SPEED-INDICATING DEVICES

ONE of the most important quantities of ordinary engineering practice is the rate of revolution of rotating parts. For instance, it is necessary to know the r.p.m. of machinery accurately in order to maintain the frequency of electrical alternating supply, to fix the most economical speed of machine tools, and for safeguarding machinery parts from excessive loads due to over-running. The angular velocity is an essential factor in the measurement of power output by dynamometer tests.

In addition to the above applications tachometers are often used to deduce the linear speed of motor vehicles and locomotives from the speed of rotation of their wheels.

---

[1] More recent observations have been recorded, *e.g.* at Montsouris, April 5 1899.

[2] Paris, *Comptes Rendus*, 1920, clxx. 334.

[3] *Ibid.* p. 609.

[4] The writer desires to record his indebtedness to Messrs. George Routledge & Sons for permission to draw upon material published in *Engineering Instruments and Meters*, in the preparation of this article.

The simplest workshop method of estimating r.p.m. is by the use of a geared counter in conjunction with a watch. Such counters are made in various forms.

§ (1) COUNTER INSTRUMENTS.—A development of the counter principle in a convenient form is found in the Hasler instrument. In this device the watch and counter are combined and the operation of pressing one button winds the watch, automatically connects the counting train to the driving shaft for a definite period of seconds, and then disconnects it. The graduation on the scale is arranged to read revolutions per minute directly. A diagrammatic sketch of the mechanism employed is shown in *Fig.* 1. The downward movement of the button depresses a rack against the pull of a small spring; when the button is released the spring drives the escapement wheel at a definite rate under the control of the balance wheel and lever. The shaft of the escapement wheel carries a cam, which after a rotation corresponding to one second releases the large ratchet wheel connected to the pointer. The wheel and pointer are then carried around by friction with the counter train for three seconds, after which interval the cam has rotated to a position in which the pawl is pressed up to arrest the motion of the ratchet wheel. The zeroising button returns the pointer to zero after use in the same manner as a stop watch. A simple gearing in the counter train causes the pointer to rotate in a clock-wise direction for either direction of rotation of the driving shaft. The instrument is very convenient and accurate in practice. It has the advantage of only needing a short interval of time for a determination of the speed.

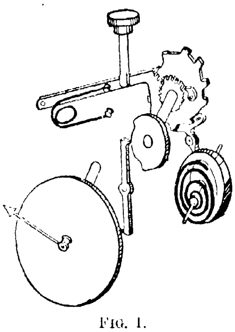


FIG. 1.

Other combinations of watch and counter are in common use, but the two units are usually kept more distinct than in the above-described instrument.

For the continuous indication of speed the tachometers employed may be broadly classified into five groups, according to the principle upon which their action is based, which may be:

(*a*) Centrifugal force.
(*b*) Chronometric action.
(*c*) Magnetic drag.
(*d*) Electrical similar to dynamo.
(*e*) Viscous drag.

In the following description no differentiation is made between tachometers and speedometers, since the only point of variation is usually the addition of an adding device or totaliser to the mechanism of the tachometer to convert it into a speedometer.

§ (2) MECHANICAL CENTRIFUGAL INSTRUMENTS.—Centrifugal tachometers depend upon the variation of the centrifugal force exerted by small masses rotating in a circle. The force tending to move each mass outward is proportional to the square of the angular velocity, multiplied by the radius of the circle in which it rotates. The masses are usually restrained by a spring, and the outward movement magnified by levers and gears.

The relation between displacement and speed varies for each particular design of instrument depending upon the configuration of the lever-system and type of spring control. The calibration cannot usually be predicted *à priori*. The chief endeavour of the designer is to make the lever arrangement such that the graduations on the scale are as nearly equidistant at all parts as possible.

In one form three equal discs are arranged on short connecting levers at equal angles around the central spindle. These discs are thrown outward by the centrifugal force against the influence of the helical spring, and the movement is transmitted through the collar to the geared quadrant which actuates the pointer wheel. The scale obtained is fairly uniform over an angular range of about three hundred degrees. The governor type of instrument is not much favoured owing to the comparatively large number of joints in the levers which are apt to wear and cause back-lash. Generally the centrifugal masses take the form of a ring pendulum or a crossbar free to swing about an axis traverse to the rotating spindle.

A typical example of a ring pendulum mechanism is shown in *Fig.* 2, and is largely used for stationary machinery. This instrument was invented as far back as 1884 by Schaffer and Budenberg. The driving shaft carries a circular disc of metal B, the position of which when the shaft is at rest is rendered

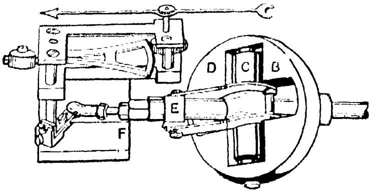


FIG. 2.

oblique to the shaft by means of a coil-spring contained in the barrel C. When the shaft rotates the centrifugal force tends to turn the ring, so that its plane is at right angles to the axis of the shaft. The movement of the ring is communicated by the rods D to the sleeve E, and thence through the ball joint F to the crank arm which actuates the quadrant and gear on the pointer spindle. The scale obtained is fairly uniform, but widens out somewhat at the middle of its range.

A lightly constructed modification of the above instrument is used on motor vehicles and aero engines. The ring is replaced by a crossbar, to which is fixed a pair of weights and a sliding muff, which gives a very simple form of construction. The control spring is a flat coiled spring, and the motion is communicated by the connecting links to a sliding muff or collar. The groove in this collar engages with a cranked pin on the gear quadrant, which in turn actuates the gear wheel on the pointer spindle. A small spring is fitted to the pointer spindle to take up back-lash. The pointer rotates through nearly one complete turn for the full range, and in some cases, in order to give a still more open scale, the graduation is arranged to start at, say, one quarter the full scale reading by putting an initial tension on the spring. Three-quarters of the scale is thus spread out to occupy the full length. Instruments of this type are not dynamically balanced at all speeds, and the couple at right angles to the driving spindle often causes the entire instrument to vibrate slightly if it is not rigidly mounted. Complete balance can be obtained by using two weighted bars disposed symmetrically on either side of the shaft.

The indications of any centrifugal instrument are the same for either direction of rotation. A few general considerations with regard to the design of such instruments might be mentioned here. The number of joints and levers should be kept as small as possible to avoid friction and slack due to wear. A hairspring should always be fitted to the pointer spindle to take up back-lash as a little play is inevitable, particularly at the groove in the sliding collar. All the moving parts, including the gear quadrant, should be statically balanced to eliminate the effect of accelerations when the instrument is used on vehicles. The inertia of the moving parts should be as small as is consistent with sufficient power to actuate satisfactorily, especially if the instrument is to be used on machinery subject to sudden fluctuations of speed. Otherwise the driving shaft, if of the flexible type, is liable to be over-stressed by the inertia forces when the speed varies rapidly. The accuracy can be within ±1 per cent under ordinary conditions, although the author has found commercial instruments of the speedometer class to be in error often by as much as 6 per cent at full scale. The temperature coefficient is almost negligible for ordinary work.

§ (3) CHRONOMETRIC INSTRUMENTS.—The name chronometric is generally applied to a class of instruments in which the number of revolutions made by the driving shaft is automatically and repeatedly counted for small equal intervals of time. The mechanism consists of a small clock escapement frictionally driven from the driving shaft, and some device operated by the clock which connects the driving shaft to the counting train for a definite time interval.

As a typical example, one of the least complicated of this class made by Van Sicklen will be described. This instrument counts the revolutions for repeated periods of one second, the pointer remaining steady at the reading for the average speed during the previous count. The mechanism is shown diagrammatically in *Fig.* 3, the trains of wheels being omitted for clearness. The rotation of the driving shaft winds up by friction a watch spring which drives the escapement wheel.

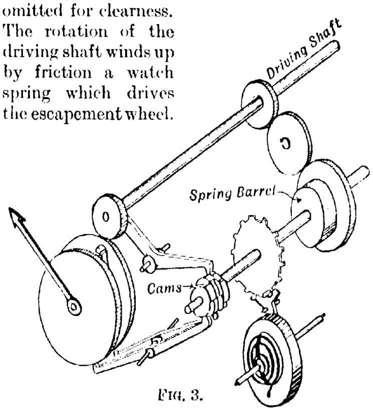


FIG. 3.

The spring barrel is fixed to the shaft which carries three cams. The escapement allows the cam shaft to make one thirty-second of a revolution suddenly every half-second. One of the cam followers moves the fine-toothed wheel on the driving shaft in or out of engagement with another wheel which is loose on the pointer spindle. The cam allows the wheels to engage for exactly one second. A similar fine-toothed wheel is fixed to the pointer, both of the wheels being controlled by hairsprings tending to return them to a zero stop. The other two cams actuate pawls working on the two wheels. The pawl on the loose wheel allows it to return to zero after each operation, so that the angle through which it is turned when thrown into gear with the driving shaft for one second is a measure of the r.p.m. of the latter. A projection on the loose wheel catches against a spring arm fixed to the pointer wheel and carries this wheel around with it. While the loose wheel is released to return to zero the first wheel is held by the pawl. Should the speed vary, the spring attached to the pointer wheel is pushed forward and the wheel takes up a new position when released by its pawl. When a decrease in speed occurs the wheel falls back until the spring reaches the projection. Thus the pointer does not return to zero after every count, but remains indicating the speed for the previous second.

Other instruments of this class have more elaborate mechanism; for example, the "Tell" instrument has three vertical parallel shafts. The first carries three sets of cams, and its speed of rotation is governed and kept constant by the escapement. These cams, of which there are six, act through six levers on to three double crown wheels mounted loosely on the second shaft. Three levers have ratchet ends which engage with the teeth on one side of the crown wheels, and lock them, whilst the other levers actually lift the crown wheels and bring them alternately in and out of mesh with pawls which are carried round by the second shaft, which is driven from the main driving spindle of the indicator, the speed of which is required to be known. Each double crown wheel drives one of three fairly large diameter wheels on which a pin projects which carries forward a bar fixed on two arms and rotating on the same axis. The second and third double crown wheels act in a similar way at equal intervals of time, in such a way that the preceding wheel is not "unlocked" until the next one has been urged forward and locked in the position to which it has been wound in the interval during which it is driven from the main spindle.

The action is as follows: The cams throw a double crown wheel into mesh with the driving spindle for a definite period of time, and at the instant of being thrown out of mesh they are locked in position. During this time the bar has been carried forward against a spring which tends to return it to a zero position. Before the first wheel is unlocked a second has gone through the same cycle and subsequently maintains the position of the bar unless the speed has altered in the interval, in which case it pushes it farther forward for an increasing speed and lets it back if the speed has decreased. The large wheels gearing with the double crown wheels are also returned to a zero position by means of a spring. The movement of the bar is recorded by the pointer through a special form of rack and pinion gearing. There are, of course, several other well-known chronometric tachometers, such as the Jaeger and Isochronous, but they do not differ sufficiently from the foregoing to need special description here.

Chronometric instruments have a perfectly uniform scale over the entire range of speed indicated, and, moreover, the scale is fixed by the dimensions of cams and wheels and can only be changed by the wearing down of the cams or changing the wheels in the counting train. The accuracy of the best class of commercial instrument is within ± ½ per cent of full scale reading. The attainable accuracy is, however, governed by the size of the teeth on the wheels, since in some instruments the width of a tooth may be equal to several r.p.m. divisions on the scale. The temperature coefficient is practically that of the time element in the mechanism and negligibly small. If a flexible driving shaft is used this should be arranged to run very steadily, otherwise a movement of the pointer occurs up and down in small jerks, rendering a true average difficult of estimation.

§ (4) CENTRIFUGAL FLUID TACHOMETERS.—Tachometers depending upon the centrifugal forces brought into play by the whirling of fluids are characterised by the simplicity of their construction and by the fact that the calibration can usually be calculated from the dimensions of the instrument without reference to a standard for comparison. This, of course, is a considerable advantage. The most elementary example of the fluid type is the rotating cup used as a rough indicator on centrifugal cream-separators, etc. When a body of liquid is rotated in an open vessel the free surface of the liquid takes the form of a paraboloid with its apex on the axis of rotation and pointing downwards. *Fig.* 4 is a sketch of the form of the liquid surface.

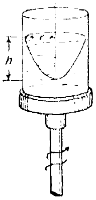


FIG. 4.

It can be shown that the depression h is given by the expressions

$$h = \frac{\omega^2 r^2}{2g}, \quad \text{or } h = \frac{\pi^2 n^2 r^2}{30g},$$

where r = radius of free surface,
n = speed in revolutions per minute,
g = acceleration due to gravity,
ω = angular velocity in radians per second.

The value of h therefore depends on the square of the speed and the device is thus most suitable for high speeds. Since the depression is also proportional to the square of the radius of the free surface, it is evident that, if the shape of the cup is other than parallel, the calibration can be made to vary from the square law, and shaped cups are sometimes used in practice.

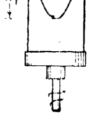


FIG. 5.

An interesting case of whirling liquids is that in which the liquid is contained in a closed cylinder, and the edge of the parabola reaches the top of the vessel after a certain speed, as shown in *Fig.* 5. It can be proved theoretically that the depth h_1 of the parabola after this critical value becomes a linear function of the speed.

That is,
$$h_1 = \omega r \sqrt{\frac{h}{2g}}$$
$$= 2\pi n r \sqrt{\frac{h}{2g}},$$

where h_1 is the depth from the top of vessel. h is the depth of paraboloid just as it touches the top.
r is the radius of cylinder.

The calibration will therefore vary as the square of the speed up to a certain value determined by the point where the liquid edge just touches the top, and above this value of the speed the depth of the apex will vary directly as the speed. This particular form of cup is the one generally used in tachometers.

§ (5) CENTRIFUGAL PUMP AND PRESSURE GAUGE INSTRUMENT.—Tachometers which utilise the dynamic head or the suction created by an elementary centrifugal pump are a well-known class of speed-measuring instruments. This principle was first used by Stroudley in 1879. In one form of instrument a small impeller with radial vanes rotates in a chamber containing liquid, and the head due to centrifugal force on the rotating fluid is transmitted to the column above the body of the instrument. A small reservoir surrounds the bottom of the tube to indicate the zero or datum line of pressure, and it is a very important point that this should not change due to leakage, etc., as any variations in level affect the height of the pressure column by an equal amount.

The calibration of the scale is very nearly proportional to the square of the speed, and the instrument is well suited for fairly constant speed machinery, since the indication can be arranged to occur at the upper or open part of the scale by the use of the appropriate gear ratio. The accuracy of fluid types of tachometers depends very largely on the construction and the conditions under which they are used.

All the above instruments are of course independent of the density, as this factor enters directly into the dynamic head produced by the pump and the hydraulic head of the measuring column. Also for the same reason they have a very small temperature coefficient, provided all the liquid is at a uniform temperature. This condition requires that self-heating in the pump should be negligible.

§ (6) AERODYNAMIC TACHOMETERS.—Instruments depending upon the centrifugal force exerted by an air column when rotated, have found some slight application. Their chief merit lies in their simplicity.

The usual arrangement is shown in *Fig.* 6. The tube A rotates about a vertical axis, and the centrifugal force on the particles of air contained in the tube creates a radial pressure gradient. The pressure at the open end of the tube being equal to that of the atmosphere, a suction is set up at the centre, and this suction is measured by a U-tube, or a sensitive vacuum gauge fixed at any convenient point. The joint between the rotating tube and the stationary tube is effected by a simple mercury seal. The suction obtained varies as the square of the speed and is, of course, very small even when the instrument runs at a high speed.

One of the disadvantages of this type of instrument is the influence of the variability in the atmospheric density due to changes in the barometer and the temperature. An error amounting to several per cent is possible from day to day due to these causes.

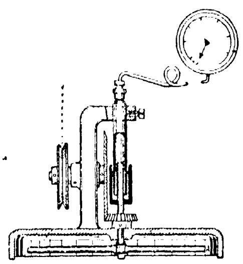


FIG. 6.

Another pattern of aerodynamic instrument which does not, however, depend upon centrifugal force is the Air-Vane type. The principle of this device is the use of two small air vanes mounted in close proximity, one rotated by the driving shaft and the other pivoted and free to rotate under the control of a spring. The torque on the pivoted vane depends approximately upon the square of speed of the driven vane and directly upon the density of the surrounding medium (air). The spring is so arranged that the controlling force increases with the deflection to counteract the increased torque, and this gives a fairly uniform scale. The disadvantage of variable density, already mentioned, limits the accuracy obtainable. Moreover, the rotation of the vane itself tends to heat up the air inside the case, which results in a change of density. This inaccuracy could be eliminated by hermetically sealing the case, but the expedient would give rise to serious practical difficulties in the construction.

§ (7) MAGNETIC TACHOMETER.—It is a well-known fact that if a magnet is rotated near a sheet of electrically conducting material, the sheet will experience a couple about the axis of rotation of the magnet due to the eddy currents induced in it by the motion of the magnet.[1] Tachometers designed on this simple principle have found extensive application. The two best known instruments of the class are the "Warner" and the "Stewart."

The Warner instrument (*Fig.* 7) has a metal drum A mounted on the axis B, so that its rim cuts the field due to the magnet C. The magnet is rotated by a shaft connected to the machine, whose r.p.m. is being measured. The eddy currents induced in the drum drag it around against the action of the spring E. The edge of the drum is graduated and viewed through a small window at the side. The divisions are equidistant at all points of the scale. The adjustment for calibrating the instrument is made by simply screwing the bearing in or out to decrease or increase the air-gap in the magnetic path and thus vary the deflection.

---

[1] This principle is frequently used in the design of a damping device in electrical instruments.

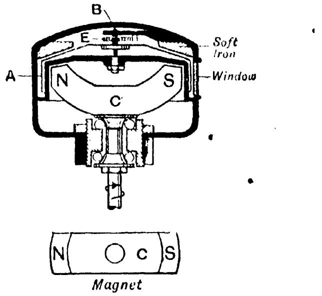


FIG. 7.

The Stewart instrument is simpler, the magnet being circular, as shown in A (*Fig.* 8). The eddy current disc, as in the Warner, is made in the form of a drum B, fitting over the magnet with one long pivot C in the axis of the magnet. In order to reduce friction no bearing is fitted at the top, where the control spring C is fastened. The graduations in the simplest form are printed directly on the edge of the drum. A simple method is employed to set the calibration. The magnet A is provided with a small soft iron plate, fitting tightly on the axis, and by rotating this plate by a very small amount relative to the magnet, with which it turns, more or less of the magnetic field can be short-circuited. The magnetic field in which the drum moves can therefore be changed and the calibration set, or corrected at any time, provided the magnet is not too weak.

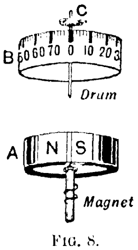


FIG. 8.

Some instruments are fitted with a mileage counter to render them suitable for motor-car work as speedometers

The temperature coefficient of this type of tachometer is usually very large, being of the order of ½ per cent per degree centigrade. To obtain as great torque as possible on the disc for a given speed the magnetic field should be strong and the disc of low electrical resistance. The first factor is limited by the size of the magnet, which must not be cumbersome or possess a large moment of inertia. The second factor, low electrical resistance, demands the use of a pure metal, such as silver, copper, or aluminium. Aluminium is the one in general use on account of its low density. These metals have temperature coefficients of resistance of the order of 0·4 per cent per degree, and to completely eliminate this factor would necessitate the use of some alloy, such as manganin, which has the attendant disadvantage of high specific resistance. If a drum of this alloy were used the torque would be reduced in the inverse ratio of the electrical resistance, *i.e.* 1 to 30 approximately. It is not, of course, difficult to devise automatic compensation for the temperature coefficient, but in commercial practice very little attention is paid to the point, cheapness of construction being a vital consideration. One continental instrument, however, has a compensating device which varies the air gap by the use of a liquid-filled capsule. Also, some models of the Warner instrument have a bimetallic strip compensator, and this is effective.

§ (8) ELECTRICAL TACHOMETERS, MAGNETO GENERATOR INSTRUMENT. This type of tachometer is widely used under conditions where the indicator has to be placed at a considerable distance from the machine whose speed is required, as, for example, on board ship. The essential part of the device is a small dynamo generating continuous or alternating current; this generally takes the form of a permanent magnet dynamo. A suitable voltmeter is employed with a scale graduated in r.p.m. A generator of the continuous current type is shown in *Fig.* 9. The armature may have from six to eighteen slots, according to circumstances, and it is desirable that it should be wound with the maximum number of turns in order to obtain a high voltage. This armature rotates in a tunnel in the soft iron shoes fitted to a permanent magnet or alternately in a tunnel ground in the ends of the magnet itself. The clearances between the rotating parts should be made very small, as this is advantageous in two ways: it helps to maintain the strength of the magnet constant, and also gives a stronger magnetic field. The commutator and brushes both require careful design and skilled workmanship. Carbon brushes need a considerable pressure and are liable to chatter under vibration. They are, moreover, susceptible to any readjustment, and must be run in before the calibration becomes consistent.

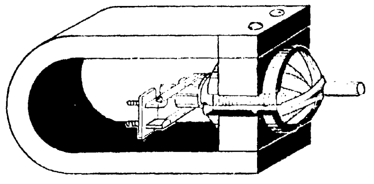


FIG. 9.

The modern tendency is towards brushes made from narrow strips or fingers, both brushes and commutator being made of silver or gold to avoid oxidation. The current taken from these generators is so small that armature reaction does not enter into consideration, and the voltage generated is almost exactly a linear function of the speed. The indicator is a standard electrical instrument of moving coil pattern, as shown in *Fig.* 10. This instrument is connected by a pair of insulated wires to the terminals of the generator, and the distance between the two units is quite immaterial, provided the electrical resistance of these leads does not become comparable with that of the instrument. The current in the circuit is usually not more than about one-hundredth of an ampere at full scale, and as the magneto generates at least five volts per thousand revolutions per minute, a large swamping resistance is employed in the circuit.

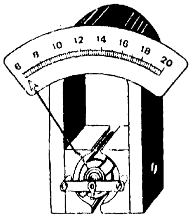


FIG. 10.

The object of generating a comparatively high voltage and then using a high resistance in the circuit is twofold.[1] In the first place it renders the variations in the resistance of the connecting wires unimportant, and, secondly, it enables the effects of changes in temperature to be rendered small. The temperature coefficient is the most serious difficulty in connection with the design of electrical tachometers. Temperature changes affect (*a*) the magnetic strength of the magnets in both instrument and generator; (*b*) the strength of the control spring in the instrument; and (*c*) the resistance of the windings of both indicator and generator. The temperature coefficients of the magnet and the spring in the indicator may, to some extent, counteract one another; but the temperature effect on the magnet of the generator remains uncompensated.

The resistance of the copper winding can be made small in value in comparison with total resistance. This artifice, however, only affords a partial solution of the difficulty, and it would be preferable to eliminate copper entirely from the windings of both the indicator coil and the generator armature and use an alloy of negligible temperature coefficient such as manganin. The writer has tried this and found it successful, the only practical difficulty being the stiffness of the manganin when winding the moving coil former. It would of course be possible to use very fine wire under these conditions, since the "swamp" resistance is not then required and need only be retained for the purposes of adjustment to secure correct range and interchangeability.

The temperature coefficient of a well-designed speed-measuring set can be made of less than 0·1 per cent per degree C., but the average instrument is found to have a temperature coefficient of more than twice this amount. It is advisable to shield the instruments, both generator and indicator, in soft iron cases to eliminate the effect of stray fields and the proximity of magnetic material.

Alternating-current tachometers were at one time used to a considerable extent on motor vehicles, and these were very simple in construction. The generator was of the simple inductor type with a stationary winding and without either brushes or slip rings. The indicator was a miniature hot-wire instrument, the indication depending on the expansion of the wire due to the heating effect of the electric current. A sketch of the complete instrument is given in *Fig.* 11. The arrangement was cheap to construct and compact in form, but no attempt was made to attain great accuracy or to reduce the temperature coefficient.

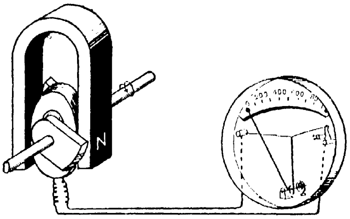


FIG. 11.

§ (9) SQUIRREL CAGE SPEED INDICATOR.—An interesting instrument, which depends upon the angular position of the resultant of two magnetic fields in the interior of a squirrel cage winding, has been invented by Mr. E. B. Brown,[1] of Melbourne. The instrument consists of a cylindrical soft-iron armature core, which can be rotated on its axis between pole-pieces attached to a permanent magnet. This armature is provided with slots or tunnels carrying a number of insulated conductors short-circuited by rings at both ends of the armature in a manner similar to the well-known squirrel cage winding used in A.C. motors. At one end of the armature the squirrel cage projects considerably beyond the armature core, as shown in *Fig.* 12, in which A is the armature core (which may be laminated), C are the conductors (which may be any number from three upwards), and R_1 and R_2 are the short-circuiting rings. The conductors C are insulated in the slots from the core, but the ring R_1 need not be insulated and may be sweated or screwed on to the core. When the armature is rotated between the pole-pieces of a permanent magnet, E.M.F.'s are set up in the conductors. If the rotation round the axis (assumed vertical) be in the direction of the arrow (*Fig.* 13), the E.M.F.'s will be upwards on the left-hand side of the armature, and downwards on the right-hand side. There will consequently be a belt of current flowing up one side of the armature and down the other. The direction of the current in the conductors is indicated in *Fig.* 13, and also the forward displacement of the line of zero current, which results from the self-induction of the armature winding.

---

[1] See "Voltmeters," Vol. II.

[1] *The Electrician*, 1917, lxxx. 117.

In *Fig.* 13 OE represents the magnetic field due to the currents in the armature, OF the field due to the permanent magnets, and

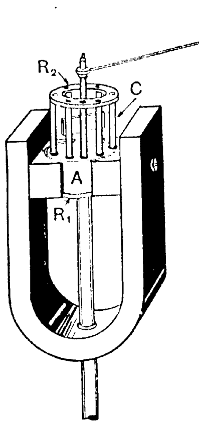


FIG. 12.

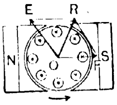


FIG. 13.

OR the resultant. The direction of OR, "the resultant field," depends on the speed of rotation of the armature, and it is only necessary to add a small pivoted vane of soft iron attached to a pointer and some form of damping mechanism, to make the apparatus a practical speed indicator. The soft iron is enclosed in a fixed tube to prevent the action of air currents (this is omitted from the diagram for clearness), and is attached to a vertical pivoted axis, which also carries an air-damping vane and a pointer moving over a graduated scale. *Fig.* 14 gives a calibration curve of an instrument of this type.

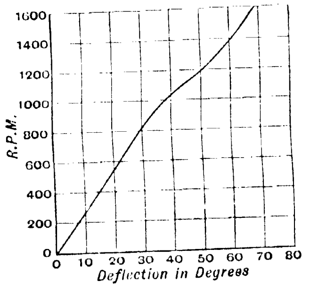


FIG. 14.

As is usual in such instruments, the moving element is balanced about its axis, so that the instrument can be used in an inclined position. No control spring is necessary, because the plate of soft iron takes up the direction of the resultant field, while moderate variations in the moment of the magnet will not affect the indications of the instrument, since they affect both initial and deflecting fields in the same ratio, and thus the direction of the resultant is unchanged.

§ (10) VISCOSITY INSTRUMENTS. (i.) *Viscosity of Mercury Tachometers.*—Tachometers making use of the variations in the viscous drag on a solid immersed in a whirling fluid have been developed in a variety of forms. In one type, which is used on aircraft, the arrangement consists essentially of a perforated circular disc fixed to the driving shaft and rotating close to a small cross-arm fixed to the pointer spindle, as shown in the sketch (*Fig.* 15). The disc and arm are contained in a cavity full of mercury. The mercury, being carried around by the disc, tends to drag the cross-arm with it against the pull of the spiral spring. The torque exerted upon the arm depends approximately upon the square of the speed, and the scale opens out in the middle, but closes in rapidly near the top, due presumably to slip between the mercury and the plate. The device is very simple and compact, the mercury chamber being only about one inch in diameter. The instrument is fairly reliable; its chief disadvantage lies in the fact that the spindle of the pointer has to pass through a hole and be made mercury-tight. The friction here must necessarily be more than if the spindle were pivoted. The temperature coefficient of viscosity is small. In the case of a well-made instrument the temperature coefficient was found to be 0·03 per cent per degree C., and this includes the temperature coefficient of the control spring.

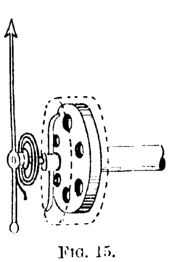


FIG. 15.

A slightly different type of mercury viscosity instrument to the above has been made for use on motor vehicles. This particular instrument employs a rotating cup containing mercury immersed in which is a small drum. The drum is fixed to the pointer spindle and controlled by spiral springs. This spring control is of a novel type in so much that an attempt has been made to equalise the spacing of the graduation by the use of three control springs of different strengths; these springs are not rigidly fixed to the spindle carrying the pointer, but come into action on contact with a stop on the spindle at a definite angular position. The controlling force is thus due to one spring only at the beginning of the scale and to the sum of three springs at the top end when the torque due to viscosity is greatest.

(ii.) *Viscosity of Air Tachometers.*—The advantages of using air as the viscous medium in a tachometer are obviously considerable, since the spindle need not pass through a liquid-tight joint. But, on the other hand, the forces obtained are very much smaller than those available with liquids, such as mercury, and consequently the workmanship must be very good to produce a satisfactory instrument.

A speedometer has been introduced recently by the Waltham Watch Co., in which the viscous drag between concentric cups is utilised. The device consists of two brass cups fixed to the driving shaft, telescoping into which are two inverted aluminium cups pivoted and controlled by a spiral spring. The arrangement is shown in sketch, *Fig.* 16. The air-gap between the adjacent walls of the rotating and stationary cups is only half a millimetre. The aluminium cups are very light, the wall being only 0·08 of a millimetre in thickness. The spindle is mounted in jewel bearings: the method of mounting is shown in sketch, *Fig.* 16.

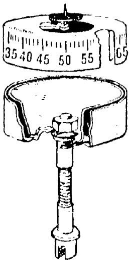


FIG. 16.

The scale is marked on the outer surface of the aluminium cups. The graduation of the scale is uniform, the deflection being proportional to the speed.

It can be shown by experiment that the drag between two surfaces close together in relative motion is of the form: Torque $\propto Vs/d$,

where V = relative linear speed of the surfaces,
s = area of surface,
and d = distance between them.

This law holds up to a critical velocity, after which the index of V increases to approximately the second power. The fluid instruments already considered work over a range of velocities above this critical velocity, since its value is very low for liquids. The air viscosity instrument is arranged to run at low speed (about 1000 r.p.m. at full scale), and the greater torque incidental to high speeds of rotation is sacrificed to retain the linear calibration. An arrangement is provided to cause frictional damping when fluctuating speeds are measured. This consists of a small disc at the top of the drum spindle, the surface of which bears a jewel on the end of a spring, so that by varying the pressure more or less friction is introduced; such damping is of course obtained at the expense of sensitivity.

§ (11) RESONANCE TACHOMETERS.—Resonance instruments of the reed type are often used in electric generating stations where machinery runs at a fairly constant predetermined speed. It is a well-known fact that if a flexible bar or reed is rigidly connected to a support which is vibrating or subject to impulses of definite frequency, the bar will vibrate in resonance with a considerably greater amplitude than that of the support if the period of the impressed oscillation is equal to, or is a definite multiple of, the natural period of vibration of the bar. Generally a piece of rotating machinery possesses a slight out of balance effect which sets the entire machine and bed-plate in vibration, and usually the oscillations set up have the same period as the speed of rotation. A reed of the appropriate period will therefore resonate if fixed to any part of the machine. In practice an instrument is made up of a series of reeds of uniformly decreasing period, as shown in *Fig.* 17. The reeds usually take the form of steel strips of varying lengths loaded at the end, the period being initially adjusted by filing down the weight. The weight of the white paint on the tips has an appreciable effect on the natural period of the reed and consequently it must not be liable to flake off or absorb moisture.

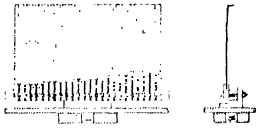


FIG. 17.

If the number of reeds cover a wide range of speed, then more than one reed will respond to a given vibration, since the reeds whose periods are multiples of the particular frequency will be set oscillating. The primary frequency can, however, be recognised as that of the reed having greatest amplitude. The case of the instrument is usually mounted direct on the machine. The amplitude of the reed is normally as shown in *Fig.* 18, but if found to be insufficient for easy observation, use can be made of a springy support, such as a steel arm, for the case; on the other hand, if the amplitude is too great, then a pad of soft material, such as felt, is interposed between the case and its support.

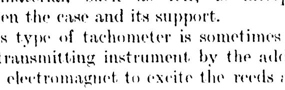


FIG. 18.

This type of tachometer is sometimes used as a transmitting instrument by the addition of an electromagnet to excite the reeds and a make and break contact on the rotating shaft; the instrument then being identical with the electrical frequency meter used on alternating-current circuits.

§ (12) MISCELLANEOUS TACHOMETERS—BOYER RECORDER.—The indication of this tachometer depends upon the pressure generated by a pump delivering oil through a variable orifice. The instrument is designed primarily as a recorder for railway work. The pump is of the well-known type, employing two meshed cog-wheels driven in opposite directions from the driving shaft. This forces oil into a cylinder, displacing a piston against the pull of a spring; the oil escapes through an adjustable slot in the side of the cylinder. Since the quantity of oil delivered by the pump varies directly as the speed, the piston must move upwards to increase the size of the slot and the pressure to a sufficient extent to permit the increased quantity to pass. The piston rod carries a pen recording on a drum, and also transmits the indications to a dial gauge by means of a chain and pulley arrangement similar in form to the simple string type of level gauge described in § (14) (ii.). The instrument's chief merit is its robustness, which fits it for the somewhat trying conditions prevailing on locomotives. The temperature coefficient cannot be made small, since the discharge of this type of pump, and the flow through the orifice, will be influenced by changes in viscosity, etc.

§ (13) CALIBRATION OF TACHOMETERS.—The calibration, or checking of tachometers, can be effected by two methods: (1) counter and watch, or (2) stroboscopic observation. The counter method needs no explanation, except to point out that the calibration gear must run very steadily over definite periods to allow of sufficient time for counting. Stroboscopic methods are very convenient and accurate, since they depend essentially on the constancy of a tuning-fork. The time of vibration of a steel fork can be approximately determined from the following formula, regarding each prong as a bar fixed at one end:

$$N = 84{,}590\ a/l^2,$$

where N = number of vibrations per second of the prime tone,
a = thickness in cm.,
l = length in cm.

The approximate nature of the formula is due to the assumption that the bar is rigidly fixed at one end, whereas in practice the bar is bent into a U, thus making the equivalent value of l uncertain. Forks can easily be tested and adjusted by comparison by ear with a standard fork. The beats in the sound become distinct when the forks are very nearly in unison. Forks can, however, be obtained commercially within about one-tenth per cent of a specified frequency. When it is desired to verify a fork in the laboratory, the usual method is to use a dropping plate. In this method a very light quill of paper is stuck on the tip of one prong and the fork supported in a position such that the quill traces a line on a sheet of smoked glass which is allowed to fall freely in front of it. The number of vibrations in a given distance can be counted, and the time interval found by calculation from the known value of acceleration due to gravity. The temperature coefficient of a steel fork amounts to 0·01 per cent per degree centigrade.

(i.) *Stroboscopic Method of measuring Speed with Slits.*—The slit fork method utilises the persistence of vision when an object is viewed intermittently. A tuning-fork is fitted with two plates at the ends of the prongs, which are perforated with narrow slits, as shown in *Fig.* 19. The vision through the slits will be interrupted twice every complete vibration of the fork. Consequently, if any rotating regular figure or circle of equidistant dots is viewed through the fork is vibrating, the first momentary view will give an impression of a stationary disc, the next coincidence of the slits will show the dots in a new position, and if the speed of rotation has certain definite values a corner or dot will have moved forward to the position occupied by the one previously observed, hence the disc will appear to remain quite stationary. The series of speeds at which this effect is apparent is given by the expression

$$n = 120f/a,$$

where n = speed of disc in r.p.m.,
f = frequency of fork (complete vibrations per second),
a = number of corners or dots.

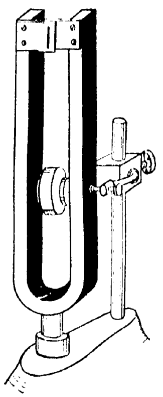


FIG. 19.

The figures repeat themselves for multiples of the speed, and with the elementary types of figures it is somewhat difficult to determine which multiple of the speed is under observation. For this reason it is usual to employ a standard disc (with a 50 D.V. fork), on which is printed a thirty-point star, a hexagon, a pentagon, and a square. Certain figures will appear to be motionless for particular speeds. All other figures will, of course, be blurred.

It will be observed that the apparatus is especially adapted for giving points at intervals over the range for calibration purposes. Intermediate speeds cannot be measured, although slightly different speeds from the fixed values can be estimated from the apparent backward or forward creep of the disc in a measured interval of time.

Tuning-forks for stroboscopic methods are usually maintained electrically by a small electromagnet between the prongs with a make

ıd break contact on the side of the prong.
two-volt battery and less than a fifth of an
npere current is usually sufficient to main-
ιin the vibration. The weight of the shutters
ı the end of the prongs affects the frequency
: the fork, and they should therefore be fitted
ɔfore the fork is adjusted.

Harrison and Abbot have devised a very
onvenient arrangement of the stroboscopic
ethod in which the disc is observed by inter-
ittent illumination. The fork is electrically
aintained, as above described, while a contact
perated by the fork is used to make and break
ie primary of an induction coil operating a
eon discharge tube. The complete arrange-
ient is shown in *Fig.* 20. The illumination obtained is a faint red glow, but is quite sufficient if the disc and tube are placed in a box with an inspection hole. The advantages over the slit fork are considerable, since both the tachometer under test and the rotating disc are under observation simultaneously. The same diagram on the disc now corresponds to half the speed for the same frequency of fork, since the flashes of light only occur once for each complete vibration, whereas the slits will register twice in each vibration.

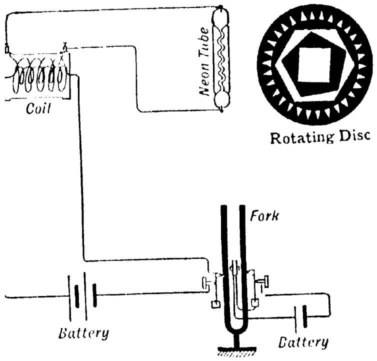


FIG. 20.

TABLE I

TABLE OF SPEEDS FOR 50 D.V. TUNING-FORK IN REVS. PER MINUTE

With Neon Tube.	30-point Star.	Other Figures.
50	Double	..
100	Single	..
150	Double	..
166·7	..	Triple Hexagon.
200	Single	Triple Pentagon.
250	Double	Double Hexagon, Triple Square.
300	Single	Double Pentagon.
333·3	..	Triple Hexagon.
350	Double	..
375	..	Double Square.
400	Single	Triple Pentagon.
450	Double	..
500	Single	Single Hexagon, Triple Square.
550	Double	..
600	Single	Single Pentagon.
650	Double	..
666·7	..	Triple Hexagon.
700	Single	..
750	Double	Single Square, Double Hexagon.
800	Single	Triple Pentagon.
833·3	..	Triple Hexagon.
850	Double	..
900	Single	Double Pentagon.
950	Double	..
1000	Single	Single Hexagon, Triple Square, Triple Pentagon.
1050	Double	..
1100	Single	..
1125	..	Double Square.
1150	Double	..
1166·7	..	Triple Hexagon.
1200	Single	Single Pentagon.
1250	Double	Double Hexagon, Triple Square.
1300	Single	..
1333·3	..	Triple Hexagon.
1350	Double	..
1400	Single	Triple Pentagon.
1450	Double	..
1500	Single	Single Square, Single Hexagon, Double Pentagon.
1550	Double	..
1600	Single	Triple Pentagon.
1650	Double	..
1666·7	..	Triple Hexagon.
1700	Single	..
1750	Double	Double Hexagon, Triple Square
1800	Single	Single Pentagon.
1833·3	..	Triple Hexagon.
1850	Double	..
1875	..	Double Square.
1900	Single	..
1950	Double	..
2000	Single	Single Hexagon, Triple Square, Triple Pentagon.
2050	Double	..
2100	Single	Double Pentagon.
2150	Double	..
2166·7	..	Triple Hexagon.
2200	Single	Triple Pentagon.
2250	Double	Single Square, Double Hexagon.
2300	Single	..
2333·3	..	Triple Hexagon.
2350	Double	..
2400	Single	Single Pentagon.
2450	Double	..
2500	Single	Single Hexagon, Triple Square.
2625	..	Double Square.
2700	Single	Double Pentagon.
2750	Double	Double Hexagon, Triple Square.
3000	Single	Single Square, Single Pentagon, Single Hexagon.

(ii.) *Synchronising Fork.*—Messrs. Leeds and Northrup, of Philadelphia, have devised a method of obtaining constant and definite speeds for the calibration of tachometers without the necessity of manual adjustment. The instruments under test are driven through a series of gears, giving the desired calibration speeds, off the shaft of a rotary converter. It is well known that an alternating-current generator and synchronous motor will react upon each other and keep in step. In the Northrup device the rotating converter supplies alternating current to a synchronous load in the form of a tuning-fork and incandescent lamp. The fork holds the generator to a constant speed such that the frequency of the alternating current is equal to the frequency of the make and break on the prong as determined by the natural period of the fork. The scheme of connections is shown in *Fig.* 21.

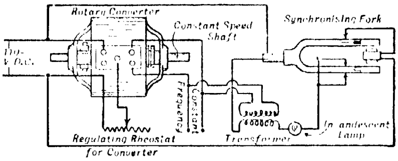


FIG. 21.

The standard fork is arranged to operate at from fifty to seventy-five vibrations per second. In order to procure this range of rate of vibration, sliding weights are provided which are mounted upon the prongs of the fork. Slight variations in the rate of vibration of the fork can be accomplished by moving heavy springs along the prongs of the fork. These springs are moved by means of a screw, and the adjustment can be made while the fork is in operation.

Flat brass plates are mounted upon the movable weights on the prongs of the fork. Slits are provided in these plates in order that it may be stroboscopically determined when the converter is running at the proper speed. There is a vibrating contact device on each prong of the fork. One of these makes contact only at one end of its travel, and is in the circuit of the electromagnet which actuates the fork. The other contact interrupts a circuit from the A.C. end of the rotary converter through an incandescent lamp. The load which is thus thrown upon the converter is used in holding the converter in synchronism with the fork. The operation is as follows:

The fork is caused to vibrate at a rate corresponding to the frequency of the A.C. current delivered by the rotary converter when running at the desired speed. This is a comparatively simple operation made by means of a telephone. Assuming that the fork and converter are in synchronism, the function of the device is to maintain this synchronism despite changes of the D.C. voltage and of the load upon the converter, either of which would tend to change the speed of the converter.

It will be seen from an inspection of *Fig.* 21 that a circuit is taken from the A.C. end of the converter through an incandescent lamp and through the double contacting device on one prong of the fork. A transformer is interposed in this circuit for two reasons. First, were it not for this transformer there would be a cross-connection of circuits in the fork, as both the A.C. current from the converter and the D.C. driving current pass through the fork and also through the same windings in the armature of the converter. Second, the transformer raises the A.C. voltage to 110 volts, or a multiple of 110, in order that lamps of a standard voltage may be used for the regulating load. Assuming, now, that the converter and fork are in synchronism, for every cycle of the A.C. voltage curve two contacts will be made at the fork, and two impulses will be sent through the lamp. These contacts may take place at any point on the voltage curve, but so long as there is no tendency to alter the speed of the converter, they will always take place at the same point. Thus a certain voltage is impressed across the lamp during a brief interval twice in each cycle. This lamp is thus a load upon the converter, the magnitude of this load depending upon the voltage impressed upon the lamp.

Now let us suppose that either a change of D.C. voltage or a change of load upon the converter should occur. The speed of the machine would tend to change. This would tend to throw the A.C. current out of synchronism with the fork. The result would be that the voltage impressed across the lamp would be different, because the contacts would be made at different points on the voltage curve. If the speed were tending to increase, the contacts would occur nearer the top of the voltage curve, impressing a greater voltage across the lamp load, and thus putting a greater load upon the machine and slowing it down. Should the speed be tending to become less, a reverse result would occur, both results acting to keep the A.C. current in rigid synchronism with the fork. Thus the speed regulation takes place automatically, until the forces tending to change the speed of the converter become excessive.

An absolutely steady source of E.M.F. is not necessary for the successful operation of the converter. Fluctuations in the D.C. voltage amounting to 5 to 7 per cent from the nominal value can be cared for.

It is sometimes necessary to use more than one lamp. In this case the several lamps are generally connected in parallel. The number of lamps which it is necessary to use depends upon the amount of change of the load, either electrical or mechanical, which is put upon the machine. For example, a machine with a normal rating of 500 watts was controlled with a load of 40 watts when the useful load varied between 330 and 270 watts. A regulating load of about 340 watts was required when the useful load upon the same machine varied between 360 and 60 watts. The speed regulation claimed is within the accuracy of the fork, that is 0·1 per cent, provided the load put on to the shaft is not sufficient to break the synchronism.

The frequency of the fork when once set can be found by stroboscopic comparison with a standard, and all the changes for calibration purposes are made by means of gears.

II. Liquid Level Indicators

The time-honoured method of indicating the level of liquid in tanks is of course the gauge-glass. This consists simply of a glass tube communicating at its upper and its lower end with the top and the bottom of the tank. The method is universal in the case of steam boilers, and has been tried on the fuel tanks of aircraft and motor vehicles, but owing to the varying inclination of the tanks, particularly in the case of aeroplanes, the indicator rarely gives an accurate estimation of the contents. Moreover, the fragile nature of glass is a serious consideration where inflammable liquids are concerned.

§ (14) Float Gauges. (i.) *Float and Rod Gauge.*—Gauges based on the use of floats have been devised in a variety of forms. Probably the simplest example is that shown in *Fig.* 22. The vertical tube contains a rod free to move up and down with a pointer at the upper end, and a float at the lower. Changes in level are transmitted directly to the pointer. This device is of course only applicable to shallow tanks, and would become very cumbersome if applied to deep tanks.

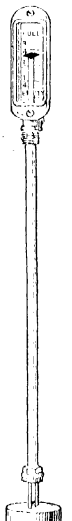


Fig. 22.

(ii.) *Float and String System.* — Another method which is almost as elementary in principle is the float and string.

The float of the gauge is suspended by means of a silk cord, which winds on the pulley of the gauge proper. The pulley is under such a tension as to nearly balance the weight of the float in air. Therefore, as the surface of the liquid rises or falls, the pointer will show the position of the float in the tank. The float is guided by an upright cylinder which keeps it from swinging about in the tank. The pointer is connected to the pulley through spur-wheels. Since the effective force producing the motion of the pointer is very small, the slightest binding of the gears may cause error. A sketch of the arrangement is shown in *Fig.* 23.

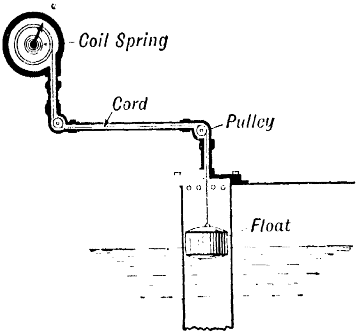


Fig. 23.

The capacity of the float used for short transmissions is usually 3 cubic inches, and for long transmissions, of 10 feet or more, the float is made larger (about 8 cubic inches). For deep tanks the pointer of the indicator is arranged in an ingenious manner to travel through several complete revolutions of a spiral curve by means of a small rack and pinion.

A diagram of the spiral mechanism is shown in *Fig.* 24. The pointer is free to slide radially in the boss of the pulley, but rotates with it. Fixed to the back of the case and projecting through the pulley is a small pinion which engages with a rack fixed to the pointer.

As the string coils or uncoils on the pulley the rack rolls round the fixed pinion, thus moving the pointer in or out radially a distance proportional to the pitch circle of the pinion for each revolution.

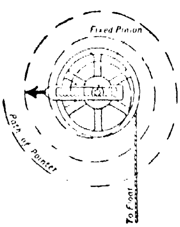


Fig. 24.

The float and string system, although very simple in theory, becomes elaborate and complicated in practice, since pulleys must be arranged at each bend, and both the tube and instrument must be air-tight.

(iii.) *Float and Eccentric Rod.* — Gauges depending upon simple mechanical arrangements for converting the movement of a float into indications on a dial fixed to the tank are also largely used. One form of this class of instrument is shown in *Fig.* 25. The rod A is free to rotate, through an arc about the pin C at the bottom, and carries the pointer or a spur-wheel at its upper end. The movement of the float F under the influence of the liquid causes the rod to turn, since the float forms a connecting link between the rod A and the fixed rod B. The angular rotation of the rod A is, of course, limited to less than 180°, but the pointer is sometimes geared to give a longer scale. This instrument is light and fairly satisfactory in use.

(iv.) *Float and Twisted Strip.*—A more elaborate form of instrument is shown in *Fig.* 26. The float is guided by two rods. A twisted pinion wire, passing through a nut in the float, converts the linear motion of the float to angular rotation of the pointer. The pointer can in this case turn through a complete circle without the use of gears. When the instrument is used in pressure tanks, where air is employed to force the liquid out, the glass front of the indicator has to be made air-tight.

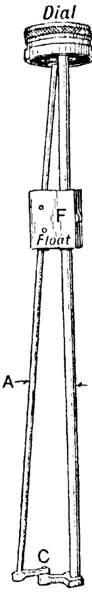


FIG. 25.

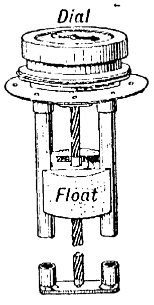


FIG. 26.

To avoid this necessity, the gauge may be worked magnetically as follows: The twisted rod carries at its upper end a magnet which rotates with the rod, and actuates a magnetised steel pointer pivoted on the other side of a thin metal partition forming the wall of the tank. This avoids all glass to metal pressure joints, but detracts somewhat from the accuracy of the indication. This magnetic device was first used by Müller in 1886, and has since found considerable application, especially in steam meters.

(v.) *Float on Pivoted Arm.*—A novel gauge differing slightly from the above and employing the magnetic pointer is shown in *Fig.* 27. This consists simply of a small float moving through a circular arc at the end of a rod. The scale is not uniform, and the float and arm require a considerable amount of space. The same disadvantage is met with to some extent in all float methods, and is particularly felt in the case of aircraft tanks which are elaborately stayed and stiffened internally. In fact, the tanks have generally to be specially designed with a view to accommodating the gauge to be used. These appliances are rarely regarded as accurate quantity-measuring devices owing to the back-lash, friction, and variable immersion of the float. Generally, they are classified as indicators graduated in fractions such as $\frac{1}{4}$, $\frac{1}{2}$, and full.

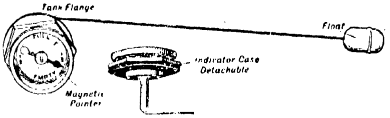


FIG. 27.

§ (15) PNEUMATIC GAUGES.—Quite a number of instruments have been devised to transmit indications of depth by the aid of air pressure.

(i.) *Air Pump Method.*—One type of instrument employs a small pressure pump to force air through the liquid by means of a pipe dipping to the bottom of the tank. The head required to effect this is measured on a pressure gauge. A sketch of the device is shown in *Fig.* 28. Each time a reading is required the hand pump is operated until the liquid is blown out of the vertical pipe in the tank, and the reading on the gauge taken immediately afterwards. The defect of this method is the inconvenience of pumping up for each reading, and the fact that the leakage back through the pump valve, etc., causes the pressure to fall rapidly. The reading has usually to be taken within a few seconds of pumping.

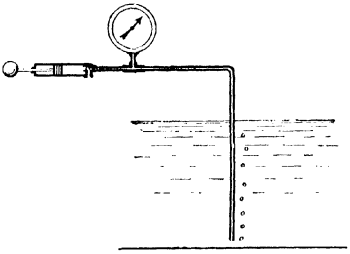


FIG. 28.

(ii.) *Tide Gauge.*—An interesting application of the above principle is the tide recorder of Field and Cust developed by the Cambridge Scientific Instrument Company. In this instrument the height and frequency of the tide at a station on the coast is continuously recorded on a chart. The recording portion of the instrument is placed at a sheltered spot inshore, and a pipe laid out and anchored in the sea where the tide record is required. A small but continuous stream of air escapes from the open end of the pipe, and the instrument on shore graphically records the pressure in the pipe, which pressure is equal to the hydrostatic head over the open end of the pipe.

A diagrammatic view of the instrument is shown in *Fig.* 29. Below the recording apparatus, and forming a stand for it, is a reservoir of air compressed to about 150 lbs. per square inch. This air is allowed to escape through a reducing valve into a pipe, one end of which leads to the top of a vessel A, containing mercury, whilst the other end is anchored on the bottom of the sea, at the point where the variations of tide are to be recorded. Air is allowed to escape slowly from the open and anchored end of the tube, a single charge of compressed air sufficing to run the apparatus for fifteen days.

As the tide rises and falls the head of water over the open end of the pipe varies. The pressure of the air in the pipe and the vessel A varies correspondingly, and forces more or less mercury into the float-chamber B, thus raising or lowering the float C. From this float a thin steel band passes over and is attached to a pulley mounted on a horizontal shaft. A second pulley on the same shaft supports by a similar band a pen carriage D and a counter-weight to the float.

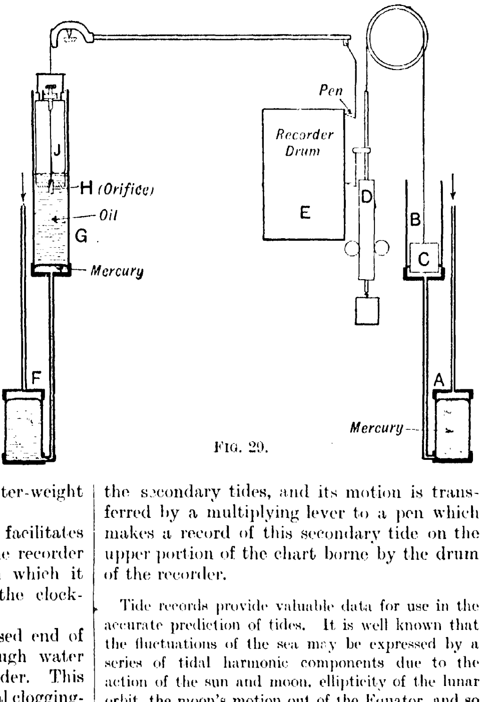


FIG. 29.

This arrangement of two pulleys facilitates the adjustment of the motion of the recorder pen to the scale of the chart on which it works. This chart is carried by the clock-driven drum E.

All the air passing to the immersed end of the pipe has first to bubble through water contained in a horizontal glass cylinder. This at once renders evident any accidental clogging-up of the under-water escape. The apparatus described above traces the long-period tide.

Superimposed on this, however, may be short-period oscillations of secondary tidal waves. A separate record of these on an enlarged scale is obtained by means of the apparatus shown to the left of *Fig.* 29.

The vessel F is also in communication with the air supply. The varying air pressures are accompanied by corresponding alterations in the level of the oil floating on the top of the mercury in the float-chamber G. The bottom of the float J in this chamber is provided with an adjustable orifice H, through which oil can flow into or out of the interior of the float, which is balanced so that it always tends to maintain its mean position.

In the case of the ordinary 12½-hour tide, the alteration of the level of the oil takes place so slowly that the liquid flows in and out of the float at a rate sufficient to enable the float to maintain its mean position. The variations of level due to the secondary tides being much more rapid, the oil is unable to pass with sufficient freedom through the orifice to keep its level the same both inside and outside the float. The latter, therefore, rises or falls in conformity with the secondary tides, and its motion is transferred by a multiplying lever to a pen which makes a record of this secondary tide on the upper portion of the chart borne by the drum of the recorder.

Tide records provide valuable data for use in the accurate prediction of tides. It is well known that the fluctuations of the sea may be expressed by a series of tidal harmonic components due to the action of the sun and moon, ellipticity of the lunar orbit, the moon's motion out of the Equator, and so on. The constants of all these tidal constituents can be determined by the harmonic analysis of the tide-gauge records for the port in question. That is to say, the amplitude of each harmonic constituent and its phase relationship with all the others can be found.

The information thus obtained can then be utilised in a tide-predicting machine to predict the tides for the port.

(iii.) *Air Vessel Method.*—Another form of pneumatic instrument, which has been tried on motor vehicles, consists of a large air vessel fixed to the base of the tank, and connected to a pressure gauge. The changes of hydrostatic pressure due to variations in liquid level are transmitted by the air column to the gauge. A sketch of the method is shown in *Fig.* 30. The air vessel must have a large

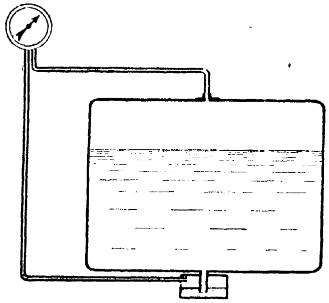


FIG. 30.

diameter, to minimise the changes of level due to the compressibility of the air. In the case of pressure tanks the gauge is arranged differentially, so as to indicate the difference of pressure between the top and the bottom of the tank. The gauge and its connecting tubes must be absolutely air-tight, but even then the air will slowly disappear by solution in the liquid, etc.

An additional source of trouble is condensation of the vapour with the formation of air-locks in the pipe.

The design of a suitable pressure gauge for any pneumatic level indicator presents practical difficulties. The maximum pressure-head in the case of the general run of tanks is only a pound or two per square inch at full scale. Bourdon tube gauges cannot be made with sufficient sensitivity, whilst silk diaphragm

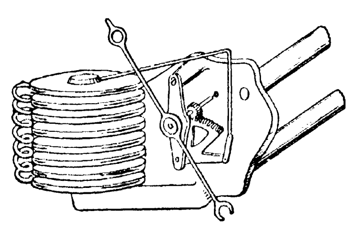


FIG. 31.

gauges invariably leak slightly. Well-made metal diaphragm gauges are probably the most satisfactory.

A typical form of low-pressure gauge is shown in *Fig.* 31. A battery of aneroid capsules is connected to the pressure pipe leading to the bottom of the tank, while the interior of the case containing the capsules is in communication with the static pipe to the top of the tank. The motion of the diaphragm, under the pressure difference, is transmitted by means of a quadrant and gear-wheel to the pointer. A peculiar feature of the instrument is the control spring; this is soldered to the edge of each capsule. The tendency of the capsules under pressure is to open out fan-wise, curving rather than elongating the spring, and by this device greater sensitivity is obtained.

The average accuracy of the instrument itself is not great, owing to the small pressure available and the friction in the mechanism. Moreover, when the gauge is allowed to remain under pressure for a period of several hours the reading tends to increase, due to elastic fatigue of the diaphragms. It will be observed that when such gauges are used differentially the glass front of the case has to be made pressure-tight. To avoid the necessity for this joint two diaphragm capsules connected differentially have been tried. It was found, however, that the sensitivity of the gauge was a function of the static pressure in the system, the variation being due to the initial distortion of the diaphragms.

§ (16) DISTANT READING TYPE OF LIQUID DEPTH GAUGE.[1]—It is a well-known fact that the cooling power of a liquid is much greater than that of a gas, and a gauge operating on this principle was designed by the writer. It consists, essentially, of a thin wire of platinum electrically heated to a temperature excess of 20° to 30° above the surrounding air. The wire is insulated and suitably protected by a tube projecting to the full depth of the tank. The portion of the wire immersed in the liquid is cooled down to practically the same temperature as the liquid, while the part above the surface is at the excess temperature.

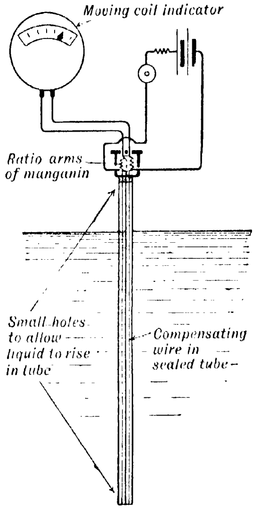
Since the temperature coefficient of the resistance of platinum is of the order of 0·4 per cent per degree, it follows that the average temperature, and hence the resistance of the wire, will depend upon its depth of immersion in the liquid. Now the most convenient method of measuring changes of electrical resistance is by means of the Wheatstone bridge, and in this gauge the wire forms one arm of a bridge, as shown in the diagram (*Fig.* 32). The influence of changes in the temperatures of the liquid and the atmosphere above are eliminated by arranging alongside a similar wire, totally sealed off from the liquid and electrically connected in the other arm of the bridge.

The changes of the resistance of the partially immersed wire, with variations of liquid level, are indicated by the deflection of the galvanometer pointer. The sensitivity of the method is such that a very small elevation of temperature of the wire suffices, and in practice a robust form of moving coil instrument is employed as indicator.

---

[1] *Proc. Phy. Soc.*, 1921, xxxiii. Part III. 171.

Since the sensitivity of any bridge arrangement is a function of the current, it is necessary to keep this constant. The customary procedure is to have an adjustable series resistance in the battery circuit, and by means of a throw-over switch replace the wire by a dummy coil totally immersed. The current in the circuit is then adjusted to give full-scale deflection; the procedure being identical with that employed with direct-reading resistance thermometers.



Moving coil indicator
Ratio arms of manganin
Small holes to allow liquid to rise in tube
Compensating wire in sealed tube

FIG. 32.

An alternative way would be to employ the same indicator with a shunt as ammeter when it is desired to adjust the battery current to a predetermined value.

The necessity for such periodical adjustments is a great disadvantage for aircraft purposes, since the attention of the pilot is fully occupied with more vital controls, and consequently the apparatus was modified to avoid this necessity. In this arrangement an iron wire ballast resistance is inserted in the battery circuit. The iron wires are sealed in a bulb similar to an electric lamp, with an atmosphere of hydrogen.

Such a resistance will maintain a current fairly steady even if the voltage of the battery changes by as much as 20 per cent, so frequent adjustment of the current is not necessary. This property of iron wire is, of course, well known, and iron wire resisters will be found in most commercial types of Nernst lamps to compensate for the negative temperature coefficient of resistance of the rare earth mixture of which the glower is composed.

As the two ratio arms of the bridge are of manganin of negligible temperature coefficient, calculation shows that a constant battery current gives a constant sensitivity for the instrument over a moderate range of temperature.

III. STEAM METERS

The metering of the steam consumption of a modern power plant presents considerable difficulty, since a more elaborate instrument than a simple flow indicator is usually desired.

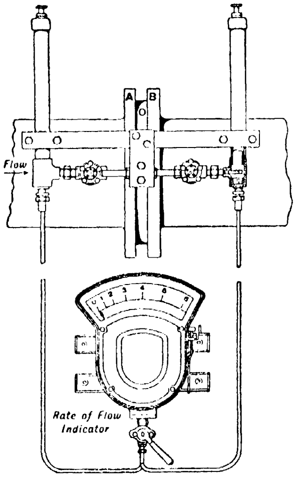
Some of the steam recorders in use at the present time are "energy" rather than "weight" recorders, in so much as they automatically take account of the variations in the steam pressure and thus give a measure of the energy being supplied in the form of steam. Considerable practical difficulties have been encountered in the operation of some of these meters, as the conditions prevailing in the boiler-house or its vicinity are somewhat severe on delicate instruments. Three typical meters are described below.

§ (17) THE HODGSON KENT STEAM METER.[1]—This meter is of the diaphragm type described in connection with gas meters, § (22). The pressure difference across an orifice is measured, and a direct indication of the rate of flow is obtained. If the pressure of the steam supply could be kept constant, this reading would be a measure of the total energy of the steam passing. In practice, however, it is necessary to make allowance for variations in the steam pressure, as with a higher pressure a greater amount of energy in the form of steam will naturally be passed through the orifice for a given difference of pressure. The meter described below automatically makes this correction, and an elaboration of this meter includes an integrator, which shows the total amount passed in a period of time.

(i.) *General Description of Meter.*—The orifice may take the form of a square-edged hole in the centre of a plate inserted in the pipe line, as shown in *Fig.* 47 (§ (25), "Coal-gas and Air Meters"), or may be a plate projecting in, as in *Fig.* 33. In order to provide a suitable difference of pressure, the size of the orifice opening must depend on the maximum velocity of the steam in the pipe. This velocity may vary over wide limits, say from 85 ft. to 260 ft. per sec. At the higher speed only a slight reduction in the area of the pipe is necessary to bring about the desired difference in pressure, and with a circular orifice the plate would be a mere ring protruding only very slightly into the bore of the pipe. Supposing that in these circumstances the plate were not very carefully positioned, or the jointing material not cut very nicely, the effects of the orifice on the flow of steam could not be relied upon. For these reasons the circular form of orifice is only adopted when a considerable restriction in the pipe is required. When the necessary restriction is less than half the cross-area of the pipe the orifice is arranged as shown in *Fig.* 49 ("Coal-gas and Air Meters"), and as further reductions in the area of the plate are required the forms shown by the dotted lines adopted. The holes which communicate the steam pressures from opposite sides of the orifice plate to the meter are both brought up as close to the plate as possible. It is assumed that the steam in the corner between the plate and its carrier has practically no movement, and that in consequence the exact form of the pressure holes is immaterial so long as they are in the proper position.

---

[1] *Engineering*, October 10, 1919.



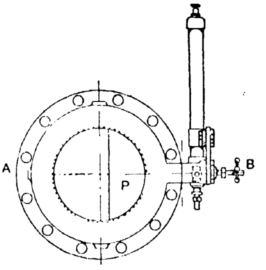


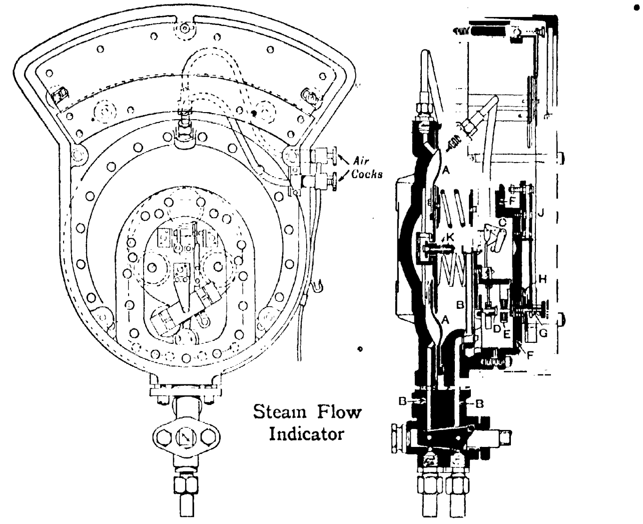
FIG. 33.

The restriction is generally designed to produce a drop in pressure of about 2 lbs. per sq. in. at full scale. Should the range of flow be greater than could be efficiently dealt with by one orifice plate, a variable orifice is used to avoid the necessity of changing the plates. This consists essentially of a short length of pipe containing a carefully constructed butterfly valve, which may be locked in various positions by means of an external sector. The over-all range of measurement which can be obtained by the use of this device goes down to one-thirtieth of the maximum flow, as compared with a range down to one-quarter of the maximum, which is all that can be accurately measured by a fixed orifice plate.

The two pressure passages in the orifice carrier communicate with a pair of condensing columns which are bracketed off the main pipe, as shown in *Fig.* 33. These columns ensure that the pipes leading to the metering instrument are kept absolutely full of water at all times, so that the readings shall not be affected by movements of the water surface employed for transmitting the pressure to the measuring apparatus.

(ii.) *Indicator Non-compensated Type.*—The rate-of-flow indicator depends for its action on the movement of a rubber diaphragm under the influence of variations in the difference of pressure on its opposite faces. The spaces on opposite sides of the diaphragm (*Fig.* 34) are in communication with the two sides of the orifice plate in the steam pipe. A set of three coiled springs controls the movement of the diaphragm, which is communicated to a spindle D, through rods and a bell crank C. This spindle carries a powerful permanent magnet E. Outside the case and in line with D, there is another spindle G, carried in jewelled bearings, to which is attached an iron armature H, and the pointer J. In this way the movement of the diaphragm is transmitted to the pointer without it being necessary to make a water-tight sliding joint. The graduations on the dial over which the pointer works can, of course, be made to show the energy of steam passing at any predetermined pressure and temperature. At the bottom of the instrument there is a plug cock, by means of which the two sides of the diaphragm can be put in direct communication for the purpose of setting the pointer to zero, while air cocks are provided at the top to ensure that the diaphragm chamber is full of water.

(iii.) *Recorder Non-compensated Type.*—When it is desired to make a permanent record of the steam flow, a modified form of instrument is used, although the orifice plate remains the same. Inside a cylindrical casing there is arranged a series of diaphragms similar to those of an aneroid barometer. The inside of these diaphragms is subjected to the lower steam pressure from the orifice, while the space outside the diaphragms is under the higher pressure. There is thus a tendency to collapse the diaphragms, which is opposed by a helical spring attached to the diaphragm spindle. The movements of this spindle are transmitted by a simple system of links to the pivot of the pen arm. The spindle which passes out through the wall of the pressure casing is kept tight by a leather-packed gland.

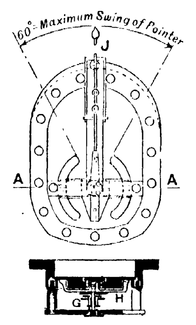


Steam Flow Indicator

Front View

Section

(iv.) *Recorder with Pressure Compensating Device.*—The meter referred to above does not take into consideration variations in the steam pressure. In cases where the pressure is liable to vary a correction must be applied to the readings if the amount of energy in the steam is to be metered. The arrangement illustrated in *Fig.* 35 automatically makes this correction over a range in pressure of about two to one.



Indicator
non-compensated type.
Pointer and magnet system.

Fig. 34.

The spindle, which is operated by the aneroid diaphragms used to measure the pressure difference across the orifice, is indicated at A. A crank and link connect A with the pointer B which pivots about C. The steam pressure in the main pipe is measured by a series of diaphragms (shown separately on the right) and is opposed by a helical spring. The rod D transmits the movement of the diaphragms to the pointer E and the lever F. We thus have two pointers, one indicating the rate of flow through the orifice and the other the actual steam pressure. The movements are combined and transmitted to a pen arm C, which consequently makes a graph expressing the energy value of the steam being metered. The combination of the two movements is effected by the link H and quadrant J.

In the positions shown in the sketch both the pointers E and B are at zero and the end of the link H is over the pivot C. The result is that the individual movement of neither of the pointers E and B will affect the pen arm G. If, however, the flow pointer B and quadrant J move round the pivot C, an increase in the steam pressure will force down F and H and produce a movement in G which will vary in proportion to the movements of E and B. The various links are, of course, so proportioned that the resultant movement of the pen arm G is a true measure of the weight of steam passing the orifice, and consequently, assuming that the steam is saturated, shows the energy being delivered. Even if the steam is superheated, the instrument can be arranged to give a direct reading, but with a variable superheat a mathematical correction must be made. An accuracy within 2 per cent is claimed for the meter at full load, and 4 per cent at one-sixth load.

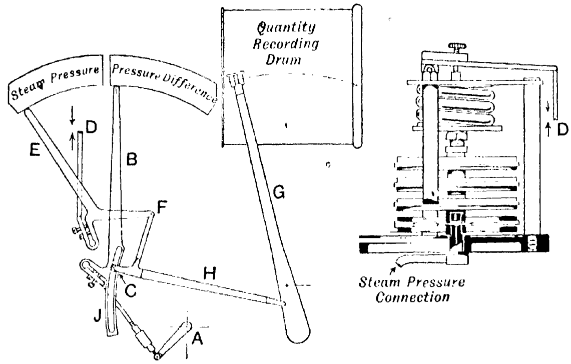


FIG. 35.

§ (18) THE B.T.H. STEAM METER.—In this steam meter a peculiar form of pressure-head is employed for determining the velocity of steam in the pipe line. This consists of a specially shaped pipe with its two openings in the path of the steam. The leading opening faces against the direction of flow, and the trailing opening faces in the direction of the flow of steam (*Fig.* 36). The two openings are connected by a vertical U-tube containing mercury.

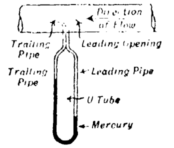


FIG. 36.

The flow nozzle has disadvantages when the feed water supplied to the boilers is dirty and the steam liable to cause scaling. Then the small passages or openings in the nozzle after a time become partially or totally closed up. Under these conditions the nozzle method is abandoned in favour of a constriction; this may be of the Venturi form or a shaped nozzle.

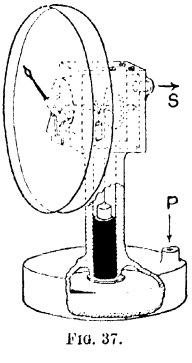


FIG. 37.

The Recorder.—The recorder (*Fig.* 37) consists of an iron casting, so designed as to form the limbs and well of a U-tube system and contains mercury. A by-pass valve is provided at the top of the meter to equalise the pressure at any time upon the U-tube system. A small iron float rests on the surface of the mercury in one limb. This float operates a horizontal shaft by means of a thread wound around a pulley, a small counterweight being provided to keep the string taut. This mechanism is totally enclosed in the casing and the movement is communicated to a horseshoe magnet mounted on the shaft. Another horseshoe magnet is mounted on pivot bearings with its poles near and parallel to the copper plate which forms the wall of this portion of the meter.

The magnet has its axis of rotation in alignment with the shaft carrying the horseshoe magnet inside the body of the meter.

This arrangement eliminates the use of a packing gland with its attendant friction. The indicating needle is attached directly to the outside magnet. The instrument is made recording by adding a pinion to the shaft carrying the outside magnet, and this pinion engages a quadrant, the shaft of which carries the recording pen. The recording chart is concentric with the indicator dial and rotated by clockwork.

§ (19) THE SARCO STEAM METER.—This meter has a novel type of indicator invented by Gehre in 1907. It consists essentially of a cast-iron reservoir filled with mercury and water, and has at one end a branch from which swings a hollow cone suspended on two springs (see *Fig.* 38). The high pressure

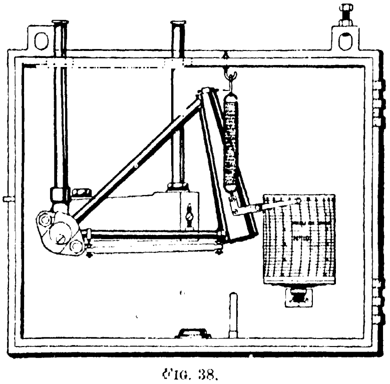


FIG. 38.

side of the throttle disc is led into a mercury reservoir which connects, through a trunnion and tube, with the lower end of the hollow conical vessel, while the lower pressure is led through a similar trunnion and tube to the upper end of the cone. The cone is suspended by helical springs and with its tubes turns about the trunnions. The higher pressure, acting through the water in the connecting tubes upon the surface of the mercury in the box, will tend to drive this out into the lower end of the cone, thus causing it to sink. On the other hand, the lower pressure will act on the mercury in the cone and tend to force it back into the reservoir, so that the difference between the two pressures will determine the position of the cone. The shape of the cone and its position are such that its fall is proportional to the square root of the pressure difference.

Since the weight of steam passing per unit time is proportional to the square root of the product of the pressure drop and the density of the steam, the scale of the indicator is a uniform one. The movement of the cone is transmitted to the recording pen by a lever mounted on an extension arm. The pen moves in a curved path. A vertical clock-driven drum is mounted behind the pen, and this carries the chart.

When it is necessary to take the account of variations of pressure in the steam main a modification of the instrument is employed in which the fulcrum of the pen lever is made to move so that the pen reading increases with the density of the steam. The high-pressure side of the throttle disc is connected with an oil cylinder in which a piston moves against the resistance of a spring. The piston-rod is connected with one arm of a lever, the other arm of which carries a curved slot that engages with the fulcrum block of the pen lever: this slot is so formed that the pen movements due to the moving fulcrum are proportional to the square root of the density of the steam, so that account is taken of the two variables. The proportions of the throttle, levers, and scale are arranged so as to give the correct readings.

§ (20) THE BAILEY STEAM METER.[1]—The Bailey meter is of the orifice type and utilises a novel form of liquid-sealed bell for recording the pressure differences.

The bell[2] is so shaped (see *Fig.* 39) that the displacement is directly proportional to the rate of flow. Further, the wall thickness is so proportioned that the volume of mercury displaced from the interior is exactly equal to the volume of the portion of the wall of the bell which is forced out from the mercury, so that the mercury level in the containing reservoir is substantially constant.

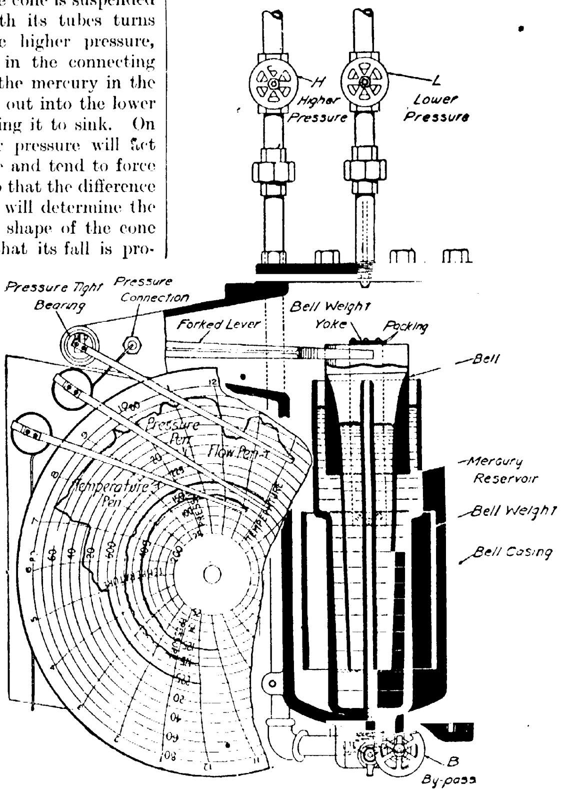


FIG. 39.

The higher and lower pressure connections terminating within and above the bell respectively are so placed that a soft packing filler piece will engage and seal either of these when an extreme position is taken.

This effectively seals either outlet, and as the entire mechanism is enclosed in a steam pressure light-casing and completely filled with water, no mercury can be blown out or the mechanism injured by subjection to excessive pressure differences.

---

[1] Ervin G. Bailey, *Amer. Soc. Mech. Eng.*, May 23, 1916.

[2] A mechanism for metering and recording the flow of fluids, etc., Ledoux, *Trans. Amer. Soc. C.E.*, 1913, lxxvi.

§ (21) THE CALIBRATION OF AIR AND STEAM METERS. (i.) *Calibration based on the Principle of Dynamical Similarity using Water as the Fluid.*—The direct calibration of the large sizes of industrial meters by volume measurement in the case of air, and weighing the condensate in the case of steam, is necessarily an expensive and troublesome undertaking. Consequently indirect methods of calibration are much favoured.

It can be proved[1] that for the turbulent flow of any two fluids in a given channel or pipe if V/η is kept constant, where V is the mean velocity, η the kinematical viscosity,[2] then the coefficient of discharge is the same for these corresponding rates of flow for the different fluids.

For example, air and water; the kinematical viscosity of air is about thirteen times the value of that for water at the same temperature. Consequently the coefficient of discharge in a meter with water flowing at the rate of one foot per second would be the same as that found for air flowing at thirteen feet per second in the same instrument. Hence this theoretical relationship affords an extremely convenient method of calibrating very large meters. The general form of the curve connecting coefficient of discharge with variations of velocity, density, and viscosity is shown in *Fig.* 40; and the coefficient of discharge being plotted as ordinate and V/η as abscissae.

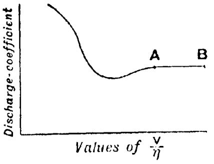


FIG. 40.

The importance of making comparisons at identical values of V/η was pointed out by Dr. Stanton, and it will be evident from a consideration of *Fig.* 40 that since the coefficient of discharge varies with the value of V/η in making comparisons between two fluids of kinematical viscosities η_1 and η_2 it is essential that $V_1/\eta_1 = V_2/\eta_2$.

It is possible, however, at high velocities, for the coefficient of discharge to remain constant for a considerable range of V, as shown by the horizontal AB of the curve, when one or two values of the coefficient will suffice for the range employed. It is essential when calibrating with water to employ a range of velocities for the water which correspond (on the V basis) with those which will be encountered in practice when the meter is employed for gas, *i.e.* the water velocities must be one-thirteenth those of the air. Experimental results confirm this generalisation.

The use of water instead of superheated steam in the calibration of steam meters is economical on the score of power consumption alone apart from considerations of capital outlay for the plant.

(ii.) *CO_2 Method of Steam Meter Calibrations.*—Ervin G. Bailey[3] has worked out a method of calibrating steam meters which is based on the principle of introducing CO_2 gas from a high-pressure cylinder into the steam line at a known and continuous rate. The gas is permitted to mix with the steam, then a sample is drawn out, and by condensing the steam the ratio of condensed steam to CO_2 gas in the sample is determined.

In the first experiments by this method the attempt was made to determine this ratio by considering the CO_2 as carbonic acid in solution and titrating to find its amount contained in the condensed steam by chemical means. Some rather erratic results were obtained from this titration method, and in an effort to determine the source of error the method was modified so that the gas was separated continuously from the condensed steam and the two measured separately so as to determine the ratio of water to gas. This ratio multiplied by the rate at which the gas was added will give the rate of flow of steam. The general diagrammatic arrangement of the apparatus used in this method is shown in *Fig.* 41.

The drum containing CO_2 gas under high pressure is supported on a scale beam so that its weight can be determined at any time, from which the rate of adding the gas is accurately determined. The gas is continuously discharged through a flexible copper tube and a governor valve into the steam line through a ⅛-in. spray tube. A sample of the mixed steam and gas is taken from the steam pipe at a point farther down the line. The sample is then passed through a condensing and separating apparatus and the condensed steam and gas are measured separately in a graduated jar and Hempel burette, respectively. The entire operation is continuous, except that these two measurements are taken simultaneously at regular intervals of four or five minutes.

An accuracy of 1 to 1½ per cent can be secured when using 1 lb. of gas to 2000 lbs. of steam. Hence it is possible to use this method on large capacities up to several hundred thousand pounds of steam per hour. Some data are given in Table II.

---

[1] Stanton and Pannell, "Similarity of Motion in relation to the Surface Friction of Fluids," *Phil. Trans.* A, 1913, ccxiv.

[2] *I.e.* density/viscosity.

[3] *Journ. Amer. Soc. Mech. Engineers*, October 1916.

TABLE II

SUMMARY OF DATA COMPARING STEAM MEASUREMENT BY CO_2 METHOD WITH ACTUAL WEIGHT AND STEAM-FLOW METER, ORIFICE TYPE

Duration of Test.	Water fed to Boiler by Actual Weight.	CO_2 fed per Hour.	Ratio Water to CO_2.	Steam by CO_2.	Steam by Meter.	Difference between CO_2 Method and	
						Actual Weight	Steam Meter.
hours.	lbs.	lbs.		lbs.	lbs.	per cent.	per cent.
4	..	8·64	2548	90,500	91,620	..	−1·22
2·5	78,515	8·53	3645	77,650	78,040	−1·10	−0·50
6	133,474	8·68	2506	130,500	129,190	−2·26	+1·01
3	54,505	14·63	1276	56,000	55,830	+2·78	+0·29
9 *	187,979	..	..	186,500	185,020	−0·78	+0·80
5	132,667	8·85	2992	132,130	133,650	−0·40	1·14

* This line gives the sums of the two seen directly above.

This method has also been tried for measuring the flow of water, the gas being injected into the feed line, and with small ratio of gas required the CO_2 is completely absorbed by the water so that a representative sample is readily obtained. The gas is then completely separated from the water and the ratio determined by the measurement of each.

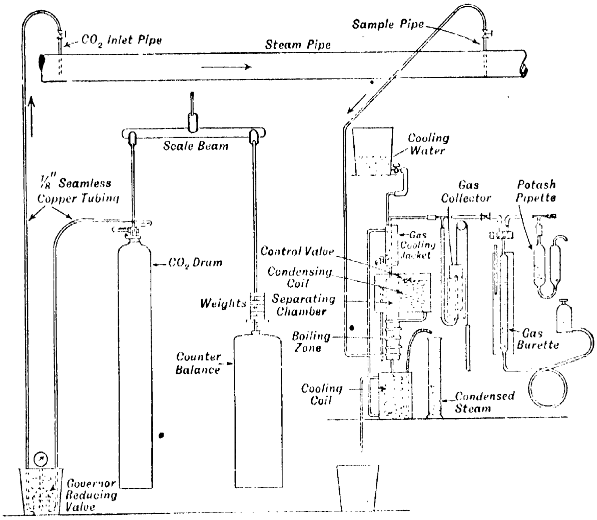


FIG. 41.

(iii.) *Calibration Apparatus of the Displacement Type.*—Probably the largest calibrating plant hitherto constructed is that installed at the Rand mines. This plant is now the standard air calibration plant for South Africa. The air to be measured is passed through a large displacement meter which is built somewhat on the lines of a steam engine. It has three vertical double-acting cylinders 36 inches in diameter, and the stroke is 27 inches. The admission and discharge of the air to the cylinders is controlled by means of piston valves which are set to cut off exactly at the top and bottom of the stroke. Each piston has two rods. The connecting rods are inverted and a three-throw crank-shaft is mounted close to the cylinder cover, as in the old type of marine engines. The valves are operated by simple link motion, since the engine does not require the usual cut-off reversing gear. The stroke volume is 95·156 cubic feet and the meter passes roughly one ton per minute of air at the maximum speed at which it is designed to run.

This displacement meter forms part of a closed air circuit, wherein the air is circulated by a Rateau fan producing a pressure difference of about 2½ lbs. per square inch. The pressure in the circuit is maintained by a small "make up" compressor connected as shown in *Fig.* 42. The air is passed through a cooler so as to obtain a steady temperature, then via the meter under test to the displacement meter. The displacement meter operates in precisely the same manner as a reciprocating engine with a small pressure drop to overcome the friction of the mechanism.

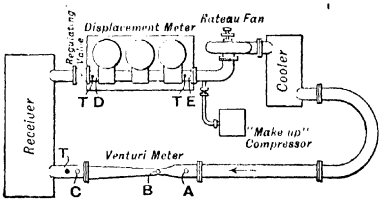


FIG. 42.

IV. COAL-GAS AND AIR METERS

§ (22) GAS METERS.—Gas meters in general use may be divided into two types—the dry and the wet meter. The former is ordinarily used to measure the consumption of gas in domestic service.

(i.) *Dry Gas Meter.*—In its simplest form this consists of a rectangular box divided into two chambers by a diaphragm (see *Fig.* 43). The diaphragm is made of a metal plate connected to the side walls of the box midway between its ends by a ring of very flexible leather;[1] the diaphragm is constrained by guides to move parallel to itself. It thus forms a piston, free from leak, and can be pushed from end to end of the box by a slight difference of pressure on its two faces. The space on one side of the diaphragm is connected to the pipe by which the gas is supplied, that on the other to the pipe feeding the burners in which it is consumed. These connections can be reversed by means of the slide valve mechanism shown in the figure. When the burners are lighted the gas is drawn away from the chamber on one side of the diaphragm and the pressure of the supply forces this up to the outer wall of that chamber; the slide valve then comes into action actuated by the motion of the diaphragm; the empty chamber is connected to the supply and the full chamber to the burners. The diaphragm then moves back; this action repeats itself continually; at each change the volume of gas filling the chamber has been supplied to the burners and this volume is known. An ordinary counting train, connected to the mechanism which moves the slide valve, records the number of times the chamber is filled and emptied and hence the volume of gas supplied. With the simple arrangement described the flow of gas would be checked each time the diaphragm reached the limit of its motion; this is obviated by having two pairs of chambers with two diaphragms and connecting them in such a way that, while one diaphragm is at the end, the other is in the middle of its path; a continuous supply is thus maintained.

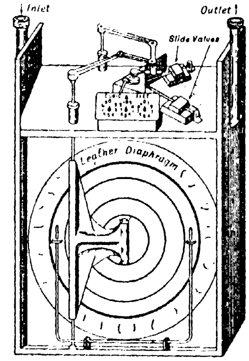


FIG. 43.

The actual quantity of gas supplied depends on the pressure and the temperature of the meter. The pressure is maintained constant by the supply authorities; changes in temperature are usually neglected, the meter being placed in a cellar or some such position where the variations are not great.

(ii.) *Wet Gas Meter.*—This consists of a drum which revolves in a cylindrical casing. The casing is rather more than half filled with water. The drum is divided by partitions into compartments, four of which are shown in the figure. One end of each partition is always below the water surface, while at the other end there is a communication between the compartment and the outer casing; the gas enters through the central shaft and passes to the burners through a pipe leading from the outer casing. In the diagram, *Fig.* 44, compartments B and C are filled with gas; C is in communication with the casing and is thus supplying the burners; communication between B and the inlet has just been cut off and gas is commencing to enter compartment A. The gas pressure causes the drum to rotate, thus opening communication between B and the outlet and filling A with gas; this gas is in its turn transferred through the casing to the burners. Thus each rotation of the drum supplies to the consumer an amount of gas equal in volume to twice the capacity of the upper part of the drum; a counter recording the number of rotations of the drum registers the volume passed. The dials of the counter, like those of the dry meter, are graduated to show the volume supplied.

---

[1] Specially treated sheepskin.

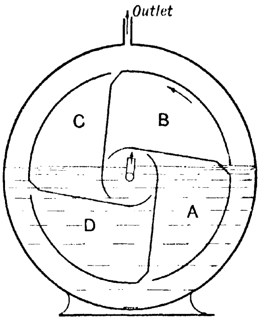


FIG. 44.

Fig. 44 represents the principle diagrammatically. Actually the meter has the compartments arranged spirally, and the inlet pipe projects into a lenticular chamber at the back of the drum. A gauge-glass with horizontal datum line is also provided, since variations in the water level inside the meter, occurring as they do at the area of maximum cross-section of each chamber, influence to a considerable extent its indications.

Station meters are usually fitted with syphon overflows to ensure a constant level. The drum operates the pointer of the recording dial through a stuffing gland and this must be reasonably frictionless, otherwise the pressure drop in the meter is considerable and this affects the capacity. Any stiffness at the gland usually manifests itself by a surging motion of the water in the gauge-glass during the rotation of the drum.

In the use of wet meters for experimental work attention should be given to the position of the pointer when the meter is read, for, although the instruments may be accurate when the quantity of gas measured is equivalent to several complete revolutions of the drum, they rarely indicate correctly for fractions of a complete rotation. Hence the meter should always be started and stopped at the same point when making a test.

§ (23) PITOT TUBE METHOD.—The Pitot tube is frequently used in scientific investigations for the measurement of the velocity of the flow of fluids in pipes. A detailed description of this instrument, together with the gauges employed with it, will be found in the article on "Friction" in Volume I.

§ (24) THE VENTURI TYPE AIR METER.—Theoretically the Venturi tube [1] is one of the most satisfactory forms of flow meters. The smooth curves of the upstream and the throat-sections ensure that the square root law is almost exactly obeyed, and the loss of head due to the insertion of the meter in the pipe line is exceedingly small. When used for gas measurement the pressure difference for the majority of practical cases does not exceed one pound per square inch. Consequently it is desirable to have a precision manometer to measure these pressure differences. One convenient form of gauge for this purpose designed by Mr. J. L. Hodgson [2] is shown in *Fig.* 45.

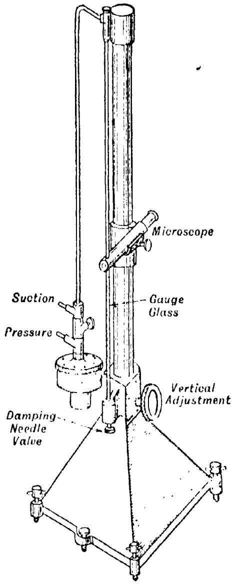


FIG. 45.

(i.) *Hodgson Gauge.*—The gauge consists of a barometer in which the cistern is made air tight and connected with the pressure side of the Venturi tube and the upper end of the column to the suction side. The cross-section of the reservoir is large compared with that of the tube, so that when pressure is applied to the liquid in the reservoir nearly the whole change of head occurs in the tube and the variations of pressure are given by multiplying the change of height by a constant depending on the ratio of the section of the tube to that of the reservoir.

The change of level of the liquid in the gauge-glass is measured by means of a travelling microscope, shown in *Fig.* 45, which is moved by a guide-screw. A graduated dial is fitted in connection with the handle by which this screw is rotated, so that readings accurate to about one-thousandth part of an inch can be taken. One of the practical difficulties met with in designing an instrument that would enable a change of liquid level to be read to anything like this degree of accuracy was the difficulty of obtaining a definite and consistent reading on the meniscus. It is overcome by using a microscope of about thirty diameters magnifying power and a gauge-glass of five-sixteenths of an inch internal diameter, and illuminating the meniscus always in the same direction. It will be seen (*Fig.* 45) that the gauge-glass passes through a tube which slides on the end of the microscope. The inside of this tube is blackened to prevent cross-reflection. High accuracy can only be obtained by using oil or alcohol as the manometer liquid, since, even with the cleanest gauge-glass, a water meniscus is apt to be sluggish.

---

[1] See "Meters," § (30).

[2] *Min. Proc. Inst. Civil Eng.*, 1917, cciv. 108.

(ii.) *Meter with Compensation for Specific Gravity Changes of the Gas.* (*a*) *Description.*—In large stations the meters required become exceptionally clumsy if of the wet type. Consequently attention has been given to the adaptation of the Venturi type of meter when these enormous volumes of gas have to be dealt with. Now the Venturi meter (see "Meters," § (30)) measures the weight of fluid passing per unit time, and, consequently, if the density of the gas varies, or the pressure, the readings obtained would not be a true measure of the actual volume passing. Hodgson has modified the Venturi meter so that the variations in the density are automatically taken account of. His form of gas meter is shown diagrammatically in *Fig.* 46.

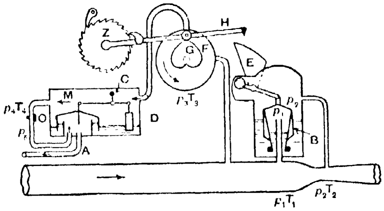


FIG. 46.

The Venturi head is measured by a water-sealed bell whose motion is transmitted by suitable means to a cam E. This is so shaped that the feeler H which comes into contact with it adds on to the counter-reading an amount proportional to the square root of the Venturi head at each revolution of the heart-shaped cam G which actuates it. If this cam were rotated at a uniform speed by F the rate of registration for a given flow in the main would be proportional to the square root of the density of the gas passing, and the meter would therefore only read correctly for gas of a particular density. The correction for variations in the density of the gas is obtained by making the speed of rotation of the heart-shaped cam depend upon the density by driving it by means of a small wet gas meter F, which is rotated continuously by gas escaping from the main through a small orifice A to the atmosphere, the pressure across this orifice being maintained constant by a specially sensitive regulating valve. Since the rate of flow through this orifice, across which the difference of pressure is maintained constant, is inversely proportional to the square root of the density of the gas, the variation in speed of the wet meter gives the exact compensation required, and the counter registers the actual volume passing. It will be observed from *Fig.* 46 that the regulating valve is compensated for changes of level of the liquid seal by means of the displacer D, and for variations in the inclination of the balance arm by the weight C. The ratio of the area of the bell to the area of the controlled orifice is made large enough to prevent variations in the pressure from affecting the accuracy of working. In practice the valve maintains the head across the orifice O correct to within ± 0·002 inch of water.

(*b*) *Mathematical Theory.*—The gaseous discharge in cubic feet per minute through a Venturi tube is given by the relation

$$D_1 = K_1\left[\frac{(p_1 - p_2)T_1}{\Delta p_2}\right]^{\frac{1}{2}}, \quad . \quad . \quad (1)$$

where T_1 is the absolute temperature and p_1 the absolute pressure at the Venturi up-stream, p_2 the absolute pressure at the Venturi throat, K_1 a numerical constant, and Δ the specific gravity of the gas relative to air. The distance between the zero position of the point of the feeler H and the surface of the cam E is made proportional to $(p_1 - p_2)^{\frac{1}{2}}$ for each position which the cam is caused to take up by the bell B. Each time the feeler is raised from the surface of the cam, an amount proportional to this distance is added on to the counter-train Z by means of a pawl and ratchet. The counter thus registers an amount proportional to

$$[p_1 - p_2]^{\frac{1}{2}} \times N, \quad . \quad . \quad . \quad . \quad (2)$$

where N is the number of revolutions per minute of the meter F.

The rate of rotation of this meter depends upon the discharge through the orifice O, which is given by the relation

$$D_4 = K_4\left[\frac{(p_4 - p_5)T_4}{\Delta p_5}\right], \quad . \quad . \quad . \quad (3)$$

where p_4 and p_5 are the pressures on the up-stream and down-stream sides of the orifice respectively, and T_4 the up-stream temperature.

If D_3 is the number of cubic feet per minute passing through the meter F at temperature T_3 and pressure p_3,

$$D_3 = D_4\frac{T_3}{T_4} \cdot \frac{p_4}{p_3}$$

$$= K_4\left[\frac{(p_4 - p_5)}{\Delta \cdot p_5} \cdot T_4\right]^{\frac{1}{2}} \cdot \frac{T_3\,p_4}{T_4\,p_3} \quad . \quad . \quad (4)$$

Putting $N = K_3 D_3$.

$$\text{quantity registered} = K(p_1 - p_2)^{\frac{1}{2}} \times K_3 D_3$$

$$= K \,.\, K_3 \,.\, K_4 (p_1 - p_2)^{\frac{1}{2}} \left[\frac{(p_4 - p_5) \,.\, T_4}{\Delta p_5} \,.\, \frac{T_3}{T_4} \,.\, \frac{p_4}{p_5} \right] \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Now if $(p_4 - p_5)$ is kept constant by means of the regulating valve M, and if, by suitably arranging the apparatus, the temperatures T_3 and T_4 are made sensibly equal to T_1, and the pressures p_3, p_4, and p_5 to p_2, equation (5) becomes

$$\text{Quantity registered} = K_1 \left[\frac{(p_1 - p_2) T_1}{\Delta \,.\, p_2} \right]^{\frac{1}{2}}, \qquad (6)$$

which is identical with equation (1).

A meter of the above type was tested against a drum-type station meter for a period of two years and the indications of the two meters always agreed within ± 1 per cent.

The disadvantage of the meter is the liability of the Venturi tube to become clogged with naphthalene, tar, etc. In order to reduce this to a minimum, the tube should be installed vertically in a by-pass with the gas entering from above, so that any liquid carried along by the gas will not collect in the meter and pressure tubes.

§ (25) THE DIAPHRAGM METHOD.—A simple device for measuring the flow of gases in pipes which is coming into extended use consists of a diaphragm orifice inserted in the pipe line with appropriate means of measuring the drop of pressure across it. The method, in general principle, is similar to the Venturi tube, but has the advantage of being cheap to construct and install. On the other hand, the Venturi tube is theoretically a more reliable instrument and is to be preferred when high accuracy is aimed at.

In its most elementary form the diaphragm is a thin plate placed symmetrically in the pipe line. The diaphragm has a square-edged hole bored through it; it is essential that the edges should be square, for it has been found that round-edged orifices are not interchangeable.

The differential pressure is obtained at two holes drilled close to the faces of the diaphragm, as shown in *Fig.* 47. The pressure is measured by the usual types of manometers and recorders already described in connection with Venturi meters.

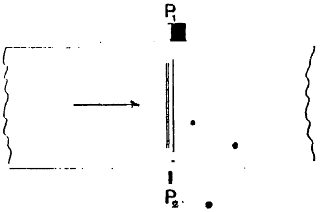


FIG. 47.

Another practical advantage of the diaphragm over the Venturi tube is the fact that one diaphragm can easily be changed for another with a hole of a different size to give the most suitable pressure difference for the particular recorder available.

(i.) *Distribution of Pressure in the Pipe Line in the Vicinity of a Diaphragm.*—The action of a diaphragm on the stream is similar to that of a submerged weir notch, and it sets up a converging stream which reaches its maximum contraction some distance down-stream from the orifice. In *Fig.* 48 some typical curves are shown for various ratios d_2/d_1 of orifice opening to pipe diameter.[1]

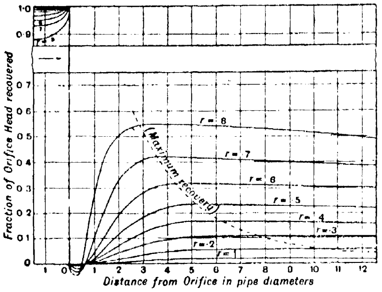


FIG. 48.

There is some difference of opinion as to the best location for the pressure holes; some workers insist that the most reliable results are obtained if these are placed as close to the plate as possible on both sides, whilst others claim that the best positions are as follows:

(*a*) On the up-stream side at the point at which normal flow is discontinued and the stream begins to converge, which point is about eight-tenths the diameter of the pipe up-stream.

(*b*) On the down-stream side at the section of maximum contraction of the jet, which occurs about four-tenths of the pipe diameter away from the orifice.

These two positions, it is claimed, give the steadiest reading of the pressure difference and also the highest value.

The effect of rounding or bevelling the edge of the orifice is at once evident when the behaviour of a contracted jet is considered, as the area of the stream at its maximum contraction must vary accordingly, and the discharge coefficients obtained by the use of sharp-edged orifices cannot apply to the case of a round-edged orifice of the same area.

The actual pressure drop is greater than the head lost by the insertion of the diaphragm, as will be seen from an inspection of the curves in *Fig.* 48. The values here shown are plotted for one particular velocity and various values of d_2/d_1, where d_1 is the diameter of the pipe and d_2 of the orifice. The recovery is greatest for large values of d_2/d_1, and the point of maximum recovery of pressure travels farther downstream as the ratio d_2/d_1 is reduced. It is worthy of note that in cases where d_2/d_1 is large, more than half the differential pressure is recovered without any special means being taken, such as the fitting of a diverging cone, to effect such recovery.

---

[1] In *Fig.* 48 the ratio d_2/d_1 is denoted by r for clearness.

The laws of flow through orifices have been investigated experimentally by a number of observers. Hodgson, in particular, has devoted much time to the development of commercial meters based on this principle. In the course of extensive experiments he found that when the diameter of the orifice was more than about three-quarters that of the main, the coefficient of discharge became very dependent upon accurate centring and smoothness of the pipe surface on the up-stream side of the orifice. This difficulty could be partially overcome by fitting two or more sets of pressure holes in a circumferential belt. In a further development of this type of meter Hodgson replaced the concentric disc constriction by a plate projecting into the pipe, the whole area of the obstruction being concentrated around the pressure holes in the form of a segment of a circle bounded by a chord, as shown in *Fig.* 49. In places where high velocities of flow had to be measured it was found more satisfactory to replace the straight chord by the arc of a circle with centre at the pressure holes.

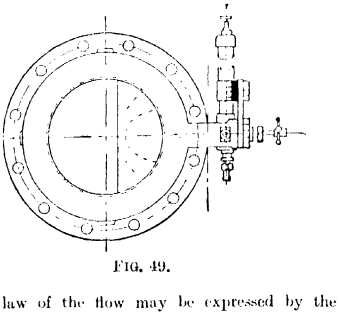


FIG. 49.

The law of the flow may be expressed by the formula

$$Q = \delta\phi A[(P_1 - P_2)W_1]^{\frac{1}{2}}, \quad . \quad . \quad . \quad (1)$$

where Q is the discharge in pounds per second. δ is defined as the "discharge intensity coefficient" for the particular type of constriction. This coefficient includes the discharge coefficient Ω, and the ratios of the area of the main to the area left by constriction n; the actual value of δ is

$$\frac{\Omega}{\sqrt{n^2 - 1}}. \quad . \quad . \quad . \quad . \quad (2)$$

ϕ is a term which allows for the compressibility of the fluid. Its value for various values of n and P_2/P_1 is shown graphically in *Fig.* 50.

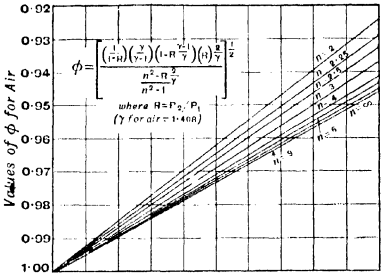


FIG. 50.

A is the area of the pipe at the up-stream pressure hole in square feet.

P_1 and P_2 are the pressures at the up-stream and down-stream pressure holes respectively in pounds per square inch.

W_1 is the density of the fluid in pounds per cubic foot at the up-stream pressure hole.

For pure dry air $W_1 = 2{\cdot}6998\ P_1/T_1$, where T_1 is the absolute temperature in degrees Fahrenheit.

For a square-edged orifice having a diameter less than three-quarters of that of the main, and pressure holes in the plane of the orifice, and for $\phi = 1$ (nearly), the equation for the flow reduces to

$$Q = 0{\cdot}608\frac{A}{\sqrt{n^2 - 1}}\left[(P_1 - P_2)\frac{P_1}{T_1}\right]^{\frac{1}{2}} \text{ lbs. per second.} \quad (3)$$

(ii.) *Variation of the Discharge Coefficient Ω, with the Diameter of the Orifice and the Head producing Discharge.*—Hodgson investigated the laws of discharge for a range of diameters varying from one-sixteenth of an inch to nine inches. In these experiments water was used on account of the ease with which it could be metered. As was shown in § (21) dealing with "The Calibration of Air and Steam Meters," the value of the discharge coefficient is the same for air and water under certain conditions.

In his experiments the orifices were in all cases geometrically similar to the pressure holes in the plane of the orifice, as shown in *Fig.* 47. Great care was taken in the preparation of these smaller orifices. They were made in hardened steel to ensure that their edges were exactly square and without the least trace of "wire edge." Their diameters were measured in four directions at 45° to one another by means of a travelling microscope. The orifices were fitted in a length of bored pipe. His results show that, for all values of d_2 within these limits, and for all values of d_2/d_1 less than 0·7, after the critical head has been reached the coefficient of discharge for any orifice will not differ by more than ± 1 per cent from the value 0·608. Below the critical head the water discharge varies as $(P_1 - P_2)^m$, where m is about 0·49.

The experimental values of the discharge coefficient, and approximate[1] values for the critical head, are given for the smaller orifices in the following table:

Diameter of Up-stream, d_1.	Diameter of Orifice, d_2.	s/d_2 (approximate).	Ω.	Critical Head of Inches of Water.
	inch			
1·5	0·0660	0·04	0·612	340
1·5	0·1256	0·02	0·605	250
1·5	0·2412	0·02	0·607	154
1·5	0·3539	0·02	0·606	87

s = thickness of orifice plate in inches.

Hence for values of d_2/d_1 which lie between zero and 0·7 the coefficient of discharge does not differ materially from 0·608.

For values of d_2/d_1 which are greater than this the discharge coefficient gradually diminishes, and is so sensitive to minute variations in the conditions under which the orifice is installed, owing to the rapid variations in the pressure immediately up-stream and down-stream of the orifice, that it is preferable to calibrate each individual orifice *in situ*.

In some other experiments in which the flow of air was studied it was found that the discharge coefficients for these square-edged orifices are identical with the water values only in the limit when P_2/P_1 is unity. As P_2/P_1 diminishes, the discharge coefficient increases according to what is apparently a straight line law, which, if assumed to hold beyond the limits of the experiment which Hodgson was able to make, would seem to indicate a value of 0·914 when the ratio P_2/P_1 is zero. The value of the discharge coefficient for various values of P_2/P_1 is given with fair accuracy by the relation $\Omega = 0{\cdot}914 - 0{\cdot}306\ P_2/P_1$, as shown graphically in *Fig.* 51, which represents experimental results for the following series of values of d_2:

d_1.	d_2.
5·995″	0·670″
5·995	1·001
5·995	1·568

The same relation holds approximately (within ± 1 per cent) for steam flows. The investigation of the value of the discharge coefficient for square-edged orifices is interesting from a laboratory point of view only, as the sharpness of the square-edge is very apt to be damaged by handling or by erosion due to the flow. The value of the discharge coefficient then increases, and the discharge through the orifice can no longer be inferred with certainty from previous tests. For this reason, although the use of square-edged orifices is to be recommended for standard work on account of the ease with which they may be reproduced accurately and the exactness with which the coefficient is known, for commercial work orifices which have slightly rounded edges are commonly used. The value of the discharge coefficient for such orifices must be determined in each individual case by actual calibration, but has the advantage that it changes far less with erosion.

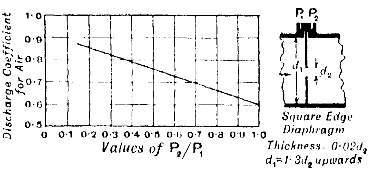


FIG. 51.

§ (26) ELECTRICAL TYPES OF GAS METERS.—Besides the above-mentioned types of gas meters a number of novel forms based on electrical measurement have been developed in recent years, and one at least of these has been made on a commercial scale.

(i.) *The Thomas Gas Meter.*[2]—This meter was invented by Professor Carl C. Thomas, of the University of Wisconsin. It is based on the measurement of the heat required to raise the temperature of the gas through a known range of temperature.

The electrical energy required to produce the change in temperature is measured, and, as is shown later, is proportional to the weight of gas flowing.

Electrical resistance thermometers are used to regulate the temperature range through which the gas is heated, because with thermometers of this type very small differences of temperature can be accurately determined.

If E is the amount of energy required to raise the temperature of Q units of weight of

---

[1] The critical head does not occur at any very well-marked point.

[2] *Journ. Frank. Instit.*, 1912, 411.

This meter is based on the same principle as the hot-wire anemometer.

Essentially a hot-wire anemometer consists of a platinum wire heated to a certain excess temperature above the surrounding atmosphere.

Experiments have shown that the heat loss from the wire is proportional to the square root of the velocity of the stream past the wire. It also depends upon the density, specific heat, and thermal conductivity of the gas.

King has developed formulae connecting these variables with the heat loss per unit length of the wire when maintained at a definite excess temperature. The hot-wire anemometer is frequently employed for measuring wind velocity.

When applied to measure the velocity of gas passing along a pipe an automatic arrangement is used for maintaining the wire at a definite excess temperature; the method employed being similar in principle to that described above in connection with the Thomas meter.

The hot wire is carried in an open framework and projects into the pipe along a diameter.

V. Liquid Meters

The meters employed in industrial measurements of liquids may be broadly classified into three types:

The displacement or positive type in which a chamber is charged and discharged alternately.

The continuous flow type which involves the measurement of the velocity of a stream and a mechanism for integrating the total flow over a time interval.

Automatic weighing machines adapted for dealing with liquids.

The majority of the meters described below have been designed for water measurement.

The primary requirements in the case of water meters are reliability and automatic action. For many purposes it is also desirable that the meter should be of such a type that it can be inserted in a pipe line carrying water under pressure and that not much head of water is absorbed for its operation. Positive type meters are generally used for this work.

§ (27) **Displacement or Positive Type Meters.** (i.) *The Kennedy Meter.*—One of the oldest meters of this class is the Kennedy meter, invented by Thomas Kennedy in 1852, and shown in *Fig. 55.* The vertical measuring cylinder is provided with a piston kept tight by a rubber ring, which rolls between the surface of the cylinder and the bottom of the wide groove in the piston, so avoiding sliding friction. The upper end of the piston rod is provided with a rack which rotates a pinion connected with the counter, and also operates the valve gear. The pinion carries an arm which catches the haft of a swinging hammer; the arm lifts the hammer until it has passed its dead centre, when the hammer falls over by gravity and strikes a finger connected with the valve gear and so reverses the motion of the piston, and similarly on the return stroke. The swinging hammer prevents the valve from stopping in its mid-position. A buffer is provided which absorbs any surplus energy in the hammer, which comes to rest with a thud.

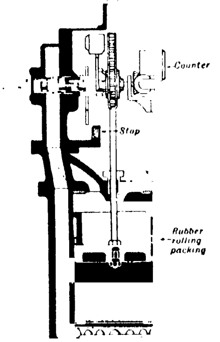


Fig. 55.

It will be observed that the indicating mechanism measures the length of the stroke, and not the number of reciprocations. This is important, because the travel of the piston is subject to accidental variations, due to speed of working, friction, etc. By means of a double ratchet, operating through little bevels, the length of the stroke is measured continuously during both the up and down motions of the piston.

If it is desired to record the rate of flow with the piston and cylinder type of meter, which is essentially an integrating meter, it becomes necessary to introduce a clock and drum, by means of which the number of operations in unit time are recorded graphically.

There are a number of other meters of this class, of which the Chadwick and Frost, and the Worthington are well-known types. The latter meter was designed primarily for use in connection with boiler plants, and is similar to the ordinary duplex double-acting water pump manufactured by the same firm.

(ii.) *Nutating Piston Meter.*—This meter belongs to the same category as the rotating pump used for charging gasometers. The action of the pump will be understood from a consideration of a simplified case.

Imagine a hollow cylinder containing a smaller cylinder capable of rolling round inside (see *Fig. 56*). A partition which can slide radially is fitted along one diameter and thus prevents passage of liquid across; on either side of this partition is situated an inlet and an outlet pipe. In the position of the cylinder shown by the strong line water is entering the portion A; that in B is being forced out through the outlet. When the cylinder has come into the position shown dotted, the water in A has been transferred to the outlet side of the diaphragm; the inlet pipe was closed as the cylinder passed its mouth and the water is just beginning to enter the space between the cylinder and the diaphragm. Each rotation of the cylinder causes the transference of a volume of water equal to that between the cylinder and its casing.

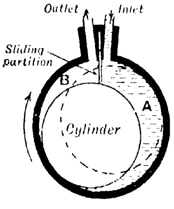


FIG. 56.

In the meter the cylinders are developed into a cone and disc inside a spherical casing. This arrangement has the advantage of permitting the meter to be double-acting.

The meter is illustrated in *Fig.* 57, whilst *Fig.* 58 is an enlarged view of the disc and casing. A disc of vulcanite is mounted on a ball working in sockets at the top and bottom of the chamber and just touches the sides of the chamber all the way round, dividing it into an upper and lower compartment. Each compartment is divided by a thin partition extending half-way across and passing through a slot in the disc. The disc does not rotate, but moves so that the axis of the ball describes a cone, while the disc tilts about its line of intersection with the diaphragm and this line moves up and down across the diaphragm.

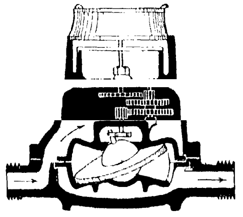


FIG. 57.

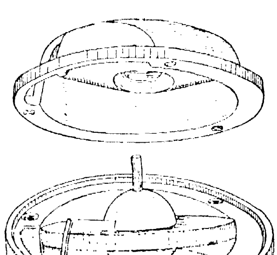


FIG. 58.

In the position shown in *Fig.* 57 the water will enter from above, as indicated by the arrow at one side of the partition, and tilt the back edge of the disc upward and the opposite edge downward, forcing water out through the lower opening at the rear side of the partition. The result of this is to bring the disc into a position in which the water now enters on its opposite side and to connect the space below the disc with the outflow; while one compartment is filling another is emptying, making the flow continuous. The end of the spindle projecting upward from the disc follows a circular path. In revolving it pushes around a little lever attached to the spindle of the gears in the middle compartment, which in turn move the hands on the register dial. Each complete movement corresponds to filling the measuring chamber once. The number of times this is done is recorded by the dials.

(iii.) *Disadvantages of the Piston-type Meter.*—Although the displacement meter is theoretically the most accurate type, there are serious practical disadvantages if the water is not free from solid matter, such as sand, etc., on account of the friction and wear of the close-fitting ports, and this defect is especially noticeable with the nutating piston form, which has no packing on the piston.

§ (28) PETROL MEASURING PUMPS.—Measuring pumps for the retailing of petrol are coming into general use, as they facilitate the storage of the liquid in underground tanks.

The pumps employed for this purpose are identical with the common piston pump, but especial attention is given to the elimination of the leakage past the valves and packing. The entire prevention of leakage is a difficult matter to achieve with a liquid like petrol, so in some types of meters the function of the pump is merely to fill a vessel of definite capacity to overflowing, the surplus liquid being returned to the tank.

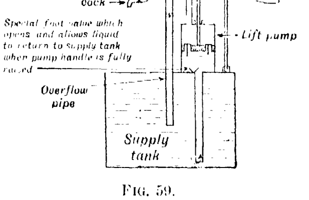

Glass measuring chamber
Delivery cock
Special foot valve which opens and allows liquid to return to supply tank when pump handle is fully raised
Lift pump
Overflow pipe
Supply tank

FIG. 59.

The principle of this system will be understood from an examination of *Fig.* 59.

Meters in which the volume of petrol supplied is determined by the length of the stroke of the piston are usually made with leather cup pistons as illustrated in *Fig.* 60, the leather being pressed into close contact with the cylinder walls by the use of a coiled spring.

In these meters the piston-rod is attached to a rack so that the operator moves the piston slowly up and down by means of a spur engaging in this rack.

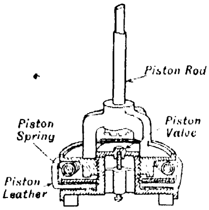


FIG. 60.

§ (29) WATER METERS OF THE CONTINUOUS FLOW TYPE.—Great accuracy of measurement is not generally required in water measurements, consequently the very simple type in which the water operates a turbine and revolution-counting mechanism is extensively used. The meters have the advantage of being fairly cheap to construct and are capable of dealing with large volumes of water. Owing to leakage they are not very accurate, but sufficiently so for most commercial requirements, and they operate best on circuits where the water is drawn off a full bore and then shut off entirely.

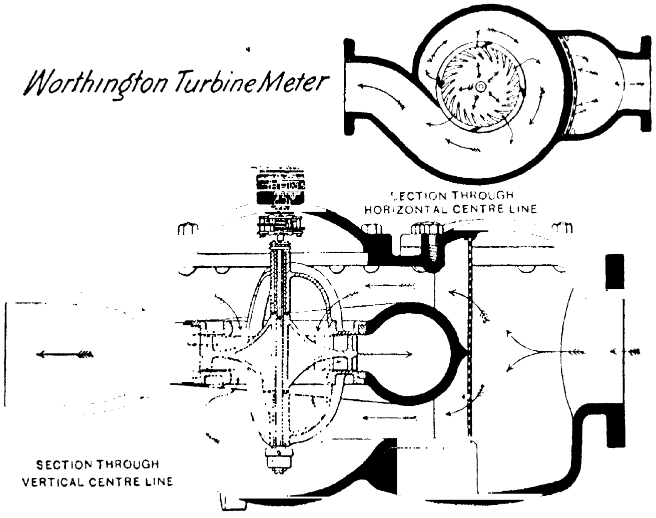

Worthington Turbine Meter
SECTION THROUGH HORIZONTAL CENTRE LINE
SECTION THROUGH VERTICAL CENTRE LINE

FIG. 61.

(i.) *Worthington Turbine Meter.*—A typical meter of this class is the Worthington turbine meter. This meter is identical in principle with the turbine pump.

Since it is assumed that the turbine wheel rotates at such a speed that there is no slip between it and the water stream, especial attention has to be given to the bearing surfaces of the moving parts to make them as frictionless as possible.

The general arrangement of the meter will be understood from *Fig.* 61. It will be seen that the water enters the main casing through the strainer. The column then divides, flowing to both sides of the double wheel, which carries two sets of vanes; thus a water balance is secured and the end thrust eliminated.

The wheel is surrounded by a chamber of volute pattern, providing at all points of the circumference the cross-sectional area necessary to handle the amount of water discharged by the wheel, at the same time conserving the speed of the water column. The wheel is of vulcanised rubber composition, of practically the same specific gravity as water, and is carried on a vertical shaft of Tobin bronze which turns in a jewelled bearing. Owing to the water balance mentioned, the friction at this point is slight, being only that due to excess weight of the wheel and shaft over that of the water displaced; the only parts which are subject to wear are the counter, gearing, and the bearing points.

(ii.) *Cone and Disc Meter.*—This meter is one of the simplest types in use at the present time and was invented by G. F. Deacon for detecting and recording waste. In it a weighted disc lies in the smaller end of an inverted cone (see *Fig.* 62). This disc is moved vertically by the flow of water, which causes an increase of the annular area between the disc and the cone proportionate to the increase of water passing through the meter. The motion of the disc is conveyed by an attached wire to a pen which records the position on a time-driven chart.

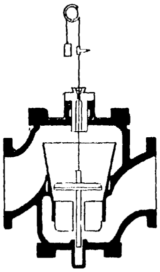


FIG. 62.

(iii.) *Integrating Mechanism.*—The late Lord Kelvin devised an exceedingly simple integrating device for use with this meter. He arranged that the meter spindle should draw a counter driven by a small vernier wheel across the centre line of a horizontal time-driven disc or plate. The reading of the counter will depend on the position of this wheel on the disc. In the actual centre of the time-driven disc there is obviously no motion, and if the vernier wheel of the counter is resting on this centre, then no movement is transmitted to the counter. When the maximum flow occurs the counter is drawn by the action of the meter from the centre to the periphery of the time-plate, so that the maximum motion of the plate is transmitted to the counter. By this interconnection of the disc and vernier wheel the distance of the latter from the centre is proportional at any point to the amount of water passing through the meter. The vernier wheel is thus enabled to run on the time disc and totalise the amount of water which has passed through the meter for any required period. The movement of the vernier wheel is communicated to the spindle of the meter through an ordinary fusee device, of form common in English lever and verge watches. This fusee is corrected by calibration to give the proper movement of the vernier wheel.

The chief friction in the meter is between the disc wire and the gland, and this, it is stated, need not exceed 2¼ oz. Added to this is the friction due to the bending of the flexible cords in the fusee. It is important that the frictional force should remain constant and of definite magnitude. The makers claim an accuracy of better than 1 per cent for this meter.

(iv.) *Accuracy of Water Meters used on Domestic Supply Mains.*—Water meters are commercially accurate instruments. Cases of meters which register correctly when installed and over-register after being in service are very rare. Any derangement of the meter from dirt entering the working parts or from other causes is likely to slow the meter down and cause it to under-register. There is usually a small amount of unavoidable leakage through the meter which causes it to under-register when very small quantities of water are passing.

§ (30) THE VENTURI METER.[1]—The application of the Venturi tube to the metering of water is due to Clemens Herschel, who constructed an instrument working on this principle in 1881.

Herschel named the meter "the Venturi" in honour of the Italian investigator who made a study of the flow of water in pipes of varying cross-section.

The Venturi meter has a sound theoretical foundation. Its operation is dependent on the fact that if a stream of water flows through a frictionless and horizontal pipe of varying section which it completely fills, the pressure of the water is smaller in the narrow sections and greater where the pipe is of large diameter. Since the same quantity of water flows through each cross-section of the pipe, its velocity must vary inversely as the area of the pipe, and its kinetic energy will therefore be greater at these cross-sections. As, by hypothesis, its total energy is unaltered through the flow, it follows that what it gains in kinetic energy it must lose in pressure energy, the sum of these two remaining constant from one end of the pipe to the other.

Expressing the same conditions mathematically, it can be shown that if A and a be the areas of the cross-section of the main pipe and the throat respectively, then the mean velocity V across the section at A is given by the expression

$$V = \sqrt{\frac{2g(P-p)}{d[(A/a)^2-1]}},$$

where d is the density of the liquid and g the acceleration due to gravity.

Hence the velocity of flow is proportional to the square root of the observed pressure difference. In practical cases viscosity of the fluid and pipe friction renders the velocity somewhat less than the calculated value, although in large pipes it may reach a value 99·5 per cent of the theoretical.

The meter finds its greatest field of application for the measurement of large supplies, and it is stated that meters are in service each unit of which deals with as much as 500,000,000 gallons per day, passing through 200-inch diameter conduits.

(i.) The general method of construction is illustrated in the diagram (*Fig.* 63). The inlet converges sharply to the throat while the outlet expands more gradually; an angle of divergence of 5° or 6° gives the best results in reconversion of kinetic to pressure energy. In practice a hollow belt is cast around the pipe at the up-stream side, where the pressure is observed, and this belt communicates with the interior of the pipe by four small holes. These holes are bushed with vulcanite to reduce incrustation. The throat is lined with a gun-metal casting, and also has an annular belt round its centre which communicates with the interior by four small holes. By careful smoothing of the curves it is possible to obtain at the throat about six-sevenths of the differential pressure given by the formula, and also ensure that the square root law is almost exactly obeyed. The actual velocity corresponding to a given fall of pressure is less than the calculated from the formula owing to the surface roughness of the pipe. Herschel found that the formula required a coefficient varying from ·94 to nearly unity, being generally between ·96 and ·99.

---

[1] See "Hydraulics," § (24), Vol. I. The denominator should be $[(A/a)^2-1]$, not as printed in Vol. I.

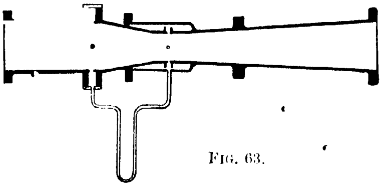


FIG. 63.

The ratio of the convergence A/a is generally 9 to 1, but of course is adjusted to meet the requirements of each case. Too high a ratio of convergence is generally inadmissible owing to the loss of head in passing the meter.

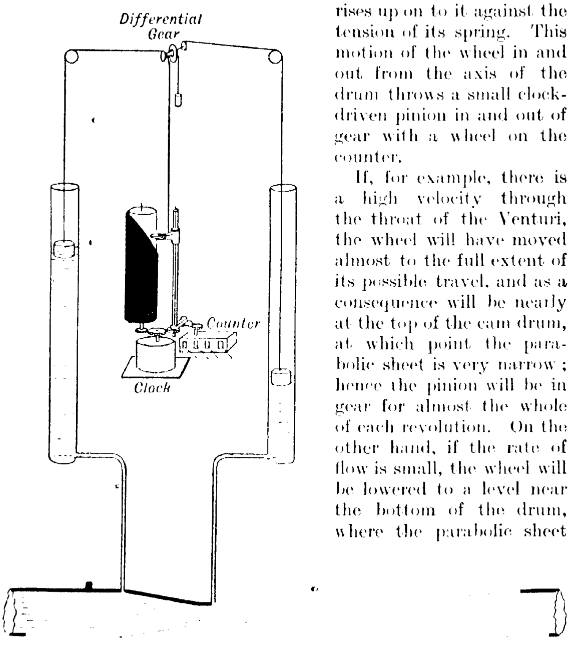


FIG. 64

(ii.) *Recording Meter for Venturi.*—In practice it is desirable to have a record of the quantity passing at any instant. In one form of meter this is effected by means of the following device. At the base of the instrument are arranged a couple of cylinders filled with water. From the bottom of each cylinder a pipe leads off to the Venturi, one being coupled to the up-stream and the other at the throat; so the height of the water column is the pressure head at the point of the Venturi to which it is coupled. In each cylinder is a float resting on the surface of the water moving with the latter. Strings lead from these floats around pulleys on a small differential gear (see *Fig.* 64) so that the angular deflection of the shaft on which the gear is mounted is proportional to the difference in the head only and independent of changes in both tubes simultaneously of equal magnitude. This deflection may be recorded on a clock-driven drum, and from this record it is possible to find the total quantity discharged in any time interval. In order to eliminate the necessity of integrating the diagrams the instrument shown in *Fig.* 64 is provided with a mechanical integrator. To effect this a drum is employed which is driven on a vertical axis by the clock at a fairly rapid rate, say six revolutions per hour. The motion of the pen arm is communicated to a small wheel mounted on a rod, and the wheel is pressed against the drum by a spring. To the surface of the drum is fastened a sheet of metal cut in the form of a parabola, so that this surface is on two levels, and as the parabolic sheet comes round, the small wheel rises up on to it against the tension of its spring. This motion of the wheel in and out from the axis of the drum throws a small clock-driven pinion in and out of gear with a wheel on the counter.

If, for example, there is a high velocity through the throat of the Venturi, the wheel will have moved almost to the full extent of its possible travel, and as a consequence will be nearly at the top of the cam drum, at which point the parabolic sheet is very narrow; hence the pinion will be in gear for almost the whole of each revolution. On the other hand, if the rate of flow is small, the wheel will be lowered to a level near the bottom of the drum, where the parabolic sheet is very wide, and as a consequence the pinion will be held out of gear for most of each revolution. The counter, being driven by this pinion, records therefore the actual quantity passed, although the difference in level of the water in the tubes varies as the square of this quantity, for the ordinate of the parabola, on which the amount recorded depends, varies as the square root of its distance from the vertex, and this distance varies as the difference in level or $(P - p)$.

(ii.) *Advantages and Disadvantages of the Venturi Meter*.—The disadvantages of the Venturi tube are (1) high initial cost of installation; (2) great length; (3) difficulty of altering the range of differential pressures without replacing the meter. On the other hand, it possesses the great advantage that the flow through Venturi tubes of large diameter can be predicted from theoretical considerations with a reasonable degree of certainty, when it would be difficult to calibrate them directly owing to large volumes which would be involved.

§ (31) NOTCH METERS.[1]—Liquid meters in which the volume discharged is determined by the height of liquid in a notch differ in principle from those previously described.

Notch meters are of considerable service when the liquid to be metered contains gritty material which would cause serious wear and tear if passed through the piston-pumps meters. The best-known notch meter is the V form. In 1861 Professor James Thomson (brother of Lord Kelvin) showed that the rate of flow over a V notch was governed by a very simple formula. For a right-angled notch—

$$Q = 2{\cdot}536H^{\frac{5}{2}} \text{ cubic feet per second.}$$

This formula was found to hold to better than 1 per cent with the results obtained in experiments with heads varying from two inches to seven inches. The formula presupposes the notch being placed in the side of an infinite reservoir, and it is not strictly valid when the stream has initial velocity. The following table gives the flow for various depths of stream in the case of a 90° V notch.

FLOW THROUGH 90° V NOTCHES

Depth in Notch.	Flow in Gallons per Hour.	Depth in Notch.	Flow in Gallons per Hour.
inches.		inches.	
1	114	9	27,796
2	648	10	36,174
3	1,783	11	45,903
4	3,661	12	56,872
5	6,394	13	69,471
6	10,086	14	83,611
7	14,829	15	99,351
8	20,706		

The formula for the flow in cases where the angle differs from 90° is not simple, but this is a point of secondary importance when the meters are empirically calibrated. In practice the height of the stream in the notch is shown by means of a float connected to a spindle. The immersion of the float being proportional to the density of the water, compensation for change of temperature can be made automatic.

(i.) *The Lea Recorder*.—The Lea Recorder has an ingenious arrangement to convert the movement of the spindle which varies as H into a movement varying as $H^{\frac{5}{2}}$ for the pen, which accordingly has a deflection proportional to the rate of flow over the notch. This is effected as follows: The float spindle is provided with a rack which gears into a small pinion upon the axis of a drum, which drum has a screwed thread upon its periphery. The contour of the thread is the curve of flow for the notch, and just as the flow through a notch increases rapidly with its depth, so the pitch of the screw increases *pro rata*. Above the spiral drum is a horizontal slider bar, supported upon pivoted rollers and carrying an arm, which is provided with a pen point in contact with a chart upon a clock-driven recording drum. As the float rises, the movement of the spiral drum is imparted to the pen-arm by the saddle-arm, which engages at its lower end with the screwed thread.

(ii.) *Integrator*.—It will be noted from the foregoing that the depth of water in the notch can be observed at any time, and that the recording pen, which moves in direct proportion to the flow, produces a diagram whose area is a measure of the total flow; and as each square inch of area represents so many pounds of water, the addition of the clock turning a cylinder having a scroll cut-in toothed wheel enables an integrating mechanism to be operated on a step-by-step method (*Fig.* 65).

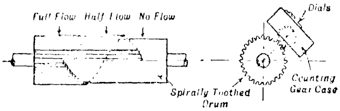


FIG. 65.

Readers acquainted with the arithmometer calculating machine[2] will recognise the principle involved.

The toothed drum rotates at a uniform rate, and the small pinion engages for a definite angular interval during each rotation dependent upon its longitudinal displacement. The counter is thus actuated for a varying time interval which is a function of the height of the water above the notch.

(iii.) *The Glenfield and Kennedy Meter*.—The Glenfield and Kennedy is another well-known form of notch meter in which a cam is employed to convert an *n*th power into a linear function of the quantity. This cam is a profiled plate. The float rotates the cam by means of a cord and gears; a small wheel bears on the cam and slides up and down vertical guides, carrying with it a wire which operates the pen recording on a clock-work driven drum. No automatic integration device is provided in this meter.

---

[1] See "Hydraulics," § (9), Vol. I.

[2] See "Calculating Machines," § (4).

(iv.) *Yorke Meter.*—In yet another form of meter, known as the Yorke, the weir is so shaped that the rate of flow is strictly proportional to the distance measured between the bottom edge of the weir and the surface of the water. The shape of this notch is such that the submerged area above the sill is proportional to the square root of the head, but as the discharge is also proportional to the same factor, it follows that the height of water in the meter, above the weir sill, is directly proportional to the rate of flow.

(v.) *Disadvantages of the Notch Type Meter.*—It will be obvious that notch type meters cannot be operated under pressure unless totally enclosed, consequently when employed for measuring the feed-water supply to boilers they are sometimes inserted on the inlet side of the feed-pumps. If the pumps leak appreciably it introduces a source of error in the readings unless allowed for. It is therefore more usual to measure the condensate with this type of meter, which procedure has the advantage that the indication varies more closely with the changes of the load on the power plant.

§ (32) Automatic Volume Measuring Meters.—Besides the piston meters described in § (27) there are others of the displacement type in which the water is metered by charging and discharging a tank of known volume automatically. In the Tippler meter there are a pair of tanks each fitted with a float and valve. The water is directed into either tank by a light shoot mounted on knife edges.

The mode of working of the meter may be briefly described as follows: The shoot directs the water to be measured into one tank, and when the level has risen to a certain height the float comes into operation and rises. This throws over a weight, tips the shoot so that the water is now discharged into the other tank, opens the outlet valve and drains the full tank. The same set of operations is repeated by similar mechanism in the other tank.

In the working of these meters care has to be taken to keep the valves in good order, and for accurate work it is advisable to fit a gauge glass and scale to check the volume at the instant of tipping.

Automatic Weighing Meters.—Meters in which the water is weighed are also in use. The general scheme of one form is to mount the tank on trunnions so that the tank overbalances when it contains a certain weight of water. The overturning is utilised to bring another tank into action while the former is emptying.

In the Avery automatic liquid weigher the principle of an equal arm beam is adopted, with the weight suspended at one end, and the weight hopper at the other. A quantity of liquid determined by the weights in the weigh-box is allowed to enter the hopper, and when the correct amount has accumulated the supply is automatically cut off. The cutting off is done gradually, so as to bring the weight of liquid to the exact amount necessary for balance. Account is taken of the liquid in the air between the valve and the hopper. Immediately after receiving its load the weigh hopper overturns and discharges its contents. The empty hopper then returns to the weighing position again and the same cycle of operations takes place. No external power is required to work the scale, and a mechanical counter is fitted which automatically counts every weighing made. The weigh hopper is so designed that it completely discharges itself without shock, and a draining compartment prevents any residue remaining. The size of the outlet can be varied to allow thick or thin liquids to completely drain before the hopper tips back. The machine can be tested at any time without difficulty, as in the case of any ordinary weighing machine.

Neither the automatic volume measuring nor the automatic weighing machines are adapted for use in circuits under pressure, but are, of course, of considerable service in experimental work, and automatic weighing machines are available for capacities of 20 lbs. to 2½ tons per discharge.

E. A. G.
M

Metric System of Units, Advantages of. See "Metrology," V. § (14).

Metric Threads (Special): table of sizes. See "Gauges," § (59).

Metrological Work, Accuracy in: measured and manufactured. See "Metrology," IX. § (33).
Present degree attainable. See *ibid.* I. § (1).

METROLOGY

I. Introductory

§ (1) Scope of Subject.—The science of metrology deals primarily with the accurate measurement of the three primitive fundamental quantities, mass, length, and time. Secondarily, it is concerned with the simple direct derivatives of these quantities, such as area, volume, density, velocity, etc. In addition, certain subsidiary measurements have frequently to be made in connection with metrological work; in particular, those of temperature, barometric pressure, thermal expansion, etc. These latter subjects are treated of under their respective headings elsewhere.

Very little thought is necessary to show that measurement of one or more of these simple fundamental quantities forms the real basis of every accurate physical determination, no matter how complicated, and the final accuracy of any physical result, therefore, depends in the last resort on the accuracy attainable in the maintenance and reproduction of standards for these quantities, and in the carrying out of comparisons with such standards. The work of the metrological laboratory is therefore of the first importance to the whole world, both of science and industry, and its operations, though essentially simple in theory, often become extremely elaborate and laborious in practice, on account of the extreme care which has to be exercised in order to avoid as far as practicable every source of even the minutest error. For the same reason the apparatus which has to be employed, in order to achieve apparently simple results, is frequently very complicated and costly. It is necessary at all times for the Standards Laboratory to keep well ahead of the accuracy demanded for external purposes, since a number of steps usually intervene between the determination of the standard and the corresponding derived physical or industrial measurement. And nowadays the demands, even of the industrial world, are calculated in many ways to tax the resources of the metrologist to the utmost. A good example of this will be found in the discussion on the use of check and reference gauges given below, § (19).

It may be of interest here to state approximately the degree of accuracy at present attainable with all precautions in a few typical operations:

Comparison of two kilogramme masses — less than 1 part in 10^8.
Comparison of two yard or metre line standards—about 1 part in 10^7.
Building up from 1 metre standard, through 4 metre bar to 24 metre surveying tape—about 1 part in 10^6.
Comparison of yard or metre line standard with corresponding end standard—about 1 part in 10^6.
Calibration of set of end gauges (not less than 1 inch in length)—about 1 part in 10^6.
Determinations of volume and density—about 1 part in 10^5.
Determinations of volume and density with extreme precautions, for very special work only (see reference to the Bureau International determination of density of water, § (9))—1 part in 10^6.

§ (2) **Fundamental Standards.** (i.) *Standards of Length and Mass.*—The standards of length and of mass are material representations of the fundamental units, and serve, under certain specified conditions of employment, as the legal and scientific bases to which ultimately all measurements must be referred. The standards may be further subdivided into a number of grades.

(*a*) *Primary Standards.*—For precision of definition, it is essential that there shall be one, and only one, material standard to represent each of the fundamental units. This is called the primary standard, and is preserved under the strictest conditions of custody, used only at very rare intervals, and then solely for purposes of comparison with the corresponding secondary standards.

(*b*) *Secondary Standards.*—These are made as nearly similar as possible in all respects to the corresponding primary standards, with which they are compared at intervals with the greatest possible care, and records of their deviations from the primary standards are preserved. The secondary standards are distributed to a number of places for safe custody, and serve a double function.

In the event of loss or damage to the primary standard necessitating its replacement at any time, a new standard would be prepared, and the various secondary standards recalled, and very carefully compared with it, and so, by means of their known errors, the deviation of the new primary standard from its predecessor would be determined, and continuity preserved.

It may be objected to this procedure that continued progress in metrological science may render such a process of reproduction inferior in accuracy to the possibilities of the time at which it may be required, and several proposals have been made in the past for definitions in terms of so-called "natural" standards, which might be supposed absolutely invariable in themselves, and which could be redetermined with ever-increasing precision as methods improved. Some such natural standards have actually had a short-lived legal existence. But in practice material representations of the standards are invariably needed in industrial, and almost invariably in scientific, applications; and so far it has been found easier to reproduce these material representations faithfully from material originals, rather than from any "natural" standard that has been proposed.

And it may be remarked that, since the present primary standards are material, the introduction of new standards in their place, whether natural or material, would involve the preliminary determination of the values of the existing standards in terms of the proposed new standards, the accuracy of which determination is in any case limited by the precision with which the existing standards are susceptible of measurement.

The secondary standards are also employed, in their turn, for occasional comparisons with the third grade, or tertiary standards.

(*c*) *Tertiary Standards.*—These actually constitute the first reference standards in the ordinary work of the metrological laboratory. The primary and secondary standards are used solely as ultimate controls, and since it is not permissible to use even the secondary standards very frequently, the tertiary standards must also be kept entirely for reference purposes, *i.e.* for comparison at intervals with the working standards.

(*d*) *Working Standards.*—These are the standards of everyday use in the metrological laboratory. Their form, as well as that of the secondary and tertiary standards, should be as nearly identical as possible with that of the primary standards, though on account of cost the material of which the lower grades are made is usually different. In practice also it is frequently useful to have a different material for some of the working standards, for quite other reasons.

For instance, in the metric system the standards both of length and mass are made of a platinum-iridium alloy, which is very dense and has a comparatively low coefficient of thermal expansion. Now, if a large number of length bars have to be compared with a working standard under conditions which make it possible to bring them easily to the *same*, but not so easily to an accurately *known*, temperature, it is a great advantage if the working standard employed is one having the same thermal coefficient as the bars to be compared with it, so that slight errors in the assumed value of the temperature at which the comparison is made shall not affect the final result required, which is the difference between the bar and the standard when both are brought to standard temperature. Similarly, in comparisons of mass, if the density of the air is uncertain, it is of great advantage to reduce the magnitude of the air-buoyancy correction in the weighings, by using a standard of similar density to the weights being compared with it. Since the majority of industrial length bars, or scales, are of steel, and the majority of weights of brass, working standards of platinum-iridium would not well meet either of these cases.

In addition to direct comparison with objects more or less similar to themselves, such as those spoken of above, the working standards—particularly those of length—serve also as the foundation for other measurements which may differ very widely indeed in type, and which have usually to be derived by one or more steps, making use of further intermediate standards.

It will thus be seen that the first information received by the general public, either scientific or industrial, must be at least four steps removed from the primary standards, and frequently more, so that the necessity for extreme precision in every operation involved is immediately obvious.

(ii.) *Standards of Time.*[1]—The standard of time is in an entirely different category from those of length and mass. The ideas presented to our minds by the words "length" and "mass" are essentially attributes of matter. This is manifestly true of mass, and if it happens that we are able to form a conception of dimensional space apart from matter, it will be found that in reality this conception depends ultimately on the idea of some kind of imaginary rigid framework, which takes the place of a material framework in the process of thought. The ideas of mass and length, also, are entirely independent of each other, and of any other attributes of the bodies to which they apply. This is not true of time. The most elementary idea of time—before and after—itself implies an "event," that is, some change in the state of the universe. Without such change time would have no meaning. And little thought is needed to show that every change which can conceivably take place implies some kind of motion, either on a small scale or a large. The idea of time, therefore, is indissolubly connected with that of motion, while motion, being itself simply change of distance, cannot be conceived apart from the idea of length. The idea of time, then, though essentially different *in quality* from those of mass and length, can only be thought of in intimate association with the latter, and its *measurement* can only be effected by reference to some uniform *motion*. In fact, a little reflection shows that the very definition of time—in a quantitative sense—is to be found in Newton's First Law of Motion,[2] which serves this purpose in just the same way as the Second Law serves for the quantitative definition of force.

The first law really contains two independent ideas—the maintenance of direction of motion, and the maintenance of uniform speed—by a body uninfluenced by external forces. With the first of these ideas we are concerned only to the extent that it serves to simplify consideration of the second. The second is, in part, a statement of the experimental fact that if a number of such freely moving bodies occupy a certain series of positions at one given moment, and at two subsequent moments are found to have moved from these positions through distances $a_1, b_1, c_1 \ldots a_2, b_2, c_2 \ldots$ respectively, etc., then

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\ldots, \text{ etc.}$$

But, in describing the velocities of the bodies as "uniform," it goes, in effect, a step further than this, and states that each of the above ratios is also equal to t_1/t_2, where t_1 and t_2 are the *times* elapsed between the first moment of observation and the two subsequent ones. This assertion really constitutes the fundamental definition of time as a measurable quantity—that is, it provides a basis which enables two different times to be compared. It does not, however, serve to define a unit. The *unit* of time is entirely independent of those of length and mass and remains stiil to be specified, but the *measurement* of time can only be effected, in the last resort, by means of comparisons of length.

---

[1] See also "Clocks and Time-keeping."

[2] Every body continues in a state of rest or of uniform motion in a straight line unless compelled to change that state by the action of some force.

Tho whole of the foregoing discussion leads to a consideration of the question whether it might not be more satisfactory to take velocity (or speed) as the third of the three fundamental quantities, rather than time—a question whose present interest is considerably augmented by the great importance assumed by the velocity of light in the recently evolved theory of relativity, according to which both length and time must be regarded as *relative* and dependent on the motion of the observer, whereas the velocity of light *in vacuo* is an *absolute* constant, independent of such motion. The theory of relativity, however, does not affect ordinary considerations concerning the applications of the fundamental units in everyday life, and it would take us too far from the practical bearings of the subject to follow up this line of thought here.

Returning, then, to the practical standards of time, we find that the ultimately accepted reference is to the results of Newton's laws of motion, as exhibited by planetary motions and recorded by astronomical methods. The unit of timo is defined by the period of rotation of the earth upon its axis, ascertained by means of sun or star transits. This period is not a natural constant (in the sense, for instance, that the velocity of light *in vacuo* is ordinarily considered to be), since the rate of rotation of the earth is affected by tidal friction, by changes in the distribution of its mass due to earthquakes and other movements, and by its general shrinkage due to cooling. The whole effect due to these various causes is, however, extremely small, and so far attempts to measure its amount by astronomical means (by comparison, for instance, with the motions of the planets in their orbits) have failed to establish it with certainty. The change probably does not exceed about 0·01 sec. per diem (roughly 1 part in 10^7) per century.

A distinction is drawn between *true solar time*, which is that given by the actual transits of the sun, and according to which the length of the day (24 hours) varies slightly at different periods of the year, and *mean solar time*, which is that registered by a uniformly running clock whose hourly rate corresponds to one twenty-fourth of the average length of a true solar day, taken throughout the year. The ordinary time of both common and scientific use is mean solar time.

Astronomers in their calculations use a slightly different standard of time, called *sidereal time*, which is that obtained from successive daily transits of fixed stars instead of from those of the sun, and which, since the earth describes a yearly orbit round the sun, differs from mean solar time by one day in 365¼. Or, more precisely, one hour of mean solar time equals one hour and 9·8565 seconds of sidereal time.

The primary standard of time is thus a natural one, and the principal apparatus concerned in the maintenance of the time standard is the transit instrument of the astronomical observatory. The place of the secondary standard is taken by the principal standard clock, which essentially is a mechanism designed to indicate with as great precision as possible the lapse of successive equal intervals of time, during the periods between successive transit observations. The principal standard clock is also housed at the astronomical observatory, near the transit instrument, and its rate is periodically checked by means of observations taken with the latter.

The time standard of the metrological laboratory is a subsidiary standard clock, whose rate is checked in its turn against signals sent out from the principal standard clock, either by means of the ordinary telegraph system, or preferably by wireless. Some form of recording chronograph is usually employed to compare the beat of the standard clock with the signals received from the observatory. An alternative method of comparing two standard clocks is by means of portable chronometers.

Before leaving this subject it may perhaps be remarked that while the accepted standard of time is based on a natural phenomenon, a material standard is not inconceivable. The water-clock, hour-glass, and pendulum clock all partake partly, but not completely, of this character, being each dependent on the value of gravity. Better examples are to be found in the chronometer, or in the tuning-fork. The former of these, being affected by wear, by conditions of oiling, and so on, could not be very satisfactory for use as a reference standard. But supposing suitable conditions of preservation and of use, and a sufficiently stable material of construction, there is no theoretical reason why the tuning-fork should not so function.

All such material standards of time, however, whether clock, chronometer, or tuning-fork, record it by a series of steps, and not continuously. In order to subdivide the steps, recourse must be had to a continuously moving body, *e.g.* a chronograph drum, on which the successive steps can be recorded, and we are thus brought back once again for the final measurement of time to comparisons of length.

§ (3) APPARATUS INVOLVED.—In addition to the actual standards of mass, length, and time, the metrologist is concerned with four more or less distinct classes of general apparatus.

(i.) *Apparatus used in Comparisons of Standards.*—This includes comparators, dilatometers, certain types of end-measuring machine, balances, chronographs, etc. Full descriptions of these, and of their methods of use, will be found elsewhere.

(ii.) Subsidiary apparatus, such as barometers, thermometers, etc., used to determine the precise conditions under which comparisons are made. Standard apparatus of such types is needed by the metrologist, but since they are not peculiar to his craft, the reader is referred to the special articles on barometry, thermometry, etc., for descriptions.

(iii.) Many articles of everyday use, both scientific and industrial, for the determination of simple physical quantities, need either verification between certain specified limits of error, or for some purposes the actual determination of their errors, before they can be regarded as fit for service. As types of such articles may be mentioned:

Engineers' scales and gauges.
Surveying tapes.
Areameters.
Barometers, mercurial and aneroid.
Chemical weights.
Hydrometers.
Volumetric glassware.
Speedometers.
Taximeters.

Tests of all these various classes of apparatus are carried out in the Metrology Department of the National Physical Laboratory, while the Standards Department of the Board of Trade, which has the custody of the British Imperial Standards, is also responsible for supervising the inspection of commercial weights and measures, including the periodical verification of the Inspector's Local Standards.

(iv.) Verifications such as these latter involve the maintenance, in addition to the simple direct standards of mass, length, and time, of many derived standards of diverse types, and frequently of considerable complexity, together with special apparatus both for controlling these standards and for comparing them with apparatus submitted for test. Descriptions of both apparatus and methods employed will be found below, under various headings.

II. General Considerations

§ (4) Material Standards.—In the preparation of material standards, three essential desiderata have to be borne in mind:

Permanence, involving (*a*) the stability of the material of which the standard is constructed; (*b*) the suitability of the material for the mechanical or other operations involved in giving the standard the desired form and finish, including the permanence of the defining features; (*c*) the ability to withstand as far as possible the effects of use—this may be a matter partly of the material used, and partly of design.

Invariability, implying the attainment of as high a degree of independence as may be practicable, of the results obtained from the use of the standard, upon slight inevitable variations or uncertainties in the exact conditions of employment. This again involves both the material and the design.

Facility in Use.—This is mainly a question of design, but may be affected also by the nature of the definition, and of the material used.

(i.) *Permanence.*—With regard to (*a*) it may be said, speaking generally, that a pure homogeneous material may ordinarily be expected to have a higher degree of secular stability than an alloy, in which molecular rearrangements may take place with greater or less rapidity. It happens, however, that the majority of pure metals are less suited in their mechanical properties—*e.g.* hardness, rigidity, susceptibility to fine polishing, etc.—than are many alloys, and as a matter of fact the primary standards of length, on both the English and Metric systems, are constructed in alloy, as is also the standard kilogramme. The imperial standard pound is of pure platinum, the standard yard of bronze, and the metric standards of platinum-iridium.

Up to comparatively recent times the only available criterion of sufficient accuracy by which the permanence of length standards could be ascertained was by establishing the relative constancy of a number of bars of different materials. If the differences in lengths between a number of such bars remain unaltered between a series of comparisons spread over a considerable period of time, there is strong presumptive evidence, though not absolute proof, that all the bars involved have remained constant during that period.

In 1892-3, however, Prof. A. A. Michelson, working at the Bureau International des Poids et Mesures, Sèvres, succeeded in determining the value of the metre in terms of the wavelengths of certain rays in the cadmium spectrum, to an accuracy approaching that with which the relative lengths of two similar bars can be determined by direct comparison under microscopes, thus affording for the first time a satisfactory approach to a "natural" standard of length. Repetition of this experiment at long intervals of time should afford a real criterion by which the absolute permanence of the material standard can be estimated.

Of pure metals, nickel is perhaps the most suitable for a length standard of the ordinary type, being hard, rigid, capable of taking a high polish and fine graduation lines, and inoxidisable. A nickel standard metre which has been in service at the National Physical Laboratory for nearly twenty years has been found, on successive comparisons with the standard metre at Sèvres, to have varied only by ±0·1 micron — one part in ten million of the total length—an amount which is no greater than the experimental error of the determination.

Under heading (*b*) come considerations of such properties as those mentioned in the last paragraph as being possessed by pure nickel. It would be useless, for instance, to construct a standard of either length or mass of any material, however suitable in other respects, if it were liable to surface tarnishing. Nor would any material be suitable for a graduated length standard if its nature were such either that fine, clean-cut lines could not be produced upon it, or that after production the edges of the lines gradually deteriorated by crumbling or flaking away. In choosing materials of construction for standards all such points have to be borne in mind.

Under heading (*c*) we come for the first time to questions of choice between standards of different types. In particular there has in the past been considerable fluctuation of opinion as to the relative merits of "line" bars and "end" bars, as ultimate standards of length, each type in turn having received recognition at various times. Both the standard metre and the standard yard of the present day are line bars, the reasons principally underlying the choice being, firstly, that since the defining marks on such a bar are merely observed through microscopes and not actually touched, during any comparison, there is no risk of any alteration due to this cause; secondly, that, in the state of development which had been attained at the time when these standards were prepared, it was practically impossible to produce end bars of such perfection of finish as would be required to enable them to be compared with each other to the same degree of accuracy as is possible in comparing line bars with each other under microscopes in a good comparator, nor were suitable end-bar comparators available for work of this accuracy; and thirdly, that to secure the necessary hardness of the contact surfaces on end bars necessitates either that they should be made of steel with hardened ends, in which case there is the risk of secular change in length, and also of rusting, or else plugs of some hard material—*e.g.* agate—must be inserted in the ends of the bar, which leads to mechanical difficulties in construction, with some uncertainty as to the permanence of the joints secured.

In view of recent developments in mechanical methods, it may be doubted whether these various objections still hold with the full force which they had when the original choice was made. Certainly, end bars can now be finished with all the accuracy required, and mechanical means of comparing them are available which are at least as accurate as, if not more accurate than, the microscope comparator method of comparing line standards. So much so that, in the writer's opinion, it is certainly easier to obtain precise determinations of fractional subdivisions of the original standard by the intercomparison of a series of end bars than by the microscopical calibration of a graduated line bar. But, in any case, since both line and end measures are required in everyday use, we could not avoid the fundamental operation of translating from line to end measure or *vice versa*, whichever type of standard were originally chosen, and this, unfortunately, is one of the most difficult operations in metrology.

It should perhaps be observed, however, that, other things being equal, the end bar provides a more *definite* standard than the line bar—that is to say that, should methods of measurement improve, the distance between two well-finished surfaces, measured at specified contact points, remains still a satisfactory reference, whereas the graduation marks on a line bar, however fine, are actually furrows of such a character that if magnification is pressed beyond a certain point irregularities in their edges become apparent, and their exact location becomes uncertain to this extent. The lines on the best existing primary standard, the international prototype metre, are of a thickness of about 0·002 to 0·003 mm., and serve for observations to an accuracy of about $\frac{1}{10}$ micron (1 micron = 0·001 mm.). But A. E. H. Tutton[1] has indicated the possibility of using a new type of line standard to about ten times this accuracy, and if this were done, questions of temperature control, and also, possibly, of the nature of surface contacts, would give rise to such difficulties that the theoretical advantage of an end bar suggested above might be of little actual importance.

Under this heading it may be desirable also to refer briefly to the question of the care which it is necessary to exercise in preserving important standards. Ordinary precautions are by no means sufficient. The lightest dusting of a standard mass, repeated many times in the course of years, is sufficient to remove minute quantities of material from its surface. The standard should, therefore, be of hard material, highly polished, and of smooth external form, so that no dust can lodge in any angle or crevice. A soft dry camel-hair brush may be passed lightly over its surface before use. No other form of duster should be employed. Similarly, any rubbing of the surface of a line standard in cleaning must be scrupulously avoided, otherwise there is a risk of shifting the apparent positions of the lines by slight burnishing of their edges. If any preservative medium is used it must be entirely inert, and such as can be completely removed by simple washing with some volatile liquid which will leave the surface of the bar perfectly dry and clean. Pure vaseline oil, free from acid, has been found suitable for use as a preservative, and may be dissolved when required by flooding with pure clean petrol.

---

[1] *Proc. R.S.* A, ccx. 459, Appendix.

(ii.) *Invariability.*—Under this head the following are typical considerations:

(*a*) *Thermal Expansibility of Material.*—This is of the greatest importance in the case of a length standard. It is in all cases necessary to define the precise temperature at which the material standard represents the fundamental unit, though if the coefficient of thermal expansion is known, it is not always essential to make observations exactly at the temperature of definition. Manifestly, however, the lower the thermal expansibility, the less exactly need the temperature of observation be ascertained, in order to achieve a specified accuracy of result. In the case of a primary standard, which is used always under conditions enabling the temperature to be completely controlled and accurately ascertained, this consideration is of less importance. But in certain other cases—*e.g.* where a standard has to be employed in air, instead of in a thermally controlled bath—it becomes of very high importance indeed.

Two cases arise. Firstly, if it is possible to ensure that two bars being compared are both brought to exactly the *same* temperature, though the absolute value of that temperature may not be known with high precision, then it is most satisfactory for both bars to be of the same material—or at any rate of materials having closely equal coefficients of expansion—so that, the properties of the standard having once been determined with accuracy, only small errors will be introduced in the computed value (at standard temperature) of the bar compared with it, due to slight uncertainty as to the exact temperature at which the comparison has been made. For this reason it is frequently desirable, where a considerable number of comparisons of importance are involved, to have a special working standard of the same coefficient as the pieces to be standardised.

On the other hand, in certain classes of comparisons it would be exceedingly difficult to ensure that both the pieces being compared were brought to the same temperature, and when this is so it becomes necessary to determine with precision the *absolute* temperature of each of them. In such a case the (working) standard may very desirably be one having a low thermal coefficient, so that errors in ascertaining its temperature during the comparison become of less account.

Two materials having very low coefficients of expansion have been discovered. These are "invar," a nickel steel alloy containing 36 per cent nickel; and fused silica. Invar is a member of a remarkable series of alloys first brought into prominence by Dr. Ch. Ed. Guillaume, Director of the Bureau International des Poids et Mesures, whose book [1] should be consulted on the subject. About the percentage composition mentioned, very slight variations in the amount of nickel present in the alloy have very marked effects on the value of its coefficient of expansion. The coefficient of an ordinary invar bar is usually about $\frac{1}{10}$th that of steel (*i.e.* about 0·000001 per 1° C.). With great care, however, it is possible to procure alloys having actually zero, or even slightly negative, coefficients. Unfortunately, while being of the highest value for certain purposes in the construction of subsidiary standards, invar, from the point of view of a primary standard, suffers from two very serious drawbacks. It gains its remarkable properties from the fact that it has a metallurgical critical point at a temperature not far removed from ordinary room temperatures, and its molecular structure is in a state of imperfect equilibrium, and tends to change with changes of temperature, the effects of the molecular changes balancing the expansion or contraction which would otherwise be associated with those of temperature. This molecular instability it is not surprising to find accompanied by exceptionally large secular growth. A newly forged invar bar is found to increase in length at the rate of about seven parts in a million per annum, the rate gradually diminishing with time, but still quite appreciable after the lapse of many years. The early part of the growth can be accelerated by suitable thermal treatment of the bar, but even so the subsequent growth is still very considerable.[2]

Secondly, there is a thermal "hysteresis," by which is meant that the bar, upon a change occurring in its temperature, does not immediately take up the final value corresponding to the new temperature, so that its length at any moment involves its past history, and cannot be computed from its instantaneous temperature, even if allowance be made for its mean rate of secular growth. Invar, therefore, is entirely unsuited for use as a fundamental standard. It is, however, largely employed for certain purposes—particularly in the construction of the tapes or wires used in geodetic surveys—where the conditions of use are such that temperatures cannot be determined with high accuracy, and a low coefficient of expansion is therefore of first importance.

---

[1] *Aciers au nickel*, pub. by Gautier-Villars, Paris. See also the article "Invar and Elinvar," Vol. V.

[2] It should be noted that Dr. Guillaume's latest researches (*Proc. Phys. Soc.* xxxii. 374) indicate the possibility of realising a form of "invar" which is practically stable. See also "Invar and Elinvar," Vol. V.

Fused silica has a coefficient of expansion even less than that of ordinary invar, being just under half a part in 1,000,000 per 1° C. The suggestion was accordingly made that this material might be made use of for the construction of a standard of length. It is free from the objection of thermal hysteresis, and being a pure material might reasonably be expected to be secularly stable. This point, however, remained to be investigated. An experimental "silica metre"[1] was therefore made at the N.P.L. and has now been kept under observation for nearly ten years (excluding the war period 1915–18), with the result that no secular change has so far been detected. It might be objected that the extreme fragility of such a material renders it unsuitable for use as a standard. To this it may be replied that such extreme care should be taken of any standard as to render this objection of small account, with the half-jocular, but none the less true, corollary that a slight accident to a metal standard might affect its length without leaving any visible cause for suspicion, whereas if the silica standard were actually broken there could be no doubt in the matter. A standard of fused silica is of course very much cheaper than one of platinum-iridium, and apart from any other purpose which it may be found to serve, the unique nature of the material, and the special peculiarities in design thereby involved, certainly render it a most interesting addition to other existing types of standard.

The choice of material for the construction of a standard of length is thus guided by a variety of circumstances. In the case of a primary standard, permanence is a first essential, and cannot be subordinated to any other consideration. But the material for a sub-standard may vary greatly according to the exact requirements of the particular case.

(*b*) The invariability of a standard may also be affected by its design and method of use. Perhaps the best example of this is to be found in the gradual evolution of the form of cross-section adopted in the modern type of line standard, where the elastic neutral axis of the bar is exposed and polished, and the graduations marked thereon, so as to avoid any stretching and compression of the plane of graduation due to the flexure of the bar under its own weight; and in the choice of two points of support for such a bar, at such a distance apart that when the bar rests upon them the distance between its principal graduations has its maximum value, whence any slight error in the setting of the supports has no influence on the measured length between the graduations.

(*c*) In the case of standards of mass, which usually have to be weighed in water, as well as in air, or *in vacuo*, in order to determine their densities, it is essential that they should be so constructed that there is no risk of water entering, and so varying their weight. Weights with screwed-in tops are consequently quite unsuitable for important work. A standard mass must be formed in one solid piece, without joints of any kind. For the same reason the material must not be porous. Further, it must be non-hygroscopic, so that its surface is unaffected by variations in atmospheric humidity.

(iii.) *Facility in Use.*—This is concerned largely with considerations arising out of the nature of the methods adopted for making observations of the standards, and since these may vary considerably from time to time with the introduction of new ideas, generalisations are hardly possible, with the exception of such obvious ones as that a standard must not be too heavy or too bulky for convenient manipulation.

There is, however, another class of consideration which perhaps comes under this heading, and this is, that certain properties of the standard, or certain features in its definition, may have a bearing on the accuracy with which it is possible to carry out the actual operations involved in its employment. For example, the greater the density of a standard mass, the smaller is its volume, and the smaller, therefore, the buoyancy correction which needs to be applied to weighings made in air in order to arrive at the true value of the mass "*in vacuo*"[1] of the standard. Since it is by no means easy to determine the actual density of the atmosphere at the time of weighing with the precision necessary to give this correction to an accuracy comparable with that of the weighing itself, it is a desideratum to keep the total amount of the correction as small as possible, so that any small error in it shall have a minimum effect on the final result.

Another question of a similar kind, which has been the subject of much discussion, is that of the best temperature of definition for standards of length. This point is fully discussed below, Part III. § (5), (i. *a*), (ii.), and (iii.).

§ (5) Conditions of Observation. (i.) *Distribution of Observations in Time.*—Suppose a number of observations are being made under conditions which are slowly changing. It is then necessary to pay careful attention to the order in which the various observations are made, and to the time intervals between them. In a great many cases the conditions will be such that the whole of the observations of one set can be completed in a sufficiently short time to justify the assumption that the rate of change has been constant during that period. If this be so, and if the rate of change is so slow that there is no danger of one phenomenon "lagging" behind another to an extent which will affect the relation which it is desired to establish between them to the order of accuracy contemplated, then it is sufficient to arrange that the "centres of gravity" in point of time of each group of observations in any one set are coincident, and then to calculate from the arithmetical means. As an example, consider the case of two standard bars being compared under microscopes in the water bath of a comparator. Suppose the temperature of the water to be gradually rising. If the above-mentioned conditions hold, and if t_1, t_2, etc., represent readings of the thermometer in the water bath (and therefore, if there be no "lag," that of the bars), and if A_1, A_2 . . . B_1, B_2, etc., represent readings of lengths of the two bars, readings may be taken in the following order:

---

[1] See "Standards of Length," § (6) (ii.).

[1] This expression, commonly used, involves a redundancy of terminology. The *mass* of any object is unvaried, whether weighed in air or *in vacuo*. Its *weight in vacuo* is the property actually determined, and is employed to ascertain the measure of its mass.

$$t_1,\ A_1,\ B_1,\ A_2,\ t_2,$$

and then, if the intervals between the successive observations were equal, we should have the result:

$$\frac{A_1+A_2}{2}-B_1$$

= difference in length between A and B at temperature $(t_1+t_2)/2$.

(ii.) *Balancing Weight of Observations.*—Such a set of observations as the above is not entirely satisfactory, since twice as many readings have been taken on A as on B. To get over this it is usual to proceed further, first stirring the water in the bath again to equalise temperatures throughout as far as possible, and then reading

$$t_3,\ B_2,\ A_3,\ B_3,\ t_4,$$

giving
$$A_3-\frac{B_2+B_3}{2}$$

= difference in length between A and B at temperature $(t_3+t_4)/2$; and so, taking the whole of the observations together,

$$\tfrac{1}{2}\left\{\left(\frac{A_1+A_2}{2}+A_3\right)-\left(B_1+\frac{B_2+B_3}{2}\right)\right\}$$

= difference of length between A and B at $(t_1+t_2+t_3+t_4)/4$.

This is now satisfactory to the extent that each bar has been observed an equal number of times, and neither bar, therefore, has more influence on the result than the other. But it is still unsatisfactory in that the "weight" attached to observations A_3 and B_1 in calculating the final result is double that given to the remaining observations A_1, A_2, B_2, and B_3, so that any error in A_3 or B_1 leads to twice as great an error in the result as it would have produced had it occurred in A_1, A_2, B_2, or B_3. Since the errors to which all the A's and all the B's are respectively liable are naturally equal in amount, this is manifestly unfair.

To overcome this objection the middle observation of each series ought to be repeated, first of course making all the customary adjustments over again so that the repeat observation is genuinely independent. We should then get the complete series

$$t_1,\ A_1,\ B_1,\ B'_1,\ A_2,\ t_2\ \mid\ t_3,\ B_2,\ A_3,\ A'_3,\ B_3,\ t_4,$$

leading to the difference

$$\frac{A_1+A_2+A_3+A'_3}{4}-\frac{B_1+B'_1+B_2+B_3}{4}$$

at temperature $(t_1+t_2+t_3+t_4)/4$, in which at last each bar, and each observation also, receives equal weight.

It would be permissible, if desired, to make an additional temperature observation between readings B_1, B'_1 and A_3, A'_3, but it should be noted that such additional temperatures are not strictly needed, and should, in fact, agree with the means of t_1 and t_2 and of t_3 and t_4, respectively, if the assumption of a uniform rate of change of temperature during each series is justified. If they do not, the cause may be either error of observation, failure in equalisation of time intervals between observations, or actual departure from a uniform rate of change in temperature.

If only the first of these causes be at work, then the additional readings may be used to improve the accuracy of the final mean temperature, giving them equal weight with the original four. The second cause should not be permitted to arise, while if the third is found to be present, steps should be taken either to improve the conditions, or to speed up the observations, until its effect becomes negligible. Should this prove impossible, other methods of observation would have to be employed.

(iii.) *Balancing Numbers of Observations:*[1] *Method of Least Squares.*—The ideas underlying the above example must be kept continually in mind in all operations where results of the highest accuracy are desired. And they apply not merely to an individual comparison, such as that described, but also to the combinations of many comparisons which are ordinarily involved in any one complete investigation.

It is naturally desirable, where very high accuracy is aimed at, to have a large number of "redundant" comparisons—*i.e.* comparisons in excess of the number actually necessary to give sufficient mathematical data to enable definite values to be calculated for the various unknowns involved. For instance, the difference between the two bars A and B would not be considered satisfactorily established by means of a comparison such as that outlined above, no matter how many series of observations had gone to its making. At least one, and usually two or more other bars would be brought in, and each compared with each. If there were four bars we should have the equations

---

[1] See also "Observations, Combinations of."

$$\begin{aligned} A-B&=a_1, & B-C&=a_4,\\ A-C&=a_2, & B-D&=a_5,\\ A-D&=a_3, & C-D&=a_6, \end{aligned}$$

or six equations in all, from which to calculate the "best" values for the three unknown differences A - B, A - C, and A - D. These best values are then computed by the "method of least squares," and comparison of the final calculated values against the a's of the observational equations gives a series of "residuals" the magnitude of which serves to indicate the degree of accuracy which has probably been attained in any one of the original comparisons; while the greater the number of redundant comparisons involved, the higher is the probable accuracy of the final computed results for the same average accuracy of any individual comparison.

It will be noted that in the above example each bar enters into the observational equations the same number of times, and has, therefore, an equal influence with all the others on the final results. It is always desirable, if possible, to arrange the various observations of any complete set in such a way as to achieve this condition.

It is important in considering any particular case of the kind to be clear what observations constitute independent comparisons. As a very simple example, suppose that we have a set of standard end gauges from 1″ to 6″ by inches, of which the value of the 6″ gauge is known, and it is desired to ascertain the lengths of each of the other gauges. This may be effected by comparisons of groups of the gauges, arranged to make up the same total lengths, in a suitable measuring machine. If we limit the comparisons to those which are possible by the addition of not more than two gauges to make up any one group, the following table exhibits all the combinations available, where each group in any one column can be compared directly with every other group in the same column. The results of the comparisons would be recorded in the form of small observed differences between the various groups of nominally equal sum.

3	4	5	6	1+6	2+6	3+6
1+2	1+3	1+4	1+5	..	..	..
..	..	2+3	2+4	2+5	..	..
..	..	..	..	3+4	3+5	4+5

Consideration of the table shows at once that there are in all 1+1+3+3+3+1+1=13 observational equations for the determination of the five unknowns, and that into these equations the 1″, 2″, 3″, 4″, and 5″ bars each enter 8 times, and the 6″ bar 6 times. It is unfortunate that there should be less observations involving the 6″ bar than the others, since the 6″ is the known reference from which the determination of the remainder is to be effected, and the final accuracy of all the results, therefore, depends on the weight attaching to the link with the 6″ bar given by the whole procedure. This, however, does not affect the fact that, since each of the other bars enters into the operations the same number of times, the probable observational error attaching to the final result of each bar will be the same, provided the comparisons are properly made.

Here arises the point which we now wish to discuss. Take, for instance, the comparisons concerned with the nominal length of 5″. The natural procedure, having set the measuring machine for dealing with comparisons of this length, would be to take first the 5″ bar, then the 1″ and 4″, and finally the 2″ and 3″, bars together, insert each group in turn between the jaws of the machine, and take the corresponding readings, a, b, and c. Then we should have

$$\begin{aligned} 5''-(1''+4'')&=a-b,\\ (1''+4'')-(2''+3'')&=b-c,\\ (2''+3'')-5''&=c-a. \end{aligned}$$

But these three equations are *not* independent. By the very process by which they have been obtained, their right-hand sides add up precisely to 0 (as of course, theoretically, they should). Had they been truly independent observational equations, however, there would have been some small observational error by which this sum would have differed from the theoretical zero. And it would not be correct to use them in the above form and still to say that each bar entered equally 8 times into the final computation.

A little consideration shows that the process by which the various differences have been derived involves two actual observations for the comparison of two groups of gauges -- leading to one observational equation -- whereas in the comparisons of three groups by the above method only three observations have been made use of to obtain three observational equations. Each equation of the latter kind is therefore entitled to receive only half the weight given to those of the first kind, in making the final calculation. If this be done, *i.e.* if half weight be assigned to each observational equation derivable from the three middle columns of the table, it will be seen that the various gauges then enter into the computation as follows: 1″, 2″, 4″, and 5″, five times each; 3″, six times; 6″, four times. The 3″ is thus determined with slightly greater precision than the others. The difference in this simple example is not great, but the more correct procedure would be to re-set the measuring machine after the comparison of the 5″ with the 1″+4″ bars, and then to make a completely independent comparison of the 1″+4″ with the 2″+3″, and similarly for the 2″+3″ with the 5″. Only so would the original conclusion as to the equality of accuracy in the final results for all the five bars be justified.

Special cases may, of course, arise where it is inconvenient, or impossible, so to arrange the observations that each of the quantities which it is desired to determine enters equally into the calculations. The method of least squares provides the means for dealing with such cases as readily as with others, but naturally the results are not of such general utility when so derived.

Other cases also occur, in which the nature of the articles being compared, or the conditions under which the comparisons have to be carried out, prevent the attainment of as high accuracy in some parts of the operation as in others. For instance, one of a number of standard bars being intercompared may have graduations of poorer quality than the rest, in which case all observations involving this particular bar will be less accurate than those in which it is not concerned. Or if several end-bars had to be determined in terms of a line standard, it would be natural to compare each independently with the line standard, and then to compare each with each in a measuring machine. These two operations are essentially dissimilar, and in such cases it becomes an exceedingly difficult matter to judge the "weight" which should be associated with each class of observations. It is naturally desirable, also, as in every other experimental investigation, to repeat all determinations, if possible, by independent methods, and a similar difficulty arises in weighting the results obtained by various methods.

Where a sufficiently large number of observations of each class are involved the method of least squares gives a theoretical means of assigning weights, based on the degree of variability found amongst the observations of any one set, but it does not seem to be generally realised what a very large number of observations are required before this theoretical criterion becomes really serviceable, and in practice it will very often be found that sufficient observations are not available. In such cases much has to be left to the judgment of the skilled observer, who may either attempt to equalise the weights of the various groups of observations by increasing the number of readings in the more uncertain cases, or by "weighting" his observational equations in accordance with his estimate of their value, before making the final computation. Naturally such a process is never felt to be completely satisfactory. But, if the observations are all reasonably good, it will generally be found that considerable variation can be made in the discretionary assignment of weights without very seriously affecting the final results.

(iv.) *Absolute, Relative, and Proportional Errors.*—It is desirable perhaps to point out here the distinction which is sometimes lost sight of by writers on fine measurement, between absolute and relative accuracy. Particular attention should be paid to this point, as it is frequently possible, and for a great many purposes is often sufficient, to be able to determine differences between two similar objects to a high degree of accuracy, under conditions which make it exceedingly difficult to ascertain the absolute value of either with precision. Accuracy of *comparison* is therefore frequently liable to be confused with accuracy of absolute *measurement*, which is a more fundamental conception. In the case we have just been considering, where a group of six bars are intercompared with a view to determining the lengths of the shorter ones in terms of the longest, which is supposed to be known, suppose we set out the results thus,

$$1'' = \tfrac{1}{6} \times 6'' + a_1,$$

$$2'' = \tfrac{2}{6} \times 6'' + a_2,$$

$$\cdots\cdots\cdots\cdots$$

$$5'' = \tfrac{5}{6} \times 6'' + a_5,$$

which is the mathematical form in which they will actually appear from the calculation. We have shown how to arrange the observations so that each bar enters into the calculations with equal weight, whence so far as this particular investigation is concerned, each should be determined with equal accuracy. In other words, the probable errors of the a's in the above equation should be the same, *in absolute magnitude*. In proportion to its length, however, this means that the value found for the 1″ gauge will have five times the probable error of that found for the 5″. It is thus important, in any investigation of this kind, to see that the method employed is of sufficient delicacy to ensure the desired *proportional* accuracy on the smallest of the articles involved.

On the other hand, any error in the value assumed for the 6″ bar is automatically reproduced, as may be seen from the equations, in the values for all the other bars determined from it, but only in proportion to their lengths. Thus, if we are aiming at a certain proportional accuracy in the final result for the shortest bar it is necessary to have at least the same proportional accuracy in the preliminary determination of the longer bar on which the whole proceeding is based. It would be useless, for instance, as far as absolute measurement even on the shortest (1″) bar is concerned, to have a machine for making these comparisons capable of indicating millionths of an inch, unless it were possible to deduce the 6″ bar from the line standard to an accuracy somewhat better than 1 part in 1,000,000.

We have already indicated that with the best means hitherto devised this is about the limit of accuracy with which the conversion from line to end measure can be effected, so that an absolute accuracy of one-millionth of an inch can only be obtained on sizes not exceeding 1″, although a sufficiently delicate machine may be able to *compare* considerably larger sizes to this accuracy. It may safely be said, in fact, that the actual attainment of an *absolute* accuracy of one-millionth of an inch over the 1″ length has only recently been achieved.[1]

In those cases, however, where the very highest accuracy on the smallest sizes is not sought, the proportionate reduction of the original error of the head-piece of a set, when the rest of the set is evaluated in terms of it, is extremely useful, since it is usually not difficult in such cases to determine the value of the head-piece to a degree of accuracy sufficient to ensure that all uncertainties due to this cause are negligible throughout the series.

Precisely analogous remarks apply, for instance, to the determination of the values of a set of sub-standards of mass, or of chemical weights, in terms of the head mass of the set.

(v.) *Symmetrical and Asymmetrical Errors.*—The theory of least squares proceeds on the assumption that all the errors which occur are equally likely to be positive or negative in sense.

Errors of this type are called "symmetrical" and are those which most commonly arise. But errors predominantly, or entirely, in one sense may, and do, occur under certain circumstances. Such errors are called asymmetrical, and may be due either to the actual conditions of the problem, to the apparatus used, or to the personal idiosyncrasies of the observer. As examples of each of these causes we may cite the following:

(*a*) Suppose it is desired to measure by means of a divided scale the distance between two parallel straight lines. Lay the scale on the paper, across the lines, and take the readings at the two lines. If there are no errors of reading, and the scale is correct, there is still one possible source of error remaining. The scale may not be *exactly* perpendicular to the lines. In that case the length measured will exceed the true distance between the lines in the ratio, $\sec \theta : 1$, where θ is the error in setting of the scale. If the operation is repeated a large number of times, different readings will be obtained, all approximating to the true value, but all, so far as the error occurs, in excess of it. And the mean of all such readings will be in excess of the true value by an amount depending on the accuracy of the means available for setting the scale square to the lines. In this case the difficulty is inherent in the problem. Its effect can be reduced, but never entirely eliminated, by improving the method of control.

(*b*) When slowly falling temperatures are being measured by means of mercury thermometers there is a well-known tendency for the reading to be slightly high, by an uncertain and variable amount, due to the sticking of the surface of the mercury thread in the capillary of the thermometer. The mercury moves with a series of small jerks instead of continuously, and under the conditions ordinarily applying to the use of mercury thermometers for recording temperatures in the water bath of a comparator the error in reading may in the worst case amount to +0·1° C., in circumstances where an accuracy of at least 0·01° C. is required.

Another case of a familiar operation in which an asymmetrical error may arise from the nature of apparatus employed is in gauging a piece of cylindrical work by means of a snap-gauge. The distance between the jaws of the gauge may be determined with the greatest delicacy, but the smallest pressure applied in use (it is hardly possible to avoid some pressure—and a clumsy workman may use a good deal) exerts a most powerful wedge action on the gauge which causes it to spring open elastically, and so pass work in excess of the nominal size.

(*c*) It is well known that individual observers have personal peculiarities which affect their observations with a certain bias. For instance, in judging the setting of a graduation mark between the two cross-wires of the micrometer eyepiece of a measuring microscope, one observer will systematically set more to the left, and another more to the right. This is true even of skilled observers, and makes it necessary to arrange either that one observer should take *all* the readings, so that on taking differences his "personal equation" is eliminated, or else, if more than one observer is employed, to arrange a systematic interchange of observers, in such a way that each takes an equal share in the total of each group of observations concerned. If there is still any question as to the complete elimination of "personal" error, the matter can only be dealt with by having the work repeated by a number of different observers, and the results compared.

Asymmetrical errors naturally are very much more troublesome than symmetrical ones, which latter can be dealt with completely by taking a sufficient number of repeat observations and then taking a mean. The larger the number of observations taken, the smaller will be the probable error of the mean—though only in proportion to the inverse square root of the number of observations, so that unless the probable error of the individual observation is itself fairly small, the attainment of a really small probable error in the final result means taking an enormous number of repeat readings. But there is no theoretical limit to the improvement obtainable by multiplying observations where symmetric errors only are involved. The case of asymmetrical errors is quite different. If the distribution of the asymmetrical errors follows any kind of law there will be a definite mean error to which all the observations tend, and the only result of multiplying readings is to approach this mean error more closely. Where any cause of asymmetrical error is known, or supposed to exist, no precaution should be omitted to eliminate it, or to minimise it as far as possible, and care should always be taken to determine an upper limit to its possible influence, and to see that this is not in excess of what can be tolerated in the particular investigation concerned. Where the error is instrumental in character it can frequently be got over by a re-design, or by the use of some alternative procedure. Where it is inherent in the problem there is nothing for it but to improve the methods of control until it has been sufficiently reduced. The mode of eliminating personal errors has already been indicated.

---

[1] See "Gauges," §§ (82)-(83).

In considering any proposed design of apparatus, or any proposed procedure in a metrological operation, therefore, the most careful attention should be directed to investigating all possible sources of asymmetrical error, and providing against them in the most suitable manner. In cases where direct single readings are required, and means are not likely to be taken, symmetrical errors have naturally to be treated on the same footing as asymmetrical ones, and equal precautions taken with regard to errors of either type. But where means of large numbers of readings are to be taken symmetrical errors of larger magnitude can be allowed.

When once the accuracy to be attained in any particular operation has been laid down, the magnitude of the permissible error due to any cause can be decided. It is naturally desirable to keep each source of error as small as practicable within the limits of cost and labour which the particular operation may justify. And this applies especially to asymmetrical errors, which tend to give definite and (algebraically) additive errors in the results. But if one particular source of error cannot, within the limits of the problem, be reduced below a certain amount, there is no great advantage in making any special effort to reduce any other source to an amount less than (say) $\frac{1}{16}$th of this, particularly where the errors considered are symmetrical in type, since a large number of such sources of smaller errors would be needed to affect appreciably the total probable error of the result, while so long as errors are symmetrical in character the mean is unaffected, and a few additional readings will suffice to restore the desired accuracy.

(vi.) *Temperature Control.*—The elimination or reduction of errors due to variation and uncertainty in temperature conditions, naturally plays a very important part in all exact metrological operations and becomes, in fact, one of the most difficult problems involved when results of the very highest accuracy are desired. We have already indicated how by suitably choosing the materials of construction of the standards to be employed in operations of different types, and by suitably arranging the order of the observations, errors due to uncertainty as to the absolute temperature conditions, or to a gradual uniform change in these conditions, may be minimised. But it still remains necessary both to control, and generally also to measure, temperatures with great precision in order to obtain satisfactory results.

Variations in temperature may usually be attributed to four main causes, viz.: to differences between the temperature at which the apparatus has to be maintained for the purpose of the investigation and that of its surroundings; to general variations of external conditions due to the weather and to inequalities of distribution due to the heating and ventilating arrangements of the building in which the experiments are carried out; to the use of small lamps for illumination, or other similar accessories, in conjunction with the apparatus itself; or to the bodily presence of the observer. And their influence may be manifested either directly by their effects upon the objects being measured, or indirectly by their effects on the apparatus used, or upon other circumstances affecting the conditions of the experiments. The circumstances that may arise are naturally very varied in character, and the detailed descriptions given under various headings below show sufficiently the kind of precautions taken in a number of typical cases. A few general remarks can, however, be made.

The nature of the control which can be effected is, of course, very largely dependent on the conditions of the particular problem to be dealt with. If it is essentially an outdoor, or "field" experiment, such as the determination of the base for a geodetic survey, any control over the external condition is naturally out of the question, and recourse must be had (for example) to the use of invar as a means of reducing the amounts of the variations produced by changes of temperature, and to the provision of appropriate means for ascertaining the actual temperature of the tape or wire employed. This is by no means easy in the open air, particularly if the sun be shining and/or a wind blowing.

In the laboratory, the control of external conditions which can be effected still depends to some extent on the nature of the problem. A mural base, intended for standardising the tapes or wires used in the field, itself occupies considerable space, which it would be difficult to control completely against variations of temperature. And here, too, the comparisons still have to be made in air. But the majority of metrological apparatus, and in particular that concerned with comparisons of the principal standards, can be accommodated in rooms which are not too large to be conveniently regulated by thermostatic devices which enable a rough preliminary control of temperature to be maintained. The proper design and location of the rooms in a suitable building also assist largely in this direction. The principal rooms of the Metrology Department of the National Physical Laboratory are all inner rooms, completely surrounded on all sides by corridors, or by other rooms, and lighted through double-glazed ceilings, over which is a roof-space below double-glazed north lights, so that no direct sunlight can enter. Artificial heating and ventilation are provided.

Protection from the effects of the observer's presence may usually be secured by the introduction of suitable screens or lagging round the important parts of the apparatus, or alternatively, as is done, for instance, with certain balances designed for the most refined weighings, provision may be made by which the observer is enabled to perform all the operations needed for the experiment, and to take all the observations, while himself remaining at a distance. Screens or lagging may also be used to reduce the effects of local heating due to small lamps, which, however, can frequently be replaced with advantage by collimators arranged to focus light on to the apparatus from a distance.

These two classes of effects can often be further minimised by the choice of suitable materials for the essential structural parts of the measuring apparatus (as distinct from the objects being measured). The judicious employment of invar may be particularly useful in this connection when the effect is direct. In other cases invar may be applied in the construction of the working standard itself, as, for example, in the pendulum rod of a standard clock. If, on the other hand, the effect to be feared is due to the *difference* of expansion of two different parts of the apparatus subject to the same thermal influence, it is usually more important to ensure that each is made of the *same* material than that either of them individually should have a low thermal coefficient.

When temperatures are varying even slowly it is extremely difficult to be sure that the thermometer readings really correspond with the temperatures simultaneously existing in the objects being compared, owing to the lag of both thermometer and object behind their surroundings, which lag may be entirely different for one and for the other, according to their external form, to the material of which they are made, and to the history of the temperature changes. This, of course, applies with greatly increased force to those cases where it is necessary for the comparisons to be made in air. As far as possible the objects to be compared, with their thermometers, should be placed together in enclosures sufficiently well lagged to prevent any but very slow changes in external temperature from penetrating, and precautions should be taken to ensure that the objects and the thermometers are in as intimate association as possible. A second and delicate thermostatic control of the interior of the lagged compartment may be desirable in many cases, and if it can be filled with water, or other liquid, and well stirred at frequent intervals, a great advantage may be gained.

Two types of thermal screen may be employed, either singly or in conjunction. Their respective advantages and applicability should be considered in each case. A non-conducting lagging has the effect of preventing the flow of heat, and so damps out the more rapid of the external fluctuations. But it does not facilitate the equal distribution of temperature throughout the whole of the internal space. A metallic envelope, on the other hand, though it lets heat pass, tends, as a conductor, to establish a uniform temperature over its whole surface, and so by radiation to equalise the temperature distribution throughout its interior. For this reason metallic cases are of great value for the better types of balance. To secure the maximum advantage from a metallic case it should preferably be polished to a highly reflecting surface externally, and coated a dull black to facilitate radiation internally.

If surrounded by an outer covering of lagging material, the polishing of the external surface is, of course, not necessary.

(vii.) *Rigidity of Apparatus and Foundations.*—This may play an important part in determining the accuracy of results. Errors may arise from vibrations of the apparatus as a whole, transmitted to it through the earth and caused generally by the motion of heavy bodies or traffic in the neighbourhood. Or they may arise from displacements due to the motion of the observer, or of other individuals close at hand; or from the necessary movements of parts of the apparatus itself leading to slight distortion of other parts.

It is hardly possible to eliminate completely all effects due to earth tremors, but they can usually be minimised by mounting the whole apparatus on a sufficiently massive foundation, which should also be made quite independent of the foundations and floors of the building, so that it may be unaffected by movements of the observers and others. Those important parts of the apparatus itself which are liable to distortion, or vibration, should be made as rigid as possible, and should be designed so that they are supported in a manner which relieves them from the influence of deformations arising from the motions of other parts. This may frequently be done by making the part in question as a separate unit, and giving it only a kinematic constraint *e.g.* by resting it on three balls arranged on the well-known hole, slot, and plane principle. A good example of such a case is to be found in the microscope girder of the 1-metre comparator (see "Comparators," § (6)).

III. SYSTEMS OF STANDARDS

§ (6) HISTORICAL AND GENERAL.—At the present day two systems of standards, the British and the Metric, are so firmly established, and each of such widespread application, that both have to be fully considered in any general discussion of this question, while all others fade into comparative insignificance, and have, at the most, local or historical interest. Of the two systems mentioned the British is of far the greater antiquity, weights and measures according to this system having been handed down, with some vicissitudes but with no violent alterations, from those in use by the Romans. Each system uses the second as the unit of time, so that the differences between them relate solely to the units of length and mass.

(i.) *British Units.*—The fundamental standard of length on the British system is the yard, a length which has been preserved almost unchanged since the days of Edward I. The popular idea that the yard was originally "the length of the King's arm" is not correct. Various attempts at legislation with a view to enforcing the use of uniform standards of length and weight have been made in almost every country from very early times. The importance of such uniformity in commercial intercourse is obvious, and is even now the most powerful argument of those who advocate the universal adoption of the metric system. Such legislation had occurred in England prior to the reign of Edward I. But the inch, the foot (derived from the Roman foot), the cubit (18″), and the "ulna," which was the predecessor of the yard, and which gave its name to the yard of Edward I., had been defined independently of each other, and in various manners, at different times. The effect of the Act of Edward I. was to correlate and unify these various pre-existent measures, and the important clauses, translated, read as follows:

"It is ordained that three grains of barley, dry and round, make an inch; 12 inches make a foot; 3 feet make an 'ulna'; 5½ ulne make a perch; and 40 perches in length and 4 perches in breadth, make an acre.

"And it is to be remembered that the Iron Ulna of our Lord the King contains 3 feet and no more; and the foot must contain 12 inches, measured by the correct measure of this kind of Ulna, that is to say, one thirty-sixth part of the said Ulna makes one inch, neither more nor less, and 5½ ulne, or 16½ feet, make one perch in accordance with the above described Iron Ulna of our Lord the King." [1]

Unfortunately the actual standard bar created by Edward I. has been lost, and the earliest authentic standard we possess is the brass yard of Henry VII., now preserved in the Standards Department of the Board of Trade.

The Act of Edward I. also specified measures of capacity, the gallon and the bushel, and apparently copies of the principal standards were made and distributed to various towns, for it continues:

"The standards of the bushel, of the gallon, and of the 'ulne' which have been sealed with the iron seal of our Lord the King are to be kept diligently and safely, under a penalty of £100. And let no measure be made in a town, unless it agrees with the measure of our Lord the King, and is sealed with the seal of the corporation of the town. If any person buys or sells with measures that have not been sealed, or have not been inspected by the mayor and the bailiffs, he will be severely punished. And all measures and 'ulne,' greater and less, are to be inspected and carefully examined twice every year. The standards of the bushel, of the gallon, and of the 'ulne,' and the seals with which they are sealed, are to be kept in the custody of the mayor and the bailiffs, and of six legally sworn citizens of the town, in whose presence all measures must be sealed." [1]

It will be seen that here we have, in theory at least, a very well developed scheme of inspection of weights and measures corresponding fairly closely to the current practice of to-day.

The history of the standards of mass is similar. At different times "pounds" of various kinds have been used and standardised for different purposes, and the case is complicated by the inclusion of money measures, owing to the minting of coins by weight. The term "sterling" is a survival of an early name of the penny, while on the other hand we still have the "pennyweight" of 24 grains, or one-twentieth of an ounce Troy. It took very much longer for the standards of mass to be co-ordinated and regulated than for those of length. Of five standards which have been variously legalised at different times we have still the systems of Avoirdupois and Troy weight in existence, but even these were not finally defined, in their present values and relationship to each other, until as late as 1824.

---

[1] Translations taken from *British Weights and Measures*, by Col. Sir C. M. Watson, K.C.M.G., C.B., M.A. Published by John Murray, London, 1910.

An Act of George IV. passed in this year repealed all the laws on the subject enacted since the time of Edward I., and ordained that all measures of length were to be based upon a standard yard which had been constructed by a Parliamentary Committee in 1758, which was in future to be called the imperial standard yard; and that all measures of weight were to be derived from the troy pound constructed by the same Committee, which was to be known as the imperial troy pound, and that the pound avoirdupois (containing 16 avoirdupois ounces) was to be exactly equal in weight to 7000 troy grains.

The Act further defined the gallon, in the form still legal, as the volume occupied by 10 avoirdupois pounds of distilled water at the temperature of 60° F. weighed in air against brass weights, with the barometer at 30 inches of mercury.

The yard constructed by the Parliamentary Committee of 1758, and legalised by the Act of 1824, was based on a brass yard made by order of Queen Elizabeth in 1587, which is also preserved at the Board of Trade, and which agrees with the present standard yard within $\frac{1}{100}$ inch. The yard of Henry VII. differs from the present legal standard by 0·037 inch.

The Act of 1824 required the yard if damaged or lost to be replaced by reference to the length (39·1393 in.) of the "pendulum vibrating seconds of mean time in the latitude of London, in a vacuum at the level of the sea," and the pound weight by reference to the weight of a "cubic inch of distilled water at a temperature of 62° F."

The present legal standards were constructed after the destruction by fire, in 1834, of the standards authorised ten years earlier. A very careful study was made of all available copies of the lost standards on which it was considered that any reliance might be placed, and an elaborate investigation was made into the conditions of the problem. In particular, new and more accurate thermometers had to be specially made and calibrated for controlling the temperatures of the length standards during measurement. The Committee entrusted with the work recommended against referring to any natural standards such as the quadrant of the earth's meridian, or the length of the seconds pendulum—the idea of comparisons with the wave-length of light by interference methods had not then been developed—and decided to safeguard against future loss by duplicating the standards and causing copies, known as "Parliamentary copies," to be deposited in safe custody with various responsible bodies. These copies were carefully compared with the imperial standards preserved in the office of the Exchequer, so that their small inevitable errors are known, and were deposited in 1854 (1) in the Houses of Parliament (immured in the New Palace at Westminster), (2) at the Royal Observatory, Greenwich, (3) at the Royal Mint, and (4) with the Royal Society. These Parliamentary copies constitute the secondary standards of the British system, and, with the exception of those immured in the New Palace at Westminster, are required by law to be intercompared once every ten years, and to be compared with the imperial standard once every twenty years.

An Act (18 & 19 Vict. c. 72) of 1855 legalised these standards, and at the same time reversed the relative positions of avoirdupois and troy weight, making the former the only legal standard for general use, and limiting the application of the latter to the weighing of gold, silver, and precious stones, and the retail sale of drugs. This Act did not nominally alter the actual magnitudes of the standards, although the standards themselves had in the meantime been lost by fire and replaced as faithfully as possible by reference to existing copies, between the years 1834 and 1855.

An Act of 1866 transferred the custody of the imperial standards from the Comptroller-General of the Exchequer to the Board of Trade, and the Weights and Measures Act of 1878, which is still in force, repealed the Standards Act of 1855, but re-enacted so much of it as particularly described the imperial standards themselves. At the same time the troy *pound* was abolished, though the troy *ounce*, of 480 grains, remains a legal unit for weighing gold, silver, and precious stones. Apothecaries' Measure is also retained for the use of druggists.

(ii.) *Metric Units.*—The history of the metric system is less involved, commencing from the time of the French revolution (1792). The standard of length on this system is the *metre*, which was originally intended to be one ten-millionth part of the quadrant of the earth's meridian. The measurement of the meridian was made in terms of the old French measure, the "toise," and from this determination, made with very high precision for its time, it was found that the theoretical length of the metre should be ·513074 toise. The original "Mètre des Archives" of France was constructed on this basis, and consisted of a flat bar of platinum, 25 mm. wide × 4 mm. thick, the metre being thenceforward defined as the distance between the centres of the end faces of this bar, at the temperature of melting ice, without further reference either to the toise or to the earth's quadrant.

The metric unit of mass, the kilogramme, was intended to be the mass of a cubic decimetre of water at its temperature of maximum density (4° C.). This was determined by careful measurement and hydrostatic weighing of a bronze cylinder of equal diameter and length (243·5 mm.), from which, by comparison, was constructed the platinum standard kilogramme.

Both these operations were conducted with extreme skill, and the accuracy obtained in both standards, as is now proved by measurements made with improved appliances and more modern methods, was appreciably higher than could have been anticipated from the means available at the time. The metre appears to have been established in terms of the toise to about 0·01 mm. (1 part in 100,000), and the most recent determination of the kilogramme in terms of the cubic decimetre of water shows it to be in error[1] by only 2·7 parts in 100,000. Such results, for the time at which they were made, must be attributed in some degree to good fortune, as well as to the skill and care of those[2] concerned on the work.

The metric system thus originated as a national system of weights and measures in France, and, represented by the two standards just mentioned, remained as such until 1889. Representations made in 1867 and 1869 by the Conference of the Geodetic Association, and by the Academy of Science of St. Petersburg, led to the appointment in 1870 of an International Metric Commission, which in 1872 reported in favour of replacing the French metric standards by new international standards which should preserve as closely as possible the values of the standards of the Archives of France, and of the formation of an International Bureau of Weights and Measures; and at the same time made detailed proposals as to the character of the new standards and the methods to be adopted in preparing them. In 1875 effect was given to these proposals by the signature at Paris of the "Convention du Mètre," by which the governments of the various contracting States undertake to maintain at common expense a permanent International Bureau for the purposes mentioned. This Bureau, known as the Bureau International des Poids et Mesures, is actually housed at the Pavillon de Breteuil, Sèvres, near Paris, and is controlled by an International Committee, acting under the general instructions of a General Conference, which meets once in six years. The present Director is Dr. Ch. Ed. Guillaume, who, with his predecessor, Dr. René Benoit, was very largely concerned in the work of preparing the new international and national metric standards.

The new international prototype standards were completed in the year 1882, and after elaborate comparisons were both declared *identical*, within the limits of error of the measurements, with the original standards of the Archives. The intercomparison of the national copies of the new standards with the prototypes, and with each other, occupied another seven years, and it was not till 1889 that these were accepted by the Conference and distributed by lot to the various nations signatory to the convention. The national copies are all as nearly as possible identical, in material and construction, with the prototypes, and are to be regarded as tertiary standards. Two standards of each type (metre and kilogramme) were also selected to serve as "temoins" or secondary standards—*i.e.* to act as controls on the prototypes, or to afford the basis for replacement if ever required.

§ (7) THE STANDARDS OF LENGTH. (i.) *British.*—*The Imperial Standard Yard* is fully defined and described in the First Schedule, Part I., of the Weights and Measures Act, 1878 (41 & 42 Vict. c. 49).

"The imperial standard for determining the length of the imperial standard yard is a solid square bar, thirty-eight inches long and one square inch in transverse section, the bar being of bronze or gun-metal; near to each end a cylindrical hole is sunk (the distance between the centres of the two holes being thirty-six inches) to the depth of half an inch, at the bottom of this hole is inserted in a smaller hole a gold plug or pin, about one-tenth of an inch in diameter, and upon the surface of this pin there are cut three fine lines at intervals of about one-hundredth part of an inch transverse to the axis of the bar, and two lines at nearly the same interval parallel to the axis of the bar: the measure of length of the imperial standard yard is given by the interval between the middle transversal line at one end and the middle transversal line at the other end, the part of each line which is employed being the point midway between the longitudinal lines, and the said points are in this Act referred to as the centres of the said gold plugs or pins; and such bar is marked 'copper 16 ozs., tin 2½, zinc 1. Mr. Baily's metal. No. 1 standard yard at 62·00° Fahrenheit. Cast in 1845. Troughton & Simms, London'" (*Fig.* 1).

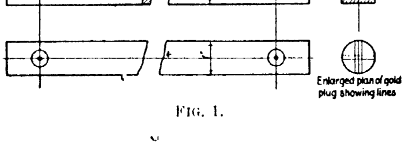


FIG. 1.

The object of sinking the graduations below the surface of the bar is twofold. In the first place it serves to protect them from accidental damage. But a more important consideration is this: if the graduations are carried on the upper surface of the bar, then, as was first pointed out by Capt. Kater, any flexure of the bar due to its own weight will produce a contraction or elongation of its fibres, except in the plane of the neutral axis of the bar, which may vary according to the mode of support. By bringing the graduations into the neutral plane this source of uncertainty is eliminated.

---

[1] See "Volume, Measurement of," §§ (1), (2).

[2] Borda, Lefevre-Gineau, and Fabbroni.

There is, however, still the possibility of a slight error due to the elasticity of the bar, since, when supported in any determined manner, the neutral axis will itself be slightly curved, and there will, therefore, be a very small difference between the length of the bar when placed on its supports and when in the free condition. To minimise this effect as far as possible a special lever frame is provided for supporting the imperial standard yard or its copies during measurement, so that the weight of the bar is equally distributed over eight rollers, diagrammatically thus:

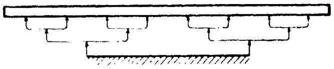


FIG. 2.

Sir G. B. Airy[1] calculated the best distance apart for any number of equally spaced supports to be

$$b = \frac{a}{\sqrt{n^2 - 1}},$$

where a is the total length of the bar (supposed of uniform section), and n the number of supports. In the case of the imperial standard $a = 38''$, $n = 8$, so that $b = 38/\sqrt{63} = 4{\cdot}79''$.

(ii.) *Metric.*—The international prototype metre is defined as the distance, at 0° C., between the centre portions of two lines graduated on the polished surface of a bar of pure platinum-iridium alloy (10 per cent iridium) of 102 cm. total length, and of cross-section as *Fig.* 3. The graduations are on the web, upper surface of the web, which contains the neutral axis of the bar. This form of section was devised by G. Tresca, and presents two advantages. Firstly, being uniform throughout the length of the bar, it permits of the whole length of the bar being graduated—for example, in millimetres—and secondly, it gives very great rigidity for the amount of metal employed, which was a consideration when over forty similar bars were required of such an expensive material as platinum-iridium.

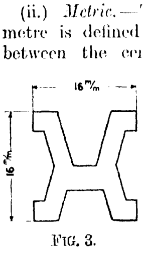


FIG. 3.

The material, platinum-iridium, was proposed by H. St. Claire-Deville, and was expected to prove extremely stable as regards secular change—a prediction which, so far as the evidence available enables an opinion to be formed, appears to have been admirably fulfilled.[2] It has the further advantages of being hard, inoxidisable, taking a high polish, and having a comparatively low coefficient of thermal expansion $(8{\cdot}65T + 0{\cdot}001T^2) \times 10^{-6}$.

The metre standard, when in use, is supported on only two supports, which, according to Airy's formula, should therefore be spaced at $102/\sqrt{3} = 58{\cdot}9$ cm. apart. Broch,[3] starting from a slightly different standpoint, found the value 57·0 cm., which is the distance actually employed. Airy's calculation is based on bringing the graduated surface of the bar at both ends into exactly the same (horizontal) plane, while Broch's is based on making the distance apart of the graduation marks at the ends a maximum, and consequently independent of slight variations in setting of the supports. It can be shown[4] that variations within the limits of the two formulae have no influence on the measurements comparable with the accuracy ordinarily obtainable in the final results. It is customary now to make use of the two-point method of support for line standard work generally, Airy's formula, however, being mostly employed.

§ (8) THE STANDARDS OF MASS. (i.) *British.*—The fundamental standard of the British Imperial System is the pound avoirdupois, which is defined as the mass of a certain cylinder of pure platinum, about 1·35 inches high and 1·15 inches in diameter, with a groove round it about 0·34 inch from the top for insertion of the prongs of an ivory fork by which it is to be lifted, and with all edges carefully rounded off, marked "P.S. 1844. 1 lb.," and now preserved at the Standards Department of the Board of Trade, 6 Old Palace Yard, Westminster (*Fig.* 4).

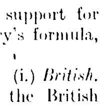


FIG. 4.

The present imperial standard pound was prepared, not from a previously existing avoirdupois standard, but from certain authenticated copies of the old brass troy pound of 1758, which, up to the time of its destruction by fire in 1834, occupied the position of principal standard.[5]

---

[1] *Phil. Trans.*, 1857, pt. iii. p. 17.

[2] The most recent report of the Bureau International indicates some very slight unexplained variations in the relative lengths of certain of the national copy metres recently recompared.

[3] *Travaux et Mémoires du Bureau International*, vii. B 62.

[4] Chree, *Proc. Phys. Soc.*, 1901, xviii. 594.

[5] H. W. Miller, *Phil. Trans.*, 1856, cxlvi.

As with the yard, Parliamentary copies of the imperial standard pound have been deposited at the Royal Mint, at the Royal Observatory, Greenwich, and with the Royal Society, and one immured in the New Palace, Westminster. The Act legalising the use of this standard proceeds :

"The said weight . . . shall be the legal and genuine Standard Measure of Weight, and shall be and be denominated the Imperial Standard Pound Avoirdupois, and shall be deemed to be the only Standard Measure of Weight from which all other Weights and other Measures having Reference to Weight shall be derived, computed and ascertained, and One equal Seven Thousandth Part of such Pound Avoirdupois shall be a Grain, and Five Thousand, seven hundred and sixty such Grains shall be, and be deemed to be, a Pound Troy.

"If at any Time hereafter the said Imperial Standard Pound Avoirdupois be lost, or in any Manner destroyed, defaced, or otherwise injured, the Commissioners of Her Majesty's Treasury may cause the same to be restored by Reference to or Adoption of any of the Copies so deposited as aforesaid, or such of them as may remain available for that Purpose."

The relation between avoirdupois and troy weight was not altered by the introduction of the new standard, but from this date the former became the primary standard, and the latter a derived standard, instead of *vice versa*, as was formerly the case.

In the work of reproducing the standard a number of weights (two of platinum and six of brass) were available, the weights of which *in air* had all been very accurately compared with that of the old standard before its loss. These weights were re-compared in 1844, and it appeared evident that the brass weights had gained in mass, as compared with the two platinum ones, whose relative values had remained unchanged, by amounts varying from 0·009 to 0·023 of a grain. It was found possible, however, by the aid of all the observations available, to re-establish the weight of the standard *in air* to an accuracy of about 0·001 or 0·002 grain (say 1 part in 5,000,000). Unfortunately, no record was to be found of any determination of the volume or density of the lost standard, and although four other troy pounds were available which had been struck with it in 1758, their densities were found to vary considerably (from 8·15 to 8·40). It was assumed, for reasons not very convincing and too long for description here, that the smaller value was more probable for the density of the lost weight, and the new standard was constructed on this assumption. The air-buoyancy correction being roughly $0{\cdot}0012/\Delta$, it will be seen that a variation of Δ from 8·15 to 8·4 represents a possible uncertainty in the reproduction of the *mass* of the standard of 4 parts in 1,000,000—*i.e.* twenty times the uncertainty of reproduction of the weight in air. It is probable that the mass was reproduced more exactly than this, but it is not possible to say so definitely.

Copies of the new standard were sent to the principal countries of the world, including the United States of America, in which the British System of Weights and Measures is still the legal standard.[1]

(ii.) *Metric.*—The primary standard of mass on the metric system is the international prototype kilogramme, which is a simple cylinder of platinum-iridium alloy (10 per cent iridium, density 21·55148), of approximately equal height and diameter, deposited at the Bureau International des Poids et Mesures, Sèvres. It was found identical in mass, within the limits of observational error, with its predecessor, the "kilogramme des Archives" of France (*Fig.* 5). Two copies of it are preserved for use as secondary standards, while further copies have been distributed for use as national standards to the various States signatory to the Convention du Mètre. All these copies are in agreement with the prototype within 1 mg. (1 part in 10^6), and have had their deviations from it determined to an accuracy probably better than 0·01 mg. (1 part in 10^8).

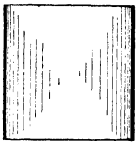


FIG. 5.

§ (9) SUBSIDIARY AND DERIVED UNITS.—For many purposes the units defined by the actual standards are not of convenient magnitude, and other units, either multiples or submultiples of the standard units, are commonly employed. In the English system we have the inch, the foot, and the mile, all at least as widely used as the yard, while as regards the metric units we have the C.G.S. (centimetre, gramme, second) system of almost universal application in scientific computations.

The following table shows the relations of the British units of length :

$2\frac{1}{4}$ inches $= \frac{1}{16}$ yard $=$ 1 nail.
12 inches $=$ 1 foot.
3 feet $=$ 1 yard.
6 feet $=$ 2 yards $=$ 1 fathom.
$5\frac{1}{2}$ yards $=$ 1 rod, pole, or perch.
4 rods $=$ 1 chain.
10 chains $=$ 1 furlong.
8 furlongs $=$ 1 mile.

The units of area commonly employed are the square inch, square foot, and square yard, but for land measure a special unit—the acre—is used. This is defined as a rectangle 4 rods wide by 1 furlong in length, and a special unit of length, known as the "chain," is used in land surveying. The chain is one-tenth of a furlong (66 ft.) in length, and is usually divided into 100 "links," each 0·66 ft. long. The acre equals 10 square chains. One quarter acre equals one rood.

---

[1] U.S. legislation refers to British standards of the time at which it was enacted, some of which have been amended since. The U.S. gallon, for example, is the old gallon of Queen Anne, and differs from the present imperial standard gallon in the ratio 0·8325 : 1.

The corresponding table for the British imperial units of mass is as follows:

Avoirdupois Weight

16 drams = 1 ounce (oz.).
16 ounces = 1 pound (lb.).
14 pounds = 1 stone.
100 pounds = 1 cental.
8 stones = 112 lbs. = 1 hundredweight (cwt.).
20 hundredweights = 1 ton.

The term "quarter," of 2 stones, should not be used in this connection, as it is the name of a legal measure of capacity.

In addition to the above, the "grain" is defined by the relation 7000 grains = 1 lb. avoirdupois, and from this troy weight, legal only for weighing gold, silver, and precious stones, is derived as follows:

24 grains = 1 pennyweight (dwt.).
20 dwts. = 480 grains = 1 ounce (troy).

The troy pound (= 12 troy ounces) is no longer a legal standard, and the troy ounce is now legally divided decimally.

The ounce of 480 grains is also legal for the retail sale of drugs, but for this purpose is differently subdivided (apothecaries' weight) as under:

20 grains = 1 scruple.
3 scruples = 1 drachm.
8 drachms = 1 ounce.

The metric system of multiples and sub-multiples is very much simpler, the factor in every case being 10, and the various subsidiary units being systematically named in such a fashion as to indicate immediately to which decimal place they correspond. The two series are as follows:

1 kilometre (km.) = 1000 metres (m.).
1 hectometre = 100 „
1 decametre = 10 „
1 decimetre (dm.) = 0·1 „
1 centimetre (cm.) = 0·01 „
1 millimetre (mm.) = 0·001 „

with the still further special units

1 micron (μ) = 10^{-6} metre.
1 Ångström unit (used in spectroscopy) = 10^{-10} metre.
1 megametre (used in astronomy) = 10^{6} metres.

and

1 kilogramme (kg.) = 1000 grammes (g.).
1 hectogramme = 100 „
1 decagramme = 10 „
1 decigramme = 0·1 „
1 centigramme = 0·01 „
1 milligramme (mg.) = 0·001 „

Areas are mostly expressed in square millimetres, centimetres, or metres. For land measure the "are" (100 sq. m.) and "hectare" (100 ares) are employed.

On both the English and metric systems special units of capacity are used, in addition to the cubic inch, foot, millimetre, centimetre, decimetre, etc., which are the correct theoretical units of volume. In each case the special unit is defined by reference to the density of pure water.

On the English system the gallon is defined as the volume of pure distilled water which, when weighed in air against brass weights, both water and air being at 62° F. and the barometric pressure 30 inches of mercury, weighs 10 imperial pounds. This definition is incomplete in so far as it does not specify the density of the brass weights, or the humidity, etc., of the air. The table of capacities is as follows:

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
2 gallons = 1 peck.
4 pecks = 1 bushel.
8 bushels = 1 quarter.
36 bushels = 1 chaldron.

Vessels constructed in accordance with this table are used for both dry and liquid measure, the smallest legal unit for dry measure being the half-pint, and the largest for liquid measure the gallon.

In addition there is the apothecaries' fluid measure, corresponding to the apothecaries' measure of weight as under:

60 minims = 1 fluid drachm.
8 fluid drachms = 1 fluid ounce.
20 fluid ounces = 1 pint.

Comparing this table with the preceding one it will be seen that there are 8 pints, and therefore 160 fluid ounces, in a gallon. Under the conditions of its definition the gallon weighs 10 pounds, so that 16 fluid ounces of pure distilled water, under these conditions, would weigh 1 pound. Thus, apart from the effects of air buoyancy on the weighings, the ounce of apothecaries' *fluid measure* corresponds to the ounce avoirdupois, whereas, as we have seen, the ounce of apothecaries' *weight* corresponds to the ounce troy—a curious anomaly.

On the metric system the unit of capacity is the "litre," defined as the volume occupied by 1 kilogramme of pure distilled water at its temperature of maximum density (4° C.) when under an atmospheric pressure of 760 mm. of mercury. As the result of an extremely careful and elaborate investigation carried out at the Bureau International[1] the volume of the litre was found to be 1·000027 cubic decimetres, a result which is probably correct within one part in 10^6.

---

[1] *Trav. et Memoires*, 1910, tome xiv.

The metric table of capacity is then:

Hectolitre = 100 litres (l.).
Decalitre = 10 ,,
Decilitre = 0·1 ,,
Centilitre = 0·01 ,,
Millilitre (ml.) = 0·001 ,,

IV. CONTROL OF WEIGHTS AND MEASURES USED IN TRADE

§ (10) THE TESTING AUTHORITIES.—In the United Kingdom, under the Weights and Measures Acts, all weights, measures, and weighing instruments used for trade must be verified and stamped by an Inspector of Weights and Measures. A fee is charged on the original verification and stamping of a weight, measure, or weighing instrument, and whenever it is presented for re-stamping. Weights, measures, and weighing instruments are also subject to periodical verification on annual inspection, for which no charge is made. The inspectors are appointed by the local authorities (County Councils or Town Councils). In Ireland, except in the Dublin townships, members of the Royal Irish Constabulary used to act as *ex-officio* inspectors of weights and measures. Every inspector before appointment must hold a certificate of qualification granted by the Board of Trade on the result of an examination. The local authorities are required to provide their inspectors with local standards sufficient for the needs of their districts, and with a suitable office and instrumental equipment. The local standards must be verified by the Board of Trade, and re-verified every five years in the case of weights and every ten years in the case of measures, either by the Board of Trade, or by the inspector himself in the presence of a Justice of the Peace, by comparison with other local standards which have themselves been verified or re-verified within the prescribed period. Local standards which have been adjusted must in all cases be re-verified by the Board of Trade.

§ (11) LEGAL DENOMINATIONS OF WEIGHTS AND MEASURES.—In order to avoid the facilitation of fraud, only weights and measures of certain specified "denominations" are permitted to be used in trade, but when it is proved to the satisfaction of the Board of Trade that any new denomination of weight or measure is reasonably required in trade, the Board may cause to be made, and legalised by Order in Council, a new standard of that denomination. The principal legal denominations at the present time are listed, with their equivalents, in § (9).

Since the year 1897—under the provisions of the Weights and Measures (Metric System) Act, 1897—the use of the Metric System has been legal in the United Kingdom for all purposes of trade. Metric denominations of weights and measures were legalised by an Order in Council dated 19th May 1898 (S.R. and O., No. 410 of 1898), and by an Order in Council of the same date (S.R. and O., No. 411 of 1898) tables of equivalents of imperial weights and measures in terms of the metric system were duly issued. By an Order in Council dated 14th October 1913 (S.R. and O., No. 1118 of 1913), new denominations of weight of the Metric Carat of 200 mg. and its multiples and sub-multiples were also authorised for use in trade.

In order to prevent confusion metric weights intended for use in trade must be of distinctive external form. The requirements with respect to the permissible denominations, form and material of construction, errors tolerated on verification and on inspection, methods of stamping, etc., as regards measures of length, measures of capacity for liquids, measures of capacity for dry goods, weights and weighing instruments, are set out in the Weights and Measures Regulations, 1907, which also include a series of instructions to inspectors as to the manner in which their various duties should be performed.

§ (12) REGULATIONS AS TO INSPECTION.—Although all sales made according to weight or measure must be made in terms of Imperial or metric weights or measures, it is not obligatory for commodities to be sold only by weight or measure, except bread and coal, which must be sold by weight, and intoxicating liquor sold in quantities not less than ½-pint (1 pint in Scotland) and not in casks or bottles, which must be sold by measure. The regulations with respect to the inspection, verification, and stamping of weights, measures, and weighing instruments are made by the Board of Trade and are the same throughout the United Kingdom. The local authorities are empowered to make by-laws for regulating the sale of coal in quantities not exceeding 2 cwt. in their respective districts, but these by-laws have first to be approved by the Board of Trade; they are in general different in different districts, and there is no model or prescribed code of by-laws.

Weights and measures must be marked with their denomination before they can be stamped by an inspector of weights and measures; and weighing instruments with their capacity, *i.e.* the maximum load they are intended to weigh. Traders can in general use whatever weights measures, or weighing instruments they choose, provided that these are stamped by an inspector of weights and measures; but weighing instruments for use in certain special trades specified in the regulations, *e.g.* dealers in precious metals or precious stones, jewellers, retail chemists, silk merchants, retail dealers in tea, coffee or tobacco, must satisfy certain specially prescribed requirements.

There are no special regulations with respect to the importation of weights and measures, but all weights, measures, and weighing instruments imported from abroad are required to be verified and stamped in the United Kingdom before they can be used for trade in this country, whether they were stamped in their country of origin or not.

Under Section 6, the Weights and Measures Act 1904, the Board of Trade have power to grant certificates of approval for patterns of weights, measures, weighing or measuring instruments, etc., which are submitted for examination and are found not to facilitate the perpetration of fraud. If the Board decline to give such certificate, no weight, etc., of such pattern shall be deemed legal and no inspector shall verify and stamp it for use in trade.

It will be noticed that measuring instruments are included among the instruments which may be submitted under this clause. Measuring instruments are, however, in an anomalous position, since although the Board of Trade have power to make regulations with respect to their verification and stamping, and to examine them under Section 6, there is no statutory provision prescribing that measuring instruments used for trade must be verified and stamped by an inspector of weights and measures. This obligation can, however, be imposed as necessity arises in particular cases by special legislation, as in the recent case of leather measuring instruments.

§ (13) Board of Trade Standards.—The standards used by the Board of Trade for the purpose of verifying or re-verifying local standards are called "Board of Trade standards." A weight or measure used for trade must be verified by the inspector by comparison with his local standard of the same denomination, and as the inspectors' local standards are all of the denomination of some Board of Trade standard, it follows that weights and measures which are not of the denomination of a Board of Trade standard cannot be stamped by an inspector of weights and measures for use for trade. The Board of Trade standards must be re-verified every five years.

The standards by comparison with which the Board of Trade standards of imperial denomination (*i.e.* not metric) are verified and re-verified are the Parliamentary Copies of the Imperial Standards of the Yard and Pound. The national metric standards of the United Kingdom, corresponding to the Imperial Standards, are the iridio-platinum line Metre No. 16 and the iridio-platinum Kilogram No. 18, obtained from the International Bureau of Weights and Measures; these are deposited at the Standards Department, and the Board of Trade standards of metric measure and weight are based on them and re-verified every five years by comparison with them. There are no "Parliamentary Copies" of metric standards. The Board of Trade standard measures of capacity are based on the Gallon and the Litre respectively; these are defined by the weights of water contained in them under specified conditions.

Board of Trade standards are provided of all the legal denominations listed in § (9), and also for the industrial standards given in § (22).

V. Comparison of British and Metric Systems

§ (14) Advantages of the Metric System.—The following advantages are claimed by the adherents of this system:

(*a*) That the definition of the standard of length by means of a measurement taken at the melting-point of ice renders the standard independent of errors in the temperature scale, and is therefore intrinsically more reliable than a definition at 62° F. which involves thermometric measurements.

(*b*) That the material of construction of the British standard has not proved completely stable, and that there is therefore a doubt as to the permanence of the standard, and whether it still represents accurately its original value.

(*c*) That the true kilogramme being (approximately) the mass of a cubic decimetre of distilled water at its temperature of maximum density affords a logical basis for the unit of mass, and one which is of great convenience in chemical computations.

(*d*) That the multiplicity of subsidiary standards, and the variability of the factors employed on the British system for forming multiples and sub-multiples of standards, are highly inconvenient, and constitute an unnecessary educational stumbling-block, contrasted with which the employment of the decimal scale of multiples and sub-multiples greatly facilitates computations of all kinds.

(*e*) That the system has received international sanction, is already legally obligatory in many countries, and optionally legal in many others, and therefore forms a proper basis for an eventual world system of standards.

With regard to these arguments in turn, the following remarks may be made:

(*a*) It is perfectly true that there is uncertainty as to the exact temperature at which the imperial standard yard was originally defined, owing to the variability of mercury-in-glass thermometers.

The thermometers employed were most carefully studied by Sheepshanks, who had charge of the work until his death in 1855.

But some of these thermometers, which have been preserved, are now found to differ amongst themselves by as much as ±0°·3 F.[1] corresponding to a possible variation of ±0·0001 inch in the length of the yard. It should be observed, however, that this is not a defect in the British standard *as it exists now.* It is precisely one of those cases in which gradually improving methods of observation permit improving precision of definition, *within the limits* of *former uncertainty.* The original uncertainty was inherent in the best known methods of the time when the standard was prepared, and the accuracy attained may fairly be said to have been as high as possible at that date. There is nowadays no difficulty in repeating measurements of temperature in the neighbourhood of 62° F. on the hydrogen (or, in conformity with the most recent decisions of the International Conference, on the absolute thermodynamic) scale of temperature to an accuracy corresponding to the requirements made. The difficulty lies rather in being sure that the temperature recorded is actually that of the bar being measured. Moreover, in actual practice, measurements are not made *at the melting-point* of ice, but at temperatures close to this, so that the uncertainty of thermometric measurement is not eliminated, but is somewhat reduced by the fact that the thermometers employed are used only over a small region of their scales, in the immediate neighbourhood of the fundamental point, whereby the possible errors of their calibration are minimised.

(*b*) There appears to be some reason for suspecting that this defect actually exists, and the Standards Department of the Board of Trade, admitting this possibility, have had made a new copy of the imperial standard, in platinum-iridium, of X section, similar to that of the metric standards. Variations have been found between the results of various periodical comparisons of the Parliamentary copies of the yard, in no case exceeding ±0·0001 inch. A study of the results does not however seem to indicate any definite directional changes, and it remains to be seen from the results of further intercomparisons in which the new platinum-iridium copy will be included, whether the variations so far observed are real, or due to observational errors. In the meantime the imperial standard yard remains the legal standard, and its variations, if any, do carry with them actual variations in the standard.

(*c*) This consideration is clearly one of scientific interest only, and from this point of view the (almost inevitable) failure to carry out exactly the intention has led to the introduction of an anomalous measure of capacity—the litre—into the metric system. The litre, defined as the volume occupied by one kilogramme of pure distilled water at its temperature of maximum density (4° C.), differs from the true measure of capacity, the cubic decimetre, by 27 parts in 1,000,000. This amount is too small to be of importance in the ordinary operations of chemical analysis, and the system accordingly has its practical utility. But the appeal to logic fails.

(*d*) This argument undoubtedly has weight. But it should be realised that it is a collateral advantage of the metric system rather than an inherent merit of the metric standards themselves. This distinction is often lost sight of in controversial statements on this subject. It is, in fact, perfectly possible to use the decimal system of subdivision in connection with any system of standards, and this is now commonly done, for example, in British engineering practice, where the inch is subdivided into tenths, hundredths, thousandths, etc., instead of into halves, quarters, eighths, and so on; on the other hand, it must also be remembered that while the decimal system, owing to the accepted use of the decimal notation in arithmetic, is a decided convenience in computation, the inability to divide by any integer between 2 and 5 without introducing a further decimal place or places in the result is often a distinct disadvantage in practical design. This is noticeable, for example, in cases where the value of the new digits introduced is such as to lead from the region of practical workshop measurement to something unattainable by ordinary workshop means.

It is undoubtedly unfortunate that the historical development of the British system has left it with a confused schedule of multiple and subdivisional units. The ideal system of subdivision is probably that incorporated in the division of the hour, or of the degree of angle, into minutes and seconds, the factor 60 being divisible by all the elementary numbers 2, 3, 4, 5, and 6. The next best, probably, would be the duodecimal system, into multiples of 12, each divisible into 2, 3, 4, or 6 and to a considerable extent this system is already to be found incorporated in the British measures—*e.g.* 12 inches = 1 foot, 12 pence = 1 shilling, 24 grains = 1 dwt. (troy). It would not be impossible to extend this scheme of division throughout the British system, and with a duodecimal notation in arithmetic such a scheme would be extremely convenient, though this proposal is hardly within the scope of practical politics.

As regards the practical utility of the two standards, there is probably nothing to choose so far as the larger measurements are concerned, but where workshop processes are involved, controlled generally by the use of a micrometer, the ten-thousandth of the inch is a practical limit of accuracy, while the hundredth of the millimetre, though amply sufficient for many purposes, is not quite adequate for others, and the thousandth of the millimetre is too small for measurement by the means usually available.

---

[1] *Memorandum on the Construction and Verification of a new Copy of the Imperial Standard Yard*, pt. 1. H.M. Stationery Office, 1905.

(*e*) This is the only argument which in the writer's opinion would be of sufficient importance to justify the attempt to substitute the metric system universally, in place of the British system, in countries where the latter at present prevails. It must be observed, however, that old measures die hard, since the general public, which has no scientific interest in the matter, is not readily made accustomed to a new order of things, while the industrial world is very deeply committed in the nature of its equipment, and by the necessity of providing interchangeable spare parts for goods already produced, to the system on which its organisation has been developed. The British system is probably more deeply rooted, owing to its antiquity, than any other, and is of the most widespread application. Even in France, the home of the metric system, where it has now for many years been the only legal standard, traces of the old French measures are (or were until recently) still to be found in use, being translated solely for purposes of sale into the legal units. It may be taken as certain that no kind of legal compulsion would be sufficient to effect the substitution of the metric for the British system against the wishes of the peoples using the latter. The only practical policy, and that which has actually been followed, is to give legal sanction to the *alternative* use of the metric system, and to trust to the processes of time to effect a gradual change. The efforts of those who desire to see the metric system in universal use would be more usefully employed in endeavouring to encourage and facilitate its voluntary adoption in this way, than in seeking to secure legal compulsion in advance of public desire.

§ (15) CONVERSION FACTORS. (i.) *British.*—Careful intercomparisons have been made of the Imperial and Metric standards, with the following results :[1]

1 metre = **39·370113** inches.
1 kilogramme = **2·2046223** lbs. (avoirdupois).

From these the following relations may be obtained by direct calculation :

1 inch = 25·399978 mm.
1 foot = 30·479973 cm.
1 yard = 0·9143992 m.
1 square inch = 6·451589 sq. cm.
1 square foot = 9·290288 sq. dm
1 square yard = 0·8361259 sq. m.
1 cubic inch = 16·38702 cu. cm.
1 cubic foot = 28·31677 cu. dm.
1 pound (avoirdupois) (7000 grains) = 0·45359243 kg.
1 ounce (avoirdupois) = 28·34953 g.
1 ounce (troy) (480 grains) = 31·10348 g.
1 grain = 64·79892 mg.

also

1 metre = 3·280843 ft. = 1·093614 yd.
1 sq. metre = 10·76393 sq. ft.
1 sq. cm. = 0·1550006 sq. in.
1 cu. m. = 35·31476 cu. ft.
1 cu. dm. = 61·02390 cu. in.

The relationship between the gallon and the litre, owing to the complexity of their definitions and the slight ambiguity involved in the definition of the gallon, cannot be determined with the same precision. It may perhaps best be arrived at thus.

The gallon weighs 10 lbs. when weighed in air against brass weights, at 62° F., under pressure 30 inches mercury. In the official computation the density of the brass weight is assumed[2] as $\delta = 8·143$. Taking the density of air half saturated with water vapour, and containing normal content (0·04 per cent) of carbonic acid, under pressure 30″ mercury as $\sigma = 0·0012175$, and the density of water at 62° F. (16°·667 C.) as $\Delta = 0·998860$ kilogrammes per litre,[3] the equation of weighing becomes

$$M\left(1-\frac{\sigma}{\Delta}\right)=10\left(1-\frac{\sigma}{\delta}\right) \text{ lbs.}$$

$$=4·5359243\left(1-\frac{\sigma}{\delta}\right) \text{ kg.,}$$

where M is the mass of 1 gallon of water at the temperature of definition, 62° F.

If G be the volume of the gallon, then

$$G=\frac{M}{\Delta}=4·5359243\frac{(1-\sigma/\delta)}{(\Delta-\sigma)} \text{ litres,}$$

whence, substituting the above values for σ, Δ, and δ, we get finally

1 gallon = 4·5459627 litres.

---

[1] These figures have been given legal sanction. Order in Council, May 19, 1898. In Vol. I. p. 580 of this Dictionary a slightly different series of values is given for these equivalents. The figures were taken from the *Computer's Handbook*, issued by the Meteorological Office in 1921, and are based on the ratio

1 metre = 39·37008 inches.

This is equivalent to

1 inch = 2·5399999 cm.,

or 1 inch = 2·54 cm. correct to 1 part in 25,000,000. This ratio, 1 inch = 2·54 cm., has been adopted in England as the basis for meteorological comparisons, and leads to the value

1 metre = 39·3700787 inches,

or 39·37008 correct to about 1 part in 30,000,000. The British legal value as stated above is 39·370113 inches, and hence we find

1 inch = 2·5399978 cm.,

or 2·54 cm. correct to about 1 part in 1,000,000. The relation between the yard and metre has recently been redetermined at the National Physical Laboratory by a new method giving very high precision. The final result is not yet available, but a provisional value, 1 metre = 39·370131 inches, has been obtained, which agrees within less than 1 part in 2,000,000 with the figure at present accepted.

[2] *Ency. Brit.*, "Weights and Measures," article by H. S. Chaney, footnote.

[3] Chappuis, *Trav. et Mémoires*, 1907, tome xiii

This figure differs slightly, owing no doubt to certain divergencies of assumption, from the legally accepted conversion, which is

1 gallon = 4·5459631 litres.

It should be remarked that since the litre itself has only with difficulty been determined in terms of the cubic decimetre to 1 part in 10^6 and the definition of the gallon is still less certain, the last two digits in the above figures are without real significance, so that the two values may be regarded as in agreement within the limits of the problem.

(ii.) *American.*—Reverting to the consideration of the two fundamental conversion factors, the case of the standards of mass presents no difficulty. In respect of the length standards, however, it has been objected that the doubt as to the values of Sheepshanks' thermometers renders the intended value of the yard uncertain to 0·0001 inch, so that the last three digits in the conversion are worthless. The argument, as indicated above, has no real weight, since what is of importance is rather the existing facts than the original intention. None the less, because of it the relationship has received legal sanction in America in the form 1 metre = 39·370000 inches. For this reason it is sometimes stated that the American inch differs from the British inch, but this does not appear to be the case legally. The ancient standard of America—dating from the times of the earliest British settlement in that country—is the yard. And in 1858 the American Government was supplied by the British Government with copies, similar to the Parliamentary copies, of the new British Imperial Standards. There does not appear to have been any legal enactment in America relating to these standards, but the Act of Congress of 1866, which authorises the optional use of metric units, does so in the following terms:

"It shall be lawful throughout the United States of America to employ the weights and measures of the metric system. . . . The tables in the schedule annexed shall be recognised . . . as establishing *in terms of the weights and measures now in use* in the United States, the equivalents of the weights and measures expressed therein in terms of the metric system. . . ."

Then follows the table commencing 1 metre = 39·37 inches. This clearly implies that the yard and inch remain unaltered, and presumably, though this is not stated, the yard of that period was in agreement with the British imperial yard, of which a copy not long before had been sent to America.

In 1893 there was what is known as the Mendenhall order, quoted thus in Circular No. 47 of the Bureau of Standards, Washington: "The Office of Weights and Measures will in future regard the international prototype metre and kilogramme as fundamental standards, in accordance with the Act of July 28th, 1866."

This order apparently has the effect of substituting the international metre for the yard as the principal standard of length in America, and in this case it would have the effect of altering the value of the yard. The order is so interpreted by the Bureau of Standards, and inch standards based on the certificates of that institution differ from our British inch to the extent that the factor 39·37 differs from the legalised value 39·370113. But the fact that there has been no Act of Congress altering the Act of 1866, and the reference to this Act in the Mendenhall order, seem to show that there is no legal authority for the variation of the yard standard, and that it would be more strictly correct to say that it is the American metre which is slightly different from the international metre. The great bulk of industrial standards in America are undoubtedly based on the original British inch. There is no doubt, however, that at the present time there is some ambiguity as to the actual position of the American standards, and the whole situation is an excellent illustration of the need for universal co-ordination.

It should be remarked that while the actual primary standards, being material bars affected by temperature conditions, have to be brought to certain exact temperatures (0° C. and 62° F. respectively) in order to serve their function in defining the two fundamental units of length, these units themselves are absolute in character, and entirely independent of temperature. It is incorrect to speak of the standard as representing the metre or yard except when at its appropriate temperature of definition. The ratio between the units is consequently a *pure number*, which, when once determined, is independent both of the standards themselves and of every other consideration.

The ratio 1 metre = 39·370113 inches represents the most accurate determination of this number so far made, and it is worth noticing that it gives rise to the very convenient relation

1 inch = 25·4 mm.,

which is accurate to within 1 part in a million. This factor besides being numerically simple has one great value in the industrial problem of the transference from the British to the metric system, since it enables screw threads of metric pitches to be cut accurately on lathes with English lead screws, by the simple interposition of a gear wheel of 127 teeth. It is unfortunate, therefore, that America should have adopted legally the ratio 1 metre = 39·37 inches, which, besides being less convenient, differs from the best determined value by rather more than three times as much as the simple ratio 1 inch = 25·4 mm.

VI. INDUSTRIAL METROLOGY

§ (16) TEMPERATURE OF ADJUSTMENT FOR INDUSTRIAL STANDARDS.—This brings us to the consideration of another point of considerable practical importance, which has been the subject of much debate. We have seen that the two primary length standards have different temperatures of definition, chosen for different reasons. The temperature of definition of the metre (0° C.) is designed to give the greatest scientific precision. That of the yard (62° F.) is intended to represent (and as far as Great Britain is concerned does fairly represent) a temperature of employment corresponding to the average conditions of use of industrial standards. Engineers' scales and gauges in Imperial measure are therefore ordinarily adjusted so as to be correct at 62° F., *i.e.* so as to agree with the primary standard when both they and it are at this temperature. On the metric system, however, since it is impracticable to use workshop appliances at 0° C., there is a certain ambiguity as to whether the industrial standards should be adjusted so as to be correct at the ordinary temperature of employment, or so as to agree with the primary standard when both are at 0° C. The latter procedure is that advocated by the orthodox metricists, and is adopted to some extent in France and Switzerland. But for the most part Continental and also American practice tends to the use of 20° C. as the temperature of adjustment for metric industrial standards. This is a somewhat higher temperature than 62° F., and its prevalence may be accounted for, no doubt, by the fact that American and Continental factories are usually kept rather warmer than English ones.

It has to be noticed in the first place that the use of 0° C. as the temperature of adjustment for industrial metric standards leads to the inconvenience that such standards, if made of varying materials, will not be mutually consistent at the actual temperature of use. A brass scale, for instance, which agrees with a similar steel one at 0° C., will exceed it in length by about 1½ parts in 10,000 when both are at 20° C. This amount is just too great to be neglected in modern workshop practice of a fairly precise nature. Consequently, if 0° C. be accepted as the temperature of adjustment for industrial standards, either the material of which such standards are to be made must be specified—an undesirable limitation, and one liable possibly to hamper progress—or else allowance must continually be made for the coefficient of expansion of the scales and gauges employed, which is a serious inconvenience in general engineering practice. It must be admitted, of course, that in certain cases—*e.g.* of very fine precision work for scientific apparatus, or for parts of heat engines destined to function at temperatures widely remote from those at which they are made—allowance has in any case to be made for thermal expansion. But such cases are a small proportion only of all that are met with in practice, and in this small class, at any rate, the necessary calculations can be completed in the drawing office and the allowances shown on the drawings, so that the workman is not affected. The variations of temperature ordinarily experienced in workshops not being excessive, it follows that in the vast majority of work sufficient precision will be obtained by the use of scales and gauges adjusted at the average temperature of use. And where a slightly higher precision is required the effects of temperature fluctuations can be further minimised if the scales or gauges employed are made of materials having approximately the same coefficients of expansion as the parts they respectively control. If all the scales and gauges are then adjusted to some common temperature of use, the various parts, even if of different materials, will then assemble and function correctly at such common temperature. There is thus a very strong case for the universal adoption of some common temperature, corresponding to average workshop conditions rather than 0° C., as the temperature of adjustment for metric industrial standards, and it is to be hoped that international agreement on the subject may not be long deferred.

With regard to the temperature to be chosen for this purpose, it would be an obvious convenience to the user of the British imperial system if the *same* temperature (62° F.) could be taken in both cases, and this would no doubt be a small factor assisting the gradual transition from the British to the metric system. But apart from this quite general consideration there is a practical factor of no less importance which bears on the case. We have seen that the conversion factor 1 inch = 25·4 mm. is correct within 1 part in 10^6, and that this simple ratio has a real utility in the very important problem of securing interchangeability of screw threads cut on lathes made according to the two systems. But this ratio is a pure number, representing the absolute relationship between the units, apart from the temperatures of definition. Consequently its direct employment in workshop practice implies that both systems are supposed to be based on standards adjusted to be correct at the *same common temperature*. And since the industrial standards of the imperial system are already adjusted at the reasonable working temperature of 62° F., while those of the metric system are still the subject of debate, it is very strongly indicated that universal agreement should be found on the adoption of 62° F. (16·67° C.) as the common temperature of adjustment for industrial standards on both systems.

§ (17) ENGINEERS' SCALES AND GAUGES. (i.) *Preliminary.*—The engineer's scale is ordinarily a comparatively thin strip of metal, usually steel, graduated fairly boldly in inches or centimetres, and subdivisions thereof. It is used only for relatively rough measurements. The vernier calliper, reading to 0·001 inch, is a somewhat more accurate application of line measurement to engineering practice, but even this accuracy is not high, and presents no difficulty in attainment. For precise measurement the engineer relies almost exclusively on micrometers and gauges; both representing forms of end-measurement.

The importance of establishing standard gauges for engineering use was first realised and acted upon by the late Sir Joseph Whitworth, who, in his workshops at Openshaw, Manchester, produced the first accurate surface planes, by the well-known method of scraping a set of three planes together until any two of them will touch each other all over. This he followed by the introduction of the Whitworth measuring machine—really a large and specially constructed micrometer—using a gravity piece to improve the delicacy of the sense of contact, whereby measurements to an accuracy of 0·00001 inch were made possible; and of accurately made end gauges and cylindrical plug and ring gauges. Further, he collected data relating to the proportions of the very heterogeneous screw threads prevalent in his day, and upon this data formulated the Whitworth series of screw threads, involving the standardisation, firstly, of the form of thread, and, secondly, of a series of suitably related pitches and diameters for general work. Finally, by establishing the manufacture of screw gauges and of accurate tools for the production of screw threads in accordance with his proposals, he gave them a practical form which has led in due course to the very extensive employment which they now enjoy. In all this work Whitworth was a pioneer, and its results were of inestimable value to the engineering industry.

(ii.) *Limit Gauges.*—The system of gauging established by Whitworth was designed to secure interchangeability of parts by the process of making the work a good fit to the corresponding gauges. It left the quality of the fit obtained to the skill of the workman, and to this extent was incomplete. For example, suppose a 1-inch shaft had to be made to fit a 1-inch hole. The 1-inch plug and ring gauges, made to fit each other as perfectly as possible, would be taken; the shaft would be made to pass the ring gauge, and the hole to take the plug gauge, and it would then be certain that the shaft would enter the hole. But the nature of the fit of the shaft in the hole would depend on how closely the workman had made the two parts to fit their respective gauges. Different qualities of fit are naturally required for different purposes, and it is left to the skill of the workman, firstly, to judge how closely the parts should fit the gauges, in order to secure a particular kind of fit in the product, and, secondly, to secure that in actual practice the fit of the work to the gauges is in accordance with this judgment. Such a system is defective in two ways. In the first place, different workmen do not aim at precisely the same results, and in the second place, having no definite limit of variation laid down, each man tends to spend far too long in getting the work as near as possible to what he judges desirable. The aggregate of all resulting work may be just as variable as if a fairly generous "tolerance" had been allowed in the first place, but the cost is much higher.

The tendency of all modern economical production is to decide in advance what difference of dimensions is needed between the parts in order to secure the desired quality of fit, and what limit of variation can be allowed on either part individually without impairing its satisfactory functioning. Suppose, for instance, that a 1-inch shaft were required to be a running fit in its hole. It might be decided that a clearance of not less than 0·001 inch was necessary for lubrication, while a clearance greater than 0·002 inch would lead to too slack a fit. The total tolerance available is the difference between 0·001 inch and 0·002 inch, which has to be distributed between the two parts. Suppose it equally divided. Then we might have the following limits:

		Gauges.	
For the hole	1·0005″	"High" "Not-Go"	(Plugs).
	1·0000″	"Low" "Go"	
For the shaft	0·9990″	"High" "Go"	(Rings).
	0·9985″	"Low" "Not Go"	

All this should be decided definitely in the drawing office, before actual construction of the parts is commenced, and the limits for each part should be shown on the drawing. When the workman comes to produce the parts, he is given a set of four gauges, two plugs for the hole, and two rings for the shaft, made as closely as possible to the sizes given above. Such gauges are called *Limit Gauges*, since they are the material representations of the limits of error between which the workman is required to work. And in working to limit gauges all the workman has to do is to reduce the shaft until it will readily enter the 0·999 inch "Go" ring gauge, taking care only that in so doing he does not overstep the mark and make it so small that the 0·9985 inch "Not-Go" gauge will also pass. Provided he secures this result the work is within the limits which it is known will be satisfactory, and he has no need to spend time carefully adjusting until he obtains what he thinks a satisfactory "feel" on the gauge. Corresponding remarks apply to the hole.

The limit gauge system thus greatly facilitates the economic production of repetition parts in bulk, and adds to what may for distinction be called the "standard" gauge system the advantage of not merely ensuring interchangeability, but at the same time, by means of the "Not-Go" gauges, maintaining a control over the quality of fit produced. It should be noticed that to secure *interchangeability* alone the "Go" gauges are in all cases sufficient. But in the absence of the "Not-Go" gauges the *fit* may be anything within the discretion of the workman.

It may be remarked that a limit gauge system is more expensive in prime cost than a standard gauge system. But where mass production is concerned, and under present-day conditions only mass production can be considered economical, the prime cost is very soon wiped out by the saving effected on each of a large number of similar parts.

§ (18) TOLERANCES ON GAUGES.—So far we have spoken as if the gauge were absolutely correct to its supposed size. Naturally, it is no more possible to make a perfect gauge than a perfect piece of work. Since the number of gauges employed is relatively small as compared with the number of parts each is used to control, it is possible to spend time and care in making the gauges to a considerably higher degree of accuracy than the work, but none the less some tolerance must be permitted for errors in manufacture of the gauge itself.

Before deciding as to this tolerance it is necessary to give careful attention to the purpose which the gauge in question is to serve. We have been considering so far only the process of producing the work in the shop. In practice it frequently happens that the work after production is subjected to independent inspection by the purchaser. In such cases it is important to observe that the limits given on the drawings are the nominal limits *for the work*. The manufacturer has no claim to have any work accepted which falls outside these limits; the inspector, on the other hand, has no right to reject any work which lies within them. Consequently, any tolerance which the manufacturer may require for the construction of his gauges must be so distributed as to fall *within* the limits for the work, while any tolerance on the inspector's gauges must be *outside* the same limits. The effect of this is that when the gauges are new the workman will actually have rather less tolerance on the work than the drawing shows, but he will be able to continue using his gauges until the "Go" gauge has worn right to the drawing dimension. And since he is working to the gauges, and not to the drawing, he will not ordinarily be conscious of the slightly increased stringency. But the system, if properly controlled, should ensure that no work which passes out of the shop is outside the drawing limits. The inspector, on the other hand, can continue to use his gauges until the "Not-Go" has worn to drawing size, and still be sure of rejecting nothing unfairly. The wear of the workshop "Not-Go" gauge, and that of the inspector's "Go" gauge, tend to improve the conditions for agreement between the parties. Naturally the wear of the "Not-Go" gauges is normally much less rapid than that of the "Go" gauges.

The whole of the above can readily be expressed in the form of a simple rule. Calling the larger gauge of each pair the "High," and the smaller the "Low," limit gauge, then the signs of the tolerances on the various gauges must in all cases be as in the following table:

Gauge.	Tolerances.	
	Workshop.	Inspection.
"High" . .	−	+
"Low" . .	+	−

If in our previous example the amount of the gauge tolerance be taken as 0·0001″ in each case, the limits of acceptability for new gauges would then be:

Gauge.	Workshop.	Inspection.
"High" plug .	**1·0005** 1·0004	1·0006 **1·0005**
"Low" plug .	1·0001 **1·0000**	**1·0000** 0·9999
"High" ring .	**0·9990** 0·9989	0·9991 **0·9990**
"Low" ring .	0·9986 **0·9985**	**0·9985** 0·9984

the numbers printed in heavier type being the nominal limits for the work.

It will be seen that in the worst case, with new gauges, the workman might be left with a tolerance of only 0·0003″, instead of the nominal 0·0005″. The inspector, on the other hand, might accept, if such were offered to him, work showing a total variability of 0·0007″. The example chosen is one where the work is being made to a fairly fine tolerance, and the influence of the tolerances needed for the gauges is relatively considerable. In a very large proportion of work made to gauges the gauge tolerance is of less importance.

In many factories the practice is for all work after leaving the shop to pass a factory inspection. In such cases, if no further outside inspection is to be anticipated, the factory inspector may make use of gauges made to inspection tolerances. But if the factory inspection is to be followed by outside inspection, the parts must be controlled right up to the moment of leaving the factory by gauges with *workshop* tolerances. If the tolerance on the work is generous there is no difficulty in arranging two series of manufacturer's gauges, one for production and one for domestic inspection, both inside the limits for the work, but the latter less stringent than the former. If, on the other hand, the work tolerances are tight there must be some give and take between the workshop and the local inspection department. A procedure frequently adopted with success in such cases is for the factory inspector to make use of "Go" gauges which have been in service for a time as workshop gauges and become slightly worn, but not outside the limits. The workman should use the worn "Not-Go" gauges.

Another case which may arise is that of a firm purchasing parts from a sub-contractor. If no final outside inspection is involved, the purchasing firm is in effect outside inspector to the sub-contractor, and may proceed accordingly. But if the finished article is to be subject to final inspection by a third party, then the purchasing firm must satisfy itself that all parts supplied by the sub-contractor are actually *within* the drawing limits for the work, and must make suitable provision in the contract, either by assigning slightly reduced limits to the sub-contractor, or otherwise, to enable this to be done.

§ (19) STANDARD, REFERENCE, CHECK, AND MASTER GAUGES.—In addition to the two classes of gauge so far discussed, there are a number of others which have to be considered. The terms "standard" and "reference" gauges are frequently employed, more or less indiscriminately, for gauges intended to represent as closely as possible the limiting dimensions of work. It is desirable to make a distinction. The term standard gauge should be reserved for simple types of gauge—*e.g.* end bars, slip gauges, such as those described in "Gauges," § (5), plain plug and ring gauges, etc.—made as accurately as possible to certain standard nominal sizes, which are used, not for the actual verification of work, but as bases of measurement in the construction and verification of other gauges. Reference gauges are simply inspection gauges, made with extra care so as to be as close as possible to the nominal limits for the work they are intended to control, and are used only in cases of doubt or dispute.

A "Check" gauge is a special gauge used in the manufacture or verification of other gauges, and stands in exactly the same relationship to the gauges as they themselves do to the work. The gauges in this case take the place of the work, and the tolerances on the check gauges must be even finer than those on the gauges they are to control, and their signs are governed according to the rôle each particular check is destined to play by precisely analogous rules to those which apply to the gauges. The check gauge, in many cases, will be practically a model of the work.

Lastly, a "Master" gauge is one specially made to assist in the manufacture or verification of a check gauge. If complete in form the master gauge bears the same relation to the check as the gauge does to the work. The tolerance on a master gauge should, if possible, be less again than that on the check. Generally speaking, however, check gauges are verified almost entirely by direct measurement, and a master gauge, when needed, often takes the form of a relatively simple auxiliary piece, used firstly in the manufacture of the check, and subsequently retained with it as an aid to measurement.

We thus see that there are several grades of accuracy necessary to the establishment of a complete system of gauge control. Taking the stages in order we have:

(*a*) The work,
(*b*) Workshop or inspection gauges,
(*c*) Check or reference gauges,
(*d*) Master gauges,
(*e*) Standard gauges,
(*f*) Ultimate standards,

each of which stages should represent an increase of accuracy over the one preceding it. On an ideal theory, the step in each case should represent at least a tenfold increase in accuracy, which, if the tolerance on the work were, say, 0·001 in., would involve making and measuring inspection gauges to 0·0001 in., check gauges to 0·00001 in., master gauges to 0·000001 in., and standard gauges to 0·0000001 in.; and finally keeping control on the ultimate standards to the equivalent of the hundred-millionth of an inch on the size involved. Neither workshop practice nor methods of measurement are yet adequate to the realisation of such high orders of accuracy. Nor would it be of any practical utility if they were, as variations of temperature would render such accuracy nugatory. Fortunately, it happens that the higher stages of the work are all carried out under expert supervision, and a much lower factor than 10 is permissible in these circumstances. But it is by no means easy, in a great many cases, to achieve as good a graduation as would be desirable, and the conditions of the problem indicate very clearly how important it is to sacrifice nothing of attainable accuracy at any stage of the work.

§ (20) PARTIAL AND UNIVERSAL INTERCHANGEABILITY OF STANDARDS.—The next point demanding attention is the distinction which has to be drawn between the requirements of a single workshop or factory and of the nation or world as a whole. To secure interchangeability between parts made in a single workshop it is sufficient if all corresponding parts are made to the same gauges, which have been suitably adjusted in relation to each other. It does not matter in the least whether the gauges are or are not strictly in accordance with the true nominal sizes they are supposed to represent. A firm, the dimensions of whose work are all consistently based on a false standard, will experience no difficulty until it comes to deal with other outside firms. But when it comes to purchasing standard tools, such as taps, drills, or reamers, or has occasion to contract out for the supply of certain components, or when its products come to be the subject of outside inspection, it may find itself seriously inconvenienced. In all such cases the parties with whom it enters into relations may be using standards different from its own, and the results may be disastrous. Consequently it is of the highest importance for any large engineering concern to assure itself, not only that its standards and gauges are consistent *inter se*, but further that they are correct to *absolute size* and consequently in agreement with those of every other firm which takes the same precaution. To achieve this some central authority is necessary by which the standards in use in industry may be controlled and verified.

The National Physical Laboratory, Teddington, Middlesex, undertakes this function as far as Great Britain is concerned.

§ (21) THEORY OF "GO" AND "NOT-GO" GAUGES.—The expression "limit" gauging implies the use of two gauges to control each dimension on the work. Each gauge is made nominally to one of the two limits of the tolerance permitted for the dimension in question, and one gauge must "go" and the other must not. Where a single dimension alone is concerned two simple gauges are sufficient for the purpose. But in many cases the form of the work is such that a number of elements have to be controlled simultaneously, not only as regards their various dimensions, but also as regards their relationships with each other. In such cases it is not sufficient to take a pair of "Go" and "Not-Go" gauges for each element separately. The "Go" gauge, to be effective, must combine in itself the "go" limits for all the elements, *in their correct mutual relationship*; otherwise it is quite possible that the work, though correct as regards each dimension taken separately, will not assemble as a whole.

On the other hand, while it is essential in order to secure satisfactory assembly that the "Go" gauge should combine in itself *all* the important elements of the work it controls, it is equally essential that the "Not-Go" gauges for each element should be separate and independent of each other. If the "Not-Go" gauge were made similarly to the "Go" gauge, in a form in which all the elements were combined, then it would be sufficient, in order to prevent the work from mating with the "Not-Go" gauge, for any one of the elements to be correct within its limits, and if this were the case, the gauge would give no indication as to the accuracy or otherwise of the remaining elements as regards their "Not-Go" limits.

The meaning of the two preceding paragraphs may best be illustrated by actual examples. As a very simple case, consider an ordinary photographic plate which has to fit into the rebate of a dark slide. To avoid fractions let us suppose it is a 4″ × 5″ plate. And let us suppose also that a perfect rectangle 4″ × 5″ in size represents the minimum clear opening of the rebate in any dark slide. The necessary and sufficient condition of interchangeability of all plates in all dark slides is then clearly that no plate shall be of such a size or shape that it will not fall into a gauge correctly representing the perfect figure of the 4″ × 5″ rectangle. Such a gauge is a correct "Go" gauge, and combines in itself not only the measure of the maximum dimensions, 4″ and 5″ respectively, allowable for the plate, but in addition provides a control on the straightness of the sides and on the angles between them.

Consider what might happen if, instead of such a complete form "Go" gauge, we were to use only two simple snap gauges, E_1 and E_2 (*Fig.* 6), to control the 4″ and 5″ dimensions. In such case we should pass without question so far as the gauges are concerned, plates of the forms shown at either A or B, where the full lines represent the minimum rebate in the slide.

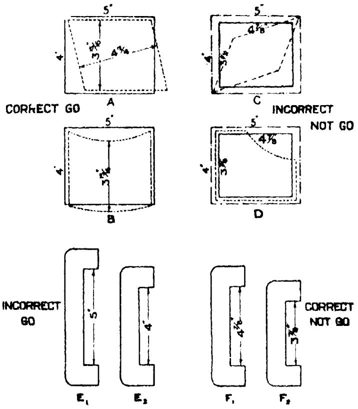


FIG. 6.

On the other hand, suppose that the ledge of the rebate is ¼ inch smaller in both directions than the recess. It is required then that the plate should in all cases cover the whole of the area of a perfect rectangle 3¾″ × 4¾″ in size. And since it is free to take up any position in the rebate, this means that the plate itself must not be less than 3⅞″ × 4⅞″. But now suppose we were to attempt to gauge this by a complete form "Not-Go" gauge similar to the 4″ × 5″ "Go" gauge. Then either the distorted plate (C) or the broken plate (D) (*Fig.* 6) would be accepted —clearly an unsatisfactory result. But if we used separate "Not-Go" gauges F_1 and F_2 to control each of these elements *independently*, both the defective plates would quite correctly be rejected.

The correct types of gauge for use as "Go" and "Not-Go" in this case are indicated in the figure, and if such gauges are used *every* plate which passes the "Go" gauge, and refuses the "Not-Go" gauges at all points to which they apply, is necessarily an entirely satisfactory plate in accordance with the specified requirements. The mere application of the gauges is a sufficient criterion, and there is no need to consider how closely the plate fits the "Go" gauge, or to observe it critically for defects in shape.

The same principle applies in every case where limit gauges are used on components in which two or more important elements are found in some definite relationship. The "Go" gauge must be unique, and complete in itself, and the "Not-Go" gauges separate and independent for each element. We shall meet another excellent example of the principle when discussing the subject of screw threads, and need not therefore labour the point further here.

The writer is indebted for the above example to Mr. W. Taylor, O.B.E., an acknowledged expert in all matters relating to limit gauging.

§ (22) **INDUSTRIAL STANDARD GAUGES.**—Standard gauges have been authorised for use in trade by Orders in Council dated as follows:

26th August 1881

DENOMINATIONS OF STANDARDS

(Whitworth Gauges)

(1) Whitworth's External Cylindrical Gauges:
    External Diameters in terms of the inch.
    Fifteen gauges from ⅛th to 1 inch, increasing by sixteenths of an inch.
    Twenty-four gauges from 1⅛th to 4 inches, increasing by eighths of an inch.
    Eight gauges from 4¼ inches to 6 inches, increasing by quarters of an inch.
    Nineteen gauges from 0·1 to 1 inch, increasing by five one-hundredths of an inch.
    Thirty gauges from 1·1 to 4 inches, increasing by tenths of an inch.
    Ten gauges from 4·2 to 6 inches, increasing by fifths of an inch.

(2) Whitworth's Internal Cylindrical Gauges:
    Internal Diameters in terms of the inch.
    (See (1).)

(3) Whitworth's External Plane Gauges:
    Thickness in terms of the inch.
    Ninety-one gauges from 0·01 to 0·1 inch, increasing by one-thousandths of an inch.

23rd August 1883

DENOMINATIONS OF STANDARDS

(Imperial Wire Gauge)

Descriptive Number.	Equivalents in parts of an Inch.	Descriptive Number	Equivalents in parts of an Inch.
No.	Inch.	No.	Inch.
7/0	0·500	23	0·024
6/0	464	24	22
5/0	432	25	20
4/0	400	26	18
3/0	372	27	164
2/0	348	28	148
0	324	29	136
1	300	30	124
2	276	31	116
3	252	32	108
4	232	33	100
5	212	34	0·0092
6	192	35	84
7	176	36	76
8	160	37	68
9	144	38	60
10	128	39	52
11	116	40	48
12	104	41	44
13	0·092	42	40
14	80	43	36
15	72	44	32
16	64	45	28
17	56	46	24
18	48	47	20
19	40	48	16
20	36	49	12
21	32	50	0·0010
22	28		

16th July 1914

DENOMINATIONS OF STANDARDS

(Birmingham Gauge)

Descriptive Number.	Equivalents in parts of an Inch	Descriptive Number.	Equivalents in parts of an Inch.
No.	Inch.	No.	Inch.
15/0 B.G.	1·000	7 B.G.	·1764
14/0 B.G.	0·9583	8 B.G.	·1570
13/0 B.G.	·9167	9 B.G.	·1398
12/0 B.G.	·8750	10 B.G.	·1250
11/0 B.G.	·8333	11 B.G.	·1113
10/0 B.G.	·7917	12 B.G.	·0991
9/0 B.G.	·750	13 B.G.	·0882
8/0 B.G.	·7083	14 B.G.	·0785
7/0 B.G.	·6666	15 B.G.	·0699
6/0 B.G.	·625	16 B.G.	·0625
5/0 B.G.	·5883	17 B.G.	·0556
4/0 B.G.	·5416	18 B.G.	·0495
3/0 B.G.	·500	19 B.G.	·0440
2/0 B.G.	·4452	20 B.G.	·0392
1/0 B.G.	·3964	21 B.G.	·0349
1 B.G.	·3532	22 B.G.	·03125
2 B.G.	·3147	23 B.G.	·02782
3 B.G.	·2804	24 B.G.	·02476
4 B.G.	·250	25 B.G.	·02204
5 B.G.	·2225	26 B.G.	·01961
6 B.G.	·1981	27 B.G.	·01745

DENOMINATIONS OF STANDARDS—*continued.*

Descriptive Number.	Equivalents in parts of an inch.	Descriptive Number.	Equivalents in parts of an Inch.
No.	Inch.	No.	Inch.
28 B.G.	·015625	41 B.G.	·00343
29 B.G.	·0139	42 B.G.	·00306
30 B.G.	·0123	43 B.G.	·00272
31 B.G.	·0110	44 B.G.	·00242
32 B.G.	·0098	45 B.G.	·00215
33 B.G.	·0087	46 B.G.	·00192
34 B.G.	·0077	47 B.G.	·00170
35 B.G.	·0069	48 B.G.	·00152
36 B.G.	·0061	49 B.G.	·00135
37 B.G.	·0054	50 B.G.	·00120
38 B.G.	·0048	51 B.G.	·00107
39 B.G.	·0043	52 B.G.	·00095
40 B.G.	·00386		

VII. SCREW THREADS

§ (23) An excellent illustration of the principle set forth in § (21) is to be found in the theory of screw-thread gauging.

(i.) *Definitions.*—The reader may be assumed to be familiar with the general characteristics of a screw thread, but in entering upon any detailed discussion of its properties and possible errors it is necessary to make use of certain technical nomenclature and definitions, which are thus given in Report No. 84, published by the British Engineering Standards Association, 28 Victoria Street, S.W.1.

Effective Diameter of a Screw.—The effective diameter of a perfect screw having a single thread is the length of a line drawn through the axis and at right angles to it, measured between the points where the line cuts the flanks of the thread (*Fig.* 7).

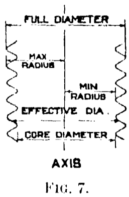


FIG. 7.

Core Diameter.—The core diameter is twice the minimum radius of a screw, measured at right angles to the axis (*Fig.* 7).[1]

Full Diameter.—The full diameter is twice the maximum radius of a screw, measured at right angles to the axis (*Fig.* 7).

Crest.—The crest is the prominent part of the thread, whether of the male screw or of the female screw (*Fig.* 8).

Root.—The root is the bottom of the groove of the thread, whether of the male screw or of the female screw (*Fig.* 8).

Flank of Thread.—The flank of the thread is the straight part of the thread which connects the crests and roots (*Fig.* 8).

Angle of Thread.—The angle of the thread is the angle between the flanks, measured in the axial plane (*Fig.* 8).

Pitch.—The pitch is the distance in inches or millimetres measured along a line parallel to the axis of the screw between the point where it cuts any thread of the screw and the point at which it next meets the corresponding part of the same thread (*Fig.* 8). *The reciprocal of the pitch* measures the number of turns per inch or millimetre as the case may be.

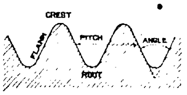


FIG. 8.

The above definitions have been the subject of considerable criticism, and the most appropriate and convenient system of nomenclature is still a question of debate. It is probable that the official definitions may be revised in the near future. But until this is definitely decided it is preferable to make use of the existing definitions as they stand rather than to introduce now certain modifications which might appear desirable. Otherwise further confusion would be introduced into the discussion of a subject already sufficiently intricate. It may be said that in the present form the definitions, in common with many other possible variants, are quite satisfactory so long as we are considering perfect threads, but that they are not directly applicable to screws in which certain types of error are present.

It is, in fact, extremely difficult, if not impossible, to frame useful definitions which will take cognisance of all the possible combinations of error which may occur. What is required is a series of definitions of sufficiently general applicability to make it possible to develop the theory of the imperfect screw more completely. But the definitions as they stand, with the aid of certain conventions referred to later, enable us to deal with the more important considerations.

(ii.) *Methods of Production.*—There are a variety of ways in which screw threads can be produced, and accuracy in pitch of screw threads, which is the first desideratum, is attained by a building-up process, in which the errors of a parent screw are successively diminished by a process either of averaging, or of definite correction, in each successive generation. The original screw was no doubt formed by hand, probably in wood, and its errors would be considerable. By embracing a number of threads of such a screw in a suitable nut, and using it as a lead-screw in a primitive lathe, it would be possible to cut a screw in which the variations of pitch, though probably no less in total magnitude than those of the original, would be much less abrupt. And so by gradual evolution a screw of reasonably uniform, but still of incorrect average pitch would be derived. The transition from wood to metal is incidental, but by affording a more suitable mechanical material enables further progress to be made. There is a simple method of correcting gradual errors of pitch by the use of a correcting cam on a suitable lathe, and so eventually we are enabled to prepare a lead screw for a lathe, of sufficient accuracy for the purpose either of direct screw-thread cutting, or of producing other tools used for this purpose. For work of the highest accuracy—such as the production of screw-thread gauges, micrometer movements, or master screws for the engines used in ruling diffraction gratings—a process of lapping the finished screw in a suitable long nut is frequently resorted to, in order to eliminate as far as possible the last traces of irregularity in pitch. This applies particularly to hardened steel screws, which are liable to distortion in the process of hardening.

---

[1] From *Fig.* 7 it will be seen that the "*core diameter*" of the male thread is measured between the roots of the thread; it should be borne in mind, however, that the "*core diameter*" of the female thread, being approximately the same dimension, is measured between the crests of the thread.

It will be found that in the last resort the production of an accurate screw by no matter what process, comes back almost always to the use of a previously formed accurate master screw. And it is remarkable that up to a comparatively recent date very few machine-tool makers appear to have appreciated this, or to have produced lead screws for their tools of anything like a suitable degree of accuracy. It would take us too far to describe in detail the various methods employed in making screw threads, but they may be summarised thus:

Cutting on lathe,
Cutting with taps and dies, either by hand, or in automatic machines,
Milling,
Rolling,
Grinding,
Lapping,

of which the last two processes apply only to hardened screws, and rolling only to very small screws (for watchmaking) or to threads formed on thin metal—*e.g.* those on Edison-type lamp caps and holders.

There are, of course, certain types of error peculiar to each process, as well as others common to several processes. In each case, however, it will be seen that the accuracy of the product depends directly, either upon that of the master screw itself, or upon that of tools produced from the master screw. Such tools are almost invariably generated on a lathe, and to secure accuracy of result when cutting screws upon a lathe it is essential to use a single-point tool, and not a chaser with several teeth. The employment of the latter type of tool leads to a conflict between the pitch of the teeth of the tool and that of the lead screw of the lathe, and since it is practically impossible to get the two into absolute agreement the result is a compromise, and its pitch usually imperfect. Further the single-point tool can be more accurately produced and controlled, as regards its thread form, than can a many-toothed chaser—the latter, in fact, has almost of necessity to be produced by the agency of the former, and is not so susceptible of correction after hardening. The initial stage in the production of accurate screws is, therefore, the generation of a simple single-point tool of the form shown in *Fig.* 9 (*a*), which can readily be done by the aid of a suitable lapping jig, and its form checked on an optical projection apparatus (see "Gauges," § (64), etc.). This tool is used to cut the groove in the thread, and also to cut the groove in the "cresting" tool, *Fig.* 9 (*b*), which is then used to form the crests of the threads, using care to see that it is correctly registered with regard to the groove already cut.

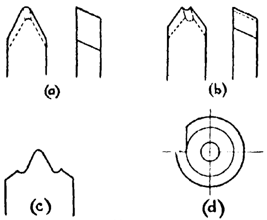


FIG. 9.

The process just described would be too elaborate and expensive for the ordinary cutting of screw threads, and is therefore employed only in the generation of other threading tools. For cutting in a lathe the profile of the tool (still of a single thread only) should be as shown in *Fig.* 9 (*c*), and this profile may be produced either on a straight bar cutter, or on a circular type of cutter such as *Fig.* 9 (*d*), by the process indicated in the last paragraph. Such a cutter as *Fig.* 9 (*d*) may be employed either on a lathe, or in the process of thread-milling, and has given excellent results in the latter mode of use. It is more usual, however, in the commercial milling of screw threads to use a hob with a sufficient number of teeth to cover the whole length of the screw to be cut, so that one revolution of the work is sufficient to complete the whole thread. In this case, of course, it is the pitch of the hob (cut as before by the aid of single-pointed tools), and not that of the milling-machine screw, which becomes the controlling factor, whereas with a tool such as *Fig.* 9 (*d*) the reverse is the case, and the milling machine must be provided with a lead screw of accurate pitch.

The process of thread milling with a multi-threaded hob introduces the possibility of a special type of error not met with in other processes, since if the machine is badly adjusted, or if springing of the parts takes place, it may happen that the end of the thread, after the work has completed its one revolution, does not fairly meet the commencement, so that there results a series of steps, one on every turn of the thread, arranged along a line parallel to the axis of the screw.

In the production of internal screws, if of sufficient size, any of the methods already described may be employed, but smaller sizes are almost always produced by taps, while for rougher work on both male and female screws, taps and dies are commonly employed. For the most accurate work the taps may be produced directly by cutting with single-point tools upon a lathe, and the dies either directly in the same manner, or at one remove by cutting with a tap so formed. A complete study of the cutting action of taps and dies has yet to be made. But it is usually found that owing to distortion in hardening, the teeth on the different lands do not follow each other perfectly, and that as a result of this and of the difficulty of effecting satisfactory guidance, and possibly also of the nature of the cutting edges themselves, taps tend to cut large and dies small, as compared with their own measured sizes. Extreme care has to be taken in every detail of the work in order to minimise this effect when fine results are required.

When taps and dies are used on automatic screwing machines, two alternative procedures are possible; either the tools may be "floating" and allowed to pick up their own pitch, or they may be fixed, and the work fed forward at a definite rate governed by the screw of the machine. In the former case there is the risk of a certain amount of variability arising from the looseness of control, while in the latter there is a contest between the pitch of the machine screw and that of the cutting tool, leading to a compromise in the result.

The process of grinding applies only to hardened screws, and is essentially similar to that of milling with a single-tooth disc cutter, which, however, is replaced by a formed abrasive wheel, or charged lapping disc. This is the only process in which generation from a single-point tool is not involved, this operation being replaced by that of forming the grinding wheel.

The process of lapping is one not of production but of finishing, employed only for fine work, and is designed to even out small residual errors left by previous operations. The laps may be designed to bear on the flanks, on the roots, or on the crests of the threads, or on all simultaneously, according to requirements. They should either be cut directly by means of single-point tools, or if this be not possible by means of taps so cut.

In the use of screw-thread gauges, one of the greatest difficulties to contend with is that of wear, and it is most important, therefore, that they should be hardened. But the hardening almost invariably introduces some distortion, accompanied as a rule by changes in pitch and diameter. These changes can be controlled within limits by careful attention to the properties of the steel employed, and to the exact conditions of hardening suited to it. And allowance can be made in advance for the average change of size to be expected. In the case of gauges, since but little reduction of size due to wear is sufficient to render them useless for their purpose, it is not necessary to have them hardened throughout; in fact, a very thin surface of hardened material is sufficient, and the difficulty of distortion can be greatly minimised by case-hardening, either by the ordinary carbonising process, or by the use of potassium cyanide. The latter method, however, as ordinarily applied, leaves so thin a hardened skin that the gauges, if at all heavy, are still liable to damage by bruising, and consequently for any but very small gauges the former method, or a combination of the two methods, is to be preferred, by which a hardened skin of sufficient thickness to support the interior plastic material against bruising is obtained. In any case there is usually sufficient uncertainty as to the result of the process to make it necessary to leave a small amount (say a few ten-thousandths of an inch) of surplus metal for final removal by lapping. When the gauge is finished by grinding, this, of course, should not be necessary. It is usual in this case to rough out the blank in carbon steel, leaving several thousandths of an inch of full metal, then to harden throughout, and finally grind to size. If, however, there is any serious pitch error in the screw before grinding, there is a tendency for the wheel to follow this, by springing elastically, and so to fail to remove it entirely. If this is the case, resort may still be had to lapping as the final finishing operation.

The lapping of a long screw to the very high accuracy needed for application to a ruling engine for diffraction gratings is a very special operation, and reference should be made to the descriptions given in papers written on the subject.[1]

Taps and dies, of course, also suffer distortion during hardening, but in this case, except for very special work, while care is taken to reduce the actual amount of the distortion as far as possible by a satisfactory control of the conditions of hardening, no attempt is usually made to correct subsequently. In fact it is found that the slight irregularities ordinarily present tend to facilitate cutting by enabling the tools to clear themselves more readily on the work, though naturally they are disadvantageous from the point of view of accuracy and uniformity of product.

---

[1] Rowland, article "Screw," *Ency. Brit.*; Sconle, *N.P.L. Collected Researches*, 1912; Grayson, *Proc. Roy. Soc.* (Victoria), 1917, xxx. 44.

§ (24) Errors of Screws.—The brief résumé of methods of production given in the last paragraph will be of assistance in understanding the following discussion of the nature and inter-relation of screw thread errors. The screw thread is a complicated geometrical form, involving eight independent elements, any one of which, either singly or in combination with some or all of the others, may be in error. The various elements in order of importance are: pitch, effective diameter, angles of flanks (two), crest diameter, root diameter, form at crest, and form at root. The possible combinations of error are almost infinite in number.

(i.) *Pitch Error.*—This, as will be seen later, has greater effect than any other error upon the mating of two threads, and is therefore considered first. Error in pitch may be of three, or possibly four, different kinds as under:

Progressive error.
Periodic error.
Drunkenness.
Irregular error.

(*a*) *Progressive error* is a gradual, but not necessarily uniform, divergence of the pitch of successive threads on a screw from their true nominal pitch. It arises usually from the presence of an original error of similar amount in the lead screw of the lathe or other tool employed to cut the screw, or from a change in length during hardening.

(*b*) *Periodic error* is an error which repeats itself at regular intervals along the thread, successive portions of the length being alternately longer and shorter than the mean. The most common cause of periodic error is a lack of squareness in the abutment of the leading screw on the lathe, whereby this screw itself is caused to move slightly backwards and forwards in the direction of its own length, once per revolution (usually corresponding to several pitches of the screw being cut). A bent lead screw with an Acme thread may produce a similar effect in the motion of its nut. Another occasional cause of periodic error is faulty centring of the various gear wheels on a lathe, while yet another cause may be the presence of periodic error in the actual leading screw itself, with its nut. It may be noticed, however, that a perfect screw in a "periodic" nut, or a "periodic" screw in a perfect nut, will not reproduce any periodic motion. It is only when both lead screw *and* nut have periodic errors that this cause may become operative.

(*c*) *Drunkenness* is an error similar to periodic error, but repeating once per turn of the thread. It may be due to the same causes as periodic error, if the pitch of the screw being cut is the same as that of the lead screw of the lathe. Or it may arise from a defective abutment on the headstock spindle, if a "live" centre is used. It is the most difficult error of all to detect and measure, but fortunately is not often present in large amount.

(*d*) *Irregular Error.*—As the name indicates, this has no specific characteristics, and correspondingly no specific cause. It may be due to a variety of circumstances—for instance, to slackness or sticking of the slide rest on the bed of the lathe; to faulty cutting, or to an actual slipping, of the tool; or to varying hardness in the material causing local changes in the depth of cut. It is almost invariably associated with error in effective diameter, from which indeed it is often difficult to distinguish it.

It should be noted that in any consideration of the effects of errors in pitch it is the *maximum relative displacement*, measured in a direction parallel to the axis of the screw, *of any two* points upon it within the length of engagement, as compared with the true nominal pitch distance between them, which is of importance.

(ii.) *Effective Diametral Error.*—This ranks next in importance to pitch error. A screw to be a satisfactory fit in a nut should make contact with it upon the flanks of the threads. Contact at the crest and root is for most purposes of less importance, and in certain circumstances may be a positive disadvantage. The error in effective diameter (other things being equal) is a measure of the possible slackness of fit.

When we come to consider any actual case of effective diameter error, we are almost sure to be confronted with the inadequacy of the definition. So long as we are dealing with a perfect screw, on which the pairs of opposite flanks are all parallel to each other and equally spaced, the length of a line intersecting the axis at right angles, and terminated by its intersections with the two opposite flanks, is the same wherever the line is drawn, and "effective diameter," so defined, is a determinate amount. But if any error in pitch (other than a uniform progressive error) be present, or if the angles of the two flanks be not equal, then the length of such a line will vary according to the position in which it is taken, and the definition, as given, ceases to be determinate.

To gain a definite conception we revert to the consideration of the methods by which screws are ordinarily made and measured, and it becomes of some importance to note that in almost all the processes of manufacture described it is the *groove*, rather than the *ridge*, of the thread which is directly produced by the tool, so that if we want our measurements of errors to throw light on the sources whence they are derived it is preferable to measure by reference to the grooves, and not by reference to the ridges. The usual methods of measuring effective diameters and pitch by means of small cylinders, or a ball-pointed stylus, do actually operate in this manner, and in considering an imperfect thread it is usual to interpret the term "effective diameter" as applying to the value measured by means of cylinders of "best" diameter—*i.e.* of such diameter as would touch the flanks of a perfect thread of the same form and nominal pitch at two points distant by one-half the pitch. In this form the definition becomes specific, even for an imperfect screw, and we can proceed to study the relations of errors in effective diameter with those which may exist simultaneously in other elements of the screw. The two forms of definition, of course, agree in the case of a perfect screw.

(iii.) *Relation of Errors in Effective Diameter and Pitch.*—It will be noticed that, in the simplest process of direct generation of a screw thread by means of a single-point tool, the effective diameter as defined by measurement with "best" size cylinders corresponds directly to the depth of the groove cut, and is entirely independent of the pitch of the thread. In other words, as is apparent from *Fig.* 10, the

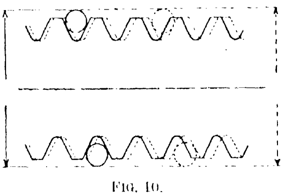


FIG. 10.

effective diameter measured by cylinders is the same whether we consider the full or the dotted outline. But it does not agree *exactly* with measurement according to the existing definition, except in the case of the perfect thread.

Thus, if the full outline of *Fig.* 11 be supposed to represent a correct screw, and the dotted outline

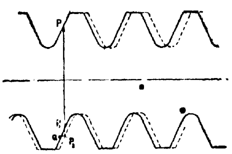


FIG. 11.

one of (uniformly) erroneous pitch, but the same depth of groove, the effective diameter of the latter, according to the original definition, would be PP_2, whereas according to the amended definition it is exactly the same as that of the correct screw, *i.e.* as PP_1. The new definition, therefore, differs from the old to the extent represented by P_1P_2. Now, considering the little triangle P_1QP_2 the distance QP_2 represents one-half of the difference in pitch per turn of thread between the two screws $= \frac{1}{2}\delta p$, say. And if α be the half angle of the thread, $P_1P_2 = QP_2 \cot \alpha = \frac{1}{2}\delta p \cot \alpha$. It may be remarked that for the angles of thread ordinarily employed $\cot \alpha$ approximates to 2, so that the difference between the two definitions amounts roughly to δp. And since the error in pitch *per turn of thread* is not usually very large, though the *total* pitch error in the whole length of the screw may be considerable, the discrepancy is not likely to be great. It would be better, however, if the definition were revised so as to eliminate it.

It is obviously advantageous to have a form of definition which corresponds to the mechanical operations of producing and measuring the screw, and keeps the two elements of pitch and effective diameter separate, so that the meaning of the errors can be interpreted. But when we come to considerations of the nature of fit to be expected when a screw and nut are assembled together, it is no longer possible to keep the ideas separate, as errors in either element contribute to the result, and interact upon each other. For example, suppose the full outlines in *Fig.* 12 to represent two portions of a correct nut, and the dotted outlines two corresponding portions of a screw of erroneous pitch which is required to fit it. Obviously the "fit" can only be effected at the two points *i.e.* at those points where the *maximum total pitch difference* exists between the screw and the nut. Let these be the portions represented in the diagram. The two threads, as shown, have the same effective diameter (according to the modified definition), and it is clear that before the screw could be made to enter the nut it would have to be reduced by pp_1 on its radius, or by $2pp_1$ on its diameter. If the maximum total error in pitch be δp, $pq = \frac{1}{2}\delta p$, and $2pp_1 = 2pq \cot \alpha$, or $\delta_1 E = \delta p \cot \alpha$, so that a total error δp in pitch has to be accompanied by a reduction $\delta p \cot \alpha$ in the effective diameter of a screw (or an increase of similar amount in the case of a nut) before it will assemble with a nut (or screw) of correct dimensions.

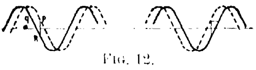


FIG. 12.

For the three angles of thread most commonly used we have the following values of cot α:

Form of Thread.	Angle $= 2\alpha$.	Cot α.
Whitworth . . .	55°	1·921
U.S.A. or S.I. . . .	60°	1·732
B.A.	47½°	2·273

The effective diameter is frequently spoken of as giving a measure of the thickness or thinness of the threads. But it is to be remarked that, on the view we have adopted, of regarding the thread for simplicity as a groove cut out by a single-pointed tool, if two similar screws be cut with equal depth of groove (and therefore equal effective diameters), but one long and the other short in pitch, then the former will have thick threads, and the latter thin ones. But if the amount of the pitch error be the same in the two cases each will require the same reduction in effective diameter to compensate for it.

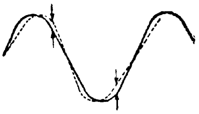


FIG. 13.

(iv.) *Errors in Angle.*—In the argument of the last paragraph we confined our attention to threads in which the only errors present were those of pitch and effective diameter. Any error in the angle of a screw thread, or in its squareness with the axis of the screw, has also to be accompanied by a reduction (or in the case of a nut by an increase) in effective diameter, as measured by means of cylinders or spheres at the half depth of the thread, in order that assembly may take place with a corresponding nut or screw of correct form and size. In *Fig.* 13 there are shown a (full) correct profile, and a (dotted) profile of the same effective diameter, but incorrect angle. It is better, instead of speaking of the whole angle of the thread, and of its squareness to the axis, to consider the half angles on the two flanks as the independent variables. And it is evident from the diagram that, whether the half angle be large or small, the change of effective diameter required to secure assembly is in the same direction. In other words, in order to obtain the total change of effective diameter required to provide compensation for the effects of angle errors, we must *add* the effects of the two half-angle errors, taken separately, *without regard to their signs.* The calculation is given in Appendix II. of B.E.S.A. Report No. 84,[1] already referred to, and the result is

$$\delta_2E = \frac{h}{\sin 2\alpha}(\delta\alpha_1 + \delta\alpha_2),$$

where δ_2E is the required change of effective diameter, h the depth of the thread corresponding to the straight portion of the flank between the points where it blends with the root and crest radii, and $\delta\alpha_1$ and $\delta\alpha_2$ the errors (supposed small) of the two half angles, in radians.

Inserting the appropriate constants this gives:

For Whitworth threads—

$$\delta_2E = 0{\cdot}0105 \times p \times (\delta\alpha_1 + \delta\alpha_2),$$

for U.S.A. and S.I. threads—

$$\delta_2E = 0{\cdot}0131 \times p \times (\delta\alpha_1 + \delta\alpha_2),$$

for B.A. threads—

$$\delta_2E = 0{\cdot}0091 \times p \times (\delta\alpha_1 + \delta\alpha_2),$$

where p is the pitch and $\delta\alpha_1$, $\delta\alpha_2$ are now measured in degrees of angle.

Of course, other errors less simple than those just considered may, and do, occur. For instance, the flanks, instead of being straight, may be curved, either concave or convex, or they may be entirely irregular in outline. In these cases there is no definite angle error to measure, and some other criterion must be applied. We shall refer to such alternative methods later.

It must be noticed, also, that the change in effective diameter needed to provide for error in angle is independent of that required to compensate error in pitch, so that if errors in both pitch and angle are present together the total change of effective diameter required is the *sum* of the changes which would be necessitated by the two errors considered separately.

(v.) *Errors in Crest and Root Diameters.*—Errors in these diameters are of less importance than those in effective diameter, firstly because they are not related in the same way to errors in other elements, and secondly because, in the majority of modern practice, it is now usual to provide a definite clearance at the roots of all threads, so that a fit between crest and root is not expected. The actual root diameter, provided the thread is deep enough to provide this clearance, is quite immaterial, except possibly in the case of threads on thin tubes, where the additional loss of material caused by cutting the thread especially deep might have an appreciable effect upon the strength. The crest diameter, however, governs the depth of engagement of the threads, and it is therefore necessary to see that it is controlled within reasonably close limits. But these limits need not ordinarily be quite so fine as those on the effective diameter, since they are not concerned in determining the quality of the fit.

(vi.) *Errors in Form of Crest and Root.*—There has been a great deal of controversy as to the relative merits of various crest and root forms—particularly of the rounded form of the Whitworth thread, as compared with the flat truncated form of the Sellers. But apart possibly from the few exceptional cases where a fit may be required over the whole surface of the thread, it is clear from the remarks of the last paragraph that the exact form of crest or root is relatively unimportant, and that considerable variation in form may be allowed. It may be said that, owing to the natural wear of the tools, the sharp angles of the Sellers or the Vee thread are never actually produced in practice, and that the tendency, particularly on screws of the finer pitches, is to approximate to the rounded form of the Whitworth or B.A. type. Provided the roots are sufficiently cleared to ensure interchangeable assembly, this natural tendency, with the geometrical limitation imposed by the general angle of the thread, is sufficient to prevent them becoming so deep or sharp as to be a source of weakness, while provided the crest, without overlapping the nominal boundary, presents a full radius and is not pared away at the sides, a sufficient bearing surface is obtained on the flank and strength is not sacrificed. These two conditions represent the only essential restrictions to be placed on the possible errors of crest and root form, and there is therefore room here for compromise between the adherents of the two systems.

---

[1] The notation of Report 84 differs from that of this article.

(vii.) *Other Miscellaneous Errors.*—In addition to the various errors already enumerated, there are a number of others which may occur, and which are not specifically related to any one element. For instance, instead of being circular, the cross-section may be elliptical, or it may be triangular, or pentagonal, in shape. The polygonal form arises sometimes in screwed holes cut by taps. It can only occur when the taps have an odd number of flutes, and the figure has the same number of sides as the tap has flutes. For an explanation of the manner in which such holes can be produced, reference may be made to the Annual Report of the National Physical Laboratory for 1919.

Another error which may be present, especially in taps or hardened screws, is that the axis, instead of being straight, may be bent, with the result that the pitch is different when measured along different generating lines of the cylinder which approximately encloses the screw, and the effective diameter, as measured at any individual position along its length, does not give the usual measure of the clearance which may be expected between the screw and a perfect nut.

Yet another type of error is eccentricity between the crest diameter and effective diameter of the screw. This is more liable to occur on flat crested screws, where the crests and the grooves are frequently finished in separate operations, than with round-crested forms, in which the whole profile is usually cut with one tool.

Again the diameters of the screw may vary in various ways along its length. It may be tapered, barrel-shaped, bell-mouthed, or otherwise irregular. And this may apply to the thread as a whole, or to one or more of the separate diameters independently of the others.

In addition to all these more or less systematic types of error, the shape of the profile, or the measurements of the various elements, may be so irregular, due to imperfections in the tools or conditions of cutting, as to be quite incapable of any systematic interpretation.

Finally, the fineness of the surface finish has also an effect upon the quality of fit of a screw in its nut. A screw of quite good form but with a roughish surface requires more clearance for assembly than a highly polished hardened screw of equal general accuracy.

§ (25) GAUGING OF SCREW THREADS.—We are now in a position to consider the application of the principles of gauging to the screw thread.

(i.) *"Go" and "Not-Go" Gauges.*—To begin with, the necessary and sufficient condition of complete interchangeability is that every screw should be capable of lying wholly within, and every nut wholly without, the same perfect theoretical profile, of correct nominal dimensions. Hence the "Go" gauges must combine in themselves *all* the essential elements of the thread in their proper mutual relationships. In other words, the "Go" gauges must be, as nearly as it is practicable to make them, complete material representations of the ideal theoretical common limit boundary between screw and nut. The "Go" gauge for the nut is, of course, a plug, and that for the screw a ring.

The "Not-Go" gauges, on the other hand, must control the individual elements separately. For the sake of clearness we will consider the gauging of a ring screw by means of plug gauges. *Mutatis mutandis* corresponding remarks will, of course, apply to the case of gauging a male screw by means of ring gauges.

The "Not-Go" gauges which are of the most importance are those for effective and crest diameter, of which, however, the latter is simply a plain cylindrical plug, which serves to ensure that there is a full depth of thread, and needs no further discussion.

The "Not-Go" gauge for effective diameter takes the form of a screwed plug, and the discussion of the interrelationships of errors given in the last section shows how important it is so to limit the action of this gauge that it takes cognizance of effective diameter only. In the first place it must have a special section of thread, such as that shown in *Fig.* 14, designed to bear only on the flanks of the threads for a very short distance near the middle of the depth. If it were of full-form profile its indications might be altogether falsified by its bearing, say, at the root of the thread, instead of on the flank, and so apparently indicating a satisfactory screw, although the effective diameter might actually be much too large; or, if the bearing surface on the flank were more than a small proportion of the total depth, then any error in angle in the screw might cause the gauge to take a bearing at either the top or the bottom of the flank, instead of in the middle, and once more a false indication would be obtained. Similarly the "Not-Go" effective diameter gauge should not comprise more than one or two turns of the thread; otherwise, if the screw had an error in pitch, the gauge might enter, but after one or two turns might pull up owing to the effect of the difference in pitch between it and the screw without this being any indication that the actual effective diameter of the latter was satisfactory.

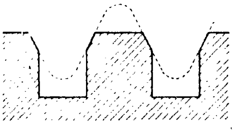


FIG. 14.

But provided the gauge is properly constructed to control effective diameter only, its indications, in conjunction with those of the "Go" gauge, and a general inspection of the appearance of the thread, are sufficient, owing to the geometrical relationships existing between the various types of error, to place definite limits on the possible errors of pitch or angle which may be present, without actual measurements of these latter being made. It is important to understand quite clearly what is implied by this statement, and for this purpose reference may be made to *Fig.* 15.

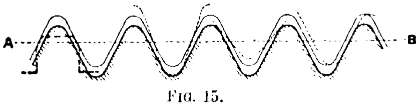


FIG. 15.

The thick full outline in this diagram represents the theoretical correct profile, and the shading indicates the material of the "Go" gauge, which follows this outline exactly. Since the "Go" gauge is assumed to have passed through the screw in the nut, it follows that the latter lies at all points completely outside this boundary. The thin full outline represents a screw of perfect form, and of such size as to be just acceptable by the "Not-Go" effective diameter gauge, which is indicated in its proper relative position by the heavy dotted outline in the left hand thread. Since the "Not-Go" gauge does not pass, the effective diameter of thread in the nut, as *measured at the half depth of thread*, can at no point exceed the value corresponding to the intersections of the thin full curve with the line AB.[1]

It does not follow from this that no point on the thread of the nut can be outside the thin full curve. If the possibility of irregular thread form were admitted there could of course be no control except at the points actually gauged. But even if the tools are known to cut threads of reasonably satisfactory form, or if actual inspection of the thread form be made (in the case of a nut this would involve taking a cast of the thread), and no obvious irregularities are observed, we may still have the conditions indicated by either of the thin dotted outlines in *Fig.* 15. In the first two of these outlines the angle, and in the third the pitch, are supposed erroneous, and in all three cases it is evident that there are portions of the thread which lie outside the nominal correct profile by twice as much as the thin full outline which represents a screw of perfect form but of the maximum permitted error in effective diameter.

The diagram *Fig.* 15 shows clearly how, if the thread form is known to be regular (but not necessarily correct) in shape, the application of the full form "Go" gauge, together with the "Not-Go" effective diameter gauge, automatically sets limits to the errors of pitch and angle which can occur, and so renders separate gauging of these elements unnecessary.

The amounts of these limits are directly calculable from the formulae of § (24) (iii.) and (iv.). In this calculation it is to be observed that in either case the full value of the limit is only available if the effective diameter is also on the extreme limit permissible for it, and that the maximum tolerance available for either pitch or angle error is directly proportional to the variation of the effective diameter from nominal size. Also, the maximum errors in pitch and angle cannot exist simultaneously. The equivalents of these two errors in effective diameter measurement are *additive*, and it is the *sum* of the equivalents of these errors which must not exceed the amount by which the effective diameter differs from standard; or, to look at the same matter from a slightly different standpoint, if either pitch or angle separately, or the two of them taken together, be in error by amounts whose combined effects are equivalent to the total tolerance allowed on the effective diameter, then this diameter *must be on the extreme limit* of its tolerance, the whole of which has been absorbed by the pitch and/or angle errors, leaving no variation possible in the effective diameter itself. It is only by so much as the combined equivalents of the pitch and angle errors fall short of the total effective diameter tolerance that any variation in this diameter is permissible, and the amount of variation which can be accepted is the amount of this difference, measured always *from* the limit, *towards* the standard size.

---

[1] If the nut should be barrel-shaped, so that a solid gauge, while refusing to enter at either end, might yet pass through the middle portion, we must suppose this part to be controlled either by some form of collapsible gauge, or by direct measurement.

(ii.) *Envelopes, Zones, Grade, and Play.*—It will be seen that the net result of the system of gauging just described is to confine the whole profile of the thread between two boundaries, one of which is the correct nominal profile, and the other a similar one separated from the first by *twice* the amount of the effective diameter tolerance. The space between two such boundaries is called a "zone" of tolerance, and it is convenient for many purposes to specify the quality of screw threads by means of such zones, rather than by separate tolerances on the individual elements. The latter procedure involves measuring the elements independently, and computing the interactions of the various errors upon each other, and cannot readily be extended to deal with any but the simpler types of error: while the former has the advantage of assigning in one operation a limit to the combined effects of all errors, of whatever type, and is particularly adapted to the method of examining screw threads by the optical projection system developed for this purpose during the war of 1914–18. In this method the image of the thread magnified 50 or 100 times is projected on to a screen, on which the limits of the zone of tolerance are indicated. (See "Gauges," §§ (64)-(70).) It is at once possible to see whether or not the profile can be made to fall at all points within the zone.

In the theoretical discussion of this method of gauging, Major P. Bishop introduced the use of the terms "Grade" and "Play," and the present writer finds it convenient to employ the word "Envelope," in the following sense:

Regarding the surface of a screw thread as a simple geometrical figure, dissociated from the material of which the screw is actually made, the *major envelope* of the thread is the geometrical surface of the minimum nut of perfect pitch and thread form into which this figure will screw, and the *minor envelope* is the geometrical surface of the maximum screw of perfect form and pitch which will screw into it. If the screw considered is defective, it will not always happen that the major and minor envelopes are similarly situated about a common axis, but this does not affect the argument. In fact it is one of the advantages of the suggestion that it enables such cases to be brought into the general treatment. It will be noted that the major envelope of a screw, and the minor envelope of a nut, lie entirely in air, while the minor envelope of the screw, and the major envelope of the nut, lie entirely within the material from which they are made. The necessary and sufficient condition for complete interchangeability is that the major envelopes of all screws should be not larger and the minor envelopes of all corresponding screwed holes not smaller than the nominal size of the thread. Making use of this idea we can then define Grade and Play as follows:

Grade is the difference between the nominal diameter of the thread, and that of the minor envelope of a screw (or major envelope of a nut).

Play is the difference between the nominal diameter of the thread and that of the major envelope of a screw (or minor envelope of a nut).

It is obvious that the smaller the play the better the fit that is to be expected with a perfect mating screw, while the smaller the difference between grade and play the more correct must be the form of the thread. For grade and play to be equal, the form would need to be perfect. By placing limits on the amounts of grade and play, the permissible tolerances on the screw can be defined in a form which enables them to be readily controlled by the optical method of gauging.

It can readily be seen also, if we limit the consideration to the three principal types of error, that if P be the play, G the grade, δ_0E the actual error in effective diameter, and δ_1E, δ_2E the equivalents in effective diameter of the errors in pitch and angle, then

$$G = \delta_0E + \delta_1E + \delta_2E,$$

$$P = \delta_0E - \delta_1E - \delta_2E,$$

or

$$\delta_0E = \tfrac{1}{2}(G + P),$$

$$\delta_1E + \delta_2E = \tfrac{1}{2}(G - P).$$

A convenient graphical method devised by Major Bishop for converting pitch, angle, and effective diameter errors into Grade and Play is described in B.E.S.A. Report No. 84, Appendix II.

If we revert now for a moment to the system of gauging with "Go" and "Not-Go" gauges, and suppose that we have a case where δ_0E and $\delta_1E + \delta_2E$ are both as great as possible, we know that in this case we must have $\delta_1E + \delta_2E = \delta_0E$, so that

$$G_{\text{max.}} = 2\delta_0E_{\text{max.}}$$

If, on the other hand, we consider a case where δ_0E is as great as possible, but δ_1E and δ_2E both zero, we get

$$P_{\text{max.}} = \delta_0E_{\text{max.}} = \tfrac{1}{2}G_{\text{max.}}$$

Provided therefore we choose the limits of G and P in accordance with these relationships, the system of gauging optically in terms of Grade and Play will give results consistent with those obtained by mechanical gauging by means of "Go" and "Not-Go" gauges.

§ (26) Tolerances on Screw Threads.—In laying down a system of tolerances for B.S.F screw threads the British Engineering Standards Association (Report No. 84) has adopted the two methods of gauging above described, as alternatives, with the relations just indicated between the various limits. The tolerances throughout this report are based on a "unit" of tolerance equal to $0{\cdot}01\sqrt{p}$ inches, it having been found, as the result of investigation of a considerable mass of available data, that this form of law gave a satisfactory basis for screws of ordinary sizes and proportions. For screws of ordinary quality two such units of tolerance are allowed on effective diameter, three on crest diameters, and four on root diameters. The wider tolerances permitted on crest and root diameters correspond to an assumption that clearance will be provided at the bottoms of the threads. Consequently, the tolerances at root and crest are not considered in deciding those on grade and play, which are, therefore, in accordance with the formulae of the last section, 4 units and 2 units respectively.

A series of tolerances for "close fits" is also laid down of half the above amounts in each case. Further, for both ordinary and close fits a minimum "allowance" of 0″·002 is specified, the maximum bolt being of nominal size, and the minimum nut this amount larger throughout. It is not improbable that this last feature of the specification may be subject to amendment.

Tolerances on similar lines, but without the provision for close fits, have also been laid down (Report CL. 7270) for B.S.W. threads. Report No. 21, dealing with B.S.P. threads, is in course of revision, and when reissued will probably show tolerances of the same kind, at any rate, on the smaller sizes, say up to 2-in. pipe. For screw threads of relatively fine pitch on large diameters, such as the larger sizes of pipe thread, it is possible that the formulae may have to be modified in the sense of taking the diameter as well as the pitch (or size of the thread) into consideration. Evidence on this point is not yet complete.

In dealing with the small B.A. screws (Report C.L. 7271) the Association has also deviated from the $\sqrt{p}$ law of tolerance.

For the actual limits of tolerance on the various types and sizes of screws reference must be made to the Reports mentioned. It may, however, be desirable to point out here one important difficulty which, owing to the very nature of the screw thread, is inherent in all attempts to control the interchangeability and the accuracy of fit of screws by specifying tolerances on the various elements.

Suppose limits are laid down, with due regard to the accuracy actually attainable in manufacture, for the maximum errors to be permitted in pitch and angle. Then it is necessary to allow on effective diameter a tolerance equal to the sum of the equivalents of these two limits *plus* a certain amount for inevitable variations in the actual depth of cut. But it *may* happen that in a certain number of screws the errors in pitch and angle are not present, and in such cases, if the effective diameter be on the extreme limit allowed, the play, and consequently the looseness of the fit when the screw is assembled, may be considerable. On the other hand, cases may also arise in which, owing to the presence of pitch and/or angle error, the play is zero, though the effective diameter is near or on the limit of tolerance. In such a case there would be no shake if the piece were mated with a perfect part, but if it should chance to be mated with a part having errors of the same nature and amount as its own, the shake might be as great as if both effective diameters were on their respective limits of tolerance, and no other errors were present.

The problem is thus, owing to the interplay of the various elements, an extremely difficult one, and it is easily seen how necessary it is, for good work, to keep the individual tolerances small, and at the same time how difficult it is from the manufacturing point of view to secure that all the elements are brought simultaneously within their respective tolerances.

§ (27) Tolerances on Screw Gauges.—The tolerances considered permissible on screw gauges of various kinds are set out in detail in the Test Pamphlet issued by the Metrology Department of the National Physical Laboratory.

It may be noticed in the first place that the tolerances on screwed work are usually so small that the gauges for it have to be of extreme accuracy; ordinary workshop and inspection gauges of average size being allowed only 0″·0006 in diameter, and reference, check, and master gauges but half this amount. The signs of the tolerances are arranged in accordance with the scheme given in § (18).

The same rules apply to the relationships of the tolerances on the various elements of the gauges as to those on the work, and when it is considered that an error of only 0″·00015 in pitch, or (in the case, say, of a 10 t.p.i. screw) of only 10′ of arc in the half-angle of each flank, is sufficient to absorb the whole of a tolerance of 0·0003 in effective diameter, and that the actual effective diameter error and the effective diameter equivalents of the errors in both pitch and angle have all to be included within this very small amount, it will readily be realised how difficult is the production of gauges to the required degree of accuracy. And it must be realised, also, that the *grade* of gauges tested under this system will be 0″·0012 for inspection or workshop gauges, or 0″·0006 for reference, check, or master gauges, although the corresponding effective diameter tolerances are only 0″·0006 and 0″·0003 respectively.

Slightly finer tolerances are possible in the case of "Not-Go" effective diameter gauges, in which the clearing of all but a small portion of the flank, and the limitation to only one or two turns of thread, make errors in pitch or angle of relatively small influence. But in the case of the full form "Go" gauges an accuracy of 0″·0003 on effective diameter may be said to represent the practical limit of present-day manufacturing processes. It is for this reason that check gauges, instead of being, as they ought, of a sensibly higher order of accuracy than the workshop or inspection gauges they serve to control, are only twice as accurate, while master gauges, which should be more accurate still, have only the same order of accuracy as the check gauges. The result of this limitation is to make the verification of check gauges, and even of workshop and inspection gauges, a matter demanding considerable skill, experience, and discretion.

And in the use of plug screw master gauges for verifying ring screw check gauges (the only purpose for which such masters are employed) a compromise has to be made as to the distribution of the tolerance. The ring check will be used for testing workshop or inspection plug screw gauges—it should, therefore, be no smaller in size than the upper limit of the gauges it is to control, or some of the gauges may be unfairly rejected. On the other hand, it should not be made deliberately larger than this limit, or oversize gauges will be passed, and these in their turn would reject work which ought to be accepted. If we could make the master gauge one stage more accurate than the check (as theoretically we ought to do) we should have no difficulty in testing the check on a proper basis. But since the accuracy of the master can only be the same as that of the check we are on the horns of a dilemma. If we make the tolerance on the master positive, then if the master be on its upper limit, the check will be forced to be not less than 0″·0003 large, and may pass gauges large to this extent. If, on the other hand, we make the tolerance on the master negative, then, if the master be on the low limit the check may be passed 0″·0003 small, and in this case will reject all gauges which are not small by at least this amount—*i.e.* by one half their nominal tolerance. In the result it is found best to divide the tolerance on the master symmetrically, making it ±0″·00015—thus aiming at getting the master as near correct size as possible, and trusting that the clearances between it and the check, and between the check and the gauges will be sufficient to prevent either gauges or work from being unduly penalised.

It may be mentioned that since plug screw gauges can readily be verified by direct measurement, it is not usual to provide ring screw checks, except where large numbers of gauges of the same type have to be verified, or where in addition to the screw proper there is a collar or some other feature on the gauge, whose concentricity or other relationships with the thread need also to be controlled. The master gauge difficulty, therefore, arises only in certain more or less exceptional cases.

In the verification of ring screw gauges, however, methods of direct measurement have only recently been devised, and are in any case much less simple and rapid than the corresponding methods of measuring plug screws. The standard method of control for ring gauges is, therefore, by means of check plugs, a full form "Go" screw plug, a "Not-Go" effective diameter plug, and a "Not-Go" plain plug for core diameter. The use of these three plugs, coupled with optical inspection of a plaster cast of the thread form in the ring, is sufficient to control all elements in the ring screw completely, in precisely the same manner as is described in § (25) for the work.

It must be noticed that the full form "Go" gauges, whether inspection or check, must be correct in *every* feature, since they are the physical representations of the nominal boundary which the work or gauge they are destined to control must in no case overlap. There is, strictly speaking, no licence on the crest and root diameters of gauges for wider tolerances, such as are permissible for the work. For although a clearance is expected on the work at the root of the thread, there is no guarantee that it is there, unless the gauge, being suitably constructed, indicates that it is so. And for this purpose the gauge must be complete and up to size at all points. In practice, however, it is found difficult, owing to the rate of wear of the tools used in producing the gauges being more rapid at their points than elsewhere, to keep the diameter at the root of the thread of the gauge within very narrow limits, and it is, therefore, customary to allow a slightly wider tolerance on this element, and to supplement the full form "Go" gauge by a plain cylindrical "Go" gauge for crest diameter only. This practice is not in accordance with the strict theory of gauging, but, owing to the fact that a clearance is aimed at in the work, is not found unsatisfactory in operation.

VIII. Cylindrical Fits

§ (28).—The question of the standardisation of tolerances on cylindrical parts for various classes of fit is one obviously of great importance to the engineer, and has received considerable attention both in England and abroad. Interchangeability of such parts in the bulk production of machinery is of the highest economic value to the manufacturer, while the ability to purchase spares which can be relied on to function properly without the need for special fitting is an inestimable boon to the user. The subject, however, is beset with difficulties of many kinds, and it cannot be said that a final and satisfactory conclusion has yet been arrived at in any country. The original Report of the B.E.S.A. on Standard Systems of Limit Gauges for Running Fits (No. 27, 1906) has not proved acceptable in practice, and is now in course of revision.

In order to discuss the subject properly we need to define a fairly ample vocabulary of special terms. Some of these have only recently come into use. Others have been generally employed for a considerable time, but often the same word is used by different persons to connote somewhat different ideas, with the result that a great deal of confusion is apt to enter into arguments on the subject.

§ (29) DEFINITIONS OF TERMS EMPLOYED.—In the present article the various terms employed are used with the significance explained below:

(i.) *Types of Work.* (a) "*Class*" *of Fit.*—Used to distinguish between different types of work, according to *function—i.e.* running, push, force, fits, etc.—without regard to accuracy or quality of workmanship.

(b) "*Grade*" *of Work.*—Used to distinguish between work made to varying degrees of accuracy in size. In particular, if a certain tolerance on work of a certain size is described as Grade 1, then work of the same nominal size but with twice the tolerance will be Grade 2; if with half the tolerance it is Grade 1/2.

(c) "*Quality*" *of Work.*—Distinguishes the nature of finish—*e.g.* whether hard or soft, machined, ground, lapped, etc., with the degree of perfection attained in any of these processes, apart from accuracy of size.

(ii.) *General Classification of Fits.* (a) *Clearance Fits.*—Fits in which the maximum dimension tolerated on the shaft does not exceed the minimum dimension tolerated on the hole.

(b) *Interference Fits.*—Fits in which the minimum dimension tolerated on the shaft is not less than the maximum dimension tolerated on the hole.

(c) *Transition Fits.*—Comprising all cases intermediate between (a) and (b), in which it is possible according to the particular sizes assumed by the shaft and hole (both within the tolerances prescribed for them) for either a clearance or an interference fit to be produced.

(d) *Allowance.*—The minimum clearance permissible in any class of clearance fit.

(e) *Obstruction.*—The minimum interference permissible in any class of interference fit.

(iii.) *Specific Classes of Fit.*—

(a) Easy running.	Clearance.
(b) Normal running.	
(c) Close running.	
(d) Sliding.	
(e) Light push, or spigot. (Maximum shaft = minimum hole.)	
(f) Push. Maximum shaft intermediate between maximum and minimum holes: minimum shaft not greater than minimum hole.	Transition.
(g) Light keying. Shaft tolerance comprised entirely within limits for hole tolerance.	
(h) Keying. Minimum shaft intermediate between maximum and minimum holes: maximum shaft not less than maximum hole.	
(j) Light driving. (Minimum shaft = maximum hole.)	Interference.
(k) Driving.	
(l) Force.	
(m) Shrink.	

In cases (a), (b), (c), (d), there is a definite "*allowance*" between the maximum shaft and the minimum hole. In cases (k), (l), and (m) there is a definite overlap, or "*obstruction*," between the minimum shaft and the maximum hole. In cases (e) and (j) the allowance and obstruction are respectively zero.

In the case of transition fits the terms "allowance" and "obstruction" are without material significance.

It must be understood that the use of the terms denoting the various classes of fit in the manner given above is intended primarily to give precise geometrical significations to a number of expressions at present somewhat vaguely employed, and not as a specific assertion that such distribution of tolerance is the most appropriate for the terms defined. Many engineers may disagree with the classification, as regards the transition fits, but it has to be remembered that in transition fits the whole tolerances must of necessity be extremely small, or the desired results cannot be attained. It may even be doubted whether the production of interchangeable work of this class is generally practicable. Assuming, however, that the tolerances can be kept sufficiently small for this purpose, the above classification will be found to correspond reasonably well with ordinary practice.

Some other definitions will be required in the course of the discussion, but these may more conveniently be left to be dealt with as they arise.

§ (30) GENERAL PRINCIPLES. (i.) *Shaft and Hole Bases.*—The first point which needs to be settled in formulating a series of tolerances for cylindrical fits is whether the shaft, or the hole, is to be taken as the basis—*i.e.* as the member which shall, as nearly as possible, remain invariable, while the different classes of fit are secured by varying the dimensions of the other. In the original report of the B.E.S.A. referred to above, a shaft basis was adopted, partly for the reason that in manufacture it is more easy to produce shafts than holes true to a prescribed dimension, but more particularly because in certain cases it is necessary to make a number of fits of differing classes on a single parallel shaft, a circumstance which does not apply to the hole. Modern improvements in methods of production have, however, very considerably minimised the importance of the first reason, while in practice it is found that in general work questions of the cost of upkeep of tools, such as drills, reamers, and broaches, and of the necessary gauges, for a large variety of holes, far outweigh the advantages gained in a limited number of cases from the second. It is no doubt as a result of this that the B.E.S.A. report has remained so largely a dead letter. It is impossible always to work on a hole basis. In certain cases, *e.g.* millwrighting, the shaft basis is essential, but for the majority of work the hole basis is to be preferred. The B.E.S.A. Committee, in reconsidering their proposals, have now agreed to the adoption of the hole basis as standard, with the facultative use of an alternative shaft basis for special requirements. The same course has been adopted in two recent reports issued by the corresponding Swiss and German Associations.

(ii.) *Unilateral and Bilateral Tolerances.*—Whether shaft or hole be taken as basis, it is natural to make the invariable member true to nominal size, though, provided the *differences* between shaft and hole for the various classes of fit are suitably maintained, it is, strictly speaking, immaterial whether the invariable member be nominal size or not. But errors of workmanship have to be provided for, and the invariable member cannot be made *exactly* nominal size. And concerning the proper distribution of the necessary tolerance on this member there is a very marked cleavage of opinion. According to one school of thought the aim should be to get the majority of the work as close to nominal size as possible, with the average, if possible, correct. As a consequence they advocate distributing the tolerance partly on one side, and partly on the other side, of the nominal. Such a distribution is termed "*bilateral*." The division of the tolerance may be either equal or unequal. In the Newall system of limits, which has considerable vogue in Great Britain, owing to the fact that gauges adapted to it have been commercially obtainable, the division is asymmetrical. An asymmetrical system may be advocated partly for reasons connected with the wear of tools and gauges. But the principal reason usually brought forward by those who favour it is that in practice the work tends to cling to the "Go" gauge, so that the average product, which they desire to see of nominal dimensions, does not fall on the mean of the two limits. In reply to this it can be said that if it be the case, then either

(*a*) The tolerances are unnecessarily wide, so that the workman has no difficulty in stopping his operations at a point after the "Go" gauge passes, but still a long way before the "Not-Go" gauge would pass; or

(*b*) He does not understand the first principle of the limit-gauge system, and is wasting his time through an instinct inherited from the procedure adopted for use with a single gauge, which makes him try to get his work only just to pass the "Go" gauge, whereas his object should be merely to get it *between* the gauges.

The other school of thought maintains that the nominal size should be, not an average on either side of which individual pieces of work may lie, but a limit such that (in the case of clearance fits) no hole shall lie below it, and no shaft above it. All the tolerance on the invariable member, therefore, must lie on one side of the nominal—positive if the hole basis be chosen, negative for the shaft basis. This is termed the *unilateral* system.

It may be said at once that while there is no conclusive argument in favour of either the unilateral or the bilateral system, on first principles the unilateral system is to be preferred. In the case of a large majority of fits the nominal dimension becomes the boundary between the hole and the shaft. The two schools of thought are, however, pretty evenly divided, and as a result the ultimate decision can only be that of a majority, and in consequence, to some extent, arbitrary.

The principal arguments adduced in favour of the bilateral system are as follows:

(*a*) That it brings the *average* work as near as possible to nominal size.

(*b*) That it is advantageous as regards life of tools.

(*c*) That existing standard products, *e.g.* ball-races, are made on this basis, and that it is desirable to maintain interchangeability with these.

(*d*) That many of the firms who at present use limit gauges already employ the system and have to be prepared to supply spare parts for products of existing types; and that it would be extremely costly to change all their tools and gauges to suit a new system.

With regard to these arguments it may be said in turn:

(*a*) There is no intrinsic virtue in the nominal size. Provided the allowances and tolerances were kept the same, an alteration of a few thousandths of an inch in the nominal size could have no effect on the functioning. This, therefore, is purely a sentimental argument. The point which does arise is this. What are the relative merits of a system in which *the same average size* is maintained, for all holes, whatever their grade (*i.e.* for all different tolerances), as against one in which the average size of the hole varies with the tolerance, but no hole is smaller than some *constant fixed limit*? The objection to the former, in the case of clearance fits (which form by far the largest number to be dealt with), is that the lower limit for the hole is less for a low-grade hole than for a high-grade one, so that a low-grade hole may give a less allowance than a high-grade one, when paired with shafts of the same class. A corresponding objection applies to the unilateral hole in the case of interference fits, but this is of less importance, as the number of cases to be dealt with is smaller, and interchangeability is not so necessary in this class of work.

(*b*) This argument depends largely on the fact that at present reamers, etc., are supplied commercially adjusted as close as possible to nominal size. If the unilateral system were adopted such tools would have to be made initially *above* nominal size, and it would be the worn and readjusted reamer that would have to be used for the highest grade holes, whereas on the bilateral system the worn reamer serves for the lower grades. But the maximum possible life for a fixed reamer is secured in either case by making it initially on the upper limit for the lowest grade hole. Then as it wears it becomes, on the bilateral system, suited first for the higher grades, and subsequently on further wear is once more suited only for the lower grades, at the other end of their tolerance. On the unilateral system it is at the end of the life of the tool that it becomes suitable for the highest grade work, but the total life depends simply on the total tolerance for the coarsest hole, and is the same in either case. If adjustable reamers are employed the argument ceases to have weight.

(*c*) The answer to this argument is simply that if a new standard were universally adopted makers of such products would fall into line.

(*d*) With regard to this, there are probably an equal number of firms at present pledged to the unilateral system, and their difficulties would be equally great if the bilateral systems were adopted as standard. There are also, however, a great number of firms still working on the old Whitworth system, to "standard" gauges. And it is evident that it would be a great convenience to such firms in adopting a new "limit" system, if the low or "Go" limit for the hole remained nominal size in all cases, as it is their present practice to make it.

As remarked before, none of these arguments is conclusive, in either direction. It is generally agreed that a universal standard, if it could be adopted, would be of immense benefit to the industry. And the desideratum, of course, is the greatest good of the greatest number. In this sense the general trend of the arguments seems to favour slightly the unilateral system, and this system has so far received approval by a small majority at all meetings of the B.E.S.A. Committees which have been dealing with the matter. It has also been adopted in the reports of the Swiss and German associations already referred to.

It is of course necessary to find some means of bridging over the transition period as easily as possible in the case of firms at present using the bilateral system. To this end it has been proposed to adopt a system of shaft tolerances giving rise to a series of shafts which, by suitable choice, should be, as far as possible, suitable for use with holes produced on either system, and to recommend the definite and immediate adoption of this series of shafts as standard, coupled with the recommendation that *for all new designs* the unilateral holes should be adopted also.

§ (31) SYSTEMS OF LIMITS. (i.) *Laws of Tolerance and Allowance.*—The different types of law governing the relation of tolerance to diameter for different classes of work may all be comprised in a single formula of the general character

$$T = a + b\sqrt{D} + cD,$$

where T is the tolerance, D is the nominal diameter, and *a*, *b*, and *c* are constants suited to the grade and class of work conceived, any one or more of which, in particular cases, may be zero.

In the original report of the B.E.S.A., which dealt with clearance fits only, the law adopted was

$$T = b\sqrt{D}$$

simply. The reason for the adoption of this formula was mainly empirical, based on actual measurements of a large number of samples of work of all kinds, which were carried out by

Mr. S. W. Attwell of the National Physical Laboratory. At the time that those measurements were made limit systems were not much in vogue, and since they were made methods of production have been very considerably improved, particularly owing to the development of grinding machinery. It does not follow, therefore, that the conclusions arrived at are necessarily suited to work produced under a limit system, or under modern conditions. None the less, all evidence seems to show that for clearance fits at any rate this type of law is well suited both for tolerances and allowances, and it has been generally adopted in most of the systems proposed.

An alternative proposal is to take merely a series of constant tolerances ($T=a$) and to make these applicable to all sizes, choosing for any particular size the tolerance appropriate to the particular class of fit desired. It may be doubted, however, if a scheme of this nature is sufficiently elastic to be adaptable for all varieties of work.

For interference fits, the law for *obstruction* should theoretically be cD simply, if the material is to be stressed to the same fraction of its elastic limit in cases of similar geometrical proportions but varying sizes. The cases which arise vary so much, however, in respect both of materials employed and of ratios of length and thickness to diameter, that it is almost impossible to lay down any series of values for the constant c which would be likely to cover all cases. The practice in different works, and for different purposes, varies between the extreme limits of the formulae $T=a$ and $T=cD$. Since in this class of fit subsequent separation and reassembly of the parts is rarely likely to be required, interchangeability is much less important than is the case with clearance fits, to which in the rest of this discussion we shall mostly confine our attention.

Before leaving the matter, however, we may remark that a great simplification of the system is effected in practice if the laws for tolerance and allowance are the same, since in this case, and only in this case, we are able to make the consecutive limit lines of the system serve as *common boundaries* between shafts corresponding to two consecutive classes of work. In the case of interference fits a tolerance of the form $T=cD$ leads either to unnecessarily large tolerances on the bigger diameters, or to inconveniently small ones on small diameters, and there is no reason in the nature of the processes of manufacture why the *tolerance* should be different for interference and for clearance fits. Consequently the $\sqrt{D}$ law appears the most suitable general law for tolerances, but if this be adopted in conjunction with a cD law of obstruction for interference fits the limit lines in this class cannot form common boundaries between the consecutive grades. This constitutes another difficulty in formulating a system of limits for interference fits.

(ii.) *Step Systems.*—The direct application of a mathematical law such as $T=b\sqrt{D}$ to the formation of a series of tolerances for practical use is not convenient, as it involves reference either to a chart or to tables of unwieldy proportions. It is customary, therefore, to approximate to the smooth curve given by the theoretical formula by means of a series of steps, the tolerance remaining constant over a certain range of sizes, then increasing by some finite amount, remaining constant over a further range of sizes, increasing again, and so on. The word "range" is used to denote any group of sizes over which the tolerance remains constant, and the amounts by which the limits change in passing from one range to the next are termed "steps."

In a chart such as that suggested in *Fig.* 16, each of the stepped lines is known as a limit line, and the tolerance permitted on work of a particular size and class lies between limits given by the two appropriate lines for the size in question. It is not essential that all the steps along any one limit line should be equal; for the larger sizes, where the whole tolerances are larger, greater steps and longer ranges are permissible. The maintenance of uniform steps along any one limit line leads, however, to a convenient simplification in practice, for, as Mr. Hedley Thompson has pointed out, if the steps are equal, then, for the $\sqrt{D}$ law, the lengths of the ranges form a simple arithmetical progression.

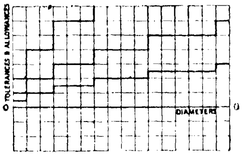


FIG. 16.

This may be proved simply as follows. Suppose we have a parabola $y=a+b\sqrt{x}$ (*Fig.* 17), and let us consider a series of ranges, starting at the origin, and of lengths given by the successive terms of the arithmetical progression, s, $2s$, $3s$, . . . ns, . . .

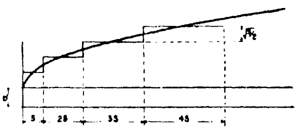


FIG. 17.

Then the sum of the 1st n ranges $=\{n(n+1)/2\}s$, and the sum of the 1st $(n-1)$ ranges $=\{(n-1)n/2\}s$.

Whence D_n, the mean diameter of the nth range, is $\frac{1}{2}n^2s$.

The ordinate at this point is

$$y_n = a + b\sqrt{\tfrac{1}{2}n^2s}$$

$$= a + b\sqrt{\frac{s}{2}}\cdot n,$$

and the difference between the nth and $(n-1)$th of such ordinates is constant and equal to $b\sqrt{s/2}$.

Putting r for $b\sqrt{s/2}$ and $k = a/r$, we may then re-write the tolerance (or allowance) on the nth range

$$y_n = a + b\sqrt{\frac{s}{2}}\cdot n$$

$$= r\cdot(k+n) \text{ simply.}$$

The factor r has been called by Mr. Thompson the "range factor"; n is the "range number"; and $(k+n)$ is known as the "size multiplier."

It will be noticed that for a strictly parabolic law, a, and therefore k, are zero. And if we had to deal with a range of sizes extending right down to nothing, we should have no option but to keep them so. It is generally felt, however, that for the smallest sizes commonly met with in practice the pure parabolic law tends to give tolerances inconveniently small, and the addition of the small constant term (represented by a and k) is made in order to increase the tolerances for small work, without appreciably affecting those on larger sizes.

When we come to consider the magnitude of the steps (*i.e.* of the range factor r) it is obviously desirable that they should be kept reasonably small, or there will be too great a disparity between the tolerances and allowances assigned to work of closely the same size, on either side of a step. On the other hand, if the step is too small, the amounts involved become difficult to measure in the workshop, and the number of ranges has to be increased, with a corresponding increase in the complexity of the tables involved. In cases where the total tolerances and allowances are greater, the steps also can naturally be increased, without introducing too marked discontinuities.

Probably the most generally acceptable step is one of 0·0002″, or some multiple thereof. This is small enough not to give sudden jumps in passing from range to range, and large enough to be measured without difficulty by a micrometer. Its multiples include all the integral thousandths of an inch, and it is conveniently translatable into metric units, as 0·005 mm. Its multiples therefore include all the hundredths and half-hundredths of millimetres, without involving smaller fractions outside the range of ordinary workshop measurement.

It should be noted, however, that although tolerances and allowances, involving only relatively small multiples of 0·0002″, can be translated into millimetres in this way, the actual diameters, and the limits of the various ranges, have to be converted by means of the factor 1 inch = 25·4 mm., and consequently do not round off automatically to convenient metric figures. If, however, these larger dimensions be rounded off to the nearest 0·005 mm., work constructed to the converted tables will in all cases be within 0·0025 mm. (0·0001 inch) of corresponding work constructed to the original tables in English units, and consequently it becomes possible to manufacture with sufficiently approximate agreement to secure general interchangeability between work made on the two systems.

A number of tolerance systems have been put forward, notably by Messrs. Newall Engineering Co. (England), C. E. Johansson (Sweden), Ludwig Loewe (Berlin), by the B.E.S.A., and by the Swiss and German Standards Associations, and a comparison of all these shows quite remarkable empirical agreement as to the general magnitude of the tolerances suitable for ordinary running fits.

If we decide on a range factor $r = 0''\cdot0002$, and make $k = 1$, then the assumption of a particular tolerance or allowance on one particular diameter is sufficient to determine the ranges, and from them all other tolerances of the same class. For example, let us suppose the tolerance on a $1\frac{1}{4}''$ normal running shaft to be $0''\cdot0012$. This will be in general conformity with the trend of the various systems mentioned. Then $0\cdot0012 = 0\cdot0002(1+n)$ so that $n = 5$, and the $1\frac{1}{4}''$ shaft must lie in the fifth range. If we suppose that it is the mean diameter of the range we have $D_n = \frac{1}{2}n^2s$, which gives us

$$1\tfrac{1}{4}'' = \tfrac{1}{2}\times 25s$$

or

$$s = \tfrac{1}{10} \text{ inch,}$$

and the ranges are, therefore, $0''\cdot1$, $0''\cdot2$, $0''\cdot3$ . . . etc.

This appears to be a convenient series of ranges. It has been objected to it that the ranges are short, and the changes of tolerance correspondingly many. It should be noted, however, that any increase in the length of the range (following the same theoretical formula) must be accompanied by an increase in the magnitude of the step, which may be objectionable in other ways. And a frequent change of tolerance with diameter is not accompanied by any increase in the number of gauges and tools required, since these are necessarily different for every nominal diameter.

We have next to consider the relation of tolerance to allowance. It is frequently argued that a somewhat wider tolerance is desirable for the hole than for the shaft, owing to the greater difficulty in its production. With modern improvements in methods of production it is doubtful whether this argument carries very much weight, and for the normal running shaft and hole the B.E.S.A. Committee has provisionally adopted equal tolerances for shaft and hole, with an allowance (or minimum clearance) of 1½ times the tolerance.

Accepting the above series of ranges, we then have for the four limits involved in this class and grade of work:

Max. hole . . .	$+0{\cdot}0002(1+n)$
Min. hole . . .	$0{\cdot}0000$
Max. shaft . . .	$-0{\cdot}0003(1+n)$
Min. shaft . . .	$-0{\cdot}0005(1+n)$

It is generally agreed also that with increasing allowances increased tolerances can be permitted, and making use of this fact, bearing in mind the desirability of maintaining common boundaries, and taking advantage of the possibility of finer tolerances on the shaft for the closer classes of fit, the following series of shafts may be employed with the normal hole, to give the results indicated.

Shaft (values of r; unit = 0·0001). Max.		Min.		Fit in Normal Unilateral Hole, Limits $r=0$, $r=+0{\cdot}0002$.	Fit in Normal Bilateral Hole, $r=+0{\cdot}0001$, $r=-0{\cdot}0001$.
+3	..	+2	..	Light driving	Driving
+2	..	+1	..	Heavy keying	Light driving
..	+1½	..	+½	Medium keying	..
+1	..	0	..	Light keying	Heavy keying
..	+½	..	½	Push	Medium keying
0	..	1	..	Light push or spigot	Light keying
..	½	..	1½	Sliding	Push
−1	..	−2	..	Easy sliding	Light push or spigot
..	1½	..	−3	Close running	Sliding
3	..	−5	..	Normal running	Running
−5	..	8	..	Easy running	Easy running

It will be seen from this table that the suggested series of shafts by suitable selection gives a reasonably satisfactory arrangement, when taken in conjunction with either the unilateral or the bilateral hole. And it would appear to be a reasonable proposal to standardise immediately some such series of shafts, and to recommend manufacturers to conform to this forthwith, and then gradually to adopt the unilateral hole for all purposes, as new designs were put into production. In this way eventual uniformity of practice might be hoped for.

IX. Design and Use of Metrological Apparatus

§ (32) Indicators and Measurers. (i.) *Their Functions.*—We have indicated in § (5) (iv.) the necessity of keeping a clear distinction between relative and absolute accuracy of measurement. In designing apparatus for metrological purposes, as indeed for any precise scientific measurement, it is important to bear this in mind, and, in fact, to carry the distinction a stage further. It is not difficult to devise a variety of types of indicator of extremely high sensitivity, but an increase in sensitivity is of no intrinsic value unless it can be associated with a machine of appropriate design to avoid the occurrence, from any cause, of errors in excess of the amount which can be indicated. Moreover, an indicator, in itself, affords no means of actually *measuring* the differences between two objects to be compared. Essentially it serves only to show that the conditions of measurement have been accurately repeated, leaving the measurement itself to be made by other means. And in this case it is clearly of no value to have an indicator whose sensitivity far exceeds the accuracy of the actual means of measurement. The measuring appliance also, whatever form it may take, has to be calibrated. And it would be equally useless to have a measuring apparatus capable of reading to extremely high accuracy unless means of calibration were available by which the values of the readings could be established to the same accuracy, and the design of the rest of the apparatus, and the conditions of working, enabled readings of this accuracy to be reproduced.

Indicators are of two kinds. In the one kind, which may be called the "fiducial" indicator, and of which a typical example is to be found in the method of observing a contact by listening to the make and break of a telephone circuit, we learn only "yes" or "no": there is no indication given as to the amount by which the parts are out of contact at any instant previous to the receipt of the signal. The gravity pieces used with the Whitworth or Pratt & Whitney types of measuring machine are another example of this kind of indicator.

In the second, and more common, kind of indicator (which we may term a "*reading*" indicator) a pointer, or spot of light, or some similar object, is made to move past a scale; a warning is given by the movement of the indicator when the conditions of repetition are approaching fulfilment, and an estimate can be made of the accuracy with which the repetition has been effected.

In many cases it is possible to ensure the repetition of the conditions of observation by mechanical or other means, more closely than can be done by the use of the indicator itself. In such cases, if suitable means of calibration are available, the rôle of the indicator may be changed into that of measurer, and the apparatus becomes a pure *comparator*, capable of measuring with high accuracy differences limited to the small amounts covered by the range of its scale.

For the measurement of larger differences, however, special *measuring* devices, as distinct from indicators, have to be employed. Such are the scale and vernier, and, by far the most generally employed, the micrometer screw. And it is worthy of remark that practically every precise scientific operation reduces itself in the end to the devising and use of suitable means and apparatus by which the phenomenon to be observed can be translated into some form of motion or record measurable on a linear scale. There are apparent exceptions to this rule, as, for example, the determination of electric current by means of the silver voltameter. But it is to be noticed that even here the weighing of the silver deposit reduces itself to observations of the movement of the pointer of a balance across its scale. The reason for this, no doubt, is threefold. Firstly, the mind more readily conceives the relative magnitudes of objects represented on a simple linear diagram than in any other way. Secondly, the linear scale is *continuous*—*i.e.* it lends itself to subdivision by simple means, and, provided circumstances are such that no sudden irregularities in the results are to be expected, this process of subdivision is a sufficient means of estimating the smaller changes in the quantity being measured, without the need of special calibration for each point on the scale that may be used. Other scales which might be employed are less advantageous in this respect. The scale of time, though in itself continuous, depends for its own representation on scales of length; for example, on the circumference of the dial of a clock, or the length of a trace on a chronograph record. The scale of mass, in its concrete manifestation at any rate, is discontinuous, consisting of a number of discrete pieces of material whose relationships with each other are known, but for interpolation between which it is necessary to build up suitable combinations by addition, and finally, for the ultimate subdivision, to refer to the movement of one of them upon a scale of length—the rider beam of a balance—or to the displacement of the beam itself. Thirdly, though this has a bearing on experimental convenience only in those cases where autographic records of results are desired, the linear scale is an essential feature of those graphical methods which so frequently offer the most convenient means of describing the results obtained. It thus happens that both indicators (of the second kind) and measuring devices are almost invariably devised with the object of reducing the quantity to be measured to some form of linear record.

The screw micrometer is for many purposes the most convenient device for measuring, and particularly for subdividing, such a record. It is, however, usually more difficult to achieve high accuracy of subdivision by any kind of *measuring* device than it is to do so (over a limited range) by means of a reading indicator. The screw micrometer, apart from unavoidable imperfections of manufacture, is affected by variations in the thickness of the oil film between the screw and its nut, and its indications, in the most favourable circumstances, are not reliable to anything closer than 0″·00001 of direct linear movement. An interesting example of an instrument combining in small compass the functions both of a fiducial indicator and measurer is to be found in the micrometer microscope, in which the fiducial setting is made by adjusting to equality the spaces between the image of a moving line, and a pair of wires fixed in a frame in the focal plane of the eyepiece, which frame is moved by a micrometer screw which serves to give the measurement of the motion of the line observed.

(ii.) *Calibration of an Indicator.*—We have next to consider the process of calibrating either a reading indicator or a measurer. If the accuracy desired in any new piece of apparatus is no higher than has already been attained by other means, it is obvious that no difficulty will arise in effecting the necessary calibration. But suppose our aim is to achieve an advance in accuracy beyond any previous attainment. Then clearly we have no preknowledge of any existing standards which will enable us to effect such calibration directly. In such case the apparatus must be made self-calibrating, and for this reliance has usually to be placed on the *continuity* of the phenomena and of their recorded effects. It is easiest to deal with this by considering a concrete case—*e.g.* the measurement of end-gauges.

The process of calibration of the 4″ slip gauge comparator ("Gauges," § (82)) to the accuracy of a millionth of an inch, will be typical. We assume in the first place that we have a series of gauges, previously standardised by other less accurate methods, whose sizes are all known, say, to one hundred-thousandth of an inch. And let them include a series 0″·1000, 0″·1001, 0″·1002 . . . 0″·1010, whose nominal sizes differ by 0″·0001. We shall assume further that none of this latter series is in error by more than 0″·00001, so that their differences are known to be correct, from the previous standardisation, within ±0″·00002. In other words, we assume that the differences of 0″·0001 are known to be defined, by means of these gauges, to an accuracy of ±20 per cent only. A set of gauges giving nominal differences of 0″·0001 would not be of much value unless the differences were accurate at least to this degree.

The scale of the indicator is capable of showing differences up to 0″·0002 as a maximum, a movement of the spot of light of roughly 3″ corresponding to 0″·0001 movement of the measuring anvil. Our first operation is to put up a temporary scale, uniformly divided—say to 1/10 inch: we then insert the 0″·1000 gauge in the machine and adjust the sliding anvil until the spot of light comes to rest on the middle division of the scale. Then the 0″·1001 gauge is substituted for the 0″·1000, and a note made of the reading of the spot. The same operation is repeated, using in turn the 0″·1001 and 0″·1002, the 0″·1002 and 0″·1003, and so on, to the 0″·1009 and 0″·1010 gauges. The mean of all these readings gives a reading corresponding to the mean of all the differences 0″·1001 − 0″·1000, 0″·1002 − 0″·1001 . . . 0″·1010 − 0″·1009—*i.e.* to one-tenth of the difference 0″·1010 − 0″·1000—and hence, in accordance with our assumption, to one-tenth of 0″·0010 + 0″·00002, or to 0″·0001 + 0″·000002. We then go through the same procedure in the reverse order, starting with the 0″·1010 gauge indicating in the middle of the scale, and reducing, instead of increasing, step by step, and so find a second mean reading corresponding with a movement of 0″·0001 in the opposite direction, also within ± 0″·000002.

If the magnitudes of these two mean readings are equal we have a first confirmation that the assumption of continuity and uniformity of scale is justifiable. The truth of this assumption can be further tested by determining the scale interval corresponding to 0″·0001 at other parts of the scale—*e.g.* from − 0″·00005 to + 0″·00005.

We then make a new scale, with three main divisions separated by intervals corresponding to the two mean readings just obtained, and we subdivide each of these intervals equally into smaller divisions each corresponding to 0″·00001, and presumably, therefore, representing this amount correctly within $0''\cdot000000_2$. One-tenth of one of these smaller divisions—*i.e.* about 0″·03—then corresponds to 0″·000001, and is easily read by the eye. If the assumption of continuity and uniformity is justified any reading taken on this scale will then be correct within the same proportionate accuracy as the two main divisions have been established—*i.e.* one part in 50—and no reading not exceeding 5 hundred-thousandths of an inch should therefore be affected by error exceeding one-millionth of an inch due to the basis of calibration.

We next make use of the machine, with the scale so prepared, to carry out a complete new intercomparison of the whole set of gauges, from 4″ downwards, in accordance with some scheme such as that described in "Gauges," § (15). We must make the assumption that the individual errors of the larger gauges are not too great to allow all the necessary intercomparisons to be made within the range (0″·0002) of the scale. Let us suppose that the larger gauges (1″ to 4″) are all correct within 1 part in 100,000 of their respective lengths, while those up to 1″ are correct within 0″·00001. Then the largest difference which would possibly need to be measured in any comparison would be + 0″·00008, which might occur in comparing the 4″ with the 1″ + 3″ gauges. This would be measured to an accuracy of $0''\cdot00008/50 = 0''\cdot000001_6$. In the mathematical working up of the results to determine the 1″, 2″, and 3″ gauges in terms of the 4″, however, only one-quarter of this error would be associated with the 1″ gauge, and three-quarters with the 3″ gauge, and similarly with each of the other observations. The 1″ gauge is thus now determined, in terms of the 4″ gauge, to an accuracy of $0''\cdot000000_4$ so far as this source of error is concerned.

If, however, we consider that 0″·000001 is the limit of accuracy of repetition and reading of the machine, this amount would have to be added to that obtained above, making the total possible observational error of the 1″ gauge, *in terms of the* 4″, $0''\cdot000001_4$. In addition we have the uncertainty as to the real value of the 4″ gauge. Our new apparatus tells us nothing about this, and we have to accept the former uncertainty, which we assumed was 0″·00001. This is transmitted, *pro rata*, to the new determination of the 1″ gauge, which finally, therefore, is known, *in absolute size*, to 0″·000004 (− 0″·000001, + $0''\cdot000002_5$), as against 0″·00001 previously.

In the remaining comparisons, with the groups of smaller gauges, no total difference between any two pairs greater than 0″·00004 can arise, and the possible error of any observed difference, due to the imperfect calibration of the scale, therefore, cannot exceed $0''\cdot000000_8$. The calculated value of any one of the series of the gauges of approximately $\frac{1}{10}$th thickness will be affected by the following possible errors:

$\frac{1}{10} \times 0''\cdot000004$ (proportion of absolute error of 1″ gauge),
$\frac{1}{10} \times 0''\cdot000000_8$ (proportion of absolute calibration error),
0″·000001 (error of observation),

making a total possible error of $0''\cdot000001_5$ in the determination of its absolute size, as compared with 0·00001 previously.

We are now in a position to redetermine more accurately the calibration of the scale, and start again. It should be noted that the observations already made do not need to be repeated. All that needs to be done is to revise the observational values in terms of the corrections to the old calibration of the scale which can be deduced from the new determination of the gauges, and repeat the calculations. If this process gives rise to changes in the calculated values of the gauges exceeding the limit of accuracy of observation, it must be repeated a second time, and so on, until finally further repetitions lead to no changes of sensible magnitude. We then have final values for the gauges, determined to the best accuracy of which the machine is capable, and are at the same time in a position to provide a final, and correctly calibrated, scale for the machine.

In all the above argument we have taken no account of the improvement in accuracy which arises from taking means of a large number of observations. We have considered only the limits of error which may enter into our results from various sources on the supposition that just sufficient observations are taken to make the solution of the problem determinate.

Let us now see what are the essential factors of the procedure.

(*a*) We must have some means of effecting a preliminary approximate calibration of the scale, by the use of quantities previously standardised to a lower order of accuracy. In the example we have taken we were able to effect this particularly well, owing to having a number of established differences of 0″·0001 of which the sum was known to the same accuracy as each individually, and the mean consequently to considerably higher accuracy. Had this not been the case, the procedure would have been the same, but we should not have achieved so good an approximation to the correct calibration at the first attempt.

(*b*) The scale must be continuous, uniform, and capable of repetition to the accuracy it is desired to attain. This is easily tested by repeating the same measurement a number of times, and at different parts of the scale. The necessity of this condition lies in the fact that by virtue of it alone are we able to subdivide our roughly calibrated scale into smaller intervals, each of which can be assumed known to the same proportional, and, therefore, to higher absolute, accuracy than the whole. If the scale be not uniform, progress can only be made if it is possible to ascertain, with sufficient accuracy, the amount of the deviations from uniformity. It is hardly necessary here to discuss in detail the additional complication which is thus introduced.

(*c*) The quantities we use for the new standardisation must be *correct* within an accuracy which enables them to be redetermined by the use of a portion only of the new scale, and so to a higher relative accuracy than that previously existing, by which the whole length of the scale was adjusted. This would have been absolutely essential to any advance had we not started with the initial advantage referred to in (*a*) above.

(*d*) For absolute accuracy, except in the case of a direct comparison with one of the three fundamental standards, we are dependent on the accuracy with which we are able by other means to effect the transition from those standards. The accuracy then attainable is of course proportional to the magnitude of the quantity being measured, up to a limit imposed by the accuracy of reading of which the apparatus is capable.

It will be found that the above conditions apply to a very large number of cases. There are, however, other cases in which the means of measurement may already be in advance of the means of indicating, and where the first condition for an increase in instrumental accuracy is the provision of a more sensitive indicator. One example of such a case which readily occurs to the mind is that of reading barometric pressure by comparing the height of a column of mercury against a scale. The accuracy with which a scale can be calibrated and read is at present far in advance of that with which the settings on the mercury surfaces can be determined.

§ (33) Measured and Manufactured Accuracy.—The third of the conditions of the last section leads to the consideration of an interesting point. In the measurement of natural phenomena—*e.g.* the wave-length of a certain spectrum line—the only limit to the accuracy which may be attained (other than that imposed by the delicacy of the means of measurement available) lies in the constancy with which it is possible to reproduce the phenomenon. The various spectral lines are known to be non-homogeneous, in varying degree, and it is not unreasonable to suppose that by varying the conditions of production—*e.g.* the temperature and pressure of the radiating molecules—a change in distribution, and consequently in the mean wave-length of the line, might be produced. In this case, by carefully specifying the exact conditions under which the radiation is to be produced, an improvement in accuracy is to be expected.

But in measuring manufactured products (and such measurement almost invariably forms a link in the chain of operations by which any natural phenomenon has to be measured) it is clearly impossible to attain increased accuracy by merely improving the means of measurement. The accuracy of the thing measured must also be increased *pari passu*. There would, for example, have been no use in constructing a machine capable of measuring slip gauges to 0″·000001 if gauges had not already been manufactured with surfaces flat and parallel within 0″·00001. Had the surfaces deviated from flatness or parallelism by more than this amount their thickness would have been so lacking in uniformity that they could hardly have been said to possess a measurement, capable of definition, to the accuracy of 0″·000001.

But there may be cases, though they are rare, in which greater accuracy is attainable by the processes of manufacture than in the operations of measurement. A case in which this condition is approached is to be found in the method of manufacturing slip gauges devised by the present writer in conjunction with Mr. A. J. C. Brookes, and described in "Gauges," § (5) (iv.). In this process 8 (or a multiple of 8) pieces are produced together, each of which is necessarily equal, within very fine limits of accuracy, to every other of the set. Theoretically the equality obtained is exact, and not the result of trial and error, or gradual approximation. This being the case, if the variations between the individual pieces of a set are not measurable by means of the apparatus available, an improved measurement of any one of them will be obtained by adding the whole 8 together, and comparing the sum of them with a standard of 8 times their nominal dimension. Dividing the result of this comparison by 8, the value of each piece is then determined, on the assumption of *manufactured* equality, to 8 times the accuracy of which the measuring machine is directly capable. And it may easily be possible, by means of a very sensitive indicator, not necessarily calibrated, or even suitable, for use as a *measurer*, to prove that the pieces are equal amongst themselves to the accuracy necessary to justify the use that has been made of this assumption.

An improvement in the processes of manufacture may thus not merely be of use as facilitating the application of measuring methods, but may itself actually afford the means for improved measurement. And we can carry the idea still further. Suppose we find by observation with a very delicate *fiducial* indicator that, by the process of manufacture, the equality obtained amongst the individual gauges constructed at one operation is better than we have any means of measuring. And suppose we carry the process of manufacture just so far that, using the same fiducial indicator, the sum of the set of 8 gauges is exactly in agreement with a piece which, according to our best available method of measurement, is correct. Then we know that, *by the process of manufacture*, we have *generated* sizes to an accuracy which, for the present, exceeds that with which we are in a position to *measure* them.

Such a reversal was, in fact, almost achieved by the method of manufacture in question. It was only by a practically simultaneous improvement in methods of measurement that the latter were enabled to hold their own. And at the present time there is little to choose in accuracy between the best processes of manufacture and measurement. Of course, exactness of manufactured size beyond existing powers of measurement would hold no practical utility. But processes of manufacture and measurement are both continually being improved, and, as we have seen, improvement in either facilitates further improvement in the other. It is interesting, however, to notice that while improvement in manufacturing accuracy normally follows only on the introduction of improved means of measurement, this is not a necessary order of development.

§ (34) DETAILS OF CONSTRUCTION. (i.) *Accuracy of Guiding Surfaces.*—In a large variety of machines measurements are made by the agency of parts movable subject to the constraint of fixed guides, and it is of importance to realise the magnitude of the errors which may be introduced by the imperfections of guides. The guide is usually assumed to be straight, and the errors which may be introduced as the result of any deviation from mathematical straightness may, according to circumstances, be of either of two types, known respectively as "sine" and "cosine" errors. In the former type, which is that of most importance, the error of measurement is proportional to the sine of the small angular error between two portions of the guide nominally in the same straight line—*i.e.* to θ; while in the latter type it is proportional to (1 the cos θ) *i.e.* to $\theta^2/2$. The factors of proportionality, of course, depend upon the particular design, and have no necessary relation to each other, but it is usually easy to attain a sufficient degree of accuracy in construction to render the "cosine" type of error negligible, and we need only concern ourselves with errors of the "sine" type.

Probably the most typical instrument, in which this type of error is of great importance, is the cathetometer, which is used to measure distant objects by comparison with the movements of an observing telescope against a scale attached to its guide. The incidence of the error is here so obvious, and its magnitude may be so great, that all such instruments are provided with means for readjusting the telescope in each position in which it is used, so that its optical axis is always brought back parallel to some fixed direction, independent of the accuracy of the guide, which serves only for rough adjustment. The fine adjustment is usually effected by means of a level. Another example where a similar type of control is effected by a slightly different method is to be found in the optical arrangements of the 50-metre mural base[1] at the National Physical Laboratory.

In many cases, however, insufficient attention is given to this point in the design of measuring apparatus. A notorious defaulter in this respect is the ordinary travelling microscope, another variant of which is the type of end-measuring machine in which there is a microscope, attached to the measuring headstock, and reading a scale placed in some convenient position alongside, for the purpose of measuring the larger movements of the headstock while the micrometer screw gives the smaller differences. In all instruments of this kind there is a sine error, proportional to the angle of bend of the guide in the plane containing the axis of the micrometer and the locus of the focal point of the moving microscope, the multiplying factor being the distance between these two lines. Many such instruments profess to give readings to 0″·00001, whereas in practice it is rare to find, and indeed extremely difficult to make, any machine of this type in which errors from this source do not amount at least to 0″·0001.

The correct solution of this difficulty is well exemplified in the type of measuring machine constructed by the Société Genevoise (see "Gauges," § (74)) in which the line of vision of the microscope lies in the prolongation of the micrometer axis. By this means any error introduced by want of straightness in the bed of the machine is reduced to one of cosine type, and readily made negligible.

In the design of measuring apparatus of all kinds this principle is fundamental, and should always be borne in mind. When dependence is placed on parallel motion along a guide, either provision should be made for checking and adjusting the maintenance of parallelism, or preferably, if this can be effected without undue complication, the motion to be measured and the means of measurement should be brought into the same straight line —*not* into two nominally parallel ones.

Two devices may be mentioned by which the accuracy of such motions may be improved. The introduction of cylindrical grinding into the modern tool-shop has made it possible to produce highly finished cylindrical surfaces of great accuracy, whose axes are necessarily straight, even though their surfaces may not be quite uniformly parallel. If a sliding piece in the form of a double vee rests on such a cylinder, with some further guide (as for instance a flat resting on a second similar cylinder placed with its axis parallel to that of the first), it should theoretically remain parallel to itself as far as motion in a plane parallel to that containing the axes of the two cylinders is concerned, in whatever position along the cylinders it is set (see *Fig.* 18), in which the line AA should be free from rotation in the plane of the paper. For many purposes the accuracy so obtained would be sufficient. But, of course, no workshop process is perfect, and it must be remembered that we have taken no account in the above argument of any bending of the cylinders after construction, either by warping of the material, under their own weight, or under the moving load of the sliding parts. For all these reasons this type of design, while frequently offering advantages over the more old-fashioned "lathe bed" type, is still only to be regarded as an approximation to the desired result.

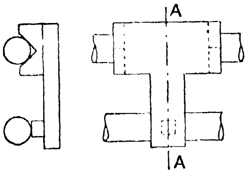


FIG. 18.

A much closer approximation to parallel motion can be obtained, when the amount of movement required is quite small, by the system of supporting the moving part on two parallel spring steel strips, as described in the article "Gauges," § (82), in connection with the moving barrel of the 4-in. "millionth" comparator. Provided side-way forces are kept within suitably small limits this type of support gives a perfectly parallel motion, though there is a slight bodily translation of the part in a direction perpendicular to the principal movement, since every point of the body moves, not in a straight line, but in a small arc of a circle.

(ii.) *"Geometric" and "Engineering" Methods of Constraint.*—It is frequently necessary, in the construction of all kinds of apparatus, to find means of locating one part definitely in relation to some other part. Two entirely distinct methods are available for achieving the desired result.

(*a*) The two parts may be carefully fitted to each other, so as to bear upon each other over considerable areas of their surfaces. This may be termed the "Engineering" method.

(*b*) The connection between the parts may be limited to the minimum that is necessary to prevent relative movement between them without imposing any restraint other than that which is mathematically required to control the various possible degrees of freedom.[2] This, theoretically, demands only the fixing of three points in the one part in relation to the other, and is termed the "Geometric" method.

A familiar example of the geometric method is found in the well-known method of locating the three feet of a tripod by means of a hole, slot, and plane, or, preferably, by means of three slots radiating from a centre. The reason for preferring the latter variant is twofold. Firstly, since the foot and its hole cannot in practice be mathematical points, we must regard them, for example, as a sphere seated in a conical cup, and making line contact along a circle, instead of point contact as required by the theory of the system. If either the sphere or the cone be slightly oval, the contact will be on two points only, and equivalent not to a point constraint, but to a line constraint such as that given by a slot. This foot then, instead of being definitely located, will have one degree of freedom remaining, of very short range, it is true, but none the less to this extent the registration of the two parts will be indefinite. Secondly, as a matter of manufacturing convenience, the three slots can all be made identical, and the three feet of the movable part can consequently be brought easily to register at the same distance from any desired reference plane on the fixed part, whereas the hole, slot, and plane have to be made independently of each other, and their different distances from the reference plane carefully adjusted—by no means an easy operation—in order to achieve the same result.

---

[1] See "Surveying Tapes and Wires," § (7).

[2] See "Scientific Instruments, The Design of."

It may be noticed that had our conical hole, instead of being nominally circular in section, taken the form of a hollow three-sided pyramid, the bearing of the spherical foot in it would have been quite definite, even had the sphere been imperfect. Such a hole, however, is considerably more difficult to make.

This brings us to an interesting mathematical relationship governing constraints of this type. When a body is free in space it has six degrees of freedom—three of translation, and three of rotation. Every point-contact made on a plane destroys one of these degrees of freedom. To fix a point involves three such contacts (as in our pyramidal hole), and the body has then left only the freedom of the three rotations about this point. The sphere resting in the V-groove or slot destroys two of these, leaving only freedom of rotation about the line joining the centres of the two spheres, and the last contact, on the plane, destroys the last possibility of movement, and completes the constraint. In the case of the three-slot arrangement, each slot destroys two degrees of freedom, making six in all.

It must be noted that in the hole, slot, and plane arrangement the slot must not be perpendicular to the line joining the feet which rest in it and in the hole respectively, while in the three-groove arrangement the grooves must not all be parallel to each other, otherwise it might be impossible to make all three feet register simultaneously, and even if they did, one degree of freedom would remain uncontrolled.

Geometrical constraints are often employed also for the control of moving parts—a typical illustration being the case of the slide on two parallel cylindrical bars referred to in the last section. The slide in this case (*Fig.* 18) has five points of contact, two between each V and the first cylinder, and one between the plane and the second cylinder. There is thus but one degree of freedom left—that of translation parallel to the axes of the cylinders.

It is necessary in the design of precision apparatus to pay careful attention to the type of constraint most suitable for any particular connection. Theoretically the geometrical constraint is ideal, and has the great advantage for certain purposes that it can very readily be removed and replaced in exactly the same relative position as before. On the other hand, it manifestly lacks the strength and resistance to wear of the "engineering" type of constraint. Where there is any considerable load on a slide the geometrical type is also very difficult to keep lubricated. The "fit" of two parts to be related by the engineering method may be a source of considerable difficulty. The bed of a lathe, for example, even if it is of uniform section to start with, is liable to wear more at one part than another, with the result that after a time it becomes impossible to adjust the saddle so as to fit equally all along the traverse. Moreover, the large bearing surfaces involved cannot be expected to make contact all over, and the fact that there are a large number of points of contact in excess of the five necessary to give complete location leads to more or less uncertainty as to the exact position which the saddle will take up on attempted repetitions of the same setting.

In the case of fixed connections the impossibility of ensuring uniform contact all over the surfaces concerned involves a certainty that the two parts, when bolted together, will be more or less strained, and there is no guarantee that after dismantling and re-assembling the bolts will be equally tightened, and the same conditions re-established. And if they were, any variation in temperature between the two parts would lead to mutual straining of uncertain nature and amount. In cases where such considerations may be of importance, the geometrical connection is to be preferred.

In the case of sliding parts the theoretical perfection of the geometrical method is somewhat off-set by the fact that the surfaces on which the sliding takes place are not of mathematical accuracy, and the contact points are likely to find minor hollows and irregularities which the surface contacts of the engineering slide would bridge over. As a matter of fact, partly for this reason and partly from considerations of wear, the nominally geometrical slide rarely depends on actual point contacts. A compromise may be made by the use of intermediate "slippers," an example of which is indicated in *Fig.* 19. Each slipper is designed to bridge the minor irregularities of a small portion of the "track"; it is free to take up whatever position on the track it desires, and the slide is definitely located from the slippers by means of the three spherical feet which rest in conical holes on the upper sides of the slippers.

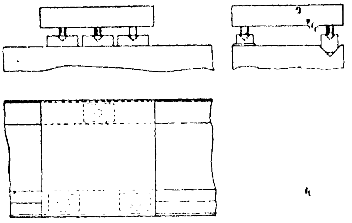


FIG. 19.

(iii.) *Friction and Frictionless Motions.*—It is often of importance, in order to ensure precision of repetition, to pay attention not merely to the geometrical properties of the constraints involved in a mechanism, but also to the elimination, as far as may be possible, of all sources of friction. Even in the absence of "backlash," friction may prevent an apparatus from recording the same reading, under nominally identical conditions, after a reversal in the direction of motion. Its effects are attributable to the variations it introduces in the forces at work, and to the elastic properties of the parts affected by these forces. The combination results in a jerky step-by-step motion, like that of a taut rope slipping round a bollard, which puts a limit to the accuracy with which any particular observation can be repeated, and at the same time defeats the desideratum of continuity to which we have referred in § (32).

Various methods of reducing the effects of friction are available. In the case of rotating parts, the frictional torque depends on the radius of the bearing, and, if no great strength is needed, can be very considerably reduced by reducing the diameter of the bearing circle—for instance, by the employment of very small conical-pointed pivots.

Another well-known device for reducing rotating friction is the friction wheel, used, for example, in Attwood's machine for measuring the acceleration due to gravity. This is capable of dealing with larger loads, and consists essentially of a wheel and axle rolling in the angles formed between two further pairs of freely turning wheels (*Fig.* 20). It depends for its success upon a double reduction of the frictional effect, by the product of the ratios of the diameters of the axles to the diameters of the wheels.

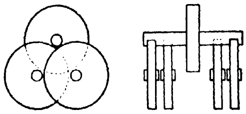


FIG. 20.

This result in its turn depends upon the fact that rolling friction is very much less than sliding friction, a fact which is also made use of in various other ways. The most obvious examples of such use are to be found in the many types of ball and roller journal and thrust bearings now on the market. These give the most satisfactory results in cases where ease of rotation is required under very heavy loads.

In the case of sliding motions, if frictionless movement is a desideratum, the engineering type of constraint becomes impossible, and we are bound to use the geometric method modified by the insertion of balls or rollers to minimise the effort at the points of contact. A typical example of how this may be effected is shown in *Fig.* 21.

The slide A moves on three balls, two resting between one vee in the upper surface of the bed and a second vee in the lower surface of the slide, and the third between a second vee in the bed and a plane on the slide. This arrangement gives complete geometrical constraint to the slide, with freedom of movement only along the bed. It is to be noted, however, that we have reverted here to actual point-and-plane contacts, so that any blemishes in the various surfaces involved will be reflected in irregularities of movement of the slide, and if the maintenance of parallelism is of importance, great attention must therefore be paid both to the perfect sphericity of the balls used, and to truth, smoothness of finish, and freedom from minute holes, of the surfaces of the vees and flat. The ordinary steel balls of commerce are fortun-

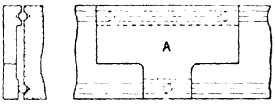


FIG. 21.

ately remarkably good as a rule, in respect of sphericity. For accurate work the surfaces should be made of hardened steel, carefully polished and lapped. Cast-iron is too liable to minute cavities. The lapping of the Vees can very conveniently be effected by means of laps in the form of long cylindrical rods, of the same diameter as the balls which it is intended to use. In this way just that portion of the surface is trued up which will eventually be employed. Only the final lapping should, however, be done in this way, as if any appreciable cylindrical groove is formed the balls will have to slip, as well as roll, and some degree of friction is again introduced.

Where only small amplitudes of movement are needed, there is an entirely different method of dealing with the problem which may be said practically to eliminate friction entirely. This is by the use of elastic suspensions. We have already mentioned (§ (29) (i.)) the method of obtaining a very accurate parallel motion by means of two parallel spring steel strips. Such a constraint is completely free from friction, except for the very small elastic hysteresis which may be present in the material of the springs, or for possible creeping of the springs in their grips. The latter may be avoided by careful workmanship, or even by soldering the springs into place after adjustment.

The same principle may also be applied where small angular movements are required, by the system of crossed spring suspension, also utilised in the "millionth" comparator ("Gauges," § (71)). Referring to *Fig.* 22, A is an arm which is required to have a slight rotation only relative to a base B. It is supported by means of the two springs S_1, S_2 rigidly attached to both arm and base in the manner shown. Rotation occurs by the bending of the springs. It is immediately apparent that the initial motions of the two points, P, Q, each half-way through the thickness of its respective spring, are in the directions P*p*, Q*q*, and hence that the instantaneous centre of rotation of the arm A is at C, the intersection of the neutral axes of the two springs. *Fig.* 23 sufficiently indicates that for a considerable amplitude of movement the motion is one of practically pure rotation about this original centre.

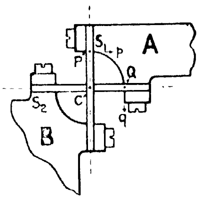


FIG. 22.

Both types of spring constraint may be made of robust construction; but the limit to the amplitude of movement possible is imposed, according to circumstances, either by the amount of variation which can be permitted in the forces actuating the movement (it is only by virtue of this variation that the flexure of the springs is effected) or by the necessity of keeping within the elastic limit of the material of the springs themselves, in order to avoid their eventual breakdown by fatigue. Naturally, the thinner and less robust the springs, the greater latitude there is in each of these directions.

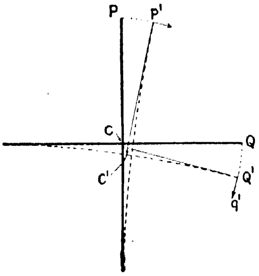


FIG. 23.

§ (35) ELASTICITY OF PARTS.—This may be a source of trouble unless due account is taken of its possible effects in design. A balance, for example, with too slender a beam, will exhibit very varying depression of its centre of gravity below the line of the centre knife-edge, consequent on its changes of flexure. In some designs of apparatus for measuring cylindrical gauges, the attempt is made to transfer from external measurements by processes involving a change in the direction of pressure of a feeler arm, with the attendant risk of a change in flexure of the feeler invalidating the results. Or moving loads may cause distortions of parts of a machine intended to remain constant, and so introduce errors.

All such matters have to be guarded against in the evolution of any new design. But although elasticity may be an enemy, we have also seen how it may be made a friend. The frictionless elastic motion, as applied to an indicating mechanism, allows us to feel absolute confidence in the *continuity* of the indications; and although the frictional resistance to movement is replaced by an elastic one varying with the amount of the movement, this variation is also continuous, and repeats itself with accuracy, so that its only disadvantageous result can be nothing worse than a slight deviation from uniformity of scale.

By a suitable proportioning of parts it is possible to make a complete mechanism of very high sensitivity, depending solely on its own internal elastic deformations. It is precisely this device which is employed in the millionth comparator.

Another and very simple device, in which the same principle is involved, is indicated in *Fig.* 24. An elastic spider, with a scale-pan at the centre of its upper surface, vertically above a suitable contact point, rests by three insulated feet upon a surface plate. A gauge to be measured is placed on the surface plate below the contact point, and the scale-pan loaded with weights until contact is just made, as indicated, for example, by sound in a telephone receiver. The gauge being replaced by a standard of known size, and the operation repeated, the difference of the weights in the pans gives a measure of the difference in thickness between the gauge and the standard. For quantitative measurement previous calibration by means of a series of standards of known size is necessary. By increasing the stiffness of the spider any desired degree of sensitivity may be attained, of course at some sacrifice of amplitude of range.

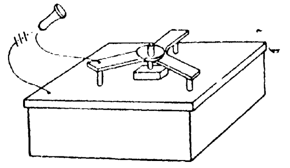


FIG. 24.

§ (36) INDICATING DEVICES. (i.) *Conversion of Linear to Angular Movement.*—This is a problem of continual recurrence, which is worthy of some attention. The diagrams of *Fig.* 25 represent three alternative possibilities. In each, the part P is supposed to have a linear movement along the axis XX, while the part Q has a rotation about an axis, perpendicular to the plane of the paper, at O. The thin lines represent the initial, or zero, position, and the thick lines a position in which Q is rotated through an angle θ, corresponding to a linear movement, x, of P. h is the distance from O to XX, which is the line in the direction of motion of P which passes through the point of contact in the zero position. Taking the cases in turn, we obtain the following relations between x and θ:

$$(a)\ x = h\tan\theta - r(\sec\theta - 1),$$

$$(b)\ x = h\sin\theta,$$

$$(c)\ x = h\sin\theta - z$$
$$= h\sin\theta - r\tan\frac{\theta}{2}\sin\theta.$$

If the motions involved are only small we may rewrite these, as far as the second or third power of θ, thus

$$(a)\ x = h\theta - \tfrac{1}{2}r\theta^2 \ .\ .\ .\ \text{(no term in } \theta^3\text{)},$$

$$(b)\ x = h\theta \ .\ .\ .\ - \tfrac{1}{6}h\theta^3,$$

$$(c)\ x = h\theta - \tfrac{1}{2}r\theta^2 - \tfrac{1}{6}h\theta^3.$$

As was to be expected, if we neglect all powers of θ above the first, we see that the ratio θ/x, for initial movement, is the same in all three cases.

All three types of mechanism are required to meet different cases that may arise. It is evident that if constancy of magnification over an appreciable range of movement is the prime desideratum, type (*b*), which does not involve any term in θ^2, is to be preferred. Types (*a*) and (*c*) lead to the same result in this respect, and the choice between them will rest on other circumstances.

If r can be made extremely small, *i.e.* if the contact is in the nature of a sharp point or knife-edge, then $\frac{1}{2}r\theta^2$ may be negligible in comparison with $\frac{1}{6}h\theta^3$. In this case type (*c*) becomes equal to type (*b*), but type (*a*) with no term in θ^3 has the advantage. But a point or knife-edge contact suffers from the disadvantage of being readily deformed elastically by the forces which may be at work, and also, from the liability to permanent damage, which is easily produced by the pressure or rubbing of such a contact.

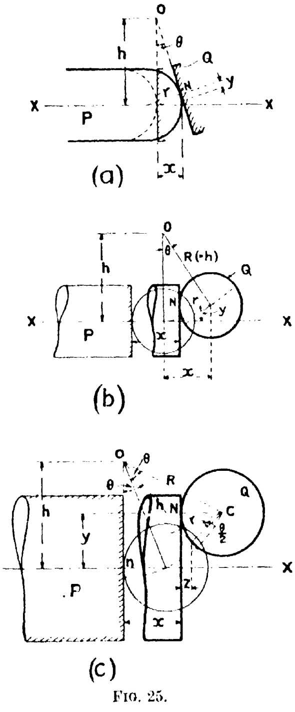


FIG. 25.

It should be noted, however, that type (*b*), while preferable to (*a*) or (*c*) on the ground of constancy of magnification, is inferior in another respect. Types (*a*) and (*c*), so far as initial motions are concerned, operate with a pure rolling contact, whereas in type (*b*) there is slipping at the surface of contact.

If we calculate y, the distances along the surface of the plane in each case, between the initial and final positions of the point of contact, we get the following results :

$$(a)\ y = r\tan\theta - h(\sec\theta - 1),$$

$$(b)\ y = h(1-\cos\theta),$$

$$(c)\ y = h(1-\cos\theta) + r\sin\theta.$$

In each case the corresponding distance along the surface of the sphere is $r\theta$. Hence, subtracting, and retaining only the first two powers of θ, we get for the distance through which the sphere has slid on the plane, in the three cases, respectively :

$$(a)\ \tfrac{1}{2}h\theta^2,$$

$$(b)\ r\theta - \tfrac{1}{2}h\theta^2,$$

$$(c)\ -\tfrac{1}{2}h\theta^2.$$

For small displacements from the zero position the amount of slip is, therefore, proportional to the movement in case (*b*), whereas in cases (*a*) and (*c*) it may be reduced to a negligible amount provided the angular displacement is sufficiently small.

The objection to any sliding motion is, of course, the fact that friction is introduced, and that thereby the action of the mechanism becomes discontinuous and uncertain in greater or less degree. Theoretically, supposing the parts were absolutely rigid, even the small slip which occurs in cases (*a*) and (*c*) would be sufficient to introduce friction. But actually the parts are not rigid, and a certain amount of elastic distortion has to take place in the neighbourhood of the point of contact before relative displacement occurs. And if the whole amplitude of motion be so small that slipping is not initiated, the mechanism as a whole behaves elastically, and the frictional difficulties do not arise.

It might appear that this was an impracticable limitation, but it is not so in practice, where very fine movements are concerned. Contact of type (*c*) is used to operate the bell-crank lever on the 4-inch " millionth " comparator. In this case the amplitude of the linear movement is $\pm 0{\cdot}0001$ in., and the dimension $h = \frac{1}{2}$ in. θ therefore varies between $\pm 1/5000$, and $\frac{1}{2}h\theta^2 = 0{\cdot}000{,}000{,}01$ in. only. This is of the order of magnitude of the diameter of a single molecule of matter, and consequently may well be supposed to lie within the limit required for such a purpose as that envisaged in the present problem. It must, however, be realised that the amplitudes of motion of which this is true are really *extremely* small, and care must be used in applying the principle.

Another type of device for converting linear into angular motion, in which greater amplitude of movement, still purely elastic in character, is possible, has been devised by Mr. E. M. Eden, and is shown in *Fig.* 26 (*a*) and (*b*). A is a fixed part, B a moving part, each rigidly connected to *c* by means of thin spring steel springs, as indicated. In the displaced position, as shown at (*b*), each of the springs, to a first approximation, is bent into the arc of a similar circle, and, if *a* be the distance between the springs, and *x* the movement of B relative to A, then it is evident that for small displacements

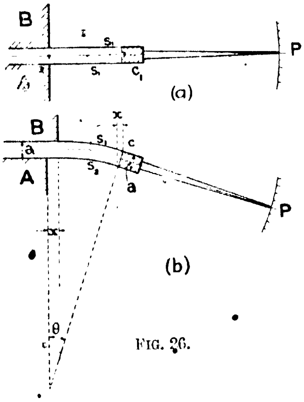


FIG. 26.

$$x = a\sin\theta.$$

This mechanism has one great advantage, in that it is possible to make *a* very small, yet definite, and so to obtain very high magnification of the motion, while the element of friction is eliminated.

In each of the three contact mechanisms illustrated in *Fig.* 25, the pivot, according to its purpose, and to the accuracy desired, may be of either journal, cone-point, or " crossed-spring " type. Or, as is frequently convenient, particularly where the rotating part is not required to transmit any force (as, for example, when it simply carries the mirror of an optical lever), it may be supported by a geometrical constraint. *Fig.* 27 (*a*) shows one method of achieving this result, and needs no further description. *Fig.* 27 (*b*) indicates a convenient variation of design for the same purpose.

(ii.) *Magnification of Linear Motions by Mechanical Levers.*—If we regard any one of the rotating elements of the pairs considered in the last section as being the short arm of a rigid lever, pivoted at the fulcrum ; and if at the end of the longer arm of the same lever (which may be either straight or bent) we provide another such mechanism—either of the same type, or of one of the other types, according to the purpose we have in view,—but arranged to function in reverse order, we can reproduce the original linear motion, magnified by a factor depending on the ratio of the lever arms, at the free end. Referring to the formulae for the conversion from linear to angular movement, we see that if mechanism (*b*) is employed, we shall invariably get a constant magnification, independent of the amplitude of motion. But if type (*a*) or type (*c*) are used, this will no longer be the case, unless r/h is the same at each end of the lever—a condition which ordinarily would involve using an inconveniently big sphere at the end of the longer arm. If, however, the amplitude is only small, types (*a*) or (*c*) give a sufficiently good approach to constant magnification, and as may be seen from the identity of their *approximate* formulae as far as the term in θ^2, are interchangeable in their effects within reasonable limits.

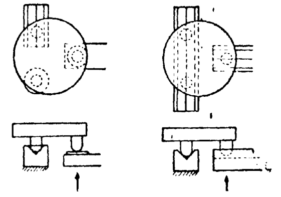

(*a*) (*b*)

FIG. 27.

Probably the best example of the application of pure mechanical lever magnification to the problem of fine measurement is to be found in the machine constructed by Dr. P. E. Shaw, of Nottingham University.[1] In this machine a series of six levers were arranged in series, to give a total mechanical magnification of 1000 : 1, the amplified movement being read by a micrometer, so that indications of the order of 10^{-7} inches could be observed.

(iii.) *The Optical Lever.*—In many cases where high magnification of small movements is required, the inertia and elasticity of mechanical parts proves a serious inconvenience. In other cases, *e.g.* galvanometers, etc., the force available for producing the angular rotation is often insufficient to permit of any mechanical means of magnification being applied. In such circumstances the optical lever may often be very advantageously employed. In this device a small mirror fixed to the rotating member reflects a beam of light. Two alternative methods of observation may be used. In the one, a scale and telescope are placed in front of the mirror, and the scale is observed through the telescope. As the mirror rotates the scale appears to move past a fixed cross-line in the eyepiece of the telescope, and a reading of the position of the rotating member is thus obtained (*Fig.* 28).

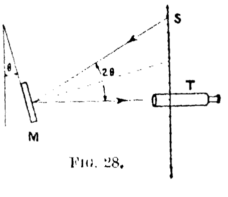


FIG. 28.

In the other variant of the method, an image of an illuminated cross-wire *c* is formed upon the scale by means of a lens L, situated between it and the mirror, and arranged so that the rays converging to form the image are intercepted and reflected by the mirror, with the result that as the mirror rotates the image moves along the scale (*Fig.* 29). The cross-wire can very conveniently be illuminated by stretching it in front of the lens of a small collimator, at the focus of which a suitable lamp is situated. The "Pointolite" tungsten arc lamps, made by the Ediswan Company, are ideal for the purpose, particularly where a brilliant image is required for reading on an opaque white scale, in daylight; but they are somewhat expensive. The rays of light then proceed as a parallel beam up to the lens L, and if this is placed at a suitable distance from the mirror, and the latter is of appropriate size, the whole of the light is used, and a bright image results.

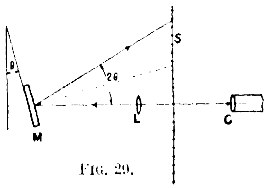


FIG. 29.

The lens L may be in any position between C and M, and as a limiting case it is possible to suppress it altogether, and to substitute for the plane mirror a concave one of suitable focal length to form an image at the desired distance of the scale. It frequently occurs also, if a silvered glass mirror is used, that a "ghost" is produced by the front surface of the glass. This is inconvenient, and can be avoided by silvering the front surface of the mirror, but if this be done the mirror is very easily damaged, and deteriorates rapidly by tarnishing. A very convenient method of avoiding the difficulty, which the writer has employed with great success, is to form an effectively concave mirror by silvering the plane surface of a small plano-convex lens, placed with its convex side towards the lamp and scale. The focal length of such a mirror is half that of the original lens, so that there is no difficulty in specifying the lens required. It should be remarked that the suppression of the separate lens L in this manner is only practicable for small deviations from the normal. If large deviations are involved the concave mirror suffers from the defect of marked astigmatism, which is avoided with the separate lens and plane mirror.

---

[1] *Phil. Proc.*, Dec. 1905, lxxvi. 350. See also "Gauges," § (75).

It should be noticed that, owing to the simple law of reflection, that the incident and reflected rays make equal angles with the normal to the mirror surface, the angular movement of the optical lever is twice that of the mirror. This in itself is an advantage, as it gives immediately double the magnification obtainable by a mechanical lever. Further advantages of the optical lever are that, being weightless, it offers no resistance to the motion being observed; the moving part can consequently be made extremely light, which for many purposes is of importance; it is instantaneous and dead beat, and since light moves in perfectly straight lines under such conditions as obtain in its employment for this purpose, it is absolutely rigid and can consequently be made with a very great length of arm, without detriment to its action, and with corresponding increase of magnification.

In using any kind of indicator it is usually necessary in some way to form an estimate of the fractional parts of the smallest subdivision of the scale. Where mechanical magnification alone is involved, a vernier can sometimes be introduced to facilitate this. With an optical lever a vernier cannot conveniently be employed, but there is an alternative method of obtaining an even more effective subdivision. Referring to *Fig.* 30, the scale is prepared with six horizontal lines, and a number of diagonal lines representing the smallest subdivisions. The spaces between the horizontal lines should be approximately equal to those between the diagonal lines. The image of the cross-line is arranged to fall across the scale in a vertical direction, and as will be seen from the figure, the tenths of the subdivisions are very clearly defined by ascertaining on which, or between which, horizontal lines of the scale the vertical cross-wire cuts the diagonal. As drawn, the reading is 0·47.

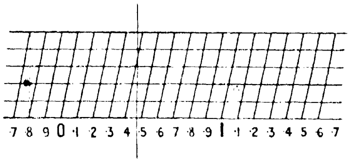
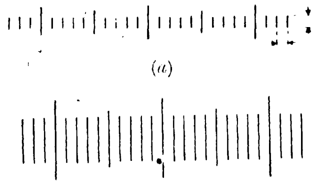


FIG. 30.

When it is considered sufficient to estimate the fractions of the subdivisions by eye, careful attention should be paid to the proportions of the scale. It is unnecessary, and in fact very undesirable, to have long lines to indicate the smallest subdivisions. *Fig.* 31 (*a*) shows a properly proportioned scale. The lines indicating the smallest subdivisions are equal in length to the space between them. The fives and tens alone are picked out with longer lines. *Fig.* 31 (*b*) shows what is known as the "palisade" effect, due to the division lines being too long in proportion to the spaces. It is very bewildering and trying to the eyes to have to take many readings on such a scale.


(*a*)

(*b*)

FIG. 31.

(iv.) *Proportionality of Scales.*—In considering the functioning of any type of indicator, another point which has to be borne in mind is the uniformity or otherwise of the magnification at different parts of the scale. It is manifestly convenient in such cases as we have been discussing if a *constant* magnification can be maintained, so that equal spaces, *throughout* the length of the scale employed, represent equal increments of the quantity measured. In some cases (*e.g.* certain types of electrical meters) it is convenient to have a variable magnification, so that, for instance, the initial movement of the indicator, corresponding to a comparatively large measurement, occupies only a short length of the scale, while over a relatively *short* range, in the region where readings are ordinarily required, a more open scale is obtained. Even in such cases, however, it is desirable that the open part of the scale should be as uniform in spacing as possible. There may also be cases in which, from the very nature of the indicating mechanism, a uniform scale is not possible. In such cases, the scale has of course to be specially calibrated at intervals more or less frequent, according to the degree of variability in magnification which may be present.

If we are dealing with a simple lever, and the angular rotation is proportional to the quantity measured, then, if a straight scale is employed, the displacement along the scale will be proportional to $\tan\theta$ for a mechanical lever, or to $\tan 2\theta$ for an optical lever. If, however, a circular scale be used, with its centre at the centre of rotation of the lever, then the displacement on the scale is proportional to θ simply, in either case, and the desired result is obtained. When very large amplitudes of movement are involved this is usually the only solution for the case of uniform angular movement of the lever. And it may be remarked that the constant length of pointer in the case of a mechanical lever, or the question of focus in the case of an optical one, makes a circular scale practically essential, except for small amplitudes.

Reverting to the three mechanisms described in section (i.) above, we may repeat the equations

$$(a)\ x = h\theta - \tfrac{1}{2}r\theta^2 \ . \ . \ . \ \text{(no term in } \theta^3\text{)},$$

$$(b)\ x = h\theta \ . \ . \ . \ - \tfrac{1}{6}h\theta^3,$$

$$(c)\ x = h\theta - \tfrac{1}{2}r\theta^2 - \tfrac{1}{6}h\theta^3.$$

If d be the deflection on a straight scale corresponding to an original motion x of the plunger, after transmission through the agency of the rotating arm of length D,

$$d = \mathrm{D}\tan\theta = \mathrm{D}\theta + \mathrm{D}\frac{\theta^3}{3} \text{ for mechanical lever}$$

$$= \mathrm{D}\tan 2\theta = 2\mathrm{D}\theta + 8\mathrm{D}\frac{\theta^3}{3} \text{ for optical lever.}$$

Hence in case (a)

$$\frac{d_1}{x} = \frac{\mathrm{D}}{h}\left(1 - \frac{r}{2h}\theta + \left(\frac{1}{2} + \frac{r^2}{4h^2}\right)\theta^2 \ . \ . \ . \right) \text{ mechanical,}$$

$$\frac{d_2}{x} = \frac{2\mathrm{D}}{h}\left(1 - \frac{r}{2h}\theta + \left(\frac{4}{3} + \frac{r^2}{4h^2}\right)\theta^2 \ . \ . \ . \right) \text{ optical.}$$

In case (b)

$$\frac{d_1}{x} = \frac{\mathrm{D}}{h}\left(1 + \frac{\theta^2}{2} \ . \ . \ . \right) \text{ mechanical,}$$

$$\frac{d_2}{x} = \frac{2\mathrm{D}}{h}\left(1 + \frac{3\theta^2}{2} \ . \ . \ . \right) \text{ optical.}$$

In case (c)

$$\frac{d_1}{x} = \frac{\mathrm{D}}{h}\left(1 + \frac{r}{2h}\theta + \left(\frac{1}{2} + \frac{r^2}{4h^2}\right)\theta^2 \ . \ . \ . \right) \text{ mechanical,}$$

$$\frac{d_2}{x} = \frac{2\mathrm{D}}{h}\left(1 + \frac{r}{2h}\theta + \left(\frac{3}{2} + \frac{r^2}{4h^2}\right)\theta^2 \ . \ . \ . \right) \text{ optical.}$$

From the above equations it is evident that, except in cases where it is possible to make $r/2h$ very small, type (b) has the advantage over types (a) and (c) as far as concerns the range over which a reasonably constant magnification is obtained. It is to be noticed that in every case the coefficient of θ^2 is essentially positive. Consequently it is not possible by any means to obtain an exactly uniform scale over any extended range.

A somewhat fairer comparison between the mechanical and optical systems of magnification is perhaps obtained if we aim at the same *total* magnification in each case. This is obtained by doubling h in the cases where the optical lever is employed. If at the same time we write $\theta = \phi/2$, then for the same total amplitude we shall have θ max. (mechanical) $= \phi$ max. (optical). We then get the amended results for optical levers:

$$(a)\ \frac{d'_2}{x} = \frac{\mathrm{D}}{h}\left(1 + \frac{r}{8h}\phi + \left(\frac{1}{3} + \frac{r^2}{64h^2}\right)\phi^2 \ . \ . \ . \right),$$

$$(b)\ \frac{d'_2}{x} = \frac{\mathrm{D}}{h}\left(1 + \frac{3\phi^2}{8}\right),$$

$$(c)\ \frac{d'_2}{x} = \frac{\mathrm{D}}{h}\left(1 + \frac{r}{8h}\phi + \left(\frac{3}{8} + \frac{r^2}{64h^2}\right)\phi^2 \ . \ . \ . \right).$$

It is thus apparent that for the same magnification and range of motion the optical system gives a slightly less distorted scale than the mechanical, even when r/h is negligibly small and the distortion depends on ϕ^2. If r/h be considerable, then, the distortion depends on the term in $r\phi/h$, and, except in type (b), the mechanical system gives four times as much deviation from a uniform scale as does the optical.

For an optical indicator a straight scale is usually convenient.

(v.) *Various Other Types of Indicator.*—Many other types of indicator may be employed in addition to those just discussed. Most of these are described, in connection with specific pieces of apparatus, below, and we need do no more than refer to them here.

(a) The Gravity Piece.—This is a purely fiducial indicator, used in connection with the Whitworth and Pratt and Whitney type of measuring machine.[1]

(b) The Tilting Level.—This is a reading indicator, used, for example, on the Newall type of measuring machine,[2] and, in a special manner, on the new sensitive gauge comparator devised by Mr. A. J. C. Brookes.[3] The tilting level is a reading indicator, and provided the bubble tube is accurately ground, gives a uniform magnification over its somewhat limited range.

(c) The Liquid Indicator, exemplified in the Prestwich gauge,[4] in which liquid contained in a receptacle closed by a flexible diaphragm is forced by the motion of the diaphragm to rise and fall in a capillary tube. The uniformity of scale in this case depends on the nature of the diaphragm. Very high magnifications can readily be obtained, as the ratio of magnification depends on the ratio of the *areas* of the diaphragm and of the capillary section of the tube—*i.e.* on the *square* of the linear ratio of their diameters. The thermal expansibility of the liquid is, however, a drawback for any purpose involving protracted time intervals between successive observations.

(d) The Telephone Contact.—Used by Dr. P. E. Shaw in connection with his multilever micrometer described above, and also in connection with his measuring machine.[1] This again is a fiducial indicator only.

---

[1] "Gauges," § (73).

[2] *Ibid.* § (72).

[3] *Ibid.* § (83).

[4] *Ibid.* § (80).

(*e*) Various forms of optical interference apparatus may be classed as indicators. See, for example, the descriptions of the Tutton wave-length comparator;[2] of the Michelson interferometer;[3] or of the Fizeau dilatometer. In this case uniformity of magnification is secured, but reading on a scale is replaced by the operations of counting interference bands as they pass some fiducial mark, and determining the initial and final fractions of a band.

(*f*) A new type of indicator, termed an "ultra-micrometer," has recently been described by Whiddington[4] and Dowling.[5] In this arrangement the effects of variation in electric capacity between a fixed and a moving plate on the frequency and amplitude of the self-sustained oscillations of a thermionic valve are made use of to measure the relative displacement of the plates. The method is interesting and its possibilities have not yet been sufficiently explored. Dowling claims a sensitivity of reading corresponding to 10^{-8} in.

X. Drawing Office Conventions

§ (37) Marking of Drawings.—There are certain points in regard to the preparation and inscription of workshop drawings to which it is desirable to refer briefly.

(i.) *Standard Sizes of Sheets.*—Considerable convenience in filing and otherwise dealing with drawings results from keeping the sheets used for various purposes to a certain limited series of sizes. There is not at present any definitely accepted series of sizes for this purpose, but a recent recommendation of the Committee on Optical Standards[6] may be quoted. This runs as follows:

"In order to facilitate the storage and handling of drawings, particularly the smaller sizes which, under modern practice, are used for the drawing of details for the shops, the committee recommends the adoption as standards of the following sizes of sheets:

Double Elephant . . .	40 × 27	inches.
Imperial	30 × 22	„
Half imperial . . .	22 × 15	„
Foolscap	13 × 8	„
Note	10 × 8	„
Index card sizes . . .	8 × 5	„
	6 × 4	„

For the last four of the above sizes, standard cabinets are available for vertical filing in folders, an arrangement which promotes compactness and accessibility."

(ii.) Another point on which there is some diversity of practice is the question of the relative positions which plans, side, and end elevations should occupy on the sheet. If the plan be supposed to occupy the centre of the sheet, some drawing offices make a practice of putting on the left-hand side the elevation seen when looking towards the object from the right, and on the right-hand side the elevation as seen from the left. In this case the front elevation is placed at the top of the sheet, and the back elevation (inverted) appears at the bottom of the sheet. In other offices the exact reverse of this practice obtains, the elevation shown on the left-hand side being that seen when looking from this side, and so on. Theoretically there is no ground for preferring one system to the other. But it would obviously facilitate the reading of drawings generally if all were prepared on the same basis. In practice the second arrangement possesses a certain advantage in that usually shorter distances are involved in transferring dimensions by means of the tee-square or set-square from one view to the neighbouring one. For this reason the Optical Standards Committee in the report already mentioned has recommended the general adoption of this system.

(iii.) *Figuring of Tolerances.*—Various methods are in vogue for figuring tolerances on drawings. The nominal dimension may be given, followed by two limits of tolerance—*e.g.*

$$1'' \begin{array}{l} +0''{\cdot}0003 \\ -0''{\cdot}0005, \end{array}$$

or, if the tolerance is all in one direction, a single tolerance figure is all that is needed, thus $1'' - 0''{\cdot}001$. In some offices this notation is abbreviated by the use of a single legend on the drawing specifying the unit of tolerance; for instance, in the above examples, the unit might be given as $0''{\cdot}0001$, in which case the dimension with its tolerances could be written

$$1'' \begin{array}{l} +3, \\ -5 \end{array} \qquad 1'' - 10,$$

respectively. Other offices again prefer to use the actual limiting dimensions in each case, and would accordingly write

$$\begin{array}{ll} 1{\cdot}0003, & 1{\cdot}000, \\ 0{\cdot}9995, & 0{\cdot}999 \end{array}$$

for the two examples given. Yet another alternative favoured by some is to have a schedule of tolerances for different ranges of dimensions and grades of work, and to give on the drawing simply a letter or symbol specifying the grade desired in accordance with the previously determined schedule. For this case the two examples might be shown, for instance, as 1″ C, 1″ Y respectively, and the drawing office and the workman would be able to interpret the symbols correctly by means of the schedule. This system has some advantage if there are a lot of parts of the same grade shown on one drawing, as it may then be possible simply to make a single legend on the drawing, such as "Grade X throughout." On the whole, however, it appears preferable that the drawing should give the workman in some form actual figures for the limiting dimensions he has to conform to; and also the difficulty of reading the drawing, in case of need, by some other party not acquainted with the system of symbols employed is an important consideration.

---

[1] "Gauges," § (75).

[2] "Comparators," § (4).

[3] "Line Standards," § (7).

[4] *Phil. Mag.*, Nov. 1920, xl. 634.

[5] *Proc. Roy. Dublin Soc.*, March 1921, xvi. (N.S.) No. 18, p. 185.

[6] *Report on Standardisation of Elements of Optical Instruments*, H.M. Stationery Office, 1920.

The system favoured by the present writer is as follows:

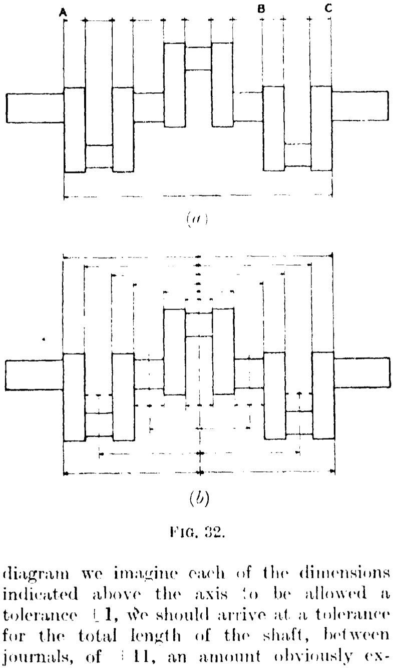
(*a*) The *preferred* size for each dimension should be given in full, and not each limit.

(*b*) The tolerances, when definite limits are of importance, should also be given in full, either ±, +, or −, as in the first example quoted above.

(*c*) Where an exact tolerance is not essential, but reasonable accuracy is none the less desired, the dimension may be quoted in decimals of an inch (or mm.), and it is then to be understood that a dimension so quoted is expected to be attained, as a general rule, within one or two units of the last decimal figure given. For example, a dimension given as 1″·05 would be expected to lie between 1″·04 and 1″·06. But if given as 1″·050 it would be expected to lie between 1″·049 and 1″·051. The addition of a + or − sign after such a dimension would imply that the tolerance taken, if any, must all be on the side of the nominal dimension indicated by the sign. A dimension given as 1″·05 − might accordingly be acceptable if between 1″·05 and 1″·03, but would not be acceptable if greater than 1″·05. The limits in such cases would not be binding, but merely a general instruction to guide to the workman. If the same limits were definitely imposed they would have to be written, respectively, as under (*b*), 1″·05 ± 0″·01, 1″·050 ± 0″·001, 1″·05 − 0″·02.

(*d*) In cases where even rougher approximations are permissible—*e.g.* over-all dimensions of non-machined surfaces—dimensions should be given in vulgar fractions, as for example $1\frac{1}{16}$″, $10\frac{1}{4}$″, etc. Dimensions so given call for no accuracy other than ordinary care in the use of a rule. This scheme, if systematically carried out, gives to the workman or other user of the drawing the maximum amount of information as to the intentions of the designer, with the minimum of labour.

(iv.) *Accumulation of Tolerances.*—It is very important when dimensioning drawings so to arrange matters that each important dimension, with its appropriate tolerance, is independently figured, and further to avoid redundant dimensioning where tolerances are involved. If the former is not properly done the addition of the tolerances on a number of separate elements, in order to ascertain that on some over-all dimension, may lead to the possibility of far too great variation being permitted on the latter. On the other hand, if the over-all dimension and tolerance are separately figured, *in addition* to those of other elements from which it may be possible to deduce them, then ambiguity is immediately introduced as to the interpretation of the tolerances indicated. For example, consider the crankshaft indicated in *Fig.* 32 (*a*). If in the upper half of the diagram we imagine each of the dimensions indicated above the axis to be allowed a tolerance ±1, we should arrive at a tolerance for the total length of the shaft, between journals, of ±11, an amount obviously excessive in relation to that allowed on the individual elements. But if, for example, we add a limit of ±2 to the over-all dimension, as indicated below the axis, *in addition* to the tolerances on the individual elements, we at once find ourselves in a dilemma. What is the tolerance, say, over the distance AB? If we go from A to B direct, adding up the individual tolerances from the left-hand side, we get ±8. If on the other hand we go from A to C and then back from C to B we get ±2 ±3 = ±5. Which of these figures are we to accept? Clearly the system, owing to the



(*a*)

(*b*)

FIG. 32.

redundant tolerancing, has become ambiguous and unsatisfactory. Moreover, the tolerance on AB ought obviously not to exceed that on AC. Yet by such a system it inevitably must.

To obtain a clear and satisfactory arrangement it is essential, first, to select some datum line in the drawing from which all dimensions constituting any one group may be figured, and secondly, to apply tolerance independently to each fundamental dimension as measured from this datum line, and not redundantly to subsidiary dimensions which can be determined by addition or subtraction of those selected as fundamental. Naturally, in assigning the datum line a certain amount of liberty of choice is available, and attention must be paid (i.) to the relative importance of the various dimensions which could be selected as fundamental, (ii.) to the methods of manufacture, and (iii.) to the methods of measurement available. Two possible satisfactory methods of dimensioning the same crankshaft are indicated in *Fig.* 32 (*b*), above and below the axis respectively, that indicated below the axis being perhaps preferable from the workshop standpoint.

J. E. S.

Michelson's Determination of the Length of the Metre in Terms of Wave-lengths of Monochromatic Light: description and results. See "Line Standards," § (7) (i.).

Micro-balances. See "Balances," § (6).

Microbarograph: an instrument, designed by Shaw and Dines, which records small fluctuations of the atmospheric pressure, to which ordinary aneroid, and even mercury, barographs are insensitive. See "Barometers and Manometers," § (3) (vii.).

Micrometer Caliper, workshop measuring instrument. See "Gauges," § (85).

MICROMETERS

Micrometric measuring apparatus consists of scales and screws, usually in combination. In general the function of the scale is to connect widely separated parts of the object measured, and of the screw to interpolate between the scale divisions. But in particular cases attempts have been made with varying success to dispense with one or other. Thus in measurements of star photographs for the Astrographic Catalogue many observatories have used scales alone; a wide scale of 5 mm. intervals and a microscopic scale with 100 divisions to the 5 mm.: the unit of measurement being the estimated tenth of the second scale, or 0·005 mm. The advantage gained is rapidity, since no time is spent in turning the screw and in reading it, and the eye is not fatigued by being withdrawn from the microscope to read the screw-head; the disadvantage is a certain loss of accuracy. On the other hand, in his pioneer measures on star photographs about 1865, L. M. Rutherfurd used a long screw only, and obtained very good measures, but he found progressive errors in the screw and his attention was diverted to the construction of a better screw. In this he was so successful that he was able to rule beautiful gratings (for spectroscopic use) of 17,000 lines to the inch; and Rowland, following him, showed how the residual errors of such a screw might be first detected and then automatically corrected during its use in ruling a grating. Michelson has carried this work to still greater perfection.[1]

But a long screw is in any case costly, and for ordinary use the mechanism is "scale + screw."

§ (1) Scales.—The divisions of a scale may be cut on metal, or glass, or photographs of a matrix of ruled lines. Divided circles are usually of the first kind, and are read by a microscope having in its focus either cross-webs or parallel webs for setting on the division. Were the division itself a clean-cut rectangle the cross-web should be the better, for the equality of its intercepts on the two edges gives a very accurate test of the centring of the cross on the division. But in practice the edges of the division are ragged under the microscope; and a pair of parallel wires, just wide enough to include the division, and set by regarding their *average* distances from the two ragged edges, has been found more satisfactory on the whole.

Glass can be ruled very clean with a diamond, and glass scales are in general use for measures of photographs. L. M. Rutherfurd turned to them when he realised the errors of his screw; and to-day, in measuring photographs of spectra with the highest accuracy, glass scales are commonly used. If a single microscope is employed the scale is placed parallel to the spectrum so that the microscope can be trained on one or other at will (it is, of course, essential that it should rotate in a strictly transverse direction without error). Or two microscopes may be clamped rigidly together, one to read the scale and the other to read the photograph.

Matrices of plate-glass silvered, the silver being cut away in accurately ruled cross-lines, are used to impress a "réseau" on star photographs. The sensitive film (either before or after exposure to the stars) is placed almost in contact with the matrix and exposed to parallel light behind it, which, of course, shines through the ruled lines only. On developing the film these lines appear on the plate together with the star images.[2] A single microscope can then refer the star images to the lines. But this convenient process is obviously not applicable to a spectrum which itself consists of lines.

---

[1] For references see *Mon. Not. R.A.S.* liii. 229, and lxii. 245.

[2] For some useful notes see *Mon. Not. R.A.S.* lix. 530.

(i.) *Virtual Scales.*—Various ingenious devices have been suggested for avoiding the use of an actual scale for spectra, especially in comparing several spectra. Thus J. Evershed[1] suggests making a positive from one of them, and superposing this on each in turn, film to film. The positive and negative lines compensate each other, but only when coincidence is exact. Variation in this coincidence in different parts of the spectra is thus easily detected and measured.

(ii.) *Errors of Scales.*—A good instrument-maker can supply scales of astonishing accuracy, but the residual errors must always be determined. An elaborate investigation of the errors of a réseau is given by Sir D. Gill.[2] The errors range from about −·005 mm. to +·005 mm. and are determined with an accuracy of ·0005 mm. In work of the highest accuracy such errors must always be determined and applied—as for instance in the parallax measures of the planet Eros.[3] The determination of the division errors of a divided circle is a long and tedious process. If there are four microscopes symmetrically placed, then by turning the circle through a quadrant at a time each quadrant may be compared with the others. Four reference points are thus fixed. Each of the intermediate arcs can be bisected by comparing the two halves, and each half can be bisected again. But as the subdivision is continued, not only does the work increase, but the accuracy diminishes owing to the accumulation of accidental error. Two important memoirs by M. Loewy and M. Fayet[4] describe a new arrangement of the work, which was found successful and which is founded essentially on a method of Bruns of Leipzig. By using divisors other than 2 a more equable distribution of probable error is attained. But to determine the errors within 0·10″ requires serious labour in any case; and the same is true for a linear glass-scale for use in spectroscopic measures.

The *permanence of the errors* was satisfactorily established by Lewis Boss of the Dudley Observatory, Albany, who showed that when his transit circle was removed from one site to another, the errors had not been altered; and the same success attended its transport to the southern hemisphere.

§ (2) SCREWS.—The screw fills in between the divisions. Usually five complete revolutions of the screw carry the wires from one scale division to the next, and the screw-head is divided into 100 parts. Reading each of these to one-tenth by estimation, we get $\frac{1}{5000}$th of the division-space. When this is 5 mm. (as in the réseaux of the Astrographic Catalogue) we therefore read to 0·001 mm. It is important to have this adjustment as close as possible (no error of "runs"), and for this purpose it should be possible to alter the length of the reading microscope, *i.e.* to separate the objective and the *wires* (the eyepiece is merely a detached reading lens), as well as to move the microscope as a whole for focussing, for which the ordinary rack-screw is convenient. The procedure then is:

(1) Focus eyepiece on wires.
(2) Focus scale in plane of wires and test "runs."
(3) If "runs" not zero, alter length of microscope.

If by oversight this third adjustment is not provided, an equivalent may sometimes be obtained by unscrewing the objective.

It is important that photographic plates should be flat (plate-glass), otherwise the "runs" correction varies in a troublesome manner. Even with plate-glass there is a slight variation.

Screws are unfortunately liable to wear with use. A good example is given by Sir D. Gill,[5] in which errors amounting to 0·50″ are found. But by arranging that two screws shall travel in opposite directions this effect of wear can be practically eliminated; and the same happy result can be obtained with one screw by reversing measures. Thus adjacent to the above paper is the description of a measuring micrometer for astrographic plates by the same author, in which screws are proposed. Instruments of this type have given very accurate results, even though the screws have worn, simply because plates are measured twice, in reversed positions in the machine, and the effects of wear have been thus largely eliminated. Alternative instruments are described in later papers.[6] In this last paper it is stated that the probable error of a setting of the screw is not so high as ± 0·0005 mm.; but the remark is added that the error of setting is only part of the error in measuring a star image, another large part being contributed by the photographic film itself.[7]

H. H. T.

MICROSCOPES, MICROMETER, for comparator use: description, sources of errors, calibration, etc. See "Comparators," § (1) (*a*).

---

[1] *Kodaikanal Bulletin*, xxxii.

[2] *Mem. R.A.S.* li. 1-27.

[3] See *Mon. Not. R.A.S.* lxvii. 175. This paper also gives some useful information on the further errors introduced when a photographic copy of a reseau is made.

[4] *Annales de l'Observatoire de Paris*, 1910, xxvii.

[5] *Mon. Not. R.A.S.* lix. 73.

[6] *Ibid.* liii. 326 and lxi. 444.

[7] See *Pub. Astr. Lab. Groningen*, pt. i. p. 81. Some conclusions on this ultimate accuracy of measurement are also given in *Mon. Not. R.A.S.* lxi. 628.

Millibar: one thousandth part of a bar (*q.v.*). It is a pressure of 1000 dynes per square centimetre. As a unit of pressure it has considerable advantages over inches and millimetres in that it is a dynamical unit of force, and not a length. The millibar has been in general use by the Meteorological Office, London, since May 1, 1914. See "Atmosphere, Physics of," § (1). See also "Atmosphere, Thermodynamics of the," § (2).

Millionaire Calculator. See "Calculating Machines," § (9).

Milne Seismograph. See "Earthquakes and Earthquake Waves," § (2).

Minimeter: instrument for testing gauges. See "Gauges," § (81).

Minimum Thermometer. See "Meteorological Instruments," § (7) (iii.).

Mirage: the image of an object which is seen displaced, usually vertically, by the refraction of the rays of light in passing through the layers of different density near the ground.

Inferior. See "Meteorological Optics," § (9).

Superior. See *ibid.* § (10).

Mock Sun Ring or Parhelic Circle: a white horizontal circle passing through the sun parallel to the horizon, and crossing the positions of the parhelia, paranthelia, and anthelion. See "Meteorological Optics," § (22).

Mock Suns or Parhelia: luminous or sometimes brilliant images of the sun which occur most frequently at an angular distance of about 22° from the sun, the distance increasing slightly with the altitude of the sun; they are also occasionally, but rarely, seen at a distance of about 44°-46°. See "Meteorological Optics," § (22) (ii.) and (viii.).

Moisture in the Atmosphere. See "Water-vapour."

Munroe Calculating Machine. See article "Calculating Machines," § (11).

N

Napier's Bones. See article "Calculating Machines," § (1) (ii.).

Nephoscope: an instrument for measuring the direction of motion and speed of clouds. See "Meteorological Instruments," § (33).

Neuhoff, O.: equations of adiabatic lines of saturated air. See "Atmosphere, Thermodynamics of the," § (21) and *Fig.* 15.

Nickel-steel Alloys: composition, their general classification. See "Line Standards," § (4) (ii.).

Night-sky Recorder. See "Meteorological Instruments," § (26).

NOMOGRAPHY

I. Aim of Nomography

§ (1) Graphical Methods.—The ordinary process of graphical computation consists in drawing a curve which represents the relation between two variables. Thus suppose it is required to exhibit in a graphical form the relation between temperature in Centigrade measure and temperature in Fahrenheit measure. If the temperature is x° C., and y° F., we have the equation $y - 32 = 9x/5$. Hence the relation is shown by taking a pair of rectangular axes x, y, and drawing the straight line $y = 1 \cdot 8x + 32$: for any given value of x, the corresponding value of y is given by the ordinate of that point on the line which has the abscissa x, and *vice versa*.

This process is at once seen to possess several disadvantages. The most important is the fact that the act of reading off the value of y corresponding to a given value of x is troublesome, and the result is inaccurate. An alternative method suggests itself. Take a straight line, and on one side of it mark off equidistant graduations to represent values of x. On the other side of the line mark off the corresponding values of y: in the present example these graduations are also equidistant. We get *Fig.* 1, a compact chart in which the reading can be carried out with ease and with accuracy.

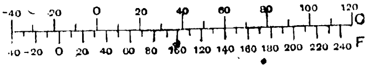


Fig. 1.

We thus see that even with only two variables the Cartesian mode of representation is not necessarily the most convenient. In many physical investigations we have to collate the values of three or even more variables. Take, for example, the formula $pv = RT$, the property of a perfect gas, R being a given constant. A single curve cannot represent the relation between the three quantities p, v, T; we have to draw a curve for p and v for each of the different values of T that occur. We get a family of rectangular hyperbolas as in *Fig.* 2, and if we wish to read off the value of T corresponding to given values of p and v, we must find the point whose co-ordinates are (p, v) and judge on which T curve it lies. In general this involves interpolating between different T curves, leading to considerable inaccuracy. Further inaccuracy is caused by the difficulty of plotting the hyperbolas themselves.

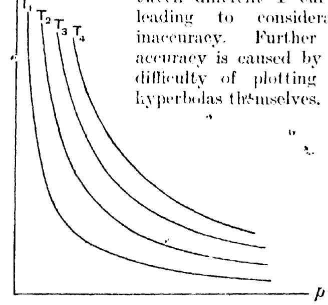


FIG. 2.

It is true that we can simplify the work by using logarithms, so that

$$\log p + \log v = \log \mathrm{T} + \log \mathrm{R}.$$

Corresponding to any given value of T we now have a straight line, and for different values of T we get a family of parallel straight lines. Thus in this instance the difficulty of plotting is eliminated; but the difficulty of reading and interpolating remains, and is in fact increased.

The method of contours for three variables has its place in such obvious and valuable applications as contour maps; for purposes of graphical computation, however, it is inadequate, and is, moreover, inapplicable to more than three variables.

§ (2).—In *many* scientific and technological investigations it is necessary *to* solve quadratic equations, and it becomes a matter of considerable importance to have a means of solving expeditiously and accurately the equation $x^2+ax+b=0$, in which a and b can assume all values within certain ranges. The fundamental graphical process would consist in plotting the curve $y=x^2+ax+b$ for the particular values of a and b that occur in each such equation, and noting where the curve cuts the line $y=0$. It is obviously impracticable to carry out the plotting for each case that has to be solved, and we soon realise in fact that this is unnecessary. If we plot the curve $y=x^2+ax$ for any given value of a, we can use it to solve the equation $x^2+ax+b=0$ for this value of a, and *any* value of b; this is done by noting where the graph is cut by the line $y=-b$. We are still left, however, with the task of plotting the family of parabolas $y=x^2+ax$, and further it is not convenient to read off where such a parabola is cut by a line $y=-b$, unless the lines $y=-b$ for the range of values occurring in the work in question are also plotted on the same graph.

We can obviate these difficulties as follows. Draw the parabola $y=-x^2$, and for given values of a, b draw the straight line $y=ax+b$. The x co-ordinates of the intersections are obviously the solutions required for these values a, b. The straight line need not be drawn at all; we can use instead a straight-edge, which is made to pass through two points which define the line for these values of a, b. Such points are, for example, $x=0$, $y=b$; $x=1$, $y=a+b$. Hence, to solve the equation $x^2+ax+b=0$ for any values of a, b, we draw the parabola $y=-x^2$ *once for all*; on the y axis we take the point b, on the line $x=1$ the point $a+b$: the join of these points cuts the parabola in points whose abscissae are the solutions required.

We have arrived at the main aim of nomography; we have an *Alignment Chart* in which the three related quantities a, b, x define three collinear points on three loci.

§ (3) METHOD OF ALIGNMENT.—The method of collinear points, as suggested in § (2), is rendered still more valuable if for any given relation between three quantities a, b, x we can devise three graduated curves so that the graduation a on one curve, the graduation b on another curve, and the graduation x on the third curve are collinear if a, b, x obey the given relation (*Fig.* 3). That such a process is a mathematical possibility is easy to see. All straight lines which pass through a given point, say x on the x curve in *Fig.* 3, belong to a family, and therefore the graduations in which any one of them cuts the a, b curves must bear some constant relationship to the particular x point. There is thus an equation connecting a, b, x; and the x graduation on any line cutting all three curves is the solution of the equation with coefficients involving the a, b graduations on this line.

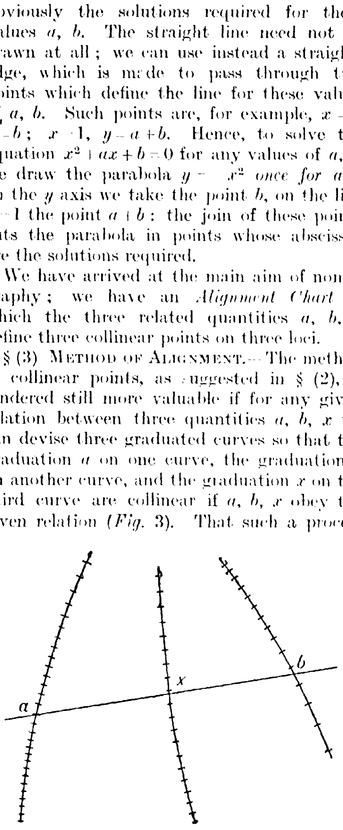


FIG. 3.

In *Fig.* 4 we have rectangular axes ξ, η. Take the line $\xi=-1$ for the a scale, and the line $\xi=0$ for the b scale: draw the hyperbola $\eta=-\xi^2/(\xi+1)$ for the x curve. Graduate the a, b scales like the η axis, and graduate the

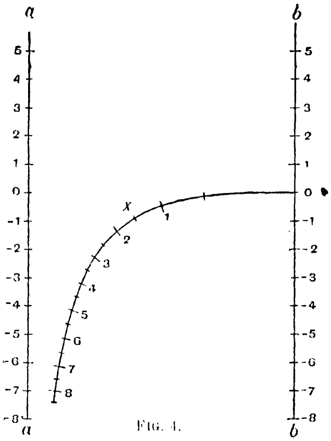


FIG. 4.

x curve so that at any point on it the value of x is the ratio η/ξ at the point. We can prove that for three collinear graduations a, b, x we have the relation $x^2+ax+b=0$. The line joining the points a, b on the a, b scales has the equation $\eta=b+(b-a)\xi$. But $x=\eta/\xi$ where $\eta=-\xi^2/(\xi+1)$. Hence at the x graduation on the hyperbola we have the equality $x=-\xi/(\xi+1)$, *i.e.* $\xi=-x/(x+1)$. Thus

$$x=b-a+\frac{b}{\xi}=b-a-b\left(1+\frac{1}{x}\right),$$

and the result follows. We shall see later

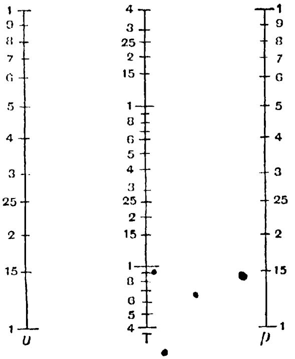


FIG. 5.

(IV.) how this construction is obtained *a priori*.

Again, consider the relation $pv=RT$ for some given value of R. In *Fig.* 5 we have three parallel logarithmic scales p, v, T, the T scale being half-way between the p, v scales, and graduated with half their unit. Also the T scale begins at the point on its line where it is intersected by the join of the point 1 on the p scale and the point R on the v scale. For three collinear graduations p, v, T we have by elementary geometry

$$\log p+\log v=2(\tfrac{1}{2}\log R+\tfrac{1}{2}\log T),$$

showing that the three graduations are connected by the relation $pv=RT$.

In both *Fig.* 4 and *Fig.* 5 we have alignment charts in which the related quantities, a, b, x in *Fig.* 4, p, v, T in *Fig.* 5, correspond to collinear graduations on their scales. Each figure is a *nomogram*, each is the graphical representation of a certain law (*nomos* in Greek) for all values of the variables that enter into the expression of the law.

II. PARALLEL UNIFORM SCALES

§ (4) ADDITION AND SUBTRACTION.—In order to grasp the fundamental idea of the subject consider *Fig.* 6. We have a, b, two

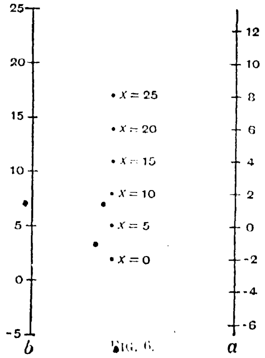


FIG. 6.

parallel lines each graduated uniformly, but not necessarily with the same unit. We shall construct a nomogram for $x=a+b$. In order to get $x=5$ we can have $a=0$, $b=5$; $a=1$, $b=4$; $a=2$, $b=3$, etc.; and, in fact, if we draw the lines joining $a=0$ to $b=5$, $a=1$ to $b=4$, $a=2$ to $b=3$, etc., we find that they all pass through the same point. Mark this point 5. Again, take $x=10$, so that we can use pairs of values $a=0$, $b=10$; $a=1$, $b=9$; $a=2$, $b=8$, etc.; we again get a common point of intersection, which we mark 10. Do the same for $x=15$, $x=20$, etc. We at once see that for any value of x we get one point, that all such points lie on a line parallel to the a, b scales, and that the graduations obtained on this line give a uniform scale.

This automatic process thus gives the

required nomogram, assuming, as suggested in § (3), that a nomogram is possible. The nomogram can be readily devised by mathematical argument, using the geometrical fact that if three parallel lines are graduated uniformly, then three collinear graduations are connected by a linear relation.

The nomogram in *Fig.* 6 gives subtraction as well as addition, since if $x=a+b$ we have also $a=x-b$.

An alternative nomogram for addition and subtraction is obtained if the a, b graduations are taken in opposite senses, instead of in the same sense as was done in *Fig.* 6. The reader can construct it for himself, either by calculation or by the automatic process.

§ (5) EXTENSION.—An exactly similar method can be used for the more general relation $x=la+mb$, in which l, m are given constants, with exactly similar results. Two nomograms are obtainable, one with the a, b scales graduated in the same sense, and one with graduations in opposite senses. If we have $x=lc+mb+n$, in which l, m, n are given constants, the method is still applicable.

The method can be at once generalised to deal with any formula such as the following, $x=la+mb+nc+pd+\ .\ .\ .\ +s$, in which l, m, n, p, . . ., s are given constants. First make a nomogram for $x_1=la+mb+s$, then construct one for $x_2=x_1+nc$, then for $x_3=x_2+pd$, and so on. At each stage we choose the one or the other of the two possible alternatives in such a way that the graduations do not become too fine for tolerably accurate reading. Note that the intermediate scales x_1, x_2, . . ., need not be graduated at all; since, in proceeding from x_1 to x_2 all we require is the geometrical point on the x_1 line which represents the value of $la+mb+s$, and not the actual value itself; the same applies in proceeding from x_2 to x_3, etc. These intermediate ungraduated lines are called *reference* or *pivotal lines*: for an example see *Fig.* 9.

§ (6) APPLICATION TO NOMOGRAM FOR MIXTURES.—For actual addition and subtraction we do not, of course, use nomograms. There is yet great value in the simple principles thus far evolved. As an example we shall work out in some detail the nomographic solution of the following problem. *It is required to mix three substances of given densities so as to produce a mixture of prescribed density; to find the percentage to be used of each constituent.* There are clearly an infinite number of possibilities, and the question is to indicate in nomographic form all the possibilities.

Consider first a particular case. Let substances a, b, c have densities 1·5, 2·5, 4 respectively, and let the prescribed mean density be 3. If we represent the percentages by the symbols a, b, c, we have $a+b+c=100$; also $1{\cdot}5a+2{\cdot}5b+4c=3(a+b+c)$. Thus we obtain $1{\cdot}5a+{\cdot}5b=c$, so that $3a+b=2c$. Eliminate c and we deduce $5a+3b=200$. Now draw a, b scales parallel to one another and graduated uniformly with the same unit in the same sense, the line joining the zeros being perpendicular to the scales. Take the mid point of this line to be the origin of co-ordinates ξ, η, the ξ axis being along this line and positive on the side of the a scale, with ξ unit such that the distance between the a, b scales is 100, while the η axis is parallel to the a, b scales and measured in the same manner as these scales. Consider a line passing through some point $(\xi,\ \eta)$ and having gradient k, the intersections of this line with the a, b scales have graduations $a=\eta+k(50-\xi)$, $b=\eta+k(-50-\xi)$. Since $5a+3b=200$, we get $200=8\eta+k(100-8\xi)$; and this is to be true for all values of k; hence we must have $\xi=12{\cdot}5$, $\eta=25$. Take the point thus found; any line through it cuts the a, b scales at graduations satisfying the conditions of the problem. The value of c corresponding to any pair of values a, b can be got by subtracting $a+b$ from 100. We can, however, work c into the nomogram conveniently and thus avoid all calculation. Take the c scale along the η axis, beginning with zero at the point $\eta=50$ and ending with 100 at the origin, so that the unit of the c graduations is half the a, b unit. It is readily verified that a line defining a, b so as to satisfy the conditions of the problem also gives c as the graduation of its intersection with the scale thus drawn. If then we let a straight-edge pass through the point (12·5, 25), and turn round this point as pivot, its intersections with the scales give all possible solutions of the particular problem enunciated.

In generalising the construction we have to consider separately two cases.

(i.) *Case* I.—If the prescribed mean density lies between the highest two of the given densities, as in the particular problem just solved, let a, b, c be arranged in ascending order of density, and let their densities be d_1, d_2, d_3; let the prescribed mean be d. Put $l=d-d_1$, $m=d-d_2$, $n=d_3-d$. We get $a+b+c=100$, $la+mb=nc$, so that we have $(l+n)a+(m+n)b=100n$. A line through the point $(\xi,\ \eta)$ with gradient k cuts the scales at the graduations given by the expressions $a=\eta+k(50-\xi)$, $b=\eta+k(-50-\xi)$, and we therefore have the following simple relation $100n=(l+m+2n)\eta+k(50\,.\,l-m-l+m+2n\,.\,\xi)$ for all values of k, *i.e.* for all lines through the definite point $(\xi,\ \eta)$. Hence we get the pivot $\xi=50(l-m)/(l+m+2n)$, $\eta=100n/(l+m+2n)$, and the solutions are given by all possible positions of a straight-edge through this pivot.

(ii.) *Case* II.—If the prescribed mean density lies between the lowest two of the given densities, let a, b, c be arranged in descending order of density, and let their densities be d_1, d_2, d_3, the prescribed mean being d. Put $l = d_1 - d$, $m = d_2 - d$, $n = d - d_3$; we get $a + b + c = 100$, $la + mb = nc$, and we can proceed exactly as in Case I.

The finding of the pivot point for any particular problem can also be facilitated by means of a graphical construction. In *Fig.* 7 the graduations on the line marked l/m are such that at any point on this line the graduation gives the ratio of the segments into which the line is divided between the zeros of the b, a scales. The graduations on the line marked $(l + m)/n$ are such that at any point on it the graduation gives twice the ratio of the segments into which this line is divided between the zeros of the c, a scales. To get the pivot point for given values l, m, n, we take the point on the l/m line whose graduation is the given value of l/m, and note the oblique line defined by this point; we then take on the $(l + m)/n$ scale the point whose graduation is the given value of $(l + m)/n$, and note the horizontal line through this point. The intersection of the two lines thus obtained is the pivot required.

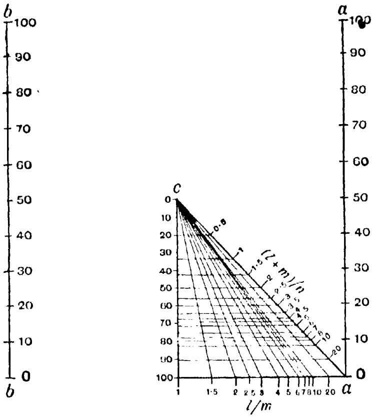


FIG. 7.

In investigations and in technological processes where the making of blends or mixtures is of importance, the nomogram of *Fig.* 7 is of great value, and a little practice with some particular cases makes the graphical computation simple and accurate.

Note the network of lines used in this nomogram; we shall see later (VI.) that this is due to the fact that we have really four independent variables, viz. a, b, and the ratios $l : m : n$.

III. PARALLEL LOGARITHMIC SCALES

§ (7) MULTIPLICATION AND DIVISION.—It is clear that if we take the nomogram for $x = a + b$, and instead of graduating each scale uniformly we graduate them logarithmically (as on the slide rule), making in each scale the 1 to 10 interval of the logarithmic graduations proportional to the unit in the uniform graduations, then we get a nomogram for $X = AB$, where X represents 10^x, etc. This figure can therefore be used for multiplication and division. As an example we have the relation $pv = RT$ for a given value of R, already considered, § (3), *Fig.* 5.

The nomogram in *Fig.* 5 is constructed without any reference to the particular ranges of the variables for which the chart is required. If these ranges are given we choose such 1 to 10 intervals in the scales as make the given ranges correspond to about equal lengths on the two scales, filling up the sheet of paper or card on which the chart is drawn.

§ (8) EXTENSION.—The extension to more general problems in multiplication and division is so obvious that we shall merely give one or two instances of actual nomograms, with a few words of explanation where necessary. In *Fig.* 8 we have a nomogram for the formula

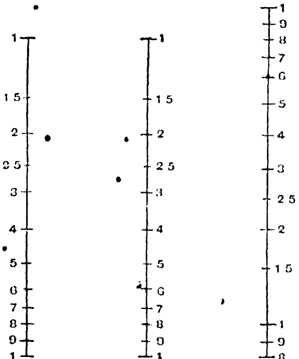


FIG. 8.

$R = SV^2/800$, which gives the resistance R in lbs. weight to a parachute of area S sq. ft., falling with a speed V ft./sec. in air of ordinary density.

When we have more than three variables, and have to multiply given powers of these variables, we introduce reference lines as explained in § (5). In this way we can deal with any formula like $X = SA^lB^mC^nD^p$ . . ., where S, l, m, n, p . . . are given constants. It is important to remember the precaution mentioned in § (5) with regard to the senses of the graduations on the successive scales so as to get the final graduations sufficiently accurate.

As an illustration, consider the problem of finding the value of the horizontal component of the earth's magnetism at any given place. If we use a steel bar of moment of inertia K, and, when it has been magnetised we observe the number n of half-oscillations it makes per second when swinging in a horizontal plane under the influence of the earth's magnetism, and also find the deflection θ it gives at distance r from a magnetometer needle in "end on" position, then the horizontal component of the earth's field is given[1] by

$$H=\pi n\sqrt{2Kr^{-3}}\tan^{-\frac{1}{2}}\theta.$$

For a given steel bar K is known once for all, and so we have H in terms of the three variables n, r, θ. Taking $K=380$, say, we get

$$H=86{\cdot}6nr^{-\frac{3}{2}}\tan^{-\frac{1}{2}}\theta.$$

The nomogram for this formula is given in *Fig.* 9. We first construct one for the quantity $r^{\frac{3}{2}}\tan^{\frac{1}{2}}\theta/86{\cdot}6$, using a logarithmic scale for r and a logtan scale for θ. The resulting scale need not be graduated, since it is a reference line as explained in § (5). We then fit on a scale for n so as to give the value of H. The cross lines in *Fig.* 9 indicate the order of the operations in using the chart.

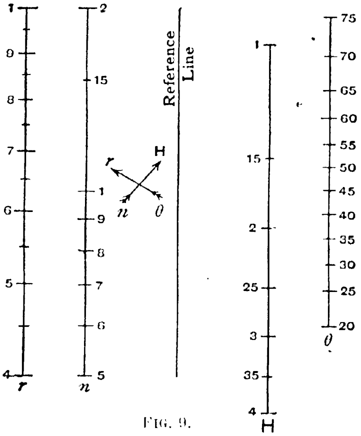


FIG. 9.

IV. PARALLEL CO-ORDINATES

§ (9).—We now proceed to the consideration of more complicated nomograms, that can be constructed by using two parallel scales, uniformly graduated, for the two variables a, b in terms of which a third quantity x is defined. The quantities a, b are said to be measured in *parallel co-ordinates*. The x scale will not in general be straight, and we shall refer to it as the x curve.

§ (10) QUADRATIC EQUATION: AUTOMATIC METHOD.—To commence with, we take the equation $x^2+ax+b=0$, in which we are given that a can have any value between 0 and 100, while b can have any value between 0 and $-10{,}000$. Take convenient units so that the a, b ranges are given by more or less equal lengths: in *Fig.* 10 we have made the a unit 100 times the b unit. Having graduated these scales, we then find the points on the x curve as follows. It is obvious that with the given ranges the positive root x can have any value between 0 and 100. (We only need to find one root, since the other is given by the fact that the sum of the roots is $-a$, and their product is $+b$.) Take $x=10$: find two equations having 10 as a root in each case, for instance $x^2+100x-1100=0$, and also $x^2+50x-600=0$. Hence for $x=10$ we can use $a=100$, $b=-1100$, and $a=50$, $b=-600$. Find the point of intersection of the lines joining these two pairs of a, b values, and we have the point 10 on the x curve. Take $x=20$; we can use the pairs of values $a=100$, $b=-2400$; $a=50$, $b=-1400$; and the intersection of the lines defined by these two pairs is the point 20 on the x curve. Proceed in this way till $x=100$. We have the x curve by drawing a smooth curve through the points thus obtained, and subdivisions can be inserted if desired. In *Fig.* 10 the graduations on the x curve appear to be equidistant, but this is a mere accident.

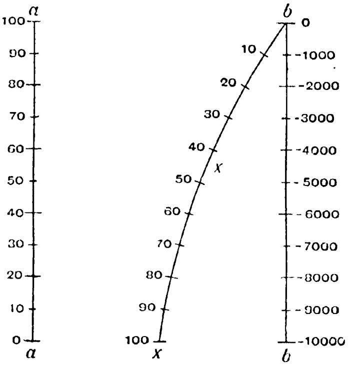


FIG. 10.

§ (11) ANALYTICAL METHOD.—The automatic method has the advantage of being easy to carry out practically, and also that the given ranges can be worked into the nomogram so as to give the best figure and the most accurate results. The analytical foundation of the process can be put in the following form.

---

[1] Glazebrook and Shaw, *Practical Physics*, p. 461.

Suppose that we have to solve nomographically an equation of the form

$$A(x)a+B(x)b+1=0,$$

where $A(x)$, $B(x)$ are given functions of x, and a, b are the variables which determine any particular root x. Using rectangular co-ordinates ξ, η, let (ξ, η) be the point x on the x curve, the a scale being the line $\xi=-1$, the b scale the line $\xi=0$, each graduated in the same manner as the η axis. Any line through the point (ξ, η) which cuts the a, b scales at the graduations a, b has the equation

$$\eta=b+(b-a)\xi,$$

i.e.

$$\frac{a\xi}{\eta}-\frac{b(\xi+1)}{\eta}+1=0.$$

If this equation is to define the graduation x at the point chosen, then the relation between a and b in this equation must be the same as that given by $A(x)a+B(x)b+1=0$ for an infinite number of pairs a, b. Hence we get

$$A(x)=\frac{\xi}{\eta},\quad B(x)=-\frac{\xi+1}{\eta},$$

so that

$$\xi=-\frac{A(x)}{A(x)+B(x)},\quad \eta=-\frac{1}{A(x)+B(x)}.$$

Thus we have the co-ordinates of the point (ξ, η) on the x curve defined in terms of the parameter x itself, and the x curve can be immediately plotted and graduated.

Returning to the quadratic equation

$$x^2+ax+b=0,$$

write it in the form $a/x+b/x^2+1=0$, and use $A(x)=1/x$, $B(x)=1/x^2$. The results $\xi=-x/(1+x)$, $\eta=-x^2/(1+x)$ follow, yielding the equation $\eta=-\xi^2/(\xi+1)$ for the x curve, the manner of graduation being indicated by the relation $x=\eta/\xi$. This is the way in which we constructed the nomogram in *Fig.* 4, due to M. d'Ocagne.

While the automatic process is best for practical construction of a nomogram, the analytical method can be used to devise new kinds of nomograms. Thus the equation $x^2+ax+b=0$ can be written $-a'x-b'/x+1=0$, in which $a'=-1/a$, $b'=-b/a$. This means that $A(x)=-x$, $B(x)=-1/x$, giving $\xi=-x^2/(1+x^2)$, $\eta=x/(1+x^2)$. We easily deduce that the x curve is the circle $\xi^2+\eta^2+\xi=0$. This is an interesting form of nomogram for the quadratic equation, due to E. T. Whittaker. It has the great advantage that the x curve can be drawn very easily and accurately: also the x graduation is simple. D'Ocagne's nomogram is, however, better in practice, because the a, b graduations are uniform in this nomogram, and therefore it is easier to use.

§ (12) CUBIC AND OTHER EQUATIONS.—The cubic equation in the reduced form $x^3+ax+b=0$ can be treated in exactly the same way as the quadratic. In the analytical method we now have $A(x)=1/x^2$, $B(x)=1/x^3$, since we can write the equation in the form $a/x^2+b/x^3+1=0$. Hence $\xi=-x/(1+x)$ and $\eta=-x^3/(1+x)$ for the point x on the x curve. The equation of the x curve is $\eta=\xi^3/(\xi+1)^2$, the graduations being given by $x^2=\eta/\xi$ (see § (15) and *Fig.* 14).

Trigonometrical equations can be treated similarly, thus $a \tan x+b \sec x+1=0$ gives $\xi=\eta^2$, with the graduations defined by $\sin x=(1-\xi)/(1+\xi)$. The a scale is along $\xi=1$, the b scale along $\xi=-1$.

If we take an equation of the type $A(x)a+B(x)b+1=0$, the process would be carried out as follows. Let the equation be, *e.g.*, $\sin x=a+bx$, x in radians, a ranging from 0 to 1, b from 0 to $-0{\cdot}5$. Take the b scale with twice the a unit, as in *Fig.* 11. Take $b=0$; then for $x=0, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°$, we get the following values of a:

$$a=0,\ 0{\cdot}17,\ 0{\cdot}34,\ 0{\cdot}50,\ 0{\cdot}64,\ 0{\cdot}77,\ 0{\cdot}87,\ 0{\cdot}94,\ 0{\cdot}99,\ 1;$$

for $b=-0{\cdot}5$ we get the values

$$a=0,\ 0{\cdot}26,\ 0{\cdot}52,\ 0{\cdot}76,\ 0{\cdot}99,\ 1{\cdot}20,\ 1{\cdot}39,\ 1{\cdot}55,\ 1{\cdot}68,\ 1{\cdot}79.$$

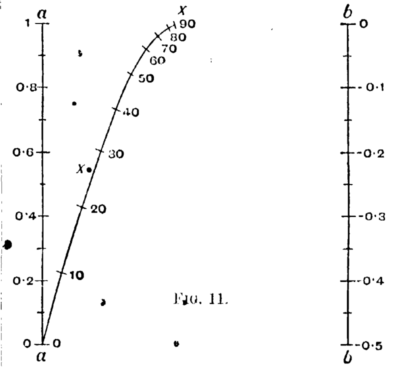


FIG. 11.

Drawing the two sets of lines thus obtained, we get the points on the x curve and the graduations. As a check we can take a third value of b and draw a third set of lines: these must pass through the points defined by the first two sets.

An equation in the form

$$a^{A(x)}b^{B(x)}=C(x)$$

is treated by taking logarithms, so that

$$A(x)\log a+B(x)\log b=\log C(x).$$

and we use parallel a, b scales graduated logarithmically. The nomograms of Part III. §§ (7) etc. are all special cases of this type. An equation of the form

$$a^{A(x)}[B(x)]^b = C(x)$$

gives, on taking logarithms,

$$A(x) \log a + \log B(x) . b = \log C(x),$$

so that we have to use parallel scales a, b, of which one is graduated uniformly and the other logarithmically. It should be pointed out that in the analytical method we can use any two lines parallel to the η axis for the a, b scales: in each case a preliminary trial would determine the best choice.

V. INTERSECTING SCALES

§ (13) ANALYTICAL BASIS.—The method of Part IV., essentially one based on tangential co-ordinates, can be applied with a, b scales not parallel. No matter what angle two lines make with one another, if a, b are the intercepts on these lines considered as axes of co-ordinates ξ, η of a line through the point (ξ, η), then we have $\xi/a + \eta/b = 1$. If, then, we have the equation $A(x)/a + B(x)/b = 1$, we must use $\xi = A(x)$, $\eta = B(x)$ in order to get the relation required between a, b and the graduation x supposed associated with the point chosen. The construction of the nomogram is now obvious.

As an example, let $A(x) = x$, $B(x) = x$, and we get $\xi = x$, $\eta = x$, so that the x curve is the straight line $\eta = \xi$. In this way we get the nomogram in *Fig.* 12 for the optical formula $1/u + 1/v = 1/f$, a nomogram which is mentioned, but not as such, in many elementary books on Light.

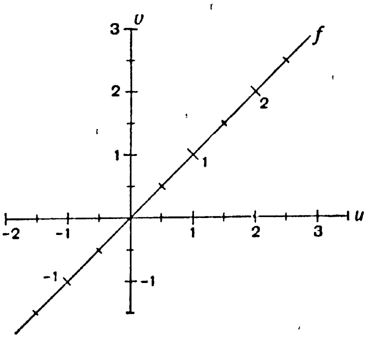


FIG. 12.

If we make the angle between the ξ, η axes 120° we find that the units are the same on all three scales, and we can deal with the addition or subtraction of any number of reciprocals, provided that some care is exercised with the choice of signs in the even operations.

§ (14) Z CHARTS.—Consider the equation $x = a^b$, so that $\log x = b \log a$. To construct a nomogram for this relation we need a process for multiplying the logarithm of any quantity a by any quantity b. Similar triangles at once suggest themselves, and *Fig.* 13 is the nomogram required: a and x are logarithmic scales, and the scale b is so graduated that the number attached to any point on it is the ratio of the segments into which the line is divided between the x, a scales. This is a segmentary scale, as already used in *Fig.* 7, § (6). The graduations x' in *Fig.* 13 give the

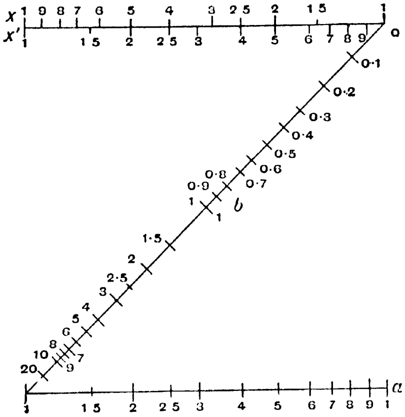


FIG. 13.

values of $x' = a^{-b}$. It is clear that Z charts can be constructed for any relation of the form $X(x) = A(a)^{B(b)}$.

VI. FOUR VARIABLES

§ (15) GENERAL CUBIC EQUATION.—In Parts IV. and V. we restricted ourselves to three variables. When there are more than three variables one of two cases can arise. Either the variables can be taken in order one after the other, as, for example, in the formula $X = SA^lB^mC^nD^p$ . . . In this case we merely have a succession of nomographic processes joined up by means of reference lines (see § (8)). It may happen, however, that the variables are not thus separable. One example has already been treated, § (6), *Fig.* 7.

We now examine the case of the general cubic equation, which we shall write in the form $x^3 + nx^2 + ax + b = 0$. The cubic equation of § (12) is a special case in which $n = 0$. We get a chart for all values of n by simply working out a series of nomograms, on the same sheet, using a number of values of n. Analytically this means that, using the method of § (11), we have the form

$$\frac{a}{x^2+nx}+\frac{b}{x^3+nx^2}+1=0,$$

so that

$$A(x)=\frac{1}{x^2+nx},\quad B(x)=\frac{1}{x^3+nx^2}.$$

Hence

$$\frac{\xi}{\eta}=\frac{1}{x^2+nx};\quad \frac{\xi+1}{\eta}=-\frac{1}{x^3+nx^2},$$

and we get

$$\xi=-\frac{x}{1+x},\quad \eta=-\frac{x^3+nx^2}{1+x}.$$

We notice that the x graduations are given by $x=-\xi/(\xi+1)$, so that, in terms of ξ, x is independent of n. In the nomogram of *Fig.* 14 the x graduations are indicated by means of straight lines parallel to the a, b scales,

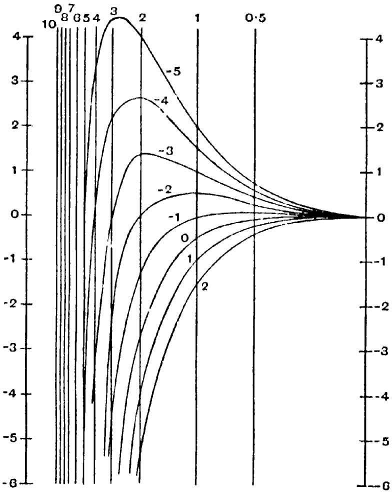


FIG. 14.

and the values of n are attached to the various x curves.

§ (16) NOMOGRAM FOR A PROBLEM IN NAVIGATION.—To find the direction in which a ship is sailing with reference to a given landmark P, observations of the bearing of P are taken from three positions A, B, C of the ship. In this way the angles APB $=\alpha$, BPC $=\beta$ are obtained; also the ratio, k, of AB/BC is known. If the angle PBC is called θ we easily find that the following equation holds: $(1+k)\cot\theta=k\cot\alpha-\cot\beta$. A little consideration shows that we can assume k greater than 1. To get a nomogram for different values of all the variables involved, we work out nomograms on the same sheet for a number of different values of k, the variables in each one being α, β, θ. We thus get *Fig.* 15, due to S. Lister.

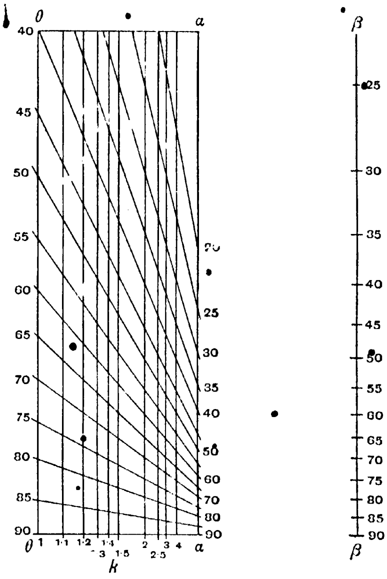


FIG. 15.

VII. OTHER METHODS

§ (17).—While referring the reader to the books mentioned below for further and more detailed information on the subject of nomography, it will be useful to indicate briefly one or two extensions of the methods described above.

§ (18) HEXAGONAL CHARTS.—Nomograms with intersecting scales suggest what are known as hexagonal charts. In *Fig.* 16, OA, OB, OC are three lines making two angles of 60° each. If from any point P we draw perpendiculars PA, PB, PC to these lines, meeting them at A, B, C respectively, then we have OA + OB = OC no matter what point P is taken. If we put logarithmic scales along OA, OB, OC we have an obvious means of carrying out multiplication and division, as in III. For purposes of reading we use a

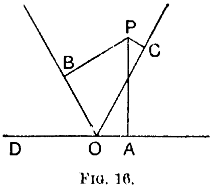


FIG. 16.

transparent sheet (celluloid) on which are drawn three lines making two angles of 60° each, and place it on the chart so that the lines on the transparency are perpendicular to the lines on the chart. A further extension is obtained if we introduce a fourth scale OD along AO produced.

§ (19) COMBINATION CHARTS.—The combination of nomograms, as used in § (8), *Fig.* 9, can be extended to the case of Z charts. This is illustrated in *Fig.* 17, where a, b, and the line AB form one Z chart, and x, c, and the line AB form another. AB is thus a reference line, and remains ungraduated. The figure leads at once to a nomogram for $x=ac/b$, with obvious extensions.

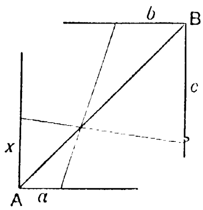


FIG. 17.

§ (20) EMPIRICAL NOMOGRAMS.—Suppose we have experimental observations connecting three quantities a, b, x, and suspect that there is a certain type of relationship between the three, we can decide the point by trying whether the observations can be worked into a nomogram suitable to the type of relation suspected. Thus if we think that $X(x)=a^lb^m$ might suit the observations, where $X(x)$ is an undetermined functional form, and l, m are unknown constants, we take parallel logarithmic scales for a, b and use the automatic process for a number of values of x: the surmise is correct if the different a, b lines for any x do actually meet in a point, and the different points are collinear. The value of l/m is then determined by the relative distances of the x line from the a, b lines, and the complete empirical formula follows by comparing the x graduations with a logarithmic scale. A similar process can be adopted if some other type of relationship is thought to be the correct one.

VIII. BIBLIOGRAPHY

The aim of this article has been to introduce the reader to the main ideas and principles of alignment charts, with some applications and extensions. The method of collinear points, which constitutes the chief beauty of the nomographic method, is due to M. d'Ocagne (1884). The standard work on the subject is d'Ocagne's *Traité de nomographie* (1899). Other books by the same author are: *Calcul graphique et nomographie* (1914); *Principes usuels de nomographie* (1920). English books on the subject are: J. B. Peddle, *The Construction of Graphical Charts* (1910); R. K. Hezlet, *Nomography* (1913); J. Lipka, *Graphical and Mechanical Computation* (1918); and S. Brodetsky, *A First Course in Nomography* (1920). Several articles in English have also appeared from time to time in technical engineering and mathematical journals.

S. B.

NORMAL EICHUNGS KOMMISSION ALCOHOL TABLES. See "Alcoholometry," § (4).

NORMAL EQUATIONS, THE SOLUTION OF: performed by the elimination between equations in pairs, or by expressing the value of any unknown as a determinant, which is to be evaluated in a schedule. See "Observations, The Combination of," § (7).

NOTCH METERS. See "Meters," § (31).

N.P.L. FOUR-METRE COMPARATOR: description and method of use. See "Comparators," § (8) (i.).

N.P.L. SUBDIVIDING COMPARATOR:
General description. See "Comparators," § (10) (ii.).
Method of use. See *ibid.* § (12).

NUTATING PISTON METER. See "Meters," § (27) (ii.).

— O —

OBSERVATIONS:
Balancing numbers of, in metrological work. See "Metrology," § (5) (iii.).
Balancing weight of, in metrological work. See *ibid.* § (5) (ii.).
Distribution of, in time. See *ibid.* § (5) (i.).

OBSERVATIONS, THE COMBINATION OF

§ (1) INTRODUCTORY.—In the following pages an attempt has been made to summarise the standard methods of performing the various kinds of computation which are met with in the so-called exact sciences.[1]

The "principle of least squares" has inevitably been adhered to throughout almost the whole article—inevitably, because the processes based upon it are the only standardised and formal methods which have been developed up to the present. At the same time an impression in the reader that the principle of least squares is "the end of all things" would be erroneous, and a section has been added at the end setting out some minor points on which the method is faulty or likely to lead to misconception.

An intelligent application of empirical formulae is always to be preferred to their mere mechanical use. Therefore wherever possible a full verbal explanation of the reasons for a given conclusion or step has been given, but mathematical analysis has been reduced to a minimum, so that the multiplicity of symbols which has at times been necessary never denotes a corresponding complication in the mathematical processes.

---

[1] They should only be applied with caution to statistical Biology. For this purpose, works devoted specially to the subject—*e.g.* those of Prof. Karl Pearson—should be consulted.

In many cases practical examples illustrate the principles. The results of all experiments of the kind considered in this article are physical magnitudes possessing dimensions, zero in certain cases.

If a certain experiment gives the result: length AB = 1·00003 yards, the truth or accuracy of this result will be influenced firstly by the errors of observation. These may cause the number 1·00003 to be slightly wrong. But the latter is also influenced by the fact that the yard used for comparison was not *the* standard yard but *a* standard yard. The error thus added to the observational error is contained in the scalar or numerical portion of the result. Hence if the other portion of a result, *i.e.* that possessing physical dimensions, contain always definite standard units, it can be regarded as a label only, and the results of all numerical experiment can be thrown into the scalar form. For instance, the above result may be stated as follows: The number of standard yards contained in the length AB = 1·00003. So it comes about that the theory of the best adjustment of errors deals in essence only with scalar quantities. Its various propositions can be developed without reference to any units, and although physical magnitudes of many different kinds occur in its practical application—sometimes even in one and the same application—they all disappear from sight and leave a purely arithmetical problem.

The existence of totally different kinds of observation in a given piece of work, each kind demanding consideration in the adjustment of the whole, calls for a personal judgment on the part of the investigator as to what constitutes an error of the same degree of badness in the several kinds of observation. This done, the problem is reduced, as stated above, to a numerical one.[1]

For example, an observer wishing to adjust his observations for the coefficient of linear expansion of a solid might decide that an error of 0·1° C. in one of his recorded temperatures would cause him as much displeasure as an error of 0·005 mm. in a single micrometer reading for length.

A surveyor might similarly fix the relationship between his base-line errors and his theodolite errors by saying that he was as likely to make an error of 10″ in a single angular measurement as he was to be wrong by 1 part in 500,000 in his base-line measurement.

§ (2) THE NATURE OF ERRORS.—It is a commonplace known to all practical experimenters to say that there is no such thing as absolute accuracy. What meaning could be attached to a statement that the distance between two fine scratches on a polished bar was *exactly* 10 cm.? The finest scratches, when closely examined, become valleys with many indentations, so that an exact apprehension of the centre of the scratch is impossible. The best chronometer in the world will make so many oscillations with its balance wheel in each mean solar day, *plus or minus a very small quantity*.

This notion of plus or minus something attaching to the statement of all magnitudes is always present in the minds of scientific men, and the facility attained in dealing with experimental data, *i.e.* in the combination of observations, is closely bound up with a proper appreciation of the way in which this plus or minus quantity *propagates itself* from one stage to another of the calculations, finally appearing as an uncertainty in the end result. To find general methods of computing from observations which shall give a smaller plus or minus quantity in the end result than any other methods may be stated to be the central object in the theory of the combination of observations.

Bearing in mind the illusory character of the "true" value of any quantity sought, we may think of *any* quantity which lies between certain close limits as the true value.

The difference between an assumed or observed value and the true value is then spoken of as the *error* of the assumed or observed value. The difference between a single observed value and one computed from a solution of the whole or part of the problem, whether this be the final solution or not, is called a *residual*.

The determination of physical quantities from observations is always burdened with errors, among which those arising from the calculation can be cut down to any desired extent so long as the computer is prepared to expend the necessary amount of labour on them. On the other hand, mistakes, whose definition here is scarcely necessary, are more likely to occur in calculation than in recording observations.

Errors of observation fall into two fairly distinct classes, *accidental* and *systematic*, though the dividing line is sometimes difficult to place. The essential characteristic of accidental errors is that any one observation is as likely to have a positive error of a given magnitude as it is to have a negative error of the same magnitude.

One of the chief limitations to accuracy of observation—a source, that is to say, of accidental errors—is the fallibility of the eye and hand of the observer, but there are many kinds of observation in which the attendant circumstances play a greater part. Every experiment is in fact attended by a number of circumstances or *conditions*, each of which is liable to have a definite effect on the result of a single observation. In so far as these conditions produce effects which change in an arbitrary manner from one observation to another, they lead to increased accidental errors, in spite of all the skill of which the observer may be possessed. For this reason, and for the reason that the number of effective conditions is generally much greater than is supposed, it is bad practice for an observer to weary himself by bestowing a maximum of energy on each observation. A large number of observations performed with caution but in an easy, flowing manner is much better than a few observations made by an observer over-anxious to use his eye to the very best advantage.

---

[1] See § (5) (vi.) and the reservations in § (7).

If one of the conditions mentioned above be imagined to persist in its influence on a long series of observations, or even on a few, in such a way that its effect no longer varies arbitrarily but according to some law, we have what is known as a *systematic error*.

The simplest of these is a *constant error*, which burdens all observations with an error constant in sign and magnitude, over and above those from other sources. Systematic errors are the experimenter's greatest enemy, and it may be truthfully said that the search for possible causes of them is nine-tenths of the battle in all physical measurements. The search is rendered a hopeful one by the fact that the error in question *must be systematic with respect to something*. Once detected it must be eliminated at all costs. Three typical methods of doing this exist. If the effect of the systematic error is quite determinate a *correction* is made for it. The records of all celebrated pieces of experimental work will be found to abound in such small correcting terms.

If the cause of the systematic error is known or suspected but the determination of its effect is impossible or inconvenient, an attempt is made to eliminate the error by making the observations *symmetrical with respect to the particular condition.* This is a still commoner practice in experimental work, and the best observer carries out his work with a maximum possible, but symmetrical, variation of those conditions not fixed by the nature of the experiment, even though there be no obvious danger of systematic error arising if the conditions are not so varied. An example of the first method is the correction for the buoyancy of the air in weighing an object whose density is much smaller than that of the weights. The second method is applied when the object and weights are interchanged on the pans so as to eliminate the systematic error arising from a possible inequality in the lengths of the balance arms.

The third possibility is the *method of substitution*, which is in many ways the most satisfactory. Before or after making the experiment to determine the unknown magnitude a known magnitude, like the former in as many respects as possible, is substituted for it, and the difference between its observed and its known value gives at once the correction, or an easy clue to its calculation, for application to the experiment on the unknown.

For instance, knowing the length of the seconds pendulum at sea-level in latitude 45°, we might wish to determine the length of the simple pendulum equivalent to a given compound pendulum by determining its time of swing. The particular station being neither at sea-level nor at 45°, the correction would best be made by substituting for the unknown pendulum another which was known to have a certain time of swing at sea-level and 45°. If the two times of swing were very nearly the same (*i.e.* those of the two pendulums) the correction could be applied directly as the discrepancy between the actual period of the standard pendulum and its period under standard conditions. Otherwise it would be given by a simple calculation.

A practical observer may himself be the cause of systematic error—the so-called personal equation. This is especially the case in experiments involving the measurement of time.

Sufficient has now been said to make it clear that the object of experimental precautions is to reduce all the errors to the accidental class. Not only the observer, but the instrument designer, should have this continually in view. The material of the succeeding pages applies, strictly speaking, only to those observations which contain nothing but accidental errors.

Much of what has been written in the present section belongs rather to a discussion on practical measurement than to one on the combination of observations. Yet the importance of a right attitude on the part of the worker to systematic error is so great that it has been thought fit to include a full statement on this part of the subject.

§ (3) PROBABILITY AND THE LAW OF ERRORS.—The discussion of probability is scarcely within the scope of the present article. It is a difficult thing to define, and the usual definition, as the numbers of ways in which an event can give rise to a certain result divided by the total number of ways in which it can happen at all, leaves the mind still seeking an antecedent definition of probability rather than an *a posteriori* one. The difficulty can be dismissed by a compromise between the innate "expectation" view of probability and the numerical definition. The two are not contradictory, but both are essential. It is natural that a search should have been made for some general mathematical formula to express the distribution of chance or "haphazard" quantities, such as accidental errors, about some value on which they appear to concentrate (zero in the case of accidental errors). There is in reality no such thing as a chance happening in nature,[1] and we can only define accidental errors as those resulting from causes too many in number and too complex in their laws to be known.

It is therefore to be expected that the desired distribution law will apply to many other things besides accidental errors. The problem and its offshoots are associated with the names of some of the greatest mathematicians of the past, such as Gauss, Laplace, Bessel, and Airy, and the law is generally known as *Gauss's Law of Errors*. It states that the probability of an accidental error lying between $x - \frac{1}{2}\delta x$ and $x + \frac{1}{2}\delta x$ is

$$\frac{h}{\sqrt{\pi}} e^{-h^2x^2}\delta x,$$

or, writing $h = 1/c$,

$$\frac{1}{c\sqrt{\pi}} e^{-\frac{x^2}{c^2}}\delta x.$$

The latter formula has perhaps a slightly more immediate significance. With the usual definition of probability the formula means that if a large number N of observations be taken a number equal to N times the above expression will have errors lying between $x - \frac{1}{2}\delta x$ and $x + \frac{1}{2}\delta x$. This will approximate more and more closely to the truth as N approaches infinity and will, for obvious reasons, be quite unreliable for very small numbers of observations. For the purposes of testing Gauss's Law of Errors by the distribution of actual sets of errors the infinitesimal width between the limits $x - \frac{1}{2}\delta x$ and $x + \frac{1}{2}\delta x$ must be increased to the practical finite limits $x - \frac{1}{2}\Delta x$ and $x + \frac{1}{2}\Delta x$. Only in this way will a reasonably smooth curve be obtained. As an example of such a curve the distribution of 174 residuals in a certain case is shown in *Fig.* 1. The residuals are the differences between observed and calculated monthly mean sea levels at three tidal observatories in the British Isles, the calculated values resting on a fairly simple formula resulting from an analysis of the records themselves. These residuals constitute one of Nature's "haphazard" groups of quantities, though they are not that particular variety, involving the human element, which we call accidental errors of observation. In this case it is the observed quantities themselves which follow a large number of complex laws, and the errors of observation were probably *nil* in comparison with the fluctuations observed. The generality of the distribution law is thus exhibited. The residuals were divided into groups having a range of 0·05 ft., so that the central group contains residuals between $-0{\cdot}025$ ft. and $+0{\cdot}025$ ft., that next on the plus side those between $+0{\cdot}025$ ft. and $+0{\cdot}075$ ft., and so on. The ordinates are the numbers of residuals occurring in each group, and the points are joined successively by straight lines.

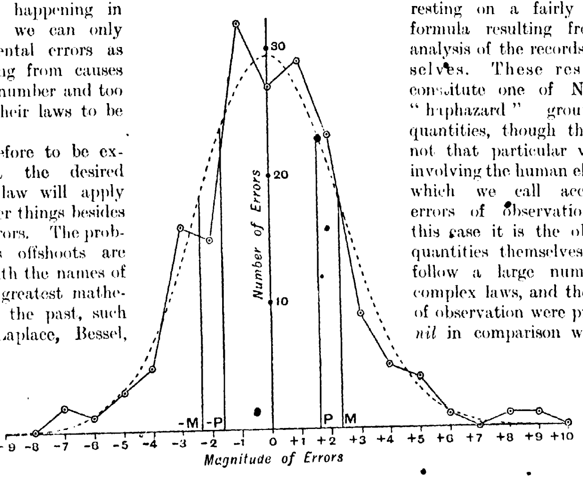


FIG. 1.

Since these distribution curves generally deal with errors, we shall speak of these instead of residuals in discussing the curves in the sequel.

What is the best Gauss's distribution curve which can be drawn through these points? An easy method of finding the constant h is as follows: Calling y the observed number of errors lying in the given group, and making the group-width equal to unity (1 unit $= 0{\cdot}05$ ft.), the group centres are at abscissae $-x$, $-x+1$, . . . -2, -1, 0, 1, 2 . . . $x-1$, x . . . The total number of errors is 174, so that the distribution formula is

---

[1] Cf. D. Brunt, *The Combination of Observations*, Camb. Univ. Press, 1917, p. 3.

$$y = \frac{174h}{\sqrt{\pi}} e^{-h^2x^2}$$

or

$$\log_e y = \log_e \frac{174h}{\sqrt{\pi}} - h^2x^2.$$

Thus if $\log_e y$ be plotted against x^2, a straight line with a slope of $\tan^{-1} - h^2$ should be obtained. The value of h so obtained may be called h_1. From the ordinate where the line cuts the axis of $\log_e y$ is obtained another value h_2, equal to $y_0\sqrt{\pi}/174$. The mean of h_1 and h_2 may be taken as the best value of h.

Before taking $\log_e y$ the + errors are added to the − errors belonging to the group of the same magnitude. This is exhibited graphically in *Fig.* 2, which shows the left-hand half of *Fig.* 1 folded over on to the right. A number of narrow closed areas is formed, and those denoting that the + errors have been more numerous than the − errors are filled in with vertical lines. Where the − errors are more numerous, horizontal lines fill in the spaces. The fundamental property of accidental errors, namely, the equal probability of a + and a − error of a given magnitude, is well illustrated. The dotted line joins points denoting the mean numbers of errors in each group irrespective of sign. In plotting $\log_e y$ against x^2 it is useless to retain groups in which y is expected to be 1 or less.

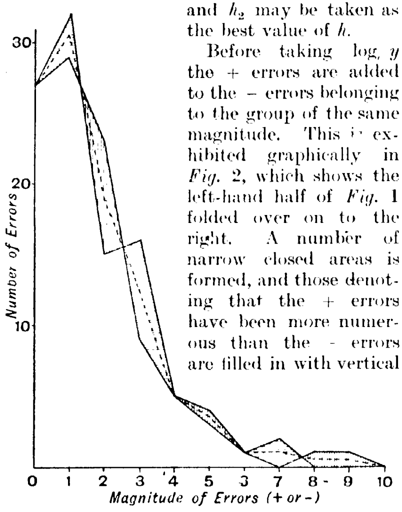


Fig. 2.

From *Fig.* 2 the values of x^2 and y, and hence $\log_e y$ are

$x^2 \longrightarrow$	0	1	4	9	16	25,
$y \longrightarrow$	27	30·5	19	12·5	5	3·5,
$\log_e y \longrightarrow$	3·30	3·32	2·94	2·53	1·61	1·25.

Fig. 3 is the graph of $\log_e y$ against x^2, and the relationship is seen to be linear. The points lying towards the right are expected to depart from their theoretical position more than the others because the addition or subtraction of even one error in these outlying groups means a large percentage change to so small a number of errors.

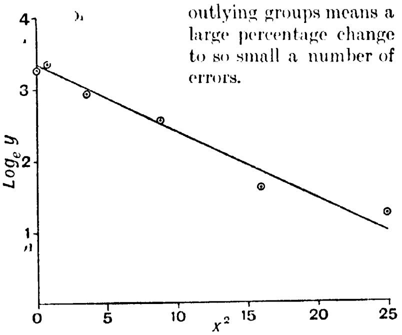


Fig. 3.

The slope of the straight line is

$$-\frac{3\cdot35 - 0\cdot99}{25} = -0\cdot0944.$$

Therefore $h_1 = \sqrt{0\cdot0944} = 0\cdot3072.$

From the zero end of the line

$$h_2 = \frac{e^{3\cdot35}\sqrt{\pi}}{174} = 0\cdot2912,$$

$$h = \frac{h_1 + h_2}{2} = 0\cdot2992.$$

A sufficiently good approximation is $h = 0\cdot3$.

The best distribution curve is therefore represented by

$$y = 174\frac{0\cdot3}{\sqrt{\pi}} e^{-(0\cdot3)^2x^2}$$

$$= 29\cdot44e^{-(0\cdot3x)^2}.$$

This curve has been superposed as a dotted line on the actual distribution graph of *Fig.* 1. The agreement is seen to be satisfactory.

The common experience that Gauss's Error Law does in practice give a close approximation to the actual distribution of errors is the justification for using it as the pedestal on which to base an extensive theory and practice of the adjustment of precise observations. For the many ingenious ways in which it has been deduced theoretically the reader should consult one of the many works dealing with the theory of errors.[1]

§ (4) DEDUCTIONS FROM THE LAW OF ERRORS.—The law is so fundamental in the theory of errors that the following conclusions or propositions to be deduced from it should be carefully noted as forming a foundation for all subsequent formulae.

---

[1] Most of the text-books mentioned in the References at the end of the article give one or more proofs. For an article devoted specially to this subject see Glaisher, *Memoirs R.A.S.* vol. xxxix.

The first 5 may be stated on inspection, while the remainder depend upon mathematical analysis. It is convenient to use the form $\frac{N}{c\sqrt{\pi}} e^{-x^2/c^2} \partial x$.

(i.) The probability of a large error is smaller than that of a small error.

(ii.) The most probable error of all is zero.

(iii.) The probability of a given + error is equal to that of a − error of the same magnitude.

(iv.) The formula contains only one parameter c. The law of errors is therefore the same for precise observations as it is for rough observations, and the parameter, which is called the *modulus*, serves to define the roughness of the observations or, in other words, the scale of the errors.[1]

(v.) From the well-known theorem that the probability of several events all happening is the product of their individual probabilities we deduce a probability proportional to

$$e^{-\left(\frac{x_1}{c_1}\right)^2 - \left(\frac{x_2}{c_2}\right)^2 - \dots}$$

for the simultaneous occurrence of a set of errors x_1, x_2, etc. Hence the most probable result in any piece of work is that which makes $(x_1/c_1)^2 + (x_2/c_2)^2 + \dots$ a minimum. This is the *Principle of Least Squares*, and is so much to the fore in the theory of combining observations that is common to find text-books with the title of "least squares."

If the errors x_1, x_2, etc., all belong to observations of the same supposed precision the condition reduces to $x_1^2 + x_2^2 + \dots$ a minimum. The quantities x_1/c_1, x_2/c_2, etc., are what we have called in § (1) "degrees of badness."

(vi.) There is a point on each side of the curve such that the probability of obtaining an error greater than the one denoted by the point is equal to the probability of obtaining one less than it, and these two points are obviously at equal distances from the ordinate denoting zero error. This error is called the *probable error* and is shown by tables of the integral of $e^{-x^2/c^2}\partial x$ to be equal to $0{\cdot}4769c$. A name such as "characteristic error" would really be more suitable, since the most probable accidental error is in all cases zero. It will be referred to as the "p.e." and is the quantity preceded by the sign ± which so often figures in the statement of numerical results.

(vii.) The *error of mean square*,[2] which is $\sqrt{\overline{x^2}}$, *i.e.* $\sqrt{[x^2]/n}$, can be shown to be equal to $c\sqrt{\tfrac{1}{2}}$ or $0{\cdot}7071c$.

It is generally written as μ.

(viii.) From (vi.) and (vii.) we obtain the important result

$$\text{p.e.} = \frac{0{\cdot}4769}{0{\cdot}7071}\mu = 0{\cdot}6745\sqrt{\frac{[x^2]}{n}}.$$

(ix.) The p.e. of any quantity A + B + C + . . . formed by the addition of quantities whose individual p.e.'s are a, b, c . . . is $\sqrt{a^2 + b^2 + c^2 + \dots}$

If $a = b = c = \dots = \epsilon$ the p.e. becomes $\epsilon\sqrt{n}$. But the p.e. of nA is na, since the same actual error enters into each of the n A's.

Therefore a further generalisation is

$$\text{p.e. of } pA + qB + rC + \dots = \sqrt{p^2a^2 + q^2b^2 + r^2c^2 + \dots},$$

and for the weighted mean

$$\frac{pA + qB + rC + \dots}{p + q + r + \dots}$$

the p.e. is $\frac{1}{p + q + r + \dots}$ times this last p.e.

Hence for the simple arithmetic mean, where $p = q = \text{etc.} = 1$ and $a = b = \text{etc.} = \epsilon$ the p.e. is $\epsilon/\sqrt{n}$.

In applying these simple rules it must be remembered that the observations giving rise to the quantity B must be *entirely independent* of those on which A or C rest.[3] Where this condition is not fulfilled and the fact is overlooked, too optimistic a view of the result is obtained, *i.e.* the p.e. is always underestimated.

It is to be noted that the methods of calculating p.e. given in (ix.) above are all *a priori* methods. That is to say, a p.e. of observation is assumed, and a final p.e. calculated. It is often necessary to judge the p.e. of observation by the *results*, that is, to use an *a posteriori* method. If the errors x could be certainly known, proposition (viii.) above would give the p.e. of a single observation. But it is necessary to make allowance for the fact that the best value of the unknown obtainable results from an adjustment of the x's themselves and is not necessarily the true value. $0{\cdot}6745\sqrt{[x^2]/n}$ would therefore always underestimate the p.e. It can be shown without difficulty that where the true value of the unknown is not available it is correct to write[4] $\text{p.e.} = 0{\cdot}6745\sqrt{[x^2]/(n-1)}$. We may also anticipate matters a little by noting a further proposition, namely, that if q unknown quantities are indirectly determined by means of n independent observations having each an actual error x and p.e. x_0, the p.e. of an observation having unit weight is [1] $0·6745\sqrt{[x^2/x_0^2]/(n-q)}$. Instead of the symbols x^2/x_0^2 the notation pv^2, *i.e.* wt. × (residual)^2, will be found in many text-books, but the former notation is retained here as being a little more expressive.

---

[1] The significance of the parameter c may be remembered by noting that it stands for "clumsiness" or "coarseness" in the observations.

[2] Unfortunately sometimes referred to as the "mean error," thus confusing it with the arithmetic mean error, irrespective of sign, which is equal to $c/\sqrt{\pi} = 0{\cdot}5642c$.

[3] A very good discussion on so-called "entangled measures" will be found in Airy, *Theory of Errors of Observation*, Macmillan, 1861. The book is now out of print, but is to be found in most scientific libraries.

[4] See, for instance, Airy, *Errors of Observation*, pp. 44-47

Returning to *Fig.* 1, proposition (vi.) above shows that the

$$\text{p.e.}=\frac{0·4769}{0·3}\text{ units }\left(\text{since } c=\frac{1}{h}=\frac{1}{·3}\right)$$
$$=1·590\text{ units.}$$

Error of mean sq.

$$=0·7071\,c$$
$$=2·357\text{ units.}$$

At the points $x=\pm1·590$ (P and $-$P) and $x=\pm2·357$ (M and $-$M) ordinates have been drawn. The ordinate at P divides the right-hand half of the area bounded by the curve and the axis of abscissae into two equal parts (cf. definition of probability, § (3), and definition of p.e. (§ (4) (vi.)), and that at $-$P does the same with the area devoted to negative errors. The actual sum of the squares of the 174 residuals is 1103·4 when expressed in units of 0·05 ft. Hence from the formula derived above

$$\text{p.e.}=0·6745\sqrt{\frac{1103·4}{173}}$$
$$=1·704\text{ units as compared with}$$
$$1·590\text{ from the ideal curve.}$$

It is to be expected that the sum of the squares taken from the irregular curve should be greater than that taken from the smooth curve.

It was noted earlier that Gauss's distribution law gives as a criterion for the best result the least squares condition. It now remains to show how this condition is to be satisfied in practice.

§ (5) THE COMBINATION OF OBSERVATIONS. (i.) *Direct Measurement of a Single Quantity.*—The simplest set of observations is that in which a single quantity is determined by direct measurement several times. Such for instance is the measurement of a length by direct comparison with a standard, or the determination of the altitude of the pole-star for latitude.

The quantity measured is constant or only varies through small amounts which may or may not be calculable. If they are calculable the method of corrections (§ (2)) is applied. For instance, corrections for small fluctuations of temperature might be made to the observed lengths in the first example above, while the small circle described about the celestial pole by the pole-star would be allowed for in the second.

Suppose that a constant quantity, whose true value X is sought, is measured directly by n observations all supposed to be equally good, and that the results are $X_1, X_2, \ldots X_n$. What is to be taken as the best value of X? The arithmetic mean $(X_1+X_2+\ldots+X_n)/n$ is the value which has at all times been taken instinctively as the best. Yet in the absence of any guide it would be conceivable to light upon such formulae as

$$\left(\frac{X_1{}^m+X_2{}^m+\ldots+X_n{}^m}{n}\right)^{\frac{1}{m}}\text{ or }(X_1X_2\ldots X_n)^{\frac{1}{n}}.$$

The principle of least squares gives a simple answer to the question. Each observed value of X, such as X_1, is burdened with an actual but unknown error $x_1=X_1-X$ (see limitations as to the "true" value of X in § (1)). We have to choose a value of X such that any other value for it will make $x_1{}^2+x_2{}^2+\ldots+x_n{}^2$ greater than the one chosen does.

If we took several values of X and found for each one the value of $[x^2]$ *i.e.* of the expression $x_1{}^2+x_2{}^2+\ldots+x_n{}^2$, and plotted the result we should obtain a curve something like *Fig.* 4, showing a minimum value of $[x^2]$ at O for a certain value of X_1, say X_0. The latter is the required value.

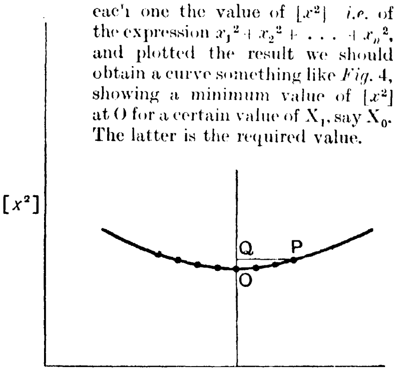


FIG 4.

Comparing the value of $[x^2]$ for a point P not far from O with that at O we find that it exceeds the latter by a very small amount OQ. Thus there are in the neighbourhood of X_0 a number of solutions which are very nearly as good as X_0, and the criterion for X_0 itself is that the alteration of X, through a very small but appreciable amount makes no appreciable difference at all to $[x^2]$. Reverting to the unknown symbol X for X_0, let ΔX be such a small alteration to X and let the new errors be called $y_1, y_2 \ldots y_n$ instead of $x_1, x_2, \ldots x_n$. Then

$$[x^2]=(X_1-X)^2+(X_2-X)^2+\ldots+(X_n-X)^2,$$
$$[y^2]=(X_1-X-\Delta X)^2+(X_2-X-\Delta X)^2+\ldots+(X_n-X-\Delta X)^2.$$

When ΔX is made a small enough quantity, we can, in view of the above statements, write $[x^2]=[y^2]$, and since $(\Delta X)^2$ may be considered negligible in comparison with $\Delta X(X_1 - X)$

---

[1] See Merriman, *Method of Least Squares*, John Wiley & Sons, 1915, 8th ed. p. 80 (London: Chapman & Hall).

$$0 = 2\Delta X(X_1 - X) + 2\Delta X(X_2 - X) + \ldots + 2\Delta X(X_n - X);$$

or, since ΔX does not itself become zero,

$$(X_1 - X) + (X_2 - X) + \ldots + (X_n - X) = 0$$

and $$X = \frac{X_1 + X_2 + \ldots + X_n}{n}.$$

The arithmetic mean is therefore not only indicated by common sense to be the most probable value, but is proved to be so by the principle of least squares. It is justifiable to reverse the order and to say that the principle of least squares, and with it the law of errors, is founded upon the rule of the arithmetic mean for the combination of direct observations on a single quantity. This was indeed the sequence of ideas in the mind of Gauss, whose name is usually associated with the law of errors.

(ii.) *Weighted Means.*—Suppose that the mean of the first four observations given by $X_m = (X_1 + X_2 + X_3 + X_4)/4$ were available, but that the four individual observations were not. It is obvious in the first place that the same result follows if we put

$$X = \frac{4X_m + X_5 + X_6 + \ldots + X_n}{4 + (n - 4)},$$

and in the second place that this is the correct thing to do. This introduces the idea of *weighted* observations. The value X_m, being the mean of 4 observations, is considered to have weight 4. Where the observations or partial values have weights w_1, w_2, etc., the weighted mean is

$$\frac{w_1X_1 + w_2X_2 + \ldots + w_nX_n}{w_1 + w_2 + \ldots + w_n}.$$

The assigning of weights to the quantities X_1, X_2, etc., is often a matter of personal judgment, especially where the quantities are actual observations. In other cases the weights are derived from the known p.e.'s of the quantities X_1, X_2, etc., for there must be some relationship such that a quantity with large p.e. is assigned a small weight w and *vice versa*. From the example given above, the first term w_1X_1 is evidently equivalent to w_1 hypothetical observations all of the same weight. From proposition (ix.), § (4), the p.e. of the mean of these w_1 observations is $\epsilon/\sqrt{w_1}$, where ϵ is the p.e. of any one of them, *i.e.*

$$w_1 = \frac{\epsilon^2}{(\text{p.e.})^2}.$$

The weight to be assigned to any partial result is therefore proportional to the reciprocal of the square of its p.e.

Since the weights may all be multiplied by any constant factor without altering the value of $w_1X_1 + w_2X_2 + \ldots/w_1 + w_2 + \ldots$, weights are only relative numbers and we may drop the symbol ϵ. The quantity $1/(\text{p.e.})^2$ is then referred to as the *theoretical weight* of the result to which it refers. The theoretical weight W of the weighted mean is easily found as follows. From proposition (ix.), § (4),

(p.e. of weighted mean)^2

$$= \frac{w_1^2(\text{p.e. of } X_1)^2 + w_2^2(\text{p.e. of } X_2)^2 + \ldots}{(w_1 + w_2 \ldots +)^2}.$$

Since, for any particular X, $w(\text{p.e.})^2 = 1$, this becomes

$$\frac{w_1 + w_2 + \ldots}{(w_1 + w_2 + \ldots)^2}$$

$$= \frac{1}{(w_1 + w_2 + \ldots)}.$$

But (p.e. of weighted mean)^2 = 1/W, hence

$$W = w_1 + w_2 + \ldots$$

The theoretical weight of a properly weighted mean is therefore the sum of the weights of the partial results.

A caution is needed against a too rigid adherence to theoretical weights where these depend upon *apparent* p.e.'s.

As will be seen later, the p.e. determined from small numbers of residuals is a very unreliable quantity, and an arbitrary assignment of weights will be much better in such cases. The least squares condition, which is $(x_1/e_1)^2 + (x_2/e_2)^2 + \ldots$ a minimum (see proposition (v.), § (4)), is now seen to be equivalent to $w_1x_1^2 + w_2x_2^2 + \ldots$ a minimum. This summation is often given in text-books with the notation $p_1v_1^2 + p_2v_2^2 + \ldots$

(iii.) *Indirect Measurement of a Single Quantity.*—Examples are the measurement of electric current by the deflexion of the needle of a galvanometer, the determination of elasticity by the measurement of extensions and forces, and the determination of the acceleration of gravity by measurement of time of swing and length of a pendulum. It will be noticed that in the first example only a single quantity, namely an angle, is measured. Now the indirect measurement of one quantity by the direct measurement of another presupposes some experiment in which a connection has been established between the two different kinds of magnitude. Further, this initial experiment must have been founded on the definition of the unit in the desired physical quantity. Electric current is defined by the reciprocal force between a coil of wire carrying a current, and a magnetic pole; so that if we could determine exactly the configuration of the coils in the galvanometer, together with the distribution of current producing a magnetic field equivalent to that of the actual control field, such determination would constitute the preliminary experiment mentioned above. But this is impracticable, and the most usual solution of the difficulty for all kinds of such units is for a very carefully executed fundamental experiment to be entrusted to some worker or committee of workers. This experiment rests directly on the definition of the unit and has for its object the determination of the unit in terms of some quantity which is easily reproducible. In the case quoted it is the amount of silver deposited by unit current in one second. Even in the event of subsequent research improving on the fundamental experiment—a not uncommon occurrence—it is usual to allow the practical unit determined by the fundamental experiment to stand good, for as stated in § (1), it then serves only as a label. In the case of the galvanometer the quantity measured, namely, an angle, is still unrelated to the required unit, for the fundamental experiment only relates, say, 1 ampere to a mass of silver and a time. A secondary experiment on the particular galvanometer is necessary. In this, observed angles of deflexion are linked up with observed masses of silver deposited in observed times. This operation is called *standardisation* or *calibration*.[1]

It will now be seen that the application of elaborate rules for the combination of observations is quite useless unless the instrument used is well standardised. If the instrument is of good performance—*i.e.* if it can be relied upon always to give the same result for the same conditions, it is doubly to be desired that no errors should be allowed to creep in through bad standardisation.

In those cases in which the observation of two or more different kinds of quantity is necessary in order to determine a single unknown it will generally be found that the observed quantities combine together to form a result of the correct dimensions, so that no preliminary experiment is implicated. For instance, the formula $g=4\pi^2 l/T^2$ shows that if we measure the length of a simple pendulum and its time of swing in the proper units we can immediately obtain the acceleration of gravity in units belonging to the same system.

As an example of the combination of observations for the indirect determination of a single unknown, suppose that a given current was passed in series through four tangent galvanometers and that the deflexions were 10°, 30°, 45°, and 70°, indicating currents C_1, C_2, C_3, and C_4 for the actual current C. Required the best value to take for C.

If $\Delta\theta$ was the p.e. of an angular measurement in all cases, the corresponding p.e. in C. for any galvanometer was

$$\Delta C = k \sec^2 \theta\, \Delta\theta, \text{ since } C = k \tan \theta.$$

The factor $k \sec^2 \theta$ or $\Delta C/\Delta\theta$ is the sensitiveness of the indicated current to changes of θ. k is a constant for a given galvanometer, but is not the same for all. In order to obtain an expression for ΔC which contains no variables but θ and $\Delta\theta$, substitute $k = C \cot \theta$.

$$\Delta C = C \cot \theta \sec^2 \theta\, \Delta\theta$$

$$= \frac{2C}{\sin 2\theta}\Delta\theta.$$

The weight of each observation is to be proportional to $1/(\text{p.e.})^2$. Dropping the constants 2C and $\Delta\theta$, which are the same for all, the weights are $\sin^2 2\theta_1$, $\sin^2 2\theta_2$, etc., and

$$C = \frac{C_1 \sin^2 (2\times 10^\circ) + C_2 \sin^2 (2\times 30^\circ) + C_3 \sin^2 (2\times 45^\circ) + C_4 \sin^2 (2\times 70^\circ)}{\sin^2 (2\times 10^\circ) + \sin^2 (2\times 30^\circ) + \sin^2 (2\times 45^\circ) + \sin^2 (2\times 70^\circ)}$$

$$= 0\cdot 05C_1 + 0\cdot 33C_2 + 0\cdot 44C_3 + 0\cdot 18C_4.$$

The sensitiveness of the observed quantity to a change in the unknown quantity is very important, both in experiment and in the combination of observations. A precise instrument allows observations highly sensitive to the unknown quantity to be made, but unless the performance of the instrument improves side by side with its sensitiveness the benefit is illusory.

The further consideration of examples of the determination of a single unknown by the simultaneous observation of two or more variables must be deferred to a later section.

(iv.) *The Simultaneous Determination of Two or More Unknowns.*—It frequently happens that observations are taken of a variable which is a linear function of several quantities, such quantities being the unknowns which it is desired to determine. This case is treated here because it is important as the basis of the general method of reduction of observations. Suppose that an observed variable X can be represented by

$$X = PA + QB + RC.$$

A, B, and C are the unknowns to be determined, and P, Q, and R are either constants or other observed quantities *whose p.e.'s of observation are supposed to be negligible in their effect on*

---

[1] Calibration implies the standardisation of a whole series of indications of an instrument or standard.

the solution when compared with those of X.[1] An example is the determination of the coefficient of thermal expansion of a body, including terms in t^2 as well as t. The equation is

$$L = L_0 + \alpha t + \beta t^2,$$

so that A, B, and C are replaced by L_0, α, and β; P, Q, R by unity, t and t^2.

In the general case above n sets of simultaneous (or as nearly simultaneous as possible) observations of X, P, Q, R are made. If these observations were all errorless the following equations would all be rigidly true:

$$X_1 = P_1A + Q_1B + R_1C,$$
$$X_2 = P_2A + Q_2B + R_2C,$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$X_n = P_nA + Q_nB + R_nC.$$

Using symbols x_1, x_2, etc., for the unknown errors, we can subtract these from the observed quantities and so obtain the following true equations:

$$X_1 - x_1 = P_1A + Q_1B + R_1C,$$
$$X_2 - x_2 = P_2A + Q_2B + R_2C,$$
$$\text{etc., etc.}$$

The principle of least squares indicates as the best values of A, B, and C those which make $x_1^2 + x_2^2 + \ .\ .\ .$ *i.e.* $[x^2]$ a minimum. We can determine how $[x^2]$ varies when any one of the unknowns is made to go through a series of values, assuming the others to be constant. The curve of $[x^2]$ against A, for instance, might reproduce *Fig.* 4, and so long as B and C had not been assigned values very far from the true ones, the lowest point of the curve would denote the most probable value of A.

Applying the same criterion as before (§ (5) (i.)), it is necessary to assume a very small change ΔA in A and to equate the change in $[x^2]$ to zero.

From the last set of equations

$$x_1 = X_1 - (P_1A + Q_1B + R_1C) = F_1 \text{ say},$$
$$x_2 = X_2 - (P_2A + Q_2B + R_2C) = F_2 \text{ say},$$
$$\text{etc., etc.}$$

If we change A to $A + \Delta A$, F_1 becomes $F_1 - P_1\Delta A$, and a new set of errors results, the sum of whose squares is

$$(F_1 - P_1\Delta A)^2 + (F_2 - P_2\Delta A)^2 + \ .\ .\ .$$

Neglecting the squares of $P_1\Delta A$, etc., in comparison with $F_1P_1\Delta A$, etc., the difference between this sum and the original $F_1{}^2 + F_2{}^2 + \ .\ .\ .$ is

$$2F_1P_1\Delta A + 2F_2P_2\Delta A + \ .\ .\ .,$$

and this is to be equated to zero without putting $\Delta A = 0$, so that the condition for least squares as far as A is concerned is

$$P_1F_1 + P_2F_2 + \ .\ .\ .\ + P_nF_n = 0$$

or $(P_1{}^2 + P_2{}^2 + \ .\ .\ .)A + (P_1Q_1 + P_2Q_2 + \ .\ .\ .)B$

$$+ (P_1R_1 + P_2R_2 + \ .\ .\ .)C = P_1X_1 + P_2X_2 + \ .\ .$$

This is usually written

$$[PP]A + [PQ]B + [PR]C = [PX].$$

Applying the same criterion to B in precisely the same way, we obtain

$$[PQ]A + [QQ]B + [QR]C = [QX],$$

and from C

$$[PR]A + [QR]B + [RR]C = [RX].$$

The principle of least squares demands that all these conditions be fulfilled. The number of conditions being equal to the number of unknowns, the equations give, when solved, the most probable values for the latter. The equations are known as *normal equations*, and their derivation is seen to be according to the following rule: "Multiply each observational equation $P_1A + Q_1B + R_1C = X_1$ by the coefficient of A in the said equation. Add the resulting equations. Perform similar operations, using the coefficient of the next unknown B in each equation as multiplier in that equation. Continue till all the unknowns have been so treated, obtaining thereby a number of equations equal to the number of unknowns."

Before giving some methods for the solution of normal equations it is necessary to treat the case in which the observed quantity is not a linear function of the unknowns.

(v.) *Reduction to the Linear Form.*—As an example of a simple but non-linear relationship between the principal observed quantity (see (iv.) above) and the unknowns, take the case of the readings of a leading screw in a lathe or measuring-machine. These might be expressible by such a formula as

$$X = tA + B \sin 2\pi(t - C),$$

where A is the unknown pitch of the screw, B the unknown amplitude of a periodic error, C its unknown phase, and t the observed number of turns of the screw.

By observing a correct scale through a microscope moved by the leading screw, X could be obtained experimentally at a number of points.

---

[1] Note on the notation used in this article. In all general statements observed quantities whose errors are to be noticed are denoted by X_1 Y_1 Z_1, X_2 Y_2 Z_2, etc. Observed quantities whose errors are not to be noticed are denoted by P_1 Q_1 R_1, P_2 Q_2 R_2, etc. Unknown constants whose values are sought are called A, B, C, . . . Wherever possible, small portions to be added to or subtracted from any quantity, whether such portions be errors of the observed quantities or corrections to tentative values for the unknowns, are denoted by the corresponding small letters. Thus, errors of X_1, Y_1, Z_1, are x_1, y_1, z_1. Corrections to A, B, C are a, b, c.

The unknowns A, B, C cannot be determined directly by the method of § (5) (iv.), but if the observations are not extremely irregular, approximate values for them should be obtainable by inspection, especially if a graphical method be employed (§ (8)).

The position is now much the same as in the last section. There is a number of equations

$$X_1 = P_1A + B \sin 2\pi(P_1 - C),$$
$$X_2 = P_2A + B \sin 2\pi(P_2 - C),$$
$$\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots$$

which probably cannot *all* be satisfied by any values of A, B, and C. Unknown errors x_1, x_2, etc., are to be subtracted respectively from the X's in such a way as to bring the equations to truth and at the same time to make $[x^2]$ a minimum.

Let the expected values of the observed quantities X_1, X_2, etc., be calculated by inserting the tentative values of the unknowns, say A_0, B_0, and C_0 in the formula. Let X_1', X_2', etc., be these calculated values. Suppose *a*, *b*, and *c* to be the small additions to the approximate values of the unknowns necessary to convert them into the true ones, *i.e.*

$$A = A_0 + a, \quad B = B_0 + b, \quad C = C_0 + c.$$

It is a well-known theorem in the differential calculus that if *a*, *b*, and *c* are sufficiently small quantities we can write down the change in X resulting from their introduction *as a linear function in a, b, and c.*

The change in X_1' resulting from the small increment "*a*" to A_0 is a_1a, where a_1 is the sensitiveness of X to A for that particular collection of circumstances. This sensitiveness is, of course, the partial differential, coefficient of X with respect to A. The total change in X_1' is $a_1a + b_1b + c_1c$. Therefore the discrepancy $X_1 - X_1'$ can be made to disappear by any additions *a*, *b*, and *c* to A_0, B_0, and C_0 which make

$$a_1a + b_1b + c_1c = X_1 - X_1'.$$

Similarly the discrepancy $X_2 - X_2'$ disappears if

$$a_2a + b_2b + c_2c = X_2 - X_2'.$$

In the absence of experimental error all of these *n* linear equations in *a*, *b*, *c* could be satisfied, but the existence of errors x_1, x_2, etc., in X_1, X_2, etc. (X_1', etc., have no errors, being numerical terms calculated on an arbitrary basis) gives a precisely similar problem to that in which X is linear from the start in A, B, and C.

Normal equations are therefore formed exactly as before and the evaluated quantities *a*, *b*, and *c* are added to A_0, B_0, and C_0.

In the above example of the leading screw the sensitiveness of X to the several unknowns, *i.e.* its differential coefficient with respect to each, is

$$a_1 = \frac{\partial X_1}{\partial A} = t_1,$$

$$b_1 = \frac{\partial X_1}{\partial B} = \sin 2\pi(t_1 - C) = S\theta_1, \text{ say, for convenience,}$$

$$c_1 = \frac{\partial X_1}{\partial C} = -2\pi B_0 \cos 2\pi(t_1 - C) = C\theta_1, \text{ say.}$$

Any other factors $S\theta_n$ and $C\theta_n$ are derived by replacing t_1 by t_n. The observational equations are therefore

a.	*b.*	*c.*	
t_1	$S\theta_1$	$C\theta_1$	$X_1 - X_1'$
t_2	$S\theta_2$	$C\theta_2$	$X_2 - X_2'$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t_n	$S\theta_n$	$C\theta_n$	$X_n - X_n'$

and the normal equations are

a.	*b.*	*c.*	
$[t^2]$	$[t.S\theta]$	$[t.C\theta]$	$[t(X - X')]$
..	$[S^2\theta]$	$[S\theta.C\theta]$	$[S\theta.(X - X')]$
..	..	$[C^2\theta]$	$[C\theta.(X - X')]$

The square brackets denote as usual summations, and the terms to the left of the leading diagonal are omitted, it being understood that they can be filled in from symmetry by inspection.

(vi.) *Successive Approximation.*—If the quantities *a*, *b*, *c* to be added to A_0, B_0, C_0 are not very small, the relation between any function $F(A_0, B_0, C_0)$ and $F(A_0 + a, B_0 + b, C_0 + c)$ will in general be no longer strictly

$$F(A_0 + a, B_0 + b, C_0 + c)$$
$$= F(A_0, B_0, C_0) + a_1a + b_1b + c_1c.$$

The reason is that a differential coefficient has to be defined in terms of infinitesimal changes of the variables. Yet the procedure will in nearly all cases give a result nearer to the true one than the tentative one first taken. That is to say, it is a step in the right direction.

In *Fig.* 5 is a geometrical analogy. The tangent AB to the curve is very nearly coincident with the curve for a short distance AA′. When too long a vector AB is drawn, B deviates appreciably from the curve AC, but within very wide limits for the length of AB, the point B is nearer to C than A is.

Although the solution of the first set of normal equations as above may not give the best values for A, B, and C, those obtained may once more be considered as tentative values and the original process may be repeated, giving further small additions a', b', c' to A_0+a, B_0+b, C_0+c.

In this way we may approach as near as we please to the theoretical "least squares" values for the unknowns, though nothing is to be gained from a slavish pursuit of the *exact* least square solution (see § (10)).

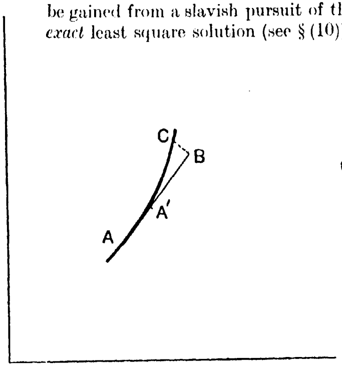


FIG. 5.

There are some cases in which the observational equations can be reduced to the linear form without the aid of the differential calculus, but it must be remembered that the normal equations (if they may be so called) derived from such reduced equations will not in general give the least square solution. For instance, the time of oscillation of a simple pendulum is

$$T=2\pi\sqrt{\frac{l}{g}}=2\pi l^{\frac{1}{2}}g^{-\frac{1}{2}}$$

or $$\log T=\log 2\pi+\tfrac{1}{2}\log l-\tfrac{1}{2}\log g.$$

Assuming that errors of observation of T influence the resulting value of g much more than those of l, the formation of normal equations from the above logarithmic equation gives a minum [error²] in log T, *i.e.* in $[(\Delta T/T)^2]$ instead of $[(\Delta T)^2]$.

If T did not assume a number of widely different values in the course of the experiment, the above solution would be quite justified, and would be used on account of its simplicity.

(vii.) *Partition of Errors among Several Kinds of Observed Quantity.*—Using the same notation as before (see footnote, p. 653), A, B, C . . . are unknown constant quantities whose values are to be determined. X, Y, Z are observed quantities.

In § (4) (v.) the condition for the most probable result was found to be that the sum of $(x/c)^2$, *i.e.* of (error/modulus)², must be a minimum. The modulus being directly proportional to the p.e. (§ (4), proposition (vi.)), we may replace it by the latter.

Hence if x, y, z be the actual errors of X, Y, Z, and x_0, y_0, z_0 the p.e.'s, the required condition is that $[(x/x_0)^2+(y/y_0)^2+(z/z_0)^2]$ is a minimum, the square bracket being understood to mean that the summation is extended to all sets of errors x_1, y_1, z_1, x_2, y_2, z_2, etc. If ϵ denotes the actual error of *any* of the observed quantities and ϵ_0 its p.e., the above condition is the same as $[(\epsilon/\epsilon_0)^2]$ a minimum.

The relationship which holds between the unknowns and the observed quantities may be written

$$F(A, B, C \;.\;.\;.\; X, Y, Z \;.\;.\;.)=0.$$

As before, let tentative values A_0, B_0, C_0 . . . for the unknowns be arrived at in some way; *e.g.* graphically or by taking only as many observations as there are unknowns and solving uniquely if this is mathematically possible.

In this way a number of results are calculated as follows:

$$F_1=F(A_0, B_0, C_0 \;.\;.\;.\; X_1, Y_1, Z_1 \;.\;.\;.),$$
$$F_2=F(A_0, B_0, C_0 \;.\;.\;.\; X_2, Y_2, Z_2 \;.\;.\;.),$$

. . . .

. . . .

These will differ slightly from zero both on account of the small quantities a, b, c which must be added to A_0, B_0, C_0 . . . in order to give the true values and on account of the errors x_1, y_1, z_1, x_2, y_2, z_2 which must be subtracted from X_1, Y_1, Z_1 . . . X_2, Y_2, Z_2 . . . in order to reduce them to ideally observed quantities.

Each of these small quantities produces its own change in F, and the total change is the sum of them all.

In any given case, say that of F_1, we can by suitably selecting a, b, c . . . and x_1, y_1, z_1 . . . reduce the expression to its correct value zero Thus,

$$\begin{aligned}0&=F(A_0+a,\ B_0+b,\ C_0+c \;.\;.\;.\\ &\qquad\qquad X_1-x_1,\ Y_1-y_1,\ Z_1-z_1 \;.\;.\;.)\\ &=F(A_0, B_0, C_0 \;.\;.\;.\; X_1, Y_1, Z_1 \;.\;.\;.)\\ &\qquad\qquad +a_1a+b_1b+c_1c-\alpha_1x_1-\beta_1y_1-\gamma_1z_1\\ &=F_1+a_1a+b_1b+c_1c-\alpha_1x_1-\beta_1y_1-\gamma_1z_1.\end{aligned}$$

The quantity a_1 is the sensitiveness of F_1 to A

or $$\frac{\partial F}{\partial A}\left\{\begin{matrix}X=X_1\\ Y=Y_1\\ Z=Z_1\end{matrix}\right\};$$

similarly for b_1 and c_1.

$$\alpha_1 \text{ is } \frac{\partial F}{\partial X}\left\{\begin{matrix}X=X_1\\ Y=Y_1\\ Z=Z_1\end{matrix}\right\}$$

or the sensitiveness of F_1 to X. β_1 and γ_1 have corresponding meanings.

The term $\alpha_1x_1+\beta_1y_1+\gamma_1z_1$ ($=f_1$, say) is the total error produced in F_1 by the errors of observation of X_1, Y_1, and Z_1. Its "probable" value is $\phi_1=(\alpha_1{}^2x_0{}^2+\beta_1{}^2y_0{}^2+\gamma_1{}^2z_0{}^2)^{\frac{1}{2}}$ (see § (4), proposition (ix.)), and it is entirely an "accidental" error. Hence we may regard F_1 as an observed quantity whose actual error is f_1 and whose p.e. is ϕ_1. In other words, we shall obtain a least squares solution by making $[(f_1/\phi_1)^2]$ a minimum.[1]

The observational equation derived from the first set of observations is

$$a_1a+b_1b+c_1c=-F_1 \text{ with p.e. } \phi_1.$$

That from the second set is

$$a_2a+b_2b+c_2c=-F_2 \text{ with p.e. } \phi_2,$$

and so on.

Before forming normal equations each of the observational equations must be divided by its p.e. in order to reduce all the equations to the same weight.

The normal equations resulting are

$$\begin{array}{cccc} \left[\frac{a}{\phi}\frac{a}{\phi}\right] & \left[\frac{a}{\phi}\frac{b}{\phi}\right] & \left[\frac{a}{\phi}\frac{c}{\phi}\right] & \left[-\frac{a}{\phi}\frac{F}{\phi}\right] \\ & \left[\frac{b}{\phi}\frac{b}{\phi}\right] & \left[\frac{b}{\phi}\frac{c}{\phi}\right] & \left[-\frac{b}{\phi}\frac{F}{\phi}\right] \\ & & \left[\frac{c}{\phi}\frac{c}{\phi}\right] & \left[-\frac{c}{\phi}\frac{F}{\phi}\right]. \end{array}$$

The solution gives the best values of a, b, and c (not to be confused with the coefficients in the above equations, all of which have actually suffixes) and enables f_1, f_2, etc., to be calculated. As a matter of interest each of these composite errors $f_1=\alpha_1x_1+\beta_1y_1+\gamma_1z_1$ may be analysed into its components so as to make $(x_1/x_0)^2+(y_1/y_0)^2+(z_1/z_0)^2$ a minimum.

(viii.) *Example.*—From the equation for a gas $PV=RNT$, where $N=$ number of gramme-molecules of the gas, it is desired to determine the constant R. It might be thought that N could be determined as an unknown side by side with R, but a moment's reflection shows that it is impossible to separate two such unknowns occurring only as a product in the formula. N must therefore be independently determined.

For an approximate value of R, suppose it is known that 1 gm. molecule at normal temperature and pressure occupies approximately 22·4 litres; and that N has been made equal to 1/22·4 by taking exactly 1 litre of gas at N.T.P.

From the formula $PV=RT$, in which the units are for convenience millibars, cubic centimetres, and degrees absolute respectively, we then have

$$1000\times1000=R_0\times273$$

or

$$R_0=3663.$$

This is the approximate value to use in preparing the coefficients in the observational equations.

Suppose that during four observations an attempt was made to keep the volume at exactly 1000 c.c., while the remaining four were made at an attempted constant pressure of 1000 millibars. Let the observations be

Const. vol. 1000 c.c.

	(1).	(2).	(3).	(4).
T. (abs.) . .	294·1	314·3	327·6	340·3
P. (millibars) .	1079	1149	1197	1249

Const. press. 1000 millibars.

	(5).	(6).	(7).	(8).
T. (abs.) . .	292·1	310·7	331·8	343·1
V. (c.c.) . .	1072	1141	1213	1254

The relationship $F(R, P, V, T)=0$ is in this case

$$F=PV-RT=0.$$

We use as before small letters to denote small changes or errors in the quantities corresponding to the capital letters. Thus r denotes the small addition to R_0 required to give the best value $R=R_0+r$. Also

$$F_1=P_1V_1-R_0T_1,$$

$$r_1=\left(\frac{\partial F}{\partial R}\right)_1=-T_1,$$

$$\alpha_1=\left(\frac{\partial F}{\partial P}\right)_1=V_1,$$

$$\beta_1=\left(\frac{\partial F}{\partial V}\right)_1=P_1,$$

$$\gamma_1=\left(\frac{\partial F}{\partial T}\right)_1=-R_0.$$

The normal equation (one only, since there is only one unknown) is to be prepared in the form of symbols before any numerical work is done. A saving of time often results in this way through cancellation.

As in the general example we have the following true equations:

$$F_1+r_1r-\alpha_1p_1-\beta_1v_1-\gamma_1t_1=0,$$

$$F_2+r_2r-\alpha_2p_2-\beta_2v_2-\gamma_2t_2=0,$$

. . . .

. . . .

That is to say, we have the following observational equations attended with the actual and probable errors shown:

Observation Equation.	Unknown Actual Error $=f$.	Calculable Probable Error $=\phi$.
$F_1+r_1r=0$	$\alpha_1p_1+\beta_1v_1+\gamma_1t_1$	$(\alpha_1{}^2p_0{}^2+\beta_1{}^2v_0{}^2+\gamma_1{}^2t_0{}^2)^{\frac{1}{2}}$
$F_2+r_2r=0$	$\alpha_2p_2+\beta_2v_2+\gamma_2t_2$	$(\alpha_2{}^2p_0{}^2+\beta_2{}^2v_0{}^2+\gamma_2{}^2t_0{}^2)^{\frac{1}{2}}$
. . . .	. . . . .	
. . . .	. . . . .	

---

[1] See § (10).

where p_0, v_0, and t_0 are the assumed p.e.'s of P, V, and T respectively. Dividing each observational equation by its p.e. in order to reduce all to the same p.e. they become

$$\frac{r_1}{\phi_1}r = -\frac{F_1}{\phi_1},$$

$$\frac{r_2}{\phi_2}r = -\frac{F_2}{\phi_2},$$

etc., etc.

Multiplying each by the coefficient of r in the usual way the normal equation becomes

$$\left(\left(\frac{r_1}{\phi_1}\right)^2 + \left(\frac{r_2}{\phi_2}\right)^2 + \ . \ . \ . \right)r$$
$$= -\left(\frac{r_1F_1}{\phi_1^2} + \frac{r_2F_2}{\phi_2^2} + \ . \ . \ . \right)$$

or $$r = -\frac{\left[\left(\frac{r_n}{\phi_n}\right)^2 \frac{F_n}{r_n}\right]}{\left[\left(\frac{r_n}{\phi_n}\right)^2\right]}.$$

The solution for r is a weighted mean of all the values of F_n/r_n, the weight being proportional to the square of the sensitiveness of F_n to R and inversely proportional to the square of the p.e. of F_n, as was found to be correct in § (5) (ii.).

The calculation is best made by forming the quantities F_n/r_n and r_n/ϕ_n for each observation.

Suppose $p_0 = 1$ millibar, $v_0 = 1$ c.c., $t_0 = 1°$ abs. Then

$$\phi_n^2 = (V_n^2 + P_n^2 + R_0^2),$$

$$r_n^2 = T_n^2,$$

since $\alpha_1 = V_1$, $\beta_1 = P_1$, $\gamma_1 = R_0$, etc.,

or $$\left(\frac{r_n}{\phi_n}\right)^2 = \frac{T_n^2}{V_n^2 + P_n^2 + R_0^2}.$$

The schedule on the following page shows the preparation of the 8 values of $(r_n/\phi_n)^2$ and $(r_n/\phi_n)^2F_n/r_n$. The lower portion is a check, showing that the solution has really made $[(f_n/\phi_n)^2]$ less than $[(F_n/\phi_n)^2]$, in accordance with the least squares criterion. The difference between the two sums of squares, 31960 and 32059, is small because the tentative value R_0 happened to be very close to the best value obtainable from the observations.

The p.e. of the result is easily found from the expression denoting the said result,

i.e. $$r = -\frac{\left(\frac{r_1}{\phi_1^2}F_1 + \frac{r^2}{\phi_2^2}F_2 + \ . \ . \ . \right)}{\left(\frac{r_1^2}{\phi_1^2} + \frac{r_2^2}{\phi_2^2} + \ . \ . \ . \right)}.$$

The p.e. of F_1 is ϕ_1, that of F_2 is ϕ_2, and so on. Therefore

$$\text{p.e. of } r = \frac{\left\{\left(\frac{r_1}{\phi_1^2}\right)^2\phi_1^2 + \left(\frac{r^2}{\phi_2^2}\right)^2\phi_2^2 + \ . \ . \ . \right\}^{\frac{1}{2}}}{\left(\frac{r_1^2}{\phi_1^2} + \frac{r_2^2}{\phi_2^2} + \ . \ . \ . \right)}$$

$$= \frac{1}{\left(\frac{r_1^2}{\phi_1^2} + \frac{r_2^2}{\phi_2^2} + \ . \ . \ . \right)^{\frac{1}{2}}}$$

or $$\left[\left(\frac{r_n}{\phi_n}\right)^2\right]^{-\frac{1}{2}}.$$

From the schedule we find at once that this quantity is

$$\frac{1}{\sqrt{0{\cdot}05176}} = 4{\cdot}4.$$

With a p.e. of this amount it is useless to express the result to two places of decimals. It is therefore written

$$R = 3662{\cdot}4 \pm 4{\cdot}4.$$

If the assumed p.e.'s p_0, v_0, t_0, etc., had been about correct ϕ_1, ϕ_2, etc., would also have been correct, and f_n/ϕ_n would have averaged about 1. Since it is in all cases less than 1 the conclusion is that the p.e.'s of the observations have been overestimated. If the ϕ's have been made k times their correct value, the best estimate of k is

$$0{\cdot}67\sqrt{\frac{\left[\left(\frac{f_n}{\phi_n}\right)^2\right]}{8-1}} \quad \text{(see § (4) (ix.))}$$

$$= 0{\cdot}67\sqrt{\frac{3{\cdot}19}{7}}$$

$$= 0{\cdot}455.$$

If the coefficients α_1, β_1, γ_1, etc., varied relatively a good deal in the different sets of observations it would be possible to tell which of the three p.e.'s of observation—p_0, v_0, t_0—had been overestimated. It is sufficient to note here that the *a posteriori* p.e. of the result is only 0·45 of that deduced from the estimated p.e.'s of observation.

On this showing the result should be stated

$$R = 3662{\cdot}4 \pm 2{\cdot}0.$$

The above example is, needless to say, only a hypothetical one. The length of the computation would not be as great as that set out above, for the schedule is practically all that is necessary. Nor is it to be taken for granted that the application of the method of least squares to 8 observations only is justified in this case. The example merely illustrates the method of dealing with larger numbers of observations.

	$P_n = s_n$.	$V_n = a_n$.	$T_n = -r_n$.	$\frac{P_n V_n}{1000}$.	$\frac{R_0 T_n}{1000}$.	$F_n = P_n V_n - R_0 T_n$.	$\frac{F_n}{r_n} = -\frac{F_n}{T_n}$.	$\frac{V_n^2}{1000}$.	$\frac{P_n^2}{1000}$.	$\frac{R_0^2}{1000}$.	$\frac{V_n^2 + P_n^2 + R_0^2}{1000} = \frac{\phi_n^2}{1000}$.	$\frac{T_n^2}{10} = \frac{r_n^2}{10}$.	$100\left(\frac{r_n}{\phi_n}\right)^2$.	$-100\left(\frac{r_n}{\phi_n}\right)^2 \frac{F_n}{r_n}$.
1	1079	1000	294·0	1079	1076·92	+2080	−7·076	1000	1164	13418	15582	8644	0·555	+3·917
2	1149	1000	314·3	1149	1151·28	−2280	+7·254	1000	1318	,,	15736	9879	0·628	−4·553
3	1197	1000	327·6	1197	1200·00	−3000	+9·158	1000	1432	,,	15850	10730	0·677	−6·198
4	1249	1000	340·3	1249	1246·52	+2480	−7·288	1000	1560	,,	15978	11580	0·725	+5·281
5	1000	1072	292·1	1072	1069·96	+2040	−6·984	1149	1000	,,	15567	8532	0·548	+3·826
6	1000	1141	310·7	1141	1138·09	+2910	−9·365	1302	1000	,,	15720	9654	0·614	+5·750
7	1000	1213	331·8	1213	1215·38	−2380	+7·174	1471	1000	,,	15889	11010	0·693	−4·971
8	1000	1254	343·1	1254	1256·78	−2780	+8·102	1573	1000	,,	15991	11770	0·736	−5·964
													→ 5·176	→ −2·912

	$r_n r$.	$f_n = P_n V_n - RT_n$.	$\frac{f_n^2}{1000}$.	$10000\,\frac{f_n^2}{\phi_n^2}$.	$\frac{F_n^2}{1000}$.	$10000\,\frac{F_n^2}{\phi_n^2}$.
1	+165	+2225	5040	3235	4326	2777
2	+176	−2104	4427	2813	5198	3302
3	+183	−2817	7936	5006	9000	5676
4	+190	+2670	7129	4461	6150	3849
5	+163	+2203	4853	3117	4162	2673
6	+174	+3084	9510	6052	8468	5387
7	+186	−2194	4813	3028	5664	3563
8	+192	−2588	6697	4188	7728	4832
				→ 31900		→ 32059

$$r = -\frac{2{\cdot}912}{5{\cdot}176} = -\ 0{\cdot}56.$$

$$R = 3663 - 0{\cdot}56 = 3662{\cdot}44.$$

$$\text{p.e.} = \frac{1}{\left[\left(\frac{r_n}{\phi_n}\right)^2\right]^{\frac{1}{2}}} = \frac{1}{\sqrt{0{\cdot}05176}} = \quad 4{\cdot}4.$$

$$\underline{R = 3662{\cdot}4 \pm 4{\cdot}4.}$$

§ (6) THE COMBINATION OF CONDITIONED OBSERVATIONS.—It sometimes happens that groups of observations are furnished with a natural check before any computation is made. They have in fact to satisfy certain *conditions*, and any solution of the whole is bound to satisfy all these conditions. For instance, when all three angles of a triangle have been measured in surveying, the sum of the true angles must be equal to 180°. That is, in the notation used previously

$$X_1 + X_2 + X_3 - 180^\circ = x_1 + x_2 + x_3.$$

Again, in geodetic levelling, the sum of the height changes observed in a complete circuit of levelling, when corrected for changes of gravity, must be equal to zero.

These examples belong to the simple type

$$X_1 + X_2 + \ldots + X_n + M = 0,$$

where M is a constant.

The relationships, or so-called *equations of condition*, may be more complicated. They can be represented by

$$F_1(X_1, X_2 \ldots) = 0,$$
$$F_2(X_1, X_2 \ldots) = 0,$$
$$\text{etc.}$$

The problem then becomes that of finding such values of X_1, X_2, etc., that the sum $x_1^2 + x_2^2 + \ldots$ is smaller than that for any other set of values of X_1, X_2, etc., which fits into the pattern, as it were, imposed by the above equations of condition. It is to be noticed that the number of conditions must be smaller than the number of observations, for if these two numbers were equal X_1, X_2, etc., might be determinate from the conditions alone.

One method of solution is obvious. If there are q equations of conditions and n unknowns, the q equations can be made use of to eliminate q of the unknowns, *i.e.* to express them in terms of the remaining $n-q$ unknowns.

The latter are then quite independent and are free to take on any values consistent with the observations.

In the case of the triangle in which all three angles had been measured the observation equations would be written

$$A = X_1,$$
$$B = X_2,$$
$$180^\circ - A - B = X_3.$$

This straightforward method is, however, very tedious in cases where the number of unknowns entering into each conditional equation and the number of conditions themselves are large. The following *method of undetermined multipliers* or *correlatives* is more convenient.

The equations of condition

$$F(X_1, X_2, \ldots) = 0, \text{ etc.},$$

are reduced to the linear form by making use of x_1, x_2, etc., instead of X_1, X_2, etc., exactly as in § (5) (v.). Thus

$$f_1 = a_1x_1 + a_2x_2 + \ldots - m_1 = 0,$$
$$f_2 = \beta_1x_1 + \beta_2x_2 + \ldots - m_2 = 0,$$
$$\text{etc.}$$

Instead of being the errors of X_1, etc., x_1, etc., may be considered to have the reverse sign, *i.e.* to be the small corrections necessary to bring the observed quantities to their most probable values. Just as in the method described more fully in § (5) (v.) the small increments a, b, c, etc., were regarded as the unknowns, x_1, x_2, etc., may be so regarded here.

In order to obtain the most probable values of x_1, x_2 . . . it is necessary to equate to zero the differential coefficient with respect to each x, not of $[x^2]$, but of $[x^2] + 2k_1f_1 + 2k_2f_2 + \ldots$:[1] k_1, k_2, etc., are the undetermined multipliers or correlatives.

This process gives

$$2x_1 + 2k_1a_1 + 2k_2\beta_1 + \ldots = 0,$$
$$2x_2 + 2k_1a_2 + 2k_2\beta_2 + \ldots = 0,$$
$$\ldots\ldots\ldots$$
$$\ldots\ldots\ldots$$

Next substitute in the reduced conditional equations the values of x_1, x_2, etc., obtained from the last equations.

$$a_1(-k_1a_1 - k_2\beta_1 \ldots) + a_2(-k_1a_2 - k_2\beta_2 \ldots) + \ldots - m_1 = 0,$$
$$\beta_1(-k_1a_1 - k_2\beta_1 - \ldots) + \beta_2(-k_1a_2 - k_2\beta_2 - \ldots) + \ldots - m_2 = 0,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$

i.e.

$$k_1(a_1^2 + a_2^2 + \ldots) + k_2(a_1\beta_1 + a_2\beta_2 + \ldots) + \ldots = -m_1,$$
$$k_1(a_1\beta_1 + a_2\beta_2 + \ldots) + k_2(\beta_1^2 + \beta_2^2 + \ldots) + \ldots = -m_2,$$

or in the usual notation for normal equations

$$\begin{array}{ccccc}
k_1 & k_2 & k_3 \ldots & & \\
[aa] & [a\beta] & [a\gamma] & \ldots & -m_1 \\
 & [\beta\beta] & [\beta\gamma] & \ldots & -m_2 \\
 & & [\gamma\gamma] & \ldots & -m_3
\end{array}$$

All the coefficients of these q equations are known, and the solution of the equations gives the values of the q correlatives k_1, k_2, etc.; x_1, x_2, etc., are then found directly from the equations

---

[1] A treatise on the differential calculus should be consulted for further information on the theory of undetermined multipliers.

$$2x_1 + 2k_1\alpha_1 + 2k_2\beta_1 + \ldots = 0,$$

$$2x_2 + 2k_1\alpha_2 + 2k_2\beta_2 + \ldots = 0,$$

etc.

Problems requiring this form of solution are not common in the laboratory. They occur pre-eminently in geodesy, in the adjustments of networks both of triangulation and levelling. For an example of the practical application the reader may consult any modern record of precise surveying or levelling.[1]

§ (7) The Solution of Normal Equations.—The solution of any set of normal equations can always be performed by successive elimination of unknowns between pairs of equations, but a judicious selection of a particular method by the computer will often save much time.

Below is a summary of some methods which are available.

(i.) *Direct elimination* between equations in pairs, the most convenient pairs being chosen by inspection. Where many of the coefficients are zero this is often the best method.

(ii.) *Solution by Determinant.*—The value of any unknown can be expressed as a determinant, which is to be evaluated in a schedule. This method is convenient when the coefficients are digits.

(iii.) *Gauss's Method.*—This is really method (i) but is reduced to a schedule and provided with checks as the work is in progress. The left-hand unknown is at each stage eliminated between the top equation and each of the others in turn. Reverting to more conventional algebraical notation, in which x, y, z denote unknowns, the solution, including the formation of normal equations from observational equations, is as follows (for 3 unknowns):

Observational equations: $a_1x + b_1y + c_1z = m_1.$ $a_2x + b_2y + c_2z = m_2.$	Operations for checks: $a_1 + b_1 + c_1 + m_1 = s_1.$ $a_2 + b_2 + c_2 + m_2 = s_2.$
1st normal equation: $[aa] \quad [ab] \quad [ac] \quad [am].$ 2nd normal equation: $[ab] \quad [bb] \quad [bc] \quad [bm].$	Check: $[aa] + [ab] + [ac] + [am] = [as].$ $[ab] + [bb] + [bc] + [bm] = [bs].$
Elimination of x between these two gives $0 \quad [bb] - \frac{[ab]}{[aa]}[ab] \quad [bc] - \frac{[ab]}{[aa]}[ac]$ $[bm] - \frac{[ab]}{[aa]}[am].$ This is denoted by symbols as follows: $[bb1] \quad [bc1] \quad [bm1].$	Check: $[bb1] + [bc1] + [bm1] = [bs1].$
Similarly by elimination between 1st and 3rd normal equations: $[bc1] \quad [cc1] \quad [cm1],$ where $[cc1] = [cc] - \frac{[ac]}{[aa]}[ac].$	Check: $[bc1] + [cc1] + [cm1] = [cs1].$

The resulting equations still possess the symmetry of the normal equations, and it can be shown that the leading coefficients $[bb1]$ and $[cc1]$ are necessarily positive, like the leading coefficients in the normal equations.

The elimination is repeated until the value of the right-hand unknown is found. The remainder are then found by substitution *in the proper equation.* The latter is always that one in which the coefficient of the particular unknown is the first term in the leading diagonal. Thus the proper equation from which to find y by substituting for z is

$$[bb1]y + [bc1]z = [bm1]$$

and for x

$$[aa]x + [ab]y + [ac]z = [am].$$

Unless the calculations are being carried to a large number of significant figures, the neglect of this precaution may lead to serious inaccuracy in the result and to a puzzling failure in the final checking back of the truth of the normal equations.

---

[1] The 28th "Special Publication" of the United States Coast and Geodetic Survey (Washington, Government Printing Office, 1915), contains a discussion by Mr. Oscar Adams on the application to triangulation.

Most of the text-books mentioned in the List of references at the end of this article contain examples of Gauss's method of solution.

(iv.) *Successive Approximation.*—Where the number of unknowns, and therefore the number of equations, is large, the foregoing methods may become very laborious. If the rough orders of magnitude of the unknowns can be found in any way it will often be seen on inspection that certain terms in each equation only contribute an insignificant amount to the value of the right-hand side; or at least that equations possessing this property can be developed by suitably combining the normal equations. The solution is first to be carried out neglecting the said terms altogether and furnishing approximate values for some or all of the unknowns. If approximate values for all the unknowns have been found the same solution is to be repeated, this time modifying the right-hand sides by correcting for the terms neglected in the previous solution, with the help of the approximate values. If approximate values for only some of the unknowns are furnished, a new series of combinations of the original equations must be found which reduces to comparative insignificance terms in unknowns other than those neglected in the first instance. The approximate values can then probably be substituted in the insignificant terms and the resulting simplified solution furnishes approximate values for another set of unknowns. By such successive approximations it is possible to approach with any desired precision the determinant values of the unknowns, and the stage at which to stop is determined either by the accuracy required in the results or by the fact that the residuals obtained on substitution in the original equations are well within the p.e.'s of the normal equations computed from the assumed p.e.'s of observation. Either of these limits may be first reached.

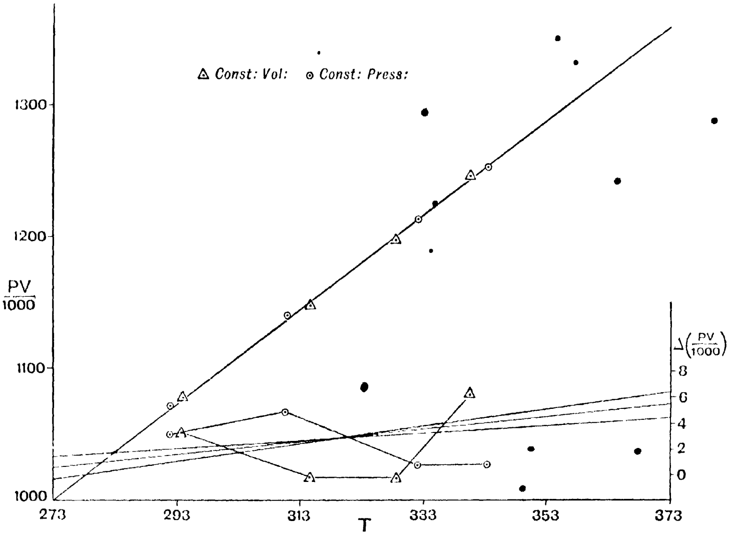


FIG. 6.

§ (6) GRAPHICAL METHODS.—The graphical representation of observations gives a clear and immediate insight into their general behaviour, especially if the laws which they are assumed to follow are not too complex. In many cases the final adjustment by purely graphical means is as reliable as any formal method such as that of "least squares."

As an example, the problem of § (5) (vii.) may be solved graphically. The several values of PV/1000 are plotted in *Fig.* 6 against T, using a different sign for points belonging to the different conditions of experiment, *i.e.* constant pressure and constant volume. Through these points the best straight line is ruled by eye. Its slope, *i.e.* the tangent of the angle, is read off as $(1362{\cdot}5 - 1001{\cdot}5)/100 = 3{\cdot}610$. If only an approximate value for R were required the above would suffice. An improvement on it may be effected as follows: With the value 3·610 found, PV/1000 is calculated for each observed temperature and is subtracted from the observed value of PV/1000. The residuals which result are then plotted on an enlarged scale of ordinates, so as to exhibit at once (1) any alteration necessary in the slope of the line and (2) *any other systematic tendency of the points.*

It is this latter opportunity which gives the graphical method its power. It may show not only whether the observations are good but also whether the law which they are supposed to obey is really obeyed.

In *Fig.* 6 the diagram of residuals for constant volume is seen to possess a concave curvature, while that for constant pressure shows a tendency to fall off at higher temperatures.

These facts *may* indicate complications in the laws which the observations follow. Before drawing this conclusion it is necessary to compute from the assumed p.e.'s of observation the probable value of the discrepancy in PV/1000 − RT/1000.

The p.e.'s assumed were $p_0 = 1$ millibar, $v_0 = 1$ c.c., $t_0 = 1°$ abs. The probable discrepancy is therefore $\frac{1}{1000}(P^2 . 1^2 + V^2 . 1^2 + R^2 . 1^2)^{\frac{1}{2}}$. Putting P = 1000, V = 1200 as an approximation to the mean condition of experiment, this becomes $\frac{1}{1000}(1000^2 + 1200^2 + 3610^2)^{\frac{1}{2}} = 3\cdot9$ about.

The irregularities observed in the graph of the residuals are mostly, if not all, within this range. The conclusion is that no complication in the law PV = RT can be assumed from the observations.

Drawing the best straight line through all the points the following correction to R/1000 is obtained:

$$\frac{5\cdot5 - 0\cdot5}{100} = +0\cdot050$$

or

$$\frac{R}{1000} = 3\cdot610 + 0\cdot050 = 3\cdot660,$$

$$R = 3660 \quad \text{(cf. value } 3662\cdot4 \pm 4\cdot4 \text{ on p. 657).}$$

Near the best straight line are drawn two others making angles with it which appear to mark the limits of the p.e. of slope. On this showing the p.e. of R would be about 1000 × 2/100 = 20, a figure very much in excess of both the *a priori* calculated p.e. (4·4) on p. 657 and the *a posteriori* p.e. (2·0) on the same page. The explanation of the discrepancy is that the assumption of the law PV = RT *constrains* the best straight line to pass through the distant zero of temperature. If we regard the diagram of residuals in this light, the p.e. of the slope of the line becomes about 1000 × 1/320 = 3·1, which lies just midway between the two former values of the p.e. of R.

Where the relationship between the observed quantities and the unknowns involves various powers of both, logarithmic plotting is very useful. An example of this has already been given in *Fig.* 3, where (error)² is plotted against log (number of errors).[1]

Since logarithmic plotting shows fractional errors x/X instead of absolute errors x, if the latter are nearly constant in probable value throughout the observation of a number of different values of X, the apparent errors may be very different in one portion of a diagram from those in another portion. This fact must be remembered in drawing numerical conclusions from the graph.

It is a good plan on every possible occasion to commence a computation with a rough graphical investigation of observations, even though formal methods of solution are eventually to be employed. The insight obtained into the general character of the results always repays the trouble.

§ (9) THE DETERMINATION OF PROBABLE ERRORS.—The simple rules given in § (4) (ix.) serve to determine the p.e. in all practical cases. Therefore if a result can be thrown into the form of an expression involving only observed quantities and calculable coefficients, and p.e.'s can be assigned to all the observed quantities, the p.e. of the result can at once be determined.

But the expressions for the p.e. which result are often intolerably complicated. In such cases a much better plan is to use symbols for the *actual* errors throughout the calculation, and only to introduce *probable* errors according to § (4) (ix.) at the last possible moment.

Example.—On a base line AB (*Fig.* 7) about 1000 yards long, and measured with a p.e. of 1 in 50,000 a triangle ABC is surveyed. The angles at A and B are about 80° and 40° respectively, and all three angles are measured with an assumed p.e. of 10″. Required: the p.e. of the point C (1) in a direction at right

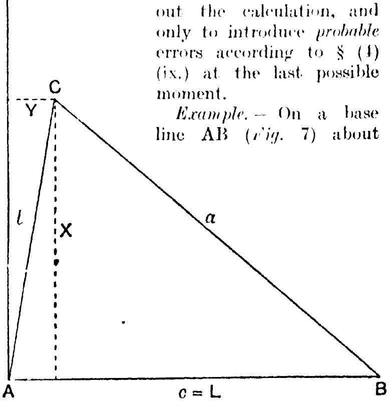


FIG. 7.

---

[1] For a good account of the graphical treatment of numbers of observational errors, see Lallemand, *Lever des plans et nivellement*, 1912, pp. 582-600 (Ch Béranger, Paris, 15 Rue des Saints-Pères).

angles to the base, and (2) in a direction parallel with the base, A being taken as origin of co-ordinates. The solution of the triangle would be according to the formula

$$b = \mathrm{L}\frac{\sin \mathrm{B}}{\sin \mathrm{C}},$$

and the two lengths whose p.e.'s are required are

$$\mathrm{X} = b \sin \mathrm{A} = \mathrm{L}\frac{\sin \mathrm{A} \sin \mathrm{B}}{\sin \mathrm{C}}$$

and $$\mathrm{Y} = b \cos \mathrm{A} = \mathrm{L}\frac{\cos \mathrm{A} \sin \mathrm{B}}{\sin \mathrm{C}}.$$

Since L influences the scale as a whole, the errors due to it can be considered afterwards, taking it for the present as errorless. Introducing small actual errors α, β, γ into the angles, sin A becomes

$$\sin(\mathrm{A}+\alpha) = \sin \mathrm{A} + \alpha \cos \mathrm{A} = \sin \mathrm{A}\,(1 + \alpha \cot \mathrm{A}).$$

Similarly for the other sines of angles: cos A becomes $\cos \mathrm{A}\,(1 - \alpha \tan \mathrm{A})$.

The value found for X is

$$\mathrm{L}\frac{\sin \mathrm{A} \sin \mathrm{B}}{\sin \mathrm{C}}\frac{(1+\alpha \cot \mathrm{A})(1+\beta \cot \mathrm{B})}{(1+\gamma \cot \mathrm{C})}$$

$$= \mathrm{L}\frac{\sin \mathrm{A} \sin \mathrm{B}}{\sin \mathrm{C}}(1 + \alpha \cot \mathrm{A} + \beta \cot \mathrm{B} - \gamma \cot \mathrm{C})$$

nearly,

so that the actual error of X is

$$\mathrm{X}(\alpha \cot \mathrm{A} + \beta \cot \mathrm{B} - \gamma \cot \mathrm{C}).$$

We must now introduce probable errors for the angles, *i.e.* probable values for α, β, and γ. The p.e.'s of the observed angles are all 10″, but an adjustment of the sum of the three observed angles to 180° would naturally precede the solution of the triangle. It is necessary to find the p.e. of an adjusted angle. The proper observational equations are (see p. 659)

$$\mathrm{A} = \mathrm{A}_0 + \alpha_0,$$

$$\mathrm{B} = \mathrm{B}_0 + \beta_0,$$

$$180° - \mathrm{A} - \mathrm{B} = \mathrm{C}_0 + \gamma_0,$$

where A_0, B_0, C_0 are true values, and α_0, β_0, γ_0 are *actual* but unknown errors of observation. The recorded value for A is $\mathrm{A}_0 + \alpha_0$, and so on.

These equations having all the same p.e. they may be combined to form normal equations as follows:

$$2\mathrm{A} + \mathrm{B} = \mathrm{A}_0 + \alpha_0 - \mathrm{C}_0 - \gamma_0 + 180°,$$

$$\mathrm{A} + 2\mathrm{B} = \mathrm{B}_0 + \beta_0 - \mathrm{C}_0 - \gamma_0 + 180°;$$

eliminating B,

$$3\mathrm{A} = 2\mathrm{A}_0 + 2\alpha_0 - \mathrm{B}_0 - \beta_0 - \mathrm{C}_0 - \gamma_0 + 180°.$$

The actual error introduced into A is therefore

$$\tfrac{1}{3}(2\alpha_0 - \beta_0 - \gamma_0).$$

Replacing actual errors by probable errors, the p.e. of A is

$$\frac{\epsilon}{3}\sqrt{2^2 + 1^2 + 1^2} \text{ (where } \epsilon = \text{p.e. of an observed angle} = 10'')$$

$$= \epsilon\frac{\sqrt{6}}{3}.$$

The p.e. of any one of the adjusted angles is therefore not 10″ but $10''\sqrt{6}/3 = 10\sqrt{6}/3 \div 206265$ radians, and the p.e. of X is

$$1000\frac{\sin 80° \sin 40°}{\sin 60°} \times \frac{10\sqrt{6}}{3 \times 206265}$$

$$\sqrt{\cot^2 80° + \cot^2 40° + \cot^2 60°} \text{ yards}$$

$$= 0{\cdot}039 \text{ yard}.$$

Proceeding in the same way it is found that the p.e. of Y is

$$1000\frac{\cos 80° \sin 40°}{\sin 60°} \times \frac{10\sqrt{6}}{3 \times 206265}$$

$$\sqrt{\tan^2 80° + \cot^2 40° + \cot^2 60°} \text{ yards}$$

$$= 0{\cdot}038 \text{ yard},$$

i.e. sensibly the same as the p.e. of X.

Next suppose that the base L has an actual error l. Then if X and Y have actual errors x and y from the triangulation, $\mathrm{X} + x$ becomes $(\mathrm{X} + x)(1 + l/\mathrm{L}) = \mathrm{X} + x + \mathrm{X}l/\mathrm{L}$, when the infinitesimal quantity xl/L is neglected.

The probable value of x is 0·039 yard and that of l/L is 2×10^{-5}. Therefore the resultant p.e. of X is

$$\sqrt{(0{\cdot}039)^2 + (731{\cdot}1)^2 \times 4 \times 10^{-10}}$$

$$\text{(since X} = 731{\cdot}1 \text{ yards)}$$

$$= 0{\cdot}042 \text{ yard}.$$

The base-line error is not very important compared with those arising from the measurements of angles. Its influence on Y is still smaller.

Similar reasoning could be applied to a succession of triangles. In this case, as in many others, the expressions would be cumbrous, and a simplification could be brought about by using none but actual numerical coefficients to multiply unknown actual errors. That is to say, the calculation leading to the solution of the original problem is repeated, using symbols for the unknown errors of all observed quantities.

These methods of calculating p.e. are all *a priori*, that is, they can be carried out before the observations are taken, and so long as (1) the assumed p.e.'s of observation are about correct, and (2) no serious systematic error enters into the results, they are quite to be relied upon.

On the other hand, the calculation of p.e.'s from residuals, *i.e.* the *a posteriori* method, is to be used with caution. As will be noticed in the next section, the formulae used for their calculation are unreliable for small numbers of observations. It is a common experience to obtain a result with a certain p.e. ϵ, calculated from the residuals, but on repeating the work subsequently, to obtain a second result differing by more than $\sqrt{\epsilon_1^2+\epsilon_2^2}$ from the first.

The formulae commonly used in determining p.e.'s from residuals are summarised in § (4) (ix.).

§ (10) THE LIMITATIONS OF THE PRINCIPLE OF LEAST SQUARES.—Gauss's law of the distribution of large numbers of accidental errors, on which the method of least squares is based, appears essentially reasonable, whether examined by means of practical tests or by a consideration of the assumptions on which its various theoretical deductions rest.

It cannot be applied as a *distribution* law to small numbers of errors, for it has no meaning in this case. Yet if the probability of any given error is proportional to e^{-x^2/c^2}, the probability of a certain set of errors, few in number, is $e^{-[x^2]/c^2}$, so that the least squares condition appears to be theoretically justified as a criterion, however small the number of observations. Whether, in view of the labour involved, it is always *practically* justified is another question.[1]

The reason of the doubtful benefit conferred by the application of the least squares condition to small numbers of observations is made clearer by the following facts.

Whenever a quantity whose true value is M has been experimentally determined, either directly or indirectly, giving a result $M+m$, say, the set of n errors of observation has a certain probability P. It can be shown that a small finite change in m produces a smaller fractional change $\Delta P/P$ for small numbers of observations than it does for large numbers. In other words, the curve of probability of different values of m is a flat-topped one for small numbers of observations, and although that value of m corresponding to the maximum ordinate for P is given by the "least squares" method, any method which gives a result somewhere near this would possess very nearly the same probability. In 1000 trials the least squares method might give, say, 520 results nearer to the true value than other more arbitrary methods, while in the remaining 480 the reverse would be the case.

There is a slight inconsistency in one application of the method of least squares. In § (5) (vii.), where the partition of errors among several kinds of observed quantity was dealt with, occurred the composite errors

$$f_1=\alpha_1x_1+\beta_1y_1+\gamma_1z_1,$$
$$f_2=\alpha_2x_2+\beta_2y_2+\gamma_2z_2,$$
$$\text{etc.},$$

where x_1, y_1, z_1, etc., were errors of observation on the different kinds of quantity, and α_1, β_1, γ_1, etc., were calculable coefficients. Since f_1, f_2, etc., are sums of accidental quantities they are themselves accidental quantities and should fall into the same distribution law as any collection of accidental errors. Or more strictly, f_1/ϕ_1, f_2/ϕ_2, etc., should be so distributed, where ϕ_1 is the probable value of f_1, *i.e.* the p.e. of F_1, and is equal to $(\alpha_1^2x_0^2+\beta_1^2y_0^2+\gamma_1^2z_0^2)^{\frac{1}{2}}$; x_0, y_0, z_0 being the p.e.'s of the observed quantities. A solution which ensures a minimum value to $[f_n^2/\phi_n^2]$ is therefore a true least squares solution. But it will be found, if f_n and ϕ_n be expressed in terms of the individual errors x, y, etc., and the p.e.'s x_0, y_0, etc., that the condition $[f_n^2/\phi_n^2]$ minimum is not in general consistent with the condition $[x_n^2/x_0^2]+[y_n^2/y_0^2]+[z_n^2/z_0^2]$ minimum, which latter might equally be made the basis of a least squares solution (see § (4) (v.)). In fact, while the latter condition represents a more faithful adherence to the principle of least squares, the two are only compatible if $\alpha_1x_0=\beta_1y_0=\gamma_1z_0$, *i.e.* if each kind of observation contributes equal partial probable errors to the result of each set of measurements.

The general method for the adjustment of mixed classes of observation outlined in § (5) (vii.) gives too much attention to those kinds of observation which produce the largest errors in the result, *i.e.* the X's if α_1x_0, α_2x_0, etc., are always larger than β_1y_0, β_2y_0, etc., respectively. Therefore, if one kind of observation produces a p.e. in the result say 3 times as large as that produced by any other kind, it will be satisfactory to adjust the observations as if that kind were the only one possessing errors. Another sphere in which the application of the method of least squares is liable to misconception is the estimation of probable errors from residuals. The formulae in § (4) (ix.), although giving *a* value for the p.e. in a given case, are not to be relied upon where only small numbers of residuals are dealt with.

It may seem pedantic to discuss "the probable error of the probable error," but there is for every class of measurement a fairly definite "expectation value" of the p.e., which could be obtained with confidence if only the observations were sufficiently multiplied. It is the departure from this definite value of values calculated from finite numbers of residuals which makes it necessary to consider the probable error of the probable error.

If a quantity has been measured three times, and the arithmetic mean of the three is M, while the actual measurements were $M+m_1$, $M+m_2$, $M+m_3$, the most probable value of the p.e. in this kind of measurement is $0{\cdot}67\sqrt{(m_1^2+m_2^2+m_3^2)/2}$, but it is only a very little more probable than a number of other values departing widely from itself. If this fact be borne in mind much disappointment at the failure of checks is avoided, and it is not long before a practical worker learns to gauge the order of a discrepancy which really calls for an investigation into the validity of a given result.

---

[1] In a suggestive article (*Phil. Mag.*, Feb. 1920) Dr. Norman Campbell has advocated the use, in a number of cases, of simplified methods depending on the replacement of the least squares condition by the "zero sum" condition, which, as he rightly points out, is a more fundamental property of accidental errors than their distribution.

REFERENCES

Amongst the extensive and valuable literature of the theory of errors and the combination of observations, the following is only a selection.

G. B. Airy, *Theory of Errors of Observation.* Macmillan, 1861. Out of print, but to be found in libraries. pp. 103.

M. Merriman, *Method of Least Squares.* 8th Ed. John Wiley (London, Chapman & Hall), 1915. pp. 230.

D. Brunt, *The Combination of Observations.* Camb. Univ. Press, 1917. pp. 219.

Helmert, *Ausgleichungsrechnung nach der Methode der kleinsten Quadraten.* 2nd Ed., 1907.

F. Kohlrausch, *Lehrbuch der praktischen Physik.* B. G. Teubner. 11th Ed., 1910. pp. 1-32.

A. de Forest Palmer, *The Theory of Measurements.* M'Graw Hill Book Co. 1912. pp. 248.

H. M. Goodwin, *Precision of Measurements and Graphical Methods.* M'Graw-Hill Book Co. 1917. pp. 104. Intended for junior students.

C. Lallemand, *Lever des plans et nivellement.* Part iii. ch. iv., Libraire Polytechnique. Ch. Béranger, Paris, 15 Rue des Saints-Pères. 2nd Ed., 1912.

J. Howard Gore, *A Bibliography of Geodesy.* App. No. 16 U.S. Coast and Geodetic Survey Report for 1887. Pub. 1889. See headings, "Least Squares" and "Errors of Observation."

Norman Campbell, "The Adjustment of Observations," *Phil. Mag.*, Feb. 1920.

H. L. P. J.

OCEAN CURRENTS:
- Bank water. See "Oceanography, Physical," § (45).
- Bjerknes' calculation of velocity. See *ibid.* § (57).
- Compensation currents. See *ibid.* § (43).
- Due to melting ice. See *ibid.* § (42).
- Due to pressure gradient. See *ibid.* § (53).
- Effect of coastal configuration. See *ibid.* § (41).
- Effect of friction of the wind on. See *ibid.* § (38).
- Effect of rotation of the earth on. See *ibid.* § (37).
- Ekman's theory of wind-drift in the ocean. See *ibid.* § (50).
- Friction on the sea bottom. See *ibid.* § (39).
- Knudsen's theorem. See *ibid.* § (58).
- Mohn's theory. See *ibid.* § (56).
- Relation between wind and current. See *ibid.* § (40).
- The reversal depth of Ekman's. See *ibid.* § (51).
- Vertical circulation. See *ibid.* § (44).
- Vertical equilibrium. See *ibid.* § (47).
- Wind-drift in an enclosed sea. See *ibid.* § (55).

OCEANOGRAPHY, PHYSICAL

INTRODUCTION.—Strictly speaking Physical Oceanography comprises the whole of our knowledge of the sea, with the exception of the part dealing with the animals and plants living in it; and some of these even cannot be excluded, since they form the coral reefs and great areas of bottom deposits.

It is customary, in this country at least, to apply the name *Hydrography* to that part of the science which concerns itself with the making of charts for the benefit of the seaman, and to group those parts which till lately appeared to be chiefly of biological or academic interest under the heading of *Physical Oceanography.* In view of the discoveries made in recent years, and in particular during the war of 1914–18, there is no doubt but that in the future the hydrographer will have to consider every branch of the science, and the distinction will cease to exist except as a matter of convenience.

I. PROPERTIES OF SEA-WATER

§ (1). COMPOSITION. Sea-water is a solution of "sea-salt" in pure water, and, speaking broadly, one specimen of it does not differ from another except in the strength of the solution, which varies from about 40 parts per thousand downwards. From the standpoint of the physical chemist it is therefore a "dilute solution." On the basis of 77 analyses made by Dittmar[1] on samples collected in many parts of the world, the composition of sea-salt is as follows, after certain rearrangements:

COMPOSITION OF SEA-WATER

	In 1000 Grms. of Sea water of 35 per Thousand Salinity.	Per Cent in Sea-salt.
Na	10·722	30·64
Mg	1·316	3·76
Ca	0·420	1·20
K	0·382	1·09
Cl	19·324	55·21
SO_4	2·696	7·70
CO_3	0·074	0·21
Br	0·066	0·19
	35·00	100·00

Since sea-water is a "dilute solution," it is very largely ionised, and tables giving the amounts of various salts such as magnesium chloride or calcium sulphate are meaningless, however useful they may be if it is required to make up an artificial sea-water by weighing out the solid constituents. The accurate determination of the various components is extremely difficult except in the case of chlorine. Modern oceanography is largely based on Knudsen's Tables (see later), which are founded on the assumption of the constancy of the ratio of the chlorine to salinity (total salts) and density at 0° C., the density of distilled water at 4° C. being the unit. The accuracy of this assumption has been attacked from time to time, and in consequence the ratio of chlorine to the other quantities and to SO_4 has been redetermined in a number of laboratories. The results have been collected by Ruppin,[1] and justify the assumption. It is probable that the question may be reopened with regard to Mediterranean water.

---

[1] *Challenger Reports: Physics and Chemistry,* i. 189 *et seq.*

§ (2) Relation of Chlorine, Salinity, and Density.—Assuming the constancy of the ratios, we may determine chlorine, salinity, or density, and from any one calculate the other two. The determinations of the total salts by direct evaporation is extremely difficult, owing to part of the chlorine being driven off as hydrochloric acid before the last traces of water are expelled, and impossible as a routine operation. It has been the rule, therefore, to determine either density or chlorine, and unfortunately the procedure has often been open to question. Density was usually determined by the hydrometer, which we now know to fall far short of its theoretical accuracy, and chlorine by titration against a standard solution of some chloride. Each observer prepared his own standard solution, and owing to slight differences of technique it is impossible to compare the results of various workers with the accuracy now required. Great confusion resulted until the end of the last century, when the International Council for the Exploration of the Sea was formed, with headquarters at Copenhagen.[2] The Council caused the ratio of chlorine to salinity and density, and the change of the latter with temperature, to be determined with the greatest possible accuracy; the results are given in Knudsen's *Hydrographical Tables, 1901*. It also prepared a "Normal Water," a natural sea-water of about average salinity, the chlorine of which was accurately determined. Secondary standards titrated against this are issued from the laboratory and used as a standard by individual workers, so that now all modern investigations are comparable among themselves, at least so far as the accuracy of the titration allows. The resulting advance in oceanography has been very great.

The "chlorine" of the "Normal Water" is not the true chlorine, but the total halogens, the bromine being considered as replaced by a *chemically equivalent* weight of chlorine; in the same way the salinity is the total weight of salts dissolved in 1000 grams of the water, the bromine being replaced by chlorine, carbonates by oxides, and all organic matter burned off.

According to the *Tables*, salinity, generally denoted by $S_{0/00}$, is equal to 0·030 + 1·8050 Cl. This means that if no chlorides were present the water would still have a salinity of $0{\cdot}03_{0/00}$. This apparently anomalous result is due to the fact that water such as this would be found at the mouth of rivers, which generally contain far more lime salts in proportion to halogens than the sea does. The third quantity given in the *Tables* is σ_0, a contraction for the density at 0°/4°; $\sigma_0 = (\text{sp. gr. } 0°/4° - 1)1000$. Its relation to "chlorine" is given by the equation

$$\sigma_0 = -0{\cdot}069 + 1{\cdot}4708\ \text{Cl} - 0{\cdot}001570\ \text{Cl}^2 + 0{\cdot}0000398\ \text{Cl}^3.$$

The *Tables* also give values of $\rho_{17\cdot5}$, which is the density at 17·5° abbreviated as before and referred to that of distilled water at the same temperature.

$$\rho_{17\cdot5} = (0{\cdot}1245 + \sigma_0 - 0{\cdot}0595\sigma_0 + 0{\cdot}000155\sigma_0^2) \times 1{\cdot}00129.$$

This is of use in much experimental work. The values are given in the tables for every 0·01 part of chlorine per thousand. The following short abstract will give some idea of them:

Chlorine, Salinity, etc.

Cl.	$S_{0/00}$.	σ_0.	$\rho_{17\cdot5}$.
2·50	4·54	3·60	3·52
5·00	9·06	7·25	6·96
7·50	13·57	10·89	10·40
10·00	18·08	14·52	13·83
12·50	22·59	18·15	17·27
15·00	27·11	21·77	20·70
17·50	31·62	25·40	24·15
20·00	36·13	29·04	27·60
22·50	40·64	32·68	31·07
19·29	34·85	28·00	26·62

The last on the list is a sea-water of about average salinity to which a great deal of experimental and tabular work is referred.

Salinities less than 30 do not generally occur except in seas such as the Baltic, or in estuaries or Polar currents. A large part of the ocean has a salinity of about 35, rising in places to 37. Salinities of 39 or more are confined to the eastern part of the Mediterranean, the Red Sea, and Persian Gulf.

---

[1] *Pub. de Circonstance, Cons. Perm. Internat. pour l'Explor. de la Mer*, No. 55. Copenhagen, 1910.

[2] Laboratoire Hydrographique, Den Polytekniske Laereanistalt, Copenhagen. The director of the laboratory at present (1922) is Professor Martin Knudsen. The laboratory supplies the *Tables*, normal water, oceanographical apparatus, and also the *Publications de Circonstance*, to which frequent reference is made in this article.

§ (3) THE CHANGE OF DENSITY WITH TEMPERATURE can be calculated with great accuracy by the following formula, if σ_0 is given:

$$\sigma_t = \sigma_0 - D,$$

where $\sigma_t = 1000(\text{sp. gr. } t^\circ/4^\circ - 1)$,

$$D = -\Sigma_t - 0\cdot1324$$
$$+ [\sigma_0 + 0\cdot1324][A_t - B_t(\sigma_0 - 0\cdot1324)].$$

Here $\Sigma_t = 1000(S_t - 1)$, where S_t is the density of distilled water at t° referred to distilled water at 4°.

$$\Sigma_t = -\frac{(t-3\cdot98)^2}{503\cdot570}\cdot\frac{t+283^\circ}{t+67\cdot26^\circ},$$

$$A_t = t(4\cdot7867 - 0\cdot098185t + 0\cdot0010843t^2)\times10^{-3},$$

$$B_t = t(18\cdot030 - 0\cdot8164t + 0\cdot01667t^2)\times10^{-6}.$$

The *Tables* contain values of Σ_t, A_t, and B_t for every tenth of a degree. In general, however, it is sufficient to know σ_t to two decimal places, *i.e.* the density to the fifth decimal place. The *Tables* give the values of D for every whole number of σ_0 and every degree of temperature from −2° C. up to 33° C.

The values of D for a few waters and temperatures are given in the table on the following page.

§ (4) THE FREEZING-POINT,[1] DENSITY AT FREEZING-POINT, etc., given below were determined in the course of drawing up the *Tables*.

The freezing-point τ is given by the equation $\tau = -0\cdot0086 - 0\cdot064633\sigma_0 - 0\cdot0001055\sigma_0^2$. The density at the freezing-point is $1 + (\sigma_\tau/1000)$, where

$$\sigma_\tau = \Sigma_\tau + (\sigma_0 + 0\cdot1324)[1 - A_\tau + B_\tau(\sigma_0 - 0\cdot1324)].$$

Σ_τ, etc., have the same meaning as before.

The change of density at the freezing-point $(d\sigma_\tau/dt)_{t=\tau}$ is obtained by differentiating the formula for σ_τ.

The temperature of greatest density, $t_{\sigma\max}$, was obtained by interpolation.

The maximum density was calculated from σ_0 and $t_{\sigma\max}$.

FREEZING-POINTS, ETC.

Cl.	S ‰.	σ_0.	τ.	σ_τ.	$(d\sigma_t/dt)t=\tau$.	$t_{\sigma\max}$.	$\sigma_{t\max}$.
1	1·835	1·400	0·099	1·39	0·062	3·589	1·556
5	9·055	7·251	−0·483	7·233	0·042	2·060	7·421
10	18·080	14·522	−0·969	14·513	0·018	0·106	14·536
14	25·300	20·324	−1·366	20·340	0·001	−1·464	20·340
15	27·105	21·774	−1·464	21·800	−0·006	−1·854	21·801
16	28·910	23·225	1·567	23·260	0·010	−2·242	23·264
17	30·715	24·676	−1·668	24·722	−0·014	−2·627	24·729
18	32·520	26·129	−1·769	26·186	−0·019	−3·008	26·198
19	34·325	27·582	1·872	27·653	0·023	3·385	27·670
20	36·130	29·037	−1·974	29·121	−0·027	−3·758	29·146
13·665	24·695	19·838	−1·332	19·852	0·000	1·332	19·852

The last water on the list is remarkable in having its greatest density at the freezing-point. A fresher water becomes lighter as it is cooled past the freezing-point, and a salter water becomes heavier.

§ (5) THE COMPRESSIBILITY OF SEA-WATER has been determined by V. Walfrid Ekman for the International Council.[2] His formula for the *mean* compressibility from atmospheric pressure, which he calls *o* bars, to *p* bars is

$$10^8\mu = \frac{4886}{1+\cdot000183p} - [227 + 28\cdot33t - 0\cdot551t^2 + 0\cdot004t^3]$$
$$+ \frac{p}{1000}[105\cdot5 + 9\cdot50t - 0\cdot158t^2] - \frac{1\cdot5p^2t}{1{,}000{,}000}$$
$$- \frac{\sigma_0 - 28}{10}[147\cdot3 - 2\cdot72t + 0\cdot04t^2 - \frac{p}{1000}(32\cdot4 - 0\cdot87t + 0\cdot02t^2)]$$
$$+ \frac{(\sigma_0 - 28)^2}{100}[4\cdot5 - 0\cdot1t - \frac{p}{1000}(1\cdot8 - 0\cdot06t)].$$

$$1 \text{ bar} = 10^6 \text{ dynes/cm.} \doteq 0\cdot987 \text{ atm.}$$

The *true* compressibility can be calculated from the above for any required pressure by the equation

$$K = \left(\mu + p\frac{d\mu}{dp}\right)\Big/(1 - p\mu).$$

In oceanography the mean compressibility is the important value; given the density at the surface of a sample of water of known salinity and temperature, it is required to calculate its density at any depth.

Ekman has drawn up tables of corrections by means of which this may be done, which are of great assistance in the hydrodynamical theory of currents in the sea.[3]

---

[1] Martin Knudsen, *Pub. de Circ.* No. 5, 1903.

[2] *Pub. de Circ.* No. 43, 1908.

[3] *Ibid.* No. 49, 1910.

VALUES OF D

$\sigma_t = \sigma_0 - D$

$t =$ \ $\sigma_0 =$	0	5	10	15	20	25	26	27	28	29	30	31	32	33
−2°	0·172	0·123	0·076	0·031	−0·012	−0·053	−0·061	−0·069	−0·076	−0·084	−0·092	−0·099	−0·107	−0·114
−1°	0·077	0·053	0·029	0·007	−0·014	−0·034	−0·038	−0·042	−0·046	−0·050	−0·054	−0·057	−0·061	−0·065
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
1°	−0·058	−0·035	−0·013	0·008	0·029	0·048	0·052	0·056	0·059	0·063	0·067	0·070	0·074	0·078
2°	−0·099	−0·054	−0·011	0·031	0·071	0·110	0·117	0·125	0·132	0·140	0·147	0·154	0·161	0·168
4°	−0·130	−0·043	0·040	0·121	0·199	0·273	0·288	0·303	0·317	0·331	0·345	0·359	0·373	0·387
6°	−0·097	0·028	0·149	0·266	0·378	0·487	0·508	0·529	0·550	0·571	0·591	0·612	0·632	0·652
8°	−0·004	0·156	0·312	0·462	0·607	0·747	0·775	0·802	0·829	0·856	0·883	0·909	0·935	0·961
10°	0·146	0·338	0·526	0·707	0·882	1·052	1·085	1·118	1·151	1·184	1·216	1·248	1·280	1·312
12°	0·349	0·572	0·788	0·998	1·201	1·399	1·437	1·476	1·514	1·552	1·589	1·627	1·664	1·701
14°	0·603	0·853	1·097	1·333	1·563	1·785	1·829	1·873	1·916	1·959	2·001	2·043	2·085	2·127
16°	0·905	1·181	1·449	1·710	1·964	2·210	2·259	2·307	2·354	2·402	2·449	2·496	2·543	2·589
17·5	1·162	1·456	1·741	2·020	2·290	2·553	2·605	2·656	2·707	2·758	2·808	2·858	2·908	2·958
18°	1·25[illegible]	1·553	1·845	2·128	2·404	2·672	2·725	2·777	2·829	2·881	2·932	2·983	3·034	3·085
20°	1·646	1·967	2·281	2·585	2·882	3·170	3·226	3·282	3·338	3·394	3·449	3·504	3·559	3·613
22°	2·080	2·422	2·756	3·080	3·395	3·702	3·762	3·822	3·881	3·940	3·999	4·057	4·116	4·173
24°	2·555	2·917	3·269	3·611	3·944	4·267	4·331	4·394	4·456	4·519	4·581	4·642	4·703	4·764
26°	3·068	3·449	3·819	4·178	4·527	4·865	4·932	4·998	5·063	5·129	5·193	5·258	5·322	5·385
28°	3·619	4·017	4·404	4·780	5·144	5·496	5·565	5·634	5·702	5·770	5·837	5·904	5·970	6·036
30°	4·206	4·622	5·025	5·416	5·793	6·158	6·230	6·301	6·371	6·441	6·510	6·579	6·648	6·715
32°	4·827	5·261	5·680	6·085	6·475	6·852	6·925	6·998	7·070	7·142	7·213	7·284	7·354	7·424

The last decimal places are only roughly accurate. Linear interpolation with respect to *t* will generally lead to inaccuracy which may affect the second decimal place.

The following table gives some values of the mean compressibility calculated from the formula above, for an average sea-water:

MEAN COMPRESSIBILITY × 10^8 OF SEA-WATER OF $\sigma_0 = 28$

p (bars).	$t = 0°$.	5°.	10°.	15°.	20°.	25°.	30°.
0	4659	4531	4427	4345	4281	4233	4195
10	4651	4523	4420	4338	4274	4226	4191
50	4620	4494	4392	4311	..	..	..
100	4582	4458	4357	4278	..	..	..
200	4508	4388	4291	..	..	..	..
400	4368	4256	..	..	..	..	..
1000	4009	3916	..	..	..	..	..

High temperatures do not occur at great depths, so blanks have been left in the table. The absolute accuracy of the results is not so great as might be supposed from the number of figures given: the figure for the total compressibility is probably not more accurate than three parts in a thousand. The accuracy with which the difference in density of two specimens of water at the same level can be calculated is considerably greater, say one part in a hundred of the difference; and it is chiefly with differences that we have to deal in oceanography.

§ (6) THERMAL EXPANSION.—The following values are given by Ekman:[1]

COEFFICIENTS OF THERMAL EXPANSION × 10^6: $e \times 10^6$ AT ATMOSPHERIC PRESSURE ($p=0$)

	−2°.	0°.	5°.	10°.	15°.	20°.	25°.	30°.
$\sigma_0 = 0$	−105	−67	17	88	151	207	257	303
5	79	−44	35	104	163	217	266	311
10	−55	−21	54	118	175	226	273	317
15	−32	0	71	132	186	235	280	323
20	10	21	88	146	197	244	287	329
25	11	40	104	159	208	252	293	333
28	23	51	114	167	214	257	297	334
30	30	58	120	172	218	260	299	336
35	48	75	131	184	228	267	304	339

CHANGE OF $e \times 10^6$ FOR A CHANGE OF 10 BARS: $p=0$

	−2°.	0°.	5°.	10°.	15°.	20°.	25°.	30°.
$\sigma_0 = 0$	3·9	3·7	3·0	2·5	2·0	1·5	1·1	0·8
5	3·8	3·5	2·9	2·3	1·9	1·4	1·0	0·7
10	3·6	3·4	2·8	2·2	1·8	1·3	1·0	0·7
15	3·4	3·2	2·6	2·1	1·7	1·3	0·9	0·7
20	3·3	3·1	2·5	2·0	1·6	1·2	0·9	0·6
25	3·1	2·9	2·4	1·9	1·5	1·1	0·9	0·6
28	3·1	2·8	2·3	1·9	1·5	1·1	0·8	0·6
30	3·0	2·8	2·3	1·8	1·4	1·1	0·8	0·6
35	2·9	2·6	2·2	1·7	1·3	1·0	0·8	0·6

EXPANSION AT 200 BARS. $e \times 10^6$

	−2°.	0°.	5°.	10°.	15°.	20°.
$\sigma_0 = 26$	74	98	152	197	238	275
28	80	105	157	202	241	278
30	87	111	162	206	245	281
32	93	117	167	210	248	283

EXPANSION AT 400 BARS. $e \times 10^6$

	−2°.	0°.	5°.	10°.	15°.
$\sigma_0 = 26$	126	147	191	229	263
28	132	152	196	233	266
30	137	157	200	236	269
32	143	162	204	240	272

---

[1] *Pub. de Circ.* No. 49.

EXPANSION AT 600 BARS. $e \cdot 10^6$

	−2°.	0°.	2·5°.	5°.
$\sigma_0 = 27$	174	191	211	228
28	177	194	213	230
29	179	196	215	232

EXPANSION AT 800 BARS. $e \cdot 10^6$

	0°.	2·5°.	5°.
$\sigma_0 = 27$	229	244	259
28	231	246	260
29	232	248	262

EXPANSION AT 1000 BARS. $e \cdot 10^6$

	0°.	2·5°.	5°.
$\sigma_0 = 27$	261	274	286
28	263	276	287
29	265	277	289

The second table gives the increase of the coefficient of expansion between 0 and 100 bars.

The *Tables* generally show how greatly the coefficient of expansion is increased by an increase of salinity, temperature, or pressure.

§ (7) SPECIFIC HEAT.—The specific heat C_p at constant pressure, under atmospheric pressure, has been calculated by Krümmel [1] for 17·5° from observations made by Thoulet and Chevallier.

S ‰.	C_p.	S ‰.	C_p.
0	1·000	25	0·945
5	0·982	30	0·939
10	0·968	35	0·932
15	0·958	40	0·926
20	0·951		

The change of C_p with temperature has not been determined. It probably does not differ greatly from that of pure water.

C_p changes with pressure according to the relation [2]

$$\frac{\partial C_p}{\partial p} = -10^6 \frac{T\partial^2 v}{J\partial t^2} = -10^6 \frac{T}{\rho J}\left(\frac{\partial e}{\partial t} + e^2\right),$$

p being pressure.

Ekman [3] gives some values for the decrease for water of $\sigma_0 = 28$ and 32 at various temperatures. For water of 0° C. and $\sigma_0 = 28$ the difference between C_p at the surface and at 1000 bars, or say 9700 metres, is about 0·0566.

The specific heat at constant volume has not been measured. It may be calculated from the formula

$$c_p - c_v = \frac{Te^2}{J\rho K},$$

T being absolute temperature, e coefficient of thermal expansion, ρ density, and K *true* compressibility. It will therefore be strongly influenced by the temperature and pressure, which affect ρ, K, and e (see above).

For water of $\sigma_0 = 28$ (*i.e.* salinity 34·85 per cent) the ratio C_p/C_v under atmospheric pressure varies from 1·0004 at 0° to 1·0207 at 30°. Pressure also increases the ratio; thus for the same water at 0° the ratio is 1·0009 at 100 bars and 1·0126 at 1000 bars.

§ (8) THE THERMAL CONDUCTIVITY has not been measured. If we assume Weber's law that the conductivities vary as the specific heats of equal volumes, and take 0·0014 C.G.S. units as the conductivity for pure water, we obtain the following values, for 17·5° C.:

THERMAL CONDUCTIVITY AT 17·5° C.

S ‰.	Conductivity × 10⁶.	S ‰.	Conductivity × 10⁶.
0	1400	30	1346
10	1367	35	1341
20	1353	40	1337

The value probably falls with increase of temperature, as in the case of distilled water, and according to the same or a very similar law, $k_t = k_0(1 - 0{\cdot}0055t)$. The figures above are only approximate, but they serve to show the vanishingly small effect of pure conductivity in the sea.

§ (9) POTENTIAL TEMPERATURE.—This is the temperature which a body of sea-water would have if raised adiabatically from the depths to the surface. See § (49).

§ (10) OSMOTIC PRESSURE AND LOWERING OF VAPOUR PRESSURE.—The osmotic pressure in atmospheres at 0° C. has been calculated by Sigurd Stenius [4] from the freezing-points, using the factor −12·08 and the change of boiling-point and vapour pressure by Krümmel.[5]

Salinity.	Osmotic Pressure.	Rise of Boiling-point.	Lowering of Vapour Pressure.
	atm.	°C.	mm.
5	3·23	0·08	2·13
10	6·44	0·16	4·23
15	9·69	0·23	6·45
20	12·98	0·31	8·47
25	16·32	0·39	10·73
30	19·67	0·47	12·97
35	23·12	0·56	15·23
40	26·59	0·64	17·55

---

[1] *Ozeanographie*, 1907 ed., i. 279.

[2] See "Thermodynamics," § (38), Vol. I.

[3] *Ann. d. Hydr.*, 1914, 342.

[4] *Öfversigt af Finska Vetenskaps-Societetens Förhandlingar*, Bd. 46, No. 6, Helsingfors, 1904.

[5] *Ozeanographie*, 1907 ed., i. 242.

A small correction for the effect of the change in heat of fusion and heat of dilution which Stenius gives separately, is included here in the figures for osmotic pressure; it is at most 0·07 atmosphere.

§ (11) EVAPORATION.—Wust[1] has calculated this from observations made at sea. Sea-water was exposed in hydrometer trial-jars with a surface of 283 sq. cm., and the loss by evaporation was determined by frequent titrations of the chlorine. Careful meteorological observations were made at the same time. He found that the height of water in millimetres evaporated in twenty-four hours could be expressed by

$$e = c \cdot f(w_2)(1 + \alpha t)(0{\cdot}98e_s - e_d),$$

where c is a constant, $f(w_2)$ expresses the wind effect, α is the coefficient of expansion of a gas, t the temperature, e_s the vapour pressure of water at the temperature in the jar, here multiplied by 0·98 to allow for the lowering caused by the presence of salt, and e_d the actual vapour pressure of the moisture in the air at the time. This led to the formula

$$e = 0{\cdot}59(1 + 0{\cdot}11w_2)(1 + \alpha t)(0{\cdot}98e_s - e_d),$$

where w_2 is in km./h. He then reduced the values for the speed of the ship, and finally to water level by means of determinations which he had previously made on the variation of wind force, humidity, and temperature from the sea surface up to 6 metres. He reached the final conclusion that the evaporation in the jars was a little over twice what it would have been at sea-level, and that the mean yearly evaporation for the whole of the sea is about 82 cm.

§ (12) SURFACE TENSION.—This has been determined by Krümmel[2] by the method of air-bubbles from a capillary tube. He found that the equation $\alpha = 77{\cdot}09 - 0{\cdot}1788t + 0{\cdot}0221\ S\ ^\circ/_{oo}$, where S is salinity, fitted the observations well. His values at 0° C., in dynes per centimetre, are:

S °/oo.	Surface Tension.	S °/oo.	Surface Tension.
0	77·09	25	77·64
5	77·20	30	77·75
10	77·31	35	77·86
20	77·53	40	77·97

The effect of salinity is therefore very small; that of temperature rather greater. For instance, the value for water of 35 salinity falls from 77·86 at 0° C. to 73·39 at 25° C.

§ (13) VISCOSITY.—This has been determined by Krümmel and Ruppin[3] relatively to distilled water. The results are given in the following table, the viscosity of pure water at 0° being put at 100:

SALINITY °/oo

	0.	5.	10.	15.	20.	25.	30.	35.	40.
0	100·0	100·9	101·7	102·5	103·2	103·9	104·5	105·2	105·9
1	96·0	96·8	97·6	98·3	99·0	99·7	100·4	101·1	101·8
2	92·6	93·5	94·3	95·1	95·9	96·6	97·3	98·0	98·7
3	89·7	90·6	91·4	92·2	92·9	93·6	94·3	95·0	95·7
5	84·7	85·5	86·3	87·0	87·7	88·4	89·1	89·8	90·5
10	73·0	73·8	74·5	75·2	75·8	76·5	77·2	77·8	78·5
15	63·6	64·3	64·9	65·6	66·2	66·9	67·5	68·2	68·8
20	56·2	56·8	57·4	58·0	58·6	59·3	59·9	60·5	61·1
25	49·9	50·4	51·0	51·6	52·1	52·7	53·3	53·9	54·5
30	44·9	45·4	46·0	46·5	47·0	47·5	48·1	48·6	49·1

The viscosity of pure water at 0° has been measured by several observers: Bingham and White,[4] 0·01797, Hasking, 0·01793[5] C.G.S. The viscosity of sea-water is of importance chiefly from the biological point of view: the frictional resistance affecting ocean currents is an entirely different quantity. The *Tables* show that viscosity is greatly decreased by rise of temperature, and slightly increased by rise of salinity. The viscosity of pure water is diminished by increase of pressure, while that of a strong salt solution is increased. The effect of pressure on sea-water has not been determined.

§ (14) ELECTRICAL CONDUCTIVITY.—Ruppin[6] measured this at 0°, 15°, and 25° C., and deduced the following formulae:

$$L_{0^\circ} = 0{\cdot}000978\ S - 0{\cdot}00000596\ S^2 + 0{\cdot}0000000547\ S^3,$$

$$L_{15^\circ} = 0{\cdot}001465\ S - 0{\cdot}00000978\ S^2 + 0{\cdot}0000000876\ S^3,$$

$$L_{25^\circ} = 0{\cdot}001823\ S - 0{\cdot}00001276\ S^2 + 0{\cdot}0000001177\ S^3,$$

where S is salinity.

---

[1] *Veroff. Inst. f. Meereskunde, Berlin*, N.F. Reihe A, Hft. 6, 1920.

[2] *Wiss. Meeresuntersuch. Kiel*, 1900, Bd. 5, Hft. 2.

[3] *Wiss. Meeresunters. Kiel*, 1906, ix. 29.

[4] *Ztsch. z. physik. Chem.* lxxx. 684.

[5] *Phil. Mag.*, 1909, i. 502, ii. 260.

[6] Krümmel, *Ozeanographie*, 1907 ed., i. 290.

It will be noticed that this gives the conductivity of pure water as zero. The temperature coefficient is considerable, about 3 per cent. Knudsen has found the following expression for it:

$$\log L_t = \log L_{15} + \epsilon(t^\circ - 15^\circ).$$

ϵ varies only slightly with salinity; according to Ruppin, at 0° it is 0·01135, at 25°, 0·00928.

CONDUCTIVITY IN RECIPROCAL OHMS

t°.	Salinity.							
	5.	10.	15.	20.	25.	30.	35.	40.
0	0·0018	0·0092	0·0135	0·0176	0·0216	0·0254	0·0293	0·0331
5	0·0055	0·0107	0·0156	0·0203	0·0248	0·0292	0·0335	0·0378
10	0·0063	0·0122	0·0178	0·0231	0·0283	0·0332	0·0382	0·0430
15	0·0071	0·0138	0·0201	0·0261	0·0319	0·0375	0·0431	0·0486
20	0·0079	0·0154	0·0225	0·0292	0·0357	0·0420	0·0482	0·0543
25	0·0088	0·0171	0·0249	0·0323	0·0391	0·0464	0·0532	0·0601
30	0·0097	0·0187	0·0273	0·0354	0·0433	0·0510	0·0585	0·0660

§ (15) REFRACTIVE INDEX.—The most recent determinations, for the D-line, air-water, are by C. Vaurabourg.[1]

t.	σ_0							
	4.	8.	12.	16.	20.	24.	28.	32.
0°	1·33507	1·33604	1·33702	1·33800	1·33897	1·33995	1·34092	1·34190
5	494	589	685	781	877	973	069	165
10	470	565	660	753	848	943	038	133
15	437	530	624	718	811	905	·33999	092
20	396	488	581	673	766	858	951	043
25	347	438	530	622	713	805	897	·33988
30	291	382	473	564	655	746	836	927

The increase of refractive index with pressure is ·000017 per atmosphere at 0° and 0·000015 at 20° for distilled water; the law for sea-water is probably similar if allowance is made for the smaller compressibility.

§ (16) TRANSPARENCY.—The larger number of observations on the transparency of sea-water have been made by noting the maximum depth at which a white disc, generally 0·5 m. in diameter, can be seen. The depth obviously depends to a large extent on the altitude of the sun and the amount of disturbance of the surface, so that the results are only roughly quantitative. The maximum depth recorded is 66 m. in the Sargasso Sea, with a disc 2 m. in diameter. In the North Sea the depth varies from 5 to 12 m. in the southern part to 20 m. or more northwards of the Dogger Bank; in the open oceans 50 to 60 m. has been often recorded.

Observations made by Regnard[2] with a selenium cell, off Monaco, showed that half the light of the particular wave-length to which it was most sensitive was absorbed in the first metre, and two-thirds in the first seven metres; beyond this depth the change was very slow. He also measured the intensity by the amount of combination caused in tubes full of a mixture of hydrogen and chlorine. The result was similar to that given by the selenium cell.

Depth (metres).	HCl formed.	Depth (metres).	HCl formed.
2	79	8	10
4	25	10	9
6	13		

Helland-Hansen[3] carried out photographic measurements at considerable depths in the North Atlantic; panchromatic plates (Wratten and Wainwright) were exposed for various lengths of time, some of them under red, green, or blue filters. In clear weather in June a plate was fogged at 1000 metres in 80 minutes, but not at 1700 metres in 120 minutes, no filter being used. It is difficult to compare the effects of the filters, since they increase the exposure necessary in the open air very considerably. A plate under a blue filter was affected in 40 minutes at 500 metres, but not under a green one; red light was found to penetrate to 100 metres. At 500 metres the light still had a definite direction downwards, a plate exposed horizontally being more affected than one exposed vertically.

---

[1] *Comptes Rendus*, clxxii., April 4, 1921, No. 14, 863.

[2] *La Vie dans les eaux*, p. 205. Paris, 1891.

[3] Murray and Hjort, *The Depths of the Ocean*, p. 251.

§ (17) ABSORBED GASES.—Oxygen and nitrogen (including argon) are absorbed according to their solubility under the particular conditions of temperature and partial pressure, while the solubility of carbon dioxide is affected by the fact that it is in equilibrium with dissolved carbonates. It has often been suggested that the amount of dissolved gases in an undercurrent could be used to determine where it was last at the surface. In the case of oxygen this is incorrect; experiment has shown that there are great bodies of water very deficient in oxygen, which must have been removed by living organisms. Nitrogen might give better results, but great uncertainties are caused by the mixing of the under layers.

(i.) *Nitrogen.* C. J. J. Fox[1] has obtained the following formula for the number of c.c., reduced to 0° and 760 mm., of nitrogen and argon dissolved by a litre of sea-water of various temperatures and chlorine contents in contact with an atmosphere assumed dry, free from CO_2 and at a pressure of 760 mm. of mercury,

$$N_2 = 18{\cdot}639 - 0{\cdot}4304t + 0{\cdot}007453t^2 - 0{\cdot}0000549t^3$$
$$- Cl(0{\cdot}2172 - 0{\cdot}007187t + 0{\cdot}0000952t^2).$$

(ii.) *Oxygen.*—The formula[1] for similar conditions is

$$O_2 = 10{\cdot}291 - 0{\cdot}2809t + 0{\cdot}006009t^2 - 0{\cdot}0000632t^3$$
$$- Cl(0{\cdot}1161 - 0{\cdot}003922t + 0{\cdot}000063t^2).$$

(iii.) *Carbon Dioxide.*—The solubility of carbon dioxide in sea-water cannot be expressed simply, since it depends partly on the excess of base over acid. It is a question of physical chemistry rather than pure physics, and this is not an appropriate place to discuss it, even if it could be done in a reasonable space. The papers by Krogh,[2] Pettersson,[3] Knudsen,[4] Fox,[5] Kurt Buch,[6] and Schulz[7] may be referred to.

II. METHODS OF OBSERVATION

Temperature and Collection of Water Samples

§ (18) SURFACE TEMPERATURE.—This is easily determined by rinsing a bucket several times in the sea and then filling it and reading the temperature at once with a quick thermometer divided to fifths or tenths of a degree. The thermometer should not have any frame or case surrounding the bulb, which should be kept in the water during the reading. Canvas buckets should not be used; the evaporation from the pores causes cooling.

§ (19) BELOW THE SURFACE: MAXIMUM-MINIMUM THERMOMETER (MILLER-CASELLA MODEL).—This is a maximum-minimum thermometer of the Six pattern with movable indices, specially constructed to withstand pressure. It has been very largely employed in the past, but is now going out of use. In waters in which the temperature continually decreases downwards, it gives results as accurate as the workmanship will allow. In coastal waters and in high latitudes generally a warmer layer is often found under a colder one, and in this case it will obviously fail to record the rise of temperature. It does not collect a sample of water.

§ (20) REVERSING THERMOMETER (*Fig.* 1).—The reversing thermometer[8] has just above the bulb a small side-branch on the capillary; above this again the capillary is much enlarged and bent into a curve for a short distance. The rest of the capillary is of the usual bore, and ends at the top in a rather large secondary bulb. It contains so much mercury that this secondary bulb is partly filled, and the amount of mercury standing above the side-branch at any time will depend on the temperature. If now the thermometer be turned upside down (reversed) the mercury breaks at the side-branch and flows to the other end; the stem is graduated to be read in this position. If, as is generally the case, the thermometer

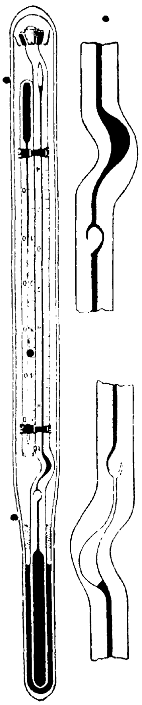


FIG. 1.

---

[1] *Pub. de Circ.* No. 41.

[2] *Meddelelser om Grønland*, Hft. 6/7, pp. 331-434. Copenhagen, 1904.

[3] *Scot. Geogr. Mag.*, 1894, pp. 300, 526, 625.

[4] *Den Danske Ingolf-Exped.* Bd. i. Hft. 1. Copenhagen.

[5] *Pub. de Circ.* No. 44.

[6] *Finnländ. hydro. biol. Untersuch.* No. 14. Helsingfors, 1917.

[7] *Ann. d. Hydr.*, 1921, 273.

[8] V. Walfrid Ekman, *Pub. de Circ.* No. 23, 1905.

becomes warmer after reversing, owing to the higher temperature of the upper layers or of the air, more mercury will expand past the side-branch; the curved enlargement is intended to trap this. In any case the broken-off thread will tend to expand and give too high a reading. To correct for this the temperature of the instrument as a whole, must be read on the small auxiliary thermometer shown in the drawing. Then if we know the "volume at 0°," that is, the volume of mercury which would be broken off at 0° C., expressed in units of the volume between two degree-lines of the reversing thermometer, and the coefficient of cubical expansion of the glass (or the apparent expansion of mercury in the glass), which quantities should be engraved on the instrument, we can calculate the correction to be applied. If a be the apparent expansion of mercury in the glass, and n be the volume of mercury broken off, *i.e.* "vol. at 0°" + $t°$, where $t°$ is the temperature shown by the reverser, then an increase of its temperature of δt will cause a rise of $an\delta t$, which must be subtracted from the reading.

Reversing thermometers are uncertain in their action, sometimes giving good results for years and then developing serious errors. For this reason they are always used in pairs. They are sealed up in a strong glass tube to protect them against pressure. An unprotected thermometer shows such a large increase of the reading due to compression that it can be used very successfully as a depth-indicator when compared with a protected one.

Reversing thermometers are used in a special frame which is caused to turn upside down at the required depth, either by a propeller or by a messenger dropped down the wire. Generally the frame is the "reversing water-bottle" described later.

§ (21) INSULATING WATER-BOTTLE AND THERMOMETER (*Fig.* 2).—The Nansen - Pettersson water-bottle[1] consists of a number of concentric cylinders, open at both ends and fixed to one another. The outer cylinder A, which alone is shown in the drawing, slides between guides B, which are really hollow tubes. The lower ends of the guides are united by a disc of metal C carrying a tap and a thick rubber washer; the upper ends are united by a cross-piece D which carries the releasing gear. A lid E with washer, similar to the bottom plate, slides on the guides above the cylinder; it carries on its upper side a strong slotted metal tube which acts as a thermometer guard and has in addition a catch which engages with the releasing gear. A delicate thermometer of the ordinary construction sealed up in an outer glass tube passes through a hole in the lid, so that its bulb projects below, and the graduations are visible above through the slots. The water-bottle is lowered to the required depth open, in the position shown in the drawing, so that the water passes freely through the cylinders. On sliding a messenger down the line the catch is released, the cylinders drop on to the bottom plate, and the lid falls and is pressed down firmly by springs contained in the guide bars. The insulating power of the various jackets of water is so great that the inner chamber will keep its temperature unchanged to a fiftieth of a degree, or less, for several minutes, up to seven, even when the water-bottle is hauled up through warmer layers of water. The insulating water-bottle cannot be used at depths greater than 400 metres with accuracy. The water enclosed expands adiabatically as the pressure is reduced, and the temperature falls. The correction may be calculated from the tables in § (49). Strictly, a correction should be made for the adiabatic cooling of the apparatus itself, but this cannot be done satisfactorily, since it is impossible to say how quickly the cooling of the outer parts affects the thermometer chamber.

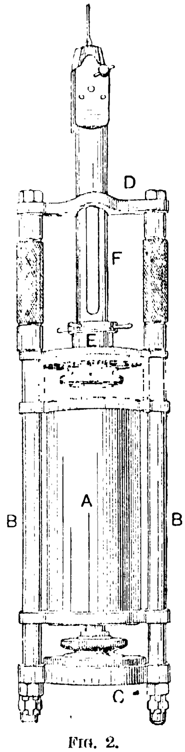


FIG. 2.

The water-bottle is extremely handy at the depths for which it is adapted, and gives the temperature and a large sample of water at the same time.

§ (22) REVERSING WATER-BOTTLE (*Fig.* 3).—This is a plain metal cylinder A revolving on trunnions in a rectangular frame B. It is provided with a cover and rubber washer C at each end, which are pulled together by a spiral spring passing down the centre of the cylinder. On the outside there are two slotted tubes D to take reversing thermometers. When set the covers are pushed away from the cylinder by rods working on cams E, so that the water-bottle is open at both ends. On releasing it by means of a messenger or a propeller the bottle turns over, the covers close down firmly, and the thermometers are reversed. Several water-bottles may be used at once at various depths on a wire, each as it closes dropping a messenger on to the next. The drawing shows the bottle in the act of reversing, having revolved about a quarter of the full distance. Messengers are much to be preferred, in spite of the time they require to reach great depths. The propeller release may reverse the bottle at 100 fathoms in a heavy swell instead of the 3000 fathoms it was hoped to reach.

---

[1] Ekman, *loc. cit.*

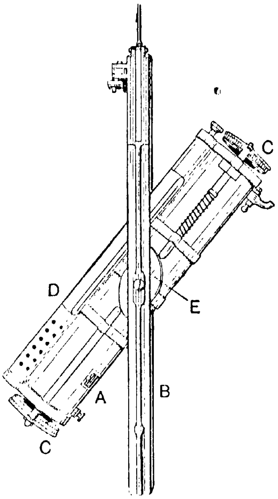


FIG. 3.

The *Richard*[1] reversing water-bottle is also adapted to carry thermometers. It is closed by two large taps instead of by rubber washers. It is both light and cheap; but great care must be used in the construction of the taps, in order to make certain that the whole of the metal is equally compressible. Any want of uniformity in this respect might lead to leakage.

A large number of water-bottles of other patterns, differing more or less from those described above, have been used from time to time. Unfortunately there is reason to believe that leakage took place with some of the earlier makes.

§ (23) STORAGE OF WATER SAMPLES.—Water can be preserved for years without change of chlorine in glass bottles if closed with rubber stoppers or washers. The milk-bottle pattern of about 6 oz. with porcelain stopper and rubber washer is suitable. Corks or ground-in stoppers should *not* be used.

Current Velocity and Direction

§ (24) THE EKMAN CURRENT METER[2] (*Fig.* 4).—This consists of a vertical spindle, suspended from a wire and turning on ball-bearings, which carries a double vane to set it head to current, a six-bladed propeller on a horizontal axle, and a means of recording the direction. The propeller is very light, and the axle runs in jewelled bearings and is connected with counting dials by a worm gear. It is lowered to the required depth locked. A messenger is dropped down the wire and releases the mechanism; a second locks it again. The time between the arrival of the two messengers, which may be five or ten minutes, with the number of revolutions recorded, give the velocity of the current by a formula which has to be determined for each instrument.

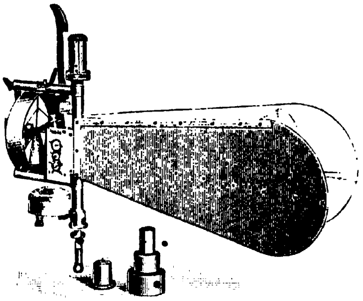


FIG. 4.

The method of recording the direction is very ingenious. Below the recording gear is a cylindrical box with its axis vertical, the bottom of which is divided into sectors of ten degrees each by raised radial partitions. In the centre is a vertical pivot on which swings a heavy compass needle, the arms of which slope slightly downwards; one arm is grooved on the upper surface. The gear-box contains a hopper full of small bronze balls, and at intervals the revolutions of the propeller cause one of these to be released and drop on to the needle. The ball runs down the grooved arm and falls into the compartment below the end of it. The mean direction of the current can be calculated from the distribution of the balls in the compartments; with a steady current they will all be found in two adjacent to one another.

The Ekman meter has done good service, but it appears to be too fragile for work in strong currents.

It obviously must be used from a ship at anchor, or from some steady support. F.

---

[1] *Bull. Inst. Océan. Monaco*, No. 374, July 25, 1920.

[2] *Pub. de Circ.* No. 24.

Nansen[1] has devised a buoy with clockwork releasing gear for use when it is impossible to anchor.

§ (25) NANSEN CURRENT METER.[2]—The Nansen current meter consists of a rectangular vertical frame within which hangs a vertical pendulum with vanes and a sharp point at the lower end. It swings over the centre of a compass card coated with wax and directed by two strong magnets. The current deflects the pendulum from the vertical by an amount which varies as the square of the velocity. At intervals the pendulum is lowered on to the compass card by clockwork and the point makes a mark in the wax. The distance of the mark from the centre of the card, as shown by concentric rings on the latter, gives the velocity, and the direction is read off directly. Three different sizes of pendulum are used to suit currents of various strengths. The meter can be used either in a tripod sitting on the bottom, or from a ship at anchor.

§ (26) THE GURLEY OR PRICE METER is employed in America for river gauging, but does not appear to have been used for oceanographical work. It is well worth a trial. It is not unlike a Robinson anemometer, having six conical cups on very short arms on a vertical spindle. A vane is provided to keep it head to current. At every revolution, or at every five revolutions, an electrical circuit is completed and gives a signal in a telephone at the surface, with which it is connected by a cable. The necessity for a cable confines its use to moderate depths.

§ (27) THE LEVEL CURRENT METER,[3] invented by Jacobsen, is adapted particularly for use on lightships. A bracket projecting over the side of the ship carries on gimbals a frame on which are two circular spirit levels, and a short tube projecting downwards from the centre for a short distance. A wire is led from a drum over a counting block and down through the tube; at the lower end is a sinker and a light cylinder which offers resistance to the current. The stray on the line, which depends on the current, deflects the tube and frame carrying the levels. The position of the bubbles in the latter gives the data for calculating the strength and direction of the current.

§ (28) EFFECT OF TURBULENCE ON CURRENT METERS.—Turbulent movement affects the various types of current meter differently. For all meters it will be found that if observations are made in turbulent water consistent results will not be obtained unless the duration of the experiment is fairly long. Comparisons made in America[4] show that the Gurley (Price) meter, which is of the cup type, registers too high in turbulent water, while meters of the screw type, on horizontal axes, read too low.

§ (29) EFFECT OF UNSTEADINESS OF THE SUPPORT.—One of the great difficulties in the use of current meters from an anchored ship has been the excess velocities set up by her swings. Obviously, a ship might be swinging violently where there was no current; the meter would be continually dragged through the water and record a high velocity. Nansen[5] has described many methods which have been used in attempts to surmount the difficulty.

§ (30) SURFACE CURRENTS can be measured by allowing some sort of floating drag, such as the old ship's log, or a tow-net properly buoyed, to carry out a marked line. The time required to carry out a known length of line is measured and the speed calculated from this. The "drift bottles," soda-water bottles closed and weighted to just float, have been used for experiments on the great ocean currents. They are thrown into the sea and picked up elsewhere, generally on the shore, at some later date. They can of course only give a very rough idea of the strength and direction of the current. There are cases where bottles appear to have circumnavigated the globe. Other methods of measuring surface currents are described by Wharton and Field.[6]

§ (31) WIRE AND WINCHES FOR USE WITH APPARATUS.—For all the gear described above a flexible steel cord of about 1000 lbs. breaking strain is amply strong. Such a cord may be composed of, say, 19 single wires twisted together, or 4 strands of 4 wires each, or if great flexibility is required of, say, 6 strands of 11 wires each with hemp hearts. The total diameter will be about 3 mm. For current meters it is better to use a smaller cord, which presents less surface to the water; 300 lbs. breaking strain is sufficient. Valuable apparatus should never be used on the single piano wire employed on sounding machines.

The wire is stored on a strong iron or steel drum, which may be driven by hand when used in depths of, say, 200 fathoms. For deeper water a power-drive is necessary. The wire paid out is measured by running it over a block with a counting device and a sheave of accurately known diameter.

Determination of Salinity and Density

There are only two methods in general use; either the chlorine or the density is determined directly and the other quantities calculated from it.

§ (32) HYDROMETERS.[7] (i.) *Common Hydrometer.*—This is no longer used, as its accuracy is not sufficient for modern work; the error on the average is ± ·2 of salinity. It is an elongated bulb with a fine stem and is weighted so that the stem is partly above the surface. The stem is graduated to read densities according to one of the many standards which have been used, and the number of these is one of the objections to its use. Scandinavian and German instruments give the density at 17·5° compared with that of distilled water at the same temperature if the reading is made at 17·5°. English hydrometers are graduated to read accurately at 15° or 15·56°, the unit of density being distilled water, sometimes at 15·56°, sometimes at 4°. If the temperature of observation is different from that for which the instrument is made, a correction must be applied. Knudsen's Tables give this for the continental form for several kinds of glass.

---

[1] *Pub. de Circ.* No. 34, 1906.

[2] *Ibid.* No. 34, 1906.

[3] *Ibid.* No. 51.

[4] Groat, *Trans. Amer. Soc. Civil Engineers*, lxxvi., lxxx.

[5] *Pub. de Circ.* No. 34.

[6] *Hydrographic Surveying.*

[7] See article "Hydrometers."

(ii.) *Buchanan's Hydrometer.*—An instrument to read all densities from 1 to 1·03 or a little more would have an inconveniently long stem if the scale were open. This difficulty may be got over either by the use of a number, each covering a short range, or by the form which was used by J. Y. Buchanan on the *Challenger*. This has a comparatively short scale, divided to millimetres, and can be made to float at a suitable height in any sea-water by the addition of small weights to the upper end of the stem. It is placed in the water under examination at an accurately known temperature t°, and weighted till it floats at a certain mark. It is then put in distilled water at the same temperature and again weighted till it floats at the same mark. The volume of water displaced is the same in each case, but the weights of these volumes are directly as the sum of the weight of the instrument and the small necessary additions. In practice the weights necessary for distilled water at various temperatures are determined beforehand, and the weight required in each case calculated by interpolation from readings on each side of the mark. It will be seen that great accuracy in the determination of the temperature is required, and that ample time must be allowed for equilibrium to be attained. A much more serious source of inaccuracy is the surface tension of the water; the slightest trace of grease, such as is almost unavoidable on a ship, causes so great a change in this, and consequently in the ring of water which rises round the stem, as to affect the readings considerably.

(iii.) *The Total Immersion Hydrometer* avoids the latter difficulty. It is an elongated bulb with a very short stem, and in use small weights are dropped over the latter and the temperature adjusted until it swims in mid-water, when obviously the density of the water and the hydrometer are equal. With care the results are correct to 5 in the sixth place of decimals.

Another form has been suggested and is now under trial; instead of weights it carries at its lower end a fine gold chain, and the length of this which is lifted off the bottom depends on the density of the water. The reading is made by bringing the top of the bulb in line with marks etched on the trial jar.

The hydrometer is not to be recommended. The stem form is uncertain, owing to changes in the surface tension, and all forms are very sensitive to temperature and tedious in use if passable results are to be obtained.

The *pycnometer*[1] need not be described here. It is very accurate when properly used with a counterpoise of similar size and shape made of the same glass, but it is far too slow for routine work. The best results are obtained by working at the melting-point of ice.

§ (33) Chemical Determination of Chlorine.—The salinity and density can, of course, be determined from the "chlorine" content by Knudsen's Tables. There are two volumetric methods in use, both of which are very accurate. Volhard's method is probably the more exact, but it is too slow and requires too much manipulation for routine work. It will be found in any good text-book of volumetric analysis.

The method almost universally employed is Mohr's. It is quick, and even when the most accurate results are required they can probably be obtained with less trouble by taking the mean of, say, four determinations than by Volhard's method.

The accurate analysis of a large number of samples without the errors due to fatigue depends so much on the smaller details that the method is described here fully.

If a solution of silver nitrate be added to a neutral or faintly alkaline solution of a chloride, such as sea-water, a white curdy precipitate of chloride of silver is thrown down, while if it be added to a solution of a chromate, such as chromate of potash, a red precipitate is formed. If the silver solution be added to a mixture of chloride and chromate, a white precipitate is formed at first, and then as soon as the last trace of chloride has been precipitated the red colour of chromate of silver appears. In order to attain the requisite accuracy special burettes and pipettes are necessary.

The pipette (*Fig.* 5) generally used is of the Knudsen pattern, with a three-way tap instead of a mark: water is sucked up through one branch and allowed to flow out by admitting air through the other. The upper stem should be long enough to allow of the pipette being held by it without heating the bulb, and the lower should reach to the bottom of the sample bottle. The capacity may be from 10 c.c. to 15 c.c.; it need not be known with the highest accuracy since the method is purely comparative. When running the water out it should be held vertically, and as soon as it is empty except for the drop at the bottom the point should be held against the side of the tumbler for some invariable length of time, say 10 seconds. The method of blowing out the last drop by warming the bulb in the hand is not to be recommended, as it introduces uncertainties into the measurement of the next sample. When clean, such a burette is accurate to about one part in four thousand. Ostwald's tap grease is the best, but there is no kind that does not soil the pipette sooner or later.

---

[1] See "Balances" § (15), Density by means of Pyknometer.

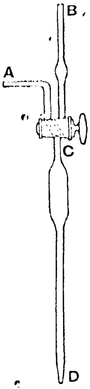


FIG. 5.

The burette (*Fig.* 6) is of the bulb form; the larger part of the capacity, which is not generally required, is expanded into a bulb at the top so as to decrease drainage errors. Instead of a zero mark there is a fine jet; each time the burette is filled a little of the silver solution is allowed to overflow into the cup. The unit of graduation is equal to 2 c.c. and the graduation is from 16 to 21·5 or from 18 to 24, corresponding to salinities of about 27·9 to 38·8, or 32·5 to 43·3. The length of a unit should be about 4 cm. and should be divided into 20 parts. The burette shown in the figure is provided with glass taps, and the lubricant is a continual source of grease and dirt. A much more convenient method is to use thick-walled rubber tubing (pressure tubing) and strong screw clips. A wooden stand is better than a metal one, and should be made to support the rubber firmly on both sides of the clip. With such an arrangement the burette will remain clean for years, except for a slight dark deposit of reduced silver which does no harm. The calibration of course should be done with the greatest accuracy.

The silver nitrate solution should be made from the fused salt, in order to avoid any trace of acidity, and should be of such a strength that when a pipetteful of the standard is titrated the burette reading is the same as the chlorine marked on the sealed tube within 0·145. The standard generally contains about 19·38 parts per thousand of chlorine; in this case, with a 10 c.c. pipette, the silver nitrate solution should contain 24·50 grm. per litre.

The indicator is a 10 per cent solution of neutral chromate of potash. The titration is made in plain thin glass tumblers; a tray is provided with compartments to hold 36 tumblers, and the compartments and tumblers are numbered.

The standard water (normal water of the International Council) is emptied from a sealed tube into bottles of the kind used to hold the samples, generally 6-oz. or 7-oz. milk bottles closed by porcelain stoppers and rubber washers held down by swing catches of wire. The bottles are allowed to stand for some hours, or better overnight, near to the burette and stock solution, so that everything takes the same temperature. Exposure to the rays of the sun or any other source of irregular heating should be carefully avoided. Four pipettefuls of standard water are measured into the first four glasses, and then the samples to be analysed. All should thus be at the same temperature. One of the standard waters is then analysed. Two drops or more of the indicator are added to the water in the tumbler, and the silver nitrate is run in with constant stirring, quickly at first, and then slowly as the red colour becomes more lasting. Finally it is added a drop at a time until the red colour is permanent for half a minute. It is better to titrate to the first slight change in colour of the precipitate, not much more than a slight soiling, rather than to a distinct red. Ten of the samples are then titrated, then a standard, and so on. The advantage of measuring out a number of standards and samples at the beginning is that it is possible to break off the work when only half done if necessary, and resume it later without causing any inaccuracy. The results of course are read in silver per volume of water. They must be calculated to weight, and this is easily done by the corrections given in Knudsen's Tables, provided that the silver nitrate has been made up of the correct strength within 0·15.

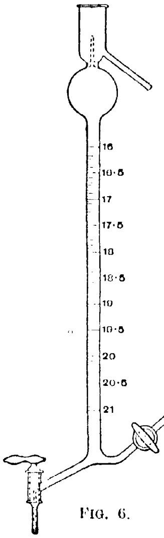


FIG. 6.

The method is very accurate when once it has become mechanical, and a difference of more than 0·01 Cl between duplicates is a sign of something wrong, generally either grease in the apparatus or a chipped point on the pipette. Sea water of low salinity, less than 20, need not be analysed with quite the same accuracy as a rule, and a good cylindrical burette of the ordinary pattern is sufficient.

§ (34) Determination of Salinity by Optical Methods.—The refractive index can be measured with a refractometer to about 0·00005, corresponding to say 0·2 salinity, which is inadequate for accurate work. The Rayleigh interferometer is capable of giving all the accuracy necessary, and since it is a comparison instrument it is almost unaffected by temperature, but in its present form it is unsuited to routine use. The optical method should be of great value if used at sea for preliminary determinations, but it cannot as yet replace the chemical method or the direct observation of density. It remains to be proved that the refractive index bears a constant ratio to chlorine and σ_0, and in any case there is considerable danger of its being affected by glass dissolved from the storage bottles.

§ (35) Determination of Salinity by Electrical Methods.—The temperature coefficient of conductivity is considerable, about 3 per cent per degree. It is possible to arrange the apparatus in a suitable bath so that differences of temperature are eliminated, but the objections to the optical method apply here with even greater force, and indeed recent experiments have shown that in some areas there are wide differences between the results of the chemical and conductivity methods, while in others the agreement is excellent.

III. The Theory of Ocean Currents

§ (36) The cause of ocean currents has been a fruitful source of controversy. Some have imagined it to be some action analogous to the forces which raise the tides, others consider differences of density the moving force, and a third school attributes them to wind friction. Otto Pettersson has attached great importance to the effect of melting ice, a special case of differences of density. In the opinion of the present writer the friction of the permanent and semi-permanent winds far outweighs the other factors. Differences of density are of less importance, except in some special areas, though they have great influence on the character of the water in any place where vertical convection currents can be set up, and are undoubtedly responsible for the slow creep of the cold Polar waters along the bottom towards the equator. Melting ice has been observed to set up strong currents in its neighbourhood, both in the open sea and in the Scandinavian fjords, but its influence is probably only local. Difference of barometric pressure between two places over the sea can give rise to currents which, however, are only transitory; equilibrium is soon reached, and the currents cease. The factors which modify a current already set up are the rotation of the earth, the frictional resistance of the water particles among themselves or against the bottom, and the coastal configuration. Finally, it must be remembered that water, unlike the atmosphere, is inextensible, and that if it is removed from any area, as, for example, by a wind-drift, other water must flow in to replace it.

§ (37) Effect of the Rotation of the Earth.—It is convenient to consider this first, since it is universal, and can be calculated accurately. According to the well-known law, any particle moving on the surface of the earth is subject to a force which tends to accelerate it to the right of its path in the northern hemisphere and to the left in southern latitudes.[1]

For unit mass

$$F = 2\omega v \sin\phi,$$

where ω is the earth's angular velocity of rotation, $2\pi/86164$ secs. $= \cdot 00007292$, v the velocity of the particle, and ϕ the latitude. The force is therefore zero at the equator and a maximum at the poles, and is independent of the direction. Consider the case of a particle moving with constant speed on the surface; its path, if otherwise undisturbed, will be curved. If r is the radius of curvature, the tangential force is v^2/r, and this is in equilibrium with the deviating force, so that

$$v^2/r = 2\omega v \sin\phi,$$

and

$$r = \frac{v}{2\omega \sin\phi}.$$

Since r changes with the latitude the path would be a series of loops, were it not for the fact that in nature the force which set the particle in motion in the first instance continues to act unchanged for an appreciable length of time. If v be 1 metre per second and r in kilometres we have the following values for the latter:

Lat.	r.	Lat.	r.
0	∞	40	10·7
2½	157·2	50	9·0
5	78·7	60	7·9
10	39·5	70	7·3
20	20·0	80	7·0
30	13·7	90	6·9

These quantities are often required in theoretical investigations. The effect of the deviating force is difficult to measure owing to other disturbing factors which have to be taken into consideration, but numerous observations of it have been made, especially in the Baltic, where the conditions are relatively simple. Dinklage[1] found that at the Adler Ground Lightship here the mean deviation of the drift was 28° to the right of the wind and that deviations to the left were uncommon. The theoretical value for an ideal boundless ocean of infinite depth is 45°, but, as will be shown later, a correction must be applied on account of the small depth which brings theory and observation into better agreement. Similar confirmation has been obtained by examination of ships' logs from the Mediterranean and Indian Ocean.

---

[1] See article "Atmosphere, Physics of the," § (8)

§ (38) Skin Friction or Tangential Pressure of the Wind on the Sea.—Colding[2] found that the relation between the wind velocity w and the height to which the water was piled up against the shore by the great storm of November 1872 in the Baltic could be expressed by a formula which in cms and secs. is

$$w = 14450\sqrt{d \sin \gamma},$$

d being the depth and γ the angle between the surface of the water and the horizontal plane. Ekman (§ (55)) has found the relation

$$\text{Sin } \gamma = \frac{3}{2}\frac{T}{\rho g d},$$

where T is the tangential force, ρ the density of the water, and g gravity. Combining the two equations,

$$T = 0{\cdot}0000032w^2,$$

which may be written

$$T = 0{\cdot}0025\rho_A w^2,$$

where ρ_A is the density of air.

G. I. Taylor[3] has obtained the value

$$T = 0{\cdot}002\rho_A w^2$$

for the friction of the wind over grass from measurements made on Salisbury Plain. The agreement of the constants is remarkable considering the great difference which there probably is between the effective roughness of grass and a shallow sea during a violent storm.

§ (39) Skin Friction of a Current on the Bottom.—Taylor's results given above were obtained at wind velocities of 6 to 30 miles per hour. According to the law of similitudes the same formula should hold good at velocities of from 6/11 to 30/11 miles per hour for the friction of an ocean current on the bottom, assumed to be of the same roughness as grass, the density of water being used instead of that of air.

§ (40) Relation between the Velocities of Wind and Current.—Nansen found the relation[4]

$$v = {\cdot}019w$$

for an ice-covered sea, where v is the velocity of the surface current and w that of the wind. H. Mohn[5] selected from the wind and current charts of the regions within 20° of the equator those cases where their directions coincided approximately, and obtained the relation

$$v = {\cdot}047w.$$

Mohn's original figures have been recalculated, using Koppen's values for the conversion of Beaufort strengths of wind to metres per second. If Ekman's theory (§ (50)) that the constants should be in the inverse ratios of the square roots of the sines of the latitude is correct, then the relations would agree if we assume that the latitudes were 82° and 3°. In any case, however, modern theory shows that the selection of the cases where the directions of wind and current are the same is inadmissible. Dinklage[6] obtained from observations at the Adler Ground Lightship in the Baltic the relation $v = {\cdot}013w$ for winds of 6·2 metres per second or less, and $v = {\cdot}014w$ for higher velocities. R. Witting[7] investigated a large number of observations made at Finnish lightships and deduced the relation $v = 0{\cdot}48\sqrt{w}$, or if Koppen's values for the reduction of the Beaufort scale are used, $v = {\cdot}44\sqrt{w}$, v and w being in cm. per sec.; he found that an expression of the form mw, where m is a constant, could not be made to fit the observations so well. Gallé[8] examined the records in ships' logs for the Indian Ocean and found that if only the wind component in the direction of the current was used the relation was $v = {\cdot}044w$. In shallow waters, such as the Baltic, the current consists of a pure wind-drift down to the bottom, while according to Ekman there is, in deep water near a coast, a wind-drift resting on another current caused by the piling up of the water. H. Thorade[9] attempted to separate the surface movement into the surface drift, which alters rapidly with change of wind, and a deep current which only changes slowly. He came to the conclusion from material contained in ships' logs that with a given wind the pure drift current varies inversely as the square root of the sine of the latitude, and that for low wind velocities the relation is

---

[1] *Ann. d. Hydr.*, 1888, p. 1.

[2] *Danske Vidensk. Selskabs Skrifter, Natur. og Math. Afd.*, 1881, i. No. 4.

[3] *Proc. Roy. Soc.* A 92, 1916, p. 196.

[4] "Oceanography of the North Polar Basin," *The Norwegian North Polar Expedition, 1893–1896*, No. 9 (Christiania, 1902).

[5] "Density, Temperature, and Currents," *Norwegian North Atlantic Expedition, 1876–1878* (Christiania, 1887).

[6] "Die Oberflachenströmungen in Südw. Tle. d. Ostsee," *Ann. d. Hydr.*, 1888, p. 14.

[7] *Ann. d. Hydr.*, 1909, p. 193

[8] *Meded. en Verhand. Kon. Ned. Met. Inst.* (Utrecht, 1910), No. 9.

[9] *Ann. d. Hydr.*, 1914, p. 379.

$$v = \frac{\cdot 259\sqrt{w}}{\sqrt{\sin\phi}},$$

for high velocities

$$v = \frac{\cdot 0126w}{\sqrt{\sin\phi}}.$$

§ (41) Coastal Configuration.—The general circulation of the ocean is broken by the land masses which confine it to relatively small areas in which the water rotates, clockwise, in the northern halves of the Atlantic and Pacific Oceans, and in the opposite direction in their southern halves. It is only in high southern latitudes that the currents find a free sweep, and here they flow eastwards round the world. The continents also influence the currents indirectly by their effect on the distribution of atmospheric pressure, and hence on the winds and wind-drifts. The most striking instance of this is found in the northern part of the Indian Ocean, where the currents are reversed twice in a year by the monsoons set up by the Asiatic continent.

On a smaller scale the influence of the coast line is generally beyond the reach of mathematical treatment on account of the many complications encountered, but on the whole it is what might be expected by any one who has watched a river flowing past piers and over falls. A current tends to flow parallel to a coast on which it impinges, and when the coast lies to its right, in the northern hemisphere, the current follows it closely, as, for instance, the East and West Greenland currents. If the current meets a coast, even at a very small angle, eddies are formed on both sides of it which rotate in opposite directions. A current meeting a coast nearly at right angles may be split into two parts, as the South Equatorial current is split on Cape San Roque. The eddies formed are often so large as to deserve the name of currents themselves; many charts are misleading on this point, and show the water as moving in smooth regular curves.

Krümmel[1] has described some interesting experiments made in tanks shaped roughly to resemble the oceans; and if due regard be paid to the law of similitude, valuable quantitative information may be obtained as to the probable effect of obstructions in estuaries. Ekman (§ (54)) has investigated mathematically some simple cases of the influence of the coast line on wind-drifts.

§ (42) Currents due to Melting Ice.—Otto Pettersson[2] has advanced a theory according to which the melting of the Polar ice is the cause of currents, not only in its neighbourhood but also in regions far removed. It is explained by *Fig.* 7, which shows one of a large number of experiments made by J. W. Sandstrom. A block of ice floats at one end of a tank of sea-water and melts in it. As the face of the block melts the water in contact with it is so much diluted that, in spite of its low temperature, it floats and flows away as a surface current. On the under side of the block the water is strongly cooled without undergoing much dilution, so that it sinks and flows away as a bottom current. Between the two a mid-water current moves towards the ice. It is highly probable that such a circulation takes place near the ice-fields, and it has been measured in detail in fjords by Sandstrom; but the total amount of ice melted in the Polar regions in each year appears to be too small to account for the great ocean currents.

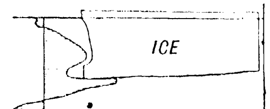


Fig. 7.

§ (43) Compensation Currents.—Since water is inextensible, it follows that if it be removed from any area, as, for instance, by the wind, other water must flow in to take its place. The compensation generally takes place horizontally, but where the wind blows away from the coast the rate of removal may be so great that water is also sucked up from the depths, and can usually be recognised by its salinity and temperature. Thus low surface salinities occur along the dry riverless south-east coast of Arabia, and low temperatures along the west coast of Africa in the Trade Wind area. On the small scale the phenomenon is well shown in the Baltic. Krümmel[3] quotes a case where, in August in the Memel Deep, after long-continued east winds, the coastal temperature fell from 19° to 8° in the course of a day while it was still 18° at four or five miles off shore.

In the same way a wind-drift impinging on the coast escapes partially downwards and carries the surface temperature to the depths, as on the west side of the Atlantic Ocean in the path of the Equatorial Currents.

Compensation currents may occur far from land. The Guinea Current is a horizontal compensation current flowing eastwards between the two westerly Equatorial Currents of the Atlantic.

One of the greatest compensation currents is found at considerable depths in low latitudes, where the cold bottom water rises towards the surface to replace that carried away by the rapid wind-drifts of the equatorial regions. On the east side of the Atlantic Ocean the 5° isotherm rises to 600 m. under the equator, while in 30° N. lat. it is found at 1500 m.

---

[1] *Ozeanographie*, 1911 ed., ii. 471.

[2] *Geogr. Journal*, 1906, p. 285; 1907, p. 279; and elsewhere.

[3] *Ozeanographie*, 1911 ed., ii. 536.

§ (44) THE VERTICAL CIRCULATION.—In addition to the vertical compensation currents we have those due to changes of density. With the possible exception of rising currents in the great depths they have their origin in an increase of the surface density either by evaporation, as in the Trade Wind areas, or by cooling. A decrease in the surface density by heating or by dilution can obviously only increase the stability, so that ascending currents of this type cannot arise except indirectly as the result of displacement by descending currents. The effect of such changes of density is well shown in the English Channel and the North Sea. In the winter the water is of nearly the same temperature and salinity at all depths. As spring advances the surface gains more heat by day than it loses by night, so that it becomes lighter and the vertical circulation is impeded. This makes it possible for the land-water to collect in the upper strata and form a layer of low salinity and density. Once such a layer is formed the vertical circulation is confined almost entirely to it, since the nightly cooling of the surface water cannot make the latter heavy enough to break through into the dense lower layer. The heating of the surface then becomes more rapid, so that in extreme cases the temperature has been found to fall at the rate of one degree per metre on passing from one layer to the other. Such a layer of rapid change is known as the discontinuity layer, thermocline, or in German as the sprungschict.

An interesting case of vertical currents due to evaporation is found in the Trade Wind area, where the descending water probably makes its way equatorwards, and there continually renews the layer of relatively high salinity which underlies the fresher surface water of the equatorial rain belt.

§ (45) THE BANK WATER is a phenomenon which is probably due to vertical circulation. Nansen[1] showed that in regions where the surface water flows from areas where rainfall is low to others where it is high, as in the north-eastern Atlantic Ocean, the salinity over shallow banks at some distance from the nearest land is often less than over the surrounding deep water. He explained this on the supposition that the natural depth of the circulation is greater than the soundings on the banks, so that rain falling on the latter is mixed with a smaller proportion of sea-water than is the case over the deep water. This explanation necessitates relatively slow horizontal movements over the banks.

§ (46) THE BOTTOM WATER of the great oceans at depths over about 2000 m. has a temperature of 3° or less, except in areas which are cut off from the general circulation by submarine ridges rising towards the surface. The bottom temperature is lowest where the connection with high latitudes, and high southern latitudes in particular, is most open. It is generally agreed that this bottom water is due to descending currents in the regions surrounding the poles, but the exact place and mode of formation is disputed. Otto Pettersson holds that it is due to cold descending currents from melting ice (§ (42)). Nansen[2] has attacked this theory on the ground that by far the larger portion of the ice is shallow pack which, in the North Polar Basin at any rate, floats in a layer of low salinity formed by the discharge from the rivers which drain into the Arctic Ocean, so that it is completely cut off from a vertical circulation reaching to the bottom. He supports this view by experiments. If ice is floated on warmer sea-water in a jar, the vertical circulation ceases as soon as a surface layer of low salinity has been formed, but if it is enclosed in a metal box, which either floats or is suspended just above the surface, the vertical circulation to the bottom of the jar continues as long as any ice is left. The last case corresponds to radiation from the sea to the sky. It is obvious, on Nansen's hypothesis, that the bottom water can be formed only where the density, apart from compression, is nearly the same at all depths. He suggests one place, for instance, east of Cape Farewell where, in autumn, the surface salinity is slightly less than that on the bottom. During the winter the salinity is raised by the formation of ice which is continually broken up by the wind so as to expose a fresh surface of water to the air. In late winter and early spring the salinity has become equal to that on the bottom and the temperature is a little lower. The dense water sinks and is warmed adiabatically by compression until its temperature is the same as on the bottom.

According to this hypothesis the bottom water is due to the formation of ice, and not to its melting. The localities suggested by Nansen for its origin are all over deep water, but the observations of the *Deutschland* in the Weddell Sea led Brennecke to the conclusion that, in southern latitudes at least, it may be formed on the shallows.

§ (47) VERTICAL EQUILIBRIUM AND STABILITY.—It has generally been the custom to discuss questions of vertical equilibrium, such as that in the Trade Wind areas (§ (44)), on the basis of σ_t only, and to neglect the effect of pressure. For instance, it was assumed that if water at the surface was slightly denser than another body of water at a greater depth, the density in each case being referred to atmospheric pressure, the former could in all cases sink below the latter. An advance was made when the adiabatic heating due to compression and the resulting decrease in density were taken into account, but for a complete discussion it is necessary to consider the effect of differences of salinity also since they affect the compressibility. The problem has been fully treated by Th. Hesselberg and H. U. Sverdrup.[1]

---

[1] "Das Bodenwasser und die Abkühlung des Meeres," *Internat. Rev. Gesamt. Hydrobiol. und Hydrographie* (Leipzig, 1912).

[2] *Loc. cit.* § (45).

At the depth z metres let salinity, temperature, pressure in decibars, and density be s, t, p, and $\rho_{s,\,t,\,p}$; and at the greater depth $z+dz$ let the corresponding quantities be $s+ds$, $t+dt$, $p+dp$, and $\rho_{s+ds,\,t+dt,\,p+dp}$. Let a water-particle A be brought down from the depth z to another particle B at the depth $z+dz$. It will be heated adiabatically by an amount $d\zeta$, and the corresponding change in density will be $(\partial\rho/\partial t)d\zeta$, so that the density will now be

$$\rho_{s,\,t,\,p+dp}+\frac{\partial\rho}{\partial t}d\zeta.$$

If A is now lighter than B it will tend to rise again; if its density is the same as that of B it will remain at rest in indifferent equilibrium, and if the density is greater it will tend to sink still deeper.

The difference of density

$$\Delta\rho=\rho_{s+ds,\,t+dt,\,p+dp}-\left(\rho_{s,\,t,\,p+dp}+\frac{\partial\rho}{\partial t}d\zeta\right)$$

is then a measure of the vertical stability between z and $z+dz$. The stability at a point can be defined as

$$\mathrm{E}=\frac{\Delta\rho}{dz},$$

$\Delta\rho/dz$ being distinguished from $d\rho/dz$, which is the change of density downwards apart from any movement in the water.

Since

$$\rho_{s+ds,\,t+dt,\,p+dp}-\rho_{s,\,t,\,p+dp}=\frac{\partial\rho}{\partial s}ds+\frac{\partial\rho}{\partial t}dt,$$

$$\mathrm{E}=\frac{\partial\rho}{\partial t}\left(\frac{dt}{dz}-\frac{d\zeta}{dz}\right)+\frac{\partial\rho}{\partial s}\frac{ds}{dz}.$$

If the salinity is constant, as is the case in great depths, the term $\partial\rho/\partial s\, ds/dz$ vanishes. Bruno Schulz[2] points out that it is more correct to multiply dt/dz by $\partial\rho/\partial t$ calculated for the mean temperature $t+\frac{1}{2}dt$. Hesselberg and Sverdrup[3] give tables to facilitate the calculation of the stability founded on Ekman's figures for compressibility and the adiabatic heating. The stability varies greatly from place to place. In the Atlantic in 28½° N., 19° W., in May, in an instance examined by the last-mentioned authors, it was negative from the surface down to a depth between 25 m. and 50 m., then rose to a very high value, after which it decreased downwards and became neutral at 5000 m. At this station the temperature and salinity were nearly constant down to 50 m. Schulz[4] gives two instances, from deep water near the Philippines and New Britain, where it was very high near the surface owing to an increase of salinity and decrease of temperature with the depth, became neutral at from 5000 m. to 6000 m., and below these depths was negative down to the bottom.

§ (48) CONDITIONS IN A PARTIALLY ENCLOSED SEA.—In a sea cut off from the general bottom circulation by submarine ridges the bottom is filled by the densest water which has access to it. This may be derived either from the surface during the winter or from the stratum of water in the surrounding sea at the greatest depth found on the ridge, and since, on the whole, temperature has more influence than salinity upon the density, it may be said that the bottom water will come from the colder of these two sources. It was at one time believed, chiefly on the ground of observations made with the maximum-minimum thermometer, that the temperature was uniform from the saddle depth to the bottom, and indeed this instrument could not have shown a rise. Later investigations with the reversing thermometer show that in many cases the temperature reaches a minimum and then rises again with increasing depth. Such conditions are found in the Arctic Ocean,[5] the Mediterranean Sea,[6] and the western Pacific Ocean.[7] This appears to be due, in part at least, to the adiabatic heating of the water by compression as it sinks (§ (49)). In the Mediterranean the heating is slightly less than is demanded by theory, in the Arctic Ocean it is slightly more, and in the remarkable depressions of the western Pacific near the Philippines and New Britain, the excess reaches 0·3° or 0·4° on the bottom. The minimum temperatures occur at the level of the bottom of the surrounding sea, and from this depth downwards, in the western Pacific, the salinity is constant and the temperature excess increases at the rate of 0·1° per 1000 m. The cause of the excess is somewhat obscure. The normal outflow of heat from the earth's crust is comparatively small and would require nearly 200 years to produce the observed effect, but it is scarcely possible that the water can have been stagnant so long without having been at the same time so completely mixed by convection currents that the increase of temperature downwards would have been obliterated. It is possible, as Schott suggests, that the excess of temperature is due to the volcanic nature of the neighbouring lands.

---

[1] "Die Stabilitätsverhältnisse des Seewassers," *Bergens Museums Aarbok*, 1914–1915, No. 15.

[2] "Die Beurteilung des vertikalen Gleichgewichts im Meere," *Ann. d. Hydr.*, 1917, p. 93.

[3] *Loc. cit.*

[4] *Loc. cit.*

[5] Fridtjof Nansen, *The Norwegian North Polar Expedition*, iii. 341 (Christiania, 1902).

[6] V. W. Ekman, *Pub. de Circ.* No. 43; and J. N. Nielsen, *Danish Oceanographical Expedition to the Mediterranean*, 1908–1910, i. 142 (Copenhagen, 1912).

[7] G. Schott, *Ann. d. Hydr.*, 1914, p. 321.

§ (49) ADIABATIC TEMPERATURE GRADIENT.—If a small volume of water-particle be compressed, by sinking towards the bottom of the sea for instance, its temperature will be raised by an amount given by the formula

$$\delta t = \frac{\mathrm{T}e}{\mathrm{J}\rho c_p}\delta p,$$

where T is absolute temperature, J the mechanical equivalent of heat, ρ the density, c_p the specific heat at constant pressure, and p the pressure; and if it be decompressed it will be correspondingly cooled. In each case it is assumed that no heat is gained or lost by conduction. If the temperature of a body of water increase downwards at such a rate that a water-particle rising from the bottom will reach each depth with the temperature that already prevails there the gradient is said to be adiabatic. W. Ekman[1] has calculated tables of the adiabatic change. These are given on the following page.

Suppose, for example, that it is wished to determine the cooling which a body of water with an original temperature of 2° and $\sigma_0 = 28$ will undergo if raised to the surface from 10,000 m. The mean cooling per 1000 m. from 10,000 m. to 8000 m. is 0·191°. It will therefore be cooled by approximately 0·4° between these depths, so that at 8000 m. its temperature will be 1·6° and at 9000 m. 1·8°. Using the last value as the mean temperature, the average cooling per 1000 m. is found to be ·189°, and the temperature at 8000 m. is 1·622°. The cooling for the other depths is calculated in the same way step by step. G. Schott[2] has rearranged Ekman's tables and also drawn graphs of δt, which are more convenient for some purposes and practically as accurate.

V. W. Ekman's Theory of Currents

§ (50) THE WIND-DRIFT IN AN IDEAL UNIFORM BOUNDLESS OCEAN.—Fridtjof Nansen[3] pointed out that a surface current due to the wind should deviate to the right of the latter (in the northern hemisphere), and that this surface current should in turn act like a wind on the layer beneath it, and set it in motion with an increased deviation and a diminished velocity so that at a certain depth the direction of the current should be reversed. V. W. Ekman[4] has investigated the question mathematically, and some of his more important results are given here.

Suppose that a steady uniform wind has been blowing over the whole surface of the ocean so long that stationary conditions have been reached. The motion of the water may, in the first instance, be considered to consist of the gliding of the layers one over the other. Let x, y be horizontal, y being 90° to the left of x, and let z be positive downwards.

u, v, w = the velocity components in the directions of x, y, z,
X, Y, Z = the components of extraneous forces per unit mass,
ρ = the density of the water,
p = the pressure,
k = the virtual frictional resistance,
t = the time.

Then, the water being regarded as incompressible and inextensible, the equations of motion are

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\\
&\qquad=\mathrm{X}-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{k}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)\\
&\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\\
&\qquad=\mathrm{Y}-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{k}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\\
&\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\\
&\qquad=\mathrm{Z}-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{k}{\rho}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\\
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0
\end{aligned}\right\}\quad . \qquad (1)$$

Then $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$, $\partial v/\partial y$, $\partial p/\partial x$, $\partial p/\partial y$, and w are all identically equal to zero.

Since no extraneous forces except gravity are taken into account, X and Y are the horizontal components of the deflecting force due to the earth's rotation,

$$\mathrm{X} = 2\omega v \sin\phi,$$
$$\mathrm{Y} = -2\omega u \sin\phi,$$

where ω is angular velocity of the earth and ϕ the latitude. The two last equations of (1) are therefore unnecessary, and the two first become

$$\left.\begin{aligned}
\frac{\partial u}{\partial t}&=2\omega v\sin\phi+\frac{k}{\rho}\frac{\partial^2 u}{\partial z^2}\\
\frac{\partial v}{\partial t}&=-2\omega u\sin\phi+\frac{k}{\rho}\frac{\partial^2 v}{\partial z^2}
\end{aligned}\right\}\quad . \quad . \quad . \qquad (2)$$

---

[1] "Der adiabatische Temperaturgradient im Meere," *Ann. d. Hydr.*, 1914, p. 340.

[2] "Adiabatische Temperaturanderungen in grossen Meerestiefen," *Ann. d. Hydr.*, 1914, p. 321.

[3] *Loc. cit.*

[4] *Arkiv for matematik, astronomi o fysik*, Bd. 2, No. 11 (Stockholm, 1905); and *Ann. d. Hydr.*, 1906, pp. 423 *et seq.*

δt = Adiabatic Temperature difference per metre × 10³.

Depth in Metres.	$\sigma_0=28$.								
	−2°.	0°.	2°.	4°.	6°.	8°.	10°.	15°.	20°.
0	0·016	0·035	0·053	0·070	0·087	0·103	0·118	0·155	0·190
1,000	·036	·054	·071	·087	·103	·118	·132	·166	·199
2,000	·056	·073	·089	·104	·118	·132	·146	·177	·207
3,000	·075	·091	·106	·120	·133	·146	·159	·188	..
4,000	·093	·108	·122	·135	·147	·159	·170	·197	..
5,000	·110	·124	·137	149	..	..	..	..	..
6,000	·127	·140	·152	·163	..	..	..	..	..
7,000	..	·155	·165	·175	..	..	..	..	..
8,000	..	·169	·178	·187	..	..	..	..	..
9,000	..	·182	·191	·198	..	..	..	..	..
10,000	..	·194	·202	·209	..	..	..	..	..

Depth in Metres.	$\sigma_0=31$.		$\sigma_0=32$.	
	12°.	14°.	21°.	22°.
0	0·138	0·153	0·202	0·208
1000	·151	·165	·210	·216
2000	·164	·176	·218	·223
3000	·175	·187	..	..
4000	·186	·196	..	..

	Depth = 0 m.			
	σ_0.			
t.	0.	10.	20.	30.
0	−0·042°	−0·014°	+0·014°	+0·040°
10	+ ·058	+ ·081	·102	·122
20	·142	·161	·177	·192

	Depth = 1000 m.			
	σ_0.			
t.	0.	10.	20.	30.
0	0·019°	+0·009°	+0·035°	+0·059°
10	+ ·076	·097	·117	·136
20	·154	·172	·187	·201

INCREASE × 10³ OF δt FOR AN INCREASE OF 1 IN σ_0

Metres.	0°.	5°.	10°.	15°.	20°.
0	0·0025	0·0022	0·0020	0·0017	0·0014
2,000	22	20	18	15	13
4,000	20	18	16	14	..
6,000	18	16	..	..	..
8,000	16	14	..	..	..
10,000	14	13	..	..	..

The last table is calculated for $\sigma_0=30$ and is not accurate if σ_0 is less than 20.

The motion has been assumed to be stationary, so $\partial u/\partial t$ and $\partial v/\partial t$ vanish. If we write

$$a=+\sqrt{\frac{\rho\omega\sin\phi}{k}},$$

equations (2) take the form

$$\frac{d^2u}{dz^2}+2a^2v=0\,;\quad \frac{d^2v}{dz^2}-2a^2u=0. \quad . \quad . \quad (3)$$

The general solution is

$$\left.\begin{aligned}u&=C_1e^{az}\cos(az+c_1)+C_2e^{-az}\cos(az+c_2)\\ v&=C_1e^{az}\sin(az+c_1)-C_2e^{-az}\sin(az+c_2)\end{aligned}\right\} . \quad (4)$$

where C_1, C_2, c_1 and c_2 are arbitrary constants.

To obtain real results ϕ is taken as being positive and the results are applicable only to the northern hemisphere. If y be drawn to the right of x the equations hold for the southern hemisphere.

If the value of z be infinite, or, practically, greater than the greatest depth to which a wind current can penetrate, *i.e.* a few hundred fathoms, C_1 is zero and equations (4) become

$$u=C_2e^{-az}\cos(az+c_2),$$

$$v=-C_2e^{-az}\sin(az+c_2).$$

Differentiating

$$\frac{du}{dz}=-a\sqrt{2}C_2e^{-az}\sin(az+c_2+45°),$$

$$\frac{dv}{dz}=-a\sqrt{2}C_2e^{-az}\cos(az+c_2+45°).$$

If T, the tangential pressure of the wind, be along the positive axis of y, C_2 and c_2 are determined by

$$k\left(\frac{du}{dz}\right)_{z=0}=0;\quad -k\left(\frac{dv}{dz}\right)_{z=0}=\text{T}.$$

If V_0 be the absolute velocity of the water at the surface, $V_0=C_2$, and

$$\left.\begin{aligned} u&=V_0e^{-az}\cos(45^\circ-az)\\ v&=V_0e^{-az}\sin(45^\circ-az)\\ V_0&=\frac{\text{T}}{ka\sqrt{2}}=\frac{\text{T}}{\sqrt{2k\rho\omega\sin\phi}}\end{aligned}\right\}\quad .\;.\;.\;.\quad(5)$$

The direction of T is the direction of the wind velocity *relative to the water.*

From equations (5) it follows that in the northern hemisphere the drift current at the surface will be directed at 45° to the right of the wind, and that the angle of deflection will increase by four right angles for an increase of depth of $2\pi/a$, while at the same time the velocity decreases to $V_0e^{-2\pi}=1/535$ of the value at the surface.

Fig. 8 shows the velocity and direction of the current down to the reversal depth for each tenth of the latter: the longest arrow denotes the surface drift, and T the direction of the wind. At great depths the velocity, and therefore the friction, are zero, and consequently the whole mass of water moves only under the influence of the wind and the earth's rotation. The conditions being stationary, these two forces must be equal and opposite, and since the deflecting force is at right angles to the direction of motion of the centre of gravity of the water, the total momentum of the water must be directed at an angle of 90° from the wind.

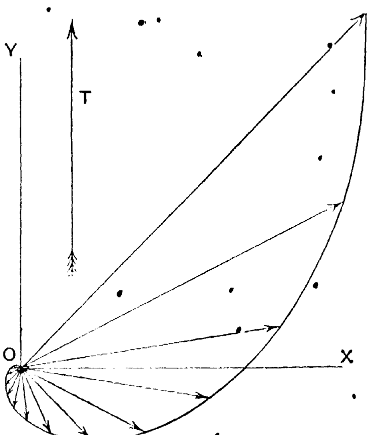


Fig. 8.

§ (51) The Reversal Depth and the Virtual Frictional Resistance.—It will be seen that Ekman's theory depends on the conception of a depth D at which the direction of the current is reversed, and to which he gave the name "Depth of Wind Friction." Its value is

$$\text{D}=\frac{\pi}{a}=\pi\sqrt{k/\rho\omega\sin\phi},$$

and it contains k, the virtual frictional resistance. Nothing was known as to k beyond the fact that its effect took the form of eddies and was far greater than the viscosity or internal resistance to regular motion; in fact the substitution of the latter quantity, 0·016 C.G.S., in the equations leads to the absurd result that no wind-drift can penetrate even to the depth of one metre. Ekman was therefore unable to give an absolute meaning to his results and was forced to make use of D as his unit of depth. It is obvious, however, that if the reversal depth could be determined the value of k could be calculated. Krümmel[1] noticed that on many occasions in the Atlantic equatorial current plankton nets lowered from the ship, drifting freely with the surface current, appeared to be dragged sideways as soon as they reached a depth of 120 to 150 metres, as if the current here were nil or in another direction. If this is assumed to be the reversal depth k can be determined. One instance which he quotes, in which in 43° N. lat. the reversal depth appeared to be at 150 metres, leads to a value for k of 295 in 10° N. lat. and 237 in 60° N. lat. Ekman assumed that k varied as the velocity, but he showed at the same time that the equations led to nearly the same results if the relation was treated as a quadratic one.

The calculations above were made for water of uniform density, apart from compression. In the sea, however, the surface waters are generally divided into layers of markedly different density, especially near coasts and in the open tropical seas, where the surface is strongly heated. Ekman investigated this point experimentally and found that layering led to a decrease of friction. It follows, therefore, that D will have less than the normal value in such regions.

Our knowledge of the nature of the frictional resistance in the sea has been considerably advanced by the researches of G. I. Taylor[2] into eddy-motion in the atmosphere. According to his theory momentum, and also heat, are transmitted from one layer of air to another by great eddies with horizontal axes. He calculated the coefficient of eddy diffusivity for various conditions from observations of the wind and temperature at different heights and found that it varied between 3000 and 100,000 cm.^{2}/sec. H. Jeffreys[3] has applied Taylor's method to the sea. He took the mean winds and currents from charts and obtained values for the coefficient varying from 4 cm.²/sec. in 40° N., 60° W., to 460 cm.²/sec. in 10° N., 40° W. The wideness of the range is probably due to differences between the disturbance of the surface at various places, and it follows that in order to arrive at a value appropriate to any given place and season far more detailed observations are necessary. Another weakness of the method lies in the fact that, as Ekman has pointed out, a pure wind-drift probably does not occur in nature; the current has to meet the resistance of other bodies of water in another state of motion or possibly at rest, so that the conditions are similar to those obtaining in the neighbourhood of a coast where the pressure gradient and the resulting current must be taken into consideration. It is interesting that Krümmel's figure is about 70 per cent of that calculated by Jeffreys for the same region.

---

[1] *Ozeanographie*, 1911 ed., ii. 461. Krümmel has made an error in the position of the decimal point, and gives 29·5 and 23·7 as the value of k.

[2] *Phil. Trans.* A, 215, 1914, p. 1; see also article "Atmosphere, Physics of the," §§ (13) and (14).

[3] *Phil. Mag.* xxxix. 578.

§ (52) WIND-DRIFT IN AN OCEAN OF UNIFORM LIMITED DEPTH *d*.—(i.) If *d* is finite but large in comparison with D, the equations for the case of infinite depth still hold practically unaltered.

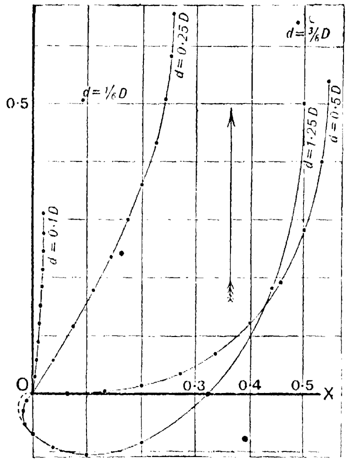


FIG. 9.

(ii.) If the depth of the water is about equal to or less than D the conditions are completely altered. The results of Ekman's investigations are shown graphically in *Fig.* 9; the current arrows are omitted for the sake of clearness and are to be imagined drawn from O to each of the points on the curves, and in this case show the conditions at each tenth of the total depth of the water, not of the reversal depth.

If *d*/D is small, say 0·1, the current flows very nearly in the direction of the wind at all depths; as its value increases the deviation becomes greater, and when it is equal to 1·25 the curve differs from that for infinitely deep water only near the bottom; the dotted line shows the curve for the latter case.

§ (53) CURRENTS DUE TO A PRESSURE GRADIENT.—If the surface of the sea be inclined to the horizontal for any reason, such as the piling up against the land of a wind-drift originating in another place, a pressure gradient will arise which can give rise to a current. Let *x* and *y* be laid on the sea surface and let the inclination be γ in such a direction that

$$X=0\,;\ Y=g\sin\gamma,$$

where *g* is the acceleration of gravity.

The equations for steady motion differ from (3) only by a gravity term

$$\frac{d^2u}{dz^2}+2a^2v=0\,;\ \frac{d^2v}{dz^2}-2a^2u+\frac{\rho g\sin\gamma}{k}=0. \quad . \quad (6)$$

The solutions are

$$\left.\begin{aligned}u&=C_1e^{az}\cos(az+c_1)+C_2e^{-az}\cos(az+c_2)+\frac{\rho g\sin\gamma}{2a^2k}\\ v&=C_1e^{az}\sin(az+c_1)-C_2e^{-az}\sin(az+c_2)\end{aligned}\right\}(7)$$

Since there is supposed to be no wind on the surface,

$$\frac{du}{dz}=\frac{dv}{dz}=0 \text{ for } z=0,$$

whence

$$C_1=C_2=\tfrac{1}{2}C\,;\quad c_1=-c_2=c,$$

where C and *c* are new arbitrary constants.

Substituting $\rho\omega\sin\phi$ for a^2k, equations (7) become

$$\left.\begin{aligned}u&=C\{\cosh az\cos az\cos c-\sinh az\sin az\sin c\}\\ &\qquad+\frac{g\sin\gamma}{2\omega\sin\phi}\\ v&=C\{\cosh az\cos az\sin c+\sinh az\sin az\cos c\}\end{aligned}\right\}(8)$$

At the bottom, where $z=d$, $u=v=0$, and consequently

$$C\sin c=\frac{g\sin\gamma}{\omega\sin\phi}\cdot\frac{\sinh ad\sin ad}{\cosh 2ad+\cos 2ad},$$

$$C\cos c=-\frac{g\sin\gamma}{\omega\sin\phi}\cdot\frac{\cosh ad\cos ad}{\cosh 2ad+\cos 2ad}.$$

Equations (8) then take the form

$$u=-\frac{g\sin\gamma}{2\omega\sin\phi}\cdot\frac{\cosh a(d+z)\cos a(d-z)+\cosh a(d-z)\cos a(d+z)}{\cosh 2ad+\cos 2ad}+\frac{g\sin\gamma}{2\omega\sin\phi}$$

$$v=+\frac{g\sin\gamma}{2\omega\sin\phi}\cdot\frac{\sinh a(d+z)\sin a(d-z)+\sinh a(d-z)\sin a(d+z)}{\cosh 2ad+\cos 2ad}.$$

Fig. 10 gives the velocities and directions of the gradient current for cases where the depth is less than D or only a little greater. The points on the curve are drawn for each tenth of the total depth, and arrows are to be imagined drawn from O to each of them. The

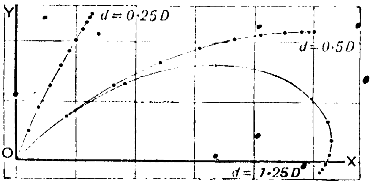


FIG. 10.

outermost point on each is at the surface. The deviation, at any depth, from OY, the direction of the pressure gradient, increases with increasing depth of the water. If d is greater than D, the current consists of a bottom current of thickness D with a component in the direction of OY, and above this up to the surface a current perpendicular to the gradient with almost uniform velocity.

$$u_0 = \frac{g \sin \gamma}{2\omega \sin \phi}; \quad v = 0.$$

The bottom acts as a wind on the current and causes a deviation to its *left*. In the above investigation it has been assumed that the water is everywhere of the same density. Ekman has also discussed the case of a sea of density increasing downwards at a uniform rate. If the sea consists of superimposed layers of different density which are uniform within themselves the conditions become too complicated for useful treatment.

§ (54) THE EFFECT OF THE COAST ON WIND-DRIFTS.—The ideal case first treated probably never occurs in nature; the flow of the wind-drift is in general affected by the resistance of continents or of other bodies of water not in motion. The stationary condition will include a pressure gradient, due to the piling up of the water, of such a magnitude that the resulting current away from the obstacle balances the flow towards it. In the following the water is assumed to be of uniform density.

Let a steady uniform wind blow everywhere outside a straight infinitely long coast, the sea being of uniform depth. The current will then be the same at all places at the same distance from the coast, so that no gradient can arise in a direction parallel to the latter; but a gradient will be formed in a direction at right angles to the coast until inflow and outflow are equal. Ekman does not give the details of the investigation, and expresses its results in curves; a few of which, for the case of a wind parallel to the coast, are reproduced in *Fig.* 11.

The current arrows are to be imagined drawn from the origin, denoted by a circle, to each point on the curve, which represent, as before, tenths of the total depth.

The motion depends on the ratio d/D and the angle between the wind and the coast.

If the depth be great there will be three distinct currents: (1) a bottom current of thickness D, flowing with a component in the direction of the gradient, with a deflection from it of 45° at the bottom and 90° at its upper boundary; (2) a mid-water current of almost uniform velocity, reaching from the bottom current to a depth D below the surface and flowing parallel to the coast; (3) an upper current the velocity of which is that of a wind-drift superimposed on the

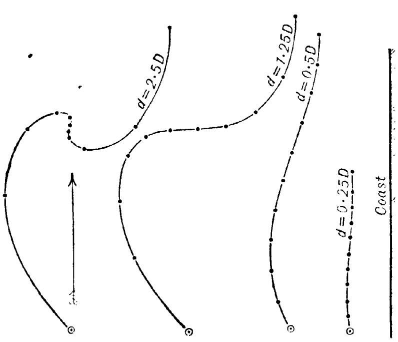


FIG. 11.

mid-water current. If the depth is less than 2D the mid-water current is absent, and the surface and bottom currents lose their characteristic form. It will be seen that in the neighbourhood of a coast a wind can produce a current down to the bottom, while in the absence of the coast the wind-drift is of limited depth. The bulk of the current is parallel to the coast and its velocity is proportional to the wind component in this direction. The direction of the wind and the side of the coast, right or left, on which it lies is of considerable importance, except when the depth is much less than the reversal depth, in which case the earth's rotation has little effect.

§ (55) THE WIND-DRIFT IN AN ENCLOSED SEA.—Ekman's investigations of this problem are of considerable interest, since many of the effects of the wind can be accurately measured. When the wind begins to blow the surface drift is at first deflected to the right until the piling up on that side gives rise to a mid-water current which carries the water to the part of the coast directly in the path of the wind. The gradient corresponding to the final conditions is almost directly opposite to the wind, with a very slight deflection to the right of the latter; it follows, therefore, that in an enclosed sea the effect of the earth's rotation is very small. Colding's

(§ (38)) observations on the great storm in the Baltic in 1872 confirm Ekman's deductions, and the latter has used them to calculate the relation between the velocity of the wind and its tangential pressure. By means of the fundamental equations the flow in the directions u and v can be calculated for currents due to wind and to a gradient. When stationary conditions have been reached the total flow in any direction must be nil, since otherwise the level of the water would be changing, and at the same time the effect of the earth's rotation may be neglected in an enclosed sea. Then by equation (5), which gives the relation between the surface velocity and the tangential pressure, Ekman deduces the formula

$$\sin\gamma = \frac{3}{2}\cdot\frac{T}{\rho g d},$$

where γ is the inclination of the surface to the horizon, and from this the value

$$T = 0{\cdot}0000032 v^2,$$

as explained in § (38).

§ (56) MOHN'S THEORY.—H. Mohn[1] in his investigations in the Norwegian Sea neglected the effect of the earth's rotation on wind-drifts but took it into account when discussing gradient currents; he also considered that friction was negligible. He therefore arrived at the conclusion that drift currents should follow the direction of the wind, while those due to an inclination of the surface should be deflected through one right angle from the direction of the gradient at all depths. Ekman's theory, on the other hand, attaches great importance to the frictional resistance and leads to a deflection of 90° for a gradient current only in the case of the upper layers of a deep sea; in the deeper strata and at all depths in a shallow sea the friction of the bottom acts like a wind and diminishes the angle.

On the above assumptions Mohn developed a method of calculating the velocity and direction of currents from observation of the distribution of density which has been much used in the past, often with considerable modifications. Consider the case of a sea, which, in the first instance, may be taken as enclosed, into which fresh water is discharged by rivers; the Black Sea may serve as an example. The fresher water near the shore will stand at a higher level and will make its way more or less directly towards the centre. The heavier water here will sink and flow along the bottom towards the shore, where it will tend to rise again. There will therefore be a layer of no motion between the two horizontal currents, to which Mohn gave the name "boundary surface," and which will in general be marked by a sudden change in the salinity and temperature. Having fixed the depth of this layer by suitable means, the mean density is determined in as many vertical columns as possible between it and the surface, and the corresponding pressure is calculated. The pressure of an arbitrary standard column is then calculated, either by taking the mean of all the columns, or by selecting the one where the pressure is greatest. Since now the surface is everywhere under equal pressure, if barometric differences be neglected, it follows that, at places where the pressure of the column is less than corresponds to the standard, the level of the sea must be slightly higher than where it is greater. The surface can therefore be mapped out by contour lines to give what Mohn called the "density surface." According to his theory the current will be everywhere deflected at right angles from the gradient, and the velocity will be

$$v = \frac{g\tan i}{2\omega\sin\phi},$$

where g is gravity, i the angle of the gradient, and ϕ the latitude.

To the velocity thus observed Mohn added that calculated from the direction and force of the wind by the formula given in § (49), the winds being taken from isobaric charts.

§ (57) CALCULATION OF CURRENT VELOCITY BY BJERKNES'S METHOD.[2]—This depends on the acceleration of the particles in a closed curve which is bound to the particles through which it originally passed.

On the analogy of Lord Kelvin's expression for the absolute circulation in a curve, neglecting rotation of the earth and friction,

$$C_a = \int v_T ds, \quad . \quad . \quad . \quad . \quad (1)$$

where v_T is the tangential component of velocity and ds is a small element of the curve. Bjerknes investigated C_r, the circulation relative to the earth, and found

$$C_a = C_r + 2\omega S, \quad . \quad . \quad . \quad . \quad (2)$$

where S is the area of the curve projected on the equatorial plane and ω is the angular velocity of rotation of the earth. He then calculated the acceleration of the circulation,[3] due to the various forces acting on the particles,

$$\frac{dC_r}{dt} = \int a_T ds. \quad . \quad . \quad . \quad . \quad (3)$$

---

[1] "Density, Temperature and Currents," *Norwegian North Atlantic Expedition, 1876–1878* (Christiania, 1887).

[2] "Über einen hydrodynamischen Fundamentalsatz," *Kongl. Sv. Vt. Akad. Handlingar*, xxxi. No. 4 (Stockholm, 1898); "Cirkulation relativ zu der Erde," *Oefversig af Kgl. Vet. Akad. Hand.* No. 10, 1901; Helland-Hansen and Sandstrom, "Über die Berechnung von Meeresströmungen," *Rep. Norwegian Fishery Investig.*, 1903, ii. No. 4, translated in *Report on Fishery and Hydrographical Investigations in N. Sea, Northern Area, 1902–1903*, S.O., 1905, cd. 2612.

[3] For a proof of this equation see Lord Kelvin's paper on "Vortex Motion," *Phil. Trans. R.S.*, Edin., 1869, xxv. 217, § 59; also *Collected Works*, iv. 49.

where the integration is taken round the whole curve and a_T is the sum of the tangential components of the accelerations due to gravity, density pressure, rotation, and friction, so that

$$a_T = g_T + p_T + d_T + f_T, \quad . \quad . \quad . \quad (4)$$

and

$$\frac{dC_r}{dt} = \int g_T ds + \int p_T ds + \int d_T ds + \int f_T ds. \quad . \quad . \quad (5)$$

Now since the work done by gravity in moving a particle completely round the curve is nil, then $\int g_T ds$ is zero and gravity is without effect.

p_T is due to pressure, and is proportional to dp/ds, and inversely proportional to the density ρ.

$$p_T = -\frac{1}{\rho}\frac{dp}{ds}; \quad . \quad . \quad . \quad . \quad (6)$$

or, using specific volumes instead of density,

$$p_T = -v\frac{dp}{ds}. \quad . \quad . \quad . \quad . \quad (7)$$

The negative sign shows that it acts in the direction of falling pressure.

$\int d_T ds$, the effect of rotation, is found from the relation given above,

$$C_a = C_r + 2\omega S. \quad . \quad . \quad . \quad . \quad (2)$$

Differentiating to get accelerations,

$$\frac{dC_a}{dt} = \frac{dC_r}{dt} + 2\omega\frac{dS}{dt}. \quad . \quad . \quad . \quad (8)$$

Now, since in the case of absolute movement the effect of rotation is nil,

$$\frac{dC_a}{dt} = \int g_T ds + \int p_T ds + \int f_T ds, \quad . \quad . \quad (9)$$

and from (5) and (9)

$$\frac{dC_a}{dt} = \frac{dC_r}{dt} - \int d_T ds, \quad . \quad . \quad . \quad (10)$$

and from this and (8)

$$\int d_T ds = -2\omega\frac{dS}{dt}. \quad . \quad . \quad . \quad (11)$$

$\int f_T ds$, due to friction, can only be calculated from (5) when all the rest is known. It can be designated by $-$R, negative because it opposes the circulation.

Finally we have

$$\frac{dC_r}{dt} = -\int v\,dp - 2\omega\frac{dS}{dt} - R. \quad . \quad . \quad (12)$$

Since $C_r = \int v_T ds$, it is of the dimensions l^2/t, and $(dC_r/dt) = (l^2/t^2)$. If C.G.S. units are used, it is expressed in cm.^{2}/sec.^{2}

Sandstrom and Helland-Hansen remark, "the first element at the right-hand side of (12) is of special importance because it contains the primary cause of the movements in the sea and in the atmosphere." The present writer does not agree with this, unless it is to be understood as acting on the sea in part by setting up winds, which does not appear to be their meaning. This is, however, no obstacle to the use of the method for the calculation of velocities due to a density gradient.

The investigation depends on the accuracy with which the specific volume v can be determined. Now at the greater depths there is always some uncertainty as to the exact level at which a sample is taken, owing to the stray on the wire, and in addition slight errors will occur in the salinity and temperature. We cannot therefore expect absolute accuracy in the specific volume in the fifth figure. This being the case, we may consider the pressure as increasing directly as the depth, at the rate of 1 decibar or 10^5dynes/cm.^{2} per metre, and lines representing equal pressures will be evenly spaced and horizontal, as in the section, *Fig.* 12.

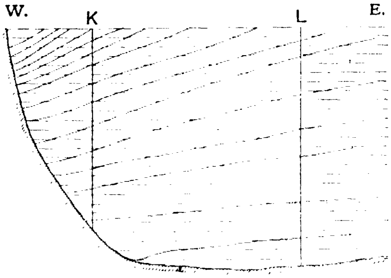


FIG. 12.

On the other hand, lines of equal specific volume will, as a rule, be curved and unevenly spaced. The figure is a section through an ideal current running southwards in the northern hemisphere, with the land on its right and lighter water near the surface; the East Greenland and Labrador currents are examples. The arrangement of the isosteres shows that the water in a vertical column at K is lighter than at L, and we should expect the water at K to rise and flow along the surface to L, and the water at the latter place to sink, so as to make the isosteres horizontal. As a matter of fact this does not occur; the conditions may be unchanged or stationary for a considerable time. The water is accelerated to the right by an amount which, for unit mass, depends on its velocity. We must therefore assume that the surface velocity is sufficiently greater than that in the depth to counterbalance the greater acceleration of the latter due to its density, and we know qualitatively that such conditions are common.

The lines of equal pressures and specific volume cut one another to form a mesh-work; if we imagine the lines extended to surfaces we obtain a series of tubes, to which Bjerknes has given the name "solenoids."

The quantitative value of one mesh in the section is obtained thus. The isosteres are drawn for differences of 0·0001 of volume, while specific volumes are observed to one-tenth of this.

Then, if v_1 and v_0 are the volumes for two adjacent lines,

$$v_1 - v_0 = 0{\cdot}0001 \frac{\text{cm.}^3}{\text{grm.}},$$

and if the pressure lines are drawn for 10-metre intervals, corresponding to 1 bar or 10^6dynes/cm.^{2},

then $$p_1 - p_0 = 10^6 \frac{\text{grm.}}{\text{cm./sec.}^2};$$

then $$-\int v\,dp = 0{\cdot}0001 \times 10^6 = 100 \frac{\text{cm.}^2}{\text{sec.}^2},$$

and the number of solenoids in a closed curve is 100 times the number of meshes.

If the number of solenoids is A, then the acceleration of the circulation is

$$\frac{dC_r}{dt} = \text{A} - 2\omega\frac{dS}{dt} - \text{R}.$$

A can obviously be determined graphically, but the arithmetical process is much quicker and more accurate. Let the closed curve be bounded by the two vertical lines K and L, and by the two horizontal lines at the surface and at some given depth. Then since the pressure is the same throughout a horizontal line (differences of atmospheric pressure are neglected), we need consider only the two vertical columns. The quantity required is the difference of the number of solenoids in the two columns,

$$\left(\int_{p_0}^{p_1} v\,dp\right)_{\text{K}} - \left(\int_{p_0}^{p_1} v\,dp\right)_{\text{L}}.$$

The pressure p increases by 10^5 units per metre; it is convenient to express it in units of 10^5 C.G.S. and specific volumes in 10^{-5} C.G.S. The required quantity A is then in C.G.S. units.

The specific volume is observed to five figures, but since we are dealing with differences only, it is convenient to reduce them to more manageable figures by subtracting a constant quantity which is most suitably the value which would be found in a sea consisting of water of $35_{0/00}$ at 0° C. Helland-Hansen and Sandstrom[1] have published tables by which this may be done easily, but they depend on Tait's values for the compressibility and are therefore slightly inaccurate. Bjerknes[2] gives improved tables, and Hesselberg and Sverdrup[3] a modification of the latter which cover a more limited range.

The tables give us $v - v_{35,0°}$. We next calculate $(v - v_{35,0°})(p_1 - p_0)$ by taking the mean value of $(v - v_{35,0°})$ for each layer of water and multiplying it by the thickness of the layer in metres. This gives us the number of solenoids in each layer at the place of observation, and the sum of these from the surface to any depth the number in the column of this depth. The same process is repeated for another vertical column of the same height in the current. The difference of these two sums will give A.

New since the vertical circulation in a current is actually very small, we may neglect it, and friction must necessarily be left out, because we know nearly nothing as to its value. Then

$$\text{A} = 2\omega\frac{dS}{dt}.$$

S is the projection of the curve on the equatorial plane. It is easy to see that if S′ is the projection on the sea surface

$$2\omega\frac{dS}{dt} = 2\omega\frac{dS'}{dt}\sin\phi,$$

where ϕ is latitude.

In *Fig.* 13 let *abdc* be a section through a current moving in the direction of the arrow, and let the velocity at the surface be V, and V′ at the level *cd*. Then after unit time *ab* will have moved to AB, but *cd* only as far as *c′d′*. Then A*a′b′*B is the increase of the projection of the curve on the surface. If S′ is the projection of the curve on the surface and L the distance between *a* and *b* in cm., then

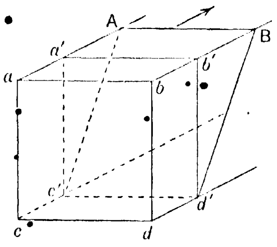


FIG. 13.

$$\frac{dS'}{dt} = (\text{V} - \text{V}')\text{L},$$

and

$$\text{A} = 2\omega\frac{dS}{dt} = 2\omega\sin\phi(\text{V} - \text{V}')\text{L},$$

so that if we know the specific volumes and pressures in the two columns *ac* and *bd*, and the distance L between them, we can calculate the *difference* of velocity between the levels *ab* and *cd* at right angles to the section. It often happens that the lowest level may be taken so deep that we can neglect its velocity, in which case we have the absolute velocity in the upper layers.

If the velocity *increased* downwards the tendency would be to drive the surface water to the left in the northern hemisphere.

In the case where one current flows over another of different origin, the surface of separation between them is at rest, but the densities may be so arranged that it is difficult to determine the position of this surface. The biological conditions will often give great help.

---

[1] *Loc. cit.*

[2] *Dynamic Meteorology and Hydrography*, Carnegie Institute.

[3] "Beitrag zur Berechnung der Druck- und Massenverteilung im Meere," *Bergens Museums Aarbok*, 1914-1915, No. 14.

Bjerknes's method has been largely used, chiefly as a result of the International Fishery Investigations, and has proved of great value. Observations made in the Labrador current by the Ice Patrol showed almost complete agreement between the observed and calculated velocities.

§ (58) KNUDSEN'S THEOREM.—The fresh water from a river flows out as a surface layer of low salinity, beneath which there is a current of salter water in the opposite direction. Martin Knudsen[1] has shown that the volumes of water transported in unit time by each layer can be expressed in terms of their salinity and the rate of discharge of the fresh water.

A and B (*Fig.* 14) are two vertical sections across the mouth of a river; i and i' are the volumes of water flowing out through them in unit time, u and u' the volumes flowing up stream, and s, s', z, z' their mean salinities. The volume discharged at B is obviously the sum of the discharge at A and the volume picked up by the surface current from the undercurrent between A and B.

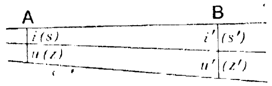


FIG. 14.

$$i' = i + u' - u,$$

$$i - i' = u - u',$$

whence

$$i' = i\,\frac{z'}{z}\cdot\frac{z-s}{z'-s'},\quad u = i\cdot\frac{s}{z},\quad u' = i\cdot\frac{s'}{z}\cdot\frac{z-s}{z'-s'}.$$

If A is so far up stream that the water is entirely fresh at all depths, s, u, and z are zero, and

$$i' = i\cdot\frac{z'}{z'-s'},\quad u' = i\cdot\frac{s'}{z'-s'}.$$

Knudsen has applied his theorem to the Baltic as a whole, making A coincident with the coast line so as to cut through all river mouths. Then s is zero, and i is the sum of the discharge from the rivers and precipitation less evaporation on the surface of the sea.

J. Gehrke[2] has shown that the method may be usefully applied in investigating the currents of the open sea where they run more or less parallel to coasts as, for instance, the branch of the European stream which flows northwards and eastwards on the west side of the British Isles.

D. J. M.

---

[1] *Ann. d. Hydr.*, 1900, p. 316.

[2] *Pub. de Circ.* No. 40.

ONE METRE COMPARATOR, at N.P.L. (typical): description, sources of error, setting up and manipulation, computation of results. See "Comparators," § (1).

OPTICAL PROJECTION APPARATUS: with a large field of view.
- Compound projection lenses for testing profile gauges. See "Gauges," § (67).
- For screw gauge testing, first type. See *ibid.* § (64).
- For testing gauges. See *ibid.* Section IV. § (64), etc.
- For testing screw gauges. See *ibid.* § (36).
- Standard horizontal machine for testing profile gauges. See *ibid.* § (68).
- Vertical type. See *ibid.* § (66).
- Vertical type projector for testing screw gauges. See *ibid.* § (69).
- "Wilson" projection comparator for testing screw gauges. See *ibid.* § (70).

OSCILLATION, THE ARC OF. See "Clocks and Time-keeping," § (16).

OSCILLOGRAPH. See "Galvanometer, Einthoven, Adaptation of," Vol. II.; "Clocks and Time-keeping," § (15).

— P —

PARALLEL RULES. See "Draughting Devices," p. 274.

PARANTHELIA. See "Meteorological Optics," § (22) (iii.).

PARANTHELIC ARCS. See "Meteorological Optics," § (22) (iv.).

PARHELIC CIRCLE OR MOCK SUN RING. See "Meteorological Optics," § (22).

PARHELIC SUN PILLAR. See "Meteorological Optics," § (20) (vii.). See also "Mock Sun."

PARHELION of 22°. See "Meteorological Optics," § (20) (ii.).
- of 44°. See *ibid.* § (20) (viii.).

PASCAL'S CALCULATING MACHINE. See "Calculating Machines," § (1) (iii.).

PENDULUM:
- Construction of, and compensation. See "Clocks and Time-keeping," § (3).
- Double. See *ibid.* § (17).
- Effect of the air on. See *ibid.* § (6).
- Free, period of. See *ibid.* § (4).
- Its function with regard to escapement and maintenance. See *ibid.* § (7).
- Measurement of gravity by the. See "Gravity Survey," § (2) (i.).

PETROL MEASURING PUMPS. See "Meters," § (28).

Pilot Balloon. A small rubber balloon used for measuring upper winds. The type of balloon commonly used measures from 18 to 30 inches in diameter when fully inflated. The balloons are filled with hydrogen, and released, their subsequent motion being followed with one or two theodolites. When one theodolite is used, it is necessary to assume that the balloon rises at a uniform rate. This is substantially true on the average, but leads to erroneous results if the balloon gets into an ascending or descending current of air. The rate of ascent of the balloon in feet per minute is given by $V=276(L^{\frac{1}{2}}/(W+L)^{\frac{1}{3}})$, where W is the weight of the balloon in grams, and L the number of grams it can lift when inflated. For a given value of W it is easy to compute the necessary lift so that V shall take any convenient value, such as 500 ft./min.

The computation of the winds at different heights is most expeditiously done by means of a slide-rule. Readings of azimuth (A) and elevation (E) of the balloon are taken at intervals of one minute. The height in feet of the balloon is tV, where t is the time in minutes since the balloon was released. The horizontal distance of the balloon is $h \cot E$. Its components along lines drawn East and North from the point of release are:

$$h \cot E \sin A, \text{ and } h \cot E \cos A.$$

These components are computed for each pair of observations of E and A, and the differences for consecutive minutes give the components of drift of the balloon during the interval. The average wind through the layer traversed by the balloon during the interval is the resultant of the component differences so derived.

If two theodolites at the ends of a measured base can be used, no assumption need be made as to the rate of ascent of the balloon, and the path of the balloon can be traced completely. The calculation of the winds at different heights is slightly more complex than for one theodolite, but the results are more reliable.

Full details of methods of computing pilot balloon results will be found in the *Computer's Handbook* (Meteorological Office) Section II., Sub-section I.

Pipettes. See "Volume, Measurements of," § (17).

Pipettes, Graduated. See "Volume, Measurements of," § (19).

Pitch (of Screw Thread): definition. See "Metrology," VII. § (23) (i.).

Platform Machines. See "Weighing Machines," § (5).

Platinum-iridium Alloy (10 per cent iridium), used for making the International Prototype Kilogramme See "Balances," § (8).

"Play" of Screw Threads: definition of. See "Metrology," VII. § (25) (ii.).

Plumb-line, Deflection of the. See "Gravity Survey," § (11).

Polar Front: a surface of discontinuity between polar and equatorial air. See "Atmosphere, Physics of," § (21).

Polarisation of Light in the Atmosphere:
- From a landscape. See "Meteorological Optics," § (15) (ii.).
- From the sky. See *ibid.* § (13) (i.).

Potential Gradient in the Atmosphere. See "Atmospheric Electricity," §§ (8)-(9).

Potential Temperature: definition of. The temperature which any mass of air would have if brought adiabatically (*i.e.* without any gain or loss of heat) to some standard pressure. See "Atmosphere, Physics of," §§ (3), (6) (ii.). See also "Atmosphere, Thermodynamics of," § (6).
- Computation of. See "Atmosphere, Thermodynamics of the," §§ (2), (6).
- Relation of, to entropy. See *ibid.* §§ (6), (19).

Precipitation—Rain and Snow:
- Forecasting of. See "Atmospheres, Physics of," § (20).
- Instruments for measuring. See "Meteorological Instruments," III. § (10), etc.

Prediction of Tides. See "Tides and Tide-prediction," § (5).

Pressure, Atmospheric:
- Changes of. See "Atmosphere, Physics of," § (20).
- Determination of height from. See *ibid.* § (4).
- Distribution of:
  - At 8000 metres. See "Atmosphere, Thermodynamics of the," §§ (8), (9), and *Fig.* 11.
  - In cyclones and anticyclones. See *ibid.* § (5), Table III.
  - In the upper air. See *ibid.* § 8. See also "Air, Investigation of the Upper," § (11).
  - Over the globe. See "Atmosphere, Thermodynamics of," § (3) and *Fig.* 7.
- Equivalents of inches, millibars, and millimetres. See *ibid. Fig.* 7.
- In millibars, connection of, with temperature and density, tabulated. See "Barometers and Manometers," § (17), Table IX.

Method of measuring. See "Atmosphere, Physics of," § (1).
Reduction of, to datum level. See "Barometers and Manometers," § (6) (iv.).
Relation of, to temperature and entropy. See "Atmosphere, Thermodynamics of the," §§ (19), (22), (23), *Fig.* 16, and Table VI.
Relation of, to temperature and height. See *ibid.* § (8).
Relation of, to temperature in dry and moist air. See *ibid.* § (18) *et seq*
Relation of wind to. See *ibid.* § (8).
Semi-diurnal wave. See "Atmosphere, Physics of," § (17).
Types of distribution. See *ibid.* § (18).
Unit of. See "Atmosphere, Thermodynamics of the," § (2).
Value of, for lowest 8 km. layer of the atmosphere. See *ibid.* § (8), *Fig.* 12.
Variation with height. See "Atmosphere, Physics of," §§ (2)-(3).
See also "Cyclone and Anticyclone."

Pressure, Measure of. (i.) *Units.*—Pressure is the force per unit area which any liquid or gas exerts on the surface in contact with it. The unit of pressure is that produced by unit force acting on unit area, and on the C.G.S. system is a force of 1 dyne per square centimetre, on the British system a force of 1 poundal per square foot. Units depending on the value of g, such as 1 gramme weight per square centimetre or 1 lb. weight per square foot, are also in common use.

(ii.) *Barometric Pressure. Bar and Millibar.*—After the introduction of the barometer, pressure came to be measured as the length of a column of fluid, usually mercury (see "Atmosphere, Physics of," § (1)); and this length was subsequently corrected for variations in the value of g and of temperature. The relation between the "mercury-inch" and the "mercury-millimetre" and the value of pressure in units of force is obtained from the equation

$$l\rho g = \text{pressure in dynes per square centimetre},$$

where l is the length of the column in centimetres, ρ is the density of mercury in grammes per cubic centimetre, g is the acceleration of gravity in cm./s.^{2}.

More recently the practice has become established of measuring the pressure of the atmosphere in units of force, and for this purpose the millibar which is equivalent to a pressure of 1000 dynes per square centimetre has come into use in the British Isles (see "Millibar"). The normal atmospheric pressure at sea-level is 1013·2 millibars, which differs little from 1 bar. In the United States the "bar" is taken as equivalent to 1 dyne per square centimetre, and the pressure of the atmosphere is measured in kilobars, which are equivalent to the English millibars.

(iii.) *Equivalents.*—

C.G.S.

1 dyne per sq. cm. = 1 microbar (=1 bar U.S.A.)
= $1{\cdot}45 \times 10^{-5}$ lbs./sq. in.
= $2{\cdot}95306 \times 10^{-5}$ mercury-inches
= $7{\cdot}50076 \times 10^{-4}$ mercury-millimetres,
1 millibar . . = 1000 dynes per square centimetre
= 1 kilobar (U.S.A.),
1 centibar . . = 10 millibars.

British Units.

Mercury-inches . = 1 inch of mercury at 32° F. in latitude 45°
= 33·8632 mb.,
Mercury-millimetre = 1 mm. at 0° C. in latitude 45°
= 1·333200 mb.,
760 mm. . . = 1013·231 mb.
1000 mb. $\left.\begin{array}{l} = 14{\cdot}496 \text{ lb./in.}^2 \\ = 2087{\cdot}424 \text{ lb./ft.}^2 \end{array}\right\}$ in London,
1 lb. sq. in. = 68,971 dynes/cm.^{2} = 70·31 gm./cm.^{2},
1 ton/sq. in. = $1{\cdot}545 \times 10^8$ dynes/cm.^{2} = 1·575 kgm./mm.^{2}.

Russian Half-lines (normal at 62° F.).

1 half-line = 1·68801 mb. = ·0498 in.,
600 half-lines = 1012·804 mb.

The standard "atmosphere" is equivalent to

760 mm. mercury at 0° C., lat. 45°, and sea-level
= 759·4 mm. mercury at 0° C. in London
= $1{\cdot}0132 \times 10^6$ dynes per cm.^{2}
= 14·7 lbs. per sq. in.
= 0·94 ton per sq. ft.

See Vol. I. "Measurement, Units of."

Pressure, Units of Measurement of: the mercury unit. See "Barometers and Manometers," § (2) (i.).

Pressure-gradient:
Anticyclonic. See "Atmosphere, Physics of," § (16).
Effect on wind. See *ibid.* §§ (9), (10).

Probability: the fraction whose numerator is the number of combinations producing an error, which is included within given limits, and whose denominator is the total number of possible combinations. See "Observations, The Combination of," § (3).

Probable Error: a numerical quantity without sign, such that when the positive sign is attached to it the number of positive errors larger than that value is about as great as the number of positive errors smaller than that value; and that, when the negative sign is attached to it, the same remark applies to the negative errors. See "Observations, The Combination of," § (9).

PROJECTION :
- Airy's, by balance of errors. See "Map Projections," § (8) (i.).
- Bonne's, or Projection du Dépôt de la Guerre. See *ibid.* § (8) (ii.).
- Cassini's (transverse simple cylindrical). See *ibid.* § (8) (iv.).
- Clarke's minimum error perspective. See *ibid.* § (8) (v.).
- Conical: a class of projections in which a set of straight lines, radiating from a common vertex, are cut at right angles by a set of concentric circular arcs described about that vertex. See *ibid.* § (5) (i.).
- Cylindrical: a special type of conical projections in which the angle of the cone is zero: this class includes Mercator's, Cassini's, and the Gauss conformal projection. See *ibid.* § (5) (iii.).
- International map. See *ibid.* § (8) (xviii.).
- Mercator's (cylindrical orthomorphic). See *ibid.* § (8) (xix.).
- Oblique conical. See *ibid.* § (5) (ii.).
- Scale value of a, at different points. See *ibid.* § (7).
- Simple conical, with one standard parallel. See *ibid.* § (8) (vi.).
- Simple cylindrical. See *ibid.* § (8) (xii.).
- The properties of. See *ibid.* § (2).

PROJECTIONS IN USE. See "Map Projections," § (8) (xxxiv.).

PROOF SPIRIT. See "Alcoholometry," § (7).

PROTRACTORS. See "Draughting Devices," p. 275.

PSYCHROMETER: an instrument, consisting of a dry- and a wet-bulb thermometer, used for measuring humidity.
- Assmann's. See "Meteorological Instruments," § (6).
- For aeroplanes. See *ibid.* § (37). See also "Humidity," §§ (4) (ii.) and (9).
- Sling. See "Humidity," §§ (4) (ii.) and (9).
- See also "Hygrometers" and "Thermometers, Wet- and Dry-bulb."

PYKNOMETER: An instrument for measuring the density (*a*) of a liquid, (*b*) of a solid, by determining the weight of a known volume. See "Balances," § (15) (i.).

PYRHELIOMETER: an instrument for the measurement of solar radiation.
- Abbot's. See "Meteorological Instruments," § (29). See also "Radiant Heat and its Spectrum Distribution," § (13).
- Ångström's. See "Meteorological Instruments," § (28).
- Callendar's. See "Radiant Heat and its Spectrum Distribution," § (17).
- Michelson's. See "Meteorological Instruments," § (30). See also "Radiant Heat and its Spectrum Distribution," § (12).
- Pouillet's. See "Radiant Heat and its Spectrum Distribution," § (6).
- Silver-disc. See "Meteorological Instruments," § (30). See also "Radiant Heat and its Spectrum Distribution," § (11).

— Q —

QUADRATIC EQUATION AND GRAPHICAL AUTOMATIC METHODS OF SOLUTION. See "Nomography," §§ (10), (11).

"QUALITY" OF WORK: definition of term. See "Metrology," VIII. § (29) (i.) (c).

QUARTZ FIBRES

"QUARTZ fibres" are filaments made by drawing out melted rock crystal, and, strictly speaking, they are not quartz any more but vitreous silica. Their interest and importance depends on the fact that they may readily be drawn out into pieces of very great length, of extreme fineness and uniformity of diameter, so as to constitute torsion threads for delicate instruments. For this purpose their very perfect elasticity, in which quality they are unique, and their high tensile strength give them great advantages over other means of support. Rods and fibres of fused silica have the further valuable property that they insulate electricity perfectly[1] or nearly so, even in an atmosphere saturated with moisture, or the fibres may be made conducting if required by a thin coat of silver. It was the want of any suitable fibre for the support of the radio-micrometer circuit[2] that led the writer to institute a research that resulted in the invention of the method of production and discovery of the properties of quartz fibres.[3]

§ (1) MAKING QUARTZ FIBRES.—Pieces of rock crystal are heated to a red heat in a fireclay crucible and allowed to cool. They will then be found broken into small angular pieces. The operator, protecting his eyes with dark spectacles and using an oxyhydrogen flame from a mixed jet—not a blow-through jet—heats the corners of one of these until, after much decrepitation, a fused corner is formed. Then holding a second piece close to the first in the flame some of the small pieces that fly off are caught by the first and melted on, and in this way an irregular stick is formed free from contamination by any other material. Broken silica ware of the transparent kind might be used instead if it is made of pure quartz. From such rough material when re-heated fine rods about one millimetre in diameter are readily drawn, for when once melted the decrepitation does not occur again. These rods are the raw material from which the fibres are drawn, or they are available at once as insulating supports.

---

[1] *Phys. Soc. Proc.*, 1889, x.

[2] *Roy. Soc. Trans.*, 1889, clxxx. 159.

[3] *Phys. Soc. Proc.*, 1887, ix.; *Phil. Mag.*, 1887, xxiii. 489; and *R. Inst. Proc.*, 1889.

In order to draw the fibres the instantaneous production of very great speed is essential, for which purpose the original bow and arrow method is still the most convenient.

The stock of the crossbow is made of two pieces of wood, each 18 × 2·5 × 1 cm., fastened together with adjacent chamfered edges forming a groove in which the arrow may lie (*Fig.* 1).

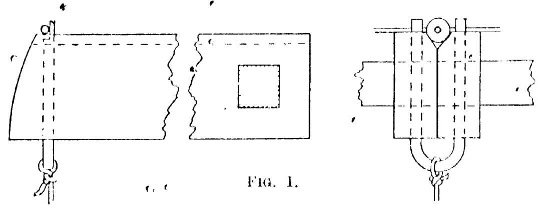


FIG. 1.

The bow is made from dry straight yellow pine, 80 × 1 × 1 cm., square in the middle and tapering towards each end, and so strung with the thinnest whipcord that the cord is tight or nearly so when the bow is straight. It should not prevent the bow from straightening. As the arrow made of straw is so light the best material for the bow is that in which the longitudinal elasticity divided by the density is greatest, or that in which the velocity of sound is greatest, and for this reason pine is chosen. The trigger is a mere loop of brass wire passing stiffly through two holes in the stock and supporting a treadle by a string. Thus when the string of the bow is resting against the flattened protruding ends of the trigger and the arrow is up against the string of the bow and the two hands are otherwise engaged the foot may be used at the exact moment to shoot the arrow. The stock of the bow is held in a vice, and the arrow is aimed along a clear passage with a smooth floor. The arrow is made from a piece of unthrashed wheat straw about 10 cm. long, a material which may be bought in bundles at millinery shops, with a half needle secured at one end with sealing-wax kept very hot to make a strong connection and carefully shaped as illustrated. A notch is cut at the tail of the arrow and strengthened with melted sealing-wax.

Fig. 2 shows the two ends of the arrow in section with the sealing-wax indicated by section lines and the quartz rod secured in position, so that it may be melted at the point marked *x*. It must be remembered that such a bow and arrow is no mere toy but a dangerous weapon. The first arrow shot in this way at a card target 90 feet from the bow was found stuck in the wall behind the card, having pierced a clean hole. Apart from the danger to any one in the passage it is essential that no one should be moving there when fibres are being shot, as the air draughts so caused would add to the difficulty of finding and winding up the fibre. It is important that the V groove, arrow, and string are so proportioned that the string rests exactly in the bottom of the notch in the arrow.

Having now the apparatus prepared as described, the operator, wearing dark spectacles and comfortably seated, holds the end of the rod in one hand and the mixed jet burner in the other hand and applies the flame a few millimetres from the nozzle to the rod at *x*. When the rod is at its hottest and before it separates into two under capillary action the trigger is drawn, *but without moving the jet away*, so that the fibre is actually drawn in the flame. Then the fibre will be found stretched from the bow to the arrow, and 20 or 30 feet is a usual length. After practice the operator can make them finer or coarser at will, and diameters from $\frac{1}{2000}$ to $\frac{1}{10,000}$ inch or $\frac{1}{50}$ to $\frac{1}{400}$ millimetre are in general the most useful. As the torsion varies as the fourth power of the diameter, this gives a range of over 600 to 1 in torsional rigidity. A silk fibre, *i.e.* half the natural double fibre, is about $\frac{1}{2000}$ inch in diameter, and ordinary spun glass $\frac{1}{1000}$ inch or more.

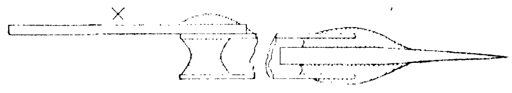


FIG. 2.

Quartz is not the only material that may be drawn into fine fibres with the bow and arrow and O.H. jet. Glass and any of the silicate minerals behave in the same way, but being so much more fusible and limpid there is no necessity to hold the end of the rod with the fingers. It is merely necessary to produce a bead about two millimetres in diameter at the high temperature, and shoot the arrow, when a fibre up to 90 feet in length (this being the extent available when the method was first tested) will be drawn out, and the bead will remain in virtue of its inertia either on the stock of the bow, or it will fall just in front. Such fibres may be drawn so that the whole length glistens with the colours of fine spider lines if seen by sunlight.

Fibres drawn from silicates or glass have rather less torsional rigidity than fibres of quartz, but they suffer from want of perfect elasticity, so that after a deflection the zero position has made a temporary movement in the direction of the deflection, from which it creeps slowly back towards, but not reaching, its former position. Or after a prolonged deflection to the right and then a short one to the left the zero position remains first at the left, then it passes to the right, and gradually settles back towards the original zero. The quartz fibre is free from this defect, and herein lies its extreme value.

It is essential that fibres when made should be gathered quickly and placed in safety. The most convenient method is to use frames of wood, as shown in *Fig.* 3, which represents one end only. Then if the cross-bar is varnished with shellac varnish and the fibre laid on the varnish, the whole length may be picked up by winding longitudinally on the frame, and be secured at one or both ends as desired on the tacky varnish. The outer edges of the end cross-bars should be rounded. A number of such frames will pack in a box which will withstand any ill-usage without causing harm to the fibres.

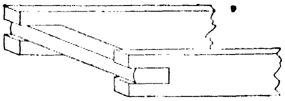


Fig. 3.

Another method of making fibres, preferable where still finer fibres are required and not more than about 30 centimetres long, depends upon the viscous drag of the flame gases upon the melted quartz when a rod is drawn into two in the O.H. flame. Under this drag tapering tails or relatively long fibres may be produced in great quantities and very quickly, but in general they are so fine that no colours are visible in them, though the white of the first order may be reached. In general they correspond with the black of the soap film, and are very difficult to see. When making fibres in this way it is well to place a retort stand or some object in the line of the O.H. jet, the burner now being fixed with the flame directed upwards and away from the operator. After drawing a rod into two a few times a number of fibres will be found on the stand to which they adhere, being for some reason electrified. Very fine and short tails so made when examined in the microscope may reach such a degree of tenuity that the end cannot be found. The method of making quartz fibres by blowing has also been practised in America by Coblentz, and an interesting account of the method there used, showing in detail some variation from that given but substantially similar, is published in the Carnegie Institution Publication for 1906, 65, Appendix V.

§ (2) Using Quartz Fibres.—The use for which quartz fibres are almost exclusively required is as torsion fibres in delicate apparatus, and for this purpose they are pre-eminent. With diminution of diameter the cross-section varies as the square of the diameter, and the strength falls off less rapidly than this. On the other hand, the torsional rigidity falls in the proportion of the fourth power of the diameter or square of the sectional area. For some classes of experiment, therefore, the angular deflection will become greater as the scale of the apparatus is reduced, and the consequent increasing delicacy is limited only by the difficulties of manipulation, construction or measurement, or by relatively increasing effects of disturbance, if such is the fact, and last by the viscosity of the air. With any but the simplest form of suspension the resistance to motion due to this cause may be such as to pass the dead-beat conditions, in which case deflections require an inordinate time, or if within the dead-beat conditions by too small an amount the determination of period is made difficult on account of the high logarithmic decrement. The easiest way to attach a quartz fibre at its ends is to use pointed wires wetted with shellac varnish, and lay the end of the fibre on the varnish, and then gently pull in the direction of the point so that the fibre lies in this direction at the point. Then on drying, with or without the aid of heat, the fibre will be held. Where there is any question as to the rigidity of such a fastening, other methods may be used. A solution of silicate of soda has been used successfully, but the writer has no experience with this. Cements of the "Caementium" type would no doubt answer very well, but the writer used in his experiments on the constant of gravitation a very secure and rigid method of fastening, effected by silvering, electro-coppering, and soldering the coppered ends to metal tags of enormous area relatively, and cementing these tags.[1]

Owing to the fourfold change of torsion with diameter a very exact choice of diameter is necessary to arrive at any particular torsional rigidity. While the frames of fibres are easily supported so that the fibres carried by them may be examined in the microscope, a quick and handy method of judging of the rigidity of fibre is often preferable. The stiffness of a fibre treated as a cantilever varies also as the fourth power of the diameter, and thus if the observer holds a piece between his finger and thumb he can judge of the rigidity in this sense in comparison with that of another fibre by observing the free end. This may project two or three inches and maintain its position, or it may only project an inch or less in a more or less limp manner, or it may not be possible to make it project at all, in which case it will be dragged upwards by the convection draught of warm air. In this way a fibre may be quickly matched or one stiffer or less stiff than another may be easily found, and very little experience of this kind will generally suffice. The coarser fibres are easily seen, but where fibres much less in diameter than $\frac{1}{5000}$ inch ($\frac{1}{200}$ millimetre) are to be handled, it is well to make them as conspicuous as possible. A table in front of a window in a room without draughts is desirable, but a dark background is essential. Black velvet or smoked glass often considered black is full of light and is useless as a background. The background used by the writer in all his work with quartz fibres was the darkness inside a drawer only opened a little way, and the darker the colour of the interior the better. Where available the silvered trumpet of Lord Rayleigh[1] might be even better, but the Egyptian darkness of the drawer has been found to be sufficient. A piece of mirror glass laid upon the table shows up fibres lying upon it very well, and this is convenient where precise lengths have to be cut and secured at the ends. The writer has also used a method of smoking the fibres over a magnesium lamp, the white soot from which is very conspicuous.

---

[1] *Phys. Soc. Proc.*, 1894, xiii.

Where observations of great accuracy are required, combined with the delicacy which quartz fibres permit, the experimentalist must remember that there is no limit in reason to the delicacy imposed by the quartz fibre. The chief difficulty, if liquids are not in question, is caused by the movements due in the main to convection currents of the air within the apparatus, which, acting on the suspension, give to it anomalous movements. Cavendish fully realised this difficulty in his apparatus for weighing the earth, and reduced it as far as the large dimensions of the apparatus permitted. In ordinary physical laboratory apparatus such as balances, galvanometers, etc., the air is never at rest, and a fine quartz fibre suspension would be in continual movement. It is essential to keep the interior dimensions as small as possible and to make the casing of thick highly conducting metal, which itself should be further screened, and all maintained in a cellar or room with as constant a temperature as possible. It is only by such precautions that the full extent of the accuracy of the quartz fibre can be attained. Baron Eötvös, who required extensive lateral dimensions in his apparatus for detecting the differential gravitation of tidal waters, maintained the necessary quiet by reducing the vertical dimensions to the utmost. Air movement in general limits the accuracy attainable in any piece of apparatus, and air viscosity limits the gain which otherwise might be attained by reduction of dimensions. In the apparatus set up by the writer in the Clarendon Laboratory at Oxford these precepts were followed to so great an extent that movements of the air past the gold balls at the rate of one inch in a fortnight would have produced a deflection greater than the disturbances observed on a particular quiet night.

All fibres seem to show increasing tenacity with diminution of diameter, as though there were a surface tenacity akin to surface tension in liquids, and the great strength of silk and spider lines is no doubt due to this cause. Such strength in fine quartz fibres is not due to the fact that the surface is a natural or vitreous surface resulting from fusion, for if a coarser fibre is made to carry a weight over a pail of water, and then a sponge dipped in hydrofluoric acid is placed against the fibre so as to dissolve it slowly—it is very slow compared to glass—the fibre will break when it is reduced sufficiently in diameter, and the one end will immediately be washed in the water. The diameter is then found to be the same as that of a whole fibre which is just broken by the same weight. Tenacities up to from 60 to 80 tons to the square inch, to use the engineer's units for comparison with metals, are found with fibres $\frac{1}{7500}$ inch or $\frac{1}{300}$ millimetre in diameter or less.

Another use for quartz fibres is as cross wires in optical instruments in the place of spider lines. They have the advantage that in damp places they are not destroyed by mould, and if by chance a spider gets into the apparatus they are not cut and destroyed as cross wires of spider thread are sure to be. While a quartz fibre with its great tenacity and moderate Young's modulus will stretch quite perceptibly, it cannot be treated to the large stretching on a fork which a spider line allows, and so the application of the quartz fibre is more difficult. The best method of securing a fibre *in a stretched state*, which is essential, is to suspend from it a load of one-quarter, or one-half of the breaking weight. Then, having the diaphragm carried on a stand, with the ruled lines to which the fibres are to be attached vertical, this is moved until the fibre lies over the selected line. A touch of shellac varnish above and below the opening will secure the fibre, but it should be left with the weight suspended until the varnish is dry. No one would use a quartz fibre in this way unless a spider line was unsuitable. It is convenient to remember that a fibre $\frac{1}{5000}$ inch ($\frac{1}{200}$ millimetre) in diameter will carry about 40 grains or nearly 3 grammes, from which a judgment can be formed of the diameter that other loads will require.

---

[1] *Roy. Soc. Proc.*, 1920, xcvii.

It is convenient to remember also that the couple in dyne centimetre units due to a fibre of this diameter and 10 centimetres long when twisted through a unit angle (57·3°) is equal to ·00018, from which a judgment can be formed of the couple due to any fibre twisted through any angle. This figure is obtained from the rigidity $n = 2·9 \times 10^{11}$ and the torsional rigidity $\tau = \pi r^4 n\theta/2l$, where θ is the angle of twist in radians and l the length in centimetres. The rigidity n has a small *positive* temperature coefficient equal to ·00013, *i.e.* it becomes very slightly stiffer with increase of temperature. The value of Young's modulus in the same units is $5·18 \times 10^{11}$ with a temperature coefficient of about the same amount $1·3 \times 10^{-4}$, while the coefficient of volume elasticity is $1·4 \times 10^{11}$.

C. V. B.

— R —

RADIANT HEAT AND ITS SPECTRUM DISTRIBUTION, INSTRUMENTS FOR THE MEASUREMENT OF

(For a list of papers on the subject, see Bibliography at the end of the article "Radiation, The Measurement of Solar, etc.")

INSTRUMENTS for the measurement of radiant energy may be broadly classified into three groups:

(i.) Meteorological appliances, designed for the recording of the intensity of the radiation received from the sun or merely the duration of the sunshine.

(ii.) Instruments for the measurement of temperature, which are known as radiation pyrometers.[1]

(iii.) Instruments for investigating the distribution of energy in the spectrum.

For routine meteorological work the instruments have to be made exceedingly robust and easy to manipulate, consequently the bulk of the recorded data on solar radiation is of somewhat empirical nature, only suitable for making deductions of a qualitative character as to climatic condition on different days throughout the year.

The quantity generally determined is the duration of sunshine, *i.e.* the number of hours during the day in which unclouded sunshine has fallen on the earth's surface.

In recent years instruments have been devised for measuring the intensity of the sun's radiation, but this measurement presents greater practical difficulty than that of the duration of sunshine.

I. SUNSHINE RECORDERS[2]

The earliest practical instrument for recording duration of sunshine was that devised by Campbell in 1853.

§ (1) CAMPBELL'S SUNSHINE RECORDER.[3]—The original instrument consisted of a spherical glass bottle filled with acidulated water and supported centrally in a white stone bowl, the globe being about 6 in. in diameter. The bowl was engraved with hour lines and its surface blackened with an oil paint or varnish. The bottle acted as a lens and the sun's heat melted the paint wherever the rays impinged upon it. The observer read off the duration of sunshine for each hour and marked it on a paper ruled for the purpose. Campbell, in his paper, recommended the employment of a wooden bowl, and also suggested the use of photographic paper.

Such an instrument was installed in 1876 in Greenwich Observatory, the sunshine being received on blackened millboard.

Three years later Stokes devised a method of fixing the cards in position by slots in a metal bowl partly surrounding the sphere.

Subsequent modifications of the Campbell instrument were made by Curtis,[4] who arranged the instrument so that the position of the bowl relative to the sphere could be varied through several degrees of arc, so that it could be used in any latitude. A modern form of the instrument is shown in *Fig.* 1. Descriptions of

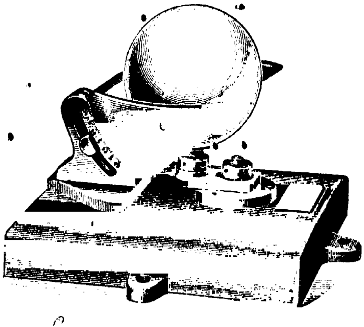


FIG. 1.

---

[1] A description of these instruments will be found in "Pyrometry, Total Radiation," Vol. I.

[2] See paper on "Instruments for the Measurement of Solar Radiation," by R. S. Whipple, *Trans. of the Optical Society*, 1914–15. (Contains an admirable résumé of the subject, and to which reference should be made for more detailed information.) See also "Meteorological Instruments."

[3] "On a New Self-registering Sun-dial," *Report of the Council of the British Meteorological Society*, May 1857, p. 18.

[4] "Sunshine Recorders and their Indications," *Quarterly Journal of the Royal Meteorological Society*, xxiv. 17.

special types of these instruments for use in the Tropics and the Arctic and Antarctic regions may be found in the following papers: "National Antarctic Expedition," 1901, *Meteorology*, Part I., p. 512, Royal Society, London; *The Observer's Handbook*, M.O. 191 (1914), Meteorological Office, London, pp. 83-94.

§ (2) JORDAN RECORDER.—An instrument working on a totally different principle is the *Jordan*[1] *Sunshine Recorder* (see *Fig.* 2). In this the intensity of the sunshine is measured by the amount of discoloration produced in a paper sensitised by the ferro-cyanide process. A strip of prepared paper is put into a semicircular box, and the sun's light allowed to pass through a small hole in the side of the box, and to fall upon the paper.

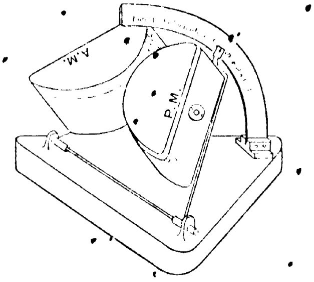


FIG. 2.

After exposure for the day, the paper is fixed by immersion in clear water, when a blue trace will be found upon its surface which roughly varies with the intensity of solar radiation.

As regards the records obtained with the Campbell-Stokes and the Jordan instruments, it will be at once recognised that there is an essential difference in their method of production. In the one case the record is produced by the thermal effect of the sun's radiation and in the other by the actinic effect. In 1896-97, R. H. Curtis, on behalf of the Council of the Royal Meteorological Society, undertook a careful examination of a year's records obtained from two of these instruments.[2] He found that, as a whole, the records agreed very well in the number of hours' sunshine recorded, but that the measurements of the Jordan curves are open to more uncertainty than are measurements of the Campbell-Stokes curves. In his paper dealing with the subject, Curtis makes some useful suggestions as to the definition of "bright sunshine," and discusses fully the mounting of sunshine recorders.

W. Marten of Potsdam Observatory has also made a comparison between the Campbell-Stokes and Jordan instruments, and is in agreement with R. H. Curtis. He states[3] that the Campbell-Stokes instrument registers earlier in the morning and later in the evening than the Jordan instrument, and is thus more suitable for winter months; also the records are easier to interpret and more permanent. The sensitivity of the Jordan instrument depends on the quality of the photographic paper. On the other hand, the Campbell-Stokes instrument is apt to over-record at midday, and is not so accurate when the sky is clouded. The cards are also liable to be spoilt by rain, but in spite of these drawbacks it is to be preferred as the more accurate of the two instruments.

§ (3) THE MARVIN RECORDER.—In America an instrument known as the *Marvin Thermographic Sunshine Recorder* has been extensively used for recording the duration of sunshine.[4] It is essentially a differential air thermometer, arranged as shown in *Fig.* 3.

A straight glass tube has cylindrical bulbs, C and D, one at each end, bulb D being smoothly coated on the outside with lampblack, and the whole enclosed in a protecting sheath A. The glass tube connecting the two bulbs is prolonged to nearly the bottom of the bulb D. The space between the inner tube with its bulbs and the protecting sheath is first, thoroughly dried, then exhausted of air and hermetically sealed. Mercury is used to separate the air in the bulbs, a small quantity with a little alcohol being placed in the bottom and stem of the lower blackened bulb D. A pair of electrical contacts is fitted at E. The instrument is mounted on a support as shown in *Fig.* 3, and the black bulb of the instrument is held downwards. When the

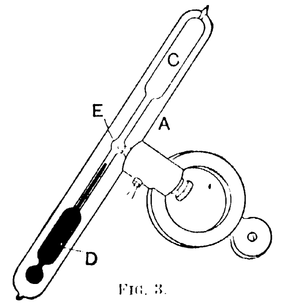


FIG. 3.

---

[1] *Quarterly Journal of the Royal Met. Soc.*, 1886, xii. 21.

[2] "Sunshine Recorders and their Indications," *ibid.* pp. 16-20.

[3] "Ergebnisse zehnjähriger Sonnenscheinregistrierungen in Potsdam," W. Marten, *Ergebnisse der meteor. Beob. in Potsdam*, 1904.

[4] *U.S. Department of Agriculture Weather Bureau*, 1914, W.B. No. 518.

black bulb is heated by radiation falling upon it, the mercury rises and closes an electric circuit between the contacts, this circuit actuating a recording counter. The inclination of the recorder is so adjusted that the mercury column keeps the circuit closed during times when the disc of the sun can just be faintly seen through the clouds.

§ (4) Dines' Recorder.—This is an ether-filled thermometer mounted on pivots. The radiation falls upon a black bulb and the expansion causes a thread of mercury to move, disturbing the equilibrium of the system. On the tipping over of the instrument the mercury closes electric contacts, completing a circuit through a chronograph.

II. The "Solar Constant"

The other group of solar instruments may be termed intensity recorders, since their function is to determine the quantity of heat received by the earth from the sun and the rate at which it is received.

§ (5) Definition of the "Solar Constant."—The quantity[1] of heat received in one minute from the sun, when at its mean distance from the earth, by one square centimetre of a perfectly absorbing surface presented normally towards the sun, and supposed to be situated just outside the earth's atmosphere, is known as the "Solar Constant."

It is probable that this quantity is not really constant, as most likely the various portions of the sun's surface presented to the earth have different radiating powers, and there may be also a periodical variation corresponding to the variation in the area of the sunspots.

On consideration of the above definition of the solar constant it will be seen that there is one outstanding difficulty in making measurements, viz. the impossibility of measuring the radiation received just outside the earth's atmosphere. The absorption by the atmosphere is very great, and is, of course, a varying quantity. It depends on the amount of dust and water vapour in the air; on the height of the observing station above sea-level; and on the elevation of the sun above the horizon. The length of the path of sunlight between two horizontal layers is proportional to the secant of the zenith distance of the sun, and, therefore, the absorption may be expected to be an exponential function of this secant. From observations taken at different hours of the day the nature of this function can be determined, and the strength of the radiation falling on the outermost layer of the atmosphere can be estimated.

It is now realised that the only way to obtain accurate data concerning the solar constant is by making a bolometric examination of the solar spectrum, and this can only be carried out with an elaborate equipment. The bolometric examination is necessary because the coefficient of absorption varies with the wave-length of the radiation.

During the past eighty years, however, numerous attempts have been made to effect determinations of the solar constant, most of the work having been carried out with comparatively simple apparatus subject to the above-mentioned sources of error.

§ (6) Pouillet's Pyrheliometer.—The first successful research on the solar constant was that made in 1837 by Pouillet, who designed the instrument since known by his name. This instrument, which he termed "Pyrheliometer," is shown diagrammatically in *Fig.* 4. It consists of a flat thin cylindrical vessel A, of which the upper face is lamp-blacked and the rest silvered. This contains water and serves as a calorimeter. It is mounted at one end of an axis D, round which it can be rotated to secure mixing of the contents, the thermometer stem lying along the axis. The part A of the instrument is first directed to a part of the sky away from the sun, and the fall in temperature noted during five minutes. It is then directed towards the sun for five minutes, and the rise noted; it is finally directed away from the sun and the fall noted during another five minutes. From a knowledge of the capacity of the calorimeter, the total heat received per unit of surface per unit of time can be determined. The results obtained by Pouillet were remarkably good for that period, when nothing was known concerning the absorption by water vapour, etc. His value, 1·7 calories per minute per square cm., compares very favourably with the latest determinations, viz. 1·93 calories.

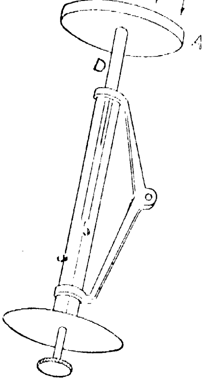


Fig. 4.

§ (7) Violle's Actinometer.—This instrument is similar in principle to Pouillet's pyrheliometer, and consists of a thermometer with a spherical bulb placed in the middle of a double-walled enclosure filled with water to maintain a constant temperature (see *Fig.* 5). An opening is provided with a shutter to allow the sun's rays to fall on the thermometer bulb. By exposing the thermometer for a definite interval of time and observing the temperature rise the value of the solar constant can be deduced.

---

[1] See "Radiation," § (1).

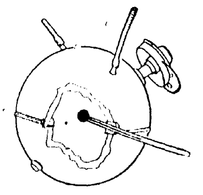


FIG. 5.

§ (8) BLACK-BULB THERMOMETER *IN VACUO*.—An instrument which has been extensively used for the rough comparison of the intensity of sunshine on different days is the black-bulb thermometer[1] (see *Fig.* 6). The construction of this instrument was first suggested by Sir John Herschel, and it consists of a sensitive maximum thermometer with the bulb and about 25 mm. of the stem coated with lamp-black. The whole is enclosed in a glass tube, of which one end is blown out into a bulb about 55 mm. in diameter, in the centre of which the bulb of the thermometer is fixed. The glass jacket is sealed and exhausted.

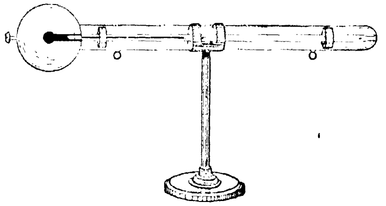


FIG. 6.

Experiments have shown that there is a relation between the intensity of radiation as indicated by the black-bulb thermometer and the number of hours of sunshine. In a discussion of graphs showing daily observations at Kew for a year, G. M. Whipple found that, taking both maxima and minima into consideration, there were seventy cases of agreement between the solar radiation and sunshine in their times of occurrence, and six of disagreement.

He was led to the conclusion that "the black-bulb thermometer is only to be considered as an indicator of the relative presence or absence of cloud from the sky at the locality, and so its use as a meteorological instrument may with advantage be set aside in favour of the sunshine recorder, which is not influenced by the element of uncertainty inseparable from the former instrument."

There are several objectionable features about the instrument. For example, the size of the bulb has a very large influence on the readings. Whipple found that a difference of 1 mm. in a bulb of 12·5 mm. in diameter or a difference of 8 per cent caused the correction required to bring the readings in accord with the standard to be more than doubled. He also found that the thickness of the blacking had considerable influence on the reading. He concluded from his experiments that it was necessary to exact from instrument makers a rigid adherence to a standard size of bulb and a definite amount of coating, as well as a constant state of exhaustion and standard size of jacket, all conditions very difficult of realisation, particularly the pressure within the jacket, since the black paint coating on the bulb slowly evolves gas, resulting in a deterioration of the vacuum with time.

§ (9) WINSTANLEY'S RADIOGRAPH.—This instrument is very similar in principle to the Dines recording sunshine receiver, except that it is intended to record the intensity of solar radiation, and not only the duration of sunshine. A differential black-bulb and unblacked bulb thermometer is poised upon knife edges so delicately adjusted that its equilibrium is disturbed by the displacement of the mercurial index, equilibrium being re-established by the whole system taking up a fresh position. The movement from one position to another is recorded by a long arm upon a sheet of smoked paper wrapped round a drum driven by clockwork.

§ (10) THE RADIO INTEGRATOR.—A simple and ingenious instrument was designed by W. E. Wilson for the measurement of total radiation received by the ground. The radio integrator, as it is named, consists of a sealed retort for the distillation of a volatile liquid, *in vacuo*, by the radiation falling upon it (see *Fig.* 7). When set for an observation, the whole of the liquid is in the upper bulb, which is exposed to the sun, while the lower limb and tube are sheltered in a white perforated box. The liquid as it evaporates is condensed in the lower bulb and trickles down into the tube. As the latent heat of vaporisation of alcohol is regarded as approximately constant over the range of ordinary air temperatures, the amount of alcohol which distils over from the upper bulb is directly proportional to the solar radiation. In the standard type of

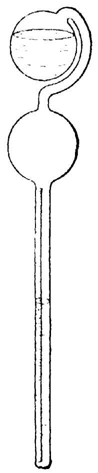


FIG. 7.

---

[1] Whipple, *Quarterly Journal Meteorological Society*, v. 142; x. 45.

instrument a cubic centimetre of alcohol passes over for 179 gram calories received by the upper bulb. Unfortunately, the instrument is sensitive to the cooling effect of the wind. As suggested by Mr. R. S. Whipple, this error might perhaps be corrected by enclosing the upper chamber of the instrument in a vacuum jacket.

§ (11) Silver-disc Pyrheliometer.—This modified form of the Pouillet pyrheliometer was developed by the authorities of the Smithsonian Astrophysical Observatory,[1] and has been largely adopted in the United States as a subsidiary standard; about 25 copies have been standardised and sent out to stations in Europe, North and South America, and other places.

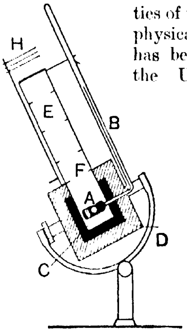


Fig. 8.

A section of the silver disc pyrheliometer is shown in *Fig.* 8. The silver disc A is bored radially with a hole to admit the cylindrical bulb of a thermometer B which is bent at a right angle.

To assist the transfer of heat from the disc to the thermometer bulb the hole is lined with thin steel and contains mercury. A soft cord soaked in shellac is forced down at the mouth of the hole to prevent the escape of mercury. The silver disc is enclosed by a copper cylindrical box C. Three small steel wires, not shown in the figure, support the silver disc. Clamping screws are provided in order that all strains may be taken off the wires during the transit. The box C is enclosed in a wooden box D to protect the instrument from temperature changes.

Sunlight is admitted through the tube E, in which a number of diaphragms are placed. The aperture in the diaphragm F nearest the silver disc is slightly smaller than the disc itself. It thus limits the cross-section of the beam whose intensity is to be measured. Shutters are provided at H. The top of the tube E carries a screen large enough to cover the wooden box. The instrument is mounted on an equatorial stand. When making an observation the instrument is pointed at the sun and the temperature rise of the thermometer during intervals of 100 seconds is observed over a period of a few minutes. From the readings obtained the intensity of the solar radiation can be deduced.

The instruments have the outstanding advantage of simplicity, and at the same time reliability. It is found that the standard scale of radiation may be reproduced in these instruments to an accuracy of 0·5 per cent.

§ (12) The Michelson Pyrheliometer (see *Fig.* 9).—In this instrument[2] the thermometer consists of a bimetallic strip A, the extension of which, P, moves when the strip is heated by the solar radiation, admitted through the aperture D. The movement of the strip is observed by means of a microscope M. The instrument is calibrated and observed in a similar manner to the Smithsonian instrument, but the sensitive element having a smaller thermal capacity, the instrument is more rapid in action. It is doubtful, however, whether it is as robust or so constant in its readings as the Smithsonian instrument.

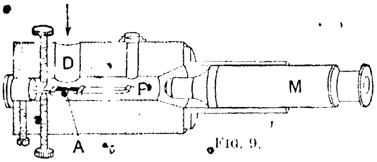


Fig. 9.

§ (13) The Absolute Pyrheliometer of Abbot and Fowle.—In 1903 Abbot and Fowle commenced the construction of their absolute pyrheliometer,[3] consisting of a "black-body" receiver combined with a flow calorimeter, the chief innovation being the adoption of a hollow chamber to receive the solar rays. Such a chamber is a nearly perfect absorber, consequently no correction is needed for the reflection of rays from the receiving surface. This pyrheliometer has been developed through various models into an accurate piece of apparatus, and the modern form is illustrated in *Fig.* 10.

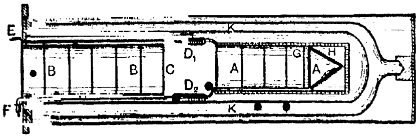


Fig. 10.

AA is a chamber of about 3·5 cm. inside diameter, with hollow walls adapted for the circulation of a stream of water. The stream enters at E, passes around the walls, the rear of the chamber, the cone-shaped receiver of rays H, and passes off at F, carrying away the heat developed by the solar rays which entered the chamber through the aperture C, the dimensions of which are accurately measured.

---

[1] *Annals of Astrophys. Observatory*, ii. 36; iii. 47.

[2] *Meteorologische Zeitschrift*, 1909, p. 246.

[3] See Abbot and Fowle, *Astrophys. Journal*, 1911, xxxiii. 191.

At D$_1$ and D$_2$ are platinum coils adapted to measure the temperature rise of the water due to heat absorption. The water flows in a spiral channel around the instrument. In order to prevent external temperature changes from vitiating the observations, the whole apparatus is enclosed in the Dewar vacuum flask KK, and to prevent the breakage of the flask it is enclosed in a brass case.

In the standard instrument the Dewar flask has been discarded in favour of a vigorous water-stirring arrangement. Reference should be made to the original paper for information as to the construction of the resistance thermometer coils, the method of maintaining the flow constant, etc. On the back of the receiving cone H, a coil of manganin wire is wound, forming a heating coil by means of which a definite quantity of electrical energy can be dissipated in the instrument. In this way a direct measurement may be made of the temperature rise in the outflowing water caused by a known quantity of electrical energy.

The published observations show that the results obtained with these instruments are remarkably concordant, the maximum divergence of the mean result of six groups derived from seventy observations being one per cent.

§ (14) ELECTRICAL TYPES OF SOLAR RADIATION INSTRUMENTS.—Several forms of pyrheliometers have been devised in which electrical resistance thermometers or thermo-elements have been employed instead of mercury thermometers, and such instruments are distinctly more reliable than the older types of pyrheliometers.

Callendar has constructed a black-bulb thermometer in which a differential resistance thermometer replaces the mercury thermometer, and the instrument has the additional advantage of being distant reading or recording.

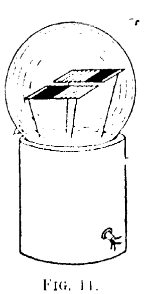


FIG. 11.

§ (15) CALLENDAR'S SUNSHINE RECORDER.—This instrument[1] is shown diagrammatically in *Fig.* 11. It consists of two pairs of resistance coils wound on mica frames, the two pairs being mounted diagonally to each other. One pair of coils is blackened by means of black glass enamel, thus avoiding any risk of deterioration of the blackened surface by age. The resistance of the two blackened coils is the same as that of the two unblackened ones when no radiation falls on the surfaces. The coils are placed in an hermetically sealed glass vessel filled with dry air. The coils are connected to a Callendar recorder. The instrument is standardised by measurements made on the radiation emitted by standard lamps. The lamps used by Callendar for this purpose emit a radiation of approximately 1 calorie per square centimetre per minute at a distance of 20 cm. when the specified voltage is applied.

§ (16) ÅNGSTRÖM'S PYRHELIOMETER.—The principle of the Ångström pyrheliometer[2] is simple. Two thin metal strips (20 mm. long, 1·5 mm. wide, and ·02 mm. thick) are alternately exposed to the radiation to be measured. When one strip is being heated by the radiation falling upon it, the other strip, which, although shielded, is close to the exposed strip, is heated to the same temperature by an electric current passing through it. Equality of temperature is indicated by a sensitive galvanometer connected to a pair of copper constantan thermojunctions, attached to the back of the receiving strips but insulated from them by thin silk paper and shellac varnish. It is then assumed that the energy expended in the electrically heated strip is equal to the radiant energy absorbed by the exposed strip.

The amount of energy absorbed by each particular instrument depends on three factors, which must be accurately known, viz. the width and resistance of the strips and the coefficient of absorption of the smoke-black film on the strips. Knowing these values, the absolute value of the radiation may be computed.

The construction of the instrument is simple, and is illustrated diagrammatically in *Fig.* 12.

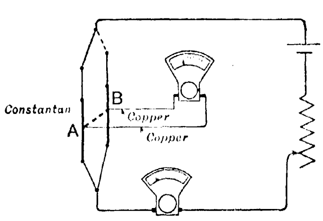


FIG. 12.

The two strips of manganin, A and B, are mounted side by side on a small circular frame about 40 mm. in diameter. On the front end of the tube carrying the strips a double-walled reversible screen is mounted, which protects one or other of the strips from external radiation. The auxiliary apparatus consists of a milliammeter, an accumulator, a rheostat, and a sensitive galvanometer.

§ (17) CALLENDAR'S RADIO-BALANCE.—The radio-balance due to Callendar[3] was developed from the radio-calorimeter described by him in the British Association Report for 1900.

---

[1] *B.A. Report*, 1900, p. 38.

[2] *Astrophysical Journal* 1899, ix. 332.

[3] *Proc. Phys. Soc.* xxiii. 10.

In this instrument a copper disc (13 mm. in diameter) was supported by two fine iron-constantan couples at right angles, either of which could be used for observing the rate of rise of temperature of the disc when exposed to radiation. It occurred to Callendar that by passing an electric current through one of the thermocouples the radiation might be directly compensated by the heat absorption due to the Peltier effect. In the first model a disc was used as a receiver, but in the later model a cup (4 mm. diameter, 10 mm. deep) was employed. This cup has been found experimentally to have an absorption coefficient of nearly unity.

The radiation is directly compensated by the heat absorption due to the Peltier effect in a thermojunction, soldered to the cup, through which a measured current is passed; and, since there is practically no temperature change, the radiation is deduced in absolute measure without a knowledge of the thermal capacity of the cup. The cup is mounted in a small tubular thermopile connected to a sensitive galvanometer for indicating the balance (see *Fig.* 13).

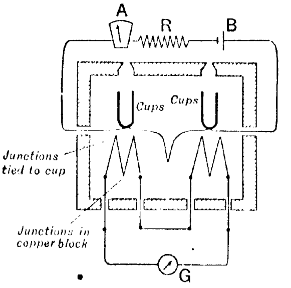


FIG. 13.

In order to compensate accurately for changes in the surrounding temperature and to avoid the necessity for a water-jacket, two similar cups with similar connections are mounted side by side in a hollow copper cylinder. Either can be connected by a double-pole switch to a circuit of the battery B, rheostat R, and milliammeter A, for supplying, regulating, and measuring the current. The thermopiles in which the cups are mounted are oppositely connected in the circuit of a sensitive galvanometer G, so as to indicate the difference in temperature of the cups. The upper junctions of each pile are bound firmly round the middle of a cup, from which they are insulated by thin paper and shellac. The lower junctions are similarly fastened to a small copper block screwed to the base of the thick copper cylinder. The thermopiles usually consist of twelve couples each. The Peltier couples are single iron-constantan couples soldered to the bottom of the cups. The four pairs of leads for the two couples and the two thermopiles are brought out separately, so that the insulation can be tested at any time.

The equation giving the value H of the radiation in absolute measure (watts per sq. cm.) is

$$aAH = PC - C^2R = PC\left(1 - \frac{C}{C_0}\right),$$

where a is the absorption coefficient of the blackened cup,

A the area of the aperture admitting radiation in sq. cm.,

H the intensity of the radiation in watts per sq. cm.,

P is the coefficient of the Peltier effect in volts,

C the current through the couple, in amperes,

$C_0 = P/R$, the neutral current giving neither heating nor cooling.

It is obvious that there is a maximum radiation which can be directly compensated by the cooling effect, but the range can be varied, and the instrument is quite suitable for the measurement of solar radiation.

When the radio-balance is employed as a pyrheliometer the thick copper cylinder containing the cups and their connections is extended by a tube containing suitable diaphragms for limiting the exposure. The whole is mounted equatorially on a levelling stand. The tube is gilded in place of being lacquered, as the gilt stands exposure to the sunshine better and also greatly diminishes the absorption of heat. A convenient feature of the direct compensation method is that the result is obtained by a single observation, but it is generally desirable to interchange the cups at each observation. The rapidity of working is a great advantage when a variable like sunshine is being measured, since an observation can be made in about one and a half minutes. The most convenient method of observation is to set the current at a suitable constant value, such as 0·2 ampere, corresponding to nearly 1 calorie per square centimetre per minute, and to observe the deflections of the galvanometer, which are proportional to the change in the intensity of radiation. The instrument is almost perfectly compensated for changes in the surrounding temperature, and it quickly reaches a final steady deflection. A disadvantage is that the increase of the Peltier effect per 1° C. for the couples employed is nearly ·0001 volt in 0·0150 volt. It is necessary, therefore, to read the instrument temperature to about 0·1° C. in order to obtain an accuracy of 1 in 1500.

III. INSTRUMENTS FOR MEASURING THE DISTRIBUTION OF ENERGY IN THE SPECTRUM.

The measurement of the distribution of energy in the spectrum of a hot object, presents considerable practical difficulty on account of the low sensitivity of all the instruments hitherto devised for the purpose. It is only in the case of the sun and the acetylene flame that it has been possible to measure the distribution right through the visible to the ultra-violet (0·36 μ); in other cases the work has been confined to the infra-red, where the energy is a maximum.

§ (18) **BOLOMETRIC APPLIANCES FOR SOLAR WORK.**—As far back as 1881 Langley invented the bolometer and applied it with masterly skill to the mapping of the energy in the solar spectrum, and his work in connection will be referred to in Section IV.

Langley's instrument was not taken up by meteorologists on account of the delicate manipulation involved, and it is only within comparatively recent years that a bolometer has been evolved capable of use in routine meteorological observations.

This development of the bolometer[1] is due to Abbott of the Mount Wilson Astrophysical Observatory, and the instrument is illustrated in *Fig.* 14.

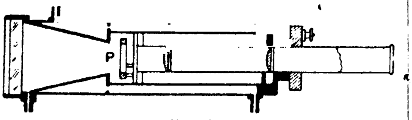


FIG. 14.

§ (19) **ABBOTT'S FORM OF BOLOMETER.**—The working part of the instrument consists of a thin strip of platinum P, about 12 mm. long, 0·06 mm. wide, and its thickness may be best estimated from the fact that its electrical resistance is about 4 ohms. For the sake of symmetry of conditions a second strip of platinum, as nearly as possible like the absorbing strip, but shielded from radiation by a diaphragm, forms the second arm of the Wheatstone bridge. The two platinum strips are blackened by camphor smoke. The third and fourth arms of the bridge consists of two equal coils.

Special precautions are necessary for the avoidance of uncertain contacts, thermal effects, and for protection from air currents, etc.

For work with the bolometer it is necessary to employ galvanometers of greater sensitivity than those ordinarily available for routine work. A suitable instrument is one which will respond to a current of 10^{-11} ampere with a period of 10 seconds and a resistance of 20 ohms.

According to Nutting a good bolometer outfit should be capable of measuring a radiation flux of one millionth watt per square centimetre, and must measure within 1 per cent a flux 100 times this amount, *i.e.* 0·0001 watt per square centimetre, which is roughly equal to the radiation from an ordinary candle at 1 metre.

Although this is a fairly bright illumination as judged by the eye, it must be remembered that the normal eye is vastly more sensitive in the visible spectrum than the most delicate bolometer.

§ (20) **THE THERMOPILE.**—The multiple junction thermo-element has been developed by Rubens and others into an instrument of considerable accuracy for spectral work.

By careful construction a multiple element radiometer can be made whose sensitivity is roughly one-tenth that of a high-class bolometer, and it has the advantage of being more convenient to use than the bolometer.

A modern type of thermopile[2] is shown in *Fig.* 15, the elements of which are bismuth and silver. The pile consists of about twenty junctions arranged in the receiver so that the junctions lie in a straight line and slightly one behind the other, so that it is impossible for radiation to pass through the pile. The cold junctions occur alternately on either side of the hot junction and are protected from external radiation.

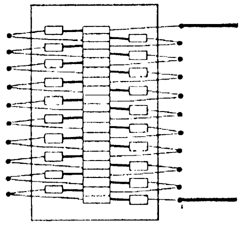


FIG. 15.

An instrument based on the same principle as the above is the Boys radio-micrometer.[3]

The active part of the instrument consists of a very light circuit composed of a single loop of fine silver wire the lower ends of which are joined by a pair of very light bars of two alloys giving a large thermo-electromotive per degree. These bars form a thermo-electric circuit and radiation is received on the lower junction. The circuit is suspended by a quartz fibre in the field of a permanent magnet. The general arrangement of a modern form is illustrated in *Fig.* 16.

---

[1] *Annals of the Astrophysical Observatory*, ii. 28.

[2] Coblentz, *Bulletin Bureau of Standards, U.S.A.*, iv. 391; ix. 7; xi. 131.

[3] See "Radio-micrometer."

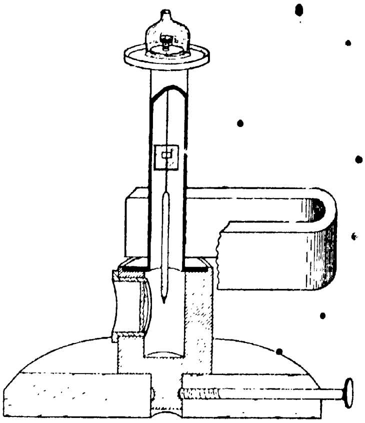


FIG. 16.

§ (21) NICHOL'S RADIOMETER.—Nichols in some of his work on the dispersion of various bodies for rays of very long wave-length employed a modified form of Crookes's radiometer.

In Nichol's instrument the vanes are suspended by a quartz fibre about 0·5 μ in diameter, and the angle through which they are turned from their position of rest is measured by the excursion of a spot of light reflected from a very small mirror.

The instrument is shown diagrammatically in *Fig.* 17. The vanes are made of the thinnest mica, 1 mm. by 4 mm., blackened. The whole suspended system weighs but a few milligrams. The heavy metal case is provided with fluorite windows. The sensitivity of the instrument is a maximum at about 0·6 mm. gas pressure and decreases rapidly for both higher and lower pressures.

Experiments show that the sensitivity of a well-made instrument is nearly as good as that of a high-class bolometer, but the period is much longer.

The radiometer gives well defined readings with little creeping at the maximum deflection and but little shift of zero. It can be moved about easily, is independent of auxiliary apparatus, and does not require skilled manipulation.

Its most serious disadvantage is the fact that the sensitivity varies with the gas pressure.

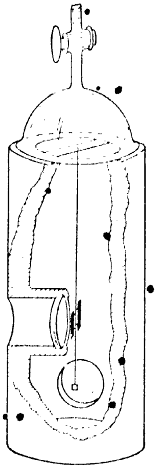


FIG. 17.

IV. METHODS OF CALIBRATION IN TERMS OF WAVE-LENGTHS

In order to be able to measure the indices of refraction of various substances in the infra-red and ascertain the wave-lengths of the Fraunhofer lines in the solar spectrum, it is necessary to have means of determining the relations between the dispersion and the true wave-length.

§ (22) LANGLEY'S METHOD.—Langley, in his work on the solar spectrum, determined the wave-lengths of the absorption bands by the following device.

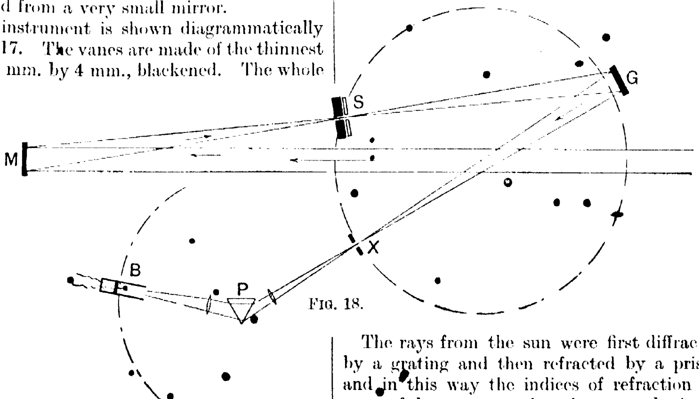


FIG. 18.

The rays from the sun were first diffracted by a grating and then refracted by a prism, and in this way the indices of refraction for rays of known wave-length were obtained; further, the prism served to separate the various superposed orders of the spectra produced by the grating. A diagram of the apparatus is shown in *Fig.* 18. A beam of sunlight from a heliostat falls on to the concave mirror M, which focusses it upon the slit S of a concave grating apparatus. This slit is protected by a plate of iron, pierced with a hole, which is only a little larger than the slit. From the slit the rays fall upon the concave grating G, which focusses the spectral rays on the dotted circle. At X, where the photographic plate, or eyepiece, is usually placed to observe the normal spectra, there is a second slit, which forms part of a prism spectrometer, of which the lenses are shown and the prism at P. The second lens focusses the image of X upon the bolometer at B; the arm carrying the bolometer rotates round a centre under the prism, and there is an arrangement for always keeping the latter in the position of minimum deviation. The readings of the angle of deviation produced by the prism can be read accurately to 1 minute of arc. The grating was one ruled by Rowland, with 18050 lines (142 to the millimetre) on a spherical mirror of 1·63 metre focus.

§ (23) Interference Method.—In 1847 an interference method was devised by Fizeau and Foucault for determining wave-lengths in the visible spectrum. About twenty years later Mouton applied a similar method to the infra-red end of the spectrum.[1]

Reference must be made to the following memoirs for a detailed description of the method: *Comptes Rendus*, 1847, xxv. 447; *Ann. Chim. Phys.*, 1879 (5), xviii. 145.

The writer desires to record his indebtedness to Mr. R. S. Whipple for permission to draw upon the material contained in his paper read before the Optical Society.

E. A. G.

RADIATION

Radiation viewed from the Aspect of Meteorology, the Determination of the Constants involved, the Absorption of Radiation by the Air and by Water Vapour, and the Meteorological Effects of Radiation.

§ (1) The Solar Constant.—The whole science of meteorology depends closely upon radiation, for without the radiant energy that comes from the sun there could be no meteorological phenomena, no climate, no sequence of weather, no rain, snow, wind, or storm; indeed a study of the weather is simply a study of solar energy as it passes through its various forms from the time it enters the earth's atmosphere as radiant heat and light until it is radiated out to space in the form of the long wave radiation of the earth and atmosphere.

Our first inquiry is naturally the amount of heat received from the sun, the value of the so-called solar constant. This is not the amount that is received at the surface of the earth, because the air intercepts some portion of the energy as it passes through and reflects back another portion, nearly half probably, into space. The solar constant may be defined as the amount falling in unit time on a unit surface that is exposed perpendicularly to the unimpeded radiation from the sun. The value now generally accepted as approximately correct is that obtained by Abbot and Fowle and is a little under 2 gramme-calories per minute per square centimetre ("Radiation, Measurement of Solar"). That is to say that if the solar heat at a place where the sun is in the zenith were entirely absorbed by a horizontal layer of water 1 cm. thick placed at the outer limit of the atmosphere it would raise that water nearly 2° C. in temperature in one minute. This is the mean value: the actual radiation varies between January and July by as much as 6 per cent because the sun is a trifle over 3 per cent nearer the earth on January 1 than on July 1, and the intensity of the radiation varies as the inverse square of the sun's distance.

There is also reason to think that the amount of the radiation is not really a constant apart from the sun's change of distance but that it may vary with the sunspot period. It has been held that the energy radiated should be greatest at the time of the sunspot maxima, but Dr. G. T. Walker has obtained the correlation coefficients between the number of sunspots and the mean temperature at 100 stations, and it appears from his work that the correlation is mostly negative. It might well happen that owing to a casual deviation the majority of the coefficients might be negative, but the mean value of the coefficients (− ·13) though small is still large enough to be significant, and 77 are negative. Now 77 out of 100 is beyond the range of a casual percentage in a sample of 100, so that the investigation shows that the earth is cooler at the times of sunspot maxima ("Correlations in Seasonal Variations of Weather," *Memoirs of the Indian Meteorological Department*, vols. xx. and xxi., by Dr. G. T. Walker, F.R.S.).

(i.) *Method of determining the Value of the Constant.*—The difficulty of determining the solar constant is readily apparent, for it is obvious that a radiometer to measure it cannot be placed at the outer limit of the atmosphere, the only place where the quantity to be measured is to be found. It follows that the value must be inferred from the quantity left after the solar rays have passed through considerable portions of the earth's atmosphere and thereby lost some part of their energy. Now if the loss of energy were simply proportional to the mass of air through which the rays had passed it would be easy to measure the radiation reaching the earth after the rays had passed through a mass of air equivalent to, say, one, two, or three atmospheres. The values so obtained would give the loss per atmosphere, and this loss added to the value at the earth's surface for a vertical sun would be the value of the solar constant, for the rays from a sun exactly overhead pass, of course, through one atmosphere in reaching the earth.

---

[1] An outline of the procedure adopted is given in Baly's *Spectroscopy*.

However the rule is not so simple as that supposed above; the same mass of air does not absorb the same quantity of heat from the solar rays passing through it in the lower part as in the higher part of the atmosphere, because in the lower part the actual stream of energy is less, some of it having been lost above. But we may assume, to begin with, that the same proportion of the energy due to a certain wave-length is always absorbed by the same mass of air, and this leads to the formula given below.

Let radiation equal to A per square centimetre be entering at right angles a layer of air of mass m per sq. cm. and let a proportion p of it be absorbed, an amount $(1-p)$A will be left. Of this $p(1-p)$A is absorbed by the next layer and $(1-p)^2$A is left. Then after n such layers have been passed the amount left, B_n let us say, will be $(1-p)^n$A. The amount absorbed will be $A-B_n$. Call this qA and note that it is absorbed by a stratum having a mass of mn per square cm. Then we have

$$qA = A - B_n = A - (1-p)^n A,$$

$$\therefore 1-q = (1-p)^n.$$

If n be a large number heavy multiplication will be required in using this formula, but it may be avoided by taking logarithms. Thus

$$\log(1-p) = \frac{1}{n}\log(1-q). \quad . \quad . \quad . \quad \text{I.}$$

Further, if the original stratum be very thin, then p will be very small and $\log_e(1-p)$ may be taken as equal to $-p$,

$$\therefore -p = \frac{1}{n}\log_e(1-q),$$

$$p = \frac{1}{n}\log_e\frac{1}{1-q} = \frac{2{\cdot}302}{n}\log_{10}\frac{1}{1-q}. \quad . \quad \text{II.}$$

Equation II. may be written in the form

$$1-q = e^{-np}$$

or

$$B_n = Ae^{-np}, \quad . \quad . \quad . \quad \text{III.}$$

where A is the incident energy and B_n is the energy that emerges after passing through a mass nm of air, provided that the pencil of radiation is of one sq. cm. cross-section; hence by suitably choosing the units we may write $B = Ae^{-mp}$, where m is the mass of air passed through.

By the use of this equation we are theoretically in a position to determine the solar constant A, for B is the amount of energy directly measured by a radiometer. We proceed thus. Make an observation when the sun is in the zenith and let B_1 be the value observed. Make a second observation when the sun is 60° from the zenith and let the amount be B_2. In the first case the rays have passed through a mass of air equivalent to one atmosphere; in the second case, as will be easily seen by drawing a figure, the rays have passed through very nearly two atmospheres—it would be exactly two if the earth were flat instead of spherical. Hence we have the two equations $B_1 = Ae^{-mp}$ and $B_2 = Ae^{-2mp}$, and since B_1 and B_2 are known, the two equations contain only two unknown quantities, namely A and the product mp, and therefore these can be determined and A the solar constant found.

But in practice the matter is not so simple and many difficulties are encountered. Apart from a further complication to be mentioned presently the value of p is not the same for different parts of the solar spectrum, and the value of A cannot be obtained as a lump sum; but A must be separated out into many parts corresponding to different wave-lengths, each part must be obtained separately and then the total added up. This involves extra observational difficulties, because a prism or a grating must be used to separate out the different wave-lengths. Then again p has not the same value for the upper as for the lower parts of the atmosphere. There is some doubt whether p does not depend on the pressure, but however that may be p depends upon the amount of water vapour present in the air, and the vapour is concentrated in the lower strata. Thus the formula $B = Ae^{-mp}$ is not rigorously exact, since it was obtained on the supposition that p is a constant. Some allowance can be made for this by using values obtained from mountain stations where the effect of the lower third of the atmosphere is cut out.

It must also be noted that observations on the solar radiation with different altitudes of the sun cannot be obtained simultaneously at the same place, and during the necessary interval of time the conditions may have altered. If one could be sure that there was no systematic change of condition with time the casual variations could be eliminated by taking the average of a large number of observations, but the value of p is known to change during the day, for the maximum of the solar radiation reaching the earth is not as well marked at noon, when the sun is highest, as it ought to be, since the curve is almost flat during the midday hours.

(ii.) *Value of the Solar Constant.*—There is not space to set out in detail how these difficulties have been met, it must suffice to say that Abbott and Fowle,[1] after long and careful work, have given the value of the constant as 1·93 gramme-calories per square centimetre per minute, and their value is generally accepted by those most competent to form an opinion as reliable. For meteorological purposes, avoiding decimals, we may take it as 2 g.c. It will be shown further on that the value of the constant can barely exceed 2 g.c.

---

[1] See "Solar Constant, Measurement of," Abbott and Fowle, *Annals of the Astrophysical Observations of the Smithsonian Institute*, Washington, 1908, ii.; 1913, iii.

To obtain the mean for the surface of the earth we must divide by 4, since the whole surface of the earth, assumed to be a sphere, is $4\pi r^2$, and the flat surface on which the same pencil of solar rays would fall perpendicularly is πr^2. Thus the mean value is ·5 g.c. per sq. cm. per minute. For meteorological purposes a more convenient unit is the day, and the value ·5 g.c. per minute gives 720 c.g. per sq. cm. per day. In other units this is 3009·6 joules per sq. cm. per day, or 34·8 milliwatts per sq. cm. The following approximate equivalents are useful, but it must be remembered that nearly half of the solar heat is reflected by the atmosphere and does no useful work.

The average solar heat received at the outer surface of the atmosphere per day is capable of raising by 1° C. the temperature of a layer of water covering the whole earth 720 cm. thick (23·62 ft.), of melting a layer of ice 9 cm. thick (3·54 in.), of evaporating at atmospheric temperatures a layer of water 12·2 mm. thick (·48 in.). It would raise the temperature of the whole atmosphere about 3° C., or if the whole supply took the form of kinetic energy it would impart to the air a velocity of 33 metres per second (73 miles per hour).

It may be added here that the maximum energy of the solar radiation occurs at a wave-length of about ·5 μ (μ = ·001 mm.).

§ (2) THE CONSTANTS OF ATMOSPHERIC RADIATION. (i.) *Effective Radiation.*—In considering the meteorological aspect of radiation it is desirable to know the quantity of radiant energy that a given mass of dry air at a definite temperature and pressure can give out. So many difficulties are met with in determining this quantity and the figures that have been given are so discordant that no attempt can be made at the present time to give authoritative values. By "dry air" the mixture of the permanent gases of the atmosphere is meant; the effect of water vapour, which is very important, must be considered separately. Of the permanent gases carbonic acid, though small as a percentage of the whole, has most effect on the radiation.

Consider, then, the radiation from one gramme of dry air. But firstly, since great confusion has arisen from neglect of this point, what is meant by radiation must be clearly defined. By the theory of exchanges a body is giving out the same radiation whatever be the nature of its surroundings, the change from an enclosure at the temperature of liquid air to one at the temperature of boiling water makes no difference, so long as the condition of the radiating body itself remains unchanged. Radiation is used in this sense in the following remarks. But the temperature of the surroundings makes a great difference to the radiant energy received by the body and therefore to its change of temperature, which depends on the difference between what it is giving out and what it is receiving. This difference has loosely been called "radiation" in some cases. It is measured by the change of temperature it causes and in the following remarks it is called "net" or "effective" radiation.

(ii.) *Radiation from the Air.*—Consider then a gramme of air and assume, as we may do with so small a unit, that the radiation given out by one part of it is not absorbed by any other part. Such a mass of air is giving out radiation equally on all sides, upwards, downwards, horizontally, and in all intermediate directions. In this respect its radiation is unlike that of the sun, where the rays that reach the earth are in one direction only. It is required to measure this radiation and there are various ways in which an attempt to do so can be made. We will consider the meteorological methods. If the gramme of air could be placed in such a position that it was losing heat by radiation alone and we could measure the rate at which its temperature was falling, we should, knowing also its temperature, be able to find the constant in question, for the mass multiplied by the specific heat at constant pressure, multiplied by the fall of temperature, gives the loss of heat, and this is equal to the net loss by radiation in the same time.

No small mass of air is precisely in this condition, but the lower strata of the atmosphere, say, for example, the stratum lying between the heights of one and two metres above the ground, cool rapidly on still, clear nights, and the rate of cooling can be measured. On the assumption made that the cooling is entirely due to radiation, an assumption that, as we shall show below, can hardly be justified, the rate of cooling gives the net radiative loss from the stratum. But to determine the constant the actual radiation is required. If the curve for a still, clear night given by an ordinary thermograph be examined, a rapid fall of temperature about the time of sunset will be noticed, and it will be seen that the curve seems to approach a horizontal asymptote. The temperature denoted by this asymptote is taken as the equivalent radiative temperature of the surrounding stratum of air, and from a knowledge of that temperature and the constant of radiation the whole radiation from the stratum can be determined. But it must be remembered that the vertical component is the only effective radiation, and this occurs from both sides of the stratum, it is directed upwards as well as downwards. The horizontal component is not effective, it is only the interchange of equal quantities of radiant energy between bodies at the same temperature, *i.e.* between different parts of the same stratum.

The method is not satisfactory (*vide* a paper by Lieutenant-Colonel Gold, *R. Met. Soc. J.* vol. xxxix. No. 168, p. 253), and the reasons for this are probably twofold.

The assumption that the air is changing its temperature under the action of radiation alone is a very doubtful one. In England at least very low temperatures at night do not occur unless there is a good covering of snow on the ground, and preferably of light fleecy snow. Thus at Benson, Oxfordshire, on February 5, 1917, at 1 A.M. the thermometer stood at 28° F. with a cloudy sky and 2½ inches of light freshly fallen snow on the ground. By 7.30 A.M. the sky had cleared and the temperature had fallen to −4° F., both temperatures being screen temperatures at 4 feet above the ground. It seems very unlikely that this unusually low temperature, which is not likely to occur once in fifty years, and the equally unusual rate of fall, were due entirely to the special radiative quality of the air on that morning; it is much more likely that it was in large part due to the snow forming a non-conducting blanket to the earth and so preventing the heat stored in the earth from maintaining the temperature of the air.

If this be so, then under ordinary circumstances part of the change of temperature in the stratum considered depends on the heat flow from the ground.

The second reason is this. If the radiating power of the atmosphere, which we may call the emissivity, is large, so also by Kirchoff's law is the absorbing power; hence if the air is giving out much radiation it is also absorbing the radiation from the ground and the neighbouring strata freely, and the result of an increased emissivity is not very marked on the net radiation. It follows that any error made in estimating the net radiation and expressed as a percentage error is largely increased when expressed as a percentage error on the emissivity.

But the method has one advantage. The amount of water vapour present in the layer of air, the radiation of which is being observed, is known, and this amount may have an important influence on the result.

There is a second meteorological method which on the whole is more satisfactory. A radiometer is used which gives the net loss of energy from a black surface to the sky at night. By the Stefan-Boltzmann law that the actual radiation is proportional to the fourth power of the temperature measured on the absolute scale, and a knowledge of Stefan's constant, the whole radiation from the blackened surface is known. Subtracting from this the net radiation that is measured by the radiometer we obtain the value of the return radiation from the sky. For England with a clear sky the average for this quantity lies somewhere in the neighbourhood of 500 gramme-calories per day. We know within about 1° C. the mean temperature of the air over England up to a height where the pressure is less than 50 mb., and the emissivity must be such that the total downward radiation may agree with the observed value. By this means an approximate value can be calculated. Mr. L. F. Richardson, using some observations by A. Ångström, made on Mount Whitney, California, on a clear dry night (Aug. 11, 1913), has calculated that a layer of dry air equivalent to an additional pressure of one millibar absorbs ·00151 of a pencil of long-wave radiation passing through it perpendicularly. His precise statement is "the absorptivity per density j may be defined by the statement that a fraction $j\rho\delta l$ of incident parallel radiation is absorbed in passing through a length δl of dry clear air, where $j = 1{\cdot}48 \times 10^{-3}$ (cm.^{2} grm.$^{-1}$)" (*Weather Prediction by Numerical Process*, Camb. Press, pp. 50 and 54).

The calculation is to some extent uncertain because the radiation is coming from all directions. The radiation for the same small solid angle falling on a surface perpendicular to the rays is least for the zenith, greatest for a horizontal direction where it reaches the full radiation from a black body at the temperature of the air at the surface.

A better way would be to make observations on the radiation from the zenith alone and base the calculation on them. Some observations have been made at the Meteorological Office Observatory at Benson, but lead to no definite conclusion.

The objection to this method is that the water contents of the atmosphere lying above the place of observation are in general unknown, and have to be inferred from the vapour pressure at the surface at the time of observation. Such inference is unsafe; it may hold for average conditions, but is doubtful for the individual case.

Much uncertainty prevails as to the value of the absorption coefficient for air containing given quantities of moisture both with regard to solar and terrestrial radiation. It is almost certain that in the case of long-wave radiation the coefficient increases with increasing moisture, and Anders Ångström in the paper quoted above found a close connection between the effective radiation at night and the humidity at the place and the time. With regard to solar radiation the readings of the black bulb thermometer *in vacuo* obtained by Scott's first expedition to the Antarctic were very high even when compared with similar instruments in the tropics. This may have been due to diffuse reflection from the snow and ice, but it seems more likely and more in accordance with other experience to suppose that the low temperature insures a low water content in the overlying air and that, therefore, more solar radiation is transmitted notwithstanding the low altitude of the sun.

It may be added that the wave-length of maximum energy for the earth's radiation is about 10 μ (·01 mm.).

§ (3) THE EFFECT OF RADIATION UPON ATMOSPHERIC TEMPERATURES. (i.) *Temperature Distribution.*—The part played by radiation in the distribution of temperature in the vertical direction in the atmosphere is more or less uncertain, as so many other factors are involved, but it must be considerable. The facts about this distribution as they have been established by observations in the last twenty years are these.

There is a steady fall[1] of temperature with height, called the lapse rate, up to a certain level, and above that level the temperature remains nearly constant up to the greatest heights that have been reached, that is to about 20 km. or 12½ miles.

The lower part of the atmosphere, called the troposphere, in which the lapse rate is found, reaches to about 10·5 km. in the latitude of England, but to 16 or more kilometres over the Equator. It is in general sharply differentiated from the upper part, called the stratosphere, which is often a few degrees warmer than the top of the troposphere. The height of the boundary varies with the latitude, being greatest in the equatorial and tropical regions, lowest in high latitudes. The temperature of the stratosphere also depends on the latitude; it is as low as -80° C. over the Equator, but 30 higher or -50° C. about over England. Thus the lowest natural temperatures ever observed are not found in the polar regions, but high up over the Equator, where solar radiation is most intense.

(ii.) *Solar Radiation.*—We can to some extent trace the radiation from the time it arrives from the sun until it is radiated back as long-wave radiation from the earth and atmosphere to space. It is estimated that from one-third to one-half of the solar energy is reflected by diffuse reflection from the clouds and air, and is therefore without effect so far as the earth is concerned. Taking the value of the solar constant as ·5 gramme-calorie per minute as the average amount that falls on 1 square centimetre parallel to the earth's surface at the outer surface of the atmosphere, this gives 720 g.c. per day, of which we may suppose 320 are reflected and the remaining 400 used in warming the air and the earth's surface. Obviously what is received from the sun must be in the long-run radiated out again, and hence we have about 400 g.c. per day radiated to space by the earth and atmosphere.

The next quantity in the course of the radiation that can be measured is the amount of solar energy falling on a square centimetre of the earth's surface. Self-recording instruments to measure this are in use at various stations, but they are not sufficiently numerous to give a mean for the whole earth. The amount naturally depends on the latitude; the value for London, which includes the scattered solar radiation from the sky, is about 200 g.c. per day (195 for a three years' average). In Washington the value is about 350. Washington is in latitude 38·53, and half the earth's surface is included between the latitude of 30 N. and 30 S. Washington ought, therefore, to receive less radiation from the sun than the general mean value which at the outer surface of the atmosphere is taken as 400. If the mean at the earth's surface is more than 350 a very small quantity is left for the amount absorbed by the air. This is one of the many discrepancies that are at present unexplained.

This amount, the heat given to the earth's surface by the sun, whatever it may be, is given back to the air, but only partly in the form of radiation. The solar rays on reaching the earth take the form of heat, and this heat passes back partly by contact with the air, for, on the whole, the mean temperature of the earth's surface is above that of the air in contact with it; partly by the evaporation of water from the surface and the return of the heat required to evaporate the water as latent heat where the vapour is condensed to form rain or snow; and partly by the net or effective radiation from the earth to the sky.

(iii.) *Radiation from the Earth.*—This latter quantity seems to have been first measured by Maurer in 1887. A more recent and very valuable determination was made by Anders Ångström at Bassour in Algeria in the summer of 1912, and in California in 1913. The following values are adapted from Ångström's paper ("A Study of the Radiation of the Atmosphere," *Smithsonian Miscellaneous Collections*, vol. lxv. No. 3, p. 16):

Observer.	Date.	Place.	Height.	Mean Value.
			feet.	g.c. per day.
Maurer . . .	June	Zurich	1,524	184
Pernter . . .	Feb.	Sonnblick	10,150	289
Pernter . . .	Feb.	Sonnblick	10,150	217
Horner . . .	Aug.	Lojosee	..	245
Exner . . .	..	Sonnblick	10,200	274
K. Ångstrom . .	May to Nov.	Upsala	609	223
Lo Surdo . . .	Sept.	Naples	98	262
A. Ångström . .	July to Sept.	Algeria	3,530	250

Net radiation from the earth to the sky on clear nights.

Considering that these values are obtained by the use of different instruments at different heights and temperatures there is very good agreement. They do not represent the general average because they were made on nights selected for their clearness; on cloudy nights the net radiation is very small; on a few occasions now and then it may even be negative. The general average for the whole earth and the whole year is about 100 g.c. per day. The radiation is going on by day just as by night, but by day if a black body is used to measure it, it is influenced by diffuse radiation from the sun. If a body like glass is used, which is pervious to those solar rays which have already passed through the atmosphere, the net long wave radiation by day between the glass and the clear sky is plainly apparent.

---

[1] See "Atmosphere, Thermodynamics of," § (5).

It has already been pointed out that this net radiation is simply the difference between that which the earth is giving out and that which it is receiving from the hemisphere of sky. It has been found that at ordinary temperatures both the earth and sea radiate very nearly as full radiators and at 283 a., the mean temperature of London, a full radiator gives out 711 g.c. per sq. cm. per day. Assuming the outward radiation to be about 100 g.c. per day, this leaves for England about 600 g.c. as the return radiation and reflection from the air. Observations made at Benson on the radiation from the sky agree closely with this value.

The further course of the radiant energy cannot be measured directly, but it may be inferred, as stated above, that about 400 g.c. per sq. cm. per day leave the outer limit of the atmosphere. Stellar and planetary radiation and the amount of heat coming from the interior of the earth are, compared with the solar radiation, so small that they may be neglected.

(iv.) *Transference of Heat.*—The question naturally arises as to what part radiation plays in the curious distribution of temperature in the vertical direction in the atmosphere, but no certain conclusion can be reached. Heat moves vertically in the atmosphere by three other means besides radiation. These means are (1) convection, by which heat is disseminated upwards. This is notably the case when the solid surface of the earth is warmed by strong sunshine and an adiabatic lapse rate established above it for a few hours in the afternoon, extending upwards to perhaps two and very rarely to three kilometres' height. It is also the case in thunder-storms and torrential rains, but these phenomena are the exception and not the rule. Secondly, by the latent heat of condensation, where the water vapour formed by evaporation of water at the earth's surface again becomes water in the form of clouds and rain. This carries the solar heat from the earth to the strata where rain is formed. In temperate climates this is from one to three kilometres' height, more in the tropics. The precise amount of heat so carried is easily calculated when the rainfall is known.

The third process is not so easy to follow, and it carries heat downwards. Owing to the action of the wind a continual interchange of masses of air between different levels is always in progress. The process has been called "mixing," "stirring," "turbulence," and, in German, "Massenaustausch." Inasmuch as the potential temperature (that is the temperature after the air has been brought adiabatically to a standard pressure) in general increases with height the interchange of air masses in a vertical direction must cause a flow of heat downwards. It is obvious that thoroughly mixing the atmosphere would produce an adiabatic lapse rate throughout, that is, would raise the temperature of the lower strata and lower that of the upper. The numerical measure of this flow of heat is uncertain; it must vary widely under different circumstances and with height: Dr. W. Schmidt of Vienna gives it as 50 g.c. per day per sq. cm. at two kilometres for Europe ("Der Massen-Austausch bei der ungeordneten Strömung in freer Luft und seine Folgen," *Akad. Wiss. Wien Ber.*, Abteilung IIa, 126. Band, 6. Heft, p. 25).

Since no systematic change of temperature is taking place in any stratum of the atmosphere the total flow of heat across concentric surfaces of equal pressure or equal altitude must for the year and for the whole earth be zero. This is probably true also for any sufficiently extended area such as Europe, and for periods such as a couple of months or so at times when the seasonal change of temperature is small. Thus, if we could sum up the total flow of heat due to the three causes enumerated above, the balance required to equate the value to zero would be the amount due to radiation. Now all three causes bring heat to the lower strata, say 0–4 km., so also does solar radiation, hence the layer of air from 0 to 6 km. must be losing more heat by its own radiation outwards than it is receiving from the radiation of the earth and the layers above it. This is confirmed by a theoretical investigation dealing with long wave radiation (see *R. Met. Soc. J.* vol. xlvi. No. 194), according to which the lower strata from 1 to 5 km. are some 20° C. above their radiative equilibrium temperature.

Above six kilometres neither convection nor the condensation of vapour can bring much heat to the atmosphere, and hence the flow of heat downwards from these strata due to mixing of masses of air must be balanced by the supply of heat by radiation, solar and long wave, to the upper strata. Since the mixing must tend towards a purely adiabatic lapse rate throughout the atmosphere, and such a rate is never reached for any extensive area or time, the effect of radiation taken alone must be towards a more isothermal condition than actually exists. Probably radiation produces the isothermal condition of the stratosphere, but why there should be so distinct a division between the troposphere and stratosphere is not apparent.

§ (4) The Effect of Radiation upon the Mean Temperature of the Earth and upon Climate. (i.) *Temperature of the Earth.*—The earth must radiate out exactly the same heat that it receives from the sun, and it is an interesting but difficult problem to ascertain at what precise mean temperature this will occur. The Stefan-Boltzmann law of radiation states that the intensity of the radiation from a full radiator, very commonly called a "black body," is proportional to the fourth power of its absolute temperature. The earth and sea have been found, by experiment, to radiate almost as freely as a full radiator, and the mean temperature of the earth's surface is not far from 288° absolute, at which temperature the radiation to the hemisphere of sky is 763 gramme-calories per square centimetre of surface per day. The solar constant cannot exceed, though it may be less than, this amount, which is equivalent to ·529 gramme-calories per sq. cm. per minute as an average for the whole earth or 2·12 g.c. per sq. cm. per minute for a surface exposed at right angles to the solar rays. Abbott and Fowle's value is just under 2 gramme-calories, actually 1·93 g.c. (see § (1) (ii.)). Strictly, we are not justified in calculating the earth's radiation as proportional to the fourth power of the mean temperature; the mean of the fourth power of the temperatures ought to be obtained, but the error so caused can hardly exceed 1 per cent. Abbott and Fowle state,[1] and their conclusion is borne out by other considerations, that, on a mean for the whole earth, a large percentage of the solar heat, probably some 37 per cent, is reflected back to space and has no effect. Thus 63 per cent or 481 g.c. per sq. cm. per day are available for warming the earth; but other observers tend to reduce this figure. If we take 400 g.c. per sq. cm. per day effective solar energy, this amount must be radiated by the earth and the equivalent radiative temperature is 245° a. (–28° C. or –18·4° F.). For 500 g.c. per cm.^{2} per day it would be 260° a. This does not mean that the mean temperature of the air is 245° a.—it is about 255°—but only that the total radiative energy that leaves the outer surface of the atmosphere, excluding reflected solar radiation, is just that which a full radiator, at 245° a. would give out. The difference between 245° and 288°, the mean temperature of the earth, is readily explained. A very thick layer of isothermal gas radiates from both its faces as a full radiator at the same temperature would do. The atmosphere is not quite thick enough to meet this condition, though it is nearly so, and it is not isothermal. The best observations[2] show, as we have seen in § (3) (iii.), that the atmosphere radiates downwards about 600 g.c., the equivalent radiative temperature of which is 271° a. (–2° C.), and this temperature we know is the mean at a height of about 3 km. The outward vertical radiation above must be much less because the upper layers are at a much lower temperature. Over England the upper fifth part of the atmosphere has a mean temperature of 222° a. Over the whole earth the mean for this upper fifth is probably well below 210° a. A mean of 245° a. is met with a little over half way through the mass of the atmosphere. Thus there is nothing unreasonable in ascribing as low a value as 400 g.c., the amount corresponding to the mean temperature of 245° a., to the outward radiation. A "black" or "grey" sphere subject to solar radiation at the distance of the earth should be at a temperature of about 284° a., and thus the temperature of the earth is not far from what might be expected from first principles.

It has been commonly asserted that the mean temperature of the earth is raised by the special property of the atmosphere which transmits readily the short-wave solar radiation and hinders the return of the long-wave radiation from the earth. This may be so, but it is somewhat doubtful. For, in the first place, the actual mean temperature, 288° a, does not differ appreciably from the value 284° a. which apparently would be attained under a solar radiation of 720 g.c. per sq. cm. per day if there were no atmosphere at all. Secondly, if we consider the effect of a layer of air in the upper strata becoming a full absorber of radiation of all wave-lengths, and therefore by Kirchhoff's law a full radiator, it is apparent that it would have a mean temperature of 284° a., for it could have no effective radiation downwards, and the temperature of the underlying layers could not on the whole be less than this. They would, in fact, under radiation alone be at a temperature of 284° a., for they would be in the position of a body inside a closed isothermal surface.

(ii.) *Radiation and Climate.*—The effect of radiation upon climate is more readily intelligible and does not present so many difficulties as the other parts of the subject.

---

[1] *Annals of the Astrophysical Observation of the Smithsonian Institute*, Washington, 1908.

[2] *Smithsonian Misc. Coll.* vol. 65, No. 3, Washington, 1915. *Über die Gegenstrahlung der Atmosphäre*, von Anders Ångström.

At either equinox on the Equator at midday, apart from absorption by the atmosphere, we have already stated that 2 g.c. per minute of solar energy fall on each horizontal square centimetre. The sun is only overhead for a short time, but it is easy to see that the average amount per minute through the 24 hours is $2/\pi$ per minute, or very nearly 916 g c. per day. For the equinox in any other latitude we must multiply 916 by the cosine of the latitude.

But the mean for the whole year in any latitude is by no means proportional to the cosine of that latitude on account of the obliquity of the ecliptic, and using a table by Hann, who quotes Angot, we get the following values of the mean daily solar radiation for each 10° latitude circle:

Latitude . .	0	10°	20°	30°	40°	50°	60°	70°	80°	90°
Mean daily solar radiation .	916	903	865	806	725	628	521	434	393	380

This takes no account of the absorption of the atmosphere or diffuse reflection. In discussing climate it can hardly matter what the absorption is, because it can matter little if the heat is communicated directly to the air by being absorbed, or indirectly by first heating the ground or sea. But the amount reflected is of consequence, and presumably it is greater when the rays are oblique.

The following mean annual temperatures on the absolute scale for each 10° of latitude are also taken from Hann's *Meteorology*. They refer to the northern hemisphere.

Latitude . .	0	10°	20°	30°	40°	50°	60°	70°	80°	90°
Temp. Abs. . .	299°	300°	298°	293°	287°	279°	272°	263°	256°	253°
Radiation per sq. cm. . . .	886	898	874	817	752	672	607	531	476	454

Below in the third line is placed the radiation per day for one square centimetre at the temperature shown in the line above. If the solar heat were strictly confined to the latitude belt in which it is received, that is, if there were no convection of heat by wind or water the equivalent radiative temperature of each belt would have to be proportional to the fourth root of the number of gramme-calories assigned per sq. cm. to that particular belt. That is, the numbers in the first table should be proportional to the corresponding radiation per sq. cm. shown in the second table. Tested by this criterion, the mean temperatures of the polar belts are distinctly too high, and if the solar heat received in any latitude were reduced to allow for absorption and reflection the excess would be increased. It is also otherwise apparent that a large supply of heat must be carried from lower to higher latitudes by currents of air and water, but we cannot use the figures given above to calculate the amount for two reasons. The amount of radiation from the earth and air to space in any locality does not depend only on the temperature of the earth's surface in that locality, the temperature of the upper layers of air and the absorbing power of the air come into the account. Also the absorbing power of the air for radiation is different for different latitudes on account of the greater water contents of the atmosphere in the hotter regions of the earth, so that the amounts directly received by the earth in each latitude belt are in error if calculated upon the assumption of a uniform absorption coefficient.

However, the dependence of climate upon latitude, which means, of course, upon solar radiation, is one of the primary facts of human knowledge, and the figures quoted above show a certain proportionality between the fourth power of the mean temperature and the mean radiation received from the sun.

The same rule holds for the change from winter to summer, but the three months of each season are hardly long enough for the temperature to adjust itself to the radiative conditions. This is notably the case near large bodies of water, and the western coasts both of Europe and America from 45° to 60° N. lat. owe their mild climate and their small change between summer and winter to the prevailing westerly winds and the large oceans to the West.

W. H. D.

RADIATION, THE MEASUREMENT OF SOLAR, SKY, NOCTURNAL, AND STELLAR

I. The Measurement of Solar Radiation

§ (1) Introductory Statement.—The measurement of thermal radiation from the sun, and from celestial bodies in general, is a field which is quite undeveloped and the importance of which cannot be foretold. Animal and vegetable life exists primarily because of thermal and photochemical activity induced by the radiations emanating from a neighbouring sun. We are therefore vitally concerned with the question of the constancy of the radiation from our sun. A continual fluctuation in the emissivity of the sun is to be expected, and, if the terrestrial atmospheric transmission were more constant, this fluctuation in solar radiation could, no doubt, be recorded to the minute. But, limited as we are in our ability to eliminate the disturbing causes in making the measurements, perhaps the best that we can expect, for the present at least, is to record great fluctuations in the value of the solar constant of radiation and determine whether or not there is any regularity or periodicity in these fluctuations. This will establish a level of values, and our successors of one hundred of more years hence (if not at an earlier date, and if interested in the subject) will be able to determine to what extent, and in what spectral radiation qualities, solar radiation differs from that of the present.

Measurements of stellar radiation show that red stars have a much greater emissivity (are losing heat faster) than stars of the solar type. It is therefore interesting to speculate on the terrestrial conditions that will ensue when (and if) the sun emits, relatively at least, more infra red and less visible radiation than obtains at present.

In the meantime the observational data accumulated may perhaps be correlated and utilised (as, indeed, is already being attempted) in predicting conditions of the weather.

§ (2) The Intensity of Solar Radiation at the Earth's Surface.[1]—The solar radiation intensities most commonly recorded in meteorological journals generally pertain to a given station, for a given date. They are useful for purposes of comparison, and may be obtained by means of simple apparatus.

One instrument frequently used for this purpose is the Ångström (4)[2] compensated pyrheliometer. This device consists of two thin blackened manganin strips, the one of which is exposed to (and hence warmed by) solar radiation. The other strip is shielded from solar radiation and is heated electrically to the temperature of the exposed strip. Equality of temperature is determined by thermojunctions attached to the manganin strips and connected with a galvanometer. From a knowledge of the area exposed to radiation and the electrical data, the value of the solar radiation intensity is obtained in absolute value, which is usually expressed in gr.-cal. per cm.^{2} per min. The Marvin pyrheliometer (9), which consists of a resistance thermometer and a heating coil embedded in a disc of silver, is used in a similar manner to measure solar radiation.

A dynamic method of observing the intensity of solar radiation consists in noting the rate of rise in temperature of a mercury in glass thermometer, the bulb of which is embedded in a disc of copper or silver such as, for example, the silver disc pyrheliometer described by Abbot (3, 5). A paper by Whipple (10) describes and gives illustrations of various types[3] of instruments for measuring solar radiation. Instructive and important data on solar radiation intensities are being obtained, and in concluding this part of the discussion it is of interest to note the great seasonal variability of the intensity of solar radiation. For the central latitude of the U.S.A. Kimball (28) shows that the maximum solar radiation at perpendicular incidence varies from 1·37 gram-calories per minute per square centimetre in January to 1·5 gram-calories in May and September. The total radiation on a horizontal surface, with a clear sky, varies from 0·77 gram-calories per minute in December to 1·55 gram-calories in June. Clouds nearly in line with the sun, but not obscuring it, increase the radiation by about 0·15 gram-calories.

The use of daylight and artificial light is increasing in dye fading and other photochemical tests. In a recent investigation by Coblentz and Kahler (13) of the component radiations from the sun and from a quartz-mercury vapour lamp, it was shown that there is no marked difference in the total ultra-violet radiation of wave-lengths less than about 0·4 μ from these two sources. However, the spectral quality of the ultra-violet (the energy distribution) is entirely different. The ultra-violet of the solar spectrum terminates at about 0·3 μ. On the other hand, in the quartz-mercury vapour lamp, the ultra-violet component of wave-lengths less than 0·3 μ is about 20 per cent of the total ultra-violet component radiation from this lamp.

§ (3) The Solar Constant of Radiation.—By solar constant is meant the intensity of solar radiation (usually expressed in gr.-cal.-cm.$^{-2}$, min.$^{-1}$) in free space, at the earth's mean solar distance. The determination of this constant involves (*a*) an accurate measurement of the solar radiation intensity at the earth's surface, and (*b*) an accurate estimation of the losses in intensity suffered by the solar rays, in passing through the earth's atmosphere.

The first step in the determination of this constant is to measure the solar radiation intensity by means of the silver disc (3, 5) pyrheliometer. This is a secondary instrument which was calibrated against a primary standard water-flow instrument (viz., a water-flow calorimeter of special design).

Simultaneously with the pyrheliometric measurements spectrobolometric energy curves are obtained of the sun. From these spectral energy curves of the sun at different altitudes the atmospheric transmission curve for all wave-lengths is determined; and from these two sets of observations it is possible to compute the solar radiation as it would be outside the terrestrial atmosphere.

---

[1] See also articles "Radiation" and "Radiant Heat and its Spectrum Distribution."

[2] References are given in the Bibliography at end of text.

[3] For an account of this see "Radiant Heat and its Spectrum Distribution."

This method of determining the solar constant of radiation was introduced by Langley, and the numerical values obtained therewith are perhaps the first to command the confidence of experimenters. The fault to be found with other determinations, which differ from each other by almost 100 per cent (16), no doubt lies in an inaccurate estimate of the amount of solar radiation lost in passing through the terrestrial atmosphere.

Recently Abbot (29) and his collaborators have worked out a new method of determining the solar constant, based upon a relation between measurements of sky brightness and the above-mentioned spectral transmission coefficients. The sky owes its brightness to scattering of the solar rays by water vapour, etc. The more hazy the sky the greater is its brightness and the less is its atmospheric transmission.

The measurement of the atmospheric transmission coefficients is a long tedious process. The measurements of sky brightness are very quickly made with a pyranometer (6). They were able to work out graphically a relation between the atmospheric transmission coefficients and sky brightness, which appears to be valid at least for the Smithsonian Station at Calama, Peru, thereby greatly shortening the time of observing and calculating the results. All that appears necessary is to make simultaneous measurements of sky brightness and solar radiation intensities, and determine from the graph the corresponding transmission coefficients.

The mean value of the solar constant of radiation for the epoch 1902–12 (696 observations) is 1·93 gr.-cal. (15° C.) per cm.^{2} per min. (*loc. cit.*, (3) p. 134). The value of the so-called "solar constant" is in reality not a constant, but is subject to variation with sunspot activity, etc. These fluctuations occur at irregular intervals and range over perhaps 8 per cent. They are thought to indicate a true variability of the sun (16) (*loc. cit.* (3), p. 117). However, no definite periodicity in variation in solar radiation intensity has yet been observed (30, 23).

§ (4) The Temperature of the Sun.—Various attempts have been made to obtain an estimate of the temperature of the effective radiating layer of the surface of the sun. The usual procedure is to apply the radiation laws of a perfect radiator, which of course cannot be exactly true for the sun.

According to these laws the energy radiated per unit area per second from the surface of a perfect radiator at absolute temperature T is equal to σT^4 ergs. Wilson (20) compared directly the radiation from the sun with that of a black body at a known temperature, using Kurlbaum's value of $\sigma = 5{\cdot}3 \times 10^{-5}$ erg. Assuming a zenith transmission at 71 per cent, he obtained a value of 5500° C. for the temperature of the sun. Poynting (26) concluded that either Wilson's estimate of zenith transmission is too high or Kurlbaum's value of the coefficient of total radiation is too small. This is very interesting; for recent work shows that Kurlbaum's original value of $\sigma = 5{\cdot}3$ is about 7 per cent too low.

The most recent value of the effective temperature of the sun's radiating layer is by Abbot (23). It is based upon the distribution of energy in the spectrum of the sun, as it would be outside of the terrestrial atmosphere. After correcting the bolographs for absorption by the spectrometer mirrors and the atmosphere, the wave-length of maximum spectral energy is at $\lambda_m = 0{\cdot}470\ \mu$. On the basis of the Wien displacement law, using the most recent value of $A = \lambda_m T = 2885$, the effective solar temperature is 6140° K., or about 5870° C. (Abbot used the old value $A = 2930$ and obtained 6230° K.)

On the basis of the fourth power law of total radiation we find that, using the coefficient $\sigma = 5{\cdot}72 \times 10^{-5}$ erg. and the above-mentioned value of the solar constant (1·922), the computed effective solar temperature is about 5740° K., or about 5470° C. Kurlbaum (24) has made measurements with an optical pyrometer, which yielded values of the order of 5500° C. on the basis of $C = 14600$ (for a black body $C = 14300$, but is higher for a selective radiator like platinum).

In view of the fact that the sun is highly selective in its spectral emission, these two methods of calculation must indicate temperatures which are lower than the true temperatures. The conclusion to be drawn is that the sun's effective radiating layer is roughly comparable with that of a black body at about 6000° K., or about 5700° C.

II. The Measurement of Sky and Nocturnal Radiation

§ (5) Radiation from the Earth.—Nocturnal radiation is the expression commonly used for describing the loss of thermal radiation from terrestrial objects to space. The name no doubt had its origin from the early experiments on the cooling of objects, which is best observed at night. Being a low temperature radiation, the wave-lengths most strongly emitted are in the region of the spectrum greater than 8 μ. In connection with the question of the incoming solar radiation it is interesting to note that the outgoing loss in nocturnal radiation may amount to 10 per cent of the solar constant. The study of the heat interchanges between the earth and the sky requires radiometers of novel construction and operation.

§ (6) THE PYRGEOMETER.—Ångström (7) has modified the original compensated pyrheliometer (4) by using polished and blackened strips of manganin. On exposure to space the dark strip cools more rapidly than the light strip, and by heating the dark strip electrically to the temperature of the bright strip a measure is obtained of the outgoing or nocturnal radiation.

In a recent paper Ångström (8) describes a new modification of the pyrgeometer for measuring sky radiation. In the new device one strip is covered with platinum black, the other is covered with magnesium oxide, which has a high diffuse reflecting power (31) for luminous rays and a low reflecting power for radiation of wave-lengths at 8 to 10 μ. The two strips will therefore have practically the same emissivity when exposed to nocturnal radiation. However, as a mechanical protection, he covers the device with a hemispherical glass vessel. When exposed for measuring sky radiation (or solar radiation) the difference in emissivity for long wave-length radiation does not enter the measurements as it does in the Callendar (12) sunshine recorder and in the pyranometer (6), in which the heating or cooling of the glass cover may introduce errors from long-wave radiation.

§ (7) THE PYRANOMETER. This instrument was devised by Abbot and Aldrich (6) for measuring sky radiation. The device consists essentially of two short strips of blackened manganin suitably mounted in the centre of a circular nickel-plated block of copper. One strip is ten times as thick as the other, and hence because of its greater thermal conductivity on exposure to radiation the temperature rise in the thick strip is less than in the thin strip. This is indicated by thermo-junctions attached to the back of the strips and connected with a galvanometer. After again shading the strips, the deflection, which was caused by the absorption of radiation, is reproduced by heating by means of an electric current which is divided between these two strips so as to produce equal heating effects in each. These receiving strips are covered with an optically figured, hollow, hemispherical screen of ultra violet crown glass which admits direct or scattered solar radiation but which prevents the exchange of long wave-length radiation between the manganin strips and the sky. On removal of the glass cover the instrument is useful at night for measuring the outgoing or so-called nocturnal radiation.

§ (8) RESULTS OF THE MEASUREMENTS.—Interesting and important data have been obtained with these instruments, although the subject is comparatively new. In the discussion of the solar constant of radiation attention was called to use of measurements of sky radiation (29) in determining the transmission coefficient of the atmosphere.

(i.) *The Cloud Effect.*—The blanket effect of clouds in preventing nocturnal radiation from the earth is a common observance. Because of this effect, Humphreys (38) arrives at the conclusion that the intensity of the earth's outgoing radiation is much greater in middle latitudes than it is in equatorial regions. The extraordinary effects that result from nocturnal radiation are well illustrated in the book of investigations by Barnes (36) on ice formation, in which it is shown that the "anchor ice" formed at the bottom of the St. Lawrence river is owing to cooling by radiation.

(ii.) *Humidity Effects.*—Interesting data on nocturnal radiation as affected by atmospheric humidity are given by Ångström (32). In a recent paper by Boutaric (37) it is shown that the intensity of nocturnal radiation may be expressed as a function of the absolute temperature of a black radiating surface and the vapour pressure in its immediate vicinity. The application of such data to horticultural problems is well illustrated in an interesting paper by Kimball (39), in which it is shown that the nocturnal or outgoing radiation from the pyrgeometer at 20° C. increased from about 0·13 gr. cal. for a vapour-pressure of 12 mm., to 0·23 gr. cal. per cm.^{2} per min. for a 2 mm. vapour-pressure.

III. THE MEASUREMENT OF STELLAR RADIATION

§ (9) METHODS AND RESULTS. — Numerous attempts have been made to measure the radiation from stars. One method of attacking the problem is by means of photoelectric photometry. The instruments and methods for this type of (selective) radiometry have been developed principally by Stebbins (40), who has been very successful in obtaining important data on the change in brightness of variable stars. By this means he has added important data to the subject of variable stars. In his earliest work he used a selenium photometer, and in his most recent work has used a potassium hydride photoelectric cell.

The second method of attack is by means of thermal radiometry with non-selective receivers. The most recent attempt, by Coblentz (43), using thermocouples, has yielded some interesting results. Measurements were made on 112 celestial objects. It was found that red stars emit from two or three times as much *total* radiation as blue stars of the same visual photometric magnitude. Measurements made through a cell of water showed that of the total radiation emitted the blue stars have about two times as much *visible* radiation as the yellow stars and about three times as much *visible* radiation as the red stars. The absolute value of the total radiation received from all the stars is estimated at less than 2×10^{-8} gr. cal. per cm.^{2} per minute. From this it appears that if the total radiation from all the stars, incident upon 1 cm.^{2}, were collected and conserved, it would require from 100 to 200 years to raise the temperature of 1 gr. of water 1° C., whereas the solar rays, which reach the earth's surface, can produce the same effect in about 1 minute.

BIBLIOGRAPHY

Radiometers, Pyrheliometers

1. *Annals Astrophys. Obs., Smithsonian Inst.*, 1902, i., S. P. Langley.
2. *Annals Astrophys. Obs.*, 1908, ii., Abbot, Fowle.
3. *Annals Astrophys. Obs.*, 1913, iii., Abbot, Fowle, Aldrich.
4. Ångström, "Compensated Pyrheliometer," *Astrophys. Jour.*, 1899, ix. 332.
5. Abbot, "Silver Disk Pyrh.," *Smithsonian Miscellaneous Coll.*, 1911, lvi. No. 19.
6. Abbot and Aldrich, "Pyranometer," *Smithsonian Misc. Coll.*, 1916, lxvi. Nos. 7 and 11.
7. Ångström, "Pyrgeometer," *Smithsonian Misc. Coll.*, 1915, lxv. No. 3, p. 28.
8. Ångström, "A New Instrument for measuring Sky Radiation," *Mo. Weather Rev.*, 1919, xlvii. 795.
9. Kimball, "The Marvin Pyrheliometer," *Mo. Weather Rev.*, 1919, xlvii. 769.
10. Whipple, "Instruments for the Measurement of Solar Radiation," *Trans. Opt. Soc.*, London, Session 1914-1915.
11. Foote, *Bulletin B.S.*, 1918, xiv. 605.
12. Callendar, "Sunshine Recorder," *Proc. Roy. Soc.*, London, 1906, lxxvii. A. 6.
13. Coblentz and Kahler, "A New Spectropyrheliometer," *B.S. Sci. Papers*, 1920, No. 16, 233.

Solar Constant of Radiation

14. Langley, *Astrophys. Jour.*, 1903, xvii. 89.
15. Abbot and Fowle, *Astrophys. Jour.*, 1911, xxxiii. 191.
16. Abbot, *Proc. Amer. Phil. Soc.*, 1911, l. 235.
17. Abbot, *Astrophys. Jour.*, 1913, xxxvii. 130.

The Temperature of the Sun

18. Warburg, *Verh. Phys. Gesell.*, 1899, l. 50.
19. Amerio, *Nuovo Cimento*, 1901, xvii. 308.
20. Wilson, *Proc. Roy. Soc.*, 1901, lxix. 312.
21. Millochau and Féry, *Compt. Rend.*, 1906, cxliii. 505 and 570.
22. Millochau, *Jour. de Phys.*, 1909, (4), viii. 347.
23. Abbot, *Astrophys. Jour.*, 1911, xxxiv. 197; also *Annals Astrophys. Obs.*, 1913, iii. 204.
24. Kurlbaum, *Sitzber. K. Preuss. Akad. Wiss.*, 1911, xxiv. 541.

Miscellaneous Papers on Solar Radiation

25. Abbot, "Bolometric Measurements on the Solar Corona," *Smithsonian Misc. Coll.*, 1908, lii. 31.
26. Poynting, "Radiation from the Solar System." *Phil. Trans. Roy. Soc.*, 1903, ccii. A, 525; and *Jahrb. der Radivak. u. Elektronik*, 1905, ii. 42.
27. Abbot, "The Sun's Energy Spectrum and Temperature," *Astrophys. Obs.*, 1913, iii. 200.
28. Kimball, "Total Radiation Received on a Horizontal Surface from the Sun and Sky," *Mo. Weather Rev.*, 1914, xlii. 474.
29. Abbot, *Mo. Weather Rev.*, 1919, xlvii. 580; and *Proc. Nat. Acad. Sci.*, 1920, vi. 4.
30. Abbot, "Periodicity in Solar Variation," *Smithsonian Misc. Coll.*, 1918, lxix No. 6.
31. Coblentz, *B.S. Bulletin*, 1913, ix. 283.

Nocturnal Radiation

32. Ångström, *Nova Acta Reg. Soc., Sc. Upsala*, 1905, (4), i. No. 2; and *Ann. der Phys.*, 1910, (4), xxxiii. 845; also *Smithsonian Misc. Coll.*, 1915, lxv. No. 3.
33. Lo Sudro, *Nuovo Cimento*, 1908, (5), xv. 253. (Gives an extensive bibliography.)
34. Melloni, *Ann. de Chim. et de Phys.*, 1844, (3), xxii. 129.
35. Langley, *Mt. Whitney Researches in Solar Heat*, Washington, 1884.
36. Barnes, *Ice Formation*, 1906.
37. Boutaric, *Compt. Rend.*, 1920, clxx. 1195; and *Mo. Weather Rev.*, 1920, xlviii. 234.
38. Humphreys, *Bulletin, Mt. Weather Obs.*, 1911, iv. 129.
39. Kimball, *Mo. Weather Rev.*, 1918, xlvi. 57.

Stellar Radiation

40. Stebbins, *Astrophys. Jour.*, 1910, xxxii. 185; 1911, xxxiii. 385; 1914, xxxix. 459; 1915, xlii. 133; 1919, xlix. 344; Kunz and Stebbins, *Phys. Rev.*, 1918, xii. 62.
41. Nichols, *Astrophys. Jour.*, 1901, xiii. 101.
42. Pfund, *Publ. Allegheny Obs.*, 1913, iii. 13
43. Coblentz, *B.S. Bulletin*, 1915, xi. 613; 1922, xvii. 725.

W. W. C.

Radiation, Solar, instruments for measurement of:

Black-bulb *in vacuo*. See "Meteorological Instruments," § (27).

Callendar radiation recorder. See *ibid.* § (31).

Pyrheliometers (Abbot's, Ångström's, Michelson's, and silver-disc). See *ibid.* §§ (28)-(30). See also "Radiant Heat and its Spectrum Distribution."

Radiation in the Atmosphere. See "Atmosphere, Physics of," § (6) (iii.).

Absorption of. See "Atmosphere, Thermodynamics of the," § (10).

Analysis of, into short and long wave. See *ibid.* § (10).

Balance of solar and terrestrial. See *ibid.* § (11).

Loss of heat by. See *ibid.* § (24).

Outflow of, from the earth. See *ibid.* § (12).

Relation of distribution of temperature to. See *ibid.* §§ (11), (12).

Radiation from the Sky: measurement of, by Ether differential radiometer. See "Meteorological Instruments," § (32).

Radiative Equilibrium in the Atmosphere, variation of temperature with height in. See "Atmosphere, Thermodynamics of the," §§ (11), (12), *Fig.* 14.

Radio Balance, Callendar's. See "Radiant Heat and its Spectrum Distribution," § (17).

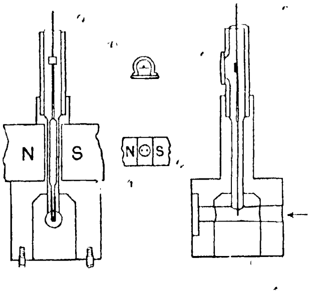
Radio Integrator. See "Radiant Heat and its Spectrum Distribution," § (10).

Radiometer, Dines', ether differential. See "Meteorological Instruments," § (32).

RADIO-MICROMETER AND SOME OTHER INSTRUMENTS DEPENDING UPON QUARTZ FIBRES

§ (1) THE INSTRUMENT.—The radio-micrometer[1] is an instrument designed and constructed by the writer for measuring minute amounts of radiation as heat, and its special feature, besides great delicacy, is its freedom from disturbance by adventitious heat or by magnetic changes outside. Its disadvantage is the necessity for a quiet and level support, so that it cannot be moved to follow a source of radiation except slowly in azimuth. Its extreme delicacy of construction requires more than ordinary skill on the part of the user.

It consists of a thermoelectric circuit suspended in a fairly strong and uniform magnetic field due to a permanent magnet. The circuit is composed of two extremely delicate thermoelectric bars soldered to a heat-receiving disc or strip of two or three square millimetres at one end and to a single long narrow loop of highly conducting copper or silver wire at the other end; the circuit so formed is attached to the end of a capillary tube of glass carrying a mirror, and the whole is supported by a delicate quartz fibre. As the thermoelectric bars may have magnetic properties, they are screened from the magnet as far as possible by being surrounded by a mass of soft iron not less than 5 millimetres distant from the magnet and contained within an exterior block of brass. The figure shows two vertical sections through the axis of the circuit, one horizontal section through the middle of the loop and one through the mirror, from which all that is essential will be made clear.



Theoretical considerations show that if magnetic damping and Peltier effect are overlooked, the best proportions are found when both the resistance and moment of inertia of the loop are equal to the corresponding values of the thermoelectric bars, and that the stronger the field the greater the sensitivity. As, however, the field of a permanent magnet concentrated by pole pieces is already more than strong enough to impose dead-beat conditions, a weaker and more extended field is necessary; for this reason, and because heat is transferred from the hot junction to the two cold ones by the thermoelectric current, the reduction of electric resistance to the uttermost is not of such importance as it otherwise would be. On the other hand, the extreme fineness of the bars is essential in order that they may not conduct heat from the hot to the cold junctions, and the tenuity of these and of the receiving surface is necessary in order that the final temperature may be reached in a few seconds. Antimony and bismuth are generally taken as the typical metals at the two ends of the thermoelectric scale, but these are not so good as certain alloys, which not only have more thermoelectric power, but are more fusible than antimony, and when the necessary skill for soldering these has been acquired are generally preferable. The following alloys have been used by the writer: To replace bismuth, 10 of Bi to 1 of Sb; another, 32 of Bi to 1 of Sb; this last is better than Bi as 10 is to 9. To replace antimony, 14 of Bi to 1 of Sn; 12 of Bi to 1 of Sn; 15 of Sb to 14 of Cd; in this last case electrolytic antimony was used, but care must be taken in handling this when fresh on account of its almost explosive change of state. There seems to be no assignable reason for the remarkable thermoelectric changes induced in metals by alloying them, and it is to be expected that further investigations, especially as so many more metals are now available, would lead to the discovery of more suitable metals or alloys. Thermoelectric power is desirable, but if ductility and less fusibility could be attained these would enable finer elements to be used and so counteract somewhat less thermoelectric power. Conductivity for heat is undesirable, and if this should be low it would be a gain more than setting off the almost inevitable low electric conductivity, as the circuit might then be used in a stronger magnetic field.

The process of constructing[2] the very small bars used by the writer is as follows: Two microscope glass slides are smoked and a fragment of the alloy is placed between the smoked surfaces with the upper plate lying loosely upon it. Two slips of microscope cover-glass of the thickness desired for the bars are cut from the usual discs by means of a writing (not a glazier's) diamond, and these are laid on the lower slide on either side of the fragment of metal. Then heat is gently applied from below, most conveniently by a small flame, and one or more interposed sheets of wire gauze. When the metal melts a warm weight is placed upon the upper glass to squeeze out the liquid metal, which in this way is made to spread out into an irregular disc of the size of a sixpence or shilling and of the thickness of the cover-glass slips. When all is cold the metal may be separated from the smoked mould and is ready for being cut into bars. For this purpose it should be laid upon a piece of plate glass and a sharp chisel or knife should be firmly and evenly pressed upon the metal to sever the whole width at once. Then successively a number of bars, much longer than will be required, can be cut of a width of half a millimetre or even less. These are similarly cut to the length desired, *e.g.* 5 to 8 millimetres. In order to make the circuit, a perfectly smooth hardwood board is used, on which the outline of the circuit should be drawn. A typical circuit would have a receiving surface of thin copper foil only one millimetre in diameter or square, if for receiving focussed heat as from a star, or two, or even three millimetres in diameter for a more diffused source, or perhaps 5 × 1 or 5 × $\frac{1}{2}$ millimetre if for focussed spectrum lines. This piece of copper cut from freshly scrubbed foil is laid upon the board where drawn and held down by a narrow strip of paper secured by pins or little weights. Then the two bars are laid side by side so as just to overlap the copper and as close together as possible, and these are similarly held in position. Then the loop of copper or silver wire of high conductivity and made from wire of No. 36 to 40 S.W.G., or $\frac{1}{5}$ to $\frac{1}{8}$ millimetre in diameter, of the form shown in the drawing, is laid so as just to overlap the ends of the bars, and a tag of wire is hooked round the loop at the far end. These also are similarly held in place. Convenient dimensions for the bars are: length 5 to 8, width $\frac{1}{2}$ to $\frac{1}{3}$, thickness $\frac{1}{10}$ millimetre; for the conducting loop a rectangle 25 · 3 millimetres. Soldering these things together is an art requiring some practice as well as skill, for, even using one of the fusible metals as solder, there is danger that one of the bars will be caught up by capillary forces and vanish instantly in the solder on the bit. The method that has been found to succeed is one in which a soldering-bit is used made of copper wire 1$\frac{1}{4}$ to 1$\frac{1}{2}$ millimetres in diameter wound near the point into two close convolutions so as to act as a heat reservoir, and then projecting 6 to 8 millimetres and filed to a point, and all held in a wooden handle. Such a bit can be heated in a bunsen or spirit lamp flame in a few seconds, and a well-directed touch at the right spot made under observation by a lens will effect a perfect soldering with an almost infinitesimal amount of solder. One milligramme should suffice for all five solderings. Each junction should be moistened with a solution of chloride of zinc before soldering, and the point of the bit should be frequently dipped in the solution. Before trying to remove the circuit it must be tested by a light touch with a camel's-hair brush to see if it is accidentally stuck to the wood at any point, when moisture applied with the brush will probably free it. The circuit must then be lifted by inserting a hook under the loop and lifting the board until the loop is hanging free. It should then be dipped in very hot water to clean off chloride of zinc. A tender circuit may be broken by capillary forces if dipped in cold water. When the circuit is dry a piece of capillary glass tube is fastened to the copper tag with shellac varnish or, better, with melted shellac and made to take a truly axial position, and the copper disc is given a thin coat of black varnish on its front face only. Then a mirror should be attached to the glass at the proper place to suit the window. It is much more convenient to have the mirror and circuit in one plane, but it may be necessary for other reasons to set the mirror in a position perpendicular to the plane of the circuit. These little mirrors are best made as follows. Find the thinnest dozen out of a box of cover glasses all of one diameter by dealing them like a pack of cards on to a table. The musical clink at once indicates the thickness, and those that give the lowest note are set apart. These are then examined for form by looking at a straight and distant window bar by very oblique reflection in the glasses laid one at a time on a support at arm's length from the eye. Those that give an undistorted reflection are retained and the rest are best destroyed. The selected glasses are then silvered in the usual way, and, when washed and dried, varnished with lacquer. From the silvered discs so prepared small mirrors about 3 millimetres square are cut with a writing diamond. One of these is attached to the glass with a speck of melted shellac or of shellac varnish. The melted shellac at one point only is best. If the shellac extends over the contiguous surfaces it is certain to bend the mirror and give a double image. Mirrors made as described are exceedingly light, and they give with sufficient illumination an image so sharp that it can be read to $\frac{1}{10}$ millimetre on a scale at a metre distance. Being plane the focussing has to be done by a lens. Instead of the usual method of a lens near the lamp and scale, the writer has generally used a plano-convex spectacle lens of about one metre focal length cut down to the required diameter and cemented by its plane face to the opening of the window, which is also ground flat. With such an arrangement the scale and wire are both at the principal focus of the lens, through which the light passes twice, just before and just after reflection from the mirror. The image of the wire on the scale is then as fine as the wire itself, but, what is more important, the plane face of the lens also reflects some light which forms an image of the wire on the scale, and this serves as a datum line from which to measure the deviations of the mirror image, a datum which belongs to the instrument and is not affected by accidental movements of instrument or scale. A suitable diameter of quartz fibre (*q.v.*) for a radio-micrometer circuit such as is described is from $\frac{1}{5000}$ to $\frac{1}{7500}$ inch, or about $\frac{1}{250}$ millimetre. The period of oscillation away from the magnet should be from 5 to 10 seconds, and the magnetic field should be so adjusted with movable pole pieces or sliding keeper that the circuit is very nearly, but not quite, dead-beat. When exposed suddenly to radiation then there is a first defined elongation requiring 2 or 3 seconds, and which for comparative purposes may be read and used as a measure of the radiation. The ultimate steady deflection will be greater to a trifling extent. The sensibility of different circuits naturally differ, and if any of the material is magnetic — quite a possible event — the magnetic control will spoil the delicacy. A circuit made by the writer for Mr. E. Wilson for his measures on a very magnified solar image gave a deflection of 80 millimetres when exposed to a candle flame at a distance of nearly 2 metres. The copper disc was 2 millimetres in diameter.

---

[1] *Roy. Soc. Phil. Trans.* A, 1889, clxxx. 159.

[2] See also Witt, *Phys. Zeit.*, 1920, xxi. 374, and *Proc. R. Soc.* A, 1922, cii. 48.

The tube within which the capillary glass tube and mirror and quartz fibre are suspended is made of glass, and the window opening is blown with a welt and ground flat. The whole instrument should be contained within an exterior casing of thick wood to protect it from stray radiation, and the light for the mirror should be allowed to enter and leave by a narrow slit in a sort of wooden chimney tube containing the glass tube. There must be no metallic connection with the outer world.

§ (2) APPLICATIONS OF THE INSTRUMENT.—The writer combined a radio-micrometer with a reflecting telescope[1] of nearly 16 inches aperture (40 centimetres), with a special altazimuth mounting designed for the purpose, in order to investigate the radiation from the moon, planets, and stars. Two observers, using a thermopile and galvanometer and working independently, believed that they had found surprisingly large amounts of radiation from the brighter stars. The delicacy of the new combination was tested by the use of a candle flame or person's face at a distance of 216 metres on which the telescope was focussed. The circuit was made with bars of BiSb 10 : 1 and BiSn 14 : 1. A more delicate circuit with the SbCd alloy was not tested in this way. These observations were made by the late Dr. W. Watson, F.R.S., and the writer. The candle flame gave a deflection repeated 20 times of from 60 to 80 millimetres. Dr. Watson's face gave only 25 or 26 millimetres, the background being a handkerchief on a frame. From the candle figures it will be seen that a candle flame at a distance of from 3 to 3¾ miles (5 to 6 kilometres) would, in a perfectly transparent air, have given a deflection of $\frac{1}{10}$ millimetre. Mercury, observed on three days, Venus, Mars, Jupiter, and Saturn, and all the brighter stars and some nebulae visible in this latitude failed to give any deflection. Arcturus, examined on the meridian on nights of perfect quiet, did not give a movement of $\frac{1}{10}$ millimetre, and so certainly did not send the heat of a candle three miles away. The moon was so violent that it was necessary to stop the telescope down to 6 inches to keep the deflection on the scale. As is to be expected, that part of the moon on which the sun is vertical, whether it is in the middle as in the full moon or limb with a half moon, sends most heat, and this falls as the solar altitude falls to nothing at the terminator. In the case of the full moon the curve is, as nearly as could be ascertained, symmetrical, *i.e.* the lunar surface acquires its temperature so quickly that there is no effect of storing heat. The west side sends no more heat than the east.

Referring to the figures again, it will be seen that there is a glass window on one side of the brass block, but that at the other side the disc of the circuit is exposed naked to the radiation indicated by an arrow. The glass window is to enable the observer, by the aid also of a reflecting prism or mirror and long eyepiece, to observe the receiving disc and watch the passage of a star image or of a spectrum line or other object to be examined. The dark heat from the eye is absolutely cut off even by the thinnest film of glass. On the other side, no window except one of rock salt or sylvine would be of any use, as the greater part of the dark heat would be cut off. Draughts, however, must be excluded, and this is done by the use of a substantial paper tube (a firework case) with a series of diaphragms of diminishing size to receive the converging beam of focussed radiation. Draughts are baffled by such an arrangement, and this, together with the telescope tube, made it possible to use the instrument in the open air on quiet nights.

§ (3) DEVELOPMENTS.—Coblentz has developed the radio-micrometer[2] by using a rock salt window and a good vacuum instead of air within. He states that he has so made it considerably more delicate than those made by the writer.

---

[1] *Roy. Soc. Proc.*, April 1890; see also *English Mechanic*, Feb. 23, 1917, p. 87.

[2] *Carnegie Inst.* Publication 65, Appendix iv.

The manipulative difficulties of soldering the fine junctions may very probably be greatly reduced by the use of a hot pencil given to the writer by Dr. Carl Benedicks of Stockholm. This is a gas burner in which the flame is formed at the end of a capillary tube of melted quartz. The gas may then be turned down gradually by means of a micrometer needle valve until the flame, if it exists at all, is of the size of a pin's point, and the only evidence of its existence is the red-hot end of the tube on the surface of which the combustion probably takes place. When so burning the gas is being consumed at the surprisingly large rate of 1 cubic foot in 5 days or thereabouts. Such a hot point would provide almost certainly the most convenient means of applying the necessary small amount of localised heat for soldering the junctions, and the solder would be in the form of small pieces cut from foil. In making fusible solder for these delicate solderings it is not necessary to prepare fusible metal according to the well-known proportions; a little bismuth and tinman's solder made of tin 2 parts, lead 1 part melted together does very well.

§ (4) THE TONO-MICROMETER.—The radio-micrometer for its development required a suspension fibre of unknown delicacy and perfection, and this led to the discovery of the quartz fibre. The telescope radio-micrometer combination by a curious accident gave rise to an instrument constructed by the writer for detecting the faintest musical sound of the pitch to which it is tuned. When calling to the observer at 236 yards, the radio-micrometer at times made violent deflections, and this was found to occur when a certain note was sounded. The telescope tube acted as a resonator to the tone, this being the fifth of the octave of its fundamental note, and the node one-third down the tube was close to the horizontal axis. The changes of pressure then set up alternating air movements past the copper disc and in and out of the glass tube, and these acted on the disc, tending to set it into the position of greatest opposition. A large resonator was accordingly made with a small opening at the node leading to a small resonator, each tuned to the same note. A radio-micrometer mirror was suspended in the neck at an angle of 45° to the direction of air movement, and a window was put in the neck so that a beam of light could enter the instrument and leave at right angles to the direction of entry. Such an instrument[1] will respond to the note to which it is tuned. The late Lord Rayleigh had previously made an instrument, but without the double resonator,[2] but he showed later[3] that the double resonator, also employed by Professor Callendar, should give an improved effect. The following figures relating to the suspension are of interest as showing the very small forces that are easily under control:

	Weight.	Cm.	Moment of Inertia. C.G.S.	Period.
Glass tube .	·009 gram	·1 diameter	·000022	10 secs.
Mirror . .	·0035 ,,	·4 square	·000044	
			·000066	

From these figures it will be found that the couple needed to give a deflection of 1 millimetre on a scale 1 metre distance is ·000,000,013 C.G.S. unit, or in gravitation and English measure a couple of 1/13,000,000,000 grain at 1 inch. A quartz fibre 10 centimetres long and ·00026 centimetre, or $\frac{1}{10,000}$ inch, in diameter gave this result, and such a fibre will carry safely fifty times the load given above.

§ (5) THE POCKET ELECTROMETER.—The pocket electrometer[4] is an instrument, made by the writer, depending on the quartz fibre, and is unlike other electrometers in that it can be carried in the pocket and set down anywhere and at any time, and when levelled the needle is found charged and it will respond to an E.M.F. of 1/100 volt. This instrument, made in 1891, is still, after thirty years, in perfect order. The needle being a disc can have no disturbing edge actions between it and the quadrants. The disc is made of foil 1 centimetre in diameter, and alternate quadrants are platinum and zinc soldered together; it is suspended by a fine quartz fibre within quadrants with a clearance of about 1 millimetre. A short fine wire axis between the disc and fibre carries a radio-micrometer mirror, and the period is 40 seconds. The air resistance due to the stationary quadrants makes it nearly dead-beat. Instead of the usual high potential of a highly charged needle the effective potential is always about ·8 volt, the contact potential of platinum and zinc. It is immaterial for the purpose of the instrument whether this is in the metal itself or in the air only on the surface of the metal. If V_1 and V_2 are the potentials of the pairs of fixed quadrants and v_1 and v_2 the corresponding effective potentials of the platinum and zinc, the couple will vary as $(V_1 - V_2)(v_1 - v_2)$, and the sensibility usually attained by high potential in this instrument depends upon the delicacy of the quartz fibre and freedom from systematic disturbance.

C. V. B.

RADIO-MICROMETER, BOYS', AS APPLIED TO SOLAR MEASUREMENTS. See "Radiant Heat and its Spectrum Distribution," § (20).

Nichol's. See *ibid.* § (21).

---

[1] *Proc. Roy. Soc.* A, ci. 391.

[2] *Phil. Mag.*, 1882, xiv.

[3] *Ibid.*, 1918, xxxvi.

[4] *The Electrician*, 1891.

RAIN:
Electric charge on. See "Atmosphere, Physics of," § (22). Also "Atmospheric Electricity," §§ (22), (24) (iv.).
Equations for formation of, in adiabatic conditions. See "Atmosphere, Thermodynamics of the," § (21).
"Equivalent" fall measured by observations of intensity of sunlight. See "Meteorological Optics," § (16) (i.) and (iii.).
Opacity of. See *ibid.* § (16) (ii.).

RAINBOW:
Effect of size of drops on colours of. See "Meteorological Optics," § (14).
Explanation on undulatory theory of light. See *ibid.* § (14).
Older theories of. See *ibid.* § (14).

RAINFALL:
Distribution of, over the N. hemisphere in July. See "Atmosphere, Thermodynamics of the," § (3) and *Fig.* 6.
Energy of. See *ibid.* § (2).

RAIN-GAUGES:
Exposure of. See "Meteorological Instruments," § (13).
Obsolete types. See *ibid.* § (12).
Self-recording:
Balance-gauges (Casella and Beckley). See *ibid.* § (14) (iii.).
Float-gauges (Dines, Fernley, and hyetograph). See *ibid.* § (14) (ii.).
Tilting-bucket gauges. See *ibid.* § (14) (i.).
Standard "Snowdon" type. See *ibid.* § (10).

RAIN MEASURE. See "Meteorological Instruments," § (11).

RANGE-FINDING BY DEPRESSION RANGE-FINDER. See "Trigonometrical Heights," § (8).

REFLECTION IN THE ATMOSPHERE, PHENOMENA DUE TO. See "Meteorological Optics," § (22).

REFRACTION:
Astronomical formula for. See "Meteorological Optics," § (4).
Atmospherical. See *ibid.* § (3).
Errors caused in surveying. See *ibid.* § (7).
Green ray at sunrise and sunset. See *ibid.* § (6).

REFRACTION, EFFECT OF, ON SURVEY WORK, exact theory of. See "Trigonometrical Heights," § (3).
Formula representing normal conditions. See *ibid.* § (5).
General account of. See *ibid.* §§ (1) and (2).

REFRACTION IN THE ATMOSPHERE, phenomena due to. See "Meteorological Optics," §§ (20), (21).

REGNAULT:
Dew-point hygrometer. See "Humidity," II. §§ (1), (2).
Formulae for hygrometry. See *ibid.* II. § (4) (ii.).
Verification of Dalton's law. See *ibid.* I.

RELATIVE HUMIDITY. The ratio of the amount of water-vapour present in any specimen of air to the maximum amount the air could contain at the same temperature; relative humidity may be measured by the ratio of the pressure exerted by the water-vapour to the saturation pressure for the same temperature. See "Humidity," I.

RESILIENCE OF THE ATMOSPHERE. See "Atmosphere, Thermodynamics of the," § (14).

REVERSIBLE CHANGES OF NICKEL-STEEL ALLOYS. See "Line Standards," § (4) (ii.), (iv.), (v.).

RIGIDITY OF APPARATUS AND FOUNDATIONS, as required for metrological work. See "Metrology," II. § (5) (vii.).

ROBERVAL BALANCE. See "Weighing Machines," § (3).

ROBINSON CUP-ANEMOMETER. See "Anemometers."

ROOT OF SCREW THREAD: definition. See "Metrology," VII. § (23) (i.).

ROTATION OF THE EARTH, effect of, on motion of air. See "Atmosphere, Physics of," § (8).

— S —

SACCHAROMETER, THE BALLING-BRIX, graduated to show directly the percentage of sugar by weight. See "Saccharometry," § (8).

SACCHAROMETRY

§ (1) INTRODUCTION.—Rapid methods of finding the proportion of sugar in solutions of this substance are essential in the sugar factory, and are often convenient elsewhere. With solutions of pure sugar the determination of the specific gravity gives the required result easily and quickly, by reference to tables which have been constructed for this purpose. On account of the importance of this determination, both in the factory and in the laboratory, a great amount of labour has been devoted to developing the method and obtaining the requisite data.

Many of the liquids dealt with in the sugar factory, however, are not solutions of pure sugar. They contain various other matters, organic and inorganic, sometimes in considerable proportion. Hence the determination of the specific gravity does not, in these instances, give the true sugar content. Moreover, since these "non-sugars" have not the same effect upon the specific gravity of the liquid as the same quantity of sugar has, the determination does not give even the total quantity of solid matters with accuracy, but only to a more or less close approximation.

Nevertheless, the specific gravity is of great importance, in technical practice, for establishing the "quotient of purity" of these impure sugar solutions. This quotient (or "coefficient," as it is also termed) is the percentage of sugar contained in the total solid matter. It is necessary to distinguish between the "true" and the "apparent" value of the quotient. The former is defined as the percentage of real sucrose in the total solid matter as determined by the method of actual drying and weighing. But to ascertain the "true" quotient of purity involves two rather troublesome and lengthy analytical determinations; hence in practice it is customary to be content with the "apparent" quotient. In obtaining this, the sugar, determined from the rotation of the plane of polarisation, is calculated as a percentage of the apparent solids ascertained from the specific gravity, or by other indirect means. That is, two relatively easy and rapid operations, neither of which gives strictly accurate results, are substituted for the more lengthy processes involved in ascertaining the "true" quotient, partly because of the saving of time secured, and partly because the data obtained, whilst admittedly approximations, are sufficiently correct for the technical purposes in view. Moreover, a certain correlation exists between the "true" and the "apparent" quotients of purity. The fact is that the nature of the non-sugars remains practically the same throughout a season's operations, so that the "true" quotient, determined once or occasionally, serves to fix its relation to the apparent coefficient for the duration of the "campaign" at any particular factory.

Another point may be mentioned here. In practice, mixtures of various sugars are met with. It happens, however, that the principal one, sucrose, has a specific gravity differing but little from that of the others in solutions having the same concentration. Thus at 20°/4° C., solutions containing 10 per cent of the following sugars have the specific gravities shown: Arabinose 1·0379, fructose 1·0385, galactose 1·0379, glucose 1·0381, lactose 1·0376, maltose 1·0386, raffinose 1·0375, sorbose 1·0381, and sucrose 1·0381. Hence if accurate tables of specific gravity values are constructed for sucrose, they may be used also for the others, or for mixtures of various sugars. Whilst not theoretically correct, the results are sufficiently accurate for practical purposes. In what follows, therefore, "sugar" always means the particular sugar sucrose, unless the contrary is stated or obviously implied by the context.

§ (2) UNITS OF VOLUME.—The standard unit of volume is the volume occupied by a mass of 1 gram of water at its temperature of maximum density, viz. 4° C., *i.e.* the unit adopted is the millilitre.[1] The standard temperature recommended for sugar work is 20° C., and at this temperature 1 ml. of water weighs 0·99718 gram against brass weights in air at 20° C. and 760 mm. pressure.

The "Mohr cubic centimetre" has been much used in the past in sugar analysis. The volume of a "Mohr's cubic centimetre" varies according to the temperature at which it is defined. The temperature 17·5° C. was commonly used, and for this temperature "Mohr's cubic centimetre" is the volume occupied at 17·5° C. by a quantity of water which weighs 1 gram against brass weights in air. This unit of volume is equivalent to 1·00235 ml. or 1·00238 c.c.

§ (3) CONTRACTION IN VOLUME OF SUGAR SOLUTIONS.—Just as in the well-known case of alcohol and water, when sugar and water are mixed the volume of the resulting solution is less than the sum of the volumes of the sugar and water taken separately. Taking, for instance, a solution of sugar containing 20 grams of sucrose per 100 c.c., its density at 15°/4° C. is 1·07608. A litre of this solution weighing (*in vacuo*) 1076·08 grams contains 200 grams of sugar and 876·08 grams of water. The specific gravity of sugar at 15° being 1·588, the volume of the 200 grams is 200/1·588 = 125·94 c.c., and the volume of the water at this temperature is 876·08 × 1·00087 = 876·84 c.c. Hence the sum of the two separate volumes, 1002·78 c.c., exceeds the volume of the mixture, 1000 c.c., by 2·78 c.c., and there has thus been a contraction in the aqueous solution of this amount, equivalent to 0·278 per cent.

The maximum amount of contraction occurs in sugar solutions of strength about 38 to 46 per cent by weight, as is shown in Table I., calculated by Sidersky from determinations made by Plato.

A glance at the table will show that the apparent density of sugar in solution, high at small concentrations, gradually decreases until, in saturated solutions, it approximates to that of crystallised sugar (1·5879). But the values expressing contraction of volume increase with the concentration up to a maximum value of 0·47 per cent, remain constant for a time, and then decrease.

---

[1] This unit of volume is often erroneously called a cubic centimetre. The cubic centimetre is, however, the volume of a cube, each of whose edges is 1 cm. in length and the accepted relation between the millilitre and the cubic centimetre is

1 ml. = 1·000027 c.c.

(See "Volume, Measurements of," § (1).)

TABLE I

DENSITIES AND CONTRACTIONS OF SUGAR SOLUTIONS
(Deduced from densities found by Plato)

Sugar. grams.		Density at 15°/4° C.	Density of Sugar in Solution.	Contraction. Per cent by Volume.
Per 100 c.c.	Per 100 grms.			
5	4·91	1·01844	1·629	0·08
10	9·64	1·03777	1·628	0·16
15	14·19	1·05694	1·627	0·23
20	18·59	1·07608	1·625	0·28
25	22·83	1·09511	1·623	0·34
30	26·93	1·11407	1·622	0·39
35	30·90	1·13208	1·620	0·43
40	34·73	1·15173	1·617	0·45
45	38·45	1·17043	1·615	0·47
50	42·06	1·18906	1·612	0·47
55	45·57	1·20763	1·609	0·47
60	48·96	1·22601	1·607	0·45
65	52·26	1·24429	1·605	0·42
70	55·48	1·26255	1·602	0·38
75	58·60	1·28063	1·600	0·34
80	61·64	1·29871	1·597	0·29
85	64·59	1·31633	1·594	0·23
90	67·48	1·33406	1·591	0·11

The same phenomenon of contraction occurs, though to a smaller degree, when a strong sugar solution is mixed with a weaker one, or with water.

Various estimations of the amount of contraction have been given, as follows:

Observer.	Sugar.	Water.	Max. Contraction.
	grms.	grms.	c.c.
Brix . .	55·42	44·58	0·9937
Gerlach .	56·25	43·75	0·9946
Zeigler . .	56	44	0·9958

§ (4) TEMPERATURE CORRECTIONS. — The specific gravity of sugar solutions decreases as their temperature rises, by reason of the expansion in volume produced. The relative amount of expansion, however, is not constant; it increases with the degree of concentration. The following values for the coefficient of cubical expansion of sugar solutions between 15° and 25° C. are given by Josse and Remy:[1]

Concentration.	Coefficient.
6·32	0·0002053
12·75	0·0002100
23·88	0·0002250
33·71	0·0002574
43·81	0·0002896
53·37	0·0003153
62·39	0·0003262
66·74	0·0003289

Alternatively, Schönrock's formula may be employed to calculate the mean coefficient of expansion.[2] Let p denote the percentage of sucrose, and r the mean coefficient of cubical expansion for temperatures t between 10° and 27° C. Then, according to Schönrock,

$$r = 0{\cdot}000291 + 0{\cdot}0000037(p - 23{\cdot}7) \\ + 0{\cdot}0000066(t - 20) - 0{\cdot}00000019(p - 23{\cdot}7)(t - 20).$$

The probable error is stated to be ±0·000006.

Having obtained the proper coefficient, and knowing the specific gravity of the sugar solution at a given temperature t, we can calculate the specific gravity at another temperature t_2 from the following equation:

$$D_{t_2} = D_{t_1} + D_{t_1}(t_1 - t_2)r.$$

Here D_{t_2} is the required specific gravity, D_{t_1} the known specific gravity, and r the coefficient of expansion.

In technical work, however, the corrections for temperature are not usually applied to the specific gravity of the sugar solution. They are used as a direct correction of the percentage of sugar itself, suitable tables being calculated out for this purpose. The one subjoined (Table II.) is from a table published by the Bureau of Standards, Washington, and calculated from Plato's data (*vide infra*). The table should be used with circumspection, and only for approximate results, when the temperature differs much from the standard temperature or from that of the surrounding air.

§ (5) RELATION BETWEEN SPECIFIC GRAVITY AND SUCROSE CONTENT. — Formulae have been calculated which indicate the relation existing between the specific gravity of a sugar solution and its content of dissolved sugar. Thus Gerlach has given the following equation:

$$y = 1 + 0{\cdot}003866x + 0{\cdot}0000141x^2 \\ + 0{\cdot}000000329x^3,$$

where x denotes the percentage of sugar, and y is the specific gravity of the solution at 17·5°/17·5° C.

Gerlach's equation has been revised and extended to sugar solutions at other temperatures by Scheibler. For 20° C., Scheibler's equation is:

$$y = 1 + 0{\cdot}003844x + 0{\cdot}0000144x^2 \\ + 0{\cdot}0000000309x^3.$$

§ (6) SPECIFIC GRAVITY OF SUGAR SOLUTIONS.—Several sets of tables are in existence, correlating the percentages of sugar in aqueous solutions of this substance with the corresponding specific gravities. Those considered

---

[1] *Bull. Assoc. Chim. Sucr. Dist.*, 1901–2, xix. 302.

[2] *Zeits. der deut. Zuckerind.*, 1900, l. 419.

TABLE II

TEMPERATURE CORRECTIONS TO READINGS OF SACCHAROMETERS (STANDARD AT 20° C.)

Temp. ° C.	Observed Percentages of Sugar.												
	0	5	10	15	20	25	30	35	40	45	50	55	60
	Subtract from Observed Percentage :												
10°	0·32	0·38	0·43	0·48	0·52	0·57	0·60	0·64	0·67	0·70	0·72	0·74	0·75
11	·31	·35	40	·44	·48	·51	55	·57	·60	·63	·65	·66	·68
12	·29	·32	·36	·40	·43	·46	·50	·52	·54	·56	·58	·59	·60
13	·26	·29	·32	·35	·38	·41	·44	·46	·48	·49	·51	·52	·53
14	·24	·26	·29	·31	·34	·36	·38	·40	·41	·42	·44	·45	·46
15	·20	·22	·24	·26	·28	·30	·32	·33	·34	·36	·36	·37	·38
16	·17	·18	·20	·22	·23	·25	·26	·27	·28	·28	·29	·30	·31
17	·13	·14	·15	·16	·18	·19	·20	·20	·21	·21	·22	·23	·23
18	·09	·10	·10	·11	·12	·13	·13	·14	·14	·14	·15	·15	·15
19	·05	·05	·05	·06	·06	·06	·07	·07	·07	·07	·08	·08	·08
	Add to Observed Percentage :												
21°	0·04	0·05	0·06	0·06	0·06	0·07	0·07	0·07	0·07	0·08	0·08	0·08	0·08
22	·10	·10	·11	·12	·12	·13	·14	·14	·15	·15	·16	·16	·16
23	·16	·16	·17	·17	·19	·20	·21	·21	·22	·23	·24	·24	·24
24	·21	·22	·23	·24	·26	·27	·28	·29	·30	·31	·32	·32	·32
25	·27	·28	·30	·31	·32	·34	·35	·36	·38	·38	·39	·39	·40
26	·33	·34	·36	·37	·40	·40	·42	·44	·46	·47	·47	·48	·48
27	·40	·41	·42	·44	·46	·48	·50	·52	·54	·54	·55	·56	·56
28	·46	·47	·49	·51	·54	·56	·58	·60	·61	·62	·63	·64	·64
29	·54	·55	·56	·59	·61	·63	·66	·68	·70	·70	·71	·72	·72
30	·61	·62	·63	·66	·68	·71	·73	·76	·78	·78	·79	·80	·80

most trustworthy are based upon the work of Plato, Domke, and Harting, and particulars of these are given farther on ; but as some of the others are still widely used it is necessary briefly to mention them. The differences between these various tables are due partly to the fact that sugar of the same degree of purity was not used as the starting-point in all the determinations, and partly to the fact that the results are expressed at different temperatures. Thus in France the "normal" temperature is taken as 15° C., and some of the tables are constructed for the temperature 15°/4° C. : in Germany the "normal" was formerly 14° R. (=17·5° C.), and one of the most widely used tables is calculated for 17·5°/17·5° C., whilst since 1890 the normal temperature adopted has been 15° C., and tables have been constructed for 15°/15° C. Finally, Plato's tables mentioned above, as also others, in the United States, have been calculated for 20°/4° C. and for 20°/20° C.

Confining the selection to those tables founded upon direct experiment, the first to be mentioned is that of *Balling* (1835), which formed the basis of the more complete table by Brix (1854). In this table, subsequently extended by *Mategczeck and Scheibler*, and still in general use on the Continent, are given percentages of sugar by weight corresponding with specific gravities taken at 17·5°/17·5° C.

Gerlach's table, expressed in the same manner,[1] was somewhat more accurate than Balling's. It did not, however, come much into use until 1890, when, the temperature 15° C. having been adopted as the "normal" in Germany, *Scheibler* revised the table, and recalculated it for the temperature 15°/15° C.[2] About a year afterwards, Scheibler published also a volume[3] containing an extended table, divided into one-hundredths of "degrees Brix," with tables of corrections for different temperatures.

In France, tables calculated for the temperature 15°/4° C., and giving grams of sugar per 100 c.c., have been published by Barbet, and also by Vivien (1873). Barbet's results agree closely with those of Plato when the two series are reduced to comparable terms ; Vivien's deviate from them rather considerably.

Plato's experiments, made at the instance of the German Normal Eichungs Kommission, were carried out on a much more comprehensive scale than those of his predecessors, and with much better equipment. The sugar used was chemically pure sucrose, prepared by Herzfeld in the following manner.[4] A saturated solution of crystallised or refined cane sugar was made, filtered from insoluble matters, and triturated with an equal volume of 96 per cent alcohol; this treatment precipitates much of the sugar in a purified form. The mixture was then filtered, the precipitated sugar washed with ether, and dried at a low temperature. Plato found the specific gravity of this sugar to be 1·58965 at 15°/4° C., and 1·59103 at 15°/15° C. Crystallised sugar prepared from it gave the results: 1·5879 at 15°/4°; and 1·5892 at 15°/15°. The specific gravity of sucrose *in solution* was found to be 1·5549 at 15°/4° and 1·55626 at 15°/15°.

---

[1] *Dingl. Polyt. J.*, 1863, clxxii. 31.

[2] *Bull. Assoc. Chim. Sucr. Dist.*, 1890–91, viii. 412.

[3] *Le Titrage des solutions sucrées*, Berlin, 1891.

[4] See Rumpler's *Ausführliches Handbuch der Zuckerfabrikation*, 1906, p. 88.

With this purified sugar two quite independent series of experiments on the specific gravity of its aqueous solutions were carried out. The first covered the range 10 to 60 per cent of sugar. Six sets of determinations were made, one at each of the concentrations 10, 20, 30, 40, 50, and 60 per cent. The second series was a much more extensive one; the lowest concentration was 2·5 per cent of sugar, the next 5 per cent: and then the series proceeded by increments of 5 up to 75 per cent. At all the strengths which were not the same as those in the first series two separate determinations were made, by different observers on different days, so that duplicate determinations were obtained for the whole range of percentages. The method adopted was that of weighing a "sinker," first in air, then in water, and then in the sugar solution, by means of an analytical balance. All the necessary corrections were introduced. The experiments included also observations upon the thermal expansion of sugar solutions; on capillarity, contraction, variation of density with change of temperature, and on the hygroscopic properties of sugar. By reason of their comprehensiveness, and the minute care bestowed upon them, they are now generally accepted as giving the most accurate values for the specific gravity of sugar solutions.[1] There are three main tables of specific gravity values, worked out in percentage of sugar by weight ("degrees Brix"). The first is calculated for the temperature 15°/15° C., and divided into tenths of percentages or degrees. A second is constructed for a range of temperatures $t°$ between 0° and 60° C. ($t°/15°$), and gives integral percentages of sugar from 1 to 70 per cent. The third, like the first, shows tenths-per-cent of sugar, and is established for the temperature 20°/4°, in accordance with the recommendations made by the Fourth International Congress of Applied Chemistry (Paris, 1900). An abridgment of this table is given here (Table III.), and a comparison of the three principal earlier tables with Plato's first table is also appended (Table IV.).

TABLE III

DENSITY OF SOLUTIONS OF SUCROSE AT 20°/4° C.

Sucrose. Per cent by Weight.	Density.	Sucrose. Per cent by Weight.	Density.
0	0·998234		
1	1·002120	51	1·235085
2	1·006125	52	1·240641
3	1·009934	53	1·246234
4	1·013881	54	1·251866
5	1·017854	55	1·257535
6	1·021855	56	1·263243
7	1·025885	57	1·268989
8	1·029942	58	1·274774
9	1·034029	59	1·280595
10	1·038143	60	1·286456
11	1·042288	61	1·292354
12	1·046462	62	1·298291
13	1·050665	63	1·304267
14	1·054900	64	1·310282
15	1·059165	65	1·316334
16	1·063460	66	1·322425
17	1·067789	67	1·328544
18	1·072147	68	1·334722
19	1·076537	69	1·340928
20	1·080959	70	1·347174
21	1·085414	71	1·353456
22	1·089900	72	1·359778
23	1·094420	73	1·366139
24	1·098971	74	1·372536
25	1·103557	75	1·378971
26	1·108175	76	1·385446
27	1·112828	77	1·391956
28	1·117512	78	1·398505
29	1·122231	79	1·405091
30	1·126984	80	1·411715
31	1·131773	81	1·418374
32	1·136596	82	1·425072
33	1·141453	83	1·431807
34	1·146345	84	1·438579
35	1·151275	85	1·445388
36	1·156238	86	1·452232
37	1·161236	87	1·459114
38	1·166269	88	1·466032
39	1·171340	89	1·472986
40	1·176447	90	1·479976
41	1·181592	91	1·487002
42	1·186773	92	1·494063
43	1·191993	93	1·501158
44	1·197247	94	1·508289
45	1·202540	95	1·515455
46	1·207870	96	1·522656
47	1·213238	97	1·529891
48	1·218643	98	1·537161
49	1·224086	99	1·544462
50	1·229567	100	1·551800

---

[1] A full account of the work is published under the title: *Die Dichte, Ausdehnung, und Capillarität von Lösungen reinen Rohrzuckers in Wasser* (Springer, Berlin, 1900); and an abridgment, with tables, is given in *Zeits. der. deut. Zuckerind.*, 1900, 1. 982, 1079.

TABLE IV

SPECIFIC GRAVITY OF SUCROSE SOLUTIONS

Sucrose, Per cent by Weight.	Balling-Brix, 17·5°/17·5° C.	Gerlach, 17·5°/17·5° C.	Gerlach-Scheibler, 15°/15° C.	Plato (Normal-Eichungs-Kommission), 15°/15° C.
0	1·00000	1·00000	1·00000	1·00000
5	1·01970	1·01969	1·01978	1·01973
10	1·04014	1·04010	1·04027	1·04016
15	1·06133	1·06128	1·06152	1·06134
20	1·08329	1·08323	1·08354	1·08328
25	1·10607	1·10600	1·10635	1·10604
30	1·12967	1·12959	1·12999	1·12962
35	1·15411	1·15403	1·15448	1·15407
40	1·17943	1·17936	1·17985	1·17940
45	1·20565	1·20559	1·20611	1·20565
50	1·23278	1·23275	1·23330	1·23281
55	1·26086	1·26086	1·26144	1·26091
60	1·28989	1·28995	1·29056	1·28997
65	1·31989	1·32005	1·32067	1·31997
70	1·35088	1·35117	1·35182	1·35094
75	1·38287	1·38331	1·38401	1·38286

D. Sidersky has extended the German tables to show grams of sugar per 100 c.c., as well as percentages by weight, for temperatures between 10° and 30° C., referred to water both at 4° C. and at 15° C. as unity; and for concentrations up to 30 per cent by weight.[1] He has also given a comparative table of the chief experimental results hitherto published, all reduced to the same terms for facility of comparison.[2] This author concludes that Plato's results are the most trustworthy of all, and recommends that they should be taken as the basis of tables used in sugar analysis, to the exclusion of all others. We give (Table V.) the portions of his table which are calculated from Plato's and Brix's experiments, omitting the others. The results are referred to the temperature, 15°/4° C., adopted in French hydrometry.

§ (7) DETERMINATION OF SUGAR-CONTENT.—In the ordinary routine work of the sugar factory, the strength of the sugar solutions is usually ascertained with a hydrometer (saccharometer, "spindle," densimeter). In the sugar laboratory, the specific gravity of solutions is determined by means of the pyknometer, or by some form of hydrostatic balance (*e.g.* the Mohr or Westphal balance), whenever it is desired to obtain more accurate results than are given by the hydrometer.

To make a determination of specific gravity which shall be trustworthy to the fifth decimal, a pyknometer must be used, with a sensitive balance and the usual precautions for precise weighing, including a correction for the weight of air displaced by the liquid. Such a determination, however, is only wanted in special cases, as for very weak sugar solutions. With the hydrostatic balance, results correct to the fourth decimal can be obtained. This suffices amply for all ordinary requirements, since four units in the fourth decimal place (0·0004) represent only 0·1 per cent of sugar. The hydrometer, even at its best, gives only the third decimal accurately, and the reading is apt to be made somewhat uncertain by the effects of surface tension.

TABLE V

DENSITIES AT 15°/4° C.

Sugar, grams.		Density at 15°/4° C.	
Per 100 c.c.	Per 100 grams.	Plato.	Balling-Brix.
0	0	0·99913	0·99913
5	4·91	1·01844	1·01888
10	9·64	1·03777	1·03824
15	14·19	1·05694	1·05753
20	18·59	1·07608	1·07672
25	22·83	1·09511	1·09582
30	26·93	1·11407	1·11481
35	30·90	1·13298	1·13380
40	34·73	1·15173	1·15262
45	38·45	1·17043	1·17113
50	42·06	1·18906	1·18997
55	45·57	1·20763	1·20855
60	48·96	1·22601	1·22692
65	52·26	1·24429	1·24512
70	55·48	1·26225	1·26350
75	58·60	1·28063	1·28160
80	61·64	1·29871	1·29962
85	64·59	1·31633	1·31746
90	67·48	1·33406	1·33522

Nevertheless, the degree of accuracy obtainable with the hydrometer suffices for ordinary routine work, whilst the simplicity and rapidity with which the results are procured have made this instrument, in one form or another, indispensable to the sugar chemist. The ordinary type of hydrometer may be used for determining the specific gravity of sugar solutions; but in general, for sugar work, a special form is employed. Either it is graduated to show the percentage of sugar direct (Balling-Brix saccharometer), or it gives readings in terms of an arbitrary scale, which by means of an appropriate conversion table, will indicate the corresponding percentage of sugar (Baumé's hydrometer). If the specific gravity is determined, whether by pyknometer, Mohr's balance, or hydrometer, the corresponding sugar-content is found from the table already given (Table III.), or a similar table constructed for the particular temperature employed.

The general theory of the hydrometer, and the methods of calibrating and comparing these instruments, are dealt with elsewhere in this work (see article "Hydrometry"). Hence a simple outline only is necessary here, referring more particularly to the form of instrument usually employed in saccharometry. This consists of a hollow glass cylinder, carrying at its lower end a bulb weighted with mercury or shot, and surmounted by a slender stem of uniform diameter. A paper scale is enclosed in the stem. When placed in a liquid, the hydrometer sinks until the weight of liquid displaced is equal to the weight of the instrument. The heavier the liquid, the less the saccharometer sinks. The dimensions and weight of the instrument may be so arranged that the hydrometer sinks to a point near the top of the stem when placed in water; to a point near the bottom of the stem in a sugar solution of known sugar content, and therefore to intermediate points in solutions containing smaller proportions of sugar. These points are thus established as those corresponding with known percentages of sugar, and intermediate divisions may be interpolated. In practice this is done by dividing the intervals between the established points into equal subdivisions; this method is not strictly accurate, but the error is considered to be less than the probable error in reading the instrument.

---

[1] *Les densités des solutions sucrées à différentes températures*, Paris, 1908.

[2] *Bull. Assoc. Chim. Sucr. Dist.*, 1919, xxxvii. 73.

Alternatively, the stem may be divided in such a way as to show the specific gravity of the liquid instead of its sugar percentage. Thus in the foregoing example, if the specific gravities of the sugar solutions used are known, the established points correspond with these specific gravities, and subdivisions can be interpolated. These subdivisions, however, are much less uniform than in the former case, since the distance between them decreases as the specific gravity increases. Twaddell's hydrometer, much used for technical work in this country, is based upon this principle, each scale division representing 0·005 as specific gravity. Thus a reading of, say, 30° Tw. denotes a specific gravity of 1·000 + 30 × ·005 = 1·150.

A third method of calibration is that adopted by Baumé for his hydrometer. He used a solution of sodium chloride containing 15 parts by weight of this salt and 85 parts by weight of water. The point to which the hydrometer sank in this solution was marked 15°, and the point to which it sank in distilled water at the same temperature (probably about 12·5° C.) was marked 0. The space between these two points was divided into 15 equal parts or degrees, and divisions of the same length were extended downwards beyond the 15° point. The Baumé divisions are therefore of an arbitrary nature, showing neither specific gravity nor percentages of sugar. They can, however, be correlated with specific gravity values, and thence with sugar-content, by means of tables calculated for the purpose. This is explained in detail further on.

§ (8) BALLING-BRIX SACCHAROMETER.—This instrument is graduated to show, directly, percentage of sugar by weight. It was first introduced by Balling, the scale being subsequently revised by Brix. "Degrees Brix," or "degrees Balling," therefore, are both understood to mean the same thing, namely, the percentage of sucrose, by weight, contained in the solution referred to. The "Brix spindle," as the instrument is commonly called, is supplied in various series to cover the range of sugar-strengths dealt with, the most generally useful being a series in which each instrument has a range of 10 degrees, subdivided into tenths of a degree. For more accurate readings, the distance between the divisions is required to be greater, and therefore the stem is of smaller diameter; hence to meet these demands there is a series in which each "spindle" has a range of only 5 degrees, with subdivisions of 0·05 degree. On the other hand, for rougher work the range of each instrument extends to 30 degrees, graduated in half-degrees, or in fifths. Brix hydrometers are calibrated at the temperature 17·5° C., and if readings are taken at a different temperature the necessary correction must be introduced (see Table II.). In some forms a thermometer is fitted into the saccharometer itself, but this is not of much advantage when, as often happens, turbid liquids are being examined, since the thermometer cannot then be read distinctly.

The Brix saccharometer is probably the instrument most generally used in technical sugar work, and the modern tendency is to employ it more and more. But the Baumé form is still widely used, both on the Continent and in America, and it is therefore necessary to discuss this instrument in some detail.

§ (9) BAUMÉ'S HYDROMETER.—There has been much confusion as to the exact interpretation of the results shown by this instrument—that is, as to the precise specific gravity which the "degrees" indicate. Quite a large number of different "Baumé scales" have been used. The scale was first proposed, and the instrument used as a *pèse-sirop*, by Antoine Baumé, a French chemist, in 1768; but the description given by him is not sufficiently precise for the accurate reproduction of the scale. The exact temperature and specific gravity of the solutions he employed (see above) are in some doubt. Hence proposals have been made to construct a "rational" Baumé scale. If the hydrometer sinks to the point 0° in water, and to a point

d° in a liquid of specific gravity s, it can be shown that $ds/(s-1)=m$, a constant. The value of this constant (or "modulus," as it is often termed) is found if, d being supposed known, s is determined by any suitable method.

Conversely, if m is known, the specific gravity corresponding with any division on the Baumé scale can be calculated, since the foregoing equation transposes to $s=m/(m-d)$.

Gay-Lussac, investigating the basis of Baumé's system of hydrometry (1822), took strong sulphuric acid as the test liquid, and 66° as the value of d. The specific gravity of the acid at 15° C. was 1·8427, whence the value of m is found to be 144·32. The Baumé scale calculated from this modulus has since then been commonly used in France and Germany. In Holland the value of m adopted was 144.

Gerlach, reinvestigating the question half a century later,[1] found the specific gravity of a 15 per cent sodium chloride solution, as used by Baumé, to be 1·11383 at 17·5° C. Putting this value for s in the above equation, and taking d as the point 15°, the value of m is found to be 146·78.

What is known as the "old" Baumé scale is the one (used in Holland) with the value of m taken as 144. The "new" or "Gerlach" scale is based upon the value found above, namely 146·78. In the United States the value officially adopted, and used by all the hydrometer makers, is 145.[2]

The subjoined comparison (Table VI.) of the "old," the "new," and the U.S. scale is published by the Bureau of Standards, Washington (*Technologic Paper* No. 115).

TABLE VI

COMPARISON OF BAUMÉ SCALES

Per cent Sucrose, or Degrees Brix.	Corresponding Degrees Baumé.		
	"New" Scale (Modulus 146·78)	"Old" Scale (Modulus 144).	U.S. Scale (Modulus 145).
0	0·0	0·0	0·00
5	2·8	2·8	2·79
10	5·7	5·6	5·57
15	8·5	8·3	8·34
20	11·3	11·1	11·10
25	14·1	13·8	13·84
30	16·8	16·5	16·57
35	19·6	19·2	19·28
40	22·3	21·9	21·97
45	25·0	24·6	24·63
50	27·7	27·2	27·28
55	30·4	29·8	29·90
60	33·0	32·4	32·49
65	35·6	34·9	35·04
70	38·1	37·4	37·56
75	40·6	39·9	40·03
80	43·1	42·3	42·47
85	45·5	44·7	44·86
90	47·9	47·0	47·20
95	50·3	49·3	49·49
100	..	..	51·73

§ (10) SPECIAL FORMS OF HYDROMETER.—Various modifications of the standard forms have been devised, chiefly with the view of simplifying corrections for temperature. Thus in one device a thermometer is included with the Brix "spindle," but the thermometer scale is graduated to show, not the temperature of the solution, but the corresponding correction to be added to or subtracted from the "degrees Brix" shown by the hydrometer scale (Volquartz).

Another form (Vosatka's) is provided with a movable scale, which after adjustment to the temperature of the sugar solution gives the true reading directly.

Other varieties are specially devised for dealing with the hot dilute sugar solutions ("sweet water") obtained in the operation of exhausting filter-press cakes. The temperature of these "waters" is usually 60° to 80° C., and the hydrometers employed are often calibrated for use at these high temperatures in order to avoid the delay consequent upon cooling the liquids down to 17·5° C. or 20° C. In addition, these hydrometers are made with a large body and a thin stem, so that the reading can easily be made to the tenth of a degree. One ingenious form (Langen's) is so constructed that, at any temperature between 30° and 70° C., the readings of the stem and of the included thermometer coincide when the instrument is placed in pure water, but differ more or less widely as long as any sugar is present in the solution. Hence to determine when the extraction of sugar is complete, it is only necessary to test samples from time to time, until the two readings, at first divergent, become coincident, showing that no more sugar is being dissolved out. For further details of these special hydrometers see Browne's *Handbook of Sugar Analysis* (Wiley).

§ (11) READING HYDROMETERS.—In using hydrometers, the liquid to be tested is placed in a clear glass cylinder of such a size as will allow the instrument to float freely without touching the sides or bottom. After mixing the liquid thoroughly by means of a stirrer, when necessary, the hydrometer, which should be clean and dry, is slowly immersed in the liquid until it just floats freely, without wetting the upper part of the stem. When the instrument is at rest, and the liquid free from air-bubbles, the scale is viewed from a point just below the plane of the liquid surface, and the eye is raised gradually until that surface, first seen

---

[1] *Dingl. Polyt. J.*, 1870, cxcviii. 315.

[2] *Vide* Chandler, *Nat. Acad. Sci.*, Philadelphia, 1881, for the origin and early history of the various Baumé scales.

TABLE VII

DEGREES BRIX, SPECIFIC GRAVITY, AND DEGREES BAUMÉ

Degrees Brix, or per cent Sugar by Weight.	Sp. Grav. at 20°/4° C.	Sp. Grav. at 20°/20° C.	Degrees Baumé (Mod. 145).	Degrees Brix, or per cent Sugar by Weight.	Sp. Grav. at 20°/4° C.	Sp. Grav. at 20°/20° C.	Degrees Baumé (Mod. 145).
0	0·99823	1·00000	0·00	51	1·23508	1·23727	27·81
1	1·00212	1·00389	0·56	52	1·24064	1·24284	28·33
2	1·00602	1·00779	1·12	53	1·24623	1·24844	28·86
3	1·00993	1·01172	1·68	54	1·25187	1·25408	29·38
4	1·01388	1·01567	2·24	55	1·25754	1·25976	29·90
5	1·01785	1·01965	2·79	56	1·26324	1·26548	30·42
6	1·02186	1·02366	3·35	57	1·26899	1·27123	30·94
7	1·02588	1·02770	3·91	58	1·27477	1·27703	31·46
8	1·02994	1·03176	4·46	59	1·28060	1·28286	31·97
9	1·03403	1·03586	5·02	60	1·28646	1·28873	32·49
10	1·03814	1·03998	5·57				
11	1·04229	1·04413	6·13	61	1·29235	1·29464	33·00
12	1·04646	1·04831	6·68	62	1·29829	1·30059	33·51
13	1·05066	1·05252	7·24	63	1·30427	1·30657	34·02
14	1·05490	1·05677	7·79	64	1·31028	1·31260	34·53
15	1·05916	1·06104	8·34	65	1·31633	1·31866	35·04
16	1·06346	1·06534	8·89	66	1·32242	1·32476	35·55
17	1·06779	1·06968	9·45	67	1·32855	1·33090	36·05
18	1·07215	1·07404	10·00	68	1·33472	1·33708	36·55
19	1·07654	1·07844	10·55	69	1·34093	1·34330	37·06
20	1·08096	1·08287	11·10	70	1·34717	1·34956	37·56
21	1·08541	1·08733	11·65	71	1·35346	1·35585	38·06
22	1·08990	1·09183	12·20	72	1·35978	1·36218	38·55
23	1·09442	1·09636	12·74	73	1·36614	1·36856	39·05
24	1·09897	1·10092	13·29	74	1·37254	1·37496	39·54
25	1·10356	1·10551	13·84	75	1·37897	1·38141	40·03
26	1·10818	1·11014	14·39	76	1·38545	1·38790	40·53
27	1·11283	1·11480	14·93	77	1·39196	1·39442	41·01
28	1·11751	1·11949	15·48	78	1·39850	1·40098	41·50
29	1·12223	1·12422	16·02	79	1·40509	1·40758	41·99
30	1·12698	1·12898	16·57	80	1·41172	1·41421	42·47
31	1·13177	1·13378	17·11	81	1·41837	1·42088	42·95
32	1·13660	1·13861	17·65	82	1·42507	1·42759	43·43
33	1·14145	1·14347	18·19	83	1·43181	1·43434	43·91
34	1·14634	1·14837	18·73	84	1·43858	1·44112	44·38
35	1·15128	1·15331	19·28	85	1·44539	1·44794	44·86
36	1·15624	1·15828	19·81	86	1·45223	1·45480	45·33
37	1·16124	1·16329	20·35	87	1·45911	1·46170	45·80
38	1·16627	1·16833	20·89	88	1·46603	1·46862	46·27
39	1·17134	1·17341	21·43	89	1·47299	1·47559	46·73
40	1·17645	1·17853	21·97	90	1·47998	1·48259	47·20
41	1·18159	1·18368	22·50	91	1·48700	1·48963	47·66
42	1·18677	1·18887	23·04	92	1·49406	1·49671	48·12
43	1·19199	1·19410	23·57	93	1·50116	1·50381	48·58
44	1·19725	1·19936	24·10	94	1·50829	1·51096	49·03
45	1·20254	1·20467	24·63	95	1·51546	1·51814	49·49
46	1·20787	1·21001	25·17	96	1·52266	1·52535	49·94
47	1·21324	1·21538	25·70	97	1·52989	1·53260	50·39
48	1·21864	1·22080	26·23	98	1·53716	1·53988	50·84
49	1·22409	1·22625	26·75	99	1·54446	1·54719	51·28
50	1·22957	1·23174	27·28	100	1·55180	1·55454	51·73

as an ellipse, appears as a straight line. The point where this line cuts the hydrometer scale is taken as the reading. The meniscus of liquid formed round the stem by capillary attraction is disregarded. The temperature of the solution must be carefully noted, and if, as usually happens, it differs from the standard temperature (15·6°, 17·5°, or 20° C., as the case may be), the proper correction must be applied. Ideally, the hydrometer, the liquid, and the surrounding atmosphere should all be at the standard temperature, but these conditions, of course, are not often obtained in practice.

§ (12) Sugar Table.—The most recent sugar table issued, showing degrees Brix, or percentage of sugar by weight, correlated with degrees Baumé and with specific gravity, is one by Bates and Bearce, and published by the United States Bureau of Standards in 1918 (*Technologic Paper* No. 115). It is based upon Plato's specific gravity determinations, and the temperature 20° C. is adopted as being the most convenient, and one widely accepted for sugar work; the Baumé scale used is calculated with the modulus 145. An abridgement of this table is subjoined (Table VII.).

§ (13) Use of Solution-factors.—Within certain limits, the proportion of sugar in a solution can be determined from the specific gravity by means of a "solution-factor," thereby dispensing with the use of tables. If, for example, 10 grams of sucrose be dissolved in water, and the solution made up to 100 c.c. at 15·5° C., the specific gravity of this solution at 15·5°/15·5° C. will be found to be 1038·6 (water = 1000). The excess specific gravity over that of water is therefore 1038·6 − 1000, or 38·6. Assuming that the increase of specific gravity produced is always proportional to the quantity of dissolved sucrose, one gram of sucrose per 100 c.c. would give an increase of 3·86, and x grams would give x times 3·86. Hence, conversely, to obtain the concentration x (grams per 100 c.c.) of the sugar in a solution the specific gravity of which is S, we have

$$x = \frac{(S - 1000)}{3{\cdot}86}$$

where S is expressed in terms of water taken as 1000. For example, if a solution of sucrose has the specific gravity 1055·4, it will contain 55·4/3·86 = 14·3 grams of sugar per 100 c.c.

This method was first used by O'Sullivan in his investigations upon the products obtained by the hydrolysis of starch.[1] Brown, Morris, and Millar subsequently determined the values of the solution-factors for other sugars than sucrose,[2] and Ling, Eynon, and Lane have repeated the investigations in respect of dextrose, laevulose, and maltose.[3] The factors given below are regarded as accurate for solutions of the sugars mentioned, at a concentration of 10 grams per 100 c.c.:

Sucrose	3·86
Dextrose	3·82
Laevulose	3·92
Invert sugar	3·87
Maltose	3·91
Mixed starch-conversion products .	3·93 to 4·01

The solution-factors, however, are not constant for all degrees of concentration: their values decrease somewhat as the concentration increases. Hence it is customary, in analysing commercial products, to work with solutions containing 10 grams of the product per 100 c.c.; or if a strong sugar solution is being dealt with, to dilute a known quantity until the specific gravity is reduced to about 1·04, before calculating the sugar-content by means of the factor. For mixtures of sugars the factor 3·86 is generally employed; and for solutions of mixed starch-conversion products, the round number 4 gives fairly good approximate results.

§ (14) Impure Sugar Solutions.—It must be borne in mind that, as already mentioned at the beginning of this article, many of the liquids dealt with in technical work contain not only sugars, but more or less mineral matter, and organic acids or salts. For a given concentration, solutions of most salts show a higher specific gravity than the sugars do. Hence, whether the tables or the factors are employed, since both are based upon the data for pure sugars, the results may show appreciable errors when applied without correction to the impure solutions in question. Browne (*op. cit.*) gives the following particulars in illustration of this:

Table VIII

Aqueous Solutions at 15° C.

Specific Gravity.	Sucrose.	Tartaric Acid.	Sodium Potassium Tartrate.	Potassium Carbonate.
	per cent	per cent	per cent	per cent
1·0039	1	0·87	0·57	0·43
1·0078	2	1·73	1·14	0·86
1·0118	3	2·62	1·71	1·29
1·0157	4	3·49	2·28	1·72
1·0197	5	4·40	2·87	2·15
1·0402	10	8·67	5·87	4·40
1·0833	20	17·52	12·16	9·00
1·1296	30	26·29	18·38	13·78
1·1794	40	35·33	24·73	18·72
1·2328	50	44·22	31·10	23·76

A further point is that when very thick syrups have been diluted with water as a preliminary to taking the specific gravity, the above-mentioned inaccuracy is enhanced, owing to the difference in the amount of contraction as between sugars and impurities when members of the two groups are dissolved in water.

---

[1] *Chem. Soc. J.*, 1876, xxx. 129.
[2] *Ibid.*, 1897, lxxi. 72.
[3] *J. Soc. Chem. Ind.*, 1909, xxviii. 730.

For many factory operations the error is of no moment, since, as already explained, it is practically constant during a campaign, and the observations are essentially comparative. In other cases a correction is introduced. One method (empirical) of doing this is to multiply the percentage of soluble ash in the sample by 0·8, and deduct the product from the specific gravity of the solution, determined at a concentration of 10 grams per 100 c.c. From the specific gravity thus corrected, the sugar-content is deduced as usual by means of the tables or by using the appropriate division.

C. S.

ABBREVIATIONS AND FULL TITLES

Bull. Assoc. Chim. Sucr. Dist.—*Bulletin de l'Association des Chimistes de Sucrerie et Distillerie de France.*

Zeits. Ver. Deut. Zuckerind.—*Zeitschrift des Vereins der Deutschen Zuckerindustrie.*

Dingl. Polyt. J.—*Dingler's Polytechnisches Journal.*

Nat. Acad. Sci.—*National Academy of Sciences*, Philadelphia.

J. Soc. Chem. Ind.—*Journal of the Society of Chemical Industry.*

SACCHAROMETRY. See "Hydrometers," § (19).

SCALE OF MERCURY BAROMETER, graduation of, in pure length units. See "Barometers and Manometers," § (2) (i.).

SCALES: methods of division of micrometer scales. See "Micrometers," § (1).

SCALES AND GAUGES (engineers'). See "Metrology," § (17).

SCALES FOR WEIGHING, self-indicating. See "Weighing Machines," § (8).

SCINTILLATION, OR TWINKLING OF STARS, explanation of. See "Meteorological Optics," § (11).

SCREW GAUGES:
- Adjustable plug for measuring pitch and effective diameters of screwed rings. See "Gauges," § (40) (v.).
- Adjustable ring for testing B.A. screws. See *ibid.* § (40) (vi.).
- Measuring attachment for lathe. See *ibid.* § (23) (iv.).
- Methods of producing and generating thread forms. See *ibid.* § (41).
- Optical measurements of. See *ibid.* § (32).
- Plug form: data for use in the measurement of effective diameter with standard wires. See *ibid.* § (63).
  - Machines for measuring effective and core diameters. See *ibid.* § (23).
  - Measurement of angle of flanks. See *ibid.* § (21).
  - Measurement of roundings at crests and roots, and examination of general form of thread. See *ibid.* § (22).
  - Mechanical measurements of. See *ibid.* § (19).
  - Test on concentricity of diameters. See *ibid.* § (20).
  - Types of machines for measuring pitch. See *ibid.* § (24) (A).
- Projection apparatus for testing. See *ibid.* § (36).
- Ring form: effective diameter, measurement by means of expanding gauges. See *ibid.* § (29).
  - Experimental machine for measuring effective diameters. See *ibid.* § (30).
  - Machines for testing pitch. See *ibid.* § (25).
  - Mechanical measurement by slip gauges and special fittings. See *ibid.* § (28).
  - Mechanical measurements, examination of threads by means of casts. See *ibid.* § (27) (ii.).
  - Mechanical measurements, plug check tests. See *ibid.* § (27) (i.).
- "Scissor" type for measuring B.A. screws. See *ibid.* § (40) (iv.).
- Special gap gauge designed by Mr. W. Taylor. See *ibid.* § (40).
- Special type for small screws. See *ibid.* § (40) (vii.).
- Taper, measurements of. See *ibid.* §§ (37), (38), (39).
- Taylor expanding effective diameter plug gauge for testing screw threads. See *ibid.* § (40) (iii.).
  - For testing effective diameter of nuts, see *ibid.* § (40) (ii.).
- Tolerances of. See "Metrology," § (27).

SCREW THREADS:
- Definitions of elements of. See "Metrology," § (23) (i.).
- Effective diametral error. See *ibid.* § (24) (ii.).
- Elements of full (or major) diameter, core (or minor) diameter, effective (or pitch) diameter, pitch, angle, radius at crest, radius at root. See "Gauges," § (18).
- Envelopes, zones, grade, and play, definitions of. See "Metrology," § (25) (ii.).
- Errors in angle. See *ibid.* § (24) (iv.).
- Errors in crest and root diameters. See *ibid.* § (24) (v.).
- Errors in form of crest and root. See *ibid.* § (24) (vi.).
- Gauging of. See *ibid.* § (25).
- Methods of production. See *ibid.* § (23) (ii.).
- Microscope apparatus for measuring. See "Gauges," § (33).
- Pitch error. See "Metrology," § (24) (i.).
- Relation of errors in effective diameter and pitch. See *ibid.* § (24) (iii.).
- Special gauges and instruments for testing. See "Gauges," § (40).
- Standard types. See *ibid.* § (43).
- Table of standard sizes. See *ibid.* § (54).
- Tolerances suitable for. See "Metrology," § (26).

Screws, Micrometer, for subdividing the space between scale divisions. See "Micrometers," § (2).

Sea Water:

- Composition of. See "Oceanography, Physical," § (1).
- Measurement of currents. See *ibid.* §§ (24)-(31).
- Measurement of temperature and collection of water samples. See *ibid.* §§ (18)-(23).
- Properties of:
    - Chlorinity, salinity, and density. See *ibid.* § (2).
    - Compressibility. See *ibid.* § (5).
    - Density and temperature. See *ibid.* § (3).
    - Electric conductivity. See *ibid.* § (14).
    - Evaporation. See *ibid.* § (11).
    - Freezing point. See *ibid.* § (4).
    - Gases absorbed by. See *ibid.* § (17).
    - Osmotic pressure. See *ibid.* § (10).
    - Specific heat. See *ibid.* § (7).
    - Surface tension. See *ibid.* § (12).
    - Thermal conductivity. See *ibid.* § (8).
    - Thermal expansion. See *ibid.* § (6).
    - Transparency. See *ibid.* § (16).
    - Viscosity. See *ibid.* § (13).
- Salinity and density, determination of. See *ibid.* §§ (32)-(36).

Secondary Disturbance of Pressure. See "Atmosphere, Physics of," § (18).

SEISMOMETRY

§ (1) **The Seismometer.**—Any instrument which measures the motion of the ground during an earthquake or other seismic disturbance is termed a *seismometer*. The term *seismograph* is, however, more generally employed, since most instruments of the kind provide a record, or *seismogram*, on a more or less enlarged scale, of the motion. The most general form of such motion involves six independent quantities, viz. displacements in three mutually rectangular directions, and rotations about three axes in these directions. The measurement of the rotations (which are always exceedingly small except, perhaps, at places near the seat of disturbance) presents difficulties, and as yet no continuous record has been obtained of them. In what follows attention will therefore be chiefly directed to the measurement of the three linear displacements. The directions in which these are generally measured are the lines drawn from the place of observation to the north, east, and the zenith. Obviously the same type of instrument will serve for both horizontal measurements.

(i.) *Description.*—The general aim in seismometry has been to arrange, by mechanical or other means, that a certain point of the apparatus shall remain relatively at rest, and then to contrive means whereby the motion of the ground relatively to this "steady point," as it is termed, shall be continuously recorded. A heavy mass, suspended by a long fine wire from a point fixed in connection with the ground, is a first approximation to the former result; for when the point of support moves with the ground the heavy mass tends to remain stationary. If there be attached to the suspended mass a pen or pencil resting on a horizontal sheet of paper which moves with the ground, the trace obtained on the paper would represent that motion. Such an arrangement, known as a pendulum seismograph, has been frequently used, examples of which are the Italian seismographs of Drs. Agamennone and Vicentini (*Fig.* 1 and *Fig.* 2). In practice it is desirable that the period of the pendulum shall be of considerable length; 10 to 60 seconds are in common use.

A modification of the simple pendulum which has the effect of lengthening the period is the duplex pendulum of Ewing (1880). This is a simple pendulum combined with an inverted pendulum, where the relative masses of the bobs may be so chosen that any degree of stability is attained. Again, consider another type represented in *Fig.* 3. A rod AB is pivoted at A and B in a rigid frame secured to the ground, and the line AB is given a very slight inclination to the vertical. From its middle point C, and at right angles to it, the rod CM projects, carrying a heavy mass M at or near its extremity. It is clear that if AB were horizontal the system would behave as a simple pendulum, while if AB were exactly vertical the mass M would be in neutral equilibrium. By inclining AB through a small angle i from the vertical, the effective value of the gravitational acceleration is reduced from g to $g \sin i$. In equilibrium, with no seismic disturbance in progress, M is at the lowest point it can reach in the course of a complete rotation about AB. This type, in which the path of oscillation is approximately in a horizontal plane, is termed a horizontal pendulum and forms the basis of most of the modern forms of seismograph. Its chief merit, when compared with the vertical type, lies in the small dimensions which may be employed. Long periods of oscillation are readily obtained by a simple adjustment of the angle i. When the ground moves in a direction having a component perpendicular to the plane containing AB and this lowest point, the mass M tends to remain at rest while the axis AB, moving with the ground, produces an inclination of the arm CM away from its equilibrium position. It must also be noted that any rotation of the ground about the approximately horizontal axis CM, or about a vertical axis, will also produce motion of the arm CM. A writing point attached to M may then trace on a sheet of paper or smoked glass a record of the motion, and if the paper or glass be given a uniform motion in the direction CM, we should obtain the seismogram of the disturbance. This arrangement—one of the forms of the horizontal pendulum seismograph—was first introduced in a practicable form by Ewing in 1880, and by it he obtained the first continuous record of earthquake motion. There are a number of forms of the horizontal pendulum, the chief difference being in the method of suspension. *Fig.* 4 is perhaps the most common form and is used in the Milne, Milne-Shaw, Omori, Mainka, etc. The pendulum is supported by a tie wire FD and a point at C.

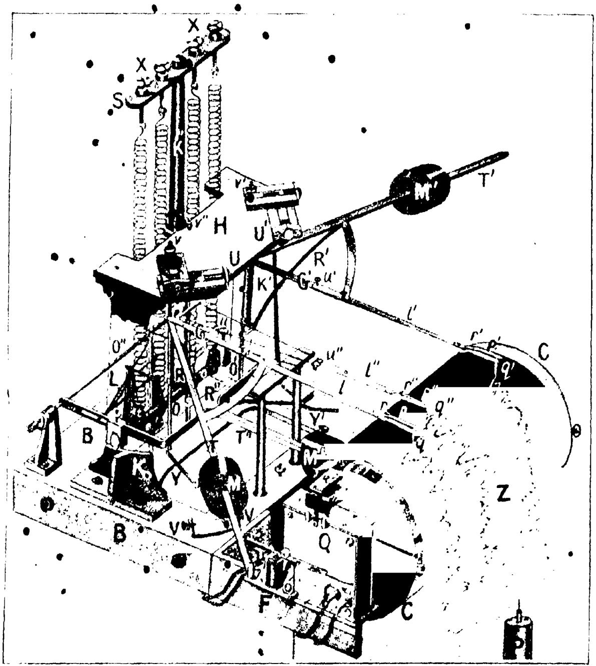


FIG. 1.

Fig. 5 is the well-known Zöllner suspension where there are two tie wires EC and DF. CD then becomes the ideal axis of rotation. The Galitzin pendulum is suspended in this manner.

Another type is the inverted pendulums of Marvin and Wiechert, where a heavy mass is carried by a stout vertical rod whose lower end terminates in a Cardan spring (two flat springs placed at right angles to each other), thus enabling the rod to oscillate in any direction. In this form the pendulum is only stable within certain limits: stability is sometimes increased by one or more delicate springs acting horizontally upon the mass; stops are also provided to arrest undue oscillation.

(ii.) *Theory.*—From the dynamical point of view the many different forms of horizontal pendulum seismograph are identical, and the following is a condensed account, on the lines of Galitzin's treatment, of the theory of their action.

We may suppose that, essentially, the instrument consists of a heavy mass, placed at one end of a rigid rod, and capable of rotation in a nearly horizontal plane about an axis passing through the other end of the rod and very slightly inclined to the vertical. Neglecting rotational effects, the equation of motion of the arm will be

$$\frac{d^2\theta}{dt^2}+2k\frac{d\theta}{dt}+n^2\theta+\frac{1}{l}\frac{d^2x}{dt^2}=0,$$

where θ is the angular deflection of the arm from its equilibrium position, l is the "reduced length" of the pendulum, x is the co-ordinate of position of the ground relative to fixed axes, and where k and n are constants. The second term is introduced in order to take account of any damping of the motion, such as might be produced by friction at the pivots, air resistance, or other artificially introduced cause. It is important to note that the assumption is made that the friction is proportional to the angular speed of the arm. When the damping is due to air resistance, or fluid friction, or to electromagnetic damping (as in the Galitzin instruments), this assumption is warranted. But where the end of the arm bears against any pivot (as in the Milne or Omori instruments), or where the nearly vertical axis is supported on two pivots (as in the bracket seismograph of Ewing), the friction is of an indefinite and possibly variable kind, and the assumption is less justifiable. The coefficient k is known as the damping coefficient. The third term involves n^2, which, for small angular displacements, is equal to g/l. We take, first, the case of the free motion of the pendulum, such as would be produced by giving the movable arm an initial angular velocity V, after which it is left to itself, the ground being at rest. We thus get

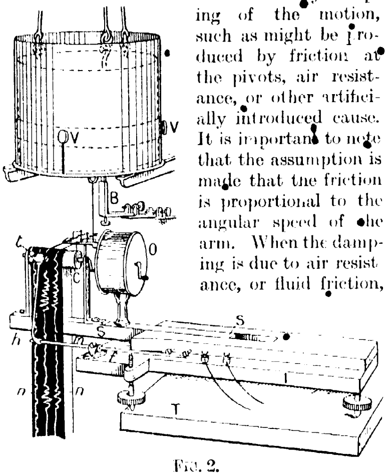


FIG. 2.

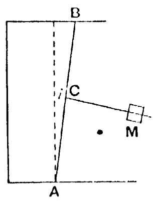


FIG. 3.

$$\theta = A_1\epsilon^{-\alpha_1 t} + A_2\epsilon^{-\alpha_2 t},$$

where

$$\alpha_1 = k + \sqrt{k^2 - n^2},\ \alpha_2 = k - \sqrt{k^2 - n^2}.$$

Three cases therefore arise, according as $k >, =$ or $< n$. In the first

$$\theta = \frac{V}{\alpha_1 - \alpha_2}[\epsilon^{-\alpha_2 t} - \epsilon^{-\alpha_1 t}],$$

and, the second term diminishing more quickly than the first, θ will always be positive, the arm swinging out to a maximum deflection and thereafter creeping back asymptotically to zero. In the third case

$$\theta = \epsilon^{-kt}[A\cos\beta t + B\sin\beta t],$$

where $\beta = +\sqrt{k^2 - n^2}$. But $A = 0$ in this case, and the deflection at time t is

$$\theta = \frac{V}{\alpha_1 - \alpha_2}\epsilon^{-kt}\sin\beta t.$$

The pendulum motion is then damped harmonic motion, the amplitude decreasing logarithmically with the time. The second is the limiting case between the first and third, and the pendulum is then said to be at the aperiodic limit. The motion is given by

$$\theta = Vt\epsilon^{-nt}.$$

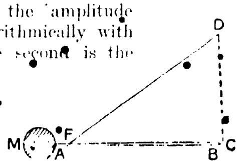


FIG. 4.

We have next to investigate the behaviour of the pendulum when the ground is subject to a given motion, and it may be supposed that this is of the simple harmonic type, so that $x = x_m\sin(pt + \delta)$ may be substituted in the fundamental equation. The solution is

$$\theta = \epsilon^{-kt}[P_1\cos\beta t + P_2\sin\beta t]$$

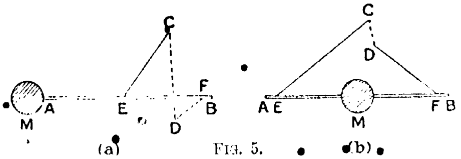
$$+ \frac{x_m}{l}\frac{p^2}{\sqrt{p^4 - 2(n^2 - 2k^2)p^2 + n^4}}\sin\{p(t - \tau) + \delta\}.$$

The first term is wholly an instrumental effect; the second is a forced oscillation, and is due to the seismic motion.

The effect of damping, or its absence, has first to be considered. If the damping be heavy, so that the pendulum is at the aperiodic limit, $k = n$ and $\beta = 0$. The second part of the instrumental term therefore disappears, and, with the coefficient ϵ^{-kt}, the importance of the first part will diminish quickly. The forced oscillation is also simplified, so that very soon after the pendulum has started moving its deflection may be represented by

$$\theta = \frac{x_m}{l}\frac{p^2}{n^2 + p^2}\sin\{p(t - \tau) + \delta\},$$

where $p\tau = 2np/(n^2 - p^2)$. The motion is there-



(a) FIG. 5. (b)

form simple harmonic, the period being the same as that of the seismic wave. The phase is retarded by an amount $p\tau$, and the amplitude is proportional to that of the seismic wave. The ratio of the maximum deflection of the pendulum to that of the seismic wave is thus $p^2/l(n^2+p^2)$.

On the other hand, with a damping coefficient of low value, so small as to approach zero, we have $k=0$, $\tau=0$, $\beta=n$, and the deflection is given by

$$\theta=[\mathrm{P}_1\cos nt+\mathrm{P}_2\sin nt]$$
$$+\frac{x_m}{l}\frac{p^2}{n^2-p^2}\sin(pt+\delta).$$

Determining the constants P_1, P_2, on the supposition that $\theta=0$ and $d\theta/dt=0$ when $t=0$, it can be shown that, if δ is supposed zero,

$$\theta=\frac{x_m}{l}\frac{p}{n^2-p^2}[p\sin pt-n\sin nt].$$

This shows that when the periods of the pendulum and the seismic wave are equal, or nearly so, resonance produces very large deflections. Undamped seismographs therefore give an entirely wrong representation of the ground motion. Further, instruments in which the damping coefficient is unknown or cannot be numerically determined with accuracy are of little use in *measuring* the actual displacement during an earthquake.

§ (2) THE RECORD.—The relation between the ground displacement and that shown on the seismogram has now to be considered. This will depend on the manner in which the record is produced. The different methods which have been employed may be classed as mechanical, optical, and galvanometric.

(i.) *Mechanical.*—In the first of these the end of the pendulum boom carries a pen or writing point which traces a record of the deflection of the boom on a suitably prepared surface which moves at constant speed parallel to the direction of the undisturbed boom. If y be the deflection on the seismogram, while the distance of the writing point from the axis is L, then $y=\mathrm{L}\theta$, and if the damping is such that the pendulum is at the aperiodic limit

$$y=\frac{\mathrm{L}x_m p^2}{l(n^2+p^2)}\sin\{p(t-\tau)+\delta\}.$$

(ii.) *Optical.*—With optical methods of registration, a mirror placed upon the axis of rotation reflects a fixed beam of light to a scale or a moving sheet of photographic paper. The angular deflection of the beam is twice that of the boom, and if the scale or photographic paper is at a distance A from the mirror, 2A takes the place of L in the foregoing expression.

(iii.) *Galvanometric (Galitzin).*—The method of galvanometric registration, introduced by Galitzin depends on entirely different principles. The boom carries at its end a horizontal coil of fine wire whose terminals are connected to a galvanometer. Permanent magnets placed above and below the coil induce a current in the coil when the boom moves, and the measurement of this current by the galvanometer provides the data required for determining the motion of the boom. The magnitude of the current will be proportional to the area and number of turns in the coil, the strength of the field, the angular speed of the boom, and inversely proportional to the total resistance in circuit. It may therefore be represented by $-a(d\theta/dt)$. The deflection ϕ of the galvanometer (usually of the moving coil type) is related to the angular displacement θ of the boom by the equation

$$\frac{d^2\phi}{dt^2}+2\mathrm{K}\frac{d\phi}{dt}+\mathrm{N}^2\phi+a\frac{d\theta}{dt}=0.$$

If therefore the ground displacement is simple harmonic, as already supposed, we have, by introducing the solution of the equation for the aperiodic horizontal pendulum moving under a seismic wave of form $x_m\sin(pt+\delta)$,

$$\frac{d^2\phi}{dt^2}+2\mathrm{K}\frac{d\phi}{dt}+\mathrm{N}^2\phi=a\frac{p^3x_m}{l(n^2+p^2)}\sin(pt+\delta_1),$$

where $\delta_1=\delta-p\tau-(\pi/2)$. It follows that θ and ϕ are related linearly and that there is a difference in phase between the two motions. The magnification can be easily obtained from the solution.

§ (3) VERTICAL SEISMOMETERS.—So far the horizontal motion of the ground has alone been considered. Instruments for the measurement of vertical motion are also in use. Generally they take the form of a boom carrying a heavy mass, and pivoted at one end while being supported horizontally by a coiled spring. The equation of motion is precisely similar to that of a horizontal pendulum, and the registration of a disturbance in the vertical direction can be carried out by any of the three methods already described.

§ (4) THE GALITZIN INSTRUMENTS.—The principles involved in seismometry find their most complete application in the instruments devised by the late Prince Boris Galitzin, and the following brief notes on the chief points in their construction may be of interest. The horizontal pendulum is built up on a metal base, plate, or frame, provided with levelling screws. From four rectangularly arranged points in the base, metal pillars, braced together, project vertically upwards so as to form a rigid framework. The suspension is of the Zollner type, and the inclination of the axis is so small that with the given pendulum length the period of oscillation is about 24 secs. The boom is a stout brass rod, 28 cm. in length, and carries a cylindrical mass of brass weighing 7 kilograms, centring at a point 14 cm. from the inner end of the boom. Beyond the brass cylinder the boom carries a horizontal flat celluloid case, enclosing four horizontally arranged coils of fine copper wire. Above and below these coils permanent tungsten steel magnets are placed. The magnets are so supported that the distance apart of opposing poles may be adjusted easily, so that the field between them may be kept constant when the strength of the magnets decreases slowly with time. The coils are so coupled together that the inductive effects of their motion in the magnetic field are added together, and their terminals are connected to two wires stretching backwards along the boom. The ends of the latter are connected to others, attached to the framework and passing to the galvanometer, by means of very fine slips of phosphor bronze. The damping is magnetic, and for this purpose the boom carries at its outer end a horizontal brass plate which swings between the poles of permanent magnets placed above and below it. The wires from the coil system pass to a mirror galvanometer of the moving coil type. A fixed beam of light is reflected from the mirror, through a horizontal semi-cylindrical lens to a sheet of photographic paper stretched on a drum rotated by clockwork. The drum also moves uniformly in the direction of its axis. Its peripheral speed is 30 mm. per minute, and time marks are made on the record by shutting off the beam of light for two seconds at the beginning of each minute. The shutter is closed by an electromagnet energised through a contact on an accurately timed and rated clock. The record thus produced is a series of curves across the sheet of photographic paper, and the time of any seismic occurrence at the place can be ascertained to within a second. The magnification for seismic waves of simple harmonic type varies with their period. For example, on the Eskdalemuir Observatory instruments, the amplitude of a wave of 20 secs. period is obtained in microns (·001 mm.) by multiplying the amplitude in millimetres on the seismogram by 1·2. This is a much higher yield in the matter of magnification than is afforded by any other type of seismograph, and in consequence the Galitzin instruments give for each seismic disturbance a wealth of detail such as cannot be otherwise obtained. For details as to the determination of the constants of these instruments, Galitzin's *Vorlesungen über Seismometrie* should be consulted. The great merit of these deservedly famous instruments lies in the fact that every detail of construction has been made the subject of careful experimental inquiry, and definite proof obtained that in their behaviour they represent an actual translation into reality of the solutions of the differential equations which express their motion.

§ (5) THE MILNE SEISMOGRAPH.—This instrument—the design of the late Professor John Milne, F.R.S.—was the one chosen of four types examined by the Seismological Committee of the British Association for a world survey of seismic phenomena commencing in the year 1897. Nineteen machines had previously been erected in Japan and the twentieth at Shidi.

The selection was made on account of its simplicity of construction and operation, freedom from friction, and its ability to record both short and long wave periods.

Its two chief defects became apparent as the science advanced, viz. it magnifies the ground movement only six times and it is almost wholly undamped.

The instrument (*Fig.* 6) consists of an extremely light boom AB of aluminium tube, 39 inches in length and averaging ½ inch in diameter. This is supported upon a cast-iron column CD, 20 inches in height, whose base is a tripod with three levelling screws at its angles. The forward screw serves for regulating the angle i to obtain the period of 18 to 20 seconds which is in general use.

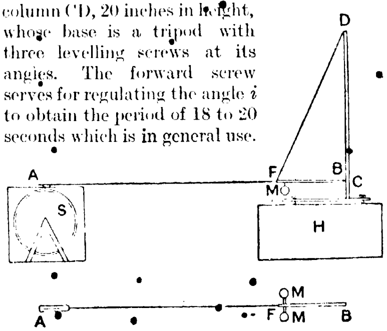


FIG. 6.

For the purpose of determining its sensitivity to tilt one of the rear screws is fitted with a lever and a divided scale; one division is equivalent to a tilt of 10 seconds of arc.

The top of the column is fitted with a winding gear for adjusting the height of the boom.

The inner end of the boom is provided with an agate cup which rests against a horizontal pointed screw at the lower end of the column. At a point about 5 inches along the boom the mass M, of balanced dumb-bell type, is pivoted. Its weight has varied from one to two or three pounds.

Just beyond the mass, at the centre of gravity of the assembled boom, a block is fitted to receive the lower end of the tie wire FD—the upper end of which terminates in a number of strands of unspun silk wound upon the winder and constitutes the upper support.

The outer end of the boom carries a strip of aluminium foil in which is cut a slit in line with the boom. This slit floats just above a second slit in the top of the recording-box cut at right angles to the upper slit.

The light of a small paraffin lamp is thrown downwards, by an angled mirror, on to the top of the recording box, and penetrates at the point of the crossed slits, and is there recorded at 240 mm. per hour upon the travelling photographic film within. Due to the drum S on which the film is wrapped possessing a lateral as well as a circular motion, the trace is in the form of a spiral - this method proved to be six times more economical than the earlier arrangement, where a 2-inch ribbon 33 feet long was employed.

The interruption of the light once per hour either by a revolving watch-hand or electric shutter marked the time upon the trace.

§ (6). THE OMORI AND WIECHERT SEISMOGRAPHS. Of the seismographs using mechanical registration the Omori, the Weichert, and Mainka are the most in use.

As previously remarked, the lack of uniformity in the friction of the writing pointers renders them quite unreliable where close comparison of records is proposed.

In a test made on an Omori type—mass 300 lbs., magnification 60, period 12 seconds—an increase of the pressure between the pointer and the smoked surface of one-third of a milligram reduced the recorded amplitude by 47 per cent.

(i.) *The Omori Seismograph.*—In this instrument (*Fig.* 7) the horizontal pendulum, varying from 30 to 40 inches long, consists of a stout rod with steel cone cup at one end to receive a hard steel point, and the bob ranging from $6\frac{3}{4}$ lbs. to 30 lbs. fixed at the other. The pendulum is supported upon a stout tapered tubular iron column from 3 feet 4 inches to 8 feet in height according to design. The bob consists of a flat cylinder of lead which is suspended by a wire from each side to a stirrup at the top of the column, where the adjustments are made both for position and period, usually 26 to 30 seconds. In some instances an inverted pendulum is poised vertically below the bob and coupled to it. This has the effect of increasing the astatic nature of the mechanism but adds to the mechanical friction.

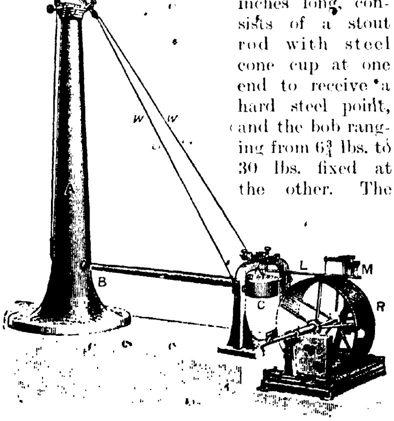


FIG. 7.

An inverted U-shaped bracket arched over the bob serves the double purpose of carrying limiting stops for the excursions of the mass and provides a support for the fulcrum of a light aluminium multiplying lever with ratio usually about 10 : 1.

The short end of the lever is a horizontal fork which engages with a vertical steel roller carried by the bob. The outer end of the lever carries a recording pointer comprised of a fine strand of glass fixed to a horizontal steel spindle pivoted in a jewelled fork, thus enabling the point of the glass fibre to rise and fall with the inequalities of the recording surface. An electric time-marker records minutes upon the trace. The record is made upon paper smoked by a paraffin flare. The drum of 36 inches circumference revolves once per hour and has a lateral traverse of one-sixth of an inch per revolution.

(ii.) *The Weichert Seismograph.*—The basis of the Weichert seismograph (*Fig.* 8) is a delicately poised inverted pendulum which records on smoked paper. It is made with masses varying from 200 lbs. to 17 tons. A heavy cast-iron tripod and recording table standing about 3 feet high carries the mechanism. The flexible end of the pendulum is attached to the base of the casting by a cardan spring connection. The pendulum has its centre of percussion one metre above the spring, and is at this point connected to two thrust arms set at right angles to each other for the purpose of recording the motion of the bob both in the N.-S. and E.-W. directions.

These arms are 25 cm. long and extend from the centre of the bob to a pair of double-ended aluminium levers whose leverage is 10 : 1. One end of these levers operate the damping devices while the other ends, through compression struts, pass on the motion to the recording levers. Depending upon the position on the recording lever where the strut is placed, so the total magnification can be made of several values between 40 and 160 times the ground movement.

The damping device is in the form of a cylinder with a piston suspended in such a position that it is just free of the sides. The opposite ends of the cylinder are connected by an adjustable valve which permits the air to pulsate more or less freely. The first lever operates the piston through a connecting rod. The cylinder can be opened out until the damping is eliminated; or it may be closed until the system becomes aperiodic.

The writing lever is 15 cm. long and has an adjustable counter-weight for regulating the pressure upon the smoked paper.

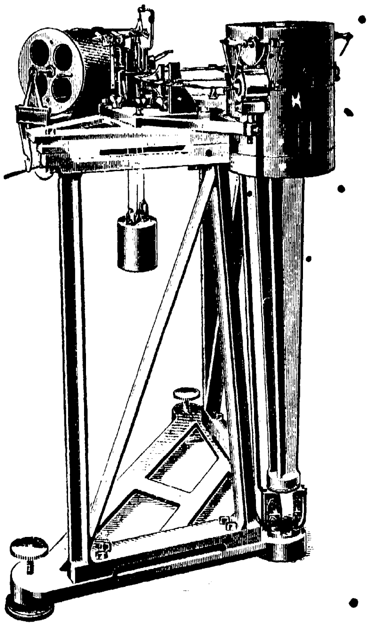


FIG. 8.

The recording sheet is pasted in the form of a ring which is hung upon the drum—a roller lies in the loop below to keep the paper taut. The registering speed is normally 10 mm. per minute but may be raised to 30 mm. if desired.

§ (7) The Milne-Shaw Instrument.—This (*Fig.* 9) is a new type of seismograph which was evolved from and now supersedes the well-known Milne apparatus. Its design was commenced just prior to the death of the late John Milne, and combines both mechanical and optical magnification, together with magnetic damping; the instrumental friction is also reduced to a minimum.

The main objects in its design are high magnification, ease in standardisation, low running cost, and simplicity in operation.

The general principle of the instrument is such as to multiply the movements of a short aluminium boom (carrying 1 lb. as a steady mass) by reflecting a beam of light from a unpivoted focal mirror coupled to the outer end of the boom by means of an iridium pivoted coupler weighing $\frac{1}{20}$ of a gram. The light passes through a vertical slit to a specially thin mirror of half-metre focal length, and thence to a horizontal plano-cylindrical lens in the recording box. The ray is here brought to a focal point upon a fine horizontal slit extending the length of the cylindrical lens. The part carrying the lens and slit hangs as a pendulum in grazing contact with the photographic film. By this means waves of one or two seconds' period are distinctly recorded upon the film with a paper speed of only 8 mm. per minute. The mass is pivoted to the boom by two pivots, which frees the boom from the torsional inertia of the mass. An electrolytic copper vane and tungsten steel magnets are used for damping.

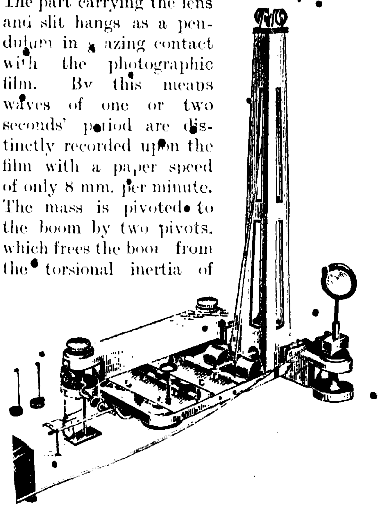


FIG. 9.

Standardisation is specially provided for. By means of a flexible cable to the calibrating screw, a solenoid to give artificial oscillation to the boom, projecting lantern, mirror, and scales, the period, damping coefficient, and tilt sensitivity may each be determined by a single observer from his fixed position at the recording box. The instrument is arranged for four standards of sensitivity, viz. either 150 or 250 magnifications of the ground movement in conjunction with either 10 sec. or 12 sec. period and a damping ratio of 20 : 1. These are the constants adopted for this apparatus. By adopting appropriate units in the calibrating scales the magnification for rapid ground movement is given by the simple formula $a \times 829/T$, where a is the amplitude in millimetres on the record and T the undamped period of the pendulum. Graphs suitable for each of the adopted sensitivities are provided for reading off the true magnifications, which vary according to the period of the ground movement.

A. C. M.
J. J. S.

Self-recording Instruments:
  General characteristics of. See "Meteorological Instruments," § (3).
  Photographic. See *ibid.* §§ (3), (9) (i.).
  See also "Thermograph," "Rain-gauge," "Anemobiagraph," etc.

Sellers or United States Standard Thread. See "Gauges," § (48).

Shadow Bands. See "Meteorological Optics," § (11).

Shaft and Hole: table of fits for a series of shafts in normal hole. See "Metrology," VIII. § (31) (ii.).

Shaft and Hole Bases of Tolerances on Cylindrical Work. See "Metrology," VIII. § (30) (i.).

Shimmering. See "Meteorological Optics," § (9).

Sidereal Day, as Standard of Time. See "Clocks and Time-keeping," § (1).

Sikes' Hydrometer: a metal hydrometer for use in conjunction with Sikes' tables for determining the strength of spirits. See "Alcoholometry," § (6).

Sikes' Light Hydrometer: a metal hydrometer for use with strong spirits. See "Alcoholometry," § (6).

Silica, Fused: a material with low coefficient of expansion. See "Metrology," § (4).

Silica Metre Standard: description, stability of. See "Line Standards," § (6) (ii.).

Sky:
  Apparent form of. See "Meteorological Optics," § (2).
  Colours of, caused by diffraction. See *ibid.* §§ (12) and (13).
  Relative brightness of. See *ibid.* § (12) (footnote).

Slide Rules. See "Draughting Devices," p. 275.

Snow: equations for formation of, in adiabatic conditions. See "Atmosphere, Thermodynamics of the," § (21).

Snow Crystals, Forms of. See "Meteorological Optics," § (18).

Snowfall, Measurement of. See "Meteorological Instruments," § (25).

Solar Constant:
  Definition of. See "Radiation," § (1). Also "Radiant Heat and its Spectrum Distribution," § (5).
  Determination of. See "Radiation," § (1) (i.).
  Value of. See *ibid.* § (1) and §§ (1) (i.) and (ii.), (4) (i.).
  Variations of. See *ibid.* § (1).

Solar Radiation. See "Radiation, Solar."
  Amount reaching the earth. See "Radiation," § (3) (ii.).
  Mean value of, for different latitudes. See *ibid.* § (4) (ii.).
  Reflection of, by the atmosphere. See *ibid.* §§ (3) (i.), and (4) (i.).
  Transmission and absorption of, by the atmosphere. See *ibid.* §§ (1) (i.), (3) (ii.), (4) (i.).
  Wave length of maximum energy of. See *ibid.* § (1).
  See also "Solar Constant."

Solar Time. See "Clocks and Time-keeping," § (1).

Solid, Density of a, determined by the hydrostatic method. See "Balances," § (16).
  Determined by the hydrostatic method: a solid heavier than water and unacted on by water. See *ibid.* § (16) (i.).
  Determined by the hydrostatic method: a solid which floats in the liquid chosen for the hydrostatic weighing. See *ibid.* § (16) (iii.).
  Determined by Nicholson's hydrometer. See *ibid.* § (16) (iv.).
  Determined by the specific gravity bottle. See *ibid.* § (16) (v.).
  Determined by the volumenometer. See *ibid.* § (16) (vi.).

Solution-factors, use of, to determine the proportion of sugar in a solution from the specific gravity. See "Saccharometry," § (13).

Specific Gravity Bottle. An instrument for determining the density of a liquid. See "Pyknometer"; "Balances," § (15) (i.).

Specific Gravity Hydrometer. See "Hydrometers," §§ (2), (7).

Specific Heat of Dry Air, Ice, Water, Water-vapour. See "Atmosphere, Thermodynamics of the," § (2). For determination of, see "Specific Heat," Vol. I.

Spirit Balances. Early forms of specific gravity balances, chiefly used in determining the strength of spirits, were termed "spirit balances." See "Alcoholometry," § (2).

Spirit Duties. For particulars of the methods adopted in determining the strength of spirits for the purpose of assessing duties in Austria, Belgium, France, Germany, Great Britain, Holland, Italy, Norway, Russia, Spain, Sweden, Switzerland, and the United States of America, see "Alcoholometry," § (9).

Spirits: determination of volume of spirits from their weight by means of Sikes' hydrometer. See "Alcoholometry," § (8).

Splines and Weights. See "Draughting Devices," p. 279.

SQUARE STANDARD THREAD. See "Gauges," § (46).

SQUARES FOR USE IN METROLOGY. See "Gauges," § (97).

STANDARD AND REFERENCE GAUGES: definition. See "Metrology," § (19).

STANDARDS, BOARD OF TRADE. See "Metrology," § (13).

STANDARDS OF LENGTH, apparatus used in comparisons of. See "Metrology," § (3) (i.).
- British: definition of imperial standard yard. See *ibid.* § (7) (i.).
- Fundamental. See *ibid.* § (2).
- Importance of thermal expansibility of material. See *ibid.* § (4).
- (Industrial), temperature of adjustment. See *ibid.* V. § (16).
- Metric: definition of international prototype metre. See *ibid.* § (7) (ii.).
- Primary, choice of material for. See *ibid.* § (4).
- Ultimate relative merits of "line" bars and "end" bars. See *ibid.* § (4).
- Working. See *ibid.* § (2) (i.) (*d*).

STANDARDS OF MASS:
- British, fundamental standard the pound avoirdupois. See "Metrology," § (8) (i.).
- Construction of. See *ibid.* § (4).
- Metric primary standard the international prototype kilogramme. See *ibid.* § (8) (ii.).

STANDARDS OF LENGTH MEASUREMENT:
- Material of. See "Metrology," § (4).
- Primary. See *ibid.* § (2) (i.) (*a*).
- Systems of, historical and general. See *ibid.* III. § (6).

STANDARDS OF TIME. See "Metrology," § (2) (ii.).

STARS:
- Cause of scintillation or twinkling of. See "Meteorological Optics," § (11).
- Displacement due to refraction. See *ibid.* § (4).

STEELYARD. See "Weighing Machines," § (4).
- Printing. See *ibid.* § (7).

STEPPED RECKONER. See article "Calculating Machines," § (4) (ii.).

STEVENSON THERMOMETER SCREEN. See "Meteorological Instruments," § (5) (v.).

STRATOSPHERE: the upper region of the atmosphere, in which there is no convection and where the temperature is sensibly constant with height.
- Definition of. See "Atmosphere, Thermodynamics of the," § (5).
- Distribution of temperature in. See *ibid.* §§ (4), (5).
- Effect of radiation on. See "Radiation," § (3) (iv.).
- Explanation of. See "Atmosphere, Thermodynamics of the," §§ (10)-(12). See also "Atmosphere, Physics of," § (6) (iii.).
- Height of. See "Atmosphere, Physics of," § (5).
- Reversal of temperature gradient in. See *ibid.* §§ (10) and (11).
- Stability of. See "Atmosphere, Thermodynamics of the," § (7).
- Variation of height and temperature of. See "Radiation," § (3) (i.).
- Variation of wind in. See "Atmosphere, Physics of," § (11).

SUGAR SOLUTIONS:
- Contraction in volume of. See "Saccharometry," § (3).
- Determination of sugar content of. See *ibid.* § (7).
- Relation between specific gravity and sucrose content of. See *ibid.* § (5).
- Specific gravity of, correlated with percentage of sugar in solution and tabulated. See *ibid.* § (6).

SUGAR TABLES, showing percentage of sugar by weight correlated with degrees Baumé and with specific gravity. See "Saccharometry," § (12).

SUN-PILLARS: columns of light stretching up about 15° or 20° from the sun. See "Meteorological Optics," § (22) (v.).
- Parhelic. See *ibid.* § (20) (vii.).

SUNRISE. Effect of refraction on time of. See "Meteorological Optics," § (5).

SUNSHINE, measurement of duration of. See "Meteorological Instruments," V.

SUNSHINE RECORDERS:
- Campbell Stokes:
  - Adjustment of. See "Meteorological Instruments," § (24) (ii.).
  - Cards for. See *ibid.* § (24) (iv.).
  - Description of. See *ibid.* § (24). Also "Radiant Heat and its Spectrum Distribution," §§ (1)-(3).
  - Errors of. See "Meteorological Instruments," § (24) (iii.).
  - Tabulation of records. See *ibid.* § (24) (v.).
- Jordan. See *ibid.* § (25).

SUN-SPOT PERIOD, variation of solar radiation with. See "Radiation," § (1).

SURFACE PLATES. See "Gauges," § (95).

SURFACE TENSION. Influence of surface tension on hydrometer readings. See "Hydrometers," §§ (10), (11), (14).

SURVEYING: errors caused by refraction. See "Meteorological Optics," § (7).

SURVEYING TAPES AND WIRES

§ (1). —Wires and tapes are used as long standards of length for surveying purposes, and although these appear to be fragile for standards, they are reasonably reliable if properly handled. It is advisable, however, to have such standards compared at short intervals with more rigid standards, such as 4 m., or 10-foot line bars, by some such means as those described below. Tapes of suitable material will maintain their length for some years, if carefully kept. For example, some steel tapes have been under observation at the National Physical Laboratory for about eight years without any change in length greater than one part in a million being noticeable. These tapes are used on the flat, that is, they are supported throughout their complete length.

The section of tapes and wires is usually small; for example, the steel tapes mentioned above have a section of $\frac{1}{4}'' \times 0''{\cdot}015$, and it is important that the tapes should be standardised and used under certain specified conditions. As the elastic elongation of such tapes amounts to nearly one part in one hundred thousand for an alteration of 1 lb. weight in the tension to which they are submitted, the standard tension must be maintained to within 0·01 lb., so that the length should not vary for this cause by more than one part in ten millions. Tapes are used on the flat for the rougher operations of surveying in which accuracy greater than a few parts in 100,000 is not required. The tension is usually controlled by the use of spring dynamometers. For measurements made along the ground in this way tapes are usually subdivided and calibrated so that odd lengths may be determined by their use. With tapes used along the ground, the irregularities in the surface of the ground prevent the attainment of the high precision now required for the best operations of surveying, consequently more accurate measurements are made with tapes or wires supported above the ground.[1] Used in this way the tape or wire is hanging in a natural catenary, the tension being applied by dead weights connected by means of suitable cord or wire which passes over ball-bearing pulleys. The standardisation is done under similar conditions but with the tape hanging in a horizontal catenary, and in this case the curve is symmetrical about the lowest point. The tension is very important, and it is advisable to have the tape standardised with the weights and straining cords that will be used in the field. If this is not done, care must be taken to find the actual tension applied by the weights, straining cords, and connecting links or swivels, and if this differs from the tension under which the tape has been standardised, a correction must be applied to the certified length, using equation (7) given below in § (3).

§ (2) The Catenary.—The theory of tapes and wires in catenary is discussed fully in a later volume,[2] but for convenience some of the equations given therein are repeated here. In *Fig.* 1

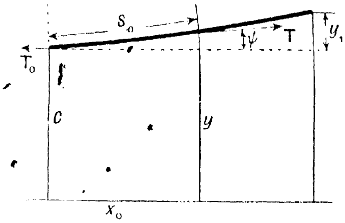


Fig. 1.

$$c=\frac{\mathrm{T}_0}{w}=\frac{\mathrm{T}\cos\psi}{w}, \quad . \quad . \quad (1)$$

$$y=\frac{\mathrm{T}}{w}=c\cosh\frac{x_0}{c}, \quad . \quad . \quad . \quad (2)$$

$$s_0=\frac{\mathrm{T}\sin\psi}{w}=c\sinh\frac{x_0}{c}, \quad . \quad . \quad (3)$$

in which w = weight of unit length of the tape or wire.

On changing the axis of x to the lowest point of the curve, (2) becomes

$$y_1+c=c\cosh\frac{x}{c},$$

or, putting this in the exponential form,

$$y_1=\frac{c}{2}(\epsilon^{x/c}+\epsilon^{-x/c}-2),$$

and on developing,

$$y_1=\frac{1}{2}\frac{w}{\mathrm{T}}x^2+\frac{1}{4!}\frac{w^3}{\mathrm{T}^3}x^4+\frac{1}{6!}\frac{w^5}{\mathrm{T}^5}x^6+\ . \ . \ . \quad (4)$$

In practice w/T is very small, and the values of the terms of (4) decrease rapidly. In a horizontal catenary y_1 gives the sag of the wire, x being half the length of the chord of the curve. If the tension is increased the curve becomes more nearly that of a parabola, that is, the second and subsequent terms of (4) can be neglected. For example, in an invar wire of 1·65 mm. diameter, with a density of 8·1, the weight per metre length is 0·01732 kg., and if T = 10 kg., the second term of (4) has a value of 0·005 mm. for a 24 m. wire.

The difference λ between the length of the curve and the chord is given in § (42), article "Surveying and Surveying Instruments," as

$$\lambda=2x\left\{\frac{1}{6}x^2\frac{w^2}{\mathrm{T}^2}\left(1-\frac{1}{4n^2}\right)^{-1}+\ . \ . \ . \right\};$$

---

[1] See article "Surveying and Surveying Instruments," § (41) (iii.), Vol. IV.

[2] See article "Surveying and Surveying Instruments," § (42), Vol. IV.

In most cases all but the first term are negligible and the equation becomes

$$\lambda = \frac{1}{24}\frac{w^2l^3}{T^2}, \quad . \quad . \quad . \quad . \quad (5)$$

in which l, the nominal length of the tape or wire, is substituted for $2x$, an approximation which will not cause any error, since the actual length is generally well within 1 part in 10,000 of the nominal.

§ (3) CORRECTIONS FOR THE ELASTICITY OF THE WIRES.—Benoit and Guillaume[1] give the modulus of elasticity of hard drawn invar wire (1·65 mm. in diameter) as 16,000 kg. per sq. mm. For rolled invar tape, 1/8″ × 1/50″ in section, the modulus is about 15,000 kg. per sq. mm. ($21·4 \times 10^6$ lbs. per sq. in.).

The elongation is proportional to the tension and follows the law $lT/E\sigma$ where E is the modulus of elasticity and σ is the section.

If the wire or tapes are used in catenary, there is also a change in the catenary curve with alteration in the tension, the sag decreasing when the tension is increased, resulting in an increase in the length of the chord. The sag is inversely proportional to the tension, and the alteration in the length follows the equation (5)

$$\lambda = \frac{1}{24}\frac{w^2l^3}{T^2}.$$

This is inversely proportional to the square of the tension. Hence the apparent elongation of a tape or wire in catenary under varying tension is very closely

$$\frac{lT}{E\sigma} - \frac{1}{24}\frac{w^2l^3}{T^2} \quad . \quad . \quad . \quad . \quad (6)$$

Taking a 24 m. tape of 1/8″ × 1/50″ in section as an example, the constants $l/E\sigma$ and $w^2l^3/24$ are 0·993 and 95·9 respectively expressed in mm., the weight per metre being 0·0129 kg. Some observations made at the National Physical Laboratory on such a tape (No. 16) gave the following results:

Tension kg. Weight.	Excess Length over 24 m.	Calculated from above Constants.	Observed Calculated.	By Least Squares.	
				Calculated.	O C.
	mm	mm.	mm.	mm.	mm
5	−6·14	−6·11	−0·03	−6·14	0·00
6·5	−3·06	−3·06	−0·00	−3·06	0·00
8	−0·79	−0·79	0·00	−0·79	0·00
10	1·73	(1·73)	..	1·74	−0·01
12	4·02	4·01	+0·01	4·02	0·00
13·5	5·67	5·63	+0·04	5·66	+0·01
15	7·24	7·22	+0·02	7·25	−0·01

The residuals given in column 4 show that the theoretical lengths calculated from the accepted length under a tension of 10 kg. weight closely follow the observed values, and this over the very large range from 5 kg. to 15 kg. Working up the same set of observations by least squares the excess of the lengths over 24 m. is given by the following:

$$\left(7·225 + 0·99936T - \frac{97·05}{T^2}\right) \text{mm.},$$

and the values and residuals calculated from this are given in the last two columns. The smallness of the residuals of the last column indicates that the observations are consistent to something better than one part in two millions, and that the constants (which were deduced from various tapes of this type) used in calculating the values of column 3 are not quite the best for this particular tape No. 16.

As a test in the straining apparatus in use at the Laboratory the following observations were made on the length of No. 16 under tensions differing by about 0·01 kg.

Tension.	Excess Lengths over 24 m.	Differences.	Calculated 1·185 δT	Observed − Calculated.
kg.	mm	mm.	mm.	mm.
10	1·730	..	..	..
10·01	1·740	0·010	0·012	−0·002
10·02	1·755	0·025	0·024	+0·001
10·035	1·773	0·043	0·041	+0·002
10·05	1·787	0·057	0·059	0·002

For small changes in tension we may differentiate equation (6) thus:

$$\frac{\Delta l}{\delta T} = \frac{l}{E\sigma} + \frac{1}{12}\frac{w^2l^3}{T^3} \quad . \quad . \quad . \quad . \quad (7)$$

Taking $l/E\sigma$ and $w^2l^3/24$ as 0·993 and 95·9 as before, and T being 10 kg.

$$\Delta l = 1·185\,\delta T,$$

and from this the calculated values given in the fourth column are deduced. The observations were made under microscopes, and the residuals indicate that the results are accurate to something better than one part in ten millions, and that the straining apparatus used (see § (11)) is satisfactory.

The first term of equation (7) shows that the greater the section the smaller will be the elastic elongation accompanying an increase in tension, and it follows that if the uncertainty of the tension is 0·02 kg. it will be necessary to have the section at least $0·02 \times 10^6/16000$, *i.e.* 1·25 sq. mm. in order that the uncertainty of the elongation may be within one part in a million.

---

[1] *La Mesure rapide des bases géodésiques.*

On the other hand, the section must not be too great, for the greater the section the greater the sag, and the greater will be the alterations in length (neglecting for the moment the elastic elongation) between the terminal graduations of the tape due to variations in the tension. Considering the second term of (7),

$$\frac{\Delta l}{\delta T}=\frac{1}{12}\frac{w^2l^3}{T^3},$$

is true, if we assume for the time being that the tape is inextensible.

From this,

$$\frac{\Delta l}{l}=\frac{1}{12}\left(\frac{wl}{T}\right)^2\frac{\delta T}{T};$$

If we keep $\Delta l/l$ within one part in two millions and $\delta T=0{\cdot}02$, whilst T is of the order of 10 kg., this equation gives $T/wl=20$ about, or T should be about 20 times the total weight (wl) of the tape. According to this rule, of the five examples given in the table below, the 48 m. wire and the 50 m. tape should be submitted to a stronger tension than 10 kg. weight. Twenty times the total weight of the 48 m. wire and of the 50 m. tape give 17 and 13 kg. weight respectively. The corrections of this table are, however, given for these examples since such tapes and wires are already in common use in the field.

The following table shows the numerical values of the effect of an increase of 0·01 kg. or 0·025 lb. in the tension in five examples of tapes and wires:

the tape itself in the same way as it will affect the force supplied by the straining masses.

The correction can be deduced from (7) above, and will be as follows:

$$\Delta l=\frac{l}{E\sigma}\left(\frac{g_2-g_1}{g}\right)T, \quad . \quad . \quad . \quad (8)$$

where g_2 and g_1 are the values of gravity at the base and at the standardising station respectively.

If, on the other hand, the tension is applied by means of spring dynamometers the effect of variations in gravity will affect the catenary curve, and not the elastic elongation. In this case, however, the greater the value of g the greater the sag, and hence the correction is opposite in sign from that considered above when the tension is applied by weights.

The effect in the length will be (see equation (7))

$$\Delta l=\frac{1}{12}\frac{w^2l^3}{T^3}\left(\frac{g_1-g_2}{g}\right)T. \quad . \quad . \quad (9)$$

For example, a 24 m. tape, section $\frac{1}{8}''\times\frac{1}{50}''$, tension 10 kg., standardised in London, will be shorter in Johannesburg, the tension being applied by the same masses at the two places. Here $g_1=981{\cdot}19$ and $g_2=978{\cdot}49$, and the correction to the standardised value, as given by (8), is $-0{\cdot}027$ mm.

On the other hand, if the same tape is used with spring dynamometers the correction as given by (9) is $+0{\cdot}005$ mm.

TABLE I

INCREASE IN LENGTH ACCOMPANYING INCREASE IN TENSION

	Tension.	Section.	Variation for 0·01 kg.		
			Elastic.	Curve.	Total.
			mm.	mm.	mm.
24 m. Wire	10 kg.	2·138 sq. mm.	0·0070	0·0034	0·0104
48 m. Wire	10 ,,	2·138 sq. mm.	0·0140	0·0276	0·0416
24 m. Tape	10 ,,	1/8" · 1/50"	0·0099	0·0019	0·0118
50 m. Tape	10 ,,	1/8" · 1/50"	0·0207	0·0171	0·0378
			Variation per 0·025 lb.		
			ft.	ft.	ft.
100 ft. Tape	20 lbs.	1/8" · 1/50"	0·000047	0·000019	0·000066

The values for the 24 m. and 48 m. wires are taken from *La Mesure rapide des bases géodésiques*, Benoit et Guillaume.

§ (4) THE EFFECT OF VARIATIONS IN g.—If a tape or wire is to be used in a country in which the value of g is different from that obtaining in the place at which it has been standardised, a correction has to be applied assuming that the same masses are used in each case for the purposes of straining the tape. The variations in g will, in this case, only affect the elastic elongation, and will not cause any variation in the catenary curve, since the change in g will affect the weight of

§ (5) EFFECT OF TEMPERATURE.—Owing to the difficulty of determining the temperature of tapes or wires in the field, steel tapes, which have a coefficient of thermal expansion of 0·000011 per 1° C., cannot be used to the high precision required in base measurements. Consequently invar tapes or wires are now almost universally used. The coefficient of thermal expansion of invar tapes of the best grade is about one-fifteenth of that of steel, whilst that of invar wires is often so small that it is practically negligible.

For a tape with a coefficient of $0{\cdot}7 \times 10^{-6}$ for 1° C. an error of 1·4° C. in the estimation of the atmospheric temperature to which the tape is submitted will result in an error of one part in a million in the length. In the laboratory, the temperature can be determined to 0·2° C. with fair certitude, and in the field an error of 1·4° C. should be unusual under reasonable conditions.

§ (6) Secular Change in Length.—Invar tapes or wires are liable to secular change of the same order as that already discussed for invar bars, and this can be reduced by suitable heat treatment as mentioned above.[1]

Some tapes belonging to the National Physical Laboratory were artificially aged in 1908. They are, however, still increasing in length at a rate of about 0·6 part in a million per annum.[2]

§ (7) Effect of coiling Invar Tapes and Wires.—It is important that the diameter of the drums should be large enough to prevent any permanent alteration in the lengths as the tapes or wires are repeatedly coiled and uncoiled. Benoit and Guillaume[3] recommend the use of drums having a diameter of 50 cm., and show that after the first few times they are coiled the lengths of wires are not further affected by continual coiling on drums of this size. These drums are of metal, and are of special design so that the "reglettes" which bear the graduations can be satisfactorily housed without fear of damage. The standard tapes of the National Physical Laboratory are coiled on drums made of three-ply wood and turned to a diameter of at least 50 cm. (see *Fig.* 2). The tapes are finished with eyelets riveted to the tape as shown, and the eyelet at one end slips over the pin P, one side of the drum being cut away to facilitate this. The width of the drum is only slightly greater than the width of the tape so that the tape coils on itself. The figure shows a light type of drum suitable for use in the field requiring no coiling apparatus. The disc D runs freely on a pivot; the operator inserts one hand under the strap S and grips the disc, whilst the coiling is done with the aid of the handle H. The coiling of a tape on these repeated over forty times has not caused any observable variation in the length of the tape.

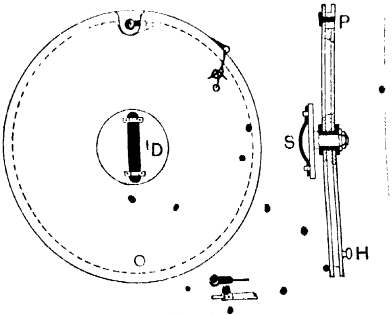


Fig. 2.

§ (8) Standardisation. The 50 m. Base at the National Physical Laboratory.—This is a mural bench connected to the wall at the back of the main rooms in the Metrology Building, and was built in 1908. Special apparatus installed by the Cambridge Scientific Instrument Company in 1909 enables the bench to be used as a large comparator. The bench is 4 ft. 8 in. above floor level of the corridor in which it is situated. It was placed at this height because it may, at some future time, be increased to 100 metres in length when this height may be required to accommodate wires in catenary with a large sag. The bench is built of brick covered with polished slate 13 in. in width. At intervals of every 4 m., and at various important points, such as 50 ft., 66 ft., 25 m., 100 ft., etc., blocks of Portland stone have been built into the brickwork, and these bear bench marks on nickel-steel blocks let into the stone. The upper surfaces of the nickel-steel blocks have been carefully levelled and are all in one horizontal plane to within 0·6 mm. Each of these surfaces has been graduated with a series of lines, a millimetre apart, set at right angles to the length of the bench, and one of these lines, specially marked with a centre dot, is the defining line of that particular length of the bench measured from the zero mark. The edges of the invar blocks have been also carefully aligned so that they are in one vertical plane to within 0·8 mm. The 4 m. lengths are determined by comparison with a 4 m. standard bar by means of apparatus described below, and as these measured lengths are finally added together to give the long lengths of the bench, it is important that the levelling and alignment should be done to a sufficient accuracy to prevent any error accruing from this cause. The bench marks are, however, very satisfactory, for if each block were 0·6 mm. out of line in both planes, the resulting error in length would not exceed one part in ten millions in the worst conceivable case.

§ (9) Measuring Apparatus.—This consists of two microscope carriages, each supplied with two microscopes and a telescope, a carriage for the 4 m. bar, a railway running throughout the length of the bench to which the three carriages can be gripped at any desired position, and two collimators, one fixed at each end of the bench.

[1] See article "Line Standards of Length," § (5).
[2] See article "Invar and Elinvar," Vol. V.
[3] *La Mesure rapide des bases géodésiques.*

A general view of one end of the bench is given in *Fig.* 3, and here can be seen the rails, one of the carriages, and one of the collimators. The rails are in 13 ft. lengths of steel rod $1\frac{1}{4}''$ in diameter, and are supported on cast-iron sleepers embedded in the slate. The sleepers are $6\frac{1}{2}$ ft. apart, and each supports the rails vertically by means of adjusting screws. Every second sleeper is large enough to support the ends of the rods where they are nearly butted together, each rod being supported by means of three screws, one underneath supporting it vertically, and two acting in opposite directions controlling the alignment horizontally and vertically. The front rail has to be carefully aligned as the carriages grip this rail, whilst they only slide on the back rail. Consequently the latter only needs lining up horizontally. Each microscope carriage is made in two parts, the lower resting on the rails. The upper part, to which are connected the microscopes and the telescope, is pivoted to the lower part at a point *c* (*Fig.* 4), and is supplied with micrometer adjustments *a* and *b*; the horizontal screw *a* giving a small rotation with reference to the lower part, whilst the vertical screw *b* gives the levelling adjustment.

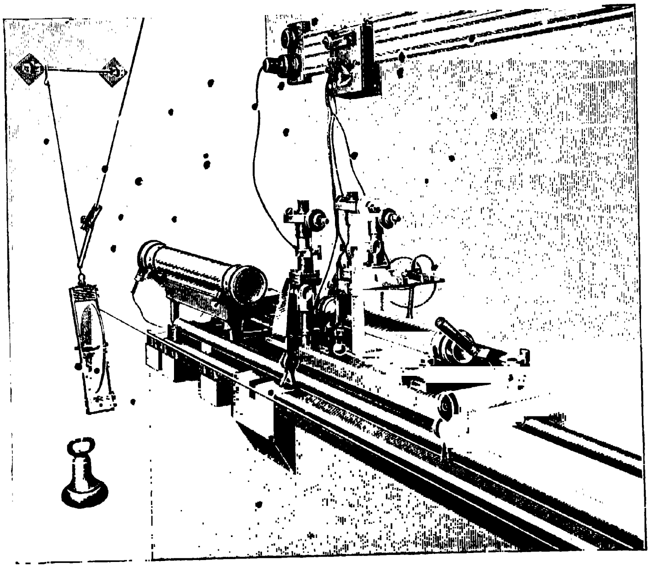


FIG. 3.

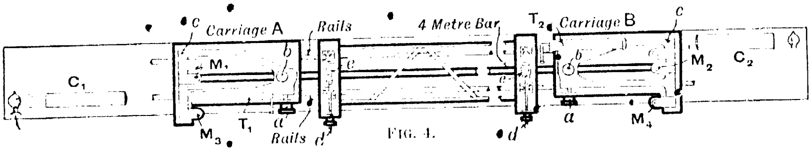


FIG. 4.

A collimator (C_1) near the front of the bench and outside the 0 mark controls the adjustment of the carriage A, whilst collimator C_2, near the back of the bench and set beyond the 50 m. mark, controls the setting of carriage B. The collimator at each end gives a beam of parallel light throughout the length of the bench, and the carriage is adjusted by means of the micrometer screws until the image of the cross-wires of the collimator is brought into coincidence with the cross-wires of the telescope (T_1). The measurement of a 4 m. length of the bench by comparison with a 4 m. standard bar is illustrated in the accompanying diagrammatical sketch (*Fig.* 4). The carriage for the 4 m. bar rests between the two microscope carriages, and the 4 m. bar is supported on rollers in two stirrups which are placed at the Airy points. The position of the bar and the focussing to the microscopes M_1 and M_2 are controlled by the adjustment screws *d* and *e*.

The carriages being collimated and the 4 m bar focussed correctly, readings are made with the four micrometer microscopes on the lines on the bar and bench marks. The carriages A and B are then interchanged, reset with reference to their respective collimators, the 4 m. bar refocussed and microscope readings again made. The optical and mechanical magnification of the microscopes is such that one division (1·5 mm. in length) on the micrometer drum represents 2 μ (0·002 mm.), and by estimation readings can be made to 0·2 μ. The two microscopes of a carriage are only approximately set up, so that the line joining the focal points is square to the length of the bench; hence the four microscopes in the first setting may be placed in plan as indicated below (*Fig.* 5A).

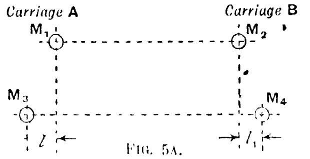


FIG. 5A.

The readings of the microscopes m_1, m_2, m_3, and m_4 will give the following equation for the length L between the two defining lines of the 4 m. step of the bench:

$$\text{L} = 4\text{ m. bar} + (l + l_1) + (m_3 + m_2) - (m_1 + m_4).$$

After interchanging the carriages (*Fig.* 5B), however, the positions of M_1 and M_3 in the second setting are parallel to the positions of the same microscopes in the first setting if the collimating is done accurately, and similarly in the case of microscopes M_2 and M_4. Hence from the second setting the following is obtained:

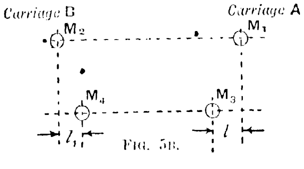


FIG. 5B.

$$\text{L} = 4\text{ m. bar} - (l + l_1) + (m_1' + m_4') - (m_3' + m_2').$$

In taking the mean of these two equations, the unknown $l + l_1$ are eliminated, and the differences between the length of the 4 m. step in the bench under standardisation and the length of the 4 m. bar can be directly expressed in terms of the eight microscope readings.

The carriages are then all moved on to the next step, and the operations repeated.

The value of $l + l_1$ can be determined in each step by the difference between the two observational equations given above, and the agreement in the values obtained in the successive steps gives a valuable check on the accuracy of the collimation. It is found that this can be done to ± 2 μ in a 4 m. length, hence the probable error in one determination of a 24 m. length will not exceed $2\sqrt{6} = 5\ \mu$ due to this cause.

§ (10) TAPE MEASUREMENT.—Since the bench is continually changing its length, the determined length is compared with one or more standard tapes or wires used in catenary suspension, immediately before and after the measurements are made with the 4 m. bar. This is done under microscopes, the field of the microscopes being large enough to give a good view both of the bench mark and also of the graduations on the tape. The tape is suspended just in front of the bench, the standard tension being applied by dead weights, the straining wires or cords running over ball-bearing pulleys, which are placed one at each end beyond the bench. One of these pulleys is seen in the view of one end of the bench (*Fig.* 3). If the full 50 m. length of the base is not in use, straining wires which are supported over ball races at intervals, are connected to the tape. The tension is thus applied horizontally, and the laboratory standard wires and tapes are standardised and used in this way. If, however, the tapes or wires are for use in the field where the tension is generally applied tangentially to the catenary curve, a small correction has to be applied.

From (1) and (2)

$$\text{T} - \text{T}_0 = w(y - c),$$

$$= \frac{1}{2}\frac{w^2x^2}{\text{T}_0} \text{ from (4).}$$

In a 48 m. wire of the ordinary type 1·65 mm. in diameter used under 10 kg. weight x becomes 24 and the value of $\text{T} - \text{T}_0$ is 0·0086 kg. The effect of this on the length can be deduced from equation (7), or more quickly from the second line of Table I above, from whence the correction is $0{\cdot}0086/0{\cdot}01 \times 0{\cdot}0416 = 0{\cdot}036$ mm. Since the tangential tension is greater than the horizontal tension, the length of this wire used under 10 kg. tangential tension will be 0·036 mm. shorter than it would be when standardised under 10 kg. horizontal tension. For a 24 m. wire the correction will be only 0·002 mm., which is practically a negligible amount.

The above method of building up a length on the bench in 4 m. steps and transferring it to standard tapes is somewhat slow, requiring some eight or nine hours' continual observations to complete. The following method of determining the length of a standard tape in a series of 4 m. catenaries by direct comparison with the 4 m. line standard is more reliable and is quicker, taking less than two hours. For this purpose four 24 m. invar tapes have been graduated at the Laboratory every 4 metres, the graduations at each point consisting of 11 lines in 5 mm. length, the lines being spaced 0·5 mm. apart. The tape under measurement and strained under a tension of 10 kg. weight, is supported on ball-bearing rollers at every 4 m. above the level of the bench so that it lies in six 4 m. catenaries, the rollers all being first carefully levelled and aligned. One 4 m. step under observation passes over the 4 m. bar, and is then supported over two small rollers placed on the neutral plane of the bar. The microscope carriages are placed with the microscopes in the central holders over the points 0 and 4 m. of the standard bar, and are locked to the rails. The bar is focussed, and readings made on the lines 0 and 4 m., the tape being temporarily raised above the bar and moved slightly on one side so that the lines of the bar can be viewed. The tape is replaced, and the bar is lowered until the graduations on the step of the tape under observation are focussed. Readings are then made on each pair of lines nominally 4 m. apart, and the mean of the deduced values is taken as the length of the particular length concerned.

The temperature of the bar is taken by means of two thermometers that lie in the channel beside the graduated surface of the line standard, and the temperature of the tape is assumed to be the same. The probable error of this assumption is small, because the length of the tape under measurement lies within the groove of the H form of the standard bar. The relative coefficient of expansion of the standard bar and tape is only $0{\cdot}4 \times 10^{-6}$ per 1° C., and that of the tape is $0{\cdot}7 \times 10^{-6}$ per 1° C. Hence if the real temperature of the tape differed by 0·1° C. from that of the bar, and also differed 0·1° C. from the assumed temperature, the error in the accepted length of the tape would not exceed 1 part in nine millions.

The straining wires are made in 4 m. lengths and one of them is detached, the 24 m. tape is drawn along until the next 4 m. length of the tape is under the microscopes; the 4 m. length of straining wire is connected up to the other end of the tape, the tension is applied, and all is ready for the measurement of the next step, and the operations are repeated until all the six steps are measured.

In this case there is no necessity for the collimation of the carriages, since both the standard bar and the tape are viewed under the same pair of microscopes. All four tapes have been measured in this way at intervals, and the results indicate that the determination of one tape is reliable to 1 part in two millions.

So that these tapes can be used as standards from which others can be compared, ball-bearing rollers have been placed in front of each bench mark, so that the tape is supported at each 4 m. length at the level of the bench, that is, the tapes are used in the same way as they have been compared with the 4 m. bar.

Tapes or wires which will be used in the ordinary way for surveying purposes are determined in simple catenary suspension by comparison with standard tapes of the same nominal length. In this case the bench is used simply as a comparator, the known and unknown tapes being compared either with the bench marks by eye, or by taking readings under the microscopes.

§ (11) Application of the Tension.—The tension is applied by dead weights acting on straining cords, or wires over two pulleys one at each end. The effect of the friction of the pulleys and the straining cords has to be eliminated when taking observations on a tape or wire. This is done by first pulling the tape until the desired lines are under the microscopes and readings made. The actual tension on the tape is slightly greater than that given by the dead weight by the force required to overcome friction. The tape is pulled a short distance, and is then pushed back until the lines are again under the microscopes. In this case the tension on the tape is less than that given by the dead weight by the force required to overcome friction. The mean of the two observations gives the length of the tape as compared with the distance between the microscopes under the tension given by the weights.

This method is used in all readings made on tapes, whether the latter are used in catenary suspension, or when supported on the flat.

At times it is more convenient to anchor the tape at one end, using a dead weight over a pulley at the other end as a means of applying the tension required, instead of the pulleys at each end. It is necessary, however, that the observations should be made under the conditions, described above, imposed for the elimination of friction. The two methods give identical results within the errors of observation.

The pulleys must be well balanced, and the turned surface must be truly circular and should be tested from time to time.

§ (12) Thermal and Secular Change of the 50 m. Bench at the National Physical Laboratory.—The lengths of the bench are continually, if slowly, changing under variations in temperatures, and possibly under change in the humidity of the atmosphere. There is also a secular change due to the tendency of the wall to flatten out under the force of gravity. As a result of this the bench lengthens.

The thermal and secular change can be seen in the upper part of the diagram (*Fig.* 6), in which the variations of the 100 ft. length from mark 50 ft. to mark 150 ft. are plotted. The temperature of the bench at the time of each measurement is also shown in this diagram, and the similarity in the two upper curves plainly indicates the connection between the variations in length and temperature.

In the lower part of the diagram a full-line curve is drawn evenly through plotted points. These points represent the upper curve after the probable variations in length, due to changes in the surface temperature, have been subtracted from the observed lengths; the amount subtracted being based on a value for the superficial expansion of the bench with surface-temperature changes, deduced from actual observations over some four years. The plotted points seldom leave the full-line curve more than 0·0002 ft. or 0·0003 ft., *i.e.* 2 or 3 parts in a million, but the curve is still somewhat irregular. This is mainly due to the summer effects, which may be attributed to general temperature changes in the body of the masonry as distinct from the surface variations, for which allowance has been made in drawing the full-line curve. Winter effects are not noticeable because the corridor is artificially warmed in the winter months to standard temperature, from which large departures are found only in hot summers.

The dotted smooth curve is drawn through the adjusted observations made at or near normal temperature, neglecting those obtained during any hot spells of weather. This curve gives approximately the secular growth of the base apart from temperature changes, and the rate of the growth amounts to 7 parts in a million per annum.

The over-all length of the bench 0 to 50 metres has increased by 2·3 mm., or 45 parts in a million during six years, and this gives an average rate of growth of between 7 and 8 parts in a million per annum, which agrees reasonably with that obtained in the length from 50 ft. to 150 ft. marks.

Owing to the pressure of war work undertaken by the Laboratory, no observations were made on the bench during 1915 to 1919, and this accounts for the gap in the diagram.

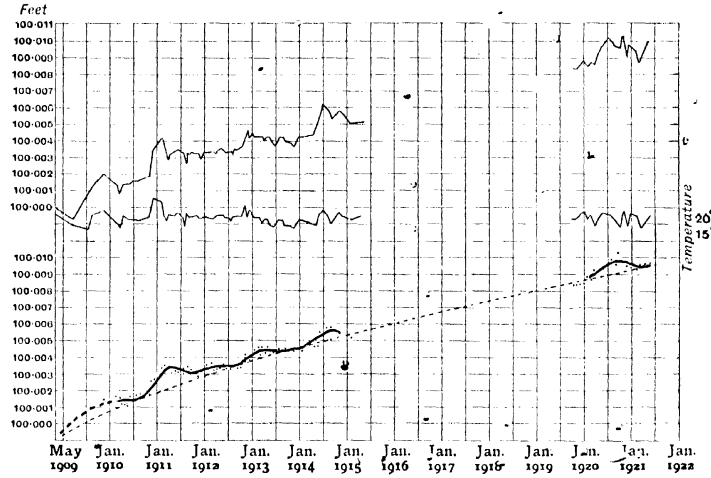


Fig. 6.

§ (13) Tapes used on the Flat.—As already mentioned, tapes supported on the flat cannot generally be used in the field to the highest accuracy owing to the irregularity of the ground, and if these tapes are made in steel there is also the difficulty of determining their temperatures, with resulting errors in the measurements obtained by their use. The thermal coefficient of expansion of steel tapes is about 0·000011 per 1° C., which indicates that the temperature must be known to a higher accuracy than 1° C., so that the error of measurement will not exceed 1 part in 100,000. It is, however, difficult to determine the temperature of the tape in the field to this accuracy, hence such tapes are not usually standardised to an accuracy greater than 1 part in 100,000. By taking precautions as to temperature, however, laboratory standard steel tapes are determined to an accuracy of the order of 1 part in a million. These are subdivided tapes, and have been calibrated, and unknown tapes are standardised by comparison with these on the 50 m. bench.

The calibration of the subdivisions of a standard divided tape is done in practically the same way in which a divided line standard is calibrated in a subdividing comparator.[1] The microscope carriages are placed a distance apart which approximately corresponds with, say, one-tenth of the total length of the tape, and these ten subdivisions are compared by bringing each successive one-tenth part of the tape for observation under the microscopes. By continuing the process in the same way as that described for a line standard, a subdivided length can be determined in terms of the over-all length of the tape. Provided that the length of the tape is less than the one-half the total length of the bench, the operation of calibration can be done fairly quickly by having the straining wire made up with suitable lengths, so that the various lengths of the tape can be viewed under the microscopes.

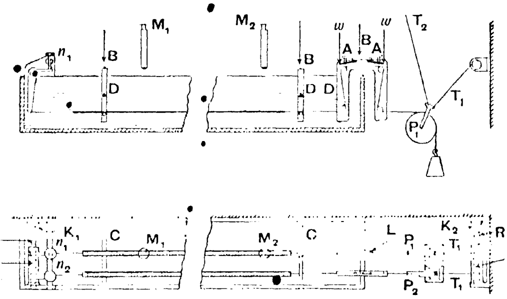


FIG. 7.

§ (14) THERMAL COEFFICIENT OF EXPANSION.—In the same corridor as the 50 m. bench but against the opposite long wall (south) a 50 m. tank has been built. This has been designed to give the relative coefficient of expansion of two tapes or wires.

The tapes or wires under observation are submerged in water which is applied hot or at ordinary temperatures. To obtain low temperatures ice is placed in the tank itself. The tank is suitably lagged, and is covered throughout with removable wooden blocks 2 in. thick and 6 in. in width. The water is stirred by a centrifugal pump, which draws water from either or both ends of the tank and discharges it at a point near the centre under a baffle plate. The temperature is taken by thermometers placed at intervals in the water, and after 15 or 20 minutes' stirring, the temperature is even throughout to within 0·1° C. about.

Microscope holders have been built into the wall at suitable points such as 0, 10 m., 50 ft., 20 m., 66 ft., 24 m., 150 ft., 50 m., so that any length in general use can be dealt with.

The tapes or wires are anchored at one end to bell-crank levers connected to a cast-iron support which can be secured to the tank at suitable points. The other ends are connected to straining wires which, after going round pulleys in two independent inverted U-pieces bridging the end of the tank, pass over ball-bearing pulleys and are attached to weights. The arrangement is shown diagrammatically in *Fig.* 7. Each of the two U-pieces is supported by two woven copper cords *w* connected to a casting bolted to beams 8 ft. above the tank. They are provided with levels (L) and plumb-bobs connected to the overhead casting, and to these the U-pieces are adjusted before any readings are made. By this means no component force, due to the weight of the U-pieces, is brought to bear on the tapes.

Since the catenary curve would cause the tape to foul the bottom of the tank, the tape is supported, at intervals of about 6 metres, by transverse supports (C) that are connected by cords to the overhead beams, and the position of each of these supports is controlled by plumb-bobs (B) so that no force (due to the weight of the supports) in the direction of the length of the tape is exerted. Points (D) on the supports and on the U-pieces are all brought to water level by levelling screws (such as A on the U-pieces) acting on levers to which the supporting cords are attached.

Adjusting screws (n_1 and n_2) acting on the horizontal arm of each of the bell-crank levers give a small adjustment to the longitudinal position of the tape.

---

[1] See article "Line Standards of Length," § (12).

In the diagram the far tape is shown in position under the two fixed micrometer microscopes (M_1, M_2). By means of the screw n_1, the tape is pulled until the lines under observation are brought into the field of the microscopes. Readings are then made. The tape is pulled further, the screw is then unscrewed until the line is again in position, and readings repeated. This operation is always done to eliminate any friction that may exist in the pulley (P_1). The two tapes are then forced towards the back of the tank, so that the second tape can be viewed under the microscopes. This is done by three movements. The bell-crank levers are connected to a sleeve (S_1) that slides on a spindle (R) fixed to the casting. The tie-bars (T_1) are attached to a similar sleeve (S_2) sliding on a bar (R_2) bolted to a transverse wall at the end of the corridor. The other pair of tie-bars (T_2) that support the pulleys (P_1, P_2), the supporting cords, and plumb-bob cords of the U-pieces are all connected to a third sleeve sliding on a bar fixed to the overhead casting, not seen in the diagram. All three sleeves are moved to stops, of which two (K_1 and K_2) are shown, and the second tape is then ready for observation.

Observations made alternately on the two tapes are repeated three or four times, and the difference in the mean readings on each tape gives the difference in the lengths of the tapes at the temperature of the water in the tank.

The water is replaced by water of a different temperature, and similar observations made on the tapes. It is usual to take observations at five or six temperatures ranging from 1° to 32° C.

§ (15) DETERMINATION OF THE ABSOLUTE COEFFICIENT OF EXPANSION.—The coefficient of expansion of the laboratory standard 50 m. tape, No. 13, has been determined absolutely, and from this the coefficients of other tape or wires have been obtained by the method described above.

If the distance between the focal points of the micrometer microscopes could be relied on not to change, it would only be necessary to take observations on a tape at different temperatures. Owing, however, to the time required in changing the water and stirring, etc., a suitable series of observations takes two days. Moreover, the south wall to which the microscopes are fixed is an outer wall, and is submitted to some considerable variations in temperature, especially on sunny days. Consequently it is necessary to correct the readings for any movement of the wall that may occur during the time required for the observations. This movement affects the measurements on the tape in two ways, by the longitudinal expansion or contraction of the wall, and by rotation of the microscope holder about an axis transverse to the length of the tape.

Tape No. 13 was tested in 24 m. lengths, and the longitudinal expansion of the wall was measured by stringing an invar 24 m. wire (1·65 mm. diameter) in catenary suspension to the microscope holders themselves, and the corrections for this movement were deduced from measurements made on the sag of the wire at the time observations were made on the tape. The wire was entirely above the tank, and only exposed to the changes in the air temperature of the corridor.

The wire was previously examined so that the corrections could be obtained from variations in the sag. This was done on the bench by observations made on the variations in the lengths and in the sag of wire under various tensions. The ratio dy/dl of the change in the sag to the change in the chord length was determined from the observations, and is plotted against the measured chord lengths of the wire in the graph (*Fig.* 8). This shows that at a tension of about 6·5 kg. $dy/dl = 15$ about, that is to say, that under this condition the measured variation in the sag is 15 times the correction to be applied to the distance between the microscopes.

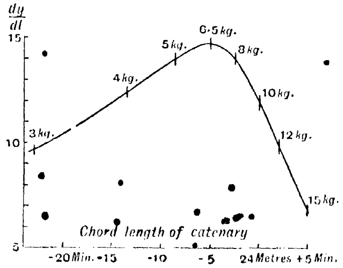


FIG. 8.

This operation can be treated mathematically as follows:

From (4)

$$y = \frac{1}{2}\frac{w}{T}x^2 \text{ (the other terms being negligible),}$$

or

$$\frac{dy}{dT} = -\frac{f}{T^2},$$

where $f = \frac{1}{8}wl^2$.

From (6)

$$l = k + \frac{lT}{E\sigma} - \frac{1}{24}\frac{w^2l^3}{T^2}, \text{ where } k \text{ is a constant,}$$

and since the second and third terms of the right side of the equation are small, we may treat l as a constant.

$$\frac{dl}{d\mathrm{T}} = a + \frac{2b}{\mathrm{T}^3},$$

where $a = l/\mathrm{E}\sigma$, and $b = \frac{1}{24} w^2 l^3$.

$$\therefore \frac{dl}{dy} = -\frac{a\mathrm{T}^2}{f} - \frac{2b}{f\mathrm{T}}.$$

Differentiating again, and equating the result with 0, dl/dy is a minimum when

$$\mathrm{T} = \sqrt[3]{\frac{b}{a}},$$

and substituting this in the last equation,

$$\frac{dy}{dl} = -\frac{f}{3a^{\frac{1}{3}}b^{\frac{2}{3}}} \text{ is at a maximum.}$$

The constants a, b, and f can be evaluated from the constants of the wire, viz. $l = 24$ metres; $\mathrm{E}_1 = 16{,}000$ kg. per sq. mm.; $w = 0{\cdot}01732$ kg. per metre; $\sigma = (\pi/4)1{\cdot}65^2$ sq. mm.; and from these it is found that at a tension of 6·45 kg. dy/dl has a maximum value of 15·1. This value confirms the practical result given in the graph (*Fig.* 8).

Having determined the connection between the sag of the wire and its length by this means, the wire was then fixed to the microscope holders at a certain tension, and suitable means of measuring the sag at any moment were arranged. The latter was done by fixing a cast-iron bracket to the wall, to which a micrometer working vertically below the wire was fitted so that the position of the wire could be measured at the lowest point of the curve. Observations were made with the micrometer at the same time that microscope readings were obtained on the graduations at each end of the tape, and these micrometer observations supplied the information by means of which the amount of lateral movement of the wall between the microscopes could be determined and allowed for. For example, observations were made on the length of the tape as compared with the distance (D_1) between the microscopes then existing, by taking microscope readings on the graduations of the tape, whilst readings were also made on the position of the wire at the lowest point of the curve, the temperature of the water in the tank and of the tape being θ_1. The water temperature was then increased to θ_2 and similar micrometer and microscope observations obtained, the distance between the microscopes being D_2. If the difference between the two micrometer readings indicated that the sag of the wire decreased by n mm., the amount of the elongation of the wall would be $n/15$ mm. The observational equations deduced from these two sets of readings may be written as follows:

$$\text{Length of tape at } \theta_1 = \mathrm{D}_1 + d_1,$$

$$\text{Length of tape at } \theta_2 = \mathrm{D}_2 + d_2 = \mathrm{D}_1 + \frac{n}{15} + d_2,$$

d_1 and d_2 being the microscope readings. Similarly the observations at various temperatures were corrected so that the relative lengths of the tape were compared in each case with the original distance D_1, and thus the lateral movement of the wall was allowed for. Small corrections also had to be applied for the change in the sag of the wire due to change in its length accompanying any change in the air temperature of the corridor. Since the coefficient of expansion of the wire was very small (less than $0{\cdot}11 \times 10^{-6}$ per 1° C.), and the range of the air temperature was less than 3° C., the corrections for this were nearly negligible.

It was necessary, however, to allow also for any rotational movement of the microscopes. This was done by fixing a fine wire at right angles to the length of the tape to each of the microscope objectives, and observing the images of these wires in two flat vessels containing mercury which were temporarily placed half-way between the wires and the focal planes of the microscopes. The position of the wire in each case was read by means of the micrometer of the microscope. These observations were made on both microscopes immediately before and after the readings were made on the tape and on the sag of the invar wire at each temperature to which the tape was submitted. If the axis of one of the microscopes rotated between two sets of observations, it is obvious that the displacement would be detected by an alteration in the reading made on the image of the wire. The corrections were obtained by calculation from the dimensions and known constants of the microscopes. As a matter of fact the movements of the wall were found to be almost entirely confined to lateral movements.

The coefficient of expansion of a 24 m. length of the tape No. 13 was determined twice by this method. During the first set of observations the sky was overcast and the atmospheric temperature did not greatly vary, whereas in the second set the sun was shining on the outer surface of the wall to which the microscopes were fixed. In the latter case the corrections for the lateral and angular movements were fairly large, as can be seen in the diagram (*Fig.* 9). In this diagram the apparent variations in the length of the tape under alterations in the temperature of the water in the tank are plotted against the temperature to which the tape is submitted. This gives the irregular curve. On adding the corrections thus giving the true length of the tape at each observation, the plotted points give the nearly straight line curve from which the true coefficient of the tape can be calculated. The coefficient was finally calculated by the method of least squares, and the β term was found to be negligible. The final results were:

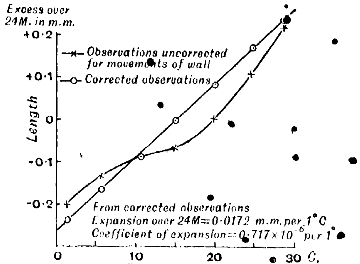


FIG. 9.

Mean coefficient No. 13, 0/24 m.	$0{\cdot}717_1 \times 10^{-6}$ per 1° C.
Mean coefficient No. 13, 26/50 m.	$0{\cdot}716_7 \times 10^{-6}$,,
Mean coefficient No. 13, 26/50 m. (repeat). .	$0{\cdot}717_1 \times 10^{-6}$,,

Hence the accepted value of the thermal coefficient of expansion of this tape has been taken as $0{\cdot}717 \times 10^{-6}$ per 1° C.

§ (16) 24-METRE COMPARATOR FOR THE INDIAN GOVERNMENT.—This comparator[1] has been designed under the supervision of the late Sir David Gill. It consists of a wall built in a corridor 110 ft. long by 10 ft. wide, and of a suitable apparatus made by the Cambridge Scientific Instrument Co.

The fundamental part of the apparatus contains seven cast-iron holders bolted to castings built into the wall. These holders are placed at intervals of 4 metres and each carries a micrometer microscope. The focal points of the seven microscopes are first arranged to be all in the same horizontal plane by focussing each pair in turn on dust particles floating on water in one of two troughs having free pipe connection. The troughs are moved along under the microscopes, one being always under a microscope already focussed, and the other microscope is then adjusted to the same focus level. The microscopes are set in the vertical plane by setting them to a stretched fine wire.

A double set of rails runs throughout the length of the wall, and is arranged so that a carriage, suitably supporting an invar 4 m. bar, can be brought into such a position that the fiducial lines 0 and 4 metre of the bar may be viewed under any consecutive pair of microscopes.

The actual distances between one microscope and the next can be thus determined from the 4 m. bar, and finally, by addition, the distance between the two end microscopes obtained. The carriage for the line standard is then run on to a 4 m. extension of the wall which acts as a siding for the carriage when the bar is not actually in use.

The tape whose length is required is then placed in position, being supported by straining wires or cords over two pulleys at the ends of the main part of the wall (about 26·5 metres in length), the tension being applied by two weights, usually 10 kg. each. There is a small space between the main wall and the siding allowing room for the weight. The pulley at this end, which has to be dismantled for the free passage of the carriage to the siding, is designed so that it can be easily and quickly replaced. The height of the pulleys above the surface of the wall is such that the tape is free to hang in a catenary curve, and the pulleys are supplied with adjusting screws so that the lines of the tape can be focussed under the end microscopes. Readings on the lines of the tape are then made, and from these the length of the tape is deduced.

The apparatus also includes a 24 m. tank which lies behind the wall, and which is used for determining the thermal coefficient of expansion of tapes. The water in the tank is connected to a circulating pump and passes through heating apparatus, so that the temperature can be quickly changed. Variations in the length of the tape under observation at different temperatures are obtained by comparing it with an invar tape at nearly constant temperature in air. The latter tape is in catenary suspension under the microscope of the end cast-iron supports 24 m. apart, as described above. Subsidiary microscopes are attached to the same castings, and these are used for observations on the tape in the tank.

Owing, however, to the effect of the heat radiating from the tank, the wall is liable to some movement during the series of observations required. A correction for the longitudinal expansion or contraction of the wall is afforded by the observation in the tape suspended in air, but further measurements have to be undertaken to correct for the possible rotational movement of the castings. A collimator is rigidly attached to the casting at one end, whilst a telescope with vertical and horizontal micrometer scales is fixed to the other casting, the axes of both instruments being parallel to the axis of the comparator, and the relative rotation of the two castings can be deduced from the readings in the telescope.

S. W. A.

SYNOPTIC CHARTS. See article "Atmosphere, Physics of," § (18).

---

[1] For more complete account see *Engineering*, Sept. 3, 1915.

— T —

TABULATING AND SORTING ENGINES. See "Calculating Machines," § (12).

TEMPERATURE, ATMOSPHERIC, linear law of vertical gradient of, as the basis of Toussaint's exponential altimeter formula. The law is

$$T = 288^\circ - 0{\cdot}0065z,$$

T being the temperature in degrees absolute at height, z metres, the temperature at sea-level being taken as 288° abs. See "Barometers and Manometers," § (17).

Conditions for low values at the surface at night. See "Radiation," § (2) (ii.).

Correlation with sun-spots. See *ibid.* § (1).

Discontinuity of, at coast-line. See "Atmosphere, Thermodynamics of the," § (5), *Figs.* 1-4.

Distribution of:

In cyclone and anticyclone. See *ibid.* §§ (5), (8), Table III.

In the upper air. See *ibid.* §§ (4) and (5), Table II.

Over the globe. See *ibid.* §§ (3), (5), (8), (10); *Figs.* 1-4.

Diurnal variation of. See "Atmosphere, Physics of," § (13).

Effect of radiation on. See "Radiation," § (3) (iv.).

Equivalent radiative. See *ibid.* §§ (2) (ii.) and 4 (ii.).

Fiducial, for a millibar barometer: the temperature at which the instrument reads true millibars under the conditions of gravity at its given station; obtained from the standard temperature by applying a small correction. See "Barometers and Manometers," § (6) (iii.).

Instruments for measurement of:

(i.) At the surface. See "Meteorological Instruments," II. § (4) *et seq.*

(ii.) In the upper air. See *ibid.* VIII. §§ 36-38.

Inversions of. See "Atmosphere, Thermodynamics of the," §§ (5) and (7).

Lapse of:

Adiabatic. See "Atmosphere, Physics of," §§ (3), (6).

Variation with height. See *ibid.* § (5).

Lapse-rate of:

Effect of water-vapour on. See *ibid.* § (6).

Normal value of. See *ibid.* § (5).

Value of, in radiative equilibrium. See *ibid.* § (11).

Mean value for different latitudes. See "Radiation," § (4) (ii.).

Observed vertical distribution of. See "Atmosphere, Physics of," § (5).

Range of (annual). See "Atmosphere, Thermodynamics of the," § (5).

Range of diurnal. See *ibid.* § (10).

Relation of, to pressure in dry and saturated air. See "Atmosphere, Thermodynamics of the," § (18) *et seq.*

Relation of, to pressure and entropy. See *ibid.* §§ (19), (23), and *Fig.* 16.

Relation of, to pressure and height. See *ibid.* § (8). See also "Air, Investigation of the Upper," § (11).

Scale of. See *ibid.* § (2) and *Figs.* 1-4 (table of equivalents).

Standard, for a millibar barometer: the temperature at which the instrument registers true millibars when stationed under standard conditions of gravity. See "Barometers and Manometers," § (6) (iii.).

For hydrometers. See "Hydrometers," § (6).

Theoretical vertical distribution in conductive, radiative, and convective equilibrium. See "Atmosphere, Physics of," § (6).

Variation of, with height:

For saturated adiabatics. See "Atmosphere, Thermodynamics of," § (22), *Fig.* 6. See also "Potential Temperature."

In radiative equilibrium. See "Atmosphere, Thermodynamics of the," §§ (11), (12), *Fig.* 14.

Vertical distribution of. See "Radiation," § (3) (i.).

TEMPERATURE OF ADJUSTMENT, suitable for industrial standards of length, gauges, etc. See "Metrology," § (16).

TEMPERATURE CONTROL AS REQUIRED FOR METROLOGICAL OBSERVATIONS. See "Metrology," § (5) (vi.).

TEMPERATURE OF THE EARTH: equivalent radiative. See "Radiation," § (4) (i.).

TEMPERATURE EFFECTS IN SURVEY WORK. See "Trigonometrical Heights," § (4). See also "Gravity Survey," § (17).

TEMPERATURE GRADIENT IN THE ATMOSPHERE: effect on wind. See "Atmosphere, Physics of," § (10).

Reversal in the stratosphere. See *ibid.* §§ (10), (11).

TEMPERATURE OF LINE STANDARDS: how measured, sources of error, etc. See "Comparators," § (1) (*b*).

TEMPERATURE VARIATIONS IN THE VALUE OF THE CONSTANT OF GRAVITATION: experiments to try to detect, by Dr. P. E. Shaw. See "Earth, Density of the," § (3).

Tercentesimal Scale of Temperature: the approximate absolute scale of temperature in Centigrade degrees taking the freezing-point at 273°. See "Atmosphere, Thermodynamics of the," § (2).

Theodolites, as used for observing pilot balloons. See "Air, Investigation of Upper," § (6) (i.), (ii.), (iii.), (iv.).

Theodorich: explanation of the rainbow. See "Meteorological Optics," § (14).

Thermal Hysteresis in Length Bar. See "Metrology," § (4).

Thermodynamics of the Atmosphere. See "Atmosphere, Thermodynamics of the."

Thermograph: a self-recording thermometer. General description of. See "Meteorological Instruments," § (9).
- Types of:
  - Bimetallic. See *ibid.* § (9) (ii.).
  - Bourdon tube. See *ibid.* § (9) (iii.).
  - Electric-recording. See *ibid.* § (9) (iv.).
  - Mercurial. See *ibid.* § (9) (i.).

Thermometer Screens:
- Arrangement of instruments in. See "Meteorological Instruments," § (8).
- Types of. See *ibid.* § (5).

Thermometers:
- Black-bulb. See "Radiant Heat and its Spectrum Distribution," § (8).
- For meteorological purposes:
  - Black-bulb *in vacuo*. See "Meteorological Instruments," § (27).
  - Dry- and wet-bulb. See *ibid.* §§ (7) (i.) and (8).
  - For upper air. See *ibid.* VIII. §§ (36)-(38).
  - Kata. See "Humidity," II. § (11).
  - Maximum. See "Meteorological Instruments," §§ (7) (ii.) and (8).
  - Minimum. See *ibid.* §§ (7) (iii.) and (8).
  - Self-recording. See "Thermograph."
- Wet-bulb:
  - Depression of, variation with speed of the wind. See "Humidity," II. § 8.
  - Effect of moving air on. See *ibid.* II. §§ (6) and (7).
  - Experimental verification of theory of, in still air and in still hydrogen. See *ibid.* II. § (5) (ii.).
  - Theory of. See *ibid.* II. § (5) (i.).
- Wet- and dry-bulb, determination of vapour pressure by. See *ibid.* II. §§ (4)-(9) and (11).

Thermopile, as used for solar measurements. See "Radiant Heat and its Spectrum Distribution," § (20).

Thomas Gas Meter. See "Meters for Measurement of Coal Gas and Air," § (26).

Threlfall and Pollock's Gravity Balance. See "Gravity Survey," § (5) (i.).

Thunder Cloud:
- Its electric field and currents within the cloud. See "Atmospheric Electricity," § (23).
- Processes connected with the development and dissipation of their electric charges. See *ibid.* § (24).

Thunder-storms, electric forces in neighbourhood of. See "Atmospheric Electricity," § (3). Also "Atmosphere, Physics of," §§ (6) (iii.), (22).

Tide-gauge: apparatus for recording the exact rise and fall of the sea. See "Tides and Tide-Prediction," § (5).

Tide-predicting Machine, devised by Kelvin to find the resultant or sum of a number of simple harmonic motions. See "Tides and Tide-Prediction," § (5).

TIDES AND TIDE-PREDICTION

§ (1) Explanation of Terms used.—The term tide is used to denote the periodic rising and falling of the water of the sea, due generally to gravitation and the relative motions of the moon, sun, and earth. The tide thus caused may be called the *astronomical tide.* A periodic rise and fall may be produced by meteorological conditions, such as day and night breezes, regular wind variations, and periodic barometric changes; such tides may be classed as *meteorological tides.* In practice, however, it is very difficult to distinguish between the tides arising from these two causes.

In mid-ocean the observed phenomenon at any point is merely a rise and fall, with practically negligible horizontal motion. Near the land, however, the effects become modified by friction and other circumstances, giving rise to *tidal currents.* A specially rapid tidal current in some localities is known as a *race.* This horizontal motion must be distinguished from the rise and fall strictly called a tide. In rivers and river estuaries such horizontal motion gives a tidal *flow* and *ebb* superposed on the seaward motion of the river water. In a river the rise and fall of the tide must be carefully distinguished from the flow and ebb of the water; at a point in a tidal river some distance from its mouth the flow upstream continues after "high water," that is after the water has reached its highest level and has commenced to fall; similarly the ebb or motion down-stream continues after low water. In a river also the interval from low to high water is less than that from high to low; the difference increases as we go up the river.

The change in water level from low to high water is called the *range* of the tide. In rivers and rapidly narrowing estuaries the range may be much augmented owing to the contraction of the channel up which the tidal wave has to pass. When a deep-water wave passes into shallow water its front becomes steeper owing to friction. In the case of some rivers with wide and shallow estuaries, owing to the combined effect of rapid or moderately rapid current, quick contraction, and friction, the rise of water in the river is exceptionally sudden and the flow violent: the tidal wave advances up the river with the appearance of a wall of water. This phenomenon is called a *bore*.

§ (2) SIMPLE EXPLANATION OF THE TIDES.—The astronomical tide is a direct consequence of the gravitational attraction between the earth and the moon and sun. In a sense we may call the tide a secondary effect of this attraction; the primary effect being the orbital motion of the moon and earth about their common centre of gravity, and of the earth and moon about the sun. As is well known, the larger part of the tide is due to the moon. Assume for the moment that the moon is in the plane of the equator. At a point in the earth's surface immediately under the moon the force of attraction is greater than the mean attraction on the whole earth: if this point is on the sea the water tends to be heaped up there. At the point antipodal to this, the point farthest from the moon, the attraction at the surface is less than the mean attraction; again the water tends to be heaped up as the earth is drawn away from it. On this crudely simple theory, as the earth rotates, there would be two high waters at any place on the equator within the day: one when the place was under the moon, the other when the place was at the point farthest from the moon. The corresponding low-waters would occur at positions 90° from these two points. The tide is thus semi-diurnal, high water occurring twice a day; the average interval is, however, not exactly 12 hours, since the moon revolves about the earth in the same direction as the earth rotates, but is about 12 hrs. 25 mins., so that high water occurs about 50 mins. later on successive days.

Similarly the sun, supposed in the equator, will produce a semi-diurnal tide, the average interval between successive high waters being exactly 12 hours. From the known masses of the sun and moon and their distances from the earth it is easy to calculate that the magnitude of the tide due to the sun is a little less than one half of that due to the moon. With sun and moon acting together the actual tide will be a combination of the tides due to each, obtained simply by adding the separate effects if the depth of the sea at the place is large compared with the tidal range. At new or full moon lunar and solar high water will coincide, and the rise will be the sum of the effects due to each: at first and third quarter lunar high water will coincide with solar low water and the rise will be the difference of the separate effects. The larger tides at or about the times of new and full moon are called *spring tides*; the smaller tides near the times when the moon is in the first or third quarter are called *neap tides*. Since the solar tide is about one-half of the lunar, the range at springs is about three times that at neaps. The interval between successive spring tides is half a lunar month; the interval from spring to neap tides is, therefore, about a week.

The sun and moon being on the equator the effect at a place north or south of the equator is generally similar, but the tide is smaller: at the poles it vanishes.

Now suppose the moon is, say, north of the equator. The tide will be greatest at the point immediately under the moon, north of the equator, and at the antipodal point, south of the equator. At any place north of the equator, moving on a parallel of latitude as the earth rotates, the high tide when the place is on the side of the earth turned towards the moon will be greater than that which occurs when the place is on the side away from the moon. The heights of the water at successive high waters will be different: the amount of the difference is called the *diurnal inequality*. As we shall see presently we may suppose this inequality to be produced by the superposition of a diurnal tide upon the semi-diurnal tide due to the combined effects of the sun and moon. Similarly the sun, when not in the equator, gives rise to a diurnal tide.

The foregoing simple theory of the most readily observed phenomena of the tides is, of course, far from exact: it indicates only the directions in which an explanation is to be sought. It assumes that the water covering the earth is capable of assuming instantaneously the form corresponding to the forces acting on it: it neglects the effects of motion and inertia. According to the simple theory the lunar high water should occur at any place when the moon is in the meridian of that place; spring tide should occur at twelve o'clock (local time) when the sun and moon are together in the meridian. Actually spring tide is later than the time of full and change of the moon by about 36 hrs. on the average. This interval is called the *age of the tide*: it varies considerably from port to port. The average interval at any port between the time of moon's transit at full or change and the succeeding high water is called the *establishment of the port*. This high water is not in general the maximum spring tide: the interval between the time of maximum spring tide and the preceding moon's transit is termed the *corrected establishment*: it differs little from the ordinary establishment. At any other state of the moon the interval between the moon's upper or lower transit and high water is called the *lunitidal interval*, or simply the interval.

For the preparation of tide tables it is of importance to fix a datum or base line from which the height of the water shall be measured. To avoid the frequent occurrence of negative quantities it is convenient that this should be approximately at, or below, the level of low water at spring tides. In British tide tables and charts the practice has usually been to refer the heights and soundings to " the mean low water of ordinary spring tides," and the depth of this datum below some datum mark fixed on the shore or in the dock at the port is given in the tables for the port. This definition is not very exact: for a datum which can be more exactly defined see below (§ (5), Harmonic Analysis).

§ (3) EARLY METHODS OF TIDE-PREDICTION.—Early tide-prediction was mainly concerned with foretelling the times and heights of high and low water. As we have seen, these depend primarily on the positions of the sun and moon. Since the sun crosses the meridian at approximately twelve o'clock (local time) the clock time at which the moon crosses the meridian indicates at once the relative positions of the sun and moon. As a first rough approximation we may assume, therefore, that for a given time of moon's transit the lunitidal interval and the corresponding heights of high and low water will be constant. It should thus be possible to construct tables or graphs in which the times and heights of high and low water are tabulated or plotted against the times of moon's transit. From such a table or graph the heights and times for any given time of moon's transit can then be computed or read off.

To obtain such tables for any port it is clearly necessary in the first place to record the times of moon's transit and the corresponding tidal heights and times over a considerable period of time in order to obtain a reliable average. Not to speak of meteorological disturbances, we have ignored in this brief statement the declination of the sun and moon and other variations in their motion relative to the earth. If, however, observations are made over a sufficient period, say a year, and a table or graph constructed, we shall be able to make rough predictions of the tides.

Tables of corrections can be prepared, to be applied to the mean values computed as explained, showing the effects of the declinations of the sun and moon and of their parallaxes (depending on their distance from the earth). Allowance must also be made for the diurnal inequality. Round the British coasts this is relatively small, but in many parts of the world it becomes of great importance, indeed at some ports, *e.g.* Aden, at neap tides, there is often only one high water in the 24 hours, the second high water becoming evanescent. The application of this method of prediction then becomes somewhat complicated, and recourse is usually had to the method based on harmonic analysis (§ (5)). It is not necessary here to consider further the procedure adopted when the preceding method is employed. It was elaborated by Sir John Lubbock and is still in use for the majority of the British ports.

§ (4) EQUILIBRIUM THEORY.—A brief account may now be given of what is known as the equilibrium theory of the tides, on which modern methods of prediction are based. The theory assumes that the earth is covered by an ocean of uniform depth, and that the water takes up at each instant the form corresponding to the gravitational forces acting upon it at that instant; in other words, it ignores the effects of motion and inertia. The importance of the theory lies in its application to the harmonic analysis of the tides and to prediction: in this application only certain main features derived from the equilibrium theory are utilised, and their use is justified, as will be seen, by the accuracy of the predictions made on this basis.

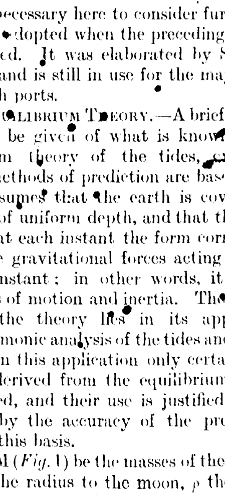


FIG. 1.

Let E, M (*Fig.* 1) be the masses of the sun and moon, r the radius to the moon, ρ the radius vector to any point P on the earth's surface, z the angle between r and ρ. Then the acceleration of the moon relative to E is $(E+M)/r^2$; that of E relative to the common C.G. is M/r^2. We shall suppose the earth's centre reduced to rest, and consider the moon's motion relative to the earth. Let the direction cosines of r, ρ be M_1, M_2, M_3; ξ_1, ξ_2, ξ_3, respectively. The component forces per unit mass required to reduce E to rest are $-M/r^2 . M_1$, $-M/r^2 M_2$, $-M/r^2 . M_3$; hence the potential at P of these forces

$$= -\frac{M\rho}{r^2}(M_1\xi_1 + M_2\xi_2 + M_3\xi_3)$$

$$= -\frac{M\rho}{r^2}\cos z,$$

since the differential coefficients of this expression with respect to the co-ordinates of P are equal to the forces given.

Again the potential at P due to M is

$$\frac{M}{\sqrt{r^2+\rho^2-2r\rho\cos z}},$$

to which may be added a constant; since we are seeking the forces which urge P relatively to E, we add the constant $-M/r$, which makes the potential at the centre of E zero, and take as the potential at P

$$\frac{M}{\sqrt{r^2+\rho^2-2r\rho\cos z}}-\frac{M}{r}.$$

If we expand this in terms of the small quantity ρ/r, and add together the terms contributing to the potential at P, ignoring the term E/ρ due to the earth's gravity, we obtain for the tide-generating potential V the expression

$$\frac{M\rho^2}{r^3}\left(\tfrac{3}{2}\cos^2 z-\tfrac{1}{2}\right)+\frac{M\rho^3}{r^4}\left(\tfrac{5}{2}\cos^2 z-\tfrac{3}{2}\cos z\right)+\ldots$$

or ignoring the higher terms

$$V=\frac{3M\rho^2}{2r^3}\left(\cos^2 z-\tfrac{1}{3}\right).$$

Let A be the point on the equator in the meridian of P, B 90° east of A; M the projection of the moon in its orbit XY. MCA is the moon's easterly local hour angle. Let λ, l be the latitude and west longitude of P, h the moon's westerly hour angle from Greenwich, δ her declination.

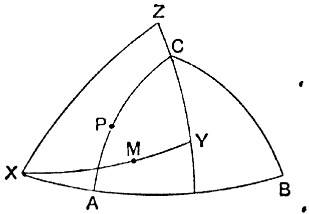


FIG. 2

Then $$MCA=l-h$$

and

$$M_1=\cos\delta\cos(h-l),\quad M_2=-\cos\delta\sin(h-l),$$
$$M_3=\sin\delta$$

$$\xi_1=\cos\lambda,\qquad \xi_2=0,\qquad \xi_3=\sin\lambda.$$

Hence $$V=\frac{3M\rho^2}{2r^3}\left[\{\cos\delta\cos\lambda\cos(h-l)+\sin\delta\sin\lambda\}^2-\tfrac{1}{3}\right],$$

which after simple transformation may be written

$$V=\frac{3M\rho^2}{2r^3}\left\{\tfrac{1}{2}\cos^2\lambda\cos^2\delta\cos 2(h-l)\right.$$
$$+\tfrac{1}{2}\sin 2\lambda\sin 2\delta\cos(h-l)$$
$$\left.+\tfrac{3}{2}\left(\tfrac{1}{3}-\sin^2\delta\right)\left(\tfrac{1}{3}-\sin^2\lambda\right)\right\}.$$

h increases as the earth rotates, changing by 360° in about 24 hours 50 minutes, while δ goes through a cycle of values in a lunar month. Hence it will be seen that V contains approximately harmonic terms of periods one-half a (lunar) day and one day (semi-diurnal and diurnal tides), as well as long-period terms depending on the lunar month. There will be similar terms due to the sun. It must be noted that r is variable owing to the ellipticity of the moon's orbit.

To proceed further we must allow for the variation in h, δ, and r as the moon moves in her orbit, the problem being to express V in terms of cosine functions, the arguments of which increase uniformly with the time. For this purpose we adopt initially a procedure somewhat different from the foregoing.

Referring again to *Fig.* 2, let $I=AXM=ZC$ denote the obliquity of the moon's orbit to the equator; $AX=\chi$; $MX=l=$ longitude of the moon in her orbit measured from the "intersection." Also write $p=\cos\tfrac{1}{2}I$, $q=\sin\tfrac{1}{2}I$. Then

$$M_1=\cos l\cos\chi+\sin l\sin\chi\cos I$$
$$=p^2\cos(\chi-l)+q^2\cos(\chi+l),$$

$$M_2=-\cos l\sin\chi+\sin l\cos\chi\cos I$$
$$=-p^2\sin(\chi-l)-q^2\sin(\chi+l),$$

$$M_3=\sin l\sin I=2pq\sin l.$$

Also

$$\xi_1=\cos\lambda,\qquad \xi_2=0,\qquad \xi_3=\sin\lambda.$$

Now

$$\cos^2 z-\tfrac{1}{3}=(\xi_1M_1+\xi_3M_3)^2-\tfrac{1}{3}$$
$$=\cos^2\lambda\frac{M_1^2-M_2^2}{2}+\sin 2\lambda M_1M_3$$
$$+\tfrac{3}{2}\left(\tfrac{1}{3}-\sin^2\lambda\right)\frac{M_1^2+M_2^2-2M_3^2}{3}.$$

And

$$M_1^2-M_2^2$$
$$=p^4\cos 2(\chi-l)+2p^2q^2\cos 2\chi+q^4\cos 2(\chi+l),$$

$$M_1M_3$$
$$=-p^3q\sin(\chi-2l)+pq(p^2-q^2)\sin\chi$$
$$+pq^3\sin(\chi+2l),$$

$$\tfrac{1}{3}(M_1^2+M_2^2-2M_3^2)$$
$$=\tfrac{1}{3}(p^4-4p^2q^2+q^4)+2p^2q^2\cos 2l.$$

Now, neglecting the perturbations due to the sun, the equation of the ellipse described by the moon is

$$\frac{c(1-e^2)}{r}=1+e\cos(l-\varpi_1),$$

where c is the moon's mean distance, ϖ the longitude of the moon's perigee in her orbit. Then

$$V=\frac{3Ma^2}{2c^3(1-e^2)^3}\{1+e\cos(l-\varpi_1)\}^3\left(\cos^2 z-\tfrac{1}{3}\right).$$

Also, by the theory of elliptic motion, if σ_1 be the moon's mean longitude measured in her orbit

$$l=\sigma_1+2e\sin(\sigma_1-\varpi_1)+\tfrac{5}{4}e^2\sin 2(\sigma_1-\varpi_1)+\dots$$

Now

$$\begin{aligned}&\{1+e\cos(l-\varpi_1)\}^3\\&=1+\tfrac{3}{2}e^2+3e\cos(l-\varpi_1)+\tfrac{3}{2}e^2\cos 2(l-\varpi_1)+\dots\\&=1-\tfrac{3}{2}e^2+3e\cos(\sigma_1-\varpi_1)+\tfrac{9}{2}e^2\cos 2(\sigma_1-\varpi_1)+\dots\\&=\mathrm{R},\text{ say.}\end{aligned}$$

$$\begin{aligned}&\mathrm{R}\cos(2l+\alpha)\\&=(1-\tfrac{15}{2}e^2)\cos(2\sigma_1+\alpha)-\tfrac{1}{2}e\cos(\sigma_1+\alpha+\varpi_1)\\&\quad+\tfrac{7}{2}e\cos(3\sigma_1+\alpha-\varpi_1)+\tfrac{17}{2}e^2\cos(4\sigma_1+\alpha-2\varpi_1)\\&\quad+\dots\end{aligned}$$

and

$$\begin{aligned}&\mathrm{R}\cos\alpha=(1-\tfrac{3}{2}e^2)\cos\alpha\\&\quad+\tfrac{3}{2}e[\cos(\sigma_1+\alpha-\omega_1)+\cos(\sigma_1-\alpha-\varpi_1)]\\&\quad+\tfrac{9}{4}e^2[\cos(2\sigma_1+\alpha-2\varpi_1)+\cos(2\sigma_1-\alpha-2\varpi_1)]\\&\quad+\dots\end{aligned}$$

$\mathrm{R}\sin(2l+\alpha)$, $\mathrm{R}\sin\alpha$ are obtained from these by putting $\alpha-\pi/2$ in place of α.

In applying these formulae we must put $\alpha=2\chi$ or -2χ, etc., and we get

$$\begin{aligned}&\text{Coefficient of }\tfrac{1}{2}\cos^2\lambda\\&=(1-\tfrac{15}{2}e^2)[p^4\cos 2(\chi-\sigma_1)+q^4\cos 2(\chi+\sigma_1)]\\&+(1-\tfrac{3}{2}e^2)2p^2q^2\cos 2\chi\\&-\tfrac{1}{2}e[p^4\cos(2\chi-\sigma_1-\varpi_1)+q^4\cos(2\chi+\sigma_1+\varpi_1)]\\&+\tfrac{7}{2}e[p^4\cos(2\chi-3\sigma_1+\varpi_1)+q^4\cos(2\chi+3\sigma_1-\varpi_1)]\\&+\tfrac{3}{2}e\,.\,2p^2q^2[\cos(2\chi+\sigma_1-\varpi_1)+\cos(2\chi-\sigma_1+\varpi_1)]\\&+\tfrac{17}{2}e^2[p^4\cos(2\chi-4\sigma_1+2\varpi_1)\\&\qquad\qquad+q^4\cos(2\chi+4\sigma_1-2\varpi_1)]\\&+\tfrac{9}{4}e^2\,.\,2p^2q^2[\cos(2\chi+2\sigma_1-2\varpi_1)\\&\qquad\qquad+\cos(2\chi-2\sigma_1+2\varpi_1)].\end{aligned}$$

Similarly we may write down the coefficients of $\sin 2\lambda$ and of $\tfrac{3}{2}(\tfrac{1}{3}-\sin^2\lambda)$.

There are also terms due to the inequalities in the moon's motion produced by the sun's attraction (evectional and variational tides): these are, however, here omitted.

It may here be noted that $e=\cdot0549$; and that when I has its mean value of 23° 27′·3, $q=\sin\tfrac{1}{2}\mathrm{I}=\cdot203$, $q^2=\cdot041$, $q^3=\cdot0084$; q^2 is thus a little smaller than e. In developing further q^2 and q^3 are treated as being of the same order as e, and the terms containing e^2 are omitted except where the coefficient is large.

We now multiply through by $(1-e^2)^{-3}$, or to the approximation adopted $1+3e^2$, and obtain an expression for $\mathrm{V}\div3\mathrm{M}a^2/2c^3$. In this the coefficient of $\tfrac{1}{2}\cos^2\lambda$

$$\begin{aligned}&=(1-\tfrac{5}{2}e^2)p^4\cos 2(\chi-\sigma_1)+(1+\tfrac{3}{2}e^2)2p^2q^2\cos 2\chi\\&\quad+\tfrac{7}{2}ep^4\cos(2\chi-3\sigma_1+\varpi_1)\\&\quad-\tfrac{1}{2}ep^2[p^2\cos(2\chi-\sigma_1-\varpi_1)\\&\qquad\qquad-6q^2\cos(2\chi-\sigma_1+\varpi_1)]^*\\&\quad+\tfrac{17}{2}e^2p^4\cos(2\chi-4\sigma_1+2\varpi_1).\end{aligned}$$

(The second term in the bracket * is small but is retained for special reasons.)

It remains to express χ, σ_1, ϖ_1, each of which increases uniformly with the time, in terms of local mean time and of the mean longitudes of the moon and of perigee.

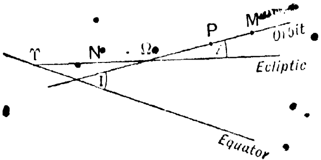


FIG. 3.

In *Fig.* 3 let M be the moon in her orbit,

- A as before,
- P the perigee,
- N the longitude of the node ☊,
- g the sidereal hour angle,
- ν the right ascension of the intersection I,
- ξ the longitude in the moon's orbit of the intersection,
- i the inclination of the moon's orbit to the ecliptic,
- ω the obliquity of the ecliptic,
- s the moon's mean longitude,
- p the mean longitude of the perigee (easily distinguished from the p used previously),
- p' the longitude of the perigee measured from ♈ in the ecliptic

Then $g=\mathrm{A}♈$, $\nu=♈\mathrm{I}$, $\xi=♈☊-☊\mathrm{I}$, $\mathrm{N}=♈☊$, $s-p=\sigma_1-\varpi_1=$ moon's mean anomaly.

By the formula for reduction to the ecliptic

$$☊\mathrm{P}=p'-\mathrm{N}+\tfrac{1}{4}\sin^2 i\sin^2(p'-\mathrm{N}),$$

$$\begin{aligned}\varpi_1&=\mathrm{IP}=\mathrm{I}☊+☊\mathrm{P}=♈☊-\xi+☊\mathrm{P}\\&=p'-\xi+\tfrac{1}{4}\sin^2 i\sin 2(p'-\mathrm{N}),\end{aligned}$$

and $\quad p=p'+\tfrac{1}{4}\sin^2 i\sin 2(p'-\mathrm{N}).$

Therefore

$$\varpi_1=p-\xi,$$

$$\sigma_1=s-\xi.$$

Also $\quad\chi=\mathrm{IA}=g-\nu=t+h-\nu,$

where $t=$ local mean solar time reduced to angle,

$h=$ sun's mean longitude.

Then $2(\chi - \sigma_1) = 2t + 2(h - \nu) - 2(s - \xi)$,

$$2\chi = 2t + 2(h - \nu),$$

$$2\chi - 3\sigma_1 + \varpi = 2t + 2(h - \nu) - 2(s - \xi) - (s - p),$$

etc.

The increase of t per hour = 15°.

The increase of h per hour = 0°·0410686 [approx. 360° ÷ (365¼ × 24)].

The increase of s per hour = 0°·5491065 [approx. 360° ÷ (27⅓ × 24)].

The increase of p per hour = 0°·0046419 (nearly 41° per annum).

ξ and ν also change slowly as N changes, but in the use of the method of harmonic analysis their variation in the course of a single year is ignored: the value assigned to them is that corresponding to the middle of the year.

The values above obtained have now to be substituted in the expression for V. We may write $V = gh$, the potential at height h, where h is the height of the tide. Then h contains the factor $3Ma^2/2c^3g$, and since $g = E/a^2$, this $= \frac{3}{2}M/E(a/c)^3a$. h is the sum of a number of cosine terms or simple harmonic motions, which may be called simple tides. In the application of this method we assume that the period of each of these forced oscillations of the water, *i.e.* the speed of each simple tide, agrees with that of the cause producing it: the water may not, however, immediately take up the equilibrium form assumed; in other words, there may be a phase difference or lag, depending on inertia, friction, the configuration of the land, etc., and the amplitude of each component may also be affected by these various causes. The values of the lag and amplitude for each component can only be obtained for any given port by actual observation and analysis of the variation of tide level at the port (see below, § (5)).

The principal terms in the theoretical tide resulting from the above substitution are given in the appended schedule (page 763). The preceding analysis has been given to show the method; the terms obtained are not all included in the schedule, but will be found in the more complete schedules of terms usually computed in the harmonic analysis of the tides. In the schedule are given also the corresponding terms due to the sun. The solar terms will contain the coefficient

$$\frac{3}{2}\frac{M_1}{E}\left(\frac{a}{c_1}\right)^3 a.$$

If we write $\tau = \frac{3}{2}M/c^3$, $\tau_1 = \frac{3}{2}M_1/c_1{}^3$, this coefficient becomes

$$\frac{\tau_1}{\tau} \times \frac{3}{2}\frac{M}{E}\left(\frac{a}{c}\right)^3 a.$$

For comparison of the relative magnitudes of the solar and lunar component tides, it is convenient to retain $\frac{3}{2}M/E(a/c)^3a$ as a universal coefficient, to be multiplied in the case of the solar tides by τ_1/τ, the numerical value of which is 0·46035 or 1/2·17226. The solar tides in the schedule are then at once distinguished by the occurrence of the factor τ_1/τ in the third column.

In addition to the tides given in the schedule may be mentioned the long-period tide whose speed is the rate of variation of N, the longitude of the moon's node, which is about 19°·34 per annum. This tide has thus a period of about nineteen years. For other components commonly evaluated and employed in prediction, reference must be made to the standard treatises on the subject.

When a tidal wave advances into shallow water its form changes, becoming steeper in front, in a manner which can be represented by the introduction of higher harmonics or *over-tides.* Thus corresponding to M_2, S_2, we have the over-tides M_4, M_6, S_4, S_6, the suffix denoting the number of periods included in (approximately) one day: the speed of M_4 is twice that of M_2. In the same circumstances the combined effect due to two simple harmonic waves is no longer simply the sum of the changes in water level due to each separately: interference occurs between two simple tides in shallow water, producing effects which may be represented by introducing additional components of speeds equal to the sum and difference respectively of the speeds of the original deep-water components. The shallow-water components thus produced are called *compound tides*: in rivers and river estuaries some of these are of sufficient importance to be allowed for in prediction and to be included on the tide-predicting machine. The most important is the quaterdiurnal tide $(MS)_4$ arising from M_2 and S_2.

Finally, mention must be made of the meteorological tides. Diurnal and annual variations of meteorological conditions affect the height of the tide, giving rise to tides S_1, S_2, S_3, . . . *Sa*, *Ssa*. . . . These may also arise directly from astronomical causes, but with the exception of S_2 are of much more importance as meteorological than as astronomical tides. In districts where regular rainy seasons occur, the annual, semi-annual, ter-annual, etc., components may be very large at riverain ports.

§ (5) HARMONIC ANALYSIS AND PREDICTION.—The preceding theory leads to the conclusion that the height of the tide at any instant above mean sea-level can be expressed as the sum of a number of terms of the form

$$R\cos(nt - \zeta).$$

n in this expression denotes the speed of the

SCHEDULE OF PRINCIPAL TIDES (HARMONIC ANALYSIS)

Universal Coefficient $= \frac{3}{2}\frac{M}{E}\left(\frac{a}{c}\right)^3 a$.

Descriptive Name.	Symbol.	Coefficient.	Mean Value of Coefficient.	Argument.	Speed in Degrees per M.S. Hour.
I. Semi-diurnal Tides: General Coefficient $=\cos^2\lambda$.					
Principal lunar	M_2	$\frac{1}{2}(1-\frac{5}{2}e^2)\cos^4\frac{1}{2}I$	·45427	$2t+2(h-\nu)-2(s-\xi)$	28·98410
Larger lunar elliptic	N	$\frac{1}{2}\cdot\frac{7}{2}e\cos^4\frac{1}{2}I$	·08796	$2t+2(h-\nu)-2(s-\xi)-(s-p)$	28·43793
Principal solar	S_2	$\frac{\tau_1}{\tau}\cdot\frac{1}{2}(1-\frac{5}{2}e_1^2)\cos^4\frac{1}{2}\omega$	·21137	$2t$	30·00000
Solar elliptic	T	$\frac{\tau_1}{\tau}\cdot\frac{1}{2}\cdot\frac{7}{2}e_1\cos^4\frac{1}{2}\omega$	·01243	$2t-(h-p_1)$	29·95893
Luni-solar (lunar portion)	K_2	$\frac{1}{2}(1+\frac{3}{2}e^2)\frac{1}{2}\sin^2 I$	·03929	$2t+2(h-\nu)$	30·08214
Luni-solar (solar portion)		$\frac{\tau_1}{\tau}\frac{1}{2}(1+\frac{3}{2}e_1^2)\frac{1}{2}\sin^2\omega$	·01823	$2t+2h$	
II. Diurnal Tides: General Coefficient $=\sin 2\lambda$.					
Lunar diurnal	O	$(1-\frac{5}{2}e^2)\frac{1}{2}\sin I\cos^2\frac{1}{2}I$	·18856	$t+(h-\nu)-2(s-\xi)+\frac{1}{2}\pi$	13·94304
Larger lunar elliptic	Q	$\frac{7}{2}e\cdot\frac{1}{2}\sin I\cos^2\frac{1}{2}I$	·03651	$t+(h-\nu)-2(s-\xi)-(s-p)+\frac{1}{2}\pi$	13·39866
Solar diurnal	P	$\frac{\tau_1}{\tau}(1-\frac{5}{2}e_1^2)\frac{1}{2}\sin\omega\cos^2\frac{1}{2}\omega$	·08775	$t-h+\frac{1}{2}\pi$	14·95893
Luni-solar (lunar portion)	K_1	$(1+\frac{3}{2}e^2)\frac{1}{2}\sin I\cos I$	·18115	$t+(h-\nu)-\frac{1}{2}\pi$	15·04107
Luni-solar (solar portion)		$\frac{\tau_1}{\tau}(1+\frac{3}{2}e_1^2)\frac{1}{2}\sin\omega\cos\omega$	·08407	$t+h-\frac{1}{2}\pi$	
III. Long-period Tides: General Coefficient $=(\frac{1}{2}-\frac{3}{2}\sin^2\lambda)$.					
Fortnightly	Mf	$(1-\frac{5}{2}e^2)\frac{1}{2}\sin^2 I$	·07827	$2(s-\xi)$	1·09803
Semi-annual	Ssa	$\frac{\tau_1}{\tau}(1-\frac{5}{2}e_1^2)\frac{1}{2}\sin^2\omega$	·03643	$2h$	0·08214

tide in degrees per mean solar hour; t is the time in m.s. hours from the epoch or instant taken as the starting-point of the observations (usually noon of the first day). In this form R and ζ are not absolutely constant, but vary with the position of the moon's node. For a period of one year, however, they may be treated as approximately constant and as having the values corresponding to the value of N at the middle of the period. For comparison of one year with another each term is written in the form

$$fH\cos(V+u-\kappa).$$

Here f is a factor which varies from year to year with the position of the moon's node: it represents the ratio of the value of the coefficient for that year to its mean value. H is now an absolute constant. V represents that part of the argument which is independent of the position of the node: $dV/dt=n$ gives the speed of the tide. u is the portion of the argument which depends on the position of the node: a mean value of this is taken for any given year. κ is then an absolute constant and κ/n represents the lag of the tide, in hours. The height of the tide at any instant is thus expressed in the form

$$h=A_0+\Sigma fH\cos(V+u-\kappa),$$

A_0 denoting the height of mean sea-level above some fixed datum. The initial value of V at the commencement of the period (V_0), and the value of u for the middle of the period are obtained from astronomical tables. Special tables for the purpose, and for the determination of f, are given in treatises on tide-prediction.

As examples of the variation of f from year to year, it may be mentioned that the extreme values for the tides M_2 and N are 1·04 and 0·96; for O, 1·18 and 0·81.

To complete the harmonic analysis of the tides of any given port it is necessary to determine the values of H and κ for the port, for the various component tides. To separate out any one component from the remainder, the principle followed is to take height measurements at time intervals equal to the period of this component. If such height measurements, taken over a long period of time, say one year, be summed, the effect of other components will average out, if their periods are incommensurable with that of the selected component, and the sum will be Nh, where h is the tide height due to this component and N the number of complete periods over which the summation extends. If we divide a single period into say 24 equal parts in the case of a diurnal tide, or 12 in the case of a semi-diurnal tide, we can obtain 24 "*hour*-ly" mean values which will enable us to construct the harmonic curve or oscillation for this single component (with its harmonics or overtones, if appreciable). The calculation is effected by the usual method for a Fourier series. If we write

$$h = A_0 + A_1 \cos nt + B_1 \sin nt + A_2 \cos 2nt + B_2 \sin 2nt + \&c.\ .$$

then

$$A_0 = \tfrac{1}{24}\Sigma h,$$

$$A_1 = \tfrac{1}{12}\Sigma h \cos nt,$$

$$B_1 = \tfrac{1}{12}\Sigma h \sin nt,$$

where h is the average height obtained as above explained and the summation extends over 24 "*hour*-ly" intervals. The term *hour* is here used to denote $\frac{1}{24}$ of the period (diurnal) which in general differs from, but approximates to, a mean solar hour. From A_1 and B_1, H and κ are readily deduced, V_0, u, f being known.

To be able to apply this method it is clearly necessary that the height of the water at the particular port should be known throughout the period. A record must thus be obtained by means of a recording tide-gauge: this is in general essential for the application of the method of harmonic analysis and prediction. Sir George Darwin has, however, given a method of deducing the principal harmonic constants from observations of the heights and times of high and low water, when a continuous record is not available.

In practice to apply the above procedure exactly as explained would involve too much labour, requiring, as it does, the measurement of the height on the tide record at "*hour*-ly" intervals differing for each component. The procedure generally adopted is to measure the height at each interval of an hour in mean solar time, and in applying the method to a component whose speed differs from 15° or 30° per hour to take, instead of the height required at any instant, the height at the nearest solar hour. An abacus has been devised by Sir George Darwin which renders mechanical the selection of the appropriate measured height for each component tide. A correction has to be applied owing to the systematic departure from the true times in the height measurements taken.

The foregoing indicates the general principles of the method, and it is unnecessary here to go into further detail.

In the preceding A_0 denotes the height of mean sea-level above the datum line: its value for the year is the mean of the hourly height measurements throughout the year. The datum line usually adopted may now be more precisely defined as being below mean sea-level by the sum of the semi-ranges of the tides M_2, S_2, K_1, O.

When the values of H and κ for all components of importance, and of A_0, have been determined for any port they can obviously be used to predict the tides at that port in future years. To effect this by calculation, however, would be too lengthy a procedure, and to enable the method to be employed Kelvin devised the tide-predicting machine, the purpose of which is to find the resultant or sum of a number of simple harmonic motions. In this machine a wire, fixed at one end, passes over a number of pulleys the centre of each of which is made to move vertically with simple harmonic motion, the portions of wire between the pulleys being vertical; the other end of the wire carries a pen moving in a vertical guide, which records on a drum rotating about a vertical axis. The rate of rotation of the drum corresponds to mean solar time, and the periods of the various components are permanently adjusted by appropriate gearing in relation to this. The amplitude fH and initial phase-angle $V_0 + u - \kappa$ can be set for each component for any given port. The summation required is then effected by the wire. From the curve obtained the heights and times of high and low water can be read off: the height of the tide at any other time can of course be obtained from the curve, if required.

The tide-machines at present in use do not in general include sufficient components to enable them to be used for riverain ports, situated on rivers where at certain seasons of the year a great rise in water level occurs owing to the rains. In such cases Lubbock's method is usually employed, tables being constructed for each month separately, connecting the mean times and heights of high and low water with the times of moon's transit in the manner explained in § (3). Corrections for the diurnal components are obtained by an empirical method, with the aid of the tide-predicter.

As an example to show the closeness of

agreement obtained in the analysis of tides for successive years, the following figures for Bombay will be of interest:

PRINCE'S DOCK, BOMBAY

Tide.	Values of H (ft.).				
	1888.	1889.	1890.	1891.	1892.
M_2	4·095	4·115	4·102	4·106	4·112
S_2	1·656	1·614	1·640	1·605	1·614
N	1·013	1·004	0·986	1·001	0·993
K_1	1·387	1·390	1·397	1·395	1·404
O	·662	·680	·651	·649	·654
P	·379	·410	·411	·403	·423
M*f*	·049	·052	·023	·057	·055
S*sa*	·130	·122	·239	·065	·134

Tide.	Values of κ.				
	1888.	1889.	1890.	1891.	1892.
M_2	328·6°	329·1°	329·6°	329·2°	329·4°
S_2	1·2	1·0	4·0	3·8	4·1
N	311·6	313·4	314·3	314·2	314·9
K_1	44·1	44·7	44·8	44·3	44·9
O	48·3	48·2	47·3	45·9	47·8
P	43·0	43·4	43·9	44·1	44·2
M*f*	344·2	345·7	10·2	36·5	5·8
S*sa*	133·9	177·9	209·0	203·1	176·2

The degree of accuracy here shown may be taken as fairly typical. It may be noted, also, that for this port the ratios of the amplitudes S_2/M_2, P/K_1 do not differ greatly from their theoretical values. The values of κ for P and K_1, whose speeds are nearly equal, are almost identical.

The age of the tide $= \kappa(S_2) - \kappa(M_2)/\text{Speed of } S_2 - \text{speed of } M_2 = 33$ hrs. approx.

As an illustration of the accuracy of the predictions made with the Indian tide-predicting machine, the following comparisons of observed and predicted times and heights for Prince's Dock for the year 1917, furnished by the Indian Survey Department, may be taken. High-water times: 43 per cent within 5 minutes, 82 per cent within 15 mins., 97 per cent within half an hour. With regard to the few occasions on which the difference exceeds half an hour it may be remarked that where the diurnal components are large the whole amount of rise and fall at neap tides may often be very small and the change in level at high and low water therefore very small. At a number of ports, *e.g.* Aden, often only one high and one low water occur in the 24 hours at neaps. High-water heights: 63 per cent within 4 inches, 92 per cent within 8 inches, 100 per cent within 1 foot; only a very few differences exceeded 1 foot. The spring range at Prince's Dock is about 14 feet.

At Bhavnagar, where the range is 31 ft., the accuracy in the same year was even greater: 100 per cent of the times were within 15 mins., 95 per cent of the heights within 8 inches. For both ports the accuracy was about the same for the low waters.

§ (6) DYNAMICAL THEORY OF THE TIDES.—The equilibrium theory of the tides is very far from giving a complete or even approximately correct solution of the motion of the sea resulting from the attraction of the moon and sun. It takes no account of the inertia of the water, of friction, or of the effect of the configuration of the land. It indicates, however, that the tides are *forced* oscillations of the water, the periods of which can be determined from the known motions of the moon and the sun. If the *free* period of oscillation in a confined ocean of finite depth should nearly coincide with the period of one of the component forced oscillations, the amplitude of that oscillation may be enormously increased. Further, if the period of the free be less than that of the forced oscillation, the maximum disturbance will tend to agree in phase with the maximum of the disturbing cause and the tide will be *direct*; if the period of the free oscillation be the greater, the phases will tend to differ by 180° and the tide will be *inverted*. The latter is to be regarded as the normal condition in respect of the tides; it follows that we may expect low water to occur under and opposite to the moon, instead of high water as in the equilibrium theory. The effect is of course complicated in the actual case by various causes.

Laplace was the first to make any serious attempt to arrive at a dynamical theory of the tides. Among the conclusions he reached the most interesting was that in an ocean of uniform depth the diurnal tide would be evanescent. This conclusion has in recent years been confirmed by Hough, who has given the most complete discussion of the dynamical theory. He examines the various types of free oscillations of the ocean, and gives examples of the effect which difference in depth may have on the theoretical tide. Thus for an ocean 29,000 ft. in depth he finds that the solar semi-diurnal tide would have at the equator a height 235 times as great as the equilibrium height and would be inverted. The general conclusion from Hough's work is that it is impossible to foresee the height of any forced tide-wave from theoretical considerations. It is unnecessary here to go into further detail.

§ (7) COTIDAL CHARTS.—A cotidal line may be defined as a line drawn through all the points on the surface of the sea at which the high water following full or change of moon occurs at the same hour of Greenwich time. If cotidal lines be drawn corresponding to the hours XII., I., II., III., a cotidal chart may be constructed from which, in certain regions, the travel of the tidal wave along the coast may be studied. Various endeavours have been made to construct cotidal charts for the different parts of the world, the most complete attempt being that of Rollin A. Harris, as given in his *Manual of Tides*, part iv. B. The difficulties, however, in preparing such charts for the larger oceans are considerable, owing to the lack of information as to the tidal movements in mid-ocean; and considerable doubt has been expressed by Sir George Darwin and others, as to the value of such charts in the present state of knowledge except in the case of certain limited areas.

For further information on this and other general questions of interest in connection with the tides, *e.g.* tidal friction and its effect on planetary motion, reference may be made to the authorities given below.

AUTHORITIES.—Sir G. H. Darwin, *Collected Papers*, i.; *Ency. Brit.*, article "Tides"; see also *The Tides and Kindred Phenomena in the Solar System* (John Murray, 2nd ed., 1901); Rollin A. Harris, *Manual of Tides*, U.S. Coast and Geodetic Survey; *Great Trigonometrical Survey of India*, xiv.; S. S. Hough, *Phil. Trans.*, clxxxix. A and cxci. A; also *British Association Reports*.

F. J. S.

TIME:

Mean solar. See "Clocks and Time-keeping," § (1). Also "Metrology," § (2) (ii.).

Sidereal. See *ibid.* I. § (2) (ii.).

The measurement of. See "Clocks and Time-keeping, § (1).

True solar. See "Metrology," I. § (2) (ii.).

TIME, MEASURE OF. The standard of time is derived from the period of the earth's rotation, and the unit of time in both metric and British systems is the mean solar second, which is equal to 1/(24 × 60 × 60), *i.e* 1/86400 mean solar day.

A true solar day is defined as the interval between successive transits of the centre of the sun's disc over a meridian, but this interval varies throughout the year, and in order that the civil day may be of uniform length, standard time is measured with reference to a "mean sun" which is supposed to revolve uniformly round the earth in a time equal to the average length of the true solar day.

(i.) The *mean solar day* on which the definition of unit time is based is therefore defined as the average interval between successive transits of the centre of the sun across any given meridian.

(ii.) The *tropical or solar year* is the average interval between successive passages of the sun across the first point of Aries (the first point of Aries is the point where the sun crosses the equator from south to north); it is the intersection of the celestial equator with the ecliptic.

(iii.) *The Civil Year.*—According to the Julian calendar the civil year contains 365 days for three successive years, the fourth year containing 366; a further correction is made by which a century year contains 365 days unless divisible by 400, when it contains 366.

The average value of the civil year

$$= \frac{365 \times 303 + 366 \times 97}{400}$$

$$= 365{\cdot}2425 \text{ days,}$$

and is accordingly approximately equal to the solar year, which contains 365·2422 mean solar days.

(iv.) The *Sidereal Day* is defined as the interval between two consecutive transits of the first point of Aries across any selected meridian, and is therefore equal to the period of rotation of the earth with reference to the fixed stars—the value is 23 hours, 56 minutes, 4·0906 seconds.[1]

(v.) The *Sidereal Year* is the time interval in which the sun appears to perform a complete revolution with reference to the fixed stars.

(vi.) *Equivalents.*—

1 tropical or solar year . . .	=365·2422166 mean solar days.
1 sidereal year .	=366·2564 sidereal days.
	=365·2564 mean solar days (epoch 1900).
1 mean solar day .	=86,400 sec.
	=0·002737909 mean solar year.
	=1·00273791 sidereal days.
	=24 hr. 3 min. 56·56 sec. in sidereal time.
1 sidereal day .	=86,164·0906 sec.
	=0·99727 mean solar day.
	=23 hr. 56 min. 4·09 sec. in mean time.
If 1 year . .	=360°.
	1 mean solar day = 0° 59′ 8·33″.
	1 week . . = 6° 53′ 58″.
	30 days . . = 29° 34′ 10″.
1 hour . . .	$=1{\cdot}140795 \times 10^{-4}$ year.
	=0° 2′ 27·847″.
1 minute . .	$=1{\cdot}90132 \times 10^{-6}$ year.
	=2·464″.
1 second . .	$=3{\cdot}168866 \times 10^{-8}$ year.
	=0·041066″.

Length of the seconds pendulum in London =39·13929 in.

---

[1] Owing to the precession of the earth's axis the true period of the earth's rotation is approximately ·01 sec. longer than the sidereal day.

(vii.) *Rotation of the Earth.*—

Relative to a star . $\omega = 0{\cdot}00007292$ r./s.
Relative to the sun . 1 hr. = 15°.
1° = 4 min.

	Revolutions.	Radians.	Degrees, etc.
Sidereal day .	1	2π	360°
Mean solar day . .	1·00273791	6·300388	360·98565°
Hour . .	$4{\cdot}178075 \times 10^{-1}$	$2{\cdot}625162 \times 10^{-1}$	15·04107°
Minute .	$6{\cdot}963458 \times 10^{-4}$	$4{\cdot}375270 \times 10^{-3}$	15·04107′
Second . .	$1{\cdot}160576 \times 10^{-5}$	$7{\cdot}292116 \times 10^{-5}$	15·04107″

See Vol. I. "Measurement, Units of."

Time Standard.—For the British Isles and the greater part of Western Europe (France, Belgium, Spain, and Portugal) Greenwich Mean Time is the standard and is known as G.M.T. or W.E.T. (Western European Time). For other countries a system of zone time has been adopted in which the time is referred to some standard meridian chosen so that the difference between the standard time for the zone and G.M.T. is a whole number of hours or half-hours. Thus zone 0 lies between 7½ W. and 7½ E. and keeps the time of meridian 0°, *i.e.* G.M.T.; zone 1 lies between 7½ W. and 22½ W., and keeps the time of meridian 15° W., *i.e.* one hour behind G.M.T.; zone −1 is between 7½ E. and 22½ E., and keeps the time of meridian 15° E. one hour in front of G.M.T. Some adjustment of the zones is made on account of political boundaries.

(i.) *Local Mean Time.*—In order to convert time in G.M.T. into local mean time add 4 minutes for each degree of longitude for places east of Greenwich, and subtract 4 minutes for each degree for places west of Greenwich.

(ii.) *Apparent Time.*—Time based on the length of the true solar day is called "apparent time," and it is this which is measured by a sundial or sunshine recorder. In order to obtain local apparent time from local mean time a correction must be applied, known as the equation of time. The value of the correction is zero on April 16 and June 15, reaches maxima of + 16 minutes 21 seconds on November 3, and + 3 minutes 49 seconds on May 14; and minima of − 14 minutes 25 seconds on February 12, and − 6 minutes 18 seconds on July 26—a positive sign meaning that the value is to be added to mean time to obtain apparent time, and a negative sign meaning that the value is to be subtracted. Accurate values of the equation of time for each day are given in the *Nautical Almanac.*

(iii.) *Sidereal Time.*—If a great circle be drawn from the pole to a star the angle this hour circle makes with the meridian is termed the *hour angle.* The hour angle west of the first point of Aries, turned into time at the rate of 15° per hour, is known as sidereal time.

(iv.) *Summer Time.*—Since 1916 it has been the practice in most countries of Western Europe to use Summer Time, which is one hour in advance of G.M.T. The period over which summer time extends varies in different countries and from year to year.

See Vol. I. "Measurement, Units of."

Time Zones. See "Clocks and Time-keeping," § (1).

Time and Azimuth, Determination of:

By equal altitudes of three or more stars. See "Latitude, Longitude, and Azimuth by Observation in the Field," § (5).

By observation in the field, the latitude being known approximately. See *ibid.* § (2).

Tolerances in Engineering Work:

On cylindrical parts, for various classes of fit, standardisation of. See "Metrology," § 28.

Shaft and hole bases. See *ibid.* § (30) (i.).

On gauges. See *ibid.* § (18).

On screw gauges. See *ibid.* § (27).

On screw threads. See *ibid.* § (26).

Unilateral and bilateral. See *ibid.* § 29 (ii.).

Tono-micrometer: an instrument, whose construction depends on the quartz fibre, for detecting the faintest musical sound of the pitch to which it is tuned. See "Radio-micrometer," etc., § (4).

Torsion Balance: a laboratory instrument for measurement of the constant of gravitation by the torsion method of balancing very small couples. See "Earth, Density of," § (2).

Tralles: alcohol tables. See "Alcoholometry," § (4).

TRIGONOMETRICAL HEIGHTS AND TERRESTRIAL ATMOSPHERIC REFRACTION

§ (1) **Effect of Refraction.**—When a series of triangulation is executed, it is usual at each station to observe the angles of elevation of all the surrounding stations with a view to determining their relative heights. The rays joining stations are not straight, but are bent into a curve by the action of atmospheric refraction. This bending takes place almost entirely in a vertical plane, and the horizontal component is in general too small to be detected except in the case of close grazing rays; although in first-class work triangular errors do not in general exceed 1″·0 or less, and a refraction effect of 1″ would be recognised. This lends support to the otherwise natural conjecture that the distribution of the atmosphere is symmetrical about the vertical, or, in other words, that the layers of equal density in the air are horizontal, for the refractive index of air diminished by unity varies as the density of the air. Some method has to be adopted by means of which the refraction in the vertical plane can be evaluated.

§ (2) THEORY. (i.) *General.*—It has been customary, for the purposes of many computations occurring in survey operations, to regard the earth as being of spheroidal form, and this is one assumption on which the generally adopted method of dealing with refraction rests. In *Fig.* 1 is shown a vertical section through two stations A, B, whose verticals cut each other at O and the sea-level spheroid at A_0B_0. The path of light between A and B is shown by the dotted line, and the tangents to this at A and B meet in T. Thus the angle of refraction at A of B is $\Omega_1 = B\hat{A}T$, and similarly $\Omega_2 = A\hat{B}T$.

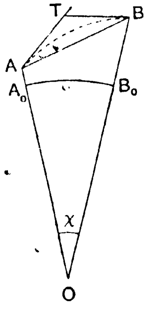


FIG. 1.

This vertical angle of B observed by theodolite at A is clearly an elevation $E = O\hat{A}T - 90°$. The distance A_0B_0 is found from the triangulation, and this combined with the known radius OA_0 allows the angle χ to be computed. The relation

$$O\hat{A}B - 90° + O\hat{B}A - 90° + \chi = 0$$

may be written

$$E_1 - \Omega_1 + E_2 - \Omega_2 + \chi = 0, \quad . \quad . \quad (1)$$

in which the only unknown quantities are Ω_1, Ω_2. Thus the sum of these two refraction angles is found if the angles of elevation at both A and B are observed simultaneously. It has generally been *assumed as a practical working method* that $\Omega_1 = \Omega_2 = \Omega$, so that $\Omega = \frac{1}{2}(E_1 + E_2 + \chi)$. It is also assumed that $\Omega = k\chi$ when k is a constant for various values of χ and is called the coefficient of refraction. These two assumptions are tantamount to saying that the path of light from the station A is always circular and of the same radius for all angular elevations. It is clear that the coefficient of refraction is given by $k = (1/2\chi)(E_1 + E_2 + \chi)$. To obtain a value of k at A observations at A and surrounding stations are made and an average of the results obtained is used. When it is not convenient to visit any station but A, a value of k may be selected based on experience and on the values of k found elsewhere.

(ii.) *Corrections.*—Some restrictions are necessary to ensure reasonable accuracy resulting from this method. Atmospheric refraction between two points is by no means a constant quantity. It was noticed about 1840 that the refraction generally had a minimum value sometime between 1 P.M. and 4 P.M.; and its values at these hours on different days vary less than the values observed at other times. In practice it is generally impossible to observe the angles at both ends of a ray simultaneously. It has accordingly become the custom to make all observations of vertical angles at or about the time of minimum refraction, whereby the equation

$$k = \frac{1}{2\chi}(E_1 + E_2 + \chi)$$

may be used with greatest chance of success. It will readily be seen that this equation has no proper application if used with values of E_1, E_2 taken at any and different hours.

(iii.) *Local Conditions.*—When a value of k has been arrived at, it is possible to write down a value $k\chi$ of the angle of refraction for any observation taken at, or about, the same time of day, and so to deduce a value of the height of any observed object whose distance is known; but it is not to be supposed that a good estimation of the refraction has been made without doubt. Atmospheric refraction is a very variable quantity and local conditions affect it largely. For example, a ray close to the ground behaves very differently from a ray fifty feet above it. The procedure given above is really only suitable to give fairly good results if used with some discretion. Cases occur in plains where no adequate allowance can be made for refraction and vertical angles in conjunction with triangulation are useless. It is to be marked, too, that the time of minimum refraction generally agrees with the time of minimum clearness; and often a low-lying object is so blurred at this time that observations on it are impossible. In other cases in hills refraction changes without warning and differs by considerable amounts at the hours of minimum refraction on successive days. It is partly on this account that in geodetic triangulation it is usual to extend observations of vertical angles over at least three days.

(iv.) *Values Found.*—Various values of k have been determined in different countries and at different heights. Clarke (Bib. 3, 4) has distinguished between rays over land and those mainly over water. As refraction depends on the density of the air, k should decrease at greater altitudes. Values collected by Montgomerie (Bib. 15) from Himalayan observations, however, show very considerable irregularities from what might be expected. Walker's values (Bib. 5) in the Punjab give an indication of variations which are liable to occur in rays close to the ground.

(v.) *Criticism of the Method.*—The procedure explained above is not fully satisfactory, considered from either the theoretical or practical standpoint. This has been recognised in practice, and when possible heights found from triangulation have been adjusted on spirit-levelled values—a method which may lead to results satisfactory enough for topographical maps. But in much of the triangulation on which topographical maps depend it is impossible to restrict observations to time of minimum refraction, and spirit-levelling is not always available; so that very considerable errors may occur in heights determined by vertical angles. For geodetic purposes the method has little to commend it, and for these it should not always be followed.

For topographical purposes a working method of finding the refraction appropriate to the time and day of observation is required, especially when the time does not coincide with the time of minimum refraction. For dealing with observations taken at times considerably different from that of minimum refraction no very satisfactory guidance can be given. At heights such as 10,000 feet the diurnal change in refraction has been found to be negligible, and at lower heights it becomes increasingly more apparent. In a series of cases it has been found that the refraction at a station varies during the day pretty closely as the temperature, and (Bib. 11) may be approximately expressed by $\Omega - \Omega_m = M(\tau_m - \tau)$, where M is a constant for a given ray for all times (probably the same for all rays proceeding to the same height) τ_m is the maximum temperature, and Ω_m is the minimum refraction. The value of M, which is an empirical quantity, would have to be found experimentally in each case, until some rules for its evaluation can be laid down; and this stands in the way of its practical application meanwhile. For geodetic purposes the whole question must be more carefully considered, which will now be done very briefly.

§ (3) A MORE EXACT THEORY.—It is to be remembered that the sea-level surface of the earth which is generally called the "geoid" does not exactly coincide with any spheroid. A closer explanation of the distinctions between spheroid and geoid will be found in the article "Gravity Survey."

The primary object of geodesy is to find the form of the geoid, which can most easily be exhibited by its deviations from some selected spheroid of reference. The geoid, being an irregular figure, does not form a convenient basis for formulae whereby computations of triangulation can be carried out; and so these are done in relation to the spheroid. In the reduction of the horizontal angles the inaccuracies brought in by applying geoidal angles to the spheroid are small. Where vertical angles are concerned the case is different. The verticals to the spheroid and geoid at a point do not in general agree. The spheroidal vertical is defined by the latitude and longitude of the point computed by triangulation, while the geoidal vertical can be obtained by astronomical observations. The difference of the two verticals is what is generally called the deflection of the plumb-line. Now all vertical angles measured by a properly levelled theodolite refer to the geoidal vertical and accordingly are not rightly introduced into a formula based on the geometry of the spheroid. To be consistent it is necessary in (1) either to apply corrections to the elevations E_1, E_2 to express them in terms of the spheroidal vertical, or else to compute χ properly for the geoid and not for the spheroid as is the usual practice. Thus if the plumb-line deflection at A towards B is δ_1 (and at B towards A is δ_2) the equation (1) may be correctly written

$$E_1 + \delta_1 + E_2 + \delta_2 + \chi = \Omega_1 + \Omega_2, \quad . \quad (2)$$

χ being computed for the spheroid. Here δ_1 and δ_2 being measured in opposite senses very often tend to cancel. But in hilly country, such as the Himalayas, large values of δ up to 1′ of arc occur, and uncancelled amounts of $\delta_1 + \delta_2$ may reach 30″, which will lead to a very considerable error in the estimation of $\Omega_1 + \Omega_2$ (which is about 6″ per mile of ray length). It is very easy to see that a 20 per cent error can easily arise in the determination of "k" on this account. The quantity δ_1 must also be introduced into the formula for the spheroidal height of B above A, which can be properly derived from the formula OB cosec $O\hat{A}B$ = OA cosec $O\hat{B}A$, in which $O\hat{A}B = E_1 + \delta_1 - \Omega_1$ and $OBA = E_2 + \delta_2 - \Omega_2$.

The resulting height has no exact connection with that derived from spirit-levelling, for this latter is the geoidal height. By the nature of the observation the spirit level is always set up parallel to the geoid, and by virtue of the large number of times of setting up follows the geoid in great detail; and at no point assumes the form of the geoid to be a spheroid (except maybe in the application of the orthometric and dynamic height corrections). The two heights of a point derivable from vertical angles (with due allowance for refraction and plumb-line deflection), and spirit-levelling, obviously differ by the same amount as the spheroid differs from the geoid; giving a measure of the height of the geoid above the spheroid and so showing what is the true figure of the geoid. It is of primary importance in deduction of values of k that the deflections δ_1, δ_2 should be introduced as in equation (2).

Now the assumption that the refraction angles of the two ends of a ray are equal, even when these are simultaneously observed, cannot be fully justified except when the two ends are at the same height. Nor is it logical to consider a coefficient of refraction the same for all angular elevations, and thus, by transference from one end of a ray to the other, the same for all heights. Refraction depends on the density of the air. It is greater in denser air than in air less dense. At one station it is greater on a descending ray than on an ascending ray. These are facts which can be readily verified, especially in the case of observations taken when the refraction is not a minimum. The idea of coefficient of refraction should be restricted to horizontal rays. Denoting this by k it may be possible to express the refraction angle on any other ray by $k(1+\beta h)\chi$, where β is a constant at any moment and h is the height above the station attained by the ray. But neither k nor β are absolute constants for different days.

§ (4) TEMPERATURE EFFECTS.—In the case of an atmosphere enclosing a gravitating sphere the atmosphere would be in mechanical and thermal equilibrium if the successive layers of equal density (and also of equal pressure and temperature) were arranged in concentric spheres; and if the adiabatic relation between pressure and density in passing from any isobar to another were maintained. This latter condition gives rise to a temperature-height gradient which is named the adiabatic gradient and amounts to fall of temperature of about 0°·54 F. per 100 feet of height for dry air. For moist air it is about 0°·3 F. per 100 feet. The atmosphere, however, is subjected to many disturbing effects of which the main is the diurnal change from night to day. This only allows the atmosphere to adjust itself somewhat nearly as laid down above, and this state is in general most nearly attained at much the same time that the air at the lower level reaches its maximum temperature. The reason of this can be seen readily if the causes are looked into. The lower air takes its temperature mostly from the surface heat of the earth. If then the layer in contact with the hot earth becomes hotter in comparison with what is above than it should be according to the adiabatic gradient, its tendency is to rise continuously through any layers in which the gradient persists. And it will continue too hot for its place during this process until it has given up its excess heat by conduction. Excessively hot air in any lower layer of the atmosphere is accordingly caused to dissipate its superfluous heat to higher layers, whose temperature it raises slightly. It will be seen, then, that so long as the lower layers are too hot there persists a tendency to adjust temperatures; and if sufficient time were allowed the adjustment would become perfect. Moreover, the temperature gradient cannot for long exceed the adiabatic gradient. If the earth is chilled by radiation, as occurs at night, the air in contact with it is also chilled to a temperature below that fitting adiabatically what is above. The tendency of the lower air would be to fall were it not precluded from doing so by the earth itself. It is only possible for it to receive heat by conduction from what lies above it, thereby chilling the air to a height which increases with time. Here then is no convection. When the earth's surface is of high radiative power this may lead even to a temperature inversion—*i.e.* a rising of temperature with height up to a certain point and later a fall of temperature—a condition of affairs brought into evidence by the hanging of smoke at a constant level. Of these two processes it is clear that the former, in which the lower layers of air become heated and strive towards equilibrium with the help, where necessary, of convection, leads to a much more uniform state of affairs than can be expected from the slow latter process.

Observations of angles of elevation of an object day after day and at stated hours show in general that greater uniformity of results occur among those taken at the time of maximum temperature, at which time the refraction is least. If observations are continued during a number of months in the year, some seasonal effect appears in the amount of the refraction at maximum temperature. True minimum refraction may be considered to be what occurs when the gradient is adiabatic; and this in general is a quantity fairly closely approximated to in case of observations taken at times of maximum temperature.

§ (5) REFRACTION FORMULAE.—Formulae may be derived which represent the refraction under normal conditions. These are based on the assumption that the air is distributed symmetrically as regards density (and consequently also pressure and temperature) about the vertical; and they require some knowledge as to the relation obtaining between the temperature at a point in the air and its height above any convenient datum. It is then necessary to make use of the following conditions:

$p = C\tau\rho$, where p = pressure,
τ = absolute virtual temperature (thus allowing for humidity),
ρ = density,
$dp = -\rho g dh$, h = height above datum,
$\mu - 1 = K\rho$, μ = refractive index of air for dominant wave length (say sodium D line),
$\mu(r+h)\sin\phi = B$, ϕ = direction measured from vertical,
C = constant,
K = constant,
B = constant.

From these it can be shown that the true "minimum" refraction corresponding to the temperature-gradient which is adiabatic (viz. $a = 0°·542$ F. per 100 feet for dry air) may be expressed:

$$\omega = \omega_1 + \omega_2,$$

where
$$\omega_1 = 3·475 \times 10^4 \left(1 + \frac{h_b}{R}\right)\left(\frac{g_o}{g_s}\right)^2 \frac{lH}{\tau^2}$$
$$\therefore 3·475 \times 10^4 \frac{lH}{\tau^2}$$

and
$$\omega_2 = -13·8 \frac{l \cot\phi}{\tau}\omega_1,$$

in which

l = sea-level length of trace of ray in miles,
h_b = height of observed point in feet,
R = radius of earth,
g_o = gravity at station A,
g_s = gravity at sea level, latitude 45°,
H = barometer reading in inches corrected for temperature of mercury only,
τ = absolute temperature (Fahrenheit),
ϕ = apparent Z.D. of object observed.

The adiabatic gradient is not usually reached, and the refraction can for the general case be represented by

$$\Omega = f_1\omega_1 + f_2\omega_2,$$

where ω_1 and ω_2 are as given above.

$$f_1 = \frac{1·869 - a}{1·327},$$

$$f_2 = \frac{(1·869 - a)(1·869 - 2a)}{1·042},$$

in which

$$a = 10^2 \frac{d\tau}{dh} = \text{fall in temperature F. per 100 feet},$$

$$b = \frac{\tau}{1·042} 10^4 \frac{d^2\tau}{dh^2}.$$

§ (6) APPLICATION OF THE FORMULAE.—The main difficulty in applying these formulae is that knowledge of $d\tau/dh$ and $d^2\tau/dh^2$ is required, which is not generally available. Perhaps the most obvious way of gaining it is by readings of pressure and temperature at two known heights. Such measurements ought certainly to be made in any case where precision is desired. Failing them, the only course is to take average values, which can hardly be expected to apply very well in all countries of widely different character and climate.

An average observed gradient of temperature in the atmosphere is 0°·6 C. per 100 metres (Bib. 18) or 3°·3 F. per 1000 feet. This agrees with Kelvin's adiabatic gradient for *moist* air (Bib. 19). It seems to be the best gradient which can be chosen for the time of minimum refraction when no special determination has been made. Taking the gradient as 3° F. per 1000 feet, the table given on the following page has been constructed from the formulae of § (5) to give the best value of k_o, the coefficient of *horizontal* refraction, appropriate to any height.

If temperature and pressure are known, or estimated, at the station of observation somewhat different from the tabular values, allowance can be made by aid of the correction columns. The table also gives the values of log $(1 - 2k_o)$, which is the quantity which actually enters the usual computations for heights.

So far a horizontal ray has been considered. It is easily shown that, if the temperature gradient is uniform along a sloping ray—it has already been assumed to be 3° F. per 1000 feet throughout—that the coefficient of refraction for the sloping ray is the coefficient of horizontal refraction for height $H_a + \frac{1}{3}\Delta h$, where H_a is the height of the observing station, and Δh is the height above this station of the point observed. The procedure accordingly is to estimate Δh roughly from a special height indicator chart, and take out k_o for the height $H_a + \frac{1}{3}\Delta h$, applying corrections for the differences of temperature and pressure tabulated for H_a and observed there. This process has (1920) given encouraging results when applied to a number of Indian observations.

For further information on this subject see Bib. 11.

§ (7) EFFECT OF SLOPING GROUND AND RAPID TEMPERATURE CHANGES. (i.) *Lallemand.*—Lallemand has proposed formulae for refraction of rays which are close to sloping ground (Bib. 12 and 13). These occur when a line of spirit-levelling is carried up a hill road. He assumes that the layers of equal air temperature in this case are parallel to the ground, which seems a very proper assumption for rays within ten feet of the ground. For the case when the level is placed half-way between the two levelling staves he obtains

$$E = -0·00108 \frac{B}{760} \frac{t_3 - t_1}{(1 + \alpha\theta)^2} \frac{L^2}{D} \phi(\delta)$$

in millimetres, where E is the error, B is the barometer reading in mm., t_1, t_2, t_3 are the temperatures in degrees centigrade of the air at points on the ray where it meets the

Height above or below M.S.L.	Barometer β.	Temperature t.	K.			Log $(1-2k)$.		
			k_0.	Correction for		Log $(1-2k_0)$.	Correction for	
				+1″.	+10°.		+1″.	+10°.
− 2,000	30″·9	85°	0·0823	+·0027	−·0031	$\bar{1}$·9219	−·0028	+·0032
0	30″·0	82	0·0804	·0027	·0030	$\bar{1}$·9239	·0028	·0031
+ 1,000	29″·0	79	0·0784	·0027	·0030	$\bar{1}$·9259	·0028	·0031
2,000	28″·0	76	0·0766	·0027	·0028	$\bar{1}$·9278	·0028	·0029
3,000	27″·0	73	0·0749	·0028	·0028	$\bar{1}$·9295	·0029	·0029
4,000	26″·1	70	0·0732	·0028	·0027	$\bar{1}$·9313	·0029	·0027
5,000	25″·2	67	0·0714	·0028	·0027	$\bar{1}$·9331	·0029	·0027
6,000	24″·3	64	0·0696	·0029	·0027	$\bar{1}$·9349	·0029	·0027
7,000	23″·5	61	0·0679	·0029	·0026	$\bar{1}$·9366	·0029	·0026
8,000	22″·6	58	0·0662	·0029	·0026	$\bar{1}$·9383	·0029	·0026
9,000	21″·8	55	0·0647	·0030	·0025	$\bar{1}$·9398	·0030	·0025
10,000	21″·0	52	0·0631	·0030	·0025	$\bar{1}$·9414	·0030	·0025
11,000	20″·3	49	0·0615	·0030	·0024	$\bar{1}$·9430	·0030	·0024
12,000	19″·5	46	0·0599	·0031	·0024	$\bar{1}$·9446	·0031	·0024
13,000	18″·8	43	0·0584	·0031	·0024	$\bar{1}$·9461	·0031	·0024
14,000	18″·1	40	0·0569	·0031	·0022	$\bar{1}$·9475	·0031	·0022
15,000	17″·4	37	0·0555	·0032	·0022	$\bar{1}$·9490	·0031	·0022
16,000	16″·8	34	0·0541	·0032	·0022	$\bar{1}$·9503	·0031	·0021
17,000	16″·1	31	0·0526	·0033	·0021	$\bar{1}$·9517	·0032	·0020
18,000	15″·6	28	0·0513	·0033	·0021	$\bar{1}$·9530	·0032	·0020
19,000	15″·0	25	0·0500	·0033	·0020	$\bar{1}$·9542	·0032	·0019
20,000	14″·4	22	0·0486	·0034	·0020	$\bar{1}$·9556	·0033	·0019
21,000	13″·8	19	0·0474	·0034	·0020	$\bar{1}$·9567	·0033	·0019
22,000	13″·2	16	0·0460	·0035	·0019	$\bar{1}$·9581	·0033	·0018
23,000	12″·6	13	0·0444	·0035	·0019	$\bar{1}$·9596	·0033	·0018

front staff, telescope, and back staff respectively, D is difference of staff readings, L the distance between staves, α is the coefficient of expansion of air, θ the mean of the three temperatures observed,

$$\phi(\delta)=\frac{\mu-\frac{1}{2}\log(1-\delta^2)}{\log(1+\delta)-\log(1-\delta)}-\frac{1}{2\delta},$$

where $\mu=0{\cdot}434$ (logarithmic modulus),

$$\tau=\frac{t_3-t_2}{t_2-t_1}=\frac{\log(1+\delta)}{\log(1-\delta)},$$

which defines δ in terms of the three temperatures. Values of $\phi(\delta)$ for various values of τ are shown in table:

τ	0·0	0·1	0·2	0·27	0·3	0·4
$\phi(\delta)$	0·000	0·040	0·063	0·066	0·065	0·059
τ	0·5	0·6	0·7	0·8	0·9	1·0
$\phi(\delta)$	0·049	0·038	0·028	0·017	0·007	0·000

The expression above is derived from the empirical temperature law, $t=a+b\log(h+c)$. The coefficients a, b, c can be found when temperatures at three levels have been observed; but this is not necessary for the application of the first formulae.

(ii.) *Cole* (see Bib. 14) has found in Egypt that the systematic error in levelling is considerably reduced if the staves are read in order back, forward, forward, back. He considers that, especially in hot countries, the air temperature near the ground is changing rapidly, and that the effect of refraction, even when the ground is level, is rapidly varying; so that when the second staff is read, refraction is perhaps greater than when the first is read. By taking the staves in the reverse order, the variation effects the results in the opposite sense, and the mean of the readings is freed from this time variation of refraction effect.

§ (8) **Range-finding by Depression Range-finder.**—The formulae given in § (5) may be applied to D.R.F. observations to take account of abnormal refraction. In this connection it is to be remarked that very extraordinary values of refraction have been observed on rays over the sea at small heights above it. The D.R.F. is usually constructed to allow for normal values of refraction and the effect of anomalous refraction could be introduced as a correction. As before, values of $d\tau/dh$, $d^2\tau/dh^2$ are required. Unless the land station is at a considerable height above the sea the term involving $d^2\tau/dh^2$ is only small, and very probably may be left out of account in most cases. The value of $d\tau/dh$ is important It is not a quantity which can be measured precisely by thermometers unless these are of a special kind, as the interval h is small and the temperature differences will be very minute reckoned in degrees. An alternative way of evaluating it depending on the dip of the horizon is available here. The dip may be represented by a'' (in seconds).

$$a'' = 56''{\cdot}33\sqrt{h'\left(1 - 15{\cdot}13\frac{\Delta\tau'}{h'}\right)},$$

in which

$$h\,(1 - {\cdot}2204) = h(1 - {\cdot}2204\mathrm{F}_1),$$

$$\Delta\tau' = \mathrm{F}_2\Delta\tau,$$

$$\mathrm{F}_1 = \frac{g}{g_s}\cdot\frac{\rho_m}{\rho_s}\cdot\frac{519{\cdot}4}{\tau},$$

$$\mathrm{F}_2 = \frac{\rho_0}{\rho_s}\cdot\frac{519{\cdot}4}{\tau}.$$

g = gravity at station,

g_s = gravity at sea level, latitude 45°,

ρ_0 = density of air at sea level,

ρ_m = an average density of air between station and sea,

ρ_s = an average standard density.

If the computed value is compared with that observed, an equation is formed from which $\Delta\tau/h = dt/dh$ can be found.

§ (9) FIELD ASTRONOMY AND ASTRONOMIC ATMOSPHERIC REFRACTION.—Bessel's or more recent refraction tables may be used with confidence for any of these observations. Refraction enters directly into those observations in which star altitudes are observed, and only indirectly in observations for time by meridian transits and azimuth by circumpolars or Polaris, or E. and W. stars. In latitude observations by Talcott method (see "Gravity Survey") it is usual to employ only stars of Z.D. < 40°. As these are in pairs N. and S. of nearly equal Z.D., the refraction, assumed symmetrical about the vertical, practically cancels in the mean. The same occurs in the circum-meridian altitude observation to a lesser degree.

In observations of altitudes of E. and W. stars for time, in topographical work the Z.D. would generally lie between 50° and 70°. If the E. and W. stars are well matched the effect of refraction, having opposite signs for E. and W. stars, will partially cancel. However, for Z.D.s not exceeding 70° the refraction can be computed from tables very satisfactorily, if the air pressure and temperature are observed. It is, of course, the temperature of the air in the open and *not* in the observatory tent that is required.

For Z.D.s up to 70° the refraction differs by < 2″ from what is derived from supposing the layers of equal density to be plane, in which case any law of variation of density with height leads to the same total refraction. By adding another term the refraction is given for Z.D.s up to 80° with an error < 1″; and the term which takes account of curvature of the isobars involves the total amount of air in a vertical column and is susceptible of accurate computation and tabulation. At greater Z.D.s other terms become appreciable in rapidly increasing degree, and these involve the actual variation of density with height which cannot be allowed for, except in an average way as done by Bessel and Argelander.

For all practical purposes of field astronomy the astronomic refraction is represented with an error < 1″ up to Z.D. 80° by

$$\psi = \psi_a - \tan\phi\sec^2\phi\,.\,\frac{\mu - 1}{r}\,.\,\mathrm{H}.$$

Where μ = refractive index of air, H is height of homogeneous atmosphere of density found at the station, r is the radius of the earth, and ψ_a is given by $\sin(\phi - \psi_a) = \mu\sin\phi$ in which ϕ is the observed Z.D. With the same precision this equation may be written in form $\psi_a = (\mu - 1)\tan\phi$. Following the general procedure of taking as standard barometer $\mathrm{H}_s = 30''$, and standard temperature $\tau_s = 459^\circ{\cdot}4 + 50^\circ$ F., and putting in numerical values, the astronomic refraction may be expressed in seconds:

$$\psi'' = 58{\cdot}4\frac{\mathrm{H}}{\mathrm{H}_s}\cdot\frac{\tau_s}{\tau}\tan\phi - {\cdot}076\left(\frac{\mathrm{H}}{\mathrm{H}_s}\right)^2\frac{\tau_s}{\tau}\tan\phi\sec^2\phi$$
$$= \psi_1 - \psi_2.$$

Values of these quantities for a few typical Z.D.s, with standard pressure 30″ and temperature 50° F., are as follows:

ϕ.	45°.	60°.	70°.	75°.	80°.
ψ_1	58″·4	1′ 41″·2	2′ 40″·6	3′ 38″·3	5′ 32″·8
ψ_2	0″·1	0″·5	1″·7	4″·3	14″·3
ψ''	58″·3	1′ 40″·7	2′ 38″·9	3′ 34″·0	5′ 18″·5
Bessel's Values	58″·2	1′ 40″·6	2′ 38″·8	3′ 34″·1	5′ 19″·2
Difference . . .	−0″·1	0″·1	0″·1	+0″·1	+0″·7

The last equation shows how refraction depends on temperature and pressure, and that the main portion of it varies as H/τ.

In geodetic latitudes only stars within 40° of Zenith are used, for which clearly the formula $\psi_a = (\mu - 1) \tan \phi$ may be considered rigidly correct, unless we are to consider the case of unequal refraction in various azimuths for which there is no means at present of allowing. Apart from purely local irregularities it is hardly conceivable that this should reach appreciable amount. In applying a correction for a pair of N. and S. stars in the Talcott observation it is clear that a differential formula may be used, $d\psi_a = (\mu - 1) \sec.^2 \phi d\phi$, which will lead to as precise a result as if each observation were corrected independently.

All formulae for astronomic refraction are based on the assumption that the layers of equal density of the air are symmetrical with respect to the local vertical. Considering the earth as a sphere the equal density surfaces are assumed to be concentric spheres. The formulae derived give values whose application to actual observations give consistent results and so support the validity of the assumption. Slight systematic errors in refraction may be conceivable as due to local peculiarities; and the form of the meridian slit of an observatory is designed with a view to lessening these. At such a slit the warmer air of the observatory enclosure meets with the cooler outer air. Such a cause might be effective in showing a false seasonal change in variation of latitude; but its amount cannot be other than small.

BIBLIOGRAPHY.

1. *Tabulae Regiomontanae Reductionum Observationum Astronomicarum.* By F. W. Bessel. 1830.
2. *Practice of Navigation and Nautical Astronomy,* Tenth Edition. By Lieut. Raper, R.N. London, 1870.
3. *Account of the Principal Triangulation (of the British Isles).* By Colonel A. R. Clarke, F.R.S. 1858.
4. *Geodesy.* By Col. A. R. Clarke. Oxford, 1880.
5. *Account of the Operations of the Great Trigonometrical Survey of India,* vol. ii. Appendix 3. By General J. T. Walker, F.R.S. Dehra Dun, 1879
6. *A Manual of Spherical and Practical Astronomy.* By William Chauvenet. Philadelphia, 1868.
7. *Astronomical Journal,* Boston, vol. xxii. No. 526. "Terrestrial Refraction and the Trigonometrical Measurement of Heights." By William Harkness.
8. *Spherical Astronomy.* By Simon Newcomb. New York, 1906.
9. *Encyclopaedia Britannica.* Eleventh Edition. Cambridge, 1911. Article "Navigation." By Captain W. R. Martin, R.N.
10. Professional Paper of the Survey of India, No. 11. "Observations of Atmospheric Refraction." By H. G. Shaw. Dehra Dun, 1911.
11. Professional Paper of the Survey of India, No. 14. "Formulae for Atmospheric Refraction and their Application to Terrestrial Refraction and Geodesy." By J. de Graaff Hunter, M.A., Sc.D. Dehra Dun, 1913.
12. *Comptes Rendus des Séances de la Commission permanente de l'Association Géodésique Internationale.* By Ch. Lallemand, 1896.
13. *Account of the Operations of the Great Trigonometrical Survey of India.* vol. xix. Appendix 4. By J. de Graaff Hunter, M.A., Sc.D. Dehra Dun, 1910.
14. Survey of Egypt, Departmental Paper, No. 35. "Systematic Error in Spirit Levelling." By J. H. Cole. Cairo, 1919.
15. Survey of India, Synoptical Volume, vii. Preface, pp. ix, x. By General Walker, F.R.S. Dehra Dun, 1879.
16. Survey of India, Synoptical Volume, xxxv. Preface, pp. viii-xviii. "Probable Errors of Heights of Peaks." By Col. Sir S. G. Burrard, K.C.S.I., R.E., F.R.S. Dehra Dun, 1909.
17. *A Sketch of the Geography and Geology of the Himalayan Mountains and Tibet,* pp. 23-30. By Col. Sir S. G. Burrard, and Sir H. H. Hayden. Dehra Dun, 1907.
18. *Phys. Soc. Proc.*, xxxii. Part v. 265. By C. H. Lees, F.R.S.
19. *Mathematical and Physical Papers,* viii. 255-260. By Right Hon. Lord Kelvin, O.M., etc.

J. DE G. H.

TROPOPAUSE: the boundary between the troposphere and the stratosphere. See "Atmosphere, Physics of," § (5). Also "Atmosphere, Thermodynamics of," § (5).

Height of:
- In cyclones and anticyclones. See "Atmosphere, Thermodynamics of the," § (5), Table III.
- Variation with latitude. See *ibid* § (5), Table II.

TROPOSPHERE: the lower region of the atmosphere in which the temperature on the average decreases with height.
- Distribution of temperature in. See "Atmosphere, Thermodynamics of the," §§ (4), (5).
- Variation of temperature in. See "Atmosphere, Physics of," § (5). Also "Atmosphere, Thermodynamics of," § (5).
- Variation of wind in. See "Atmosphere, Physics of," § (10).

TURBULENCE IN THE ATMOSPHERE. See "Atmosphere, Physics of," §§ (12), (13), (14).
- Transference of heat by. See "Radiation," § (3) (iv.).

TUTTON WAVE-LENGTH COMPARATOR: description and method of using. See "Comparators," § (4).

TYNDALL, PROFESSOR. Apparatus for the measurement of volume of water-vapour present in the air. See "Humidity," § (13) (ii.).

— U —

- Ullage: the volume of liquid in a partially filled cask. See "Volume, Measurements of," § (6).
- Ulloa's Ring (or Fog-bow): a white rainbow. See "Meteorological Optics," §§ (14) and (15) (iii.).
- Units of Measurement (in meteorology). See "Atmosphere, Thermodynamics of the," § (2).
- Units of Volume. Volumes are measured in terms of two kinds of units: (1) those based on units of length, *e.g.* the cubic foot; (2) those defined as the volume of a definite mass of liquid, *e.g.* the gallon. See "Volume, Measurements of," § (1).
- Units of Weights and Measures:
  - British imperial. See "Metrology," § (6) (i.).
  - Metric international. See *ibid.* § (6) (ii.).
  - Subsidiary and derived. See *ibid.* III. § (9).
- Upper Air:
  - Distribution of pressure in. See "Atmosphere, Thermodynamics of the," § (7), *Figs.* 11 and 12.
  - Distribution of realised entropy in. See *ibid.* § (6), *Figs.* 9, 10.
  - Distribution of temperature in. See *ibid.* §§ (3), (4), (5), (7), (11), (12); Tables II., III., VI.
  - Distribution of water-vapour in. See *ibid.* § (5), *Fig.* 9.
  - High conductivity of and its consequences. See "Atmospheric Electricity," § (17).
  - Instruments for measurement of meteorological elements. See "Meteorological Instruments," VIII. § (36), etc.
  - Relation between temperature and height. See "Air, Investigation of the Upper," § (11). Also "Atmosphere, Physics of," § (2).
- U.S.A. National Coarse Thread: table of sizes. See "Gauges," § (60).
- U.S.A. National Fine Thread: table of sizes. See "Gauges," § (61).
- U.S.A. National Pipe Thread: table of sizes. See "Gauges," § (62).

— V —

- V-Notch: table of rates of flow for 90° notch. See "Meters," § (31).
- V-shaped Depression. See "Atmosphere, Physics of," § (18).
- Vapour-pressure in the Atmosphere:
  - Distribution of, over the globe. See "Atmosphere, Thermodynamics of the," *Fig.* 5.
  - Formulae for calculation of, from readings of dry- and wet-bulb thermometers. See *ibid.* II. § (4) (ii.).
  - Methods of measurement. See "Humidity," II. § (1).
  - Reduction of, to sea-level. See "Atmosphere, Thermodynamics of the," *Fig.* 5.
  - Saturation or maximum:
    - Dalton's law, experimental verification of. See "Humidity," I.
    - Definition of. See *ibid.* I.
    - Effect of change of state on. See *ibid.* I.
    - Values of. See *ibid.* I. and II. § (15). See also "Atmosphere, Thermodynamics of the," § (2), Table I.
  - Variation of, with height. See "Atmosphere, Thermodynamics of the," § (11).
- Vapours, Determination of the Densities of. See "Balances," § (19).
- Velocity of Wind, Measurement of, with a manometer. See "Barometers and Manometers," § (22) (ii.).
- Ventilation and Humidity. See "Humidity," § (14).
- Venturi Air Meter. See "Meters," § (24).
- Venturi Water Meter. See "Meters," § (30).
- Vernier Caliper. See "Gauges," § (84).
- Vernier Time: method of subdividing the second. See "Clocks and Time-keeping," § (15).
- Vibrating and Accelerating Machines. See "Weighing Machines," § (5).
- Violles Actinometer. See "Radiant Heat and its Spectrum Distribution," § (7).
- Visibility, effect of the atmosphere on. See "Meteorological Optics," § (15) (v.).
- Volume (see also "Volume, Measurements of"). The unit of volume is based on the unit of length, but in many cases the legal unit has been defined as the space occupied by a certain weight of a standard liquid—usually water—under standard conditions.

  (i.) *Metric.*—An attempt was made by the founders of the metric system to correlate the two units by defining the unit of mass as the mass of a quantity of water which at its temperature of maximum density occupied 1 cubic decimetre; the litre or unit of volume could then be

defined as the space occupied by a kilogramme of water at its maximum density, or as the space occupied by a cube with side 10 centimetres. Further experiments (see "Volume, Measurements of," § (2)) have proved that this relation is not exact. The experimental relation now accepted is that 1 kilogramme of water at 4° C., and pressure 760 mm., occupies 1000·027 c.c. In 1872 the unit of mass was redefined as the mass of the International kilogramme in its actual state, and in 1901 the definition adopted for the *litre* was the space occupied by a kilogramme of water at its maximum density and under normal atmospheric pressure (760 mm.).

(ii.) *British.*—In British units the *gallon* is the unit of volume, and is defined as the space occupied by 10 lbs. weight of distilled water weighed in air against brass weights at a pressure of 30 in. and temperature 62° F. Units based on the unit of length are also in common use.

(iii.) *Equivalents.*—

(*a*) *Metric Units.*

1 c.c. = ·0610 c. in.
1 litre = 1000·027 c.c.
= ·03531 c. ft.
= 1·7598 pint.
= ·2200 gal.

(*b*) *British Units.*

1 c. in. = 16·387 c.c.
1 c. ft. = 28·317 litres = 28317 c.c. = 6·22882 gal.
1 c. yd. = 0·7645 m.^{3}.
1 pint = ·5682 lit.
1 gallon = 4·5460 lit.

See Vol. I. "Measurement, Units of."

Determination of volumes from linear measurements. See "Volume, Measurements of," § (5).

VOLUME, MEASUREMENTS OF

§ (1) UNITS OF VOLUME.—The volume of a body is the amount of space which it occupies, and in order to state the magnitude of any given volume it is necessary first to adopt some clearly defined unit in terms of which the volume in question can be expressed.

There are a limited number of solid figures which have the property that by continuous repetition they may be extended so as completely to fill space without leaving any interstices. Of these the simplest is the cube, and a cube of definitely specified dimensions is a simple space element in terms of which to express the volume of (*i.e.* the space occupied by) any given substance or solid figure.

A cube is completely defined when the length of one edge is stated. Hence in any system of units a cube, each of whose sides is of unit length, is a unit of volume which makes an instant appeal to one owing to its fundamental relation to the unit of length. Thus we have the cubic centimetre, the cubic foot, etc., as widely used units of volume.

It is, however a matter of extreme difficulty to determine to a high degree of accuracy the external volume of a solid, or the internal capacity of a hollow vessel, from measurements of their linear dimensions.

Moreover, volumetric measurements are most extensively employed in the case of fluids, and the direct application to such measurements of units of volume based on units of length is manifestly inconvenient.

The weight of a liquid can, however, be determined very simply. Consequently legal units of volume have in many cases been defined as the space occupied by a definite weight of a standard liquid. Thus, for example, the gallon was defined as the space occupied by the quantity of water which under certain specified conditions weighed 10 lb.

Sooner or later, however, the need arises for determining the relation between units of volume defined in terms of the space occupied by a mass of liquid and units of volume derived from units of length.

The correlation between the two types of units has been most accurately carried out in the case of the metric units. The work done in this connection is worthy of detailed consideration, not only because the experimental work itself is a notable example of extreme skill and refinement in physical measurements, but also because it is necessary for a clear understanding of the metric units themselves, and forms the basis of similar correlations on other systems of units.

(i.) *The Cubic Decimetre.*—The founders of the metric system endeavoured to secure co-ordination between the two methods of defining units of volume by specifying that the unit of mass, *i.e.* the kilogramme, should be the mass of a quantity of water which at its temperature of maximum density occupied a space of 1 cubic decimetre. The unit of volume, the litre, could hence be defined alternatively as the space occupied by a kilogramme of water at its temperature of maximum density, or as the space occupied by a cube each of whose sides is 10 cm. in length.

A provisional standard kilogramme was prepared by Lavoisier and Haüy, and later Lefèvre, Gineau, and Fabroni were entrusted with the task of constructing the fundamental standard.

It is worth while to try to form an idea of the magnitude of the task imposed on these distinguished physicists. The underlying principle is perfectly simple. Given a solid of simple geometrical form, its volume can be calculated in terms of its linear dimensions. By weighing such a solid in air and also in water the mass of water which it displaces can be determined. Hence the mass of a quantity of water is determined whose volume is known in terms of units based on the standards of length.

Suppose, however, that a cube whose sides are 10 cm. in length were to be used for the above purpose, and that the weighing in water were carried out at 15° C. Then an error of 1 micron (*i.e.* one-thousandth of a millimetre) in the measurement of the distance between opposite faces of the cube would produce an error of 30 milligrammes in the standard kilogramme, and an error of 0·1° C. in the determination of the water temperature would introduce an error of 15 milligrammes.

The above figures show clearly that although in principle the experiment is extremely simple, yet to carry it out to a high degree of accuracy calls for the highest refinements of measurement.

When it is remembered that Lefèvre, Gineau, and Fabroni prepared the "Kilogrammes des Archives" at the end of the eighteenth century, and that, as a result of the latest researches, their standard has been found to differ by only 27 milligrammes from its theoretical mass, and even admitting that this accuracy may partly be attributed to compensating errors, one must realise that their work was carried out with extraordinary skill and success.

Unfortunately, no detailed account of their work has been left by the authors themselves, the only source of information being a report by Tralles.[1] Partly, perhaps, owing to this fact and also to the discordant results of subsequent observers a feeling of doubt developed as to the accuracy with which the standard kilogramme fulfilled its theoretical definition. Consequently, in 1872, the "Commission internationale du Mètre" discussed the matter fully in all its bearings and arrived at the following decision:

"Considering that the simple relation, established by the authors of the Metric System, between the unit of mass and the unit of volume is represented by the actual kilogramme in a manner sufficiently accurate for the ordinary purposes of industry and commerce, and even for the greater part of the ordinary needs of science;

"Considering that the exact sciences have not the same need of a simple numerical relation, but only of a determination as accurate as possible of this relation;

"Considering finally the difficulties which would be brought about by a change in the actual metric unit of mass—

"It is decided that the International Kilogramme shall be deduced from the Kilogramme des Archives in its actual state."

The kilogramme is hence the mass of a particular standard weight, and its original definition in terms of the mass of 1 cubic decimetre of water has no present significance.

(ii.) *The Litre.*—Further, the definition of the litre was revised in 1901,[2] and it is now defined as follows:

"*The unit of volume, for determinations of high precision, is the volume occupied by a mass of 1 kilogramme of pure water, at its maximum density and under normal atmospheric pressure; this volume is termed the 'litre.'*"[3]

Hence the definition of the litre has no reference to the units of length, and the original intention that it should be equal to 1 cubic decimetre has been quite abandoned. However, the litre as defined above differs so slightly from the cubic decimetre that for ordinary purposes the litre may be regarded as equivalent to 1000 c.c.

§ (2) RELATION BETWEEN THE LITRE AND THE CUBIC CENTIMETRE.—It is obviously, however, a matter of prime importance in accurate work that the litre should be known as accurately as possible in terms of cubic centimetres.

The determination of this relation was included in the programme of research for the Bureau International for a long time, but it was not until 1910 that the results were finally published.[4]

The account of this classical research, which was carried out with the utmost refinement and skill, should be consulted for a full description of the work. Only a brief indication of the methods employed can be given here.

As stated previously, the problem resolves itself into measuring the dimensions of a body of simple shape and then determining the mass of water which it displaces.

Three independent determinations were carried out at the Bureau International, and whilst the mass of water displaced by measured solids was determined similarly in each case by means of weighings in air and in water, the methods of determining the linear dimensions of the bodies used differed fundamentally.

---

[1] "Rapport de M. Tralles à la Commission sur l'unité de poids du Système métrique décimal d'après le travail de M. Lefèvre Gineau," 30th May 1799 (*Base du système métrique décimal*, t. iii. 558).

[2] "Comptes Rendus des Séances de la Troisième Conférence Générale des Poids et Mesures réunie à Paris en 1901," *Trav. et mém.*, 1902, t. xii.

[3] The reference to "normal atmospheric pressure" perhaps requires a word of explanation. The volume of any given quantity of water varies with the pressure to which it is subjected owing to the slight compressibility of water. Hence, in order to be precise, the pressure to which the water is to be subjected must be introduced into the definition of the litre.

[4] *Trav. et mem.*, 1910, t. xiv.

(i.) M. Guillaume used cylinders made from hard bronze, and their dimensions were determined by comparison with a standard scale. The principle of the method used may be gathered from the following figure :

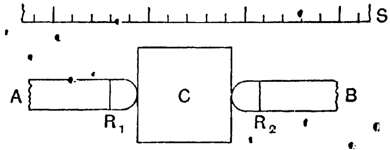


Fig. 1.

The cylinder is represented by C; A and B are two contact pieces carrying reference marks R_1 and R_2, and S is a standard scale. The whole is mounted on a carriage which runs beneath two independently mounted microscopes. With the cylinder mounted as shown, one microscope is set on R_1 and the other on R_2. The carriage is then brought forward, so that the scale is under the microscopes and the distance between R_1 and R_2 is thus determined. Then the cylinder is removed and A and B are brought into contact with each other, and the distance R_1R_2 measured as before. The difference between the two measurements gives the length or diameter of the cylinder as the case might be.

Three cylinders were used which had volumes of about 780 c.c., 1300 c.c., and 2000 c.c. respectively. To reduce their weight they were made hollow.

(ii.) M. Chappuis, in an independent series of observations, used three cubes of glass whose sides were approximately 4 cm., 5 cm., and 6 cm. in length. The linear dimensions of the cubes were determined by means of a Michelson interferometer. The cube was mounted in front of and very close to a glass plate, as shown in *Fig.* 2.

The plane surfaces A and B were adjusted parallel to each other, and their distance apart determined by the method employed by Michelson[1] in measuring his "étalons" used for determining the metre in terms of light wave-lengths. The thickness of the air film between A and C was determined by observations on the fringes formed by the film. A second series of measurements gave the distance from A to B. For the purposes of this measurement the face C was not made strictly parallel to B but inclined to it at an angle of from 8″ to 12″. The difference between the measurements AB and BC gives the thickness of the cube.

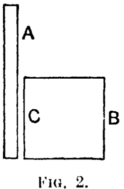


Fig. 2.

(iii.) The third determination was the work of MM. Lepinay, Buisson, and Benoit. Two cubes of quartz were used, the length of whose edges were 4 cm. and 5 cm. respectively. Their dimensions were determined by observing, in the first place, the circular fringes produced by the interference between two beams of light, one reflected from the front face of the cube and the other from the back face (see *Fig.* 3).

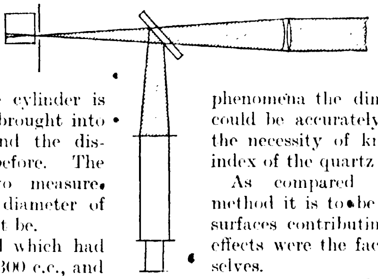


Fig. 3.

Secondly, the Talbot fringes formed by the interference between two portions of a beam of light, one of which had traversed the cube and the other an equal thickness of air, were observed (see *Fig.* 4). By combining the results obtained from observations on both sets of interference phenomena the dimensions of the cube could be accurately determined without the necessity of knowing the refractive index of the quartz used.

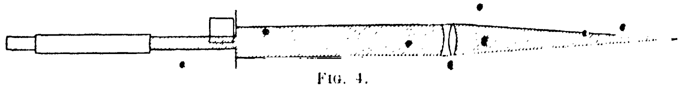


Fig. 4.

As compared with M. Chappuis' method it is to be noted that the only surfaces contributing to the interference effects were the faces of the cubes themselves.

The final results obtained by the different observers were :

M. Guillaume	1 litre = 1000·029 c.c.
M. Chappuis	1 litre = 1000·026 c.c.
MM. Lepinay, Buisson, and Benoit	1 litre = 1000·027 c.c.

In view of the diversity in the methods employed and the difficulty of the work, the excellent agreement between the above results shows with what care and skill the experiments were conducted.

In his final résumé of the whole observations M. Benoit gives

1 litre = 1000·027 c.c.

as the most probable value, and states that any inaccuracy in this figure probably does not exceed one unit in the last decimal place.

The litre and the cubic centimetre are hence clearly defined units of volume whose relation one to the other has been accurately determined. Nevertheless, misconceptions as to the exact significance of the terms are somewhat prevalent.

---

[1] *Trav. et mém.* t. xi.

These are probably mainly due to the formerly extensive use of vessels for volumetric analysis based on Möhr's system of units. On this basis a litre flask, say, is one which contains at its standard temperature, *e.g.* 15° C., a quantity of water whose apparent weight in air is 1 kilogramme. The capacity of such a flask would differ by approximately 2 c.c. from the litre as defined in § (1) (ii). Further vessels graduated on the basis of one-thousandth of Möhr's "litre" as unit were marked "c.c." Thus the terms litre and cubic centimetre became applied to volumes which were very different from the correct units.[1]

A less serious cause of uncertainty is the fact that at both the Bureau of Standards at Washington and the Reichsanstalt at Charlottenburg vessels marked "c.c." are tested on the assumption that the unit used is the millilitre, *i.e.* one-thousandth part of the litre, and not the cubic centimetre.

In view of the varying interpretations which have from time to time been placed on the terms litre and cubic centimetre it is important to emphasise the true meaning of the terms, viz. that the cubic centimetre is simply the space occupied by a cube each of whose sides is 1 cm. in length, and that the true definition of the litre is that given in § (1) (ii.).

§ (3) British Units of Volume.—On the British system of units we have the cubic inch, the cubic foot, the cubic yard, etc., based on the units of length.

The fundamental unit defined in terms of a quantity of water is the gallon, which is defined as follows:

"The gallon contains 10 lbs. weight of distilled water weighed in air against brass weights with the water and the air at the temperature of 62° Fahr., the barometer being at 30 inches."

The definition is not free from ambiguity. The density of the brass weights to be used is not specified, no standard humidity of the air is fixed, and the phrase, "the barometer being at 30 inches," is capable of varying interpretations.

The official Board of Trade equivalents of Metric and Imperial units of volume are given in the tables below.

Cubic Measure

1 Cubic inch	= 16·387 c.c.
1 Cubic foot (1728 cu. in.)	= 0·028317 cubic metre.
1 Cubic yard (27 cu. ft.)	= 0·764553 cubic metre.

Measures of Capacity

1 Gill	= 1·42 decilitres.
1 Pint (4 gills)	= 0·568 litre.
1 Quart (2 pints)	= 1·136 litres.
1 Gallon (4 quarts)	**= 4·5459631 litres.**
1 Peck (2 gallons)	= 9·092 litres.
1 Bushel (8 gallons)	= 3·637 dekalitres.
1 Quarter (8 bushels)	= 2·909 hectolitres.

Apothecaries' Measure

1 Minim	= 0·059 millilitre.
1 Fluid scruple	= 1·184 millilitres.
1 Fluid drachm (60 minims)	= 3·552 millilitres.
1 Fluid ounce (8 drachms)	= 2·84123 centilitres.
1 Pint	= 0·568 litre.
1 Gallon (8 pints or 160 fluid oz.)	**= 4·5459631 litres.**

It should, however, be noted that the litre is defined as follows:

"The litre is represented by the capacity at 0° C. of the cylindrical brass measure marked 'Litre, 1897.'"

This measure is deposited with the Board of Trade.

A factor often required is the weight of 1 cubic foot of water. This may be arrived at as follows. Starting with the legal equivalent

1 yard = 0·914399 metre,

we have 1 cu. ft. = 0·0283167 cubic metre
= 28·31599 litres.

Also 1 gallon = 4·5459631 litres,

and hence 1 cu. ft. = 6·22882 gallons.

Hence the weight of 1 cubic foot of water is 62·2882 lbs. under the same conditions that a gallon of water weighs 10 lbs.

§ (4) American Units of Volume.—The American customary units bearing the same names as the Imperial units have somewhat different values, as will be seen by comparing the table given below with the preceding tables:

American Customary Units.	Metric Equivalents.[1]
1 Fluid dram	= 3·69661 millilitres.
1 Fluid ounce	= 29·5729 millilitres.
1 Liquid Quart	= 0·946333 litre.
1 Liquid Gallon	= 3·785332 litres.
1 Bushel	= 3·52383 dekalitres.

§ (5) Determination of Volumes from Linear Measurements.—It is not proposed to give here detailed proofs of the formulae whereby the volumes of various bodies may be calculated from measurements of their linear dimensions, as these may be found in many standard mathematical text-books. It will suffice to give a general account of the problem and to quote the results obtained in special cases of common occurrence.

---

[1] The disadvantages of Möhr's system are dealt with exhaustively by Schloesser, *Z. angew. Chem.*, 1903, xvi. 953.

[1] Circular No. 47, Bureau of Standards, 1914.

Let ABC (*Fig.* 5) be the section in the plane of the paper of any solid figure. Consider an element of the solid cut off by the two parallel planes PP and P_1P_1, which are perpendicular to the horizontal axis OX and separated by the very small distance δx. If x is the distance of PP from O measured along the axis OX the area of the section PP may be represented by

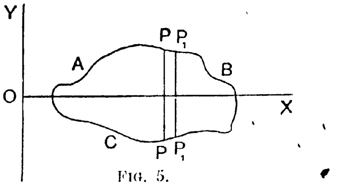


FIG. 5.

$$f(x)$$

and the volume of the above element by

$$f(x)\delta x.$$

The volume of the whole solid is given by

$$\int_a^b f(x)dx,$$

where a and b are suitably chosen limiting valves of x.

The following is a simple particular case. Let OX (*Fig.* 6) be the axis of a circular cone of which

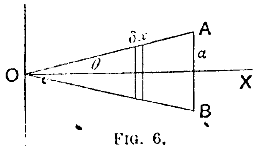


FIG. 6.

OAB is the section in the plane of the paper, and a is the radius of the base. Consider the element of the cone cut off by two planes parallel to the base and distant x and $x+\delta x$ respectively from O. The volume of this element is

$$\pi(x \tan \theta)^2 \delta x$$

or

$$\frac{\pi a^2 x^2}{h^2}\delta x,$$

where h is the height of the cone.

The volume of the whole cone is

$$\frac{\pi a^2}{h^2}\int_0^h x^2 dx$$

$$=\tfrac{1}{3}\pi a^2 h.$$

The results obtained in a number of special cases of common occurrence are given below :

(i.) *Rectangular Parallelepiped.*—The volume is equal to the product of the lengths of three mutually perpendicular edges.

(ii.) *Parallelepiped.*—If a, b, and c are the lengths of three edges which meet at a corner, and A, B, and C are the angles between these edges then the volume is given by the product

$$abc \sin A \sin B \sin C,$$

i.e. the volume is equal to the area of any face multiplied by its perpendicular distance from the parallel face.

(iii.) *Prism or Cylinder.*—

Volume = area of base × height

(iv.) *Pyramid or Cone.*—

Volume = area of base × $\frac{1}{3}$ height.

(v.) *Frustum of Pyramid or Cone.*—

$$\text{Volume} = \tfrac{1}{3}h(A_1 + \sqrt{A_1A_2} + A_2),$$

where h is the perpendicular distance between the two ends and A_1 and A_2 are the areas of each end respectively.

In the case of the frustum of a cone of height h, the radius of the base being R and of the parallel face r, the above expression becomes

$$\tfrac{1}{3}h\pi(R^2 + Rr + r^2).$$

(vi.) *Sphere.*—

$$\text{Volume} = \tfrac{4}{3}\pi r^3,$$

where r = radius of sphere.

(vii.) *Segment of a Sphere* (*Fig.* 7).—

$$\text{Volume} = \pi h^2(r - \tfrac{1}{3}h)$$

or

$$\text{Volume} = \frac{\pi h}{2}(r_1^2 + \tfrac{1}{3}h^2).$$

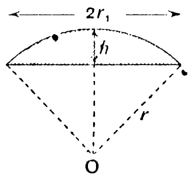


FIG. 7.

(viii.) *Sector of a Sphere* (*Fig.* 8).—

$$\text{Volume} = \tfrac{2}{3}\pi r^2 h$$

or

$$\text{Volume} = \tfrac{2}{3}\pi r^2(r - \sqrt{r^2 - r_1^2}).$$

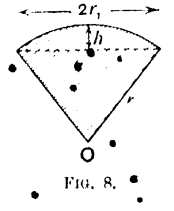


FIG. 8.

(ix.) *Section of a Sphere* formed by two parallel planes (*Fig.* 9).

Let $2h$ = distance between the planes,
d = perpendicular distance from centre of sphere to a plane parallel to the two bounding planes and midway between them,
r = radius of sphere.

Then

$$\text{Volume} = 2\pi h\left[r^2 - d^2 - \frac{h^2}{3}\right].$$

The values of r and d, however, cannot be directly measured on such a solid. They may, however, be expressed in terms of the radii of the two circular faces, which are easily measured. Thus

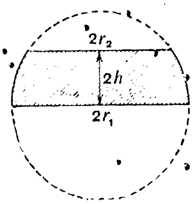


FIG. 9.

$$d = \frac{r_1^2 - r_2^2}{4h}$$

and $$r^2 = r_2^2 + (d + h)^2,$$

where r_1 = radius of larger circular face
and r_2 = radius of smaller circular face.

(x.) *Anchor Ring*, *i.e.* the solid traced out by the rotation of a circle (of radius a, say) about an axis in the plane of the circle and distant b, say, from its centre, b being greater than a.

$$\text{Volume} = 2\pi^2 a^2 b.$$

(xi.) *Spheroids.*—A *prolate* spheroid is the solid traced by the rotation of an ellipse about its major axis. Its volume is given by the expression

$$\tfrac{4}{3}\pi ab^2,$$

where $2a$ = length of major diameter
and $2b$ = length of minor diameter.

An *oblate* spheroid is the solid traced by the rotation of an ellipse about its minor axis, and its volume is given by

$$\tfrac{4}{3}\pi ba^2.$$

(xii.) *Paraboloid.*—The volume of the portion of a paraboloid included between the apex and a circular cross-section of radius r and distant h from the apex is

$$\tfrac{1}{2}\pi r^2 h.$$

(xiii.) *Cask.*—Assuming that the staves are bent in the form of an arc of a parabola the capacity is given by

$$\tfrac{1}{15}\pi h(8R^2 + 4Rr + 3r^2),$$

where h = depth of cask,
R = radius of central cross-section,
r = radius of ends,
the measurements being all internal.

Assuming the cask to be the middle frustum of a prolate spheroid, the capacity is given by

$$\frac{\pi}{12}L[2B^2 + H^2],$$

where L = internal length of cask,
B = maximum internal diameter,
H = internal diameter at end.

(xiv.) *Solid of Revolution.*—The volume of the solid traced by the revolution of any plane figure about an axis in its plane, but not intersecting it, is equal to the product of the area of the figure and the length of path of its centre of gravity, *i.e.*

$$\text{Volume} = 2\pi r A,$$

where A = area of the figure
and r = distance of its centre of gravity from the axis of rotation.

§ (6) GAUGING OF CASKS AND BARRELS.—The determination of the capacity of casks and barrels is of importance in assessing duty on their contents, in stock-taking, etc.

In gauging casks and barrels the three dimensions specified below are the basis on which the capacity is calculated:

(i.) Bung diameter, *i.e.* the internal diameter of the cask at its maximum cross-section.

(ii.) Head diameter, *i.e.* the internal diameter at the end.

(iii.) Length, *i.e.* distance between the inside surfaces of the ends.

The bung diameter is determined by means of a bung rod, which is 48 inches long and has a square cross-section. Two opposite faces of the rod each carry inch scales, the zeros of which coincide with the tip of the rod, which is bevelled (see *Fig.* 13), at this end. Scales are also engraved on the other two faces of the bung rod. These will be referred to later. A metal slide, provided with a flange perpendicular to the length of the rod, is movable from end to end of the rod. To measure the bung diameter the cask is first fixed by means of wedges so that it lies with its axis horizontal and with the bung at the highest point of the cask. The bung rod is inserted vertically through the bung until it touches the opposite side of the cask. The metal flange is then pushed gently down until it touches the outside of the cask at the bung. The rod is then withdrawn, care being taken not to move the metal slide, and the internal diameter read off on one of the inch-scales by noting the position of the slide. Lines are engraved on the slide below the flange, enabling an allowance to be made for the thickness of the stave at the bung.

The diameter of the widest portion of the cask is also measured in a direction perpendicular to that given by the bung rod. This is effected by means of cross callipers, which are shown diagrammatically in *Fig.* 10. The

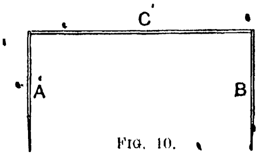


FIG. 10.

central portion C is composed of two wooden rods, which can be moved relatively to each other, and so vary the distance between the arms A and B. The arms are brought into contact with the sides of the cask at opposite ends of the diameter to be measured, the length of which is then read off on the central portion C, allowance again being made for the thickness of the staves.

The head diameter is measured by means of a "head rod," which consists of two wooden rods, 44 inches long, joined together at the ends in such a way as to leave a space between the two rods in which a third rod can slide backwards and forwards. The sliding rod carries a vertical brass cock, and a second brass cock is fixed to the end of the head rod.

The method of using the head rod is shown in *Fig.* 11. The fixed brass cock is inserted

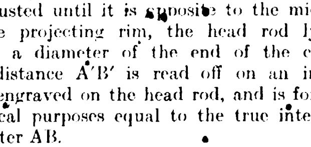


FIG. 11.

into the angle formed by the head of the cask and the projecting rim. The movable cock is adjusted until it is opposite to the middle of the projecting rim, the head rod lying across a diameter of the end of the cask. The distance A′B′ is read off on an inch-scale engraved on the head rod, and is for all practical purposes equal to the true internal diameter AB.

The length is measured by means of "long callipers," shown diagrammatically in *Fig.* 12.

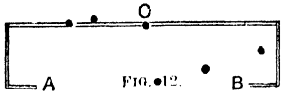


FIG. 12.

The central portion C is made of two pieces of wood which slide relatively to each other, thus varying the distance AB. The points A and B are brought into contact with the ends of the cask and their distance apart is then read off on the central portion C. An allowance for the thickness of the end staves must be made.

A variety of callipers of special design are used for measuring the thickness of end and longitudinal staves.

To compute the capacity from the observed measurements use is made of the head rod, which is constructed to serve as a calculating rule as well as for measuring head diameters.

The standard shape for a cask is taken to be the central frustum of a prolate spheroid, the capacity of which is given by

$$\frac{\pi}{4}L\left[\frac{2B^2+H^2}{3}\right],$$

where B = bung diameter,
H = head diameter,
L = length.

If we write

$$D^2=\frac{2B^2+H^2}{3},$$

then D is the diameter of the cylinder of length L, which has the same capacity as the cask.

The head rod with its central sliding portion is operated in the same way as an ordinary slide rule. The first step in the computation is to find the value of the mean diameter D.

On one side of the brass projection, which serves as a zero mark, a scale is drawn on the sliding portion of the head rod called the "spheroidal line." The above zero mark is placed opposite the point on an inch-scale engraved on one of the fixed sides of the rod, which corresponds to the value found for the head diameter. The point on the spheroidal line which is now opposite to the point on the inch-scale corresponding to the bung diameter is noted. This gives the value in inches to be added to the head diameter to give the mean diameter D.[1] The addition is effected by means of a second inch-scale engraved on the sliding portion, the zero of which is coincident with the point previously set on the head diameter. The number noted on the spheroidal line opposite to the bung diameter is read off on the above scale of inches. The mark opposite to this point on the first inch-scale obviously gives the mean diameter.

Having obtained D the equivalent in gallons of $\pi/4x D^2L$ remains to be computed. If D and

---

[1] This procedure is not strictly accurate, as for any given difference between B and H it gives the same value to be added to H irrespective of the absolute values of B and H. The spheroidal line, however, gives results which are quite sufficiently accurate for practical purposes.

L are in inches the equivalent in gallons of $\pi/4x\mathrm{D}^2\mathrm{L}$ is

$$\frac{\mathrm{D}^2\mathrm{L}}{(18\cdot79)^2},$$

since a cylinder 1 inch long and 18·79 inches diameter has a capacity of 1 gallon. The evaluation of $\mathrm{D}^2\mathrm{L}/(18\cdot79)^2$ is quickly performed by means of logarithmic scales. One such scale on the slide is numbered to cover the range of diameters usually met with, the spacing of the scale being proportional to the logarithms of the diameters. Adjacent to this scale on the fixed portion of the head rod parallel to that engraved with the inch-scale previously referred to, is a scale numbered with the lengths usually met with, and spaced proportionally to the logarithms of the lengths. The radius of this logarithmic scale is, however, half that employed for the logarithmic scale of diameters. With these two scales the capacity is found by placing 18·79 on the scale of mean diameters opposite to the value found for L on the length scale, and then reading the mark on the length scale which is opposite to the observed value of D on the scale of mean diameters. The value thus obtained on the length scale is the capacity of the cask in gallons.

When casks vary from the standard spheroidal shape, a suitable correction is applied to the measured length of the cask before carrying out the above calculation.

The scales hitherto referred to are all on the front of the head rod. On the back of the head rod are scales for computing the quantity of liquid, *i.e.* the "ullage," in a partially filled cask.

A detailed description of these and their manipulation would occupy too much space, but the underlying theory may be briefly indicated.

The depth of liquid in a cask when it is lying horizontal with the bung at the highest point is referred to as the "wet inches" in the cask.

For casks of standard shape the ratio of the bung diameter to the "wet inches" will bear the same relation as the ratio of the total capacity to the ullage, irrespective of the size of the cask.

If the ullages corresponding to various ratios of bung diameter to wet inches for a cask of known total capacity, say 100 gallons, are determined, then the data obtained form a basis on which the ullage of any other cask of known total capacity may easily be calculated from measurements of the bung diameter and the "wet inches."

The scales on the back of the head rod include one based on the ullages of a 100-gallon cask, and in conjunction with the other scales ullages for casks of varying total capacity may be computed from the measurements of their bung diameter and their "wet inches."

The capacity of small casks may be checked by means of the bung rod alone. A scale is marked on the bung rod based on the principle that the volumes of similar solid figures vary as the cubes of their corresponding linear dimensions. The scale applies only to casks of standard shape. The rod is inserted as shown in *Fig.* 13, and the scale read at the centre of the bung on the level of the inside of the cask. The scale gives directly the capacity of the cask in gallons.

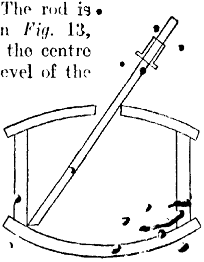


FIG. 13.

The fourth face of the bung rod bears a scale which gives the capacity, in gallons, of cylinders 1 inch in depth. The mark on this scale corresponding to, say, 20 inches on either of the inch-scales gives the volume in gallons of the liquid contained in a cylinder of this diameter when filled to a depth of one inch. The total capacity of the cylinder is obtained by multiplying this value by the length of the cylinder in inches.

§ (7) GRAVIMETRIC DETERMINATIONS OF CAPACITY. (i.) *By Means of Water.*—A convenient and accurate method of obtaining the capacity of a vessel is to determine the weight in air of its water content. From this determination the volume of the vessel may be obtained as follows: taking as an example the case of a glass vessel calibrated in terms of the cubic centimetre.

Let M gms. = apparent weight in air of water *i.e.* mass *in vacuo* of the weights which are in equilibrium with the water,[1]

Δ gms./ml. = density of weights used,

σ gms./ml. = density of air at time of weighing

T° C. = standard temperature of the vessel,[2]

V c.c. = capacity of vessel at T° C.,

α = coefficient of cubical expansion of glass per ° C.,

K = conversion factor ml. to c.c. *i.e.* 1 ml. = K c.c.,

t° C. = temperature of water (assumed also to be the temperature of the containing vessel),

ρ gms./ml. = density of water at t° C.

---

[1] Weights are adjusted to have a mass equal to the nominal value marked on them. Hence the mass *in vacuo* of any weight (assuming it to be accurately adjusted) is simply the nominal value marked on it.

[2] The standard temperature of a vessel is the temperature at which it is intended to be used and for which its capacity is adjusted. At other temperatures the capacity will, of course, differ from that at its standard temperature, owing to the thermal expansion of the material of which the vessel is made.

Then from equilibrium existing at time of weighing we have

$$M\left(1-\frac{\sigma}{\Delta}\right)=\frac{1}{K}V\{1+\alpha(t-T)\}(\rho-\sigma). \quad (1)$$

It is often necessary to check the accuracy of graduation of volumetric apparatus to a degree of accuracy which involves using the above relation without introducing approximate values. The time involved by such calculations is so great when many tests have to be carried out that it is convenient to have tables to facilitate the work.

Convenient tables may be obtained as follows. In the case of vessels graduated in terms of cubic centimetres the value of M will always be numerically not very different from V. We may therefore conveniently write

$$M+C=V,$$

where C is a small correction to be added to the observed weight in gms. to give the volume of the vessel in c.c. From (1) we have:

$$M=\frac{V[1+\alpha(t-T)](\rho-\sigma)\Delta}{K(\Delta-\sigma)},$$

and hence

$$C=V\left[1-\frac{[1+\alpha(t-T)](\rho-\sigma)\Delta}{K(\Delta-\sigma)}\right]. \quad . \quad (2)$$

The usual value for T° C. in this country is 15° C., and the variation in Δ with temperature is negligible. Hence, assuming T = 15° C. for any given value of V, the only variables on the right of (2) are t, ρ, and σ. If we further assume that all weighings are made in air under the following standard conditions, viz. temperature = 15° C., barometric height = 760 mm. of mercury at 0° C. at sea-level in latitude 45° (g = 980·62), humidity = two-thirds saturation, and CO_2 content = 0·04 per cent by volume, then the variables reduce to t and ρ.

Denoting the values of C for the above standard conditions of the air by $C^{15^\circ C.}_{760\text{ mm.}}$ and the density of the air under the same conditions by σ_s, we have

$$C^{15^\circ C.}_{760\text{ mm.}}=V\left[1-\frac{[1+\alpha(t-15)](\rho-\sigma_s)\Delta}{K(\Delta-\sigma_s)}\right]. \quad (3)$$

From this relation we can tabulate values of $C^{15^\circ C.}_{760\text{ mm.}}$ for various values of t.

Table A has been constructed on this basis, and for many purposes it is alone sufficient, as the additional correction required owing to the air not being under standard conditions at the time of weighing is comparatively small.

The following values were used in calculating the tables:

V = 1000 c.c., *i.e.* the tables apply to vessels of nominal capacity 1000 c.c., and when used for vessels of other capacities proportional values must be taken.

TABLE A

VALUES OF C IN MILLIGRAMMES

	·0.	·1.	·2.	·3.	·4.	·5.	·6.	·7.	·8.	·9.
5	1368	1367	1367	1366	1366	1365	1365	1365	1365	1366
6	1366	1367	1367	1368	1369	1371	1372	1373	1375	1377
7	1379	1381	1383	1385	1388	1391	1393	1396	1399	1403
8	1406	1409	1413	1417	1421	1425	1429	1433	1438	1443
9	1447	1452	1457	1462	1468	1473	1479	1484	1490	1496
10	1502	1509	1515	1522	1528	1535	1542	1549	1556	1564
11	1571	1579	1587	1595	1603	1611	1619	1628	1636	1645
12	1654	1663	1672	1681	1690	1700	1709	1719	1729	1739
13	1749	1759	1769	1779	1790	1800	1811	1822	1833	1844
14	1855	1867	1878	1890	1901	1913	1925	1937	1949	1962
15	1974	1987	1999	2012	2025	2038	2051	2064	2077	2091
16	2104	2118	2132	2146	2160	2174	2188	2202	2217	2231
17	2246	2261	2276	2291	2306	2321	2336	2352	2367	2383
18	2399	2414	2430	2447	2463	2479	2496	2512	2529	2545
19	2562	2579	2596	2614	2631	2648	2666	2683	2701	2719
20	2737	2755	2773	2791	2810	2828	2847	2865	2884	2903
21	2922	2941	2960	2980	2999	3018	3038	3058	3078	3098
22	3118	3138	3158	3178	3199	3219	3240	3261	3281	3302
23	3323	3344	3366	3387	3408	3430	3452	3473	3495	3517
24	3539	3561	3584	3606	3628	3651	3673	3696	3719	3742
25	3765	3788	3811	3835	3858	3881	3905	3929	3953	3976
26	4000	4024	4049	4073	4097	4122	4146	4171	4196	4220
27	4245	4270	4295	4321	4346	4371	4397	4422	4448	4474
28	4499	4525	4551	4577	4604	4630	4656	4683	4709	4736
29	4763	4790	4817	4844	4871	4898	4925	4953	4980	5008
30	5035	5063	5091	5118	5146	5175	5203	5231	5259	5288

$K = 1{\cdot}000027$ (*Trav. et Mém.*, 1910, t. xiv.).

$a = 0{\cdot}000026$ per ° C.

ρ = values taken from table given by P. Chappuis (*Trav. et Mém.*, 1907, t. xiii.).

$\sigma_s = 0{\cdot}0012201$ (from Table IV., p. 138).

$\Delta = 8{\cdot}3$ gms./ml. (A mean value obtained in the routine testing of large numbers of brass weights at the National Physical Laboratory.)

Further, the values of $C^{15^\circ\text{ C.}}_{760\text{ mm.}}$ given by (3) have been multiplied by 1000, so that Table A gives for various water temperatures the weight in milligrammes to be added to the observed weight in gms., as obtained by weighings against brass weights in air, of the water content of the vessel, to give the capacity of the vessel in cubic centimetres at 15° C.

TABLE B.

CORRECTION TO STANDARD PRESSURE.

Temp. C.	730 mm.	735 mm.	740 mm.	745 mm.	750 mm.	755 mm.	760 mm.	765 mm.	770 mm.	775 mm.	780 mm.	785 mm.	790 mm.
5	− 3	+ 4	+ 12	+ 19	+ 26	+ 34	+ 41	+ 48	+ 56	+ 63	+ 70	+ 78	+ 84
6	− 7	0	+ 8	+ 15	+ 22	+ 29	+ 37	+ 44	+ 51	+ 59	+ 66	+ 73	+ 81
7	− 11	− 4	+ 4	+ 11	+ 18	+ 25	+ 33	+ 40	+ 47	+ 55	+ 62	+ 69	+ 76
8	− 15	− 8	− 1	+ 7	+ 14	+ 21	+ 29	+ 36	+ 43	+ 50	+ 58	+ 65	+ 72
9	− 19	− 12	− 5	+ 3	+ 10	+ 17	+ 24	+ 33	+ 39	+ 46	+ 53	+ 61	+ 68
10	− 23	− 16	− 9	− 1	+ 6	+ 13	+ 20	+ 28	+ 35	+ 42	+ 49	+ 56	+ 64
11	− 27	− 20	− 13	− 5	+ 2	+ 9	+ 16	+ 23	+ 31	+ 38	+ 45	+ 52	+ 59
12	− 31	− 24	− 17	− 9	− 2	+ 5	+ 12	+ 19	+ 26	+ 34	+ 41	+ 48	+ 55
13	− 35	− 28	− 21	− 13	− 6	+ 1	+ 8	+ 15	+ 22	+ 30	+ 37	+ 44	+ 51
14	− 39	− 32	− 24	− 17	− 10	− 3	+ 4	+ 11	+ 18	+ 25	+ 33	+ 40	+ 47
15	− 43	− 35	− 28	− 21	− 14	− 7	0	+ 7	+ 14	+ 21	+ 28	+ 35	+ 43
16	− 46	− 39	− 32	− 25	− 18	− 11	− 4	+ 3	+ 10	+ 17	+ 24	+ 31	+ 38
17	− 50	− 43	− 36	− 29	− 22	− 15	− 8	− 1	+ 6	+ 13	+ 20	+ 27	+ 34
18	− 54	− 47	− 40	− 33	− 26	− 19	− 12	− 5	+ 2	+ 9	+ 16	+ 23	+ 30
19	− 58	− 51	− 44	− 37	− 30	− 23	− 16	− 9	− 2	+ 5	+ 12	+ 19	+ 26
20	− 62	− 55	− 48	− 41	− 34	− 27	− 20	− 13	− 6	+ 1	+ 8	+ 15	+ 22
21	− 66	− 59	− 53	− 45	− 38	− 31	− 24	− 17	− 10	− 3	+ 4	+ 11	+ 18
22	− 70	− 63	− 56	− 49	− 42	− 35	− 28	− 21	− 14	− 7	0	+ 7	+ 14
23	− 73	− 66	− 60	− 53	− 46	− 39	− 32	− 25	− 18	− 11	− 4	+ 3	+ 10
24	− 77	− 70	− 63	− 57	− 50	− 43	− 36	− 29	− 22	− 15	− 8	− 1	+ 5
25	− 81	− 74	− 67	− 60	− 54	− 47	− 40	− 33	− 26	− 19	− 12	− 6	+ 1
26	− 85	− 78	− 71	− 64	− 58	− 51	− 44	− 37	− 30	− 23	− 17	− 10	− 3
27	− 89	− 82	− 75	− 68	− 61	− 55	− 48	− 41	− 34	− 27	− 21	− 14	− 7
28	− 92	− 86	− 79	− 72	− 65	− 59	− 52	− 45	− 38	− 31	− 25	− 18	− 11
29	− 96	− 90	− 83	− 76	− 69	− 62	− 56	− 49	− 42	− 35	− 29	− 22	− 15
30	− 100	− 95	− 87	− 80	− 73	− 66	− 60	− 53	− 46	− 40	− 33	− 26	− 19

As stated previously, the values which are given in Table A assume that the weighings were made in air under the standard conditions specified. Such conditions will only rarely be accurately realised in practice, and when the highest accuracy is desired the variation in air density must be taken into account.

Let us write

$$C = C^{15^\circ\text{ C.}}_{760\text{ mm.}} + c.$$

Then c is a small correction to be added to $C^{15^\circ\text{ C.}}_{760\text{ mm.}}$ to give the correct value for the air conditions at the time of weighing. From (2) and (3) we have

$$c = V\left[1 - \frac{[1 + a(t-15)](\rho - \sigma)\Delta}{K(\Delta - \sigma)}\right] - V\left[1 - \frac{[1 + a(t-15)](\rho - \sigma_s)\Delta}{K(\Delta - \sigma_s)}\right],$$

i.e.

$$c = \frac{V[1 + a(t-15)]\Delta(\Delta - \rho)}{K(\Delta - \sigma_s)(\Delta - \sigma)}[\sigma - \sigma_s],$$

and since $[\sigma - \sigma_s]$ is small we may use approximate values for the terms outside the bracket, and we thus get for V = 1000 c.c.

$$c = 880[\sigma - \sigma_s]. \quad . \quad . \quad . \quad . \quad (4)$$

In calculating c from (4) the various values of σ were obtained from Table IV., p. 138, *i.e.* the air is assumed to contain 0·04 per cent by volume of CO_2 and to be two-thirds saturated. The values of c have been tabulated in Table B for the various values of σ corresponding to the air temperatures and barometer readings given in the table, the barometer readings being expressed in terms of mm. of mercury at 0° C. at sea-level in latitude 45°, ($g = 980{\cdot}62$). The values of c given by (4) have been multiplied by 1000, so that the corrections given in Table B are expressed in the same units, viz. milligrammes, as the values in Table A.

(ii.) *Example of Use of Tables A and B.*—Suppose that the apparent weight in air of the water contained in a 1000 c.c. flask was found to be 997·705 gms., the water being at the temperature 17·5° C., the air at 18° C., and the barometer reading (corrected to standard conditions) 745 mm.

Under the temperature 17·5° C. in Table A we find the value 2321, and opposite 18° C. and under 745 mm. in Table B the value −33.

Hence the capacity of the flask in c.c. at 15° C. is

$$997·705 + 2·321 - 0·033, \quad \textit{i.e.}\ 999·993 \text{ c.c.}$$

(iii.) *By Means of Mercury.*—The use of mercury is particularly suitable for the determination of small capacities, *e.g.* in the case of gas burettes which are often furnished with a graduated tube of narrow bore divided to 1/100ths c.c.

Let M gms. = apparent weight of mercury in air,
Δ gms./c.c. = density of weights used,
σ gms./c.c. = density of air,
t° C. = temperature of mercury,
V = volume of mercury at t° C.

Then $$M\left(1-\frac{\sigma}{\Delta}\right)=V\rho\left(1-\frac{\sigma}{\rho}\right),$$

or $$V=\frac{M}{\rho}\left[1-\sigma\left(\frac{1}{\Delta}-\frac{1}{\rho}\right)\right].$$

The term $\sigma(1/\Delta-1/\rho)$ is so small that we may use the following average values:

$$\sigma = 0·00122 \text{ gms. per c.c.},$$
$$\Delta = 8·3 \text{ gms. per c.c.},$$
$$\rho = 13·56 \text{ gms. per c.c.},$$

whence $$\sigma\left(\frac{1}{\Delta}-\frac{1}{\rho}\right)=0·000057,$$

and hence $$V=M\times\frac{0·999943}{\rho}.$$

Values of $0·999943/\rho$ have been tabulated for each ·1° C. from 5° C. to 30·9° C. in Table C, the values of ρ being taken from Table II., p. 131.

Hence to obtain the volume in c.c. of any weighed quantity of mercury its observed weight in air must be multiplied by the figure given in Table C under the observed temperature of the mercury.

Table C

° C.	·0.	·1.	·2.	·3.	·4.	·5.	·6.	·7.	8.	·9.
5	·0736182	·0736196	·0736209	·0736222	·0736235	·0736249	·0736262	·0736275	·0736289	·0736302
6	315	329	342	356	369	382	396	409	422	435
7	449	462	475	490	502	516	529	542	556	569
8	582	596	609	622	636	649	663	676	690	703
9	716	730	743	757	770	783	797	810	823	836
10	850	864	877	890	903	917	930	943	957	970
11	984	997	·0737010	·0737024	·0737037	·0737050	·0737064	·0737077	·0737091	·0737104
12	·0737117	·0737131	144	157	170	184	198	211	224	237
13	251	264	278	291	304	317	331	344	357	370
14	384	398	411	424	437	451	465	478	491	504
15	518	531	545	558	571	585	598	611	625	638
16	652	665	678	691	705	718	732	745	758	771
17	785	799	812	825	838	852	866	879	892	905
18	919	933	946	959	972	986	·0738000	·0738013	·0738026	·0738039
19	·0738053	·0738066	·0738080	·0738093	·0738106	·0738119	133	146	160	173
20	186	200	213	226	240	253	267	280	293	306
21	320	334	347	360	373	387	401	414	427	440
22	454	467	481	494	508	521	534	547	561	574
23	588	601	614	628	641	654	668	681	695	708
24	721	735	748	761	775	788	802	815	828	842
25	855	868	882	895	909	922	935	949	962	975
26	989	·0739002	·0739016	·0739029	·0739042	·0739056	·0739069	·0739082	·0739096	·0739109
27	·0739123	136	150	163	176	189	203	216	230	243
28	257	270	283	297	310	323	337	350	364	377
29	391	404	417	430	444	457	471	484	498	511
30	525	538	551	564	578	591	605	618	632	645

By the use of Table C the volume of the mercury at the temperature of the observations is obtained. This is equal to the capacity of the glass vessel at the same temperature, assuming that precautions were taken to have the mercury and the vessel, whose capacity was to be determined, at the same temperature.

It is more convenient, however, to know the capacity of the glass vessel at some standard temperature, and since 15° C. is perhaps the most widely used standard temperature for volumetric apparatus the following table of corrections has been drawn up. The table D gives in $\frac{1}{1000}$ths of a c.c. the correction to be added to the determined capacity of a vessel at any of the tabulated temperatures in order to obtain its capacity at 15°. The table is for a vessel of nominal capacity 1000 c.c. at 15° C., and is based on a cubical coefficient of expansion of glass of 0·000026 c.c. per c.c. per ° C.:

TABLE D

Temp. ° C.	Correction ($\frac{1}{1000}$ths c.c.).	Temp. ° C.	Correction ($\frac{1}{1000}$ths c.c.).
5	+260	18	− 78
6	+234	19	−104
7	+208	20	−130
8	+182	21	−156
9	+156	22	−182
10	+130	23	−208
11	+104	24	−234
12	+ 78	25	−260
13	+ 52	26	−286
14	+ 26	27	−312
15	0	28	−338
16	− 26	29	−364
17	− 52	30	−390

Table E is a similar table for temperatures expressed in degrees on the Fahrenheit scale.

TABLE E

Temp. ° F.	Correction ($\frac{1}{1000}$ths c.c.).	Temp. ° F.	Correction ($\frac{1}{1000}$ths c.c.).
40	+274	65	− 87
41	+260	66	−101
42	+246	67	−116
43	+231	68	−130
44	+217	69	−144
45	+202	70	−159
46	+188	71	−173
47	+173	72	−188
48	+159	73	−202
49	+144	74	−217
50	+130	75	−231
51	+116	76	−246
52	+101	77	−260
53	+ 87	78	−274
54	+ 72	79	−289
55	+ 58	80	−303
56	+ 43	81	−318
57	+ 29	82	332
58	+ 14	83	−347
59	0	84	−361
60	− 14	85	−376
61	− 29	86	−390
62	− 43	87	−404
63	− 58	88	−418
64	− 72	89	−433
65	− 87	90	−448

The tables D and E may be used also in conjunction with tables A and B to obtain the capacity of a vessel whose capacity at 15° C. has been obtained by means of tables A and B.

§ (8) DISPLACEMENT METHODS OF DETERMINING VOLUMES.—A method of determining the volume of solid bodies, which is sometimes convenient, is by means of the apparatus shown diagrammatically in *Fig.* 14. The volume of the apparatus between the two marks A and B must be accurately known. The receiver R is put in communication with the atmosphere by means of the tap T, and by raising the tube C the mercury surface in AB is brought accurately to the mark A. Then T is closed and C lowered until the liquid surface falls to B, and the difference in height, p say, of the mercury surfaces in AB and C respectively, is accurately measured. The tap is next opened, and the solid whose volume is required is introduced into R, C is adjusted until the mercury surface is once more at A, and the tap is then closed and the same procedure followed as before. It may readily be shown that the required volume V of the solid is given by

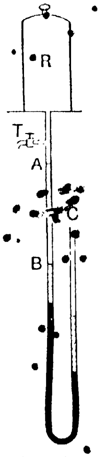


FIG. 14.

$$V = v\left(\frac{B-p}{p} - \frac{B'-p'}{p'}\right),$$

where B is the barometer reading during the first operation, B′ during the second, p and p' the two observed differences in the heights of the mercury surfaces, and v the volume of the apparatus between the marks A and B.

The method is useful for determining the volume of the solid portion of porous substances, but to obtain completely reliable results it is necessary that all the cavities in the substance should be in communication with its surface. As this is hardly likely to be perfectly achieved, the results obtained by this method should be interpreted with caution.

A more generally useful way of determining the volume of a solid is by determining its weight in air and also in water, and in passing it may be noted that all methods of determining the density of solid bodies (*q.v.*) serve also to determine their volume, provided that their mass is known.

A simple displacement method has been found useful at the National Physical Laboratory for testing butyrometers. From our present point of view the essential part of a butyrometer is a narrow-bore tube of capacity about 1 c.c., and graduated usually into 80 equal parts by volume. The butyrometers are filled to the lowest graduation mark with a 50 per cent alcohol-water mixture. Then a brass cylinder equal in volume to one-fifth the nominal total capacity of the graduated portion of the butyrometer is dropped into the instrument. The new position of the liquid surface on the scale is noted, a second cylinder is then added, and so on, until the liquid surface has occupied successive positions along the whole of the graduated scale. The volume of the cylinders being known, and the readings of the successive positions of the liquid surface having been noted, the errors in graduation are at once obtained.

The method is very quick, but demands considerable care in manipulation and extreme accuracy of manufacture of the cylinders.

§ (9) VOLUMETRIC GLASSWARE.—Very large quantities of volumetric glassware are used in chemical and physical laboratories in connection with industrial work, and in technical and educational institutions. It is important that such apparatus should be of reliable accuracy. In order that apparatus whose accuracy has been tested might be available to users, and also to foster and maintain a reliable standard of output on the part of manufacturers, arrangements have been made in several countries for testing volumetric glassware.

The National Physical Laboratory at Teddington has carried out such tests on a small scale for the past fifteen years, and recently a new building, specially equipped for dealing with such tests on a large scale, has been built at the laboratory. Similar tests are also carried out at the Bureau of Standards, Washington, and at the Reichsanstalt, Charlottenburg, and its associated institutions.

The details given below are mainly based on work carried out at one or other of the above institutions. The tolerances given, except where otherwise stated, are those in force at the National Physical Laboratory, as being the ones applicable to apparatus tested in this country. The Class A tolerances on capacity do not differ materially from those of the Bureau of Standards or of the Reichsanstalt. The Class B tolerances are somewhat less stringent, being intended to represent simply the standard of accuracy below which the general output of apparatus for commercial use should not fall. The Class A limits, like the tolerances allowed by the Bureau of Standards and the Reichsanstalt, are, on the other hand, intended for precision apparatus.

§ (10) GENERAL CONSIDERATIONS. (i.) *Cleaning.*—In order to obtain reliable results with glass volumetric apparatus either in testing or in use, especially in the case of vessels used to deliver measured quantities of liquid, cleanliness is essential. The chief sources of trouble are minute traces of grease, which are sometimes very persistent and difficult to remove. The conditions for obtaining consistent results with apparatus used for delivery—*e.g.* burettes, etc.—is that when the vessel is emptied its interior surface should remain wetted with a uniformly distributed film of liquid. The liquid remaining on the inside walls should not collect together into drops or streaks. When a vessel fulfils this condition it is regarded as clean.

A good criterion by which to judge the cleanliness of a burette or pipette is obtained by filling it slowly with water through the delivery jet, the vessel being held vertical. If the vessel is clean the water meniscus will rise in the tube without any change of shape. Should the rising meniscus come into contact with a slightly greasy portion of the vessel it immediately appears to crinkle up. Further, if the vessel is quite clean, a thin film of water may be seen travelling up the sides in front of the main water surface. The front edge of this film is clearly visible and forms an excellent indicator as to the cleanliness of the vessel. In quite clean vessels the edge of the film advances at the same rate as the water meniscus, keeping a uniform distance in front of it. Should the vessel be slightly dirty at any point the front edge of the film is retarded and the water meniscus overtakes it, and may in turn crinkle up when passing the contaminated surface. A pipette or burette which fills up through the jet, so that the front edge of the film keeps in advance of the water meniscus throughout, may be relied upon to have a uniform film of liquid left on the walls when emptied. The advantage of knowing for certainty when filling a vessel that it may be emptied without fear of error due to irregular wetting of the walls is obvious.

Many methods of cleaning apparatus have been employed. If the apparatus is not wanted for immediate use a good way of cleaning it is to fill the vessel with a previously prepared mixture, in equal parts by volume, of saturated potassium bichromate solution and concentrated sulphuric acid and leave it to stand for several hours. An alcoholic solution of caustic soda may be similarly employed. A freshly made solution of potassium permanganate in sulphuric acid acts more quickly, and fuming sulphuric acid is sometimes used for rapid cleaning. A rapid and efficient method of cleaning is to shake up a little absolute alcohol thoroughly in the vessel and then empty it out and allow the vessel to drain for a short time. Then, if a little strong nitric acid is shaken up in the vessel and afterwards thoroughly washed out with water, the vessel will be quite clean. Strong soap solutions also provide an effective means of cleaning and may be used hot in obstinate cases. A good method of mechanical cleaning is to place a number of small pieces of filter-paper in the vessel, partially fill it with water, and then shake vigorously.

If the vessels are to be used for delivery it is sufficient to rinse thoroughly with water after using one or other of the above methods of cleaning. If not required for immediate use after cleaning, the vessels should be completely filled with water until wanted. If left lying about empty and wet the vessels soon become contaminated.

Content vessels after being cleaned require to be dried before testing. A very quick method is to rinse out with alcohol and then with ether and dry by means of an air-blast. Pure alcohol should be used, not methylated spirit. If duty has to be paid on the alcohol, however, the cost is prohibitive where much cleaning has to be done. In such cases acetone will be found an excellent substitute for the alcohol and ether.

In the case of vessels which are to be used with mercury trouble frequently arises if they are dried out by using the above liquids. The most satisfactory procedure is to clean the vessel and rinse it out thoroughly with distilled water, and then dry it by drawing a current of dry air through the apparatus.

Vessels—*e.g.* gas burettes—which have been in use with mercury for some time frequently develop black or dark-coloured stains. These are very difficult to remove by the ordinary cleaning agents. They quickly disappear, however, under the action of zinc dust and dilute hydrochloric acid, being reduced to metallic mercury, which can then be readily dissolved in nitric acid.

(ii.) *Method of Reading.*—The almost universally adopted convention in reading the position of a liquid surface on a scale, or in setting a liquid surface on a graduation mark, is to read the lowest point of the meniscus in the case of liquids such as water, and the highest point of a mercury meniscus.

When a water, or similar meniscus, is viewed in ordinary illumination—*i.e.* in daylight or artificial light without using any screening device—the reflections and refractions which occur at the liquid surface and the glass surfaces render the exact location of the lowest point of the meniscus a matter of some difficulty.

The following extremely simple device is very effective in showing up the outline of the meniscus. A strip of opaque black paper, such as is used for wrapping photographic plates in, is folded round the glass tube just below the meniscus. The top edge of the strip is cut clean and straight, and should be placed not more than 1 mm. below the bottom of the meniscus. The paper should be so folded that the top edges of the two ends of the strip where they meet after encircling the tube are exactly in line. The strip can then be held in position by means of a detachable paper clip.

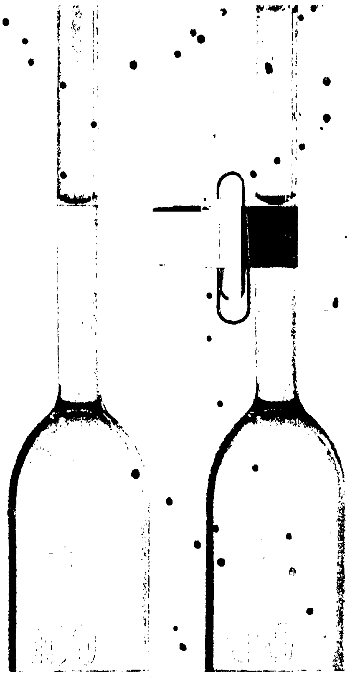


Fig. 15.

If the meniscus so shaded is viewed against a white background the bottom of the meniscus appears quite black and its outline is very sharply defined against the white background.

A piece of black-rubber tubing, slit down one side and fitted round the glass tube, may be used to serve the same purpose as the strip of black paper. The paper is, however, more easy to manipulate.

The first photograph, reproduced in *Fig.* 15, shows a water meniscus in a 100 c.c. pipette viewed against a white background. The second photograph shows the same meniscus shaded as described above. The difference in clearness of the outline of the meniscus is evident in the photographs, and is much more marked when viewing the apparatus itself.

The above method of screening gives much more satisfactory results than the more widely used methods of simply placing a white screen behind the meniscus, or a screen the top half of which is white and the bottom half black.

Further, in the case of apparatus—*e.g.* some burettes on which the graduation marks are engraved on the front of the instrument only—

the paper screen serves a double purpose. By placing the eye so that the top edge of the strip on the front of the burette coincides with the top edge of the strip at the back of the burette, errors in reading due to parallax are practically eliminated.

In the case of a mercury meniscus the opaque screen should be placed immediately above the meniscus, which is then viewed against a white background as before.

When using strongly coloured liquids, such as potassium permanganate solutions, it is difficult to see the bottom of the meniscus clearly, and readings are therefore made on the top edge of the meniscus.

§ (11) TEMPERATURE CONTROL. (i.) *In testing or calibrating Volumetric Glassware.*—No elaborate temperature control is necessary, whether gravimetric or volumetric methods are employed.

The whole operation of testing a vessel, including filling, adjusting, and weighing, can be completed in a comparatively short time. Hence, if the test liquid—*i.e.* in general either water or mercury—and the vessels to be tested or calibrated have been in the same room long enough to attain approximately room temperature, changes in temperature which may take place during the test will be small and of negligible effect. With tables such as those given previously the exact temperature at which the test is carried out is immaterial.

Similarly, when volumetric methods are used, it is only necessary to adopt the same precautions. The fact that the working temperature is different from the standard temperature of the vessel of known capacity is compensated for by the fact that the vessel to be calibrated will have expanded by the same amount as the standard vessel. This presumes of course that the standard vessel and the one to be calibrated both have the same standard temperature.

(ii.) *In using Volumetric Glassware.*—If glass vessels are used at temperatures other than their standard temperature the following correction table may be used. Suppose that a 1000 c.c. flask correct at 15° C. is filled to the mark with water at one of the tabulated temperatures, both water and flask being at the same temperature, then Table F gives for various temperatures the correction which must be added to the observed volume of 1000 c.c. in order to obtain the actual volume of the water when cooled (or heated) to 15° C.

Conversely, by subtracting the corrections from 1000 c.c., the volume which must be measured at the tabulated temperature in order to obtain 1000 c.c. at 15° C., is obtained.

For volumes other than 1000 c.c. proportionate corrections may be used.

A cubical expansion of glass of 0·000026 per degree centigrade and Chappuis' values for the expansion of water have been used in constructing the table.

TABLE F

° C.	·0.	·5.
5	−0·605	−0·608
6	−0·608	−0·603
7	−0·595	−0·583
8	−0·568	−0·549
9	−0·527	−0·501
10	−0·472	−0·439
11	−0·403	−0·363
12	−0·320	−0·275
13	−0·226	−0·174
14	−0·119	−0·061
15	0·000	+0·064
16	+0·130	+0·200
17	+0·272	+0·347
18	+0·425	+0·506
19	+0·589	+0·675
20	+0·764	+0·855
21	+0·949	+1·046
22	+1·146	+1·248
23	+1·352	+1·459
24	+1·569	+1·681
25	+1·796	+1·913
26	+2·032	+2·154
27	+2·278	+2·405
28	+2·534	+2·666
29	+2·799	+2·931
30	+3·074	+3·210

The above table may be used for dilute aqueous solutions having approximately the same coefficient of expansion as water. More accurate results may be obtained in the case of the solutions stated below by increasing the numerical values of the corrections given above by the percentages stated in the following table:[1]

Solution.	Normality.		
	N.	N/2.	N/10.
Nitric Acid . .	50	25	6
Sulphuric Acid . .	45	25	5
Caustic Soda . .	40	25	5
Caustic Potash . .	40	20	4

It will be seen that the corrections required amount to between 0·01 per cent and 0·02 per cent of the total volume per degree centigrade.

In the case of ordinary volumetric analysis therefore, if the standard solutions are made up at temperatures within a degree or so of 15° C., the correction will for most purposes be negligible. It will generally be found more convenient to adjust the temperature of the solutions to approximately 15° C. when preparing standard solutions, rather than to apply corrections.

---

[1] Circular No. 19, Bureau of Standards, 1916.

Further, if all the solutions employed in a titration are given sufficient time to assume room temperature before titration, then the necessity for applying corrections is avoided, since practically the same percentage correction would be required on each volume measured and the net correction to the equivalence of the solutions would be negligible.

So far as volumetric analysis is concerned, therefore, only very ordinary precautions need be taken as regards temperature control.

§ (12) Measuring Flasks. (i.) *Construction and Tolerances.*—Flasks should be made so that they will stand firmly on a level table. The Bureau of Standards also requires that the base must be of such a size that the flasks will stand on a plane inclined at 15° to the horizontal.

The graduation mark should be made by means of a fine clean line permanently etched into the glass. The line should be carried completely round the neck of the flask, should lie in a plane perpendicular to the axis of the neck, and should be horizontal when the flask is standing on a level table. The mark should be on the cylindrical portion of the neck, not on the conical portion where the neck joins the bulb, and also should not be too near the top of the neck. The Bureau of Standards requires that the mark must not be nearer the ends of the cylindrical portion of the neck than specified below:

Capacity.	Distance from Upper End.	Distance from Lower End.
	cm.	cm.
100 c.c. or less . .	3	1
More than 100 c.c. .	6	2

The neck should be cylindrical above the graduation mark and not show a marked taper towards the top. The internal diameter of the neck at the mark should not exceed the values given below.[1]

Capacity	Diameter.	Capacity.	Diameter.
c.c.	mm.	c.c.	mm.
10	6	1000	18
25	8	1500	20
50	10	2000	25
100	12	3000	30
250	14	4000	35
500	16	5000	40

The nominal capacity of the flask and its standard temperature should be permanently marked on it. The flask should also be marked with the letters "C" or "D," or the words "*To Contain*" or "*To Deliver*," in order that the user may know at once on which basis the flask has been calibrated.

Flasks are sometimes made with two graduation marks on the neck, so that when filled to one mark they contain a definite volume, and when filled to the other and then emptied they deliver the same volume. In such cases a letter "C" should be etched below the lower line and a letter "D" above the upper line. This mode of graduating should only be employed in cases where the distance between the two lines is not less than 1 mm.

The tolerances allowed on the capacity of flasks are:

Class A.			Class B.		
Capacity, c.c.	Tolerance ± c.c.		Capacity, c.c.	Tolerance ± c.c.	
	For Content.	For Delivery.		For Content.	For Delivery.
10	0·008	0·016	10	0·02	0·04
25	0·015	0·03	25	0·03	0·06
50	0·03	0·06	50	0·06	0·12
100	0·05	0·10	100	0·10	0·2
250	0·08	0·16	250	0·15	0·3
500	0·15	0·30	500	0·25	0·5
1000	0·20	0·40	1000	0·3	0·6
1500	0·25	0·50	1500	0·4	0·8
2000	0·35	0·70	2000	0·6	1·2
3000	0·5	1·0	3000	0·8	1·6
4000	0·8	1·6	4000	1·2	2·4
5000	1·0	2·0	5000	1·5	3·0

§ (13) Testing of Flasks. (i.) *Gravimetrically.*—The gravimetric method of testing a flask graduated for content will be described in detail, as it serves as a good example of determining capacity of a vessel by means of weighing its water content.

The illustration (*Fig.* 16) shows one of the balances used at the National Physical Laboratory for testing flasks. The balance will carry a maximum load of 2000 gms. on each scale pan. The sensitivity may be varied and is usually adjusted so that a difference in the loads on the two scale pans of 10 mgms. produces one whole scale division change in the position of the rest point. For this sensitivity the balance has a period of approximately 20 seconds for one complete to-and-fro oscillation when loaded with 1000 gms. on each side of the balance. Similar balances ranging in maximum load from 3000 gms. to 250 gms. are in regular use for testing volumetric glassware, and balances of 10 kilo and 50 kilo loads are also available for unusually large capacities.

The illustration shows a 1000 c.c. flask on each scale pan, the one on the right hand pan being filled to the mark with water, and the one on the left being empty. Above the empty flask a kilogramme weight may be seen in the photograph. This is supported on a second platform, and with this arrangement it is possible to accommodate a bulky glass vessel on the lower platform and brass weights on the upper one, and still have the scale pan hanging vertically from its supports. A wooden shelf across the balance case at the level of the upper platform serves to prevent accidents due to weights being dropped on to the lower platform or floor of the balance case.

---

[1] For intermediate capacities the value for the next larger tabulated capacity is to be taken. This applies to all other tables of tolerances unless otherwise stated.

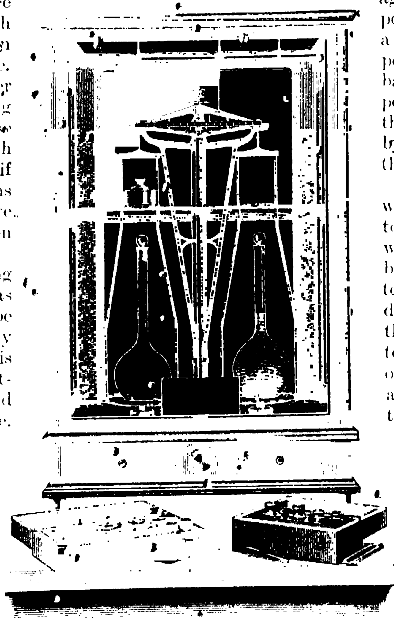


FIG. 16.

A piece of card is mounted on the front glass slide of the balance case, which is pierced with a hole placed centrally in front of the pointer scale. Readings on the pointer are taken by looking through this hole and so avoiding errors which might otherwise arise if the observer had no means of always bringing his eye into the same position when taking readings.

The procedure in testing a flask for content is as follows. The flask to be tested, having previously been cleaned and dried, is placed on the lower platform of the right hand scale pan of the balance. Then gramme weights from a standardised set of weights are placed on the upper platform of the same scale pan, the weights taken being numerically equal to or slightly in excess of the capacity of the flask in c.c. A similar flask is placed on the lower platform of the left hand scale pan and gramme weights are placed on the upper platform until the pointer of the balance swings symmetrically about the zero mark of the scale, the final adjustment being made by means of a rider with the balance case closed. The weights used on the left-hand scale pan need not be of a high degree of accuracy, and should be distinctive in appearance from the standard weights used on the other scale pan. When the balance has been counterpoised as just described the weights on the right-hand scale pan and the position of the rider are noted.

The weights are then removed from the right-hand scale pan, and the flask to be tested is also removed and filled with distilled water to about 1 mm. above the mark. The lowest point of the water meniscus is then set on the graduation mark by withdrawing water through a glass tube of fine bore, the meniscus being shaded as described previously. The carefully filled flask is returned to the lower platform of the right-hand scale pan, and small weights are added from the standard set until an exact balance is once more attained, the final adjustment being made by means of the rider as before. When this adjustment has been made the weights on the right-hand scale pan and the position of the rider are again noted. The air temperature is then read on a thermometer kept permanently inside the balance case. The temperature of the water in the flask is then obtained by means of a second thermometer.

It has been found that with the precautions taken to obtain the distilled water at room temperature before use, the change in temperature which occurs during weighing is so small that it is quite sufficient to take the temperature of the water immediately after weighing, an additional reading before completely filling the flask being unnecessary. The reading of the barometer completes the observations necessary to obtain the capacity of the flask, and the following scheme of recording the observations will serve as a summary to the whole procedure, the correction in milligrammes being obtained from Tables A and B.

Nominal capacity in c.c. at T°	1000	..
“C” or “D” . . .	“C”	..
Standard temperature, T° .	15° C.	..
Barometer reading in mm. of Hg. at 0° C.	759·9	760·1
Air temperature, ° C. . .	18·7	20·6
Water temperature, ° C. .	18·1	19·9
Weight on right-hand scale pans when flask is empty (grammes)	1001·005	1016·000
Weight on right-hand scale pans when flask contains water (grammes)	3·415	18·709
Weight in air of water contained in flask (grammes)	997·500	997·291

Amount to be added to weight to obtain volume in c.c. at T° (milligrammes)	2400	2697
Volume in c.c. of water contained in flask at T° C	999·990	999·988
Mean volume in c.c. of water contained in flask at T° C.	999·989	..

The figures given above are taken from an actual test carried out at the National Physical Laboratory, the two sets of observations being by different observers. It is interesting to note that the difference in the amounts to be added to the observed weight in air to obtain the capacity of the flask at 15° C., arising from different temperature and air conditions, is greater than the maximum error allowed (± 0·2 c.c.) for a Class A flask of the above capacity.

Flasks intended to be used for delivery are tested by first filling them to the mark and then emptying them into a glass vessel. The volume of water delivered into the receiving vessel is determined by the counterpoise method of weighing as described above for a "content" flask.

It is important that flasks used for delivery should always be emptied in the same manner.[1] When tested at the National Physical Laboratory they are emptied by gradually inclining them, until, when the continuous stream of water has ceased, they are very nearly vertical. In this position they are allowed to drain for half a minute, and the edge of the neck is then touched against the inside of the receiving vessel, in order to remove the drop of liquid which collects together on the edge of the neck.

(ii.) *Volumetric Testing of Flasks.*—Flasks may be tested by filling them to the mark from a vessel which has been calibrated for delivery. The main advantage of such volumetric methods of test over gravimetric ones is that they may be carried out in considerably less time. If carefully carried out they may be made extremely accurate, but where the highest accuracy is required gravimetric methods should be used.

Volumetric methods are of supreme importance, however, because they are very largely used by manufacturers in calibrating apparatus.

The apparatus shown diagrammatically in *Fig.* 17 has been used at the Bureau of Standards, Washington,[2] for testing flasks of 100 c.c. capacity, on which their tolerance is ± 0·08 c.c.

The flasks are filled from a standard pipette A whose lower stem is graduated to enable a direct reading of the volume delivered to be taken. The standard pipette is inserted into a heavy rubber connection C. The delivery nozzle D meets the connecting tube B in a ground joint. The pipette is filled through a glass nozzle G which is connected to a water tap and directed into the top of the pipette.

"The pipette is filled to the top, the nozzle is swung aside, and by opening the stopcock the meniscus is slowly lowered to the zero mark. Excess of water is then removed from the outflow nozzle, and the flask to be tested is placed on the platform E immediately under the outflow nozzle. The platform is raised by turning the wheel[3] F until the outflow nozzle is just inside the neck of the flask. The stopcock is opened wide and the flask rotated to wet the entire neck, and the flask is then raised until the nozzle is 1 to 2 cm. above the mark. Before completing the filling of the flask it is removed and the contents shaken as directed in the rules for manipulation. The filling is completed with the tip in contact with the wetted wall 1 to 2 cm. above the mark. The meniscus in the flask is finally brought to the mark by breaking contact of the tip with the wetted surface.

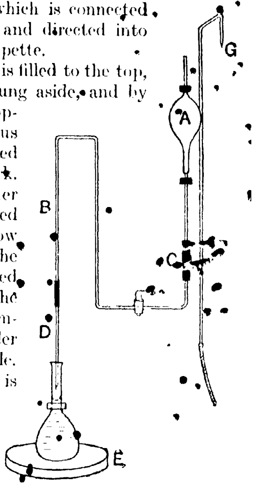


FIG. 17.

"The standard pipette is read at the end of its normal outflow time plus 15 seconds. The pipette reading plus the instrumental correction is the capacity of the flask at the standard temperature of the pipette—that is, 20° C. The object of shaking the water is to disperse the contaminations and thus produce a meniscus of normal volume. The manipulation reproduces the conditions of ordinary use. If the test is to merely ascertain whether the capacity is within the allowed limits of error, this detail is omitted unless the error is too near the limit to allow discrimination, in which case a re-test is made. . . . The pipettes used for this purpose at this bureau are interchangeable, it being only necessary to employ a nozzle of the proper size in order to use any pipette with the holder and outflow tube."

A slightly different procedure is now

---

[1] For experimental data on this point see Schloesser, *Zs. für anal. Chem.*, 1907.

[2] N. S. Osborne and B. H. Veazey, *Bull. Bureau of Standards*, 1908, iv. 590.

[3] Not shown in *Fig.* 17.

followed at the Bureau of Standards.[1] A series of standard pipettes are employed, each standard having a capacity slightly less than that of the flask to be tested. The standard pipette is emptied into the flask and the final setting is made by adding water from a finely graduated burette. The capacity of the flask is found from the known volume delivered by the pipette and the additional volume taken from the burette.

In passing, it should be noted that at the Bureau of Standards the whole of the inside of the neck of the flask is wetted when making a test, whereas at the National Physical Laboratory and the Reichsanstalt the portion of the neck above the graduation mark is kept dry.

A somewhat different type of apparatus from that previously described is shown in *Fig.* 18. The tube at the top of the graduated pipette is drawn off into a fine jet which is ground off square and polished smooth. The pipette is filled through the side tube A, and when it is partially filled the stopcock B is opened so that some water runs out at the delivery jet, when B is again closed. The tip of the jet is then touched on to the surface of some water contained in a beaker, or other convenient vessel, to remove any drop of water adhering to the jet. Water is then allowed to enter once more through the side tube until it overflows at the top of the pipette into the flask inverted over the top of the pipette as shown in the figure. The flask is provided with a hole C to allow air to enter freely, and with a side tube D to carry away the water overflowing from the pipette. When the water is overflowing freely the stopcock A is gradually shut off, leaving the pipette filled quite to the top of the overflow jet, and ready for use.

The flask to be tested is placed on a platform E which is mounted on a vertical shaft and can be readily raised or lowered. The motion of E should be easily controllable and the apparatus well made so that the platform remains horizontal throughout the whole of its traverse.

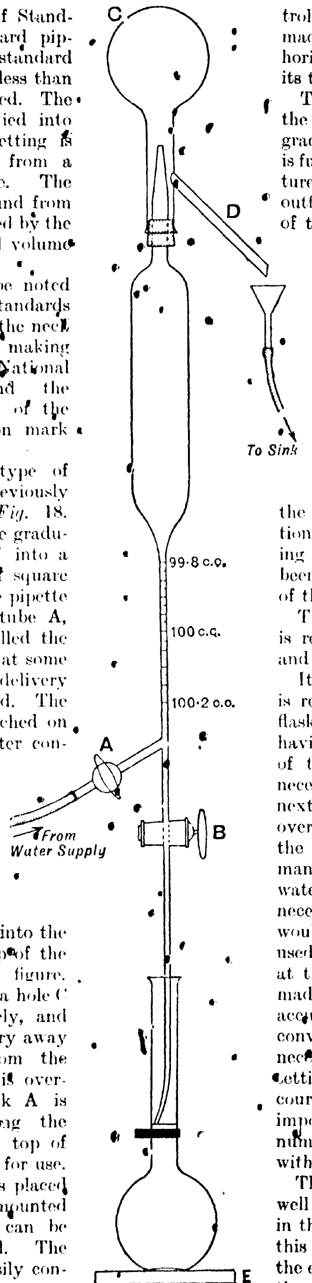


FIG. 18.

The flask is raised until the tip of the delivery jet is just above the graduation mark. The stopcock B is fully opened and the slight curvature of the delivery jet directs the outflowing water on to the inside of the flask just below the graduation mark, thus avoiding splashing and the formation of air bubbles The stopcock is kept fully open until the surface of the water in the pipette has reached the top of the graduated portion, when the stopcock is closed. The filling is completed by running out small quantities of water at a time until the water surface in the neck of the flask is exactly on the graduation mark after the last drop adhering to the jet of the pipette has been touched off on to the surface of the water in the flask.

The volume of water delivered is read off on the graduated scale and the test is complete.

It may be remarked that the jet is ready for commencing the next flask, the last drop adhering to it having been removed at the end of the previous test. All that is necessary before commencing the next test is to fill the pipette to overflowing, then close A and put the next flask in position. No manipulation of B, nor removal of water from the delivery jet, is necessary in between two tests as would be the case if a zero line were used instead of the overflow point at the top of the pipette. Well-made overflow jets give quite accurate results and are a great convenience in eliminating the necessity for making an accurate setting on a zero line, and also, of course, in thereby saving time, an important consideration where large numbers of flasks have to be dealt with.

The apparatus just described is well adapted to calibrating flasks in the course of manufacture. For this purpose a single line defining the exact capacity required replaces the graduated scale. All that is necessary is to empty the pipette down to this mark into the dried flask, which is then ready for pointing.

---

[1] Circular of the Bureau of Standards, 1916, No. 9, 8th Ed.

The two examples given will serve as types of apparatus suitable for testing and calibrating flasks. A number of variations are possible,[1] but in all forms of apparatus the following considerations are of fundamental importance.

The volume of liquid delivered by any given piece of apparatus varies considerably with the rate at which it is emptied. Again if the volume is to be read off on a graduated scale the reading obtained will depend upon how long the apparatus has been draining before taking the reading.

The exact conditions of working should be carefully determined, *i.e.* the length of time during which the apparatus is emptying freely with the stopcock fully open, the length of time during which it is emptied slowly in making the setting on the graduation mark of the flask, and the time which elapses between finally closing the tap and taking the reading on the graduated scale of the pipette should all be considered. A perfectly definite routine procedure should be laid down for the apparatus, and strictly adhered to whenever the apparatus is used. Also when calibrating the apparatus gravimetrically in the first place the conditions must duplicate the actual conditions of use. Neglect of these precautions may very easily lead to the introduction of errors greatly in excess of the tolerances with which the flasks are required to comply.

In conclusion, volumetric methods of test have the following advantages. Provided that the standard apparatus and the flasks to be tested or calibrated have approximately the same coefficient of expansion, then if the apparatus, flasks, and water used in the process are all at the same temperature no temperature observations need be recorded. Corrections for temperature and air buoyancy have always to be taken into account when using gravimetric methods. Hence fewer observations have to be made in volumetric than in gravimetric tests, and the final result is arrived at more directly, the only calculation necessary being the addition of the previously determined corrections to the observed readings of the standard apparatus. Distilled water is not necessary for volumetric methods, and hence only a storage tank in which the ordinary water supply can attain room temperature is required. Finally, volumetric methods require considerably less time than gravimetric ones, and particularly is this so in the case of calibrating blank flasks in the course of manufacture.

§ (14) CONTAMINATION OF FLASKS.—As the water surface rises in the bulb of a flask as it is being filled the surface is liable to become contaminated by slight traces of "dirt" which still remain after the ordinary methods of cleaning have been employed. When the flask is filled to the graduation mark the surface contaminations are all collected on to the comparatively small area of the water surface in the neck of the flask. Such contamination reduce the surface tension as compared with that of pure water. Consequently the volume of the meniscus (*i.e.* the volume of liquid above a horizontal plane tangential to the lowest point of the meniscus) is less than is the case with pure water, and the total volume of liquid contained in the flask is reduced by a corresponding amount.

Osborne and Veazey[2] carried out a series of experiments to investigate the errors arising from the above cause. They arrived at the conclusion that the surface tension of the water meniscus in volumetric apparatus is liable to be only 0·5 times the value for pure water, and that this causes an error of from 1 to 4 parts in 10,000 in the case of ordinary measuring flasks.

On the basis of the above diminution in the value of the surface tension the authors obtained the following values of the resulting errors for flasks of various capacities:

Capacity.	Internal Diameter of Neck.	Possible Error due to Contamination.	Internal Diameter of Neck.	Possible Error due to Contamination.
c.c.	mm.	c.c.	mm.	c.c.
50	10	0·018	6	0·003
100	12	0·032	8	0·008
200	13	0·041	9	0·012
300	15	0·058	10	0·018
500	18	0·085	12	0·032
1000	20	0·105	14	0·049
2000	25	0·155	18	0·085

It was further found that if, when filling a flask, the contents are thoroughly shaken just before completing the filling, the surface contaminations become disseminated, and by this means the errors arising from such contaminations may be eliminated.

§ (15) USE OF LIQUIDS OTHER THAN WATER.—The difference in the volume of liquid contained in a flask when it is filled with water and when it is filled with some other liquid is simply the difference between the volume of the meniscus in the two cases.

The table on following page[3] gives the difference in c.c. between this volume in the case of water and liquids of varying capillary constant a^2, where a^2 is defined by

---

[1] See also H. N. Morse and T. L. Blalock, *Amer. Chem. Jour.*, 1894, xvi. 479.

[2] *Bureau of Standards Bull.*, 1908, iv. 567-574.

[3] Osborne and Veazey, *Bureau of Standards Bull.*, 1908, iv. 582.

$$a^2 = \frac{2T}{gs},$$

T being the surface tension of the liquid in dynes per cm., s the density of the liquid, and g the acceleration due to gravity.

Flasks with graduated necks are sometimes useful; for example, the flasks graduated in $\frac{1}{10}$ths c.c. from 125 c.c. to 135 c.c. used in oil estimations.

Milk-test bottles of the Babcock and Leffmann-Beam type are small flasks with graduated necks, the graduations indicating percentages of milk fat.

Tube Diameter.	Capillary Constant in mm.²										
	14.	13.	12.	11.	10.	9.	8.	7.	6.	5.	4.
mm 4	·000	·000	·000	·000	·000	·000	·001	·001	·001	·001	·001
5	·000	·000	·000	·001	·001	·001	·001	·002	·002	·003	·003
6	·000	·001	·001	·001	·002	·002	·003	·004	·004	·005	·007
7	·000	·001	·001	·002	·003	·004	·005	·006	·007	·009	·012
8	·001	·001	·002	·003	·005	·006	·008	·010	·012	·015	·018
9	·001	·002	·004	·006	·007	·010	·012	·015	·018	·023	·027
10	·001	·004	·006	·008	·011	·014	·018	·022	·027	·032	..
11	·002	·005	·008	·012	·016	·020	·024	·030	·036	..	..
12	·003	·007	·011	·016	·021	·025	·032	·040	..	..	..
13	·004	·009	·014	·020	·026	·033	·041	..	..	..	..
14	·005	·011	·017	·025	·033	·042	..	..	..	..	..
15	·006	·013	·022	·030	·041	..	..	..	..	..	..
16	·007	·016	·027	·037	..	..	..	..	..	..	..

Hence if the capacity of a flask is known for water, then its capacity for any other liquid may be determined by subtracting the value given in the above table corresponding to the internal diameter of the neck of the flask used, and the capillary constant of the liquid, from the capacity as determined for water. The above table is strictly true for a temperature of 20° C. only, and is based on the value $a^2 = 14{\cdot}821$ mm.² for water.

Flasks calibrated with water for delivery should not be used where precision is required, for liquids which differ greatly in viscosity from water, without recalibration for the particular liquid to be used.

§ (16) VARIETIES OF FLASKS.—In addition to the most widely used type of flask with a single graduation mark on the neck various other forms are used for special purposes.

Flasks with two lines on the neck which determine two slightly different volumes, *e.g.* 100 c.c. and 110 c.c., are useful in sugar analysis where it is required to add a small volume of basic lead acetate solution to a measured volume of sugar solution.

When two successive volumes which differ considerably have to be measured, flasks with two bulbs, one above the other, are used. One graduation mark is placed on the short cylindrical tube connecting the bulbs and the other on the neck of the flask above the upper bulb. Flasks made in this manner, but with subdivisions marking $\frac{1}{10}$ c.c. intervals, both on the connecting tube and the neck, are largely used in determining the specific gravity of cement.

§ (17) PIPETTES. (i.) *Test Regulations.*—In order to obtain satisfactory results with ordinary one-mark delivery pipettes it is essential that the instruments should be made to an exact specification and calibrated for a definitely specified method of use. The various details which have to be considered are illustrated by the following account of the regulations relating to pipettes which are in force at the present time at the National Physical Laboratory:[1]

(1) The graduation mark must be made by means of a fine clean line carried completely round the suction tube and lying in a plane perpendicular to the axis of the tube.

(2) Pipettes of capacity greater than 5 c.c. must have a delivery tube below the bulb. Pipettes of 5 c.c. capacity or less may have the bulb itself drawn out into a jet if so desired.

(3) The top of the suction tube must be ground off square, and the ground surface must be smooth.

(4) The delivery jet must be made with a gradual taper. A sudden constriction at the orifice is not allowed. The end of the jet must be ground off true and the ground-off surface must be smooth.

(5) The outlet must be of such a size that the time occupied by the outflow of water, as defined in paragraph (8) below, conforms with the times given in the following table:

---

[1] Taken from the Nov. 1919 edition of a pamphlet (*Volumetric Tests on Scientific Glassware*) issued by the National Physical Laboratory.

Capacity.	Minimum Delivery Time allowed.	Maximum Delivery Time allowed.
c.c.	secs.	secs.
2	5	10
5	10	20
10	15	30
50	20	40
100	30	60
250	45	90
500	60	120

NOTE.—For capacities not tabulated the delivery times are those of the next larger tabulated capacity.

(6) The time of outflow and the drainage time (15 secs.) must be marked on all pipettes. For example, a suitable inscription for a 50 c.c. pipette would be:

50 c c.
D. 15° C.
(30 + 15) secs.

The actual time of outflow must be within the limits given in the preceding paragraph, and also must not differ from the time etched on the pipette by more than the amounts given in the following table:

Marked Time of Outflow.	Maximum Difference allowed between the Marked Time of Outflow and the Actual Time of Outflow.
secs.	± secs.
30	2
60	4
90	6
120	8

NOTE.—For marked times of outflow not given in the above table the tolerances are the same as for the next larger tabulated times.

(7) The distance from the tip of the jet to the line above the bulb is measured on all pipettes submitted for test. This dimension expressed in mm. is etched on each pipette which passes the tests.

(8) Ordinary pipettes are clamped vertically for test, and filled with water to a short distance above the mark. Water is run out until the meniscus is on the mark and the outflow is then stopped. The drop adhering to the tip is removed by bringing the surface of some water contained in a beaker into contact with the tip and then removing it without jerking. The pipette is then allowed to deliver into a clean weighed vessel held slightly inclined so that the tip of the pipette is in contact with the side of the vessel. The pipette is allowed to drain for ¼ minute after outflow has ceased, the tip still being in contact with the side of the vessel. At the end of the draining time the receiving vessel is removed from contact with the tip of the pipette, thus removing any drop adhering to the outside of the pipette. To determine the instant at which the outflow ceases, the motion of the water surface down the delivery tube of the pipette is observed, and the delivery time is considered to be complete when the meniscus comes to rest slightly above the end of the delivery tube. The ¼ minute draining time is counted from this moment.

(9) The tolerances allowed on pipettes are:

Class A.		Class B.	
Capacity.	Tolerance. For Content or Delivery.	Capacity.	Tolerance. For Content or Delivery.
c.c.	± c.c.	c.c.	± c.c.
2	0·006	2	0·012
5	0·01	5	0·02
10	0·015	10	0·03
20	0·02	20	0·035
30	0·025	30	0·045
50	0·035	50	0·06
100	0·05	100	0·08
150	0·07	150	0·10
250	0·08	250	0·12
500	0·15	500	0·25

NOTE.—For capacities not given in the above table the tolerances are the same as those given for the next larger tabulated capacity.

(ii.) *Methods of using Pipettes.*—When a liquid such as water is allowed to run out from a pipette the pipette does not completely empty itself. Apart from the film of liquid which remains wetting the walls, a small quantity of liquid collects in the jet. Many methods of using pipettes have been put forward from time to time, but the large majority fall into one or other of the two following classes:

(*a*) Those methods in which the drop of liquid which collects in the jet is allowed to remain there.

(*b*) Those in which this drop is ejected.

With regard to the second methods the drop remaining is sometimes ejected by blowing down the pipette. An alternative way is to close the top of the pipette with one finger and to clasp the bulb with the other hand. The resulting expansion of the air inside the pipette expels the drop of liquid from the jet.

Methods involving the ejection of the drop of liquid from the pipette are the less satisfactory of the two classes.

To be satisfactory any method of use must be such that it is easily reproducible by different observers, and for a given pipette must give consistent results.

Schloesser[1] found as a result of a series of experiments that the difference between the results obtained with the same pipette vary amongst themselves by a greater amount when the pipette is blown out than when the drop which collects in the jet is not ejected. This was true of various sizes of pipettes and for different observers.

---

[1] *Zs. für angewandte Chemie*, 1903.

The writer recently found, as a result of a large number of determinations by different observers, that when the drop was ejected by warming the bulb with the hand as previously described, the lack of concordance between the results obtained was much greater than when the pipettes were emptied as described above.

These results are only what might be expected on general grounds. If the drop is ejected by warming the bulb with one hand the time which elapses before the drop is expelled,[1] and also the force with which it is expelled, differ with different observers, and also depend on how tightly the bulb is clasped. The method thus involves conditions which are not accurately reproducible, and the same remark applies more forcibly if the drop remaining is ejected by blowing into the pipette.

The main advantage claimed for the methods involving ejection of the drop is that by such methods better agreement is obtained between the volumes of different liquids delivered by the same pipette.

The amount of liquid which remains in the jet of a pipette will depend on the physical properties of the liquid, and vary with different liquids. It therefore appears an obvious method of equalising the results for different liquids to eject the drop and include it in the volume delivered by the pipette.

The gain is more apparent than real. Far more important than the variation in the volume of the drop of liquid which collects in the jet is the variation in the amount of liquid which remains, wetting the whole interior of the pipette. To eject the drop from the jet by no means secures consistent results with different liquids, and the possible small gain in this direction is more than counterbalanced by the disadvantages stated previously.

It should be stated, however, that throughout the last thirty years or so methods involving ejection of the drop of liquid from the jet have had their adherents, and still have at the present time, who maintain that such methods give the most consistent results.

On the other hand, the National Physical Laboratory, the Bureau of Standards, and the Reichsanstalt have never adopted such methods for standardisation purposes. The National Physical Laboratory and the Reichsanstalt adopt the method given in (ii.) (*a*) above. The Bureau of Standards allows the pipette to empty freely until the water surface enters the delivery tube. The jet is then brought into contact with the wet surface of the receiving vessel and kept there until the emptying is complete.

The difference between the volume of liquid delivered by a pipette using free outflow and that delivered when the jet is kept in contact with the inside surface of the receiving vessel is negligible.

A method of using pipettes is sometimes applied in volumetric analysis which differs from those already considered. The pipette is calibrated to contain its nominal volume instead of to deliver that amount. When used it is filled to the mark with the solution to be measured, and then allowed to empty through the delivery jet. It is then thoroughly rinsed out with distilled water and the rinsings added to the solution previously delivered. The total volume of solution obtained in this way is of course variable, but the whole of the solution initially filling the pipette is transferred to the receiving vessel. Hence the total weight of reagent present in the solution originally taken is transferred to the receiving vessel, and this being the case the subsequent dilution is immaterial provided the concentration of the original solution and the content capacity of the pipette are known.

The advantage of the method is that the whole of the solution is obtained from the pipette, and so the variations with different liquids owing to varying amounts of liquid being left on the walls of the pipette when it is emptied without rinsing are eliminated. The differences arising from the different meniscus volumes of different liquids are not eliminated by the above method. In tubes of the diameter used for the suction tubes of pipettes, however, the differences in meniscus volumes are practically negligible.

The objection to the method is that it requires much more care in manipulation and also requires more time than the ordinary methods.

(iii.) *Delivery Time and Drainage Time of Pipettes.*—It will be seen from the account given above that the National Physical Laboratory regulations require that each pipette submitted for test must have its delivery time marked on it. The actual delivery time must lie within specified limits and must not differ from the marked delivery time by more than the small tolerances stated. In addition, the period to be allowed for drainage, viz. 15 secs., must be marked on each pipette.

The regulations are therefore based on the following conclusions:

---

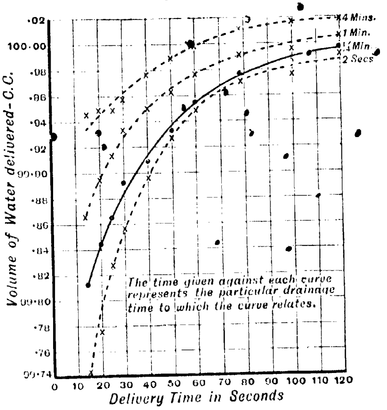
[1] In the case of pipettes of small capacity it is sometimes a matter of difficulty to expel the drop by this method.

(1) It is necessary that the delivery time of each pipette should be known.

(2) The delivery time of a pipette should lie between certain limits depending on its capacity.

(3) A definite interval must be allowed for drainage.

It will be convenient to consider these points in the order given by reference to the



The time given against each curve represents the particular drainage time to which the curve relates.

FIG. 19.

particular case illustrated by the graphs shown in *Figs.* 19 and 20.

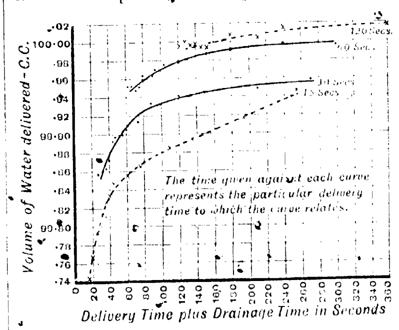
The experimental data on which the graphs were based were obtained with a 100 c.c. pipette whose dimensions were :

Length of bulb	164 mm.
Length of delivery tube	182 mm.
Length of suction tube	177 mm.
Distance of graduation mark from top end of delivery tube	149 mm.
External diameter of bulb	32 mm.
External diameter of delivery tube	7·4 mm.
Internal diameter of suction tube	7·3 mm.

Initially the delivery time of the pipette was 120 secs. For this delivery time the volume of water delivered, allowing various drainage times, was determined by weighing. The different drainage times employed were 2, 5, 10, 15, 20, 25, 30, 45, 60, 120, 180, and 240 secs. When these observations were completed the tip of the pipette was carefully ground away until the delivery time was reduced to 100 secs., and for this delivery time also the volume of water delivered for each of the drainage times given above was determined. The delivery time was reduced by successive grindings to the values 80, 60, 50, 40, 30, 25, 20, and 15 secs., and for each delivery time the same series of volumes for various drainage times was determined. The total change in the absolute capacity of the pipette due to grinding was approximately 0·01 c.c., an amount which is small in comparison with the observed changes in the volume of water delivered.

The data on which the accompanying graphs are based were taken from the more extensive series of observations just outlined.

The first point which we have to consider is the necessity for the delivery time for which a pipette was calibrated being known to the user. The full black line in *Fig.* 19 shows the relation between the volume of water delivered by the 100 c.c. pipette when the standard drainage time of 15 secs. is allowed. Suppose that the pipette had been calibrated for 60 secs. delivery time, but that the delivery time was subsequently reduced to 30 secs., *e.g.* by repairing a slight damage to the jet. Then the amount of water delivered for 30 secs. drainage time and 15 secs. drainage time would be—as may be seen from the graph—0·07 c.c. less than the volume delivered in 60 secs. and 15 secs. drainage time. The change is thus greater than the whole Class A tolerance, viz. 0·05 c.c., on a pipette of this capacity. Even if four minutes drainage time were allowed in each case the change would still be 0·04 c.c., as may be seen from the top curve in *Fig.* 19. The times 30 secs. and 60 secs. are the limits between which the delivery times of standard 100 c.c. pipettes must lie. The above figures show clearly that it is not sufficient for the delivery time of a pipette to be within the limits laid down, but the exact delivery time for which it was calibrated must also be known.



The time given against each curve represents the particular delivery time to which the curve relates.

FIG. 20.

The data given relate to 100 c.c. pipettes only, but the same thing holds good for pipettes of other capacities, as is shown by the following table, in which Δv is the change produced in the volume of water delivered by a pipette of the stated capacity when its delivery time is changed from the maximum to the minimum value allowed, a drainage time of 15 secs. being allowed in each case.

Capacity of Pipette.	Limits on Delivery Time.	Δv.	Class A. Limit on Capacity.
c.c.	secs.	c.c.	c.c.
100	30-60	$0{\cdot}07$	$0{\cdot}05$
50	20-40	$0{\cdot}04_6$	$0{\cdot}035$
25	20-40	$0{\cdot}03_0$	$0{\cdot}025$
10	15-30	$0{\cdot}02_7$	$0{\cdot}015$
5	10-20	$0{\cdot}01_8$	$0{\cdot}010$
2	5-10	$0{\cdot}01_4$	$0{\cdot}006$

Hence the change Δv is in each case greater than the maximum error tolerated on a Class A pipette, and there is thus good reason that the actual delivery time of each individual pipette should be marked on the instrument. This inscription, together with the distance from the tip of the pipette to the graduation mark, which is measured at the time of test and engraved on the pipette, enables the user to ascertain with certainty whether the pipette is in the same condition as when tested.

Turning now to the question of limits between which the delivery times of pipettes of any given capacity should lie, the graphs shown in *Fig.* 20 furnish information bearing on this point. The data for the graphs were obtained from the same series of experiments as the data for the graphs in *Fig.* 19. The curves marked 15 secs., 30 secs., 60 secs., and 120 secs. show the volume of water delivered by the 100 c.c. pipette for these particular delivery times with varying draining times.

All the curves start at a point where drainage has already been going on for 2 seconds. It will be seen from the curves that when the delivery time is 15 secs. 0·045 c.c. of water drains out of the pipette in the first 5 secs. shown on the curve. When the delivery is 60 secs., however, a drainage period of 98 secs. is required for the same volume of water to drain out, and with 120 secs. delivery time only 0·027 c.c. drained out in the whole 4 minutes interval over which the observations extended.

The fast rate of drainage which is thus seen to occur when a quick delivery time is used means that a comparatively large volume of water has been left behind on the walls of the pipette. In such cases much greater differences will be found between the volumes delivered when different liquids are used than when a long delivery time is used. Further, in order to obtain consistent results with the same liquid, the period allowed for draining must be very accurately timed if a short delivery time is employed owing to the rapid rate of drainage.

There are thus serious disadvantages attending the use of pipettes which have short delivery times.

If the rate of outflow is too slow, however, the use of the pipette becomes unduly tedious and absorbs too much of the user's time. Further, with very fine jets there are more likely to be variations in the volume of liquid which is detached against the side of the receiving vessel.

It is therefore necessary to fix both an upper and a lower limit to the delivery time.

The lower limit should be so chosen that the rate of drainage is not too rapid. The following table shows the error Δv which would arise in the volume of liquid delivered by pipettes of varying capacity if the time allowed for drainage differed by 5 secs. from the standard time of 15 secs., assuming that the delivery time of each pipette had the minimum value allowed by the regulations given previously.

Capacity of Pipette.	Minimum Delivery Time allowed.	Δv.	Class A. Tolerance on Capacity.
c.c.	secs.	c.c.	c.c.
100	30	$0{\cdot}00_9$	$0{\cdot}05$
50	20	$0{\cdot}00_8$	$0{\cdot}035$
25	20	$0{\cdot}00_3$	$0{\cdot}025$
10	15	$0{\cdot}00_2$	$0{\cdot}015$
5	10	$0{\cdot}00_1$	$0{\cdot}010$
2	5	$0{\cdot}00_0$	$0{\cdot}006$

The errors are thus seen to be small and, moreover, are based on the assumption that the 15 secs. drainage period is not strictly adhered to, and are therefore capable of being diminished by careful manipulation.

The maximum delivery times allowed are twice the minimum values given above, and give ample margin to the manufacturer and are not unduly long for the user.

Lastly, the desirability of fixing a definite period of drainage remains to be considered.

Some authorities are of the opinion that it is preferable to stipulate that no period for drainage should be allowed, but that the delivery should be regarded as complete immediately the meniscus comes to rest near the bottom of the delivery tube, *i.e.* at the end of the natural delivery time of the pipette.

One objection to this is that if the drainage period is eliminated the impression is apt to be created that it is immaterial whether an observer allows a short time for drainage or not. The data previously given show that this impression is erroneous. Again, from the point of view of reproducing in calibration the conditions of use, it is more difficult to allow absolutely no drainage time than to allow a definitely specified interval. Moreover, a variation of a few seconds in the drainage time immediately after the delivery time is ended is more serious than a variation of the same amount in observing a drainage time of

15 secs., since the initial rate of drainage is appreciably greater than it is 15 secs. later. There is also the consideration that one is less likely to obtain consistent results by removing the jet from contact with the side of the receiving vessel immediately the delivery time is ended than by removing it somewhat later, when the conditions are more steady.

There appears, therefore, to be sufficient reason to justify the practice of allowing a definite period for drainage, and the period of 15 secs. is one which does not make an inordinate demand on the user's time and yet is ample to secure satisfactory results.

(iv.) *Use of Liquids other than Water.*—Experiments on this point were carried out by W. Schloesser.[1]

The lowest point of the meniscus was set on the mark in all cases except milk, in which case the top of the meniscus was read. The pipettes were emptied with free outflow, touched against the side of the receiving vessel to remove the drop adhering to the jet at the end of the outflow time, and no period of waiting for drainage was allowed. The dimensions of the pipettes used are not stated. The results are summarised in the following table in which the value Δv is the difference: volume of liquid delivered minus volume of water delivered, and is expressed in c.c. The column headed *t* secs. gives the delivery times of the pipettes for the liquids specified.

Liquid.	Concentration.	Temp. of Observation.	50 c.c. Pipette.		25 c.c. Pipette.		10 c.c. Pipette.	
		°C.	*t* secs.	Δv c.c.	*t* secs.	Δv c.c.	*t* secs.	Δv c.c.
Water	..	18	39	..	32	..	20	..
Nitric Acid . . .	N/1	17·5	39	−0·005	32	+0·006	20	0·000
Sulphuric Acid . .	N/1	17·7	39·5	+0·003	31·8	−0·003	..	..
Hydrochloric Acid .	N/1	18·2	39	+0·017	..	..	20·7	+0·006
Oxalic Acid . .	N/1	17·9	39·5	+0·013	32	+0·005	..	..
Sodium Carbonate* .	N/1	18·0	42	−0·035	..	..	20·5	−0·003
„ „ .	N/2	17·5	39·2	0·014	..	..	21·2	0·004
Caustic Soda . .	N/1	18·4	40	−0·015	..	..	19·5	0·000
Caustic Potash . .	N/1	17·0	39·4	−0·017	32	+0·005	..	..
Ammonia . . .	N/1	18·0	39·4	+0·001	..	..	20·4	−0·005
Fehling's Solution I. .	..	18·1	39·8	−0·012	..	..	20·3	−0·005
„ „ II.*	..	18·8	46·2	−0·198	..	..	28·5	−0·013
Barium Chloride . .	N/1	17·6	39·5	−0·014	32·2	−0·005	..	..
Sodium Thiosulphate .	N/10	16·8	40	−0·005	31·4	−0·005	..	..
Potassium Bichromate	N/1	16·7	39	0·005	31·6	−0·003	20·4	−0·007
Sodium Chloride . .	N/10	16·4	39·2	−0·004	..	..	20·8	+0·002
Ammonium Sulphocyanide . . .	N/1	18·0	38·8	0·000	32·2	+0·004	20·7	−0·007
Potassium Permanganate . . .	N/10	18·4	40	+0·010	..	..	20·4	0·000
Ferric Chloride* . .	1 c.c. = 0·012 g. Fe	17·2	39·8	−0·035	..	..	20·8	0·000
Uranium Solution .	1 c.c. = 0·005 g. P_2O_5	16·7	39·4	0·006	32·0	+0·006	..	..
Mercuric Nitrate . .	1 c.c. = 0·01 g. Urea	17·6	39·6	+0·004	31·6	+0·003	..	..
Silver Nitrate . .	N/10	17·8	40·2	+0·005	32·2	+0·012	21·2	+0·004
Sugar Solution . .	1%	18·8	40	0·015	..	..	18·8	−0·003
Indigo Solution (Kubel and Tiemann) . .	6·3 c.c. = 0·001 g. N_2O_5	17·6	39·3	−0·003	32·0	0·000	..	..
Iodine* . . .	N/10	17·7	39·0	+0·025	..	..	19·5	−0·006
Alcohol	99·8%	19	41·6	−0·142	33·5	−0·067	..	..
„	50·5%	19·0	44·8	−0·229	37·4	−0·088	23·0	−0·059
Sulphuric Acid (Conc.)	96%	19·6	64	−0·442	..	..	52·4	0·085
Nitric Acid . . .	30%	19·5	40	−0·004	..	..	21·6	+0·007
Caustic Potash (Conc.)	15%	18·1	40·4	−0·038	..	..	21·2	0·007
Milk [1]	..	19·3	41·6	0·174	..	..	20·8	0·052
Milk [2]	..	19·3	41·6	−0·092	..	..	20·8	−0·022

[1] Setting made on lowest point of water meniscus and highest point of milk meniscus.

[2] Setting made on highest point of meniscus for both milk and water.

It will be seen from the above table that of the solutions above the horizontal line dividing the table into two parts the differences between the volumes of liquid and of water delivered are almost negligible except in the four cases marked with an asterisk. The concentrations given are the greatest ordinarily used in volumetric analysis, and hence with very few exceptions it may be said that pipettes calibrated with water may be used interchangeably for solutions ordinarily employed in volumetric analysis.

---

[1] *Chemiker Zeitung*, 1906, xxx. 1071.

In the case of the liquids in the lower portion of the table, with the single exception of the 30 per cent nitric acid solution, the values of Δv are excessive. For liquids such as those given whose physical properties differ considerably from those of water it is necessary to calibrate the pipettes for the particular liquid for which they are to be employed.

Ext. Diam. of Bulb of Pipette.	Delivery Time (Seconds).		Volume (c.c.) delivered with 2 secs. Drainage.			Volume (c.c.) delivered with 15 secs. Drainage.		
	For Water.	For Milk.	Water.	Milk.	Diff.	Water.	Milk.	Diff.
mm.								
15·7	3·6	3·7	9·881	9·762	−0·119	9·933	9·879	−0·054
15·4	5·3	5·5	9·903	9·813	−0·090	9·927	9·865	−0·062
15·7	7·9	8·3	9·947	9·865	−0·082	9·953	9·894	−0·059
14·9	12·3	12·8	9·972	9·922	−0·050	9·976	9·926	−0·050
14·7	16·5	17·5	10·016	9·981	−0·035	10·019	9·986	−0·033
16·2	23·1	25·7	10·019	9·985	−0·034	10·020	9·989	−0·031
15·1	27·2	28·3	11·010	10·977	−0·033	11·012	10·985	−0·027

The case of milk is of particular importance since the various methods, *e.g.* the Gerber, the Babcock, etc., of estimating the percentage of fat in milk, which are extensively used commercially involve measuring a quantity of milk by means of a pipette. The above table furnishes interesting data relative to the volume of water and of milk delivered by 10 c.c. pipettes.

The external diameters of the bulbs of the pipettes used are given above; the delivery tubes were approximately 170 mm. long (210 mm. in the case of the 11 c.c. pipette) and 5·5 mm. external diameter; the graduation marks were 20 mm. to 40 mm. above the bulb, and the internal diameter of the suction tubes 4 mm. The bottom of the meniscus was set on the line for both liquids. This is quite practicable in the case of milk if the surface is viewed against a bright background, *e.g.* a window.

It will be seen that for the last three pipettes the difference between the volume of milk and of water delivered is practically constant and equal to 0·03 c.c. both for 2 secs. drainage and for 15 secs. drainage. Further, the difference between the volume of milk delivered with 2 secs. drainage and with 15 secs. drainage is small in the case of the same three pipettes.

For the pipettes with shorter delivery times the differences between the volumes of water and of milk delivered are larger and variable. Note also that in the case of the pipette with 3·6 secs. delivery time[1] the difference in the volume of milk delivered with 2 secs. and 15 secs. drainage times respectively amounts to as much as 0·12 c.c., corresponding to 0·04 per cent. error in the estimation of the fat in a sample containing 4 per cent of fat. The use of milk pipettes with short delivery times is therefore not to be recommended.

(v.) *Effect of Temperature on the Volume of Liquid delivered by Pipettes.*—The available experimental data on this point is meagre. W. Schloesser[2] gives the following values for the quantity of water remaining on the walls of pipettes when emptied at various temperatures:

10 c.c. Pipette. Delivery Time 20 secs.		100 c.c. Pipette. Delivery Time 39 secs.	
Temp.	Volume of Residue.	Temp.	Volume of Residue.
C.	c.c.	C.	c.c.
4·3	0·067	5·5	0·260
11·5	0·067	10·8	0·224
21·3	0·061	20·3	0·216
39·9	0·062	30·6	0·197

The above results indicate that, apart from the effect due to the expansion of the glass, the effects of change in temperature are negligible for temperatures not far removed from the standard temperature 15° C.

Dimensions of Pipettes. The amount of liquid left on the inside of a pipette when it is emptied, the rate of drainage and the difference between the volumes of various liquids delivered by the same pipette will obviously depend on the dimensions of the pipette. In the accounts of work relating to the behaviour of pipettes the dimensions of the instruments used are often not given, and the value of the results is not so great as it otherwise might be. There is great diversity in the dimensions of pipettes of the same capacity made by different manufacturers. It would be an advantage if greater uniformity could be attained, and the Committee on the Standardisation of Laboratory Glassware appointed by the Society of Chemical Industry recommend the following dimensions.[1]

---

[1] Milk pipettes of 11 c.c. capacity for the Gerber method are frequently made with about 3 or 4 secs. delivery time.

[2] *Zeit. anal. Chem.*, 1907, xlvi. 413.

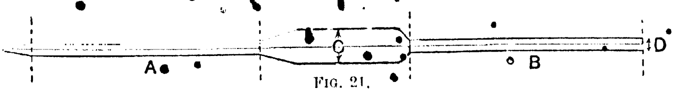


FIG. 21.

Capacity.	A.	B.	C (Int.).	D (Int.).	Distance of Mark above Bulb.
c.c.	mm.	mm.	mm.	mm.	about mm.
1	95	110	5·5-6·5	2	25
2	100	115	7-7·5	2	35
4	120	120	10-10·5	3	35
5	120	120	11·5-12	3	35
10	190	160	13·5-14	4	40
25	200	200	17-18	5	50
50	200	200	27-28	5	50
75	200	200	30-31	5	50
100	165	200	35-36	7	50

(vi.) *Calibrating Pipettes.*—An ingenious arrangement for calibrating pipettes, similar in principle to the well-known method of Ostwald for calibrating burettes, is due to S. English.[2] The apparatus used is shown in *Fig.* 22. A Woulff's bottle A, provided with two necks and a tubulure near the base, may be put in communication with a raised water supply B by means of the tap in the tube C. The pipette D to be calibrated is inserted in an inverted position in the other neck of the bottle as shown in the figure. A pipette E, provided with a capillary stem and a three-way tap, is connected to A by means of a tube which passes through a rubber cork in the tubulure of the Woulff's bottle.

The apparatus is used as follows: the Woulff's bottle is completely filled with water, care being taken not to trap any small bubbles of air. The tap C is opened and water is forced from A into the pipette D until the pipette is full, any small drop of liquid remaining on the jet being wiped off. During this procedure the pipette E is empty and not in communication with A. When D is full C is shut off and the three-way tap opened so that water flows from D into E and when the rising liquid surface reaches the mark on the capillary stem the three-way tap is closed. The standard period for drainage is then allowed to elapse and the position of the water surface marked on the suction tube of the pipette D.

The rate at which E fills is controlled by a constriction at F. The pipette E must be filled and allowed to empty through its delivery jet, *i.e.* in a definite time, before commencing a series of calibrations, and similarly emptied in between each pipette calibrated.

A pipette of the same type as those to be calibrated and known from gravimetric determinations to be of the required accuracy is used to calibrate the pipette E. This is done by filling E from the pipette of known accuracy in exactly the same way that the apparatus is used in subsequent calibrations.

The blank pipettes should be made of approximately the same dimensions as the standard pipette used to calibrate E, and should have jets which give a delivery time approximately equal to that of the same standard pipette.

Troubles arising from small traces of grease are said to be lessened if dilute acetic acid is used in the apparatus instead of water.

(vii.) *Types of Pipettes.*—By far the most usual form of pipette is that shown in the figure accompanying the table of dimensions which follows *Fig.* 21.

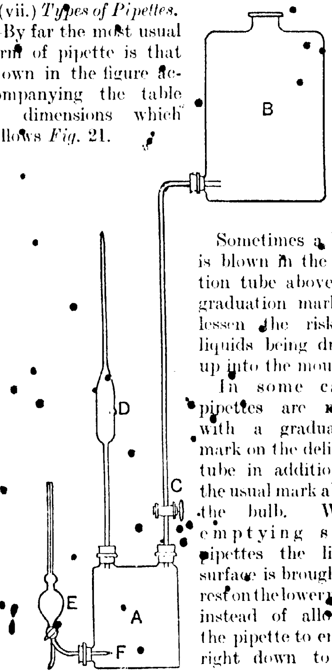


FIG. 22.

Sometimes a bulb is blown in the suction tube above the graduation mark to lessen the risk of liquids being drawn up into the mouth.

In some cases pipettes are made with a graduation mark on the delivery tube in addition to the usual mark above the bulb. When emptying such pipettes the liquid surface is brought to rest on the lower mark instead of allowing the pipette to empty right down to the jet. Such pipettes, however, are not in common use and the ordinary form is preferable in many ways.

Graduated pipettes will be dealt with after burettes, to which they are closely allied, have been considered.

Automatic Pipettes.—The simplest form o automatic pipette is the Stas pipette. It is similar to the ordinary pipette but has a short capillary tube replacing the ordinary long suction tube and a short delivery tube also, of narrow bore. Both ends of the pipette are drawn out in the form of a jet and ground-off square. The pipette is filled from below by means of a rubber tube until liquid overflows at the top. The overflowing liquid is collected in a bowl-shaped glass vessel with a hole in the bottom fitted with an india-rubber cork through which the top of the pipette projects. When the pipette is filled the top is closed by placing one finger over the top jet. The rubber tubing is removed from the bottom jet and any liquid adhering to the outside of the jet wiped away, and then the pipette is allowed to empty by removing the finger from the top.

---

[1] *Jour. Soc. Chem. Ind.*, 1919, xxxviii. 283 R.

[2] *Jour. Soc. Glass. Technology*, 1918, ii. 216-219.

The pipette is simple in construction, easy to clean, and gives consistent results if carefully used.

A second type of automatic pipette, which also has an overflow jet replacing the upper graduation mark, is fitted with a two-way tap below the bulb. In one position the tap connects the bulb with a supply bottle for filling and in the other with a delivery jet for emptying. The overflow jet projects into an arrangement similar to that shown in *Fig.* 18 to carry away surplus liquid when filling.

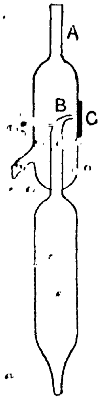


FIG. 23.

The pipette shown in *Fig.* 23 is filled by applying suction through the tube A until liquid overflows at the jet B. The jet B is then closed by pressing a rubber pad C, which covers a hole in the outer tube, against the end of the jet. The pipette is emptied by releasing the pressure on the rubber pad.

§ (18) BURETTES. (i.) *Graduations.*—All burettes accepted for the Class A Tests of the National Physical Laboratory or for tests at the Bureau of Standards or the Reichsanstalt must have their graduation marks carried out as shown in *Fig.* 24.[1] Every tenth line is carried completely round the tube and numbered. The shortest graduation marks extend half-way round the tube, and the lines midway between the numbered marks are intermediate in length. The tap should be sealed to the burette in such a position that when the burette is placed in a stand with the tap in the usual position (*i.e.* so that it is operated by the right hand) the ends of the shortest marks lie on a line running centrally down the front of the burette. The graduation marks and the numbers then appear as shown in *Fig.* 24, when the burette is viewed from the front.

This method of graduating burettes has met with considerable criticism, and it is therefore necessary to consider the matter in detail.

When reading a burette an observer should perform three distinct operations, viz.:

1. Bring his eye exactly to the level of the liquid surface.
2. Note the reading.
3. Assure himself that his eye is still at the right level.

A good observer performs these operations automatically and without conscious analysis of his actions.

With burettes graduated as previously described the above operations can be carried out with extreme ease and rapidity. By placing his eye so that the front and back portions of the line nearest to the liquid surface are seen to coincide, the observer at once brings his line of vision into the correct position. The reading is then noted and a final glance at the graduation mark suffices to ensure that the eye is still at the correct level.

Errors due to parallax can thus be practically eliminated with the minimum of trouble when using burettes graduated in the specified manner. Moreover, the feeling of absolute certainty on the part of the observer that this has been achieved is not to be underestimated.

A very large number, probably the majority, of the burettes manufactured, however, are not graduated in the above manner, but simply have short graduation marks down the front of the tube. Every fifth and tenth line is somewhat longer than the rest, but with none of the graduation marks is it possible to eliminate errors due to parallax in the manner described above.

In chemical literature and standard textbooks numerous devices, *e.g.* floats, attachable mirrors, and reading telescopes, burettes with silvered backs, burettes with enamelled backs, etc. etc., are described for use with burettes graduated with short lines only. The object of the devices is to obtain increased accuracy in reading, and many of them aim at, and others are erroneously supposed to achieve, the elimination of errors due to parallax. The very existence of these devices is a condemnation of the type of burette for which they are designed.

There is only one objection to carrying the lines round the tube which can be urged with any serious claim to consideration. This is that the length of the lines gives the burette a confused appearance which is trying to the eyes. This is no doubt true if one glances casually at such a burette, but if, as should be the case when the burette is in use, one's attention is concentrated on the graduation marks in the immediate neighbourhood of the liquid surface, no sense of confusion arises, in fact quite the reverse. The fact of being able to bring the front and back portion of the line nearest the liquid surface into coincidence creates a sense of certainty rather than confusion.

---

[1] In addition to illustrating the method of graduating, the two photographs also show the effect of using the method of screening the liquid surface previously described (see p. 789). The photograph on the left was taken against a white background. The other photograph was taken immediately afterwards on the same meniscus and against the same background, but with the paper screen in position.

The objection raised that the burettes are much more difficult to manufacture when the lines are carried round the tube instead of merely being engraved on the front is simply a question of using suitably designed dividing engines. Dividing engines are obtainable which will carry out graduations in the specified manner just as readily as short graduation marks.

Finally, it is not without significance that in the case of flasks and pipettes where it is extremely easy to carry the line completely round the tube no one has suggested that a short line on the front only would serve equally well.

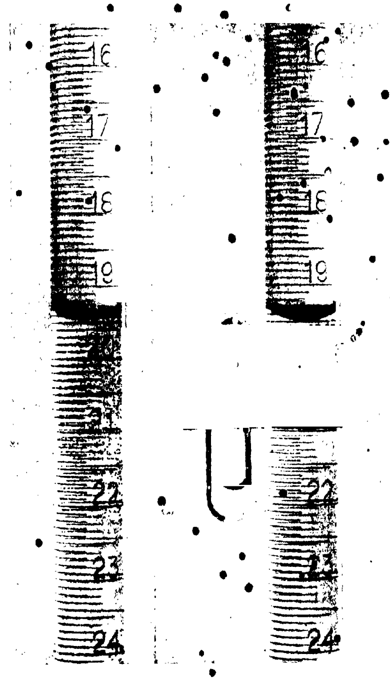


Fig. 24.

(ii.) *Methods of using Burettes.* — When a burette is emptied say from the 0 c.c. mark to the 50 c.c. mark, the volume of liquid which is delivered will vary with the rate at which the burette is emptied. Again, for any given rate of delivery the reading of the final position of the liquid surface will vary with the amount of time which elapses between closing the tap and taking the reading. This is due to the drainage of liquid from the walls causing the liquid surface to rise in the tube.

It is therefore obvious that the actual volume of liquid delivered by a burette can only be in agreement with the volume indicated by the burette when definite conditions of use are adhered to.

In order to be in a position to define conditions of use which will prove satisfactory, a necessary preliminary is to obtain exact information concerning the effect on the volume delivered of altering the delivery time[1] and also the effect of drainage in altering the burette readings.

Very valuable information on these points has been obtained at the Bureau of Standards, to which reference will be made later (see p. 807). The following account of a recent investigation carried out at the National Physical Laboratory will serve, by concentrating on a particular case, to bring out clearly the great variations in the volume of water delivered, and in the burette readings obtained, which may be obtained by varying the conditions of use. The results also indicate equally clearly the means by which such variations may be minimised in actual use.

A 50 c.c. burette was used for the experiments, and the length of the graduated portion from the 0 c.c. mark to the 50 c.c. mark was 534 mm. The first series of experiments to be described provide data relating to the drainage of water down the walls of the burette.

The burette was of the type for use with a jet attached by means of an india-rubber tube. A series of six jets was used, and the diameters at the outlets were such that the times required to empty the burette from the 0 c.c. mark to the 50 c.c. mark were respectively 21 secs., 40 secs., 73 secs., 115 secs., 161 secs., and 222 secs.

The burette was mounted vertically in front of a reading telescope provided with a micrometer eyepiece, and so situated that the 50 c.c. mark on the burette was somewhat below the centre of the field of view. The burette was allowed to empty freely through one of the jets, the outflow being stopped when the meniscus was nearly at the 50 c.c.

---

[1] By the "delivery time" or "time of outflow" of a burette is meant the time occupied by the unrestricted outflow (*i.e.* free delivery with tap fully open) of water from the zero mark to the lowest graduation mark. By "drainage time" is meant the time which elapses between closing the tap and the moment of reading the position of the liquid surface.

mark and therefore in the field of view of the telescope. The movable horizontal cross wires of the micrometer eyepiece were set on the bottom point of the meniscus as soon as possible after the outflow was stopped. Successive readings of the position of the meniscus were taken over a period of approximately half an hour, the readings being taken at 30-second intervals, for the first 8 minutes or so, and thereafter somewhat longer intervals were allowed to elapse between the readings.

The results obtained are shown in the accompanying graph (*Fig.* 25), in which the drainage time is plotted horizontally and the volume of water drained from the walls is plotted vertically. The time given at the end of each curve is the time occupied in emptying the burette from the 0 c.c. mark to approximately the 50 c.c. mark before commencing the observations on the drainage recorded on the curve.

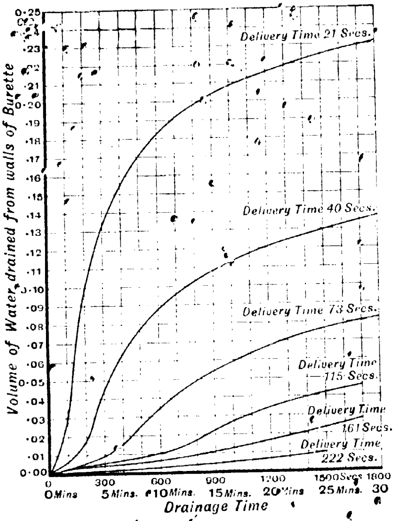


FIG. 25.

The following points may be noted from even a casual examination of the curves.

(1) The total amount of drainage is large, and in the early stages the rate of drainage is rapid when the delivery time is short. For example, for a delivery time of 21 secs. the total drainage in half an hour was 0·23 c.c., and the rate of drainage at the end of 3 minutes was about 0·07 c.c. in 100 secs. Compare these volumes with the National Physical Laboratory tolerance in the Class A tests of a 50 c.c. burette, viz. ± 0·04 c.c.

(2) When the time of delivery is increased the total amount of drainage and the rate of drainage is notably decreased. For example, for the delivery times 115 secs., 161 secs., and 222 secs., corresponding to the three lowest curves, the total drainage in 10 minutes does not exceed 0·01 c.c.

(3) Drainage persists for a considerable time. Even at the end of half an hour the curves have not become horizontal.

The above points summarise briefly the main facts relating to drainage, but before considering their bearing on the method of using burettes it is necessary to consider the effect of variations in delivery time on the volume of liquid actually delivered from the burette.

The required information may, however, be derived from the curves already given. If the observations had been continued until all the curves had become horizontal, then the difference between the ordinates of any two curves in this region would be the difference in the total amount of drainage which had taken place in each case. This difference is obviously the difference between the total amounts of liquid left in the burette when emptied in the delivery times corresponding to the two curves in question. But the difference between the amounts of liquid left behind in each case is clearly equal to the difference in the volume of liquid delivered, since the original volume in the burette when filled to the 0 c.c. mark was the same in each case.

The curves are sensibly parallel on the right-hand side of *Fig.* 25, and the difference in the ordinates at say 1500 secs. drainage will be approximately equal to the differences when the curves are prolonged until they become horizontal.

Assume that the burette delivered exactly 50 c.c. in 222 secs. Then if we subtract from 50 c.c. the volume corresponding to the final distance of the curve for, say, 161 secs. delivery time, above the curve for 222 secs. delivery time we shall obtain the volume of water delivered in 161 secs. Proceeding in this way for each of the delivery times given, data can be obtained for a curve showing the relation between delivery time and volume delivered.

The curve shown in *Fig.* 26 was obtained in the manner just described.

At this stage it is obvious that a check on the accuracy of the microscope determinations can be obtained by weighing the quantity of water delivered by the burette with jets of various diameters. The points indicated by crosses are based on such observations, and it will be seen that they fall reasonably close to the curve deduced from the microscope observations.[1]

The essential points to notice about *Fig.* 26 are the steepness of the initial portion of the curve and the gentle slope of the latter portion. Thus, for example, changing the delivery time from 21 secs. to 42 secs. produces 0·1 c.c. change in the volume delivered. A change from 110 secs. to 220 secs., *i.e.* the same percentage change as before, only produces a difference of 0·03 c.c. in the volume delivered. A change from 110 secs. to 131 secs., *i.e.* the same absolute change as in the first case, produces only 0·01 c.c. difference in the volume delivered.

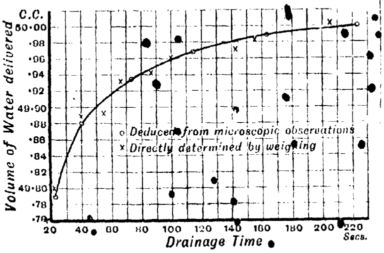


FIG. 26.

We may now consider the bearing of the above facts on the method of calibrating and using burettes.

In the first case consider the above 50 c.c. burette to have been calibrated for a quick delivery time, *e.g.* 21 secs. In calibrating the burette the time of delivery would have to be rigidly adhered to owing to the changes produced in the volume delivered by small changes in the delivery time. The magnitude of the changes in volume delivered from the 0 c.c. mark to the 50 c.c. mark produced in this way may be seen from *Fig.* 26. The slope of this curve in the neighbourhood of 21 secs. corresponds to a change volume of 0·03 c.c. for 5 secs. change in delivery time.

Again, the burette reading would have to be taken at some definitely specified time after closing the tap, because the rate of drainage is rapid. For example, at the end of 1 minute drainage time the rate of drainage is approximately 0·05 c.c. per minute.

If, therefore, a burette is calibrated for a quick delivery time, rigid conditions of use must be specified which cannot be departed from without introducing serious errors.

Next, by way of contrast, let us consider the calibration of the above burette for a long delivery time, *e.g.* 222 secs. The delivery time may be varied by about 45 secs. without changing the volume delivered from the 0 c.c. mark to the 50 c.c. mark by more than 0·01 c.c. Again, from *Fig.* 25 it will be seen that the amount of drainage which takes place in the first ten minutes is such that it would produce no visible change in the burette reading.

Hence, suppose a 50 c.c. burette to be calibrated for 222 secs. delivery time and no drainage time, then when the instrument is actually in use these conditions may be varied quite appreciably to suit the convenience of the user without affecting the accuracy of the results.

The advantage of burettes calibrated for long delivery times is clearly established by the above considerations.

If the delivery is unduly long, however, the burette becomes tedious to use. It is clearly necessary though, in order to ensure satisfactory result, that minimum delivery times must be fixed. These minima should be as small as is consistent with accuracy in order to effect economy of time.

The Bureau of Standards[2] fixed the minimum delivery times for burettes admitted to their tests on a thoroughly sound basis. The minimum delivery times chosen were such that for burettes of customary dimensions the maximum drainage from *any interval* emptied which occurs during the first two minutes after stopping the outflow amounts to about 0·05 mm. change in position of the water surface.

The minimum delivery time permitted for the 50 c.c. burette used in determining the data for *Figs.* 25 and 26 is 105 secs. according to the Bureau of Standards regulations. It will be seen from the figure that the above condition as to drainage is amply fulfilled in the case of the 50 c.c. interval considered.

The minimum delivery times specified thus practically eliminate all errors due to change in position of the liquid surface caused by drainage, provided that the reading is taken any time within a few minutes after the required amount of liquid has been run out of the burette. This allows a valuable latitude to the user without loss of accuracy.

Further, with the delivery times specified small variations in the time of outflow caused by not having the tap fully open when the burette is in use are of far less serious consequence than with burettes calibrated for short delivery times.

---

[1] Actually the burette was found by weighing to deliver 49·941 c.c. in 222 seconds. All the values found by weighing were increased by (50 – 49·941) c.c. to bring them into line with the assumption that 50 c.c. were delivered in 222 seconds, which was made in deducing *Fig.* 26 from *Fig.* 25. This assumption, made for the sake of simplicity, and the consequent adjustment of the results obtained by weighing, in no way affects the validity of any of the conclusions drawn from the curves.

[2] *Bull. Bureau of Standards*, 1908, iv. 583.

The delivery times specified by the Bureau of Standards are given below, along with the National Physical Laboratory times.

Length, Graduated.	Bureau of Standards.		N.P.L. Class A Tests.		N.P.L. Class B Tests.	
	Minimum Time of Outflow.	Maximum Time of Outflow.	Minimum Time of Outflow.	Maximum Time of Outflow.	Minimum Time of Outflow.	Maximum Time of Outflow.
cms.	secs.	secs.	secs.	secs.	secs.	secs.
15	30	180	30	60	20	60
20	35	180	40	80	30	80
25	40	180	50	100	35	100
30	50	180	60	120	45	120
35	60	180	70	140	50	140
40	70	180	80	160	55	160
45	80	180	90	180	60	180
50	90	180	100	200	70	200
55	100	180	110	220	75	220
60	120	180	120	240	80	240
65	140	180	130	260	85	260
70	160	180	140	280	90	280
75	..	..	150	300	100	300

NOTE.—For lengths not tabulated, the times allowed are those corresponding to the next larger tabulated length.

The minimum times specified by the Bureau of Standards and for the National Physical Laboratory Class A Tests are practically the same. The maximum times differ, but the exact value of the maximum is unimportant provided it is not so large as to make the burette too tedious in use.

The minimum times fixed for the N.P.L. Class B Tests are approximately 30 per cent less than the corresponding minima for the Class A Tests. This is rendered possible because a less high degree of accuracy is aimed at in the Class B Tests.

The delivery times given above are longer than those in general use. One reason for this is that the official German regulations specify much shorter times. The times specified are:

Length of Graduated Portion.		Minimum Time of Outflow.	Maximum Time of Outflow.
Greater than	But not greater than		
mm.	mm.	secs.	secs.
..	200	25	35
200	350	35	45
350	500	45	55
500	700	55	70
700	1000	70	90

It will be seen that in general the *maximum* German outflow times are less than even the N.P.L. *Class B minimum* times.

Standard German burettes are tested for the above delivery times, and allowance is made for 30 secs. drainage. Corrections are certified to 0·01 c.c. If the conditions of test are strictly adhered to, these values may be repeated. Small variations in the conditions, such as are practically found to occur when the burettes are in use, introduce errors not only in excess of the degree of accuracy to which corrections are certified, but in excess of the total error tolerated. For example, a delivery time of 55 secs. would be considered satisfactory for the 50 c.c. burette to which *Figs.* 25 and 26 relate. Yet this burette delivers the following amounts in excess of the volume delivered in 55 secs.:

For 70 secs. delivery time 0·02 c.c.
,, 90 ,, ,, ,, 0·04 c.c.

In addition to this the rate of drainage is by no means negligible for the minimum times of outflow permitted by the German regulations.

This official sanction of short delivery times combined with the fact that the uncertainties attending the use of such burettes are not generally recognised has led to the general use of burettes with quick delivery times, the delivery times being in many cases considerably less than even the specified German minimum. A more general use of burettes with longer delivery times would be a desirable reform.

The exact method of test adopted at the N.P.L. for burettes is given in the following section. By virtue of the length of delivery time specified the only demand made on the user in order that the values found in the test may be reproduced in actual use is that when emptying the burette the tap should be kept fully open. Even small quantities at a time may be emptied by sharply turning the tap fully on and then off again. With Class A burettes the sum of the volumes obtained by emptying a given interval in successive stages differs only slightly from the volume delivered when the whole interval is emptied at once, provided the above condition is fulfilled.

It is, of course, not feasible to keep the tap fully open when nearing the end point of a titration. The last c.c. or so must be added slowly. In effect, however, this simply amounts to a small increase in the drainage time which has been shown not to introduce serious errors. Moreover, a similar condition exists in the method of test (see (iii.)), as the rate of outflow is reduced when the water surface is about 1 cm. from the mark to be tested in order to control the motion of the meniscus and obtain an accurate setting on the mark to be tested.

The final reading of the burette may be taken at the user's convenience within a reasonable time after the required volume has been delivered. For reasons given previously it is unnecessary to observe a definite drainage time.

If readings other than the 0 c.c. mark are taken as the initial reading, no serious errors are introduced.

(iii.) *Testing of Burettes.*—The following account of the regulations governing the testing of burettes at the National Physical Laboratory is based on the test pamphlet previously referred to.

The only burettes admitted to the Class A tests are those with a tap permanently sealed to the instrument and graduated as shown in *Fig.* 24. In addition to these, however, burettes with short lines, burettes for use with detachable jets, and also "enamel back" burettes are admitted to the Class B tests.

The time of outflow must be marked on the tubes of all burettes, and the actual time of outflow must be within the limits, which have already been given in (ii.). Further, the marked time of outflow must not differ from the actual time of outflow by more than the amounts given in the following table:

Marked Time of Outflow.	Maximum Difference allowed between the Marked Time of Outflow and the Actual Time of Outflow.
secs.	± secs.
50	4
100	8
150	12
200	16
300	20

The above provision enables a damaged delivery jet to be replaced by one giving the same delivery time and consequently delivering the same volumes of liquids. The method of test adopted is as follows:

Burettes are clamped vertically for test, and filled with water to a short distance above the zero mark. Water is then slowly run out until the meniscus is exactly on the zero mark. The drop adhering to the tip is then removed by bringing the tip into contact with the inside of a glass beaker. The instrument is then allowed to deliver freely, *i.e.* with the stopcock fully open, into a clean weighed vessel. It is necessary, however, to arrest the full flow of liquid in time to obtain control over the final movement of the water surface and to bring the meniscus to rest accurately on the line to be tested. The instrument is therefore allowed to deliver freely until the water surface is approximately 1 cm. from the line to be tested. The rate of outflow is then reduced and the motion of the water surface brought under control so that an accurate setting can be made on the line in question. No period of waiting for drainage is allowed. The drop adhering to the tip after the setting has been made is removed by bringing the side of the receiving vessel into contact with the tip.

The zero mark is always taken as the starting-point of the intervals tested.

The volume of the water delivered in the above manner is determined by weighing, using the counterpoise method previously described in detail in connection with flasks.

The tolerances allowed are:

Total Capacity.	Maximum Error allowed at any Point and also Maximum Difference allowed between the Errors at any Two Points.	
	Class A.	Class B.
c.c.	± c.c.s.	± c.c.s.
2	0·01	0·015
10	0·02	0·035
30	0·03	0·05
50	0·04	0·07
75	0·06	0·10
100	0·08	0·14
200	0·15	0·25

Volumetric methods of testing burettes are not employed at the National Physical Laboratory or any of the similar institutions in other countries. Such methods, *e.g.* that of Ostwald, which is very frequently quoted, are not to be relied upon to the same extent as gravimetric methods. In the latter the conditions are strictly comparable with those existing when the burette is in use. With volumetric methods the conditions, particularly the time of outflow, the importance of which has been shown previously, may depart widely from those under which the burette is used.

§ (19) Graduated Pipettes.—Graduated pipettes consist of a glass tube drawn out to a delivery jet at the bottom and open at the top. The internal diameter at the top is such that the pipette can be easily closed and the outflow of liquid controlled by pressing one finger on the top of the pipette.

The pipettes are graduated in cubic centimetres and fractions of a cubic centimetre as in the case of burettes. In some cases the pipettes are arranged to deliver their total capacity when emptied down to the jet, and in others the lowest graduation mark is some distance above the jet. The mode of graduation shown for burettes in *Fig.* 24 is well adapted for graduated pipettes also.

The same considerations as to delivery time and drainage time apply to graduated pipettes as have been dealt with in the case of burettes. The same tolerances on capacity and regulations as to delivery time are employed at the National Physical Laboratory for graduated pipettes as for burettes. It is specified, however, that the graduated portion must not exceed 35 cm. in length. This provision is to exclude graduated pipettes which, being unduly long, are difficult to manipulate.

There is not such complete control in the manipulation of graduated pipettes as in the case of burettes. They are mainly useful in cases where variable volumes of liquid have to be measured to a somewhat less high degree of accuracy than is usually demanded of burettes of the same capacity. Graduated pipettes of small total capacity, *e.g.* 1 c.c. or less, are more frequently used than ones of larger capacity.

§ (20) **Graduated Measuring Glasses.**—*Graduated* measuring glasses, both cylindrical and conical, are extensively used for measuring volumes of liquid rapidly and to a moderate degree of accuracy.

Conical measures are mostly graduated in terms of imperial units, viz. fluid ounces, etc., and cylindrical ones are more frequently graduated in terms of metric units. Cylindrical measures are the more reliable because the regularity in diameter gives them a uniformly spaced scale which can be calibrated and read more accurately than the contracted scale of a conical measure.

The internal diameter of cylindrical measures, particularly when they are mould blown, is liable to considerable variation near the base of the instruments. For this reason it is recommended by the National Physical Laboratory that the graduations should be omitted from the bottom portions of the cylinders for a length corresponding to one-tenth of the total capacity.

Graduated measures should stand firmly when placed on a level table, and the Bureau of Standards also require that the base must be of such a size that the measures will stand on a plane inclined at 15° to the horizontal.

Graduated measures are calibrated in some cases for "content" and in others for "delivery," and should bear an inscription indicating the method of use for which they are intended.

When calibrated for "content" cylinders may be tested by weighing the water which they contain when filled to the graduation mark to be tested, or by filling them from a carefully calibrated delivery apparatus similar to those described in connection with flasks.

Measures calibrated for "delivery" are provided with lips, and in testing are emptied by gradually inclining them, until, when the continuous stream of liquid has ceased, they are nearly vertical. In this position they are allowed to drain for half a minute, and the lip is then stroked against the inside of the receiving vessel to remove any drop of water adhering to the lip.

The tolerances allowed by the National Physical Laboratory are given in the table below, only cylinders graduated for "content" being admitted to the Class A tests. In the case of cylinders submitted for the Class A tests the graduation marks must be carried round the tube as described previously in the case of burettes intended for the Class A tests.

Total Capacity.	Maximum Error allowed at any Point and also Maximum Difference allowed between the Errors at any Two Points.		
	Class A.	Class B.	
	For Content.	For Content.	For Delivery.
c.c.	±c.c.	±c.c.	±c.c.
5	0·04	0·06	0·08
10	0·06	0·10	0·12
25	0·10	0·15	0·20
50	0·15	0·25	0·30
100	0·25	0·40	0·50
250	0·50	0·8	1·0
500	1·0	1·5	2·0
1000	1·5	2·5	3·0
2000	2·5	4·0	5·0

The report of the committee on the standardisation of laboratory glassware [1] gives the following as suitable dimensions for graduated cylinders:

Capacity.	Diameter.	Height Overall, Stoppered.	Height Overall, Unstoppered.	Diameter of Foot.
c.c.	mm.	mm.	mm.	mm.
5	13	..	110	35
10	15	..	125	35
25	20	200	200	45
50	24	240	200	50
100	31	290	240	60
250	41	380	330	75
500	52	480	380	95
1000	67	510	440	115
2000	82	610	500	130

§ (21) **Standard Brass Measures.**—The Board of Trade have standard measures of capacity constructed of brass. They are very solidly made cylinders provided with stout handles. The base of the cylinder is closed and finished off parallel to the top, so that when the measure stands on a level surface the top of the measure is horizontal. The volume contained by the measure is determined by means of a glass "strike." This is a glass plate with a small hole in the centre. To obtain an exact quantity of water in the measure it is filled until the water surface is slightly above the level of the rim, the liquid being prevented from overflowing by the action of surface tension. The glass "strike" is then firmly pressed on one edge of the measure, and pushed across the top, thus sweeping away the excess of water in front of it, until when the strike is in the position shown in *Fig.* 27, the measure is exactly full of water.

---

[1] *Jour. Soc. Chem. Ind.*, 1919, xxxviii. 285 R.

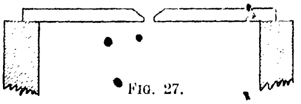


FIG. 27.

§ (22) MEASUREMENTS OF VOLUMES OF GASES.—In scientific work the measurement of volumes of gases mainly arises in connection with the analysis of gases, and in the analysis of substances by measuring the quantity of gas evolved when a known weight of the substance is made to undergo a suitable chemical reaction.

A large variety of different types of apparatus have been designed for particular purposes, but it is beyond the scope of the present work to deal with these in detail. We shall confine ourselves solely to the actual measurement of volumes of gases, leaving the reader to refer to such books as those quoted below[1] for full information as to the manipulation of the various types of apparatus and the particular operations to which they are adapted.

For most purposes it is sufficient to assume that gases obey the well-known law

$$\frac{p_1v_1}{T_1}=\frac{p_2v_2}{T_2}.$$

Owing to the large variation which takes place in the volume of a given mass of gas with change of pressure and temperature, it is necessary to state in conjunction with the volume of a gas the particular pressure and temperature for which the given volume is correct. In many cases it is desirable to reduce the observed volume of a gas to the volume which the gas would occupy under standard conditions of temperature and pressure, viz. a temperature of 0° C. and a pressure equal to that of 760 mm. of mercury at 0° C. and under standard gravity. Hence tables have been prepared to facilitate this reduction, which may be found in many books of physical and chemical tables.[2]

There are two distinct methods of measuring the volume of a gas, viz.:

(i.) Observing the volume in a graduated vessel, the temperature and pressure being noted but remaining practically constant throughout a series of observations.

(ii.) Observing the pressure exerted by a gas when it occupies a space of known volume. In this case also the temperature must be noted.

The simplest form of apparatus for measuring volumes of gases in the first manner consists of two cylindrical glass tubes connected together at the bottom by means of a length of india-rubber pressure tubing. One tube is graduated and closed at the top by means of a tap, and the other is open at the top. Mercury fills a portion of the apparatus, and by raising the open tube and opening the tap of the graduated one the latter can be completely filled with mercury. The gas to be measured is drawn in through the tap by lowering the open tube, and, on closing the tap, the volume of gas enclosed is read off after first adjusting the height of the open tube, so that the mercury surfaces in each tube are at the same level. The pressure of the gas is then equal to the prevailing barometric pressure.

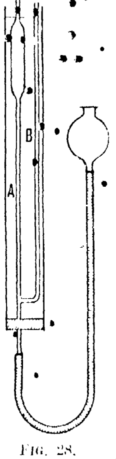


FIG. 28.

More accurate results are obtained if the open tube remains fixed in position and the mercury levels are adjusted by means of a mercury reservoir connected as shown in *Fig.* 28. The gas burette A, and the pressure tube B can then be enclosed in a water-bath and so maintained at a uniform temperature.

If the tubes A and B are of the same internal diameter no correction is required for the capillary action at the mercury surfaces in the two tubes. If the tubes are of different diameters a suitable correction must be applied.

The apparatus just described can obviously be used for making measurements by the second method if the mercury surface is always brought to the same mark on the tube A. The difference in level of the mercury surfaces is then measured to obtain the difference between the pressure of the enclosed gas and that of the atmosphere.

It is more usual, though, when using the second method to arrange that the tube B acts as a barometer, *i.e.* has a vacuum above the mercury surface. The top of B is made so that it can be closed, *e.g.* by means of a ground-in stopper with a mercury seal. The mercury reservoir is raised, both A and B being open at the top, until B is completely filled with mercury. The top of the tube B is then closed, and on lowering the mercury reservoir the mercury level falls in B, leaving a vacuum above the mercury surface.

The gas to be measured is drawn into A by lowering the mercury reservoir, A being initially full of mercury. The tap at the top of the gas burette is then closed and the mercury reservoir adjusted until the mercury surface in A stands at the selected mark. The difference in level of the mercury surfaces in A and B then gives directly the pressure of the gas enclosed in A.

---

[1] See such books as *Study of Gases*, by Dr. M. W. Travers; *Methods of Gas Analysis*, by Dr. W. Hempel; and *Methods of Air Analysis*, by Dr. J. S. Haldane, etc.

[2] *E.g.* "Smithsonian Tables," etc. A convenient form of table is that compiled by Dr. G. Barr and given in *The Chemist's Year Book*, 1920 edition.

When using the above "constant volume" method the graduated tube A may conveniently be replaced by a bulb of convenient capacity sealed to a capillary tube carrying a graduation mark on which the mercury surface is always set when making a measurement. Another alternative is to use a cylindrical bulb which has small pieces of opaque glass sealed to the inside and bent downwards so that the tip, which is drawn out to a point, is vertical. Several such indicators may be placed at different heights within the bulb. When making a measurement the mercury surface is adjusted until the point chosen as the reference mark coincides with its image in the mercury surface.

The necessity for making observations of temperature and pressure may be eliminated if absolute volumes are not required, but merely a series of volumes which are comparable with each other, *e.g.* in determining the percentage composition of a gas it is sufficient to know that the measured volumes of the constituents are in the same ratio that they would bear to each other if reduced to standard conditions. Observations of temperature and pressure may in such cases be avoided by the use of a compensating bulb, the principle of which is shown in *Fig.* 29. The tube A contains air and is connected to one limb of a manometer M, the other limb of which is connected to the tap at the top of the gas burette B. Before reading the volume of gas in B the height of the mercury reservoir R is adjusted so that the mercury surfaces in the manometer occupy definite positions defined by lines engraved on the manometer tube. The volume of air enclosed in A is thus kept constant, even though the temperature of the water-bath and the pressure of the atmosphere may vary. The gas in B is always under the same conditions as to temperature and pressure, when readings are being taken, as the air enclosed in A. Since, however, the conditions of temperature and pressure are always such that the air in A occupies a constant volume, then it follows that the volumes measured in B on different occasions are strictly comparable, without applying any correction for temperature or pressure.

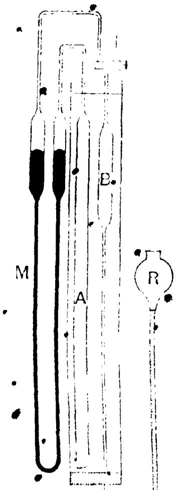


FIG. 29.

Obviously, if the quantity of air in A is adjusted initially so that the mercury levels in the manometer are opposite the lines on the manometer tube when the air in A is at 0° C. and under a pressure of 760 mm., then the volumes subsequently read off in B will be automatically corrected to these standard conditions.

A compensating bulb is employed in Dr. Haldane's apparatus, the potash absorption pipette being constructed in such a way as to serve also as the manometer of the compensating tube.

In cases where the gases to be measured are not dry it is important to ensure that they shall be saturated with water vapour when taking measurements of their volume. Corrections can then be applied by using the known values of the vapour pressure of water at various temperatures. Saturation is ensured by placing a small quantity of water above the mercury surface in the gas burette. In such cases if a compensating bulb is employed this should also contain sufficient water to ensure saturation of the enclosed air. Similarly a small quantity of water may with advantage be placed above the mercury surface in the barometer tube of "constant volume" apparatus.

§ (23) CALIBRATION OF GAS BURETTES.—Gas burettes are calibrated before assembling them in the complete apparatus. A two-way tap is sealed to the lower end of the burette. With the tap in one position the burette can be filled with mercury from a conveniently placed reservoir. By turning the tap into its other communicating position mercury can be run from the burette. A volume of mercury corresponding to the interval to be verified is run out from the gas burette in the above manner, and weighed, the temperature of the mercury being also noted. Tables for calculating the volume of the mercury from the observed weight have been given previously (see Table C, p. 786).

It is important that the tap for delivering the mercury from the burette should be sealed directly to the bottom of the gas burette. The use of rubber connections should be avoided, as they are liable to lead to uncertainties owing to the expansion of the india-rubber under the pressure of the mercury.

GASHOLDERS.—The methods of measurement hitherto considered apply only to comparatively small volumes of gas. Where large quantities of gas have to be measured a useful form of apparatus is that shown diagrammatically in *Fig.* 30. The apparatus is constructed on the principle of the gasometer. In order to drive gas out of the holder A the weight W is adjusted so that the holder descends. The amount of gas expelled is determined by reading the difference between initial and final readings of the scale S against a fixed point P. The scale S may be calibrated by successive transferences of a known volume of air from a standard of convenient size, *e.g.* one cubic foot. Such a standard is shown, again diagrammatically, in *Fig.* 31. The end A is in communication with a water reservoir which can be raised or lowered, and the end B may be put into communication either with the gasholder or the atmosphere. The vessel is filled with water to a fixed point C by raising the reservoir, B being in communication with the atmosphere. The reservoir is then lowered until the water surface is at a second fixed point D. The gasholder is next connected to B and the reservoir raised until the water surface is again at C, thus transferring an exact volume of air to the gasholder. This process is repeated until the holder is completely calibrated throughout the length of the scale S. The pressure inside the gasholder must of course be atmospheric during the calibration.

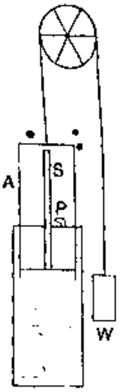


Fig. 30.

When using the gasholder to deliver measured volumes of gas it is often necessary to maintain the pressure constant inside the holder during the whole of the time during which gas is being expelled. With the apparatus as shown in *Fig.* 30 the pressure would vary as the holder descended owing to the increased immersion of the walls of the holder in the water contained in the outer vessel.

A simple way of maintaining a constant pressure is to hang a chain from a wheel concentric with the one shown in the figure and rotating with it. The chain is arranged to uncoil as the holder ascends and to wind on the wheel when the holder descends. By choosing a suitable diameter of wheel and weight of chain the change in pressure due to variation in the amount of the holder immersed may be counterbalanced.

A constant pressure may also be obtained by hanging a counterpoise weight from a cord which lies in a groove on the outside of a curved plate fixed to the axle of the wheel. The shape of the plate is such that as the holder descends the counterpoise weight acts at a less distance from the axis of rotation of the wheel. The shape of the curve and the mass of the counterpoise weight are adjusted to compensate for the variable immersion of the holder.

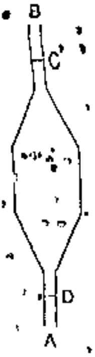


Fig. 31.

Gasholders constructed on the principle described, and fitted with suitable compensating devices on the lines indicated, are used in the testing of gas meters.

V. S.

Volume Measuring Meters, Automatic. See "Meters," § (32).

Volumetric Glassware. See "Volume, Measurements of," § (9).

WATCHES AND CHRONOMETERS, RATING OF

§ (1) Historical. The rating, or determination of daily rate, of ships' chronometers has been carried out, at Greenwich, on behalf of the Admiralty, for a very long period, but it is only since 1884 that any regular system of the examination of pocket watches for manufacturers or for the public has been in existence in this country.

For many years previous to this date the Swiss and French watch manufacturers had the advantage of being able to secure regular tests of their watches at the Besançon, Neuchatel, and Geneva Observatories, and there is no doubt that this facility has largely contributed to the steady advance in performance of Swiss watches.

In 1884 the Kew Committee of the Royal Society began the testing of watches at Kew Observatory, the trials being based upon the Swiss systems, and it continued to be carried out there till 1912, when it was transferred to the National Physical Laboratory at Teddington, where increased facilities and improved apparatus were provided.

§ (2) The Testing Equipment.—The tests are carried out in three rooms, each about 10 ft. by 8 ft., in the basement of Bushy

House, where the diurnal range of temperature is comparatively small, the rooms being known as "cold," "middle temperature," and "hot."

The "cold" room is mainly occupied by a large wooden-cased "refrigerator", lagged throughout with flake charcoal, and divided by zinc linings and air-spaces, into three compartments, the two exterior ones containing blocks of ice on grids with necessary drainage. The middle one, furnished with sliding trays, is for the chronometers under trial. The temperature is maintained at about 45° F. This chamber also contains dishes of potassic chloride to absorb the moisture, and maximum and minimum thermometer for the registration of temperature changes.

Below this is a small steel safe, similarly fitted up, where pocket watches are kept at a temperature of 42° F.

The "middle temperature" room is also fitted with shelves for chronometers, and a steel safe for the watches, and is maintained at a temperature of about 70° F. by means of three electric radiator lamps controlled by a sensitive thermostat and relay, the temperature being recorded on a thermograph.

The "hot" room is on similar lines; the temperature, about 95° F., is obtained by means of a Fletcher hot-air gas stove, the supply of gas being controlled by a toluene thermostat, and a record of the temperature changes recorded by the thermograph. The range of 50° F. between the "cold" and "hot" rooms is sufficient for all ordinary "rating" purposes.

The standard mean-time clock with which all comparisons are usually made is bolted to the substantial wall of the "middle temperature" room. The clock is of the "dead-beat" (or Graham) type of escapement, with high numbered pinions and jewelled in sapphire throughout. It is weight-driven, the weight falling in a separate channel, and is furnished with a Riefler pendulum—invar rod with a brass lenticular bob—a syphon barometer and thermometer. The whole is rigidly fixed in an iron air-tight case, with a stout plate-glass front. The pressure is normally kept at about 27 inches, any alteration of clock rate required being effected by slightly increasing or decreasing the pressure.

The dead-beat clock by French, London, which was the standard clock at Kew Observatory, is also bolted to the same wall, and serves as a subsidiary reference clock.

Both clocks are furnished with a delicate electric contact which consists of a phosphor-bronze wire bent in the form of a semicircle and attached to the bottom of the invar rod of the pendulum. To the two horns of this a strip of platinum foil is fixed in tension. The system is pivoted and moves very easily and lightly in a vertical plane, the amount of drop being controlled by a detent. Below it a "gold" balance-wheel, finely pivoted and revolving freely and delicately, is fixed on a small adjustable carriage. As the thin platinum strip is carried to and fro by the pendulum it rubs very lightly over a small segment of the balance-wheel, thus making a contact of which the duration and intensity can be closely adjusted.

As a check upon the small daily rate of the clock a time-signal is received daily from Greenwich through the G.P.O. at 10 A.M., and recorded direct on a Morse recorder.[1]

In addition to the Greenwich signals, a small "wireless" receiving set is also installed to receive the "wireless" time-signals from the Paris Observatory, and by means of a six-valve radio-frequency amplifier these signals can be recorded upon the Morse recorder in the same way as the Greenwich signals.

§ (3) METHOD OF COMPARISON.—Having received the time-signals and determined the small error of the reference clock, the daily comparison between the various timepieces under trial and the clock can be made.

This comparison can be carried out in several ways, and different observatories and testing laboratories adopt differing methods, but those most generally employed are:

(i.) By eye and ear comparison.
(ii.) By photographic comparison.
(iii.) By chronographic comparison.

(i.) The first method is often adopted where a large number of comparisons have to be made, and where a very high degree of accuracy is not aimed at.

(ii.) The "photographic" method is theoretically the most satisfactory, as by it the personal element is eliminated, since the clock face and the dials of the watches to be compared are all brought into the field of the camera and exposed simultaneously. The method, however, is not very largely used, as photographic troubles and failures are not unknown, and also some time must necessarily elapse before the final positives are ready for examination.

(iii.) The "chronographic" system is the one most generally in use in observatories, and by it the seconds marks of the standard clock and the corresponding time marks of the watches, etc., under trial are clearly defined on a paper sheet, and can be readily tabulated or read off.

The chronograph employed at the N.P.L., designed by Mr. J. E. Sears, Junr., Superintendent of the Metrology Division, consists of a drum 50 cm. long and 60 cm. in circumference, which is driven by a motor through worm-reduction gearing, at a uniform speed of two revolutions per minute, the motor itself running at 1800 r.p.m. The speed of the motor is controlled by a fairly heavy flywheel, on the face of which a small inertia bob is mounted at the end of a spring arm. As the speed rises this bob swings out, and eventually makes contact against a stop, thus short-circuiting a small resistance in the field of the motor, which accordingly tends to slow up until the contact is again released. The spring is so adjusted that at the required speed the contact is being continually made and broken. The control is in principle, though not in design, essentially similar to that described by von Helmholtz. In order to record the time there are two pointed tappers mounted together on a small carriage which traverses parallel to the axis of the drum through a distance of 4 mm. per revolution. These tappers strike through a red and black typewriter ribbon, and are actuated by means of relays, the one from the standard clock, and the other by the observer, through a tapping key. The chart is thus covered with a series of red dots at 2 cm. intervals, representing the seconds of the standard clock, while interspersed between these are black dots representing the times registered by the various movements under observation (see *Fig.* 1),

---

[1] These signals are sent hourly from the Royal Observatory to the G.P.O., London, and thence distributed by means of the "chronopher" to the London district at each hour and to country districts at ten hours and thirteen hours, the ten hours' one having the largest distribution.

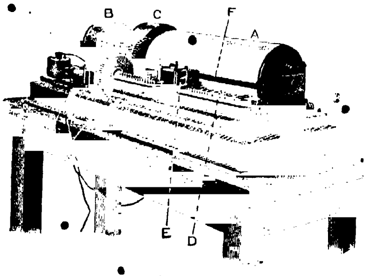


FIG. 1.

where A is the drum, driven by
B the motor, and controlled through
C the controller;
D is the lead-screw, carrying forward
E the carriage for the pointed tappers,
F typewriter ribbon.

The comparison is made when the seconds hand of the chronometer or watch lies exactly over the fifth, twenty-fifth, and forty-fifth seconds line on the dial. These three positions are thus symmetrically arranged round the dial.

The mean of the three comparisons is taken (to the nearest 0·01 second), thus reducing the chance of observational error, and eliminating any effect due either to eccentricity of the seconds wheel or irregular dividing of the dial.

§ (4) DURATION OF TRIALS.—The trial for marine or ships' chronometers is divided in five periods as below:

10 daily rates at a temperature of 70° F.
10 " " " 45°
10 " " " 70°
10 " " " 95
10 " " " 70°

Ships' chronometers are always used in the dial up or horizontal position, and are carefully swung and suspended in gimbals for this purpose.

Hence with these instruments the tests are carried out in the horizontal, dial up, position, and the trial has reference only to *variation* of the daily rate, and to *alterations* of daily rate due to the changes of temperature, and not to positional errors.

Unlike marine chronometers, the pocket watch is liable to be used in several positions, the commonest being (i.) "horizontally," out of pocket, with the dial either up or down, and (ii.) "vertically," with the pendant or winding knob approximately upright in pocket use. Hence, in addition to being timed and adjusted for steadiness of daily rate, and for alteration of rate with change of temperature, a high-class watch has also to be adjusted to have its rates as equal as possible in all the positions in which it is normally placed.

The trials for watches therefore include tests for positional adjustments.

There are two classes of trials, known as class A and class B, designed to meet both the highest grade and those of a more ordinary type of movement. The class A, which has always had the largest number of entries, is divided into 8 sections as below:

(1) 5 daily rates, in vertical position, with pendant up, at 67°.
(2) Do. in vertical position, with pendant right, at 67°.
(3) Do. in vertical position, with pendant left, at 67°.
(4) Do. in horizontal position, dial up, at 42°.
(5) Do. in horizontal position, dial up, at 67°.
(6) Do. in horizontal position, dial up, at 92°.
(7) Do. in horizontal position, dial down, at 67°.
(8) As No. 1, 5 daily rates, in vertical position, pendant up, at 67°.

The class B test, intended for a less highly finished watch, embraces 3 sections as below.

(1) 14 daily rates, in vertical position, pendant up, at 67°.
(2) Do. horizontal position, dial up, at 67°.
(3) 1 daily rate, dial up, at temperatures of 42°, 67°, 92°.

Marks.—In order to readily ascertain the relative performance of the class A watches, a system of marks is in use, based upon the scale 0—100.

Under this scheme the absolutely perfect watch would receive 100 marks, and lesser degrees of excellence are marked accordingly.

The 100 marks awarded to the theoretically perfect watch would be made up as follows:

40 for a complete absence of any variation of the daily rate.
40 for absolute freedom from change of rate with change of position.
20 for perfect compensation for effects of temperature changes.

Of course, the 100 will never be reached, but steadily and persistently the degree of performance has improved, until in 1920 the wonderful total of 96·9 was obtained.

For the past 10 years the highest number of marks awarded has been:

1910	93·2	1915	95·7
1911	94·8	1916	95·2
1912	96·1	1917	96·2
1913	95·0	1918–19	95·5
1914	94·0	1919–20	96·9

E. G. C.

WATER, DENSITY OF, in grammes per millilitre, tabulated. See "Balances," Table II.

WATER METERS. See "Meters," § (27), etc.

WATER-VAPOUR (see also "Air, Moist"):
- Addition of, to the atmosphere. See "Humidity," II. § (14). See also "Vapour Pressure."
- Density and pressure of, for saturation. See "Atmosphere, Thermodynamics of the," § (2). Table I.
- Effect of, on hygroscopic substances. See "Humidity," II. § (13).
- In the atmosphere:
  - Absorption of terrestrial and solar radiation by. See "Atmosphere, Thermodynamics of the," § (10).
  - Amount of, calculated from upper air temperatures. See *ibid.* § (12).
  - Distribution of, in the upper air. See *ibid.* § (5), *Fig.* 8.
  - Distribution of, over the globe. See *ibid.* §§ (3), (10): *Fig.* 5.
  - Effect of, on lapse-rate of temperature. See *ibid.* § (6).
  - Variation of, with height. See *ibid.* § (11). See also "Atmosphere, Physics of," § (6).
  - See also "Humidity," II. § (15).
- Latent heat of. See "Atmosphere, Thermodynamics of the," § (2). For determinations of, see "Latent Heat," Vol. I.
- Measurement of:
  - Dew-point hygrometers. See "Humidity," II. §§ (1)-(3).
  - Gravimetric method. See *ibid.* II. § (12).
  - Hair hygrometer. See *ibid.* II. § (10).
  - Volumetric method:
    - Shaw's apparatus. See *ibid.* II. § (13) (i.).
    - Tyndall's apparatus. See *ibid.* II. § (13) (ii.).
  - Wet- and dry-bulb hygrometers. See *ibid.* II. §§ (4)-(9) and (11).
- Physical constants of. See "Atmosphere, Thermodynamics of the," § (2).
- Specific heat of. See *ibid.* § (2). For determinations of, see "Specific Heat," Vol. I.
- Weight of, in saturated air. See "Atmosphere, Thermodynamics of the," Table V.

WAVE-LENGTH OF LIGHT: determination of the metre at the Bureau International des Poids et Mesures. See "Metrology," § (4); also "Line Standards," § (7).

WAVE-LENGTH RULINGS: suggestions for refining line-standards. See "Line Standards," § (1) (vi.).

WAVE-LENGTHS OF LINES IN THE INFRA-RED SPECTRUM. Langley's method of determination. See "Radiant Heat and its Spectrum Determination," § (22).

WAVE-MOTION IN THE ATMOSPHERE. See "Atmosphere, Physics of," § (17).

WAVES:
- In the interior of the earth, transmitted by brachistochronic paths in accordance with the recognised principles of wave-motion. See "Earthquakes and Earthquake Waves," § (7).
- On an elastic solid. See *ibid.* § (6).
- Primary and secondary, in an earthquake: the two distinct parts into which the preliminary tremor, as recorded on a seismogram, can be separated. See *ibid.* § (4).

WEATHER:
- Forecasting. See "Atmosphere, Physics of," § (20).
- Types of. See *ibid.* § (19).

WEIGHBRIDGES. See "Weighing Machines," § (5).
- Combination. See *ibid.* § (6).

WEIGHING MACHINES

FROM the earliest known times, weighing instruments[1] have been used to determine contracts between buyer and seller; there is ample evidence of the use of the even-armed balance in the ancient Egyptian epoch, whilst the Roman steelyard by its name reminds us of its origin. The principle of weighing by the lever is at once the most ancient as it is the most accurate. The sensitive balances of the present day differ only in refinements of construction from their prototypes. The evolution of weighing instruments therefore offers little scope for reference, and one can properly embark at once upon a description of modern appliances.

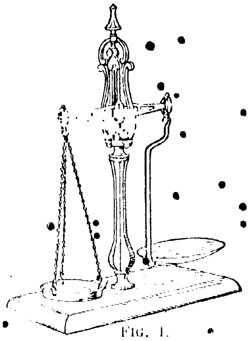


FIG. 1.

The weighing instruments here dealt with are those commonly used in the British Isles, and typical varieties only are described and illustrated.

§ (1) BEAM-SCALES.

Beam-scales are the most sensitive weighing instruments, and are invariably used when weighings to a fine degree of accuracy are required. When especial care is taken in their design and construction they can be made sensitive to the addition of a weight equal to one two-millionth part of the load weighed. There are many types, ranging from the delicate apparatus used by chemists or for physical research, including the comparison of the Imperial Standards of Weight, down to those ordinarily employed by tradesmen.

A beam-scale consists essentially of an equal-armed lever provided with a knife-edge pivot at its centre. The pivot is supported on a bearing in the pillar of the balance, or in a shackle suspended from an arm projecting from the pillar. Similar pivots are fixed at each end of the lever and equidistant from the centre knife-edge. These carry, respectively, the pans in which the weights and goods to be weighed are placed. The pivots generally take the form of triangular or pear-shaped prisms, and are fixed rigidly in the beam, projecting from each side in a horizontal plane at right angles to its axis. The centre one has its knife-edge at the lower angle of the prism, whilst the end pivots have their edges uppermost. A pointer at right angles to the beam and projecting vertically above or below the fulcrum knife-edge, as the case may be, indicates the movement of the beam from the horizontal. Types of beam-scales are illustrated in *Figs.* 1 and 2.

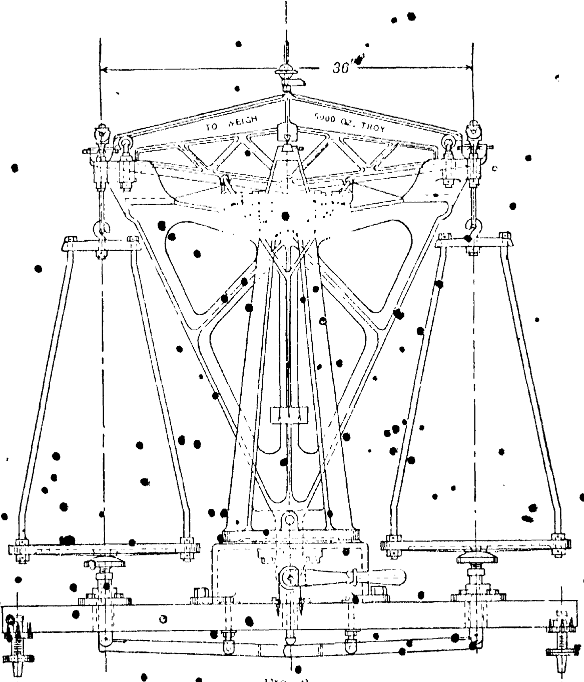


FIG. 2.

Various methods of fitting the knife-edges and bearings are adopted and have given rise to terms which specify the type. For example, there is the "box-end" beam (*Fig.* 3), in which the knife-edges pass through a box-like or bifurcated end of the beam; the "agate-box" beam (*Fig.* 4), in which agate bearings are fitted in brass or iron boxes; the "Dutch-end" beam (*Fig.* 5), in which case the end bearings are fixed inside plates bolted together across the beam to form a shackle; and the "swan-neck" beam (*Fig.* 6), now disallowed in the smaller capacities, in which the ends are curved and slotted, the bottom of the slot forming a knife-edge. Lastly, there is the "continuous knife-edge" beam (*Fig.* 7), which has the distinctive feature that the knife-edges bear along their whole length, and which is used for the most sensitive balances.

---

[1] For an account of weighing machines of high accuracy see the article on "Balances."

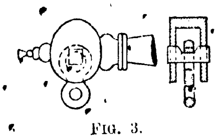


FIG. 3.

Fig. 2 represents a commercial balance of the highest degree of accuracy, constructed to weigh bullion up to 5000 oz. Troy. It is sensitive to the addition of 2 grains to either pan when fully loaded, that is, when weights approximating to 3 cwt. are suspended from each end of the beam. The beam is of gun-metal, open pattern, and cored out in the middle so as to allow the saddle at the top of the stand to pass through the beam and afford a continuous bearing for the fulcrum knife-edge. The end knife-edges are also of the continuous type and have flat steel bearings in the upper part of the suspension links. The flat bearing is the simplest as it is also the most desirable form of bearing. Its use allows free rotation of the knife-edge on its bearing, and rolling friction rather than sliding comes into play. To relieve the knife-edges from wear when the balance is not in use a triangular frame is provided and is lifted and lowered by a cam action operated by a handle at the front of the balance. The frame moves vertically on roller guides fixed on the stand. On the upward movement of the frame the vertical crutches near its ends first lift the end bearings off their knife-edges. As the frame continues its upward motion the upright pins near its ends engage in cups in the studs which project horizontally from the beam, and lift the fulcrum knife-edge from its bearing. In addition to relieving all the knife-edges when not in use, this arrangement serves to prevent the accidental overthrowing of the beam from its support, a provision which is very necessary when flat bearings are employed. The end knife-edges are adjusted and retained in position by set screws which bear against the oblique sides of the triangular prisms. A small weight on a vertical stem above the centre knife-edge can be raised or lowered to adjust the centre of gravity of the beam, whilst the small vane can be revolved to compensate slight differences in the "balance" of the instrument. Pan rests, operated concurrently with the sliding frame, and levelling feet are other refinements in the construction of this balance.

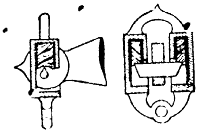


FIG. 4.

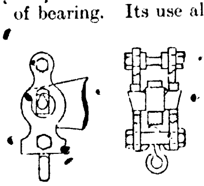


FIG. 5.

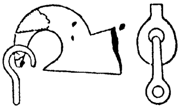


FIG. 6.

The knife-edges and bearings of beam-scales, and for that matter of all weighing appliances used in trade, are required by regulation to be of hard steel or agate. The reason for this is twofold: (*a*) To maintain the relative distance apart of the knife-edges, the alteration of which seriously affects the multiplication of the beam or levers, resulting in errors in weighing; and (*b*) to eliminate, as far as practicable, the friction between knife-edge and bearing which militates against the sensitiveness of the instrument. Agate is used much less than steel, and by custom is restricted to instruments of comparatively low capacity. Agate bearings are most frequently used in conjunction with steel knife-edges. Agate knife-edges are only inserted in chemical balances and in balances of low capacity used in scientific investigations: their brittle nature renders them unsuitable for use in trade appliances.

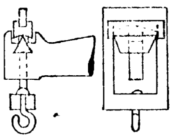


FIG. 7.

§ (2) REQUISITES OF A GOOD BALANCE.—The conditions which must be satisfied in the construction of a beam-scale, and which apply, *inter alia*, to every type of lever weighing instrument, are:

(i.) *Truth*, *i.e.* the beam, when at rest, must be horizontal both when unloaded and when equal masses are suspended from its terminal knife-edges.

(ii.) *Stability*, *i.e.* the beam, if deflected from its position of equilibrium, must return to the original position of rest.

(iii.) *Sensitiveness*, *i.e.* a small difference between the loads suspended from the terminal knife-edges must cause an appreciable deviation of the beam from the horizontal.

To satisfy the conditions of truth, it is necessary that the moments of the weights of the arms of the beam around the fulcrum should be equal, and that the terminal knife-edges of the beam, from which the pans are suspended, should be equidistant from the fulcrum.

The quality of stability is secured when the point of suspension (*i.e.* the fulcrum) is above the centre of gravity. If the centre of gravity is coincident with the centre knife-edge of the beam, then its equilibrium will be neutral, and the beam will remain in any position in which it is placed. Such a balance is wanting in proper action. If, on the other hand, the centre of gravity of the beam, when horizontal, is immediately *above* the fulcrum, the beam will be unstable, and will overturn upon the slightest displacement from the horizontal. The attainment of equipoise in such a case would be extremely difficult. But if the centre of gravity of the beam, when horizontal, is immediately *below* the fulcrum, the beam will rest in that position, and if disturbed therefrom will vibrate and at last return to the original position of rest.

The sensitiveness of a balance depends upon several factors, which may be conveniently demonstrated as follows: Let AFB (*Fig.* 8) be the beam of a balance, and W its weight, concentrated at its centre of gravity G. Let L be the length of the arms, assumed equal; and let the points A, F, and B, *i.e.* the three knife-edges, be all in the same plane. The beam has free movement about its fulcrum F, and the centre of gravity is at a distance d from F. Let a small weight p be added to the pan suspended from the knife-edge B; the balance will turn through an angle α, and, assuming the scale pans equal, the equation of moments will be

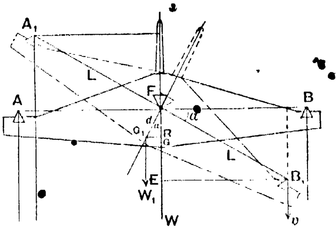


FIG. 8.

$$p \, . \, B'E = W \, . \, G'R.$$

But $$B'E = L \cos \alpha,$$

and $$G'R = G'F \sin \alpha = d \sin \alpha.$$

$$\therefore \tan \alpha = \frac{p}{W} \frac{L}{d}.$$

Now $\tan \alpha / p$ is a measure of the sensitiveness which is therefore equal to L/Wd. The equation thus proves that the sensitiveness of a balance is proportional to the length of the arms, and inversely proportional to the distance of the centre of gravity from the fulcrum, and to the weight of the beam.[1]

In practice it is usual to construct a balance in such a way that when unloaded the fulcrum knife-edge is slightly below the line joining the other two. The beam bends slightly under the load, thus lowering the outer knife-edges and bringing the three more nearly into line. When this condition is reached the sensitiveness is independent of the load.

Summarising the above statements it is found that the requirements of a good balance are:

(i.) The arms of the unloaded beam must have equal moments around the fulcrum and the pans must be of equal weight.

(ii.) The distance between each terminal knife-edge and the fulcrum must be equal.

(iii.) The centre of gravity must be below the fulcrum.

(iv.) The smaller the distance between the centre of gravity and the fulcrum the greater the sensibility.

(v.) The longer the beam the greater the sensibility.

(vi.) The lighter the beam the greater the sensibility. But care should be taken not to make the beam so long or so light as to materially affect its rigidity.

(vii.) The fulcrum should be in, or somewhat below, the line of the terminal knife-edges.

(viii.) There should be as little friction as possible between the bearing surfaces at the three points of suspension.

From what has been said, it would appear that if the arms of a balance be unequal correct weighings cannot be made thereon; but as the establishment and maintenance of exact equality in the arms is a matter of difficulty, one of two methods of avoiding all risk of error from this cause may be adopted. One is known as Gauss's method, and consists of repeating the weighings, the weights and substance to be weighed being interchanged in the pans. Then if P and Q be the apparent weights of a body of true weight W when weighed in the right and left pan respectively, l and r being the length of the arms, we have

$$Wr = Pl,$$

$$Wl = Qr,$$

$$\therefore W^2 = PQ, \qquad W = \sqrt{PQ}.$$

Thus the square root of the product of the two readings gives the true weight. In the second, Borda's method, the substance to be weighed is placed in one pan and the scale balanced by any handy material in the other pan. The substance to be weighed is then removed and weights are substituted until equilibrium is restored. The weights represent the weight of the substance, the inequality of the arms of the beam, if any, having affected both operations in a like manner.

---

[1] For a more complete discussion of the action of a balance see "Balances," § (2).

§ (3) COUNTER MACHINES.—These are equal-armed weighing instruments of a capacity suitable for counter use, and with pans above the beam. They are largely employed for trade purposes in retail shops. From the standpoint of utility, and without reference to questions of accuracy or sensitiveness, the counter machine possesses an advantage over the ordinary beam-scale because the scale pans of the former are unencumbered by chains, etc., and articles can be placed on the goods-pan, or the weights manipulated on the weights pan with greater rapidity. But this advantage is secured by a loss of sensitiveness, because it is evident that some means must be adopted to keep the pans in a horizontal position. Further, the means provided must permit of rotation of the parts, and as a consequence the increased number of knife-edges and bearing surfaces introduces additional friction. *Fig.* 9 is a perspective view of such a machine showing the stand in section and exposing the parallel motion mechanism.

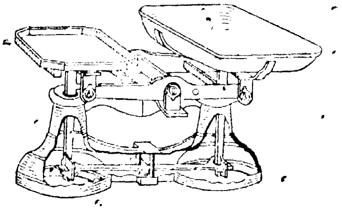


FIG. 9

(i.) *The Roberval Balance.*—The counter machine in general use in this country is evolved from a balance invented by Professor Roberval, a French mathematician, in 1670. The original model was similar to the sketch in *Fig.* 10. The balance consists of a parallelogram ABDC, hinged at its corners, and movable in the vertical plane as shown by dotted lines in the sketch. Each of the horizontal sides is supported at its centre by fulcra F and F′, resting in bearings in the vertical stand. The arms AF and BF are equal, and are also equal to CF′ and DF′ respectively. Similarly, the vertical sides AC and BD are each equal to the distance FF′, and it therefore follows that whether the beam is horizontal or deflected at an angle to the horizontal, the sides AC and BD remain vertical and parallel to FF′. To each of these vertical sides is rigidly fixed a horizontal bar EH and KI. Then, if equal weights P and Q be suspended from *any* two points H and K respectively, the balance, if originally in equilibrium, will remain so, with the arms AB and CD horizontal.

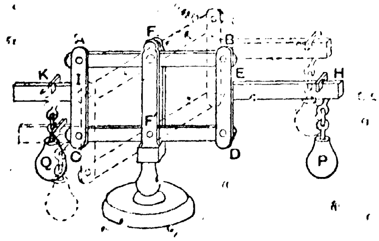


FIG. 10.

For let P and Q be weights which, when suspended from any two points H and K, maintain equilibrium, suppose the balance slightly disturbed as shown by the dotted lines in the diagrammatic figure, *Fig.* 11, and let p and q be the displacements of H and K, the points of application of the weights P and Q each measured in a vertical direction.

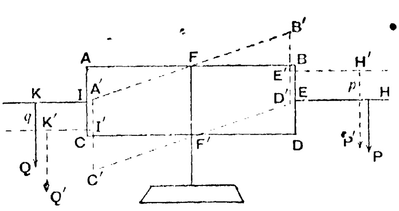


FIG. 11.

The work done on P in rising a distance p is Pp; the work done by Q in falling a distance q is Qq, and since the balance is in equilibrium, these are equal. Thus $Pp = Qq$.

But since the arms FA, FB, F′C, F′D are all equal, p is equal to q, each being equal to the vertical displacement of the extremities of the equal arms.

Therefore $P = Q$ or the balance remains in equilibrium from whatever points the weights are hung if they are equal.

In the counter machine as evolved from the original Roberval balance, the horizontal arms projecting from the vertical sides of the parallelogram are replaced by horizontal pans fitted above the end knife-edges of the beam. This construction has the effect of minimising the magnitude of the tensions and thrusts to which the beam and stay may be subjected. The extent to which the pan projects over the vertical side of the parallelogram corresponds to the length of the arm EH or KI, and the force resulting from a weight placed near the edge has the same effect as P or Q in the proof just given.

The essential condition for accuracy in a counter machine is that the beam, stay, legs, and distance between the fulcra shall form parallelograms. If this is not so, the indications of the instrument, when a load is placed on the pan in any position other than vertically above the terminal knife-edge of the beam, will be inaccurate.

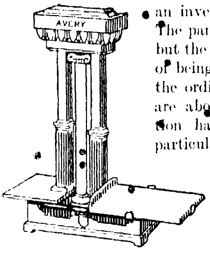


FIG. 12.

A well-known variation of the Roberval balance is called the Inverted, or Imperial machine (*Fig.* 12). It is, as its name implies, an inverted counter machine. The pans are above the beam, but the legs and stays, instead of being below the beam as in the ordinary counter machine, are above it. This construction has several advantages, particularly in machines of high capacity. The pans are nearer the floor or counter than in the ordinary pattern, thus facilitating the weighing of heavy loads; and the vertical distance between the beam and stay can conveniently be much longer than in the ordinary pattern, without affecting the height of the scale-pans from the floor or counter. The two outer pillars serve as "legs" to the goods and weights pan in this type of machine.

From the above explanation of the principle of the Roberval balance it will be seen that certain thrusts and tensions, sometimes of considerable magnitude, are introduced along the beam and stay when the weights are placed in certain positions on the scale-pans. These thrusts and tensions tend to grind the knife-edges on their bearings, a condition which is not conducive to accuracy nor favourable to the long life of the machine. In order to avoid this defect several constructions have been devised, but the only variation in use in this country is the Beranger balance.

(ii.) *Beranger Balance.*—The mechanism of this machine is, in appearance, more complicated than that of the Roberval balance, but in reality is not so. The principle of placing the pans above the beam is retained, but all loads on the pans are transferred to subsidiary beams arranged below the main beam, which, in their turn, transmit a pull, always vertical and equal to the load, to the knife-edges of the main beam. By this arrangement the introduction of lateral forces is avoided. *Fig.* 13 illustrates the principle of the construction of this balance.

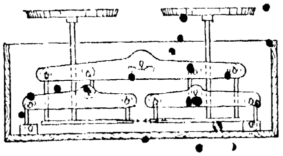


FIG. 13.

§ (4) THE STEELYARD, or unequal-armed beam, is a type of weighing instrument which in point of antiquity vies with the equal-armed beam scale. Excellently constructed specimens, found at Pompeii, vary but slightly in general design and proportion from the instrument of present-day manufacture. The steelyard finds favour among butchers for weighing carcases, and in other trades for weighing bulky articles which are easily suspended and which cannot be placed conveniently upon a pan or platform. It is a compact and portable instrument, and does not require the use of loose weights. Steelyards are not, in general, so accurate as equal-armed balances, for the following reasons: (i.) A small weight on the long arm of the steelyard balances a heavy load suspended from the short arm, and consequently a very small error in the counterpoise weight causes a multiplied error in the apparent weight of the goods weighed. (ii.) An extremely small error or change in the length of the short arm makes a sensible error in the apparent weight of the goods weighed. One type of steelyard is shown in *Fig.* 14

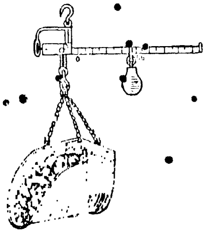


FIG. 14.

When equilibrium is established the forces acting on either arm are inversely proportional to their distance from the fulcrum. In practice the load is suspended from the goods hook, which is at constant distance from the fulcrum, and the poise weight is moved along the longer arm until the steelyard is brought into a horizontal position. The amount of the load is read from the numbered graduation at the position on the long arm at which the movable poise rests. The short arm being of constant length and the travelling poise of constant weight, it follows that the length of the long arm, that is, the distance from the fulcrum to the position of rest of the poise, is proportionate to the load suspended from the goods hook, and, therefore, the long arm can be equally divided and graduated to represent the amount of the load which produces equilibrium for any position of the poise. Occasionally steelyards are provided with a knife-edge with weights pan suspended from the longer arm, and loose weights are placed in the pan to balance the load. The disadvantage of this system is that unless the loose weights are accurate and consistent among themselves, the error in the indications of the instrument is considerable on account of the great leverage they exert. As to the relative positions of the fulcrum, the terminal knife-edges (the notch or graduation on the long arm at which the poise rests corresponds to a terminal knife-edge in a beam-scale), and the centre of gravity, the same laws apply to steelyards as to equal-armed beam-scales, which see above.

§ (5) PLATFORM MACHINES AND WEIGHBRIDGES.—Platform machines and weighbridges are types of weighing machines which are similar in principle, and differ from one another only by a capacity line arbitrarily fixed, and by the consequential differences in construction necessitated by the particular purpose for which they are designed. They are used in the main for weighings above 1 cwt., and are composed of a goods platform, either at floor level or only slightly above the ground, and a steelyard or other form of indicator disposed at a convenient height, from which the weight of the goods is read. The platform machines used at railway stations for weighing luggage are a familiar sight, and hardly less so are the weighbridges with platform sunk to road level and used for weighing cart-loads of coal, and those fitted with rails on which railway rolling-stock is weighed.

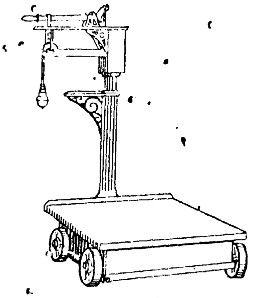


FIG. 15.

(i.) *Types of Machines.*—Types of platform machines are illustrated in *Figs.* 15 and 16.

The perspective view in *Fig.* 16 shows that this type of instrument consists, in general, of a box or frame in which are suspended two or more levers. These have their fulcra bearing in shackles (*a* in *Fig.* 16) suspended from the framing. The platform on which the goods are placed to be weighed is supported, generally at four points, on knife-edges fitted into the levers (see *b* in *Fig.* 16). One of the levers projects from the box or frame, and is connected at its outer end by a connecting rod to a steelyard which carries a poise weight or weights to balance the load on the platform. By this combination of levers a high mechanical advantage is obtained, and consequently a heavy load can be weighed against a comparatively light counterpoise weight. In the designing of platform machines a mechanical advantage of 112 is frequently chosen, so that a counterpoise weight of 1 lb. will balance a load of 1 cwt. In the case of weighbridges a mechanical advantage of 1500 is the average, although as high as 8000 is reached in some patterns.

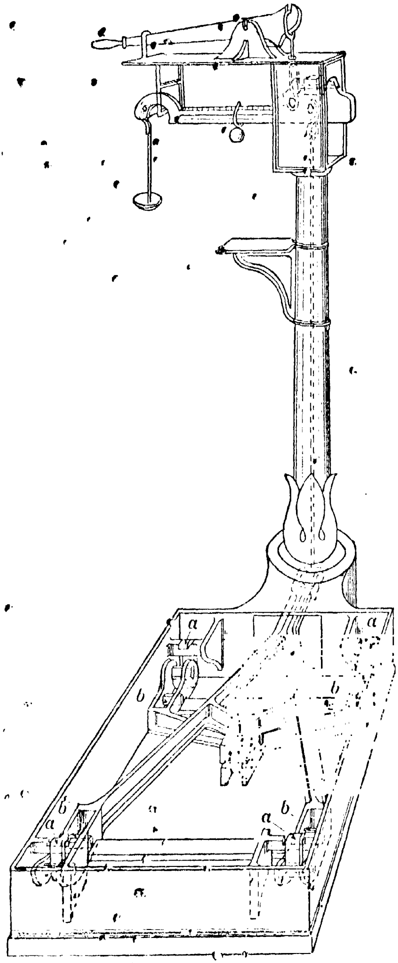


FIG. 16.

As in the counter machine, the practical consideration to be borne in mind is that the true weight of a load upon the platform of the machine shall be indicated, whatever may be its position thereon. This condition is arrived at in platform machines and weighbridges by so proportioning each lever that the pull on the connecting rod is the same whichever lever, or combination of levers, is brought into play.

For example, in *Fig.* 17, which is a diagrammatic sketch of the levers of a platform machine, FAB and *fab* are two levers, whose fulcra are at F and *f* respectively. The distance $Fc = fa$, and $FA = fb$. The lever *fb* connects with the lever FB by a link at A. The downward pressure at B exercises a pull on the "load" knife-edge E of the steelyard, the fulcrum of which is F, and which carries the poise weight P on the graduated arm FD, and counterpoise weights W suspended from the knife-edge D. Suppose, for illustration, that an article weighing 100 lbs. is placed on the platform. A portion of the 100 lbs. will act at *c*, and the remainder at *a*. If the distance $FA = 5Fc$, and $fb = 5fa$, then the force at A will be $\frac{1}{5}$ of the force at *c*; and the force at *b* will be $\frac{1}{5}$ of the force at *a*. Thus the whole effect is the same as if $\frac{1}{5}$ of the article weighed were suspended at A. This result is quite independent of the position on the platform of the articles weighed. The $\frac{1}{5}$ of the load at A will produce a downward force at B, which, if, for example, FB be equal to 3FA, will be equivalent to $\frac{1}{15}$ of the total load. The resulting tension in BE is balanced by the poise weights P and W, and the load is represented by the position of the poise weight P and by the magnitude of the weights W suspended from D. The levers FAB and *fab* are triangular in form, and being placed apex to apex they provide at the opposite corners four points of support for the platform, and no legs and stays are required to maintain the platform in the horizontal plane. The usual arrangement of levers in a weighbridge is shown in *Fig.* 18. The platform is carried on the knife-edges A, B, C, D. Two "equal power" triangular levers, having their fulcrums at A_1B_1 and C_1D_1 respectively, transmit that proportion of the load which is borne by each to a third lever with its fulcrum in the frame at E; this connects at its outer end to a rod suspended from the shorter arm of the steelyard of the machine. In many cases an intermediate lever is introduced at the bottom of the steelyard pillar. The object of this is to still further increase the mechanical advantage of the machine, and to reduce the weight of the poise necessary to balance loads on the platform.

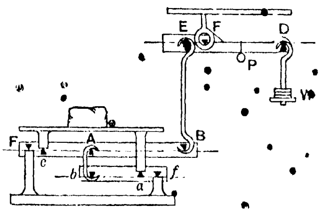


FIG. 17.

(ii.) *Sensibility.*—The degree of sensibility which weighbridges attain is remarkable when one remembers the loads applied and the number and mass of the parts which form the lever system. It is not uncommon for a new weighbridge loaded with 50 tons to give a correct indication, and to be sensitive to the addition of 2 lbs. to the load. This accuracy would not be maintained, but with reasonable usage a weighbridge is a very durable machine. One factor which contributes to its reliability is the almost infinitesimal movement of the platform and load when weighings are taken. The vertical movement of the "indicating" end of the steelyard does not, in general, exceed one inch, and this movement is, according to the leverage of the instrument, somewhere between one and nine thousand times that of the platform.

(iii.) "*Vibrating and Accelerating Machines.*"—All of the types of weighing instrument dealt with above, *i.e.* beam-scales, counter machines, steelyards, now admitted to verification are constructed on what is known as the "vibrating" principle, that is to say, they are in equilibrium when the beam or steelyard is in a horizontal position of rest, midway between the upper and lower stops by which its movement is restricted, and return to this position when slightly disturbed. They are therefore stable machines. But the establishment of equilibrium in a machine of this type is an operation which is not conducive to rapid action, and having regard to the fact that platform machines and weighbridges are rarely, if ever, used where the question of determining exact weight to any great nicety is involved, and are frequently employed where rapid weighings are an advantage, these machines are, in this country, very commonly made unstable, or "accelerating." This

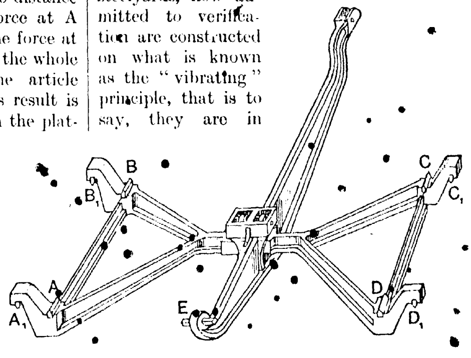


FIG. 18.

quality is secured by one of two methods. Either the centres of gravity of the levers and steelyard are located above the fulcrum knife-edges or the line of the terminal knife-edges is arranged above the fulcrum knife-edges. The effect of both these arrangements is to make the balance unstable and to cause the lever or steelyard when once moved from the horizontal position to continue to move with "accelerated" velocity until it is brought up by its stop. It is the practice to so arrange the steelyard that it is horizontal when on its lower stop, and consequently has movement one way only from the horizontal. An accelerating machine indicates a "correct" weight when the steelyard gently rises from the lower stop to the full extent of its movement. Obviously such an instrument is not adapted for ascertaining the exact weight of an article, but where, as in the case of weighbridges, the degree of accuracy attainable is predetermined by the graduation of the steelyard, the divergence from absolute "balance" is of no great moment. Nevertheless, it must be added that in continental countries, in America, and in many English colonies all weighing instruments used for trade are required to be on the vibrating principle, and there can be no question that in regard to the construction of such instruments this principle is the proper one.

(iv.) *Methods of indicating the Load.*—The arrangement and design of the lever mechanism under the platform of platform machines and weighbridges is practically stereotyped, and the variations encountered are confined to details. In the method of indicating the load, however, one meets with three main divisions, known respectively as the loose-weight type, the no-loose-weight type, and the self-indicating type. In the first of these patterns the load on the platform is balanced by counterpoise weights suspended from the end of the steelyard in the manner illustrated in *Figs.* 15 and 16, each weight representing a major unit or multiple of a major unit of weight. Intermediate loads are counterpoised by the jockey weight, which at its greatest distance from the fulcrum balances one major unit. For instance, a load of 30 cwts. would be balanced on a weighbridge provided with "1 ton" counterpoise weights, by suspending one of the counterpoise weights from the end of the steelyard and moving the travelling poise along to the "10 cwts." graduation on the steelyard. The "no-loose-weight" type is an extension of the principle involved in the intermediate weighings on the loose-weight type. Here a larger travelling poise balances, at its extreme distance from the fulcrum, the whole of the load which the machine is constructed to weigh. The load on the platform is balanced by moving the large poise along the steelyard and engaging the knife-edge, or nib, in one of the several notches cut on the top edge of the steelyard. Intermediate loads are balanced by a smaller travelling poise which operates along a bar attached to the steelyard.

(v.) *Self-indicating Mechanisms.*—Self-indicating mechanisms may be divided roughly into two classes. They are not, generally, so accurate as those requiring manual operation, but serve a useful purpose where quickness in weighings is a desirable factor. The demand for this type of instrument was for many years, and is now, to some extent supplied by indicators operating on the hydrostatic principle, that is, a cylindrical body suspended from the steelyard is partly immersed in a tank of water. On the application of a load to the platform the cylinder is lifted, thus displacing less water; the downward pull it exerts on the steelyard, being the difference between its weight and the weight of water displaced, increases and the steelyard comes to rest in such a position that this increase just balances the effect of the weight on the platform. The movement of the steelyard is communicated by a cord wrapping round a spindle to an indicating finger attached thereto. The finger traverses a graduated dial and indicates the load on the platform. Some makers use a mercury cup instead of the water tank, but neither of the arrangements can be said to be reliable. In the second, and modern, type of self-indicating machine of this class a pendulum resistant to the load is employed. A heavy weight is suspended rigidly from the steelyard, and upon the application of a load to the platform moves to such a position that the forces about the fulcrum are in equilibrium.

§ (6) COMBINATION WEIGHBRIDGE.—With the introduction into this country of long bogie trucks for carrying merchandise on railways the necessity arose for the weighing of mixed trains, that is, trains made up of long bogie wagons and the usual two-axle wagons. To weigh these on a long single machine was a time-wasting and therefore costly process which involved uncoupling the wagons, weighing and recoupling. To obviate this work, the combination weighbridge was designed. In its modern form it consists of two or more weighbridges placed end to end, with, in some cases, a certain distance between them. All are connected, so that an indication of the total weight on the platforms, or the weight on each separate platform, or any combination of the platforms, is obtainable. Even the two unit machines did not completely satisfy the demands arising from the complication of wheel-base dimensions, and the three-unit combination soon followed. An arrangement of levers in the latest type of triple weighbridge is shown diagrammatically in *Fig.* 19. Each of the weighing units transfers its load by a connecting rod to the corresponding intermediate lever. The dead weight of the platform and levers being compensated by the attached balance lever, only the pull due to the load is transferred to the box lever, and from thence by the connecting rod in the pillar to the steelyard. To disconnect any one of the bridges the appropriate handle underneath the steelyard is pulled over. The cam attached thereto depresses the right-hand end of the balance lever, lifting up the left-hand end, which communicates its movement through chain links to the left-hand end of the intermediate lever. The end knife-edge of the latter is lifted off the bearing shackle suspended from the box lever, and the steelyard movement is then independent of any load which might be on the disconnected weighbridge.

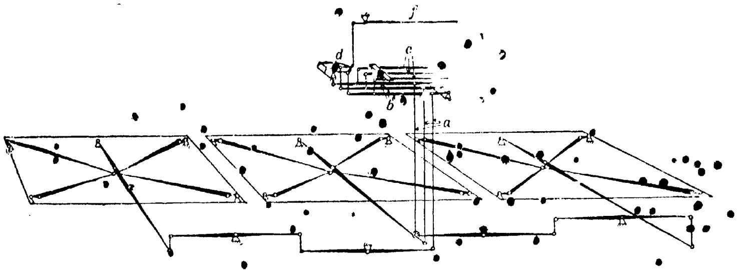


FIG. 19.

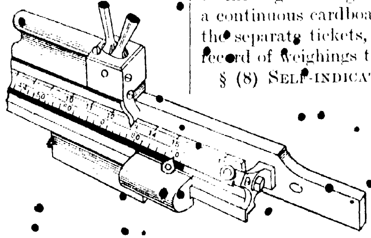


FIG. 20.

§ (7) MODERN REFINEMENTS. (i.) *Multiple Standards.*—Among the refinements of construction in platform machines and weighbridges the two outstanding are the introduction of mechanism enabling readings of weight to be taken in two or more standards, and the attachment of recording apparatus. One method of securing an indication of weight in three standards, *e.g.* English, Metric, and Egyptian, is illustrated in *Fig.* 20. A polygonal bar having notches, and graduated according to the different systems on its various faces, is supported by conical bearings alongside the steelyard. When it is desired to change from one standard of weight to another, a thumb-screw is loosened and the "polygonal bar" is revolved until the appropriate notches are underneath the nib of the travelling poise. The bar is then secured in position, and the balanced steelyard indicates, by the position of the large and small poises, the weight of the goods on the platform, the reading being in any one of the systems desired.

(ii.) *Printing Steelyards.*—Provision for printing a record of the weight of goods is provided in some steelyards. In one form, a series of gun-metal figures corresponding to the various graduations is attached to the underside of the steelyard. To obtain a record of weight a ticket is inserted in the slot in the travelling poise, and by the pulling over of the handle immediately beneath the slot, is impressed against the metal figures, which cut a record of the weight upon the cardboard ticket. In another printing device, gun-metal discs, having figures engraved on their periphery, are caused to revolve by movement of the travelling poises. When equilibrium is established a ticket is inserted in the slot and is immediately over the figures corresponding to the notches in which the several poises rest. A pull over of the handle presses the ticket on to the engraved figures. In some instruments a continuous cardboard tape is used instead of the separate tickets, and provides a complete record of weighings taken in a given period.

§ (8) SELF-INDICATING SCALES.—A noticeable modern development in weighing machine construction is that of the self-indicating instrument. The prototype of this class is the spring balance, a familiar departure from the leverage principle upon which weighing instruments are, in the main, constructed. Self-indicating mechanisms applied to weighbridges have been dealt with above, and the remarks following concern only the smaller capacity instruments for counter use. These comprise a weighing lever or levers associated with a pair of springs, or with a pendulum weight, as the final resistant. The latter is the more numerous class, and the one which calls for explanation. The principle is illustrated in *Figs.* 21 and 22. In *Fig.* 21 a simple Roberval balance mechanism is connected with a pendulum resistant. On the application of a load to the goods pan the force transmitted to the short arm of the bell-crank lever causes the latter to take up a position such that the pendulum bob has an equal moment around the fulcrum. The indicator is rigidly fixed to the bell-crank lever, and traverses a graduated dial to register the load weighed. Equal increments to the load in the pan produce equal increments of moment to the pendulum, but the consequential unequal radial movements of this member, which are projected directly on the graduated arc by the indicating finger, involve unequal divisions of the scale. This inequality is avoided in many of the types of self-indicating scale by transmitting the pull of the final weighing lever to the pendulum lever by means of a flexible metal band passing over a cam attached to the lever arm (see *Fig.* 22), the contour of the cam being arranged to give a decreasing moment arm as the load is increased to compensate for the decreasing radial movements of the pendulum. With some instruments of this type is associated a price-computing device. A vertical rack is attached to the end of the weighing lever and imparts its motion through the medium of a pinion on a horizontal shaft, to a light and carefully balanced drum whose axis is the before-mentioned horizontal shaft. On the outer surface of the drum are printed the range of weight values corresponding to the capacity of the instrument, and the money value of goods corresponding to different rates per pound. The side of the casing which is nearer to the seller is pierced centrally by two slots, one a vertical slot through which the weight is read on the drum, and the other a horizontal slot, half of it on each side of the vertical slot, through which the money values of the goods, corresponding to the different rates per pound, are read. The weight of the goods and the money value are recorded by an indicating wire stretched taut in the casing and fixed in front of the drum. On the side of the casing nearer to the buyer there is a vertical slot, through which the weight of the goods can be read on the drum.

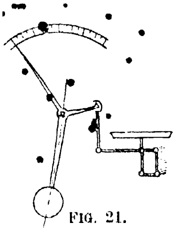


FIG. 21.

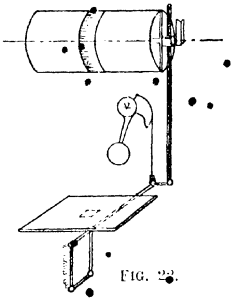


FIG. 22.

§ (9). SPECIAL DEVICES.—Many and varied special types of weighing instruments have been introduced in recent years. Particularly noticeable are the efforts which have been made to expedite the operation of weighing by the introduction of machinery, more or less complicated, which renders the instruments to a great extent self-acting. Among these may be mentioned those which have for their object the weighing out of quantities of goods in great numbers of the same weight; those constructed to weigh many loads of varying weight in succession, and present the total weight at the end of a day's work; and those which determine the amount of material carried by a continuously travelling band in any given period, as, for example, the amount of coal passed into bunkers, or from bunkers to boilers. The adaptation of weighing instruments to other uses than the indication of weight has produced instruments to indicate the number of loads weighed; to indicate the weight of a quantity of articles from the weight of one; to indicate the percentage constituents in an alloy; and to indicate the number of articles in an unknown quantity placed upon the scale. Considerations of space prevent more than a passing mention of these appliances, but all embody the physical principles involved in the simpler instruments.

G. A. O.

◆

WEIGHING MACHINES, MULTIPLE STANDARD, for weighing in two or more units. See "Weighing Machines," § (7).

WEIGHING METERS FOR LIQUIDS (AUTOMATIC). See "Meters," § (6).

WEIGHTS, ADJUSTMENT OF. See "Balances," § (12).
  Coated. See *ibid.* § (9).
  Material for making standard. See *ibid.* § (8).
  National Standard, recompared with the international prototype kilogramme and found, in half the cases examined, to agree with the initial value at the time of formal issue, to within ±0·000010 gramme, *i.e.* to within 1 part in 10^8. See *ibid.* § (8).
  Of rock-crystal (quartz) made by J. Nemetz (Vienna) and Laurent (Paris). See *ibid.* § (9).
  Shape and design of. See "Balances," § (11).
  Uncoated polished brass, variation of mass of, tabulated. See *ibid.* § (8).

WEIGHTS AND MEASURES:
  Legal denominations of. See "Metrology," § (11).
  Used in trade, control of. Testing authorities. See *ibid.* § (10).

WET- AND DRY-BULB THERMOMETERS. See "Humidity," II. § (4), "Meteorological Instruments," II. § (7).

WHITWORTH STANDARD THREAD. See "Gauges," § (44).

WIND:
- Condition for no variation with height. See "Atmosphere, Physics of," §§ (10), (11).
- Effect of, on dew-point hygrometers. See "Humidity," II. §§ (2) and (3).
- Effect of rotation of earth on. See "Atmosphere, Physics of," § (8).
- Relation of, to distribution of pressure. See "Atmosphere, Thermodynamics of the," § (8).
- Relation of horizontal to vertical, for typical distributions of velocity. See *ibid.* § (16). See also "Atmosphere, Circulation of the."
- Variation with height:
  - Effect of eddy-motion on. See "Atmosphere, Physics of," §§ (12), (13), 14.
  - Effect of temperature-gradient on. See *ibid.* § (10).
  - Formulae for. See *ibid.* §§ (12), (14).
  - In the stratosphere. See *ibid.* § (11).
  - In the surface layers. See *ibid.* § (14).
  - In the troposphere. See *ibid.* §§ (10), (12).
- See also "Anticyclone," "Cyclone," "Cyclostrophic Wind," "Geostrophic Wind," "Gradient Wind."

WIND, DIRECTION OF:
- Measurement of:
  - At the surface. See "Meteorological Instruments," IV. § (21), etc.
  - In the upper air. See *ibid.* § (35).
- Self-recording instruments for: Baxendell, Beckley, Casella, Dines, Halliwell, Munro-Rooker. See *ibid.* § (22) (ii.) (vii.).

WIND-VANES. See "Meteorological Instruments," § (21).

WIND VELOCITY:
- Measurement of, at the surface. See "Meteorological Instruments," IV. § (16), etc.
- Measurement of, in the upper air, by Darwin-Hill mirror. See *ibid.* § (35).
- Relation to atmospheric pressure and density. See *ibid.* § (16).

— Z —

ZONES AS USED IN CONNECTION WITH SCREW THREADS. See "Metrology," VII. § (25) (ii.).

INDEX

(Numbers refer to pages)

Abakanowicz: Abdank-Abakanowicz integraph, "Integration, Mechanical Methods of," 456

Abbot: bolometer, "Radiant Heat and its Spectrum distribution," 706; effective solar temperature, "Radiation, The Measurement of," 717
  (Aldrich and) pyranometer, "Radiation, The Measurement of," 718
  (Fowle and) diminution of solar and terrestrial radiation, "Atmosphere, Physics of the," 27, "Atmosphere, Thermodynamics of the," 69, 70; pyrheliometers, "Meteorological Instruments," 514, "Radiant Heat and its Spectrum Distribution," 703, 704, "Radiation, The Measurement of," 716; solar constant, "Radiation," 708, 709, 714, "Radiation, The Measurement of," 717
  (Harrison and) "Meters," 543

Abdank: -Abakanowicz integraph, "Integration, Mechanical Methods of," 456

Abel: equation, "Earthquakes and Earthquake Waves," 290

Abercromby: pressure distribution, "Atmosphere, Physics of the," 39; principles of forecasting, "Forecast," 295

Abraham, H.: timing experiment, "Clocks and Time-keeping," 221, 222

Adams, O.: "Observations, The Combination of," 660 *n.*

Adie: statical hydrometer, "Alcoholometry," [illegible]

Agamennone, Dr.: seismograph, "Seismometry," 735

Airy, Sir G. B.: "Airy's Table" 9, "Barometers and Manometers," 179; clock, "Clocks and Time-keeping," 204, 211 *n.*, 213; pendulum theorem, *ibid.* 204; conditions for correct maintenance, *ibid.* 208, 210, 211; colliery experiments and results, "Earth, The Mean Density of the," 280, 285; on isostasy theory, "Gravity Survey," 412; points of support, "Line Standards of Length," 467, "Metrology," 587; projection, "Map Projections," 481, 486; rainbows, "Meteorological Optics," 524, 525; errors of observation, "Observations, The Combination of," 647, 649 *n.* 3

Aitken: direction of depression, "Atmosphere, Physics of the," 40; method of counting ions, "Atmospheric Electricity," 94

Akerblom: eddy diffusivity, "Atmosphere, Physics of the," 34, 35; virtual friction, *ibid.* 34

Albers: projection, "Map Projections," 482

Aldrich: (Abbot and) pyranometer, "Radiation, The Measurement of," 718

Alluard: modified Regnault hygrometer, "Humidity," 415

Alpe, H. J.: (Fowler, Taylor, and) gauge for plug screws, "Gauges," 343, 344

Ames, B. C., Messrs.: dial indicator, "Gauges," 390, 391

Amsler: planimeter, "Integration, Mechanical Methods of," 452, 453, 454; integrometers, *ibid.* 455

Angot: "Radiation," 715

Ångström, A.: "Balances," 1[illegible]0; float method of determining density, *ibid.* 129; unit, "Length, Units of," 465, "Metrology," 589; compensating pyrheliometer, "Meteorological Instruments," 513, 514, "Radiant Heat and its Spectrum Distribution," 704, "Radiation, The Measurement of," 716, 718; radiation from sky, "Radiation," 714; radiation from earth, *ibid.* 712; pyrgeometer, "Radiation, The Measurement of," 718; atmospheric humidity, *ibid.* 718
  (Pettersson and) float method of determining density, "Balances," 129

Ångstrom, K.: radiation from the earth, "Radiation," 712

Apjohn: formulae for hygrometry, "Humidity," 419, 423

Arago: polarisation of light from the sky, "Meteorological Optics," 523
  (Biot and) constant of astronomical refraction, "Meteorological Optics," 519

Archibald: "Air, The Investigation of the Upper," 1

Argelander: "Trigonometrical Heights," 773

Armstrong: scale, "Draughting Devices," 268

Arndt: determination of density of gases, "Balances," 137; work by, "Barometers and Manometers," 192

Arnold: (Earnshaw and) chronometer escapement, "Clocks and Time-keeping," 213

Arrowsmith: projection, "Map Projections," 483

Assmann: psychrometers, "Humidity," 420, 424, 425, "Meteorological Instruments," 491

Aston: micro-balance, "Balances," 121, 137

Attwell, S. W.: measurement of limits of work, "Metrology," 615

August: formulae for hygrometry, "Humidity," 419, 423

Avery: automatic liquid weigher, "Meters," 570

Babbage, C.: differencing engine, "Calculating Machines," 195; analytical engine, *ibid.* 195

Babcock: milk-test bottle, "Volume, Measurements of," 796

Babinet: projection, "Map Projections," 481; neutral point, "Meteorological Optics," 523

Bagay: logarithm tables, "Latitude, Longitude, and Azimuth," 459

Bailey, E. G.: steam meter, "Meters," 553, 554; calibration, *ibid.* 554

Baille: (Cornu and) apparatus and investigations, "Earth, The Mean Density of the," 281, 285

Baily: Cavendish experiment repeated by, "Earth, The Mean Density of the," 281, 284, 285

Bain, A.: electric control, "Clocks and Time-keeping," 226

Baldwin, F. S.: Monroe machine, "Calculating Machines," 201

Ball, Dr. J.: on prismatic astrolabe, "Gravity Survey," 409
  (Knox-Shaw and) graphical reduction, "Latitude, Longitude, and Azimuth," 461; tables, *ibid.* 463

Balling: table, "Saccharometry," 727; degrees, *ibid.* 730
  (Brix and) experiments, "Saccharometry," 729; saccharometers, *ibid.* 729, 730

Ballot, Buys: "Atmosphere, Thermodynamics of the," 64; "Buys Ballot's Law," 193

Barbet: tables of specific gravity of sugar solutions, "Saccharometry," 727

Barnes: ice formation, "Radiation, The Measurement of," 718

Barr, Dr. G.: tables for reduction of gas volumes to standard conditions, "Volume, Measurements of," 811 *n.* 2
Basevi, Capt.: (Heaviside and) observations by, "Gravity Survey," 399
Bates: saccharometer, "Hydrometers," 436, 437
  (Bearce and) table, "Saccharometry," 732, 733
Bauer, L. A.: (Swann and) land and ocean values, "Atmospheric Electricity," 95, 96
Baumé, A.: hydrometer, "Alcoholometry," 20, "Hydrometers," 432, "Saccharometry," 729, 730, 731
Baxendell, J.: Fernley rain-gauge, "Meteorological Instruments," 498; wind-direction recorder, *ibid.* 508, 509
Beam: Leffmann-Beam milk-test bottles, "Volume, Measurements of," 796
Bearce: (Bates and) table of specific gravity of sugar solutions, "Saccharometry," 732, 733
  (Osborne, M'Kelvy, and) specific gravity and temperature, "Alcoholometry," 14; density investigations, "Balances," 130
Bebber, van: paths of cyclones, "Atmosphere, Physics of the," 39
Beccaria, G.: measurement of electric field, "Atmospheric Electricity," 88
Beck: hydrometer, "Alcoholometry," 20
Becker, Messrs. Christian: chainomatic balance, "Balances," 118
Beckett, E. (*see* Grimthorpe, Lord): escapement, "Clocks and Time-keeping," 213
Beckley, R.: balance gauge, "Meteorological Instruments," 498; Robinson-Beckley cup anemometers, *ibid.* 501, 502, 503, 507; wind-direction recorder, *ibid.* 509
Beckmann: manostat, "Barometers and Manometers," 192
Bemmelen, W. van: temperature in the upper air, "Atmosphere, Thermodynamics of the," 56, 57
Benedicks, C.: capillary gas-burner, "Radio-micrometer and some other Instruments," 723
Benndorf: law for waves in the interior of the earth, "Earthquakes and Earthquake Waves," 290
Bennecke: bottom water, "Oceanography, Physical," 682
Benoit, R.: international metric standards, "Metrology," 586
  (Fabry, Perot, and) metre determined in terms of wave-lengths, "Line Standards of Length," 477
  (Guillaume and) longitudinal comparator, "Comparators," 231; metric standards, *ibid.* 251, "Metrology," 586; corrections for elasticity of wires, "Surveying Tapes and Wires," 745; effect of coiling tapes, *ibid.* 747
  (Lepinay, Buisson, and) determination of litre, "Volume, Measurements of," 778
Béranger: balance, "Weighing Machines," 821
Berget, A.: float method of determining density, "Balances," 129
Bernacchi: variations of atmospheric potential, "Atmospheric Electricity," 92
Berndt, G.: annual variations of atmospheric potential, "Atmospheric Electricity," 91
Berson, A.: temperature in the upper air, "Atmosphere, Thermodynamics of the," 56, 57
Bessel: "Observations, The Combination of," 647; refraction tables, "Trigonometrical Heights," 773
Besson, L.: nephoscope, "Meteorological Instruments," 514; on halos, "Meteorological Optics," 529, 530, 531, 532, 533
Bezold, von: potential temperature, "Atmosphere Thermodynamics of the," 62; hail stage and snow stage, *ibid.* 79
Billy: specific gravity bottle, "Balances," 133
Biltz: (Meyer and) "Balances," 137
Bingham: (White and) viscosity of pure water, "Oceanography, Physical," 671
Biot: (Arago and) "Meteorological Optics," 519
Birkeland: constants of wet and dry bulb thermometer, "Humidity," 422, 424
Bishop: ring, "Meteorological Optics," 523, 528
Bishop, P.: gauging, "Metrology," 609
Bjerknes: surface of discontinuity and formation of cyclones in the atmosphere, "Atmosphere, Physics of the," 38, 41; bar unit of pressure, "Bar," 139; calculation of current velocity, "Oceanography, Physical," 689, 690, 691, 692
Blackman, P.: density of a gas, "Balances," 137
Blagden, Dr. C.: report on excise matters by, "Alcoholometry," 10, 11, 16
Block, W.: pyknometers, "Balances," 128
Bodenstein: manometer, "Barometers and Manometers," 189
Boethius: "Calculating Machines," 194
Boistissandeau: "Calculating Machines," 195
Bollée, L.: direct multiplying machine, "Calculating Machines," 195
Boltzmann: Stefan-Boltzmann law, "Radiation," 711, 714
Bond, H.: formula for Mercator's projection, "Map Projections," 484
Bonne: projection, "Map Projections," 479, 481, 485, 486
Borda: metre, "Length, Units of," 464; kilogramme, "Mass, Measure of," 487; method of weighing, "Weighing Machines," 819
Börnstein: (Landolt and) tables of vapour pressure, "Humidity," 416
Bort, Tesserenc de: stratosphere, "Atmosphere, Thermodynamics of the," 59
Boss, L.: errors of scales, "Micrometers," 634
Bottlinger, K. F.: absorption of gravitation by matter, "Gravity Survey," 413
Boucher: calculator, "Draughting Devices," 261
Bouguer: mountain observations, "Earth, The Mean Density of the," 279, 285; plateau, "Gravity Survey," 411, 412; halo, "Meteorological Optics," 526
Bourdon: tube thermograph, "Meteorological Instruments," 494
Bousfield, W. R.: pyknometer, "Balances," 128
  (Lowry and) "Balances," 130
Boutaric: humidity and nocturnal radiation, "Radiation, The Measurement of," 718
Bowie, W.: (Hayford and) calculations, "Gravity Survey," 412
Boyer: recorder, "Meters," 542
Boyle: hydrometer, "Alcoholometry," 10, 14, 15
Boys, C. V.: Cavendish experiment modified by, "Earth, The Mean Density of the," 281, 282; results, *ibid.* 282, 285; and Braun's experiments, *ibid.* 282; quartz fibre, *ibid.* 281, 282, "Quartz Fibres," 695, 697, 698; equation, "Gravity, Measure of," 397; radio-micrometer, "Radiant Heat and its Spectrum Distribution," 706, 707, "Radio-micrometer and some other Instruments," 720, 721; tono-micrometer, "Radio-micrometer and some other Instruments," 723; pocket electrometer, *ibid.* 723
Brackenbury, H. I.: adjustable ring gauge, "Gauges," 342, 343
Bradley: astronomical refraction formula, "Meteorological Optics," 519
Braun, Dr. C.: apparatus and determinations of "G," "Earth, The Mean Density of the," 282, 283, 285
Bravais: on halos, "Meteorological Optics," 529, 531, 532, 533
Bremen: volumenometer, "Balances," 135
Brensing: projection, "Map Projections," 481
Brewster: neutral point in polarisation, "Meteorological Optics," 523
Briggs: pipe thread, "Gauges," 348, 349
Brill: microbalance, "Balances," 120
Brix: Tralles' table of sugar density recalculated, "Alcoholometry," 12; contraction of sugar solutions, "Saccharometry," 726; table, *ibid.* 727; degrees, *ibid.* 730; spindle, *ibid.* 730
  (Balling and) experiments, "Saccharometry," 729; saccharometer, *ibid.* 729, 730
Broch: formula for points of support of a bar, "Line Standards of Length," 467, "Metrology," 587
  (Regnault and) table of vapour pressure, "Humidity," 416
Bromwich, T. J. I'A.: scattering of waves, "Meteorological Optics," 527 *n.* 1
Brookes, A. J. C.: end-gauge comparator, "Gauges," 387, 388, 389, 390, "Metrology," 630
  (Glazebrook, Sears, and) method of making block gauges, "Gauges," 301 *n.*, "Metrology," 621
Brookes, J. W.: draughtsman's curves, "Draughting Devices," 264, 265
Browett, T.: adjustable plug gauges, "Gauges," 341
Brown: *Handbook of Sugar Analysis*, "Saccharometry,' 731, 733
  (Morris, Millar, and) solution-factors, "Saccharometry," 733
Brown, E. B.: speed indicator, "Meters," 539

Brunner: comparator, "Comparators," 244; gravimetric method, "Humidity," 426
Bruns: errors of scales; "Micrometers," 634
Brunt, D.: eddy diffusivity, "Atmosphere, Physics of the," 35; tables of wind direction and cloud amount, *ibid.* 40 *n.*; on equation between temperature and water-vapour, "Atmosphere, Thermodynamics of the," 70
Buch, K.: CO_2 in sea-water, "Oceanography, Physical," 673
Buchanan, J. Y.: hydrometers, "Hydrometers" 436, "Oceanography, Physical," 677
Budenberg: (Schaffer and) tachometers, "Meters," 534, 535
Budgett, H. M.: experiments on wrung block gauges, "Gauges," 300
Buijsen, van: halo, "Meteorological Optics," 533
Buisson: (Lepinay, Benoit, and) determination of litre, "Volume, Measurements of," 778
Bullock, J.: "Calculating Machines," 194
Bunge: vacuum balance, "Balances," 120
Burgess: "Earth, The Mean Density of the," 280; apparatus and results, *ibid.* 283, 285
Burney: halo, "Meteorological Optics," 533
Burrard, Sir S. G.: on isostasy, "Gravity Survey," 412, 413
Burroughs: adding machine, "Calculating Machines," 199, 200

Callendar: sulphuric acid manometer, "Barometers and Manometers," 189; electrical recorder, "Meteorological Instruments," 494, 495; radiation recorder, *ibid.* 511, "Radiant Heat and its Spectrum Distribution," 704; black-bulb thermometer, "Radiant Heat and its Spectrum Distribution," 701; sunshine recorder, *ibid.* 704; radio-balance, *ibid.* 704, 705; tonomicrometer, "Radio-micrometer and some other Instruments," 723
Cambridge Scientific Instrument Co.: microscope screw-measuring machine, "Gauges," 336
Campbell: sunshine recorder, "Radiant Heat and its Spectrum Distribution," 699, 700; Campbell-Stokes sunshine recorder, "Meteorological Instruments," 488, 510, 511, 512, 513, "Radiant Heat and its Spectrum Distribution," 699
Campbell, Dr. N.: "Observations, The Combination of," 664 *n.*
Capstick: density of a gas, "Balances," 137
Carlini: experiments and results, "Earth, The Mean Density of the," 280, 285
Carman: volumenometer, "Balances," 135
Cartier: hydrometer, "Alcoholometry," 19
Casella: wind-direction recorder, "Meteorological Instruments," 509; Miller-Casella maximum-minimum thermometer, "Oceanography, Physical," 673
Cassini, C. F.: projection, "Map Projections," 480, 481, 483, 486
Cave, C. J. P.: "Air, The Investigation of the Upper," 5, 30, 31; wind and height, *ibid.* 30, 31, 32; (Shaw and) formula, *ibid.* 32
Cavendish, H.: apparatus, experiments, and results, "Earth, The Mean Density of the," 280, 281, 285, "Quartz Fibres," 698
Chadwick: (Frost and) liquid meter, "Meters," 564
Chapman: formula for variation of wind with height, "Atmosphere, Physics of the," 32
Chappuis, P.: density of water, table, "Balances," 130; high-precision work, "Barometers and Manometers," 190; determination of litre, "Volume, Measurements of," 778
Charcot: variations of potential observed by, "Atmospheric Electricity," 92
Chevalier: (Thoulet and) specific heat of sea-water, "Oceanography, Physical," 670
Clairaut: theorem, "Gravity Survey," 411
Clarke: ellipsoid, "Geodetic Measures," 396; metre, "Length, Units of," 464; projection, "Map Projections," 479, 480, 485, 486; coefficient of refraction, "Trigonometrical Heights," 768, 769
(James and) determinations, "Earth, The Mean Density of," 285
Clarke: hydrometer, "Alcoholometry," 15, 16
Clarke, G. A.: mock-sun ring and mock suns, "Meteorological Optics," 529
Claude: (Driencourt and) prismatic astrolabe, "Gravity Survey," 409, "Latitude, Longitude, and Azimuth," 463
Clifton, Prof.: "Earth, The Mean Density of the," 282
Close, Sir C. F.: Mollweide projection [illegible], "Map Projections," 484
(Cox and) *Topographical Surveying* by, "Latitude, Longitude, and Azimuth," 459, 463, 464
Coblentz: method of making quartz fibres, "Quartz Fibres," 697; stellar radiation, "Radiation, The Measurement of," 716; radio-micrometer, "Radio-micrometer and some other Instruments," 723
(Kahler and) solar radiation and quartz mercury vapour lamp, "Radiation, The Measurement of," 716
Cody: kite, "Air, The Investigation of the Upper," 2
Coffin: planimeter, "Integration, Mechanical Methods of," 453
Colding: observations on storm in Baltic, "Oceanography, Physical," 680, 688, 689
Cole, J. H.: temperature and refraction, "Trigonometrical Heights," 772
Conyngham, Sir G. L.: Indian Survey apparatus, "Gravity Survey," 399, 400, 401
Cook: mirage, "Meteorological Optics," 520
Cornu: (Baille and) apparatus and investigations, "Earth, The Mean Density of the," 281, 285
Cottingham, E. T.: crutch, "Clocks and Time-keeping," 212; gravity escapement, *ibid.* 213, clock, *ibid.* 213, 228
Cox, E. W.: (Close and) *Topographical Surveying* by, "Latitude, Longitude, and Azimuth," 459, 463, 464
Craig, J. I.: Gauss conformal projection, "Map Projections," 483; retro-azimuthal projection, *ibid.* 485
Crookes, Sir W.: vacuum balance, "Balances," 120
Crova: condensation hygrometer, "Humidity," 418
Cunynghame, Sir H.: escapement, "Clocks and Time-keeping," 214; on hygrometers, "Humidity," 429
Curtis, R. H.: sunshine recorders, "Radiant Heat and its Spectrum Distribution," 699, 700
Cussons, G., Messrs.: floating micrometer screw gauge, "Gauges," 324; ring screw measuring machine, *ibid.* 327; vertical projection apparatus, *ibid.* 360
Cust: (Field and) tide recorder, "Meters," 546, 547

Dalton: water-vapour pressure law, "Atmosphere, Thermodynamics of the," 71, "Humidity," 416; dew-point, "Humidity," 417
Daniell: dew-point hygrometer, "Humidity," 417; wet bulb thermometer, *ibid.* 422
Darras, M. A.: "Calculating Machines," 195
Darwin, Sir G.: "Tides and Tide-prediction," 764, 766
Darwin, Sir H.: temperature compensation device, "Gravity Survey," 401; Darwin-Hill mirror, "Meteorological Instruments," 515
Darwin, Major L.: projection tables, "Map Projections," 485
Deacon, G. F.: cone and disc meter, "Meters," 567
De l'Isle: projection, "Map Projections," 482
Dent: clock, "Clocks and Time-keeping," 204, 211 *n.*, 213; pendulum, *ibid.* 204, 205; temperature and change of rate, *ibid.* 230
Desaguliers: spirit balance, "Alcoholometry," 10, 15
Descartes, R.: rainbow, "Meteorological Optics," 524
Dicas: hydrometer, "Alcoholometry," 15, 16
Dines, J. S.: self-recording theodolite, "Air, The Investigation of the Upper," 5; variation of wind with height, *ibid.* 30, 32
Dines, W. H.: the upper air in England, "Atmosphere, Thermodynamics of the," 55, 58, 59, 60, "Barometers and Manometers," 180, 181; temperature, pressure, and density tables, "Barometers and Manometers," 181; recording rain-gauge, "Meteorological Instruments," 498; anemometer experiments, *ibid.* 501; tube anemometer, *ibid.* 503, 504, 505, 506; wind-direction recorder, *ibid.* 510; differential radiometer, *ibid.* 514; sunshine recorder, "Radiant Heat and its Spectrum Distribution," 701
(Shaw and) microbarograph, "Barometers and Manometers," 148, "Microbarograph," 633
Dinklage: ocean currents, "Oceanography, Physical," 680
Dittmar: analysis of sea-water, "Oceanography, Physical," 665

Dobrowolski: snow crystals, "Meteorological Optics," 529, 530, 531
Dobson, G. M. B.: variation of wind with height, "Atmosphere, Physics of the," 31, 32, 34; altimeter errors, "Barometers and Manometers," 180; barothermograph, "Meteorological Instruments," 547
Dollfus, Dr.: density of alcohol, "Alcoholometry," 11
Dominus, Antonius de: rainbow, "Meteorological Optics," 524
Domke, J.: density of H_2SO_4, "Balances," 128, 130; tables of sugar densities, "Saccharometry," 727
(Reimerdes and) work by, "Hydrometers," 441
Doroshevski: (Rozhdestvenski and) density of liquids, "Balances," 130
Douglas, C. K. M.: thunderstorm data, "Atmosphere, Physics of the," 41
Douglas, H. P.: screen for prismatic astrolabe, "Gravity Survey," 409
Dowling: (Whiddington and) ultra-micrometer, "Metrology," 631
Driencourt: (Claude and) prismatic astrolabe, "Gravity Survey," 409, "Latitude, Longitude, and Azimuth," 463
Drinkwater: determinations of spirit strength by, "Alcoholometry," 12, 17
Drucker: manostat, "Barometers and Manometers," 1[illegible]2
Duffield, W. G.: measurement of gravity at sea, "Barometers and Manometers," 192, "Gravity Survey," 403, 404, 412 *n.*, 413
Dumas: method of measuring densities of vapours, "Balances," 137
Dutheil: halo, "Meteorological Optics," 533

Earnshaw: (Arnold and) chronometer escapement, "Clocks and Time-keeping," 213
Ebert, H.: theory of fine weather electricity, "Atmospheric Electricity," 93, 96; apparatus, *ibid.* 94
Eden, E. M.: experimental machine, "Gauges," 330, 331, 332; optical examination of screw threads, *ibid.* 352, 353; measuring machine, *ibid.* 377, 378, 379, 380, 381; special end measuring comparator, *ibid.* 385; measuring scale, *ibid.* 390; device for converting linear into angular motion, "Metrology," 627
Edwards: determination of density of gases "Balances," 137
Egerton: vapour density determination, "Balances," 137
Egli, H.: Millionaire machine, "Calculating Machines," 200
Einstein: relativity, "Gravity Survey," 413
Einthoven: oscillograph, "Clocks and Time-keeping," 221
Ekama: Kern's arc, "Meteorological Optics," 53[illegible]
Ekholm, N.: chart of pressure changes, "Atmosphere, Physics of the," 40
Ekman, V. W.: virtual friction, "Atmosphere, Physics of the," 33; compressibility of sea-water, "Oceanography, Physical," 667; thermal expansion, *ibid.* 669, 670; specific heat, *ibid.* 670; reversing thermometer, *ibid.* 673; current meter, *ibid.* 675; skin friction of wind on sea, *ibid.* 680, 688, 689; theory of currents, *ibid.* 680, 684, 685, 686, 687, 688, 689; coast line and wind-drift, *ibid.* 681, 687; adiabatic temperature change, *ibid.* 684; depth of wind friction, *ibid.* 686
Ellicott, [illegible]: double pendulum, "Clocks and Time-keeping," 223, 225
Elliott-Fisher: adding machine, "Calculating Machines," 200
Elster, J.: (Geitel and) emanation content of the air, "Atmospheric Electricity," 95; potential gradient, *ibid.* 98; electricity of rain, *ibid.* 101; theory of field of thunderstorm, *ibid.* 105
Emden: radiative equilibrium, "Atmosphere, Physics of the," 26, 28, "Atmosphere, Thermodynamics of the," 68, 71
English, S.: calibration of pipettes, "Volume, Measurements of," 803
Eötvös, Baron: torsion balance determination of "G," "Earth, The Mean Density of the," 283, 285, "Quartz Fibres," 698; gravity balance, "Earth, The Mean Density of the," 283, "Gravity Survey," 404, 405, 406, 410
Erismann, T.: absorption of gravitation by matter, "Gravity Survey," 413
Eve, A. S.: ionisation of the air, "Atmospheric Electricity," 95
Evershed, J.: scale for spectra, "Micrometers," 634
Ewing: duplex pendulum, "Seismometry," 735, 736; bracket seismograph, *ibid.* 737
Exner, F. M.: conductive equilibrium, "Atmosphere, Physics of the," 25; radiative equilibrium, *ibid.* 26, 27, 28, "Atmosphere, Thermodynamics of the," 68, 69, 70; waves in surface of discontinuity, "Atmosphere, Physics of the," 38; *Meteorologische Optik*, "Meteorological Optics," 518; secondary diffraction, *ibid.* 523; polarisation from landscape, *ibid.* 523; radiation from the earth, "Radiation," 712
Eynon: (Ling, Lane, and) solution-factors, "Saccharometry," 733

Faber: log-log rule, "Draughting Devices," 274
Fabroni: (Lefèvre-Gineau and) standard kilogramme, "Volume, Measurements of," 776, 777
Fabry: (Benoit, Perot, and) metre determined in terms of wave-lengths, "Line Standards of Length," 477
Fayet: (Loewy and) scale errors, "Micrometers," 634
Ferrié, Gen.: lag-measuring devices, "Clocks and Time-keeping," 221
Feuillée: halo, "Meteorological Optics," 533
Field: (Cust and) tide recorder, "Meters," 546, 547
(Wharton and) "Oceanography, Physical," 676
Fineman: nephoscope, "Meteorological Instruments," 514
Fisher: Elliott-Fisher adding machine, "Calculating Machines," 200
Fizeau: dilatometer, "Metrology," 631
(Foucault and) interference method of determination of wave-lengths, "Radiant Heat and its Spectrum Distribution," 708
Flamsteed: Sanson-Flamsteed projection, "Map Projections," 485
Fortin: barometer, "Barometers and Manometers," 142, 143, 144, 145, 146, 147, 149; correction for temperature, *ibid.* 152, 153, 155, 156; possible errors, *ibid.* 162, 163; verticality, *ibid.* 164; accuracy and permanence, *ibid.* 164, 165
Foucault: -Hipp butterfly, "Clocks and Time-keeping," 227
(Fizeau and) interference method of determination of wave-lengths, "Radiant Heat and its Spectrum Distribution," 708
Fougner: calculator, "Calculating Machines," 195, 198
Fowle: (Abbot and) diminution of solar and terrestrial radiation by the atmosphere, "Atmosphere, Physics of the," 27, "Atmosphere, Thermodynamics of the," 69, 70; pyrheliometers, "Meteorological Instruments," 514, "Radiant Heat and its Spectrum Distribution," 703, 704, "Radiation, The Measurement of," 71[illegible]; solar constant, "Radiation," 708, 709, 714, "Radiation, The Measurement of," 717
Fowler, Sir H.: (Taylor, Alpe, and) gauge for plug screws, "Gauges," 343, 344
Fownes: determinations of spirit densities by, "Alcoholometry," 12
Fox, C. J. J.: nitrogen absorbed by sea-water, "Oceanography, Physical," 673
Franklin, Benjamin: upper air experiments, "Air, The Investigation of the Upper," 1
Frivold, O. [illegible]: determination of density, "Balances," 129
Frodsham: clock, change of rate and temperature, "Clocks and Time-keeping," 230
Frost: (Chadwick and) liquid meters, "Meters," 564
Fujiwhara: (Oti and) Parry's arc, "Meteorological Optics," 532 *n.* 2
Fuller: slide rule, "Draughting Devices," 278

Galileo: isochronism of pendulum, "Clocks and Time-keeping," 204
Galitzin, Prince: seismic instruments, "Earthquakes and Earthquake Waves," 286, 288, "Seismology," 736, 737, 738, 739; surface velocity of long waves, "Earthquakes and Earthquake Waves," 287; locating an earthquake, *ibid.* 288; galvanometric registration, "Seismometry," 738
Galle: arcs of Lowitz, "Meteorological Optics," 532 *n.* 2, 3; relation between the velocities of wind and current, "Oceanography, Physical," 680

Gauss: formula, "Integration, Mechanical Methods of," 451; projection, "Map Projections," 480, 482, 483; law of errors, "Observations, The Combination of," 647, 648, 650, 651, 661; solution of normal equations, *ibid.* 660; method of weighing, "Weighing Machines," 819
Gay-Lussac: alcoholic treatise and tables, "Alcoholometry," 12, 19; apparatus for measuring densities of vapours, "Balances," 137; and Baumé scale, "Saccharometry," 731
Gehre: indicator, "Meters," 552, 553
Gehrke, J.: ocean currents, "Oceanography, Physical," 692
Geiger, L.: (Zoeppritz, Gutenberg, and) surface reflected waves, "Earthquakes and Earthquake Waves," 292, 293
Geitel, H.: (Elster and) emanation content of the air, "Atmospheric Electricity," 95; potential gradient, *ibid.* 98; electricity of rain, *ibid.* 101; theory of field of thunderstorm, *ibid.* 105
Gent and Co., Messrs.: electric control system, "Clocks and Time-keeping," 228
Gerdien, H.: measurement of conductivity, "Atmospheric Electricity," 93, 94; theories of Ebert type criticised, *ibid.* 97; electricity of rain, *ibid.* 101, 104
Gerlach: contraction of sugar solutions, "Saccharometry," 726; specific gravity and sucrose content, *ibid.* 726; table, *ibid.* 727; scale, *ibid.* 730
Germann: density of gases, "Balances," 136
Gersten: "Calculating Machines," 195
Gibson: manometer, "Barometers and Manometers," 189
Gill, Sir D.: clock, "Clocks and Time-keeping," [illegible]3; four-metre comparator, "Comparators," 247; errors of scales, "Micrometers," 634; errors of screws, *ibid.* 634; 24-metre comparator, "Surveying Tapes and Wires," 755
Gilpin, G.: tables devised by, "Alcoholometry," 11, 16, 20
Glaisher: upper air experiments, "Air, The Investigation of the Upper," 1; factors, "Humidity," 420; screen, "Meteorological Instruments," 490
Glareanus: "Map Projections," 486
Glazebrook, Sir R. T.: (Sears, Brookes, and) method of making block gauges, "Gauges," 304 *n.*, "Metrology," 621
Glenfield: (Kennedy and) notch meter, "Meters," 569, 570
Gold, E.: theory of radiative equilibrium, "Atmosphere, Physics of the," 26, "Atmosphere, Thermodynamics of the," 68; geostrophic and observed wind, "Atmosphere, Physics of the," 29, 31; barometer gradient and wind force, *ibid.* 37; pressure distribution, *ibid.* 39, "Radiation," 711
Graham, G.: pendulum and escapement, "Clocks and Time-keeping," 204, 211, 212
Grant: (Steele and) micro-balances, "Balances," 120, 121
Gravêt: rule, "Draughting Devices," 276
Gray: (Ramsay and) quartz micro-balance, "Balances," 121, 137; density of radium emanation, *ibid.* 121, 137
Grayson, H. J.: dividing engine, "Line Standards of Length," 468, 469
Gräber: thermo-alcoholometer, "Alcoholometry," 16
Grey: (Poynting and) torsion balance used by, "Earth, The Mean Density of the," 283
Grimthorpe, Lord (*see* Beckett, E.): escapement, "Clocks and Time-keeping," 213
Gruner, P.: diffraction calculations, "Meteorological Optics," 522
Guillaume, C. E.: nickel steel alloys, "Clocks and Time-keeping," 203; "Line Standards of Length," 472, 473, 474, "Metrology," 576; Director of Bureau International des Poids et Mesures, "Metrology," 586; determination of litre, "Volume, Measurements of," 778
(Benoit and) longitudinal comparator, "Comparators," 251; metric standards, *ibid.* 251, "Metrology," 586; corrections for elasticity of wires, "Surveying Tapes and Wires," 745, 746; effect of coiling tapes, *ibid.* 747
Gunter, E.: straight logarithmic scale, "Draughting Devices," 275
Gurley: current meter, "Oceanography, Physical," 676
Gutenberg, B.: (Zoeppritz, Geiger, and) surface reflected waves, "Earthquakes and Earthquake Waves," 292, 293
Gutkowski: (Mendelejeff and) capillary depression of mercury, "Barometers and Manometers," 159 *n.* 2

Hahn: "Calculating Machines," 195
Haldane, J. S.: measurement of volumes of gases, "Volume, Measurements of," 811 *n.* 1, 812
Halden: calculex, "Draughting Devices," 262
Halliwell: modified Dines recorder, "Meteorological instruments," 505, 506; wind-direction recorder, *ibid.* 510
Hamann, Ch.: Mercedes-Euklid machine, "Calculating Machines," 200
Hammer: projection tables, "Map Projections," 486
Hammond: calculating typewriter, "Calculating Machines," 200
Hann: formula for vapour pressure and height, "Atmosphere, Physics of the," 27; equation for distribution of water-vapour, "Atmosphere, Thermodynamics of the," 69; tables, "Radiation," 715
Hanzlik: anticyclones, "Atmosphere, Physics of the," 37
Hare: determination of density, "Balances," 129
Hargreave: box kite, "Air, The Investigation of the Upper," 2
Harris, R. A.: cotidal chart, "Tides and Tide-prediction," 766
Harrison, J.: spring maintenance, "Clocks and Time-keeping," 204, 214, 216
(Abbot and) "Meters," 513
Harting: sugar tables, "Saccharometry," 727
Hartmann: automatic comparator, "Gauges," 377
Hartshorne: hygroscopic curves, "Humidity," 428
Hasking: viscosity of pure water, "Oceanography, Physical," 671
Hasler: speed counter instrument, "Meters," 534
Hastings, C. S.: ice crystals, "Meteorological Optics," 531, 532, 533
Haüy: (Lavoisier and) standard kilogramme, "Volume, Measurements of," 776
Haworth: humidifying system, "Humidity," 430
Hayford, J. F.: theory of isostasy, "Gravity Survey," 411, 412, 413
(Bowie and) calculations, "Gravity Survey," 412
Heaviside, Capt.: (Basevi and) observations of gravity by, "Gravity Survey," 399
Hecker, O.: hypsometer observations, "Gravity Survey," 403, 412
Helland-Hansen: transparency of sea-water, "Oceanography, Physical," 672, 673
(Sandström and) calculation of current velocity, "Oceanography, Physical," 690, 691
Hellman: formula for variation of wind with height, "Atmosphere, Physics of the," 32
Helmert, F. R.: formulae, "Clocks and Time-keeping," 207, "Gravity, Measures of," 396, "Gravity Survey," 411, 412; mean polar quadrant, "Geodetic Measures," 396; metre, "Length, Units of," 464
Helmholtz: conditions at a surface of discontinuity, "Atmosphere, Physics of the," 28, 41
Herbert, A., Messrs.: floating micrometer screw gauge, "Gauges," 321; optical measuring machine, *ibid.* 236, 237
Hergesell, H.: temperature in the upper air, "Atmosphere, Thermodynamics of the," 55
Herglotz, G.: earth waves equation, "Earthquakes and Earthquake Waves," 290
Hero of Alexandria: "Clocks and Time-keeping," 203
Herschel, C.: Venturi meter, "Meters," 567, 568, 569
Herschel, Sir J.: black-bulb thermometer, "Radiant Heat and its Spectrum Distribution," 702
Hersey: on "creep" in aneroids, "Barometers and Manometers," 174
Hertz, H.: isentropic equations for air, "Atmosphere, Thermodynamics of the," 78
Herwig: compression of vapours, "Humidity," 416
Herzfeld: pure sucrose, "Saccharometry," 727, 728
Hesselberg, Th.: (Sverdrup and) eddy diffusivity, "Atmosphere, Physics of the," 34, 35; stability of sea-water, "Oceanography, Physical," 683; tables, *ibid.* 691
Heuse: (Scheel and) manometer, "Barometers and Manometers," 187

Hicks, W. M.: temperature variation, "Earth, The Mean Density of the," 284
Hildebrandsson, H. H.: "Atmosphere, Thermodynamics of the," 55, 58
Hilger: constant deviation prism, "Comparators," 244; projection apparatus, "Gauges," 362
Hill, A. V.: Darwin-Hill mirror, "Meteorological Instruments," 515
Hill, L.: Kata-thermometers, "Humidity," 425, 426
Hinks, A. R.: work by, "Map Projections," 484
Hipp: Foucault-Hipp butterfly, "Clocks and Time-keeping," 227
Hipparchus: orthographic projection, "Map Projections," 484; stereographic, *ibid.* 485
Hirth: minimeter, "Gauges," 384, 385
Hoake: spirit balance, "Alcoholometry," 10
Hobson: (Taylor, Taylor, and) floating micrometer measuring machine, "Gauges," 319
Hodgson, J. L.: manometer gauge, "Meters," 557, 558; meter, *ibid.* 558, 559, 560, 561
Hodgson-Kent: steam meter, "Meters," 549, 550
Hoffmann: roller gauges, "Gauges," 303, 315
Hollerith: tabulating engine, "Calculating Machines," 271
Holmes, J.: contraction results, "Alcoholometry," [illegible]
Horner: radiation from the earth, "Radiation," 712
Hough, G. W.: chronograph, "Clocks and Time-keeping," 220; dynamical theory of tides, "Tides and Tide-prediction," 765
Howson: barometer, "Barometers and Manometers," 117
Hudson: horse-power computing rule, "Draughting Devices," 272
Humphreys, W. J.: theory of radiative equilibrium, "Atmosphere, Physics of the," 26, "Atmosphere, Thermodynamics of the," 68; temperature in upper air, tables, "Atmosphere, Thermodynamics of the," 55, 58; *Physics of the Air*, "Meteorological Optics," 518; apparent form of sky, *ibid.* 518; anthelion, *ibid.* 533; nocturnal radiation, "Radiation, The Measurement of," 718
Hutton: (Maskelyne and) mountain observations, "Earth, The Mean Density of the," 279, 285
Huyghens, Chr.: pendulum principles, "Clocks and Time-keeping," 204
Hyatt: hydrometer, "Alcoholometry," 16

Irving, J. R.: (Lovi and) Lovi's beads, "Alcoholometry," 16

Jackson: glass manometer, "Barometers and Manometers" 189
Jacobsen: level current meter, "Oceanography, Physical," 676
Jaeger: manometer, "Barometers and Manometers," 187, 188
Jakeman: experiments with hygrometers, "Humidity," 419
James, Sir H.: mountain observations of "G," "Earth, The Mean Density of the," 279; projection, "Map Projections," 484, 485
Jaquerod: (Tourpaian and) determination of density of gases, "Balances," 135
Jeffreys, H.: on isostasy, "Gravity Survey," 413; frictional resistance in sea, "Oceanography, Physical," 686, 687
Johansson, Messrs. C. E.: block gauges, "Gauges," 299, 300, 301, [illegible]; tolerance system, "Metrology," 316
Jolly, von: horizontal balance beam determinations, "Earth, The Mean Density of the," 284, 285
Jones, F. H.: electric clocks, "Clocks and Time-keeping," 228
Jones, R. L.: electric control, "Clocks and Time-keeping," 226
Jordan: sunshine recorder, "Meteorological Instruments," 488, 510, 513, "Radiant Heat and its Spectrum Distribution," 700
Josse: (Remy and) sugar table, "Saccharometry," 726

Kahler: (Coblentz and) solar radiation and quartz-mercury vapour lamp, "Radiation, The Measurement of," 716
Kater, Capt.: pendulum, "Gravity Survey," 398; graduation of standards, "Metrology," 587
Kelvin, Lord: water dropper, "Atmospheric Electricity," 88; effect of pendulum suspension, "Clocks and Time-keeping," 223, 224; integrating device, "Meters," 567; hydrodynamic equation for circulation in a curve, "Oceanography, Physical," 689; tide-predicting machine, "Tides and Tide-prediction," 764; adiabatic gradient, "Trigonometrical Heights," 771
Kennedy, T.: liquid meter, "Meters," 564
  (Glenfield and) notch meter, "Meters," 569, 570
Kern: arc, "Meteorological Optics," 532
Kimball: solar radiation in U.S.A., "Radiation, The Measurement of," 716; humidity effects, *ibid.* 718
King, L. V.: electrical gas meter, "Meters," 563, 564
King, W. F.: pendulum, "Clocks and Time-keeping," 206; chronometer escapement, *ibid.* 213, 214; change of rate and temperature, *ibid.* 230
Kirchoff: law, "Atmosphere, Physics of the," 27
Knott: waves in the interior of the earth, "Earthquakes and Earthquake Waves," 290
Knox-Shaw, H.: (Bell and) graphical reduction, "Latitude, Longitude, and Azimuth," 461; tables, *ibid.* 463
Knudsen, M.: hydrographical tables, "Balances," 130, "Oceanography, Physical," 666, 667, 673, 677; temperature coefficient of conductivity, "Oceanography, Physical," 672; current theorem, *ibid.* [illegible]
Köppen: Beaufort scale values, "Oceanography, Physical," 680
Kövesligethy, R. von: earth waves equation, "Earthquakes and Earthquake Waves," 289
Kreiling: (MacInnes and) vapour density determinations, "Balances," 137
Kriger-Menzel: (Richarz and) horizontal balance beam determinations, "Earth, The Mean Density of," 284, 285
Krogh: CO_2 in sea-water, "Oceanography, Physical," 673
Krümmel: specific heat of sea-water, "Oceanography, Physical," 670; lowering of vapour pressure, *ibid.* 670; surface tension, *ibid.* 671; effect of coastal configuration, *ibid.* 681; compensation currents, *ibid.* 681; reversal depth, *ibid.* 686, 687
  (Ruppin and) viscosity of sea-water, "Oceanography, Physical," 671
Kurlbaum: coefficient of total radiation, "Radiation, The Measurement of," 717; effective solar temperature, *ibid.* 717

Lagrange: "Map Projections," 486
La Hire: projection, "Map Projections," 484, 485
Lallemand, Ch.: projection, "Map Projections," 484; work by, "Observations, The Combination of," 662 *n.*; formulae for refraction of rays close to sloping ground, "Trigonometrical Heights," 771, 772
Lamb: (Lee and) float method, "Balances," 129
Lamb, Dr. H.: theory of waves in surface of discontinuity, "Atmosphere, Physics of the," 38
Lambert, J. H.: projections, "Map Projections," 480, 481, 482, 483, 486
Landolt: (Börnstein and) tables, "Humidity," 416
Lane: (Eynon, Ling, and) solution-factors, "Saccharometry," 733
Langen: hydrometer, "Saccharometry," 731
Langevin, P.: ions in atmosphere, "Atmospheric Electricity," 98
Langley, S. P.: bolometer, "Radiant Heat and its Spectrum Distribution," 706, 717; calibration in terms of wave-lengths, *ibid.* 707, 708
Laplace: "Observations, The Combination of," 647; dynamical theory of tides, "Tides and Tide-prediction," 765
Larmor, Sir J.: on isostasy, "Gravity Survey," 413; temperature and gravitation, *ibid.* 413
Laurent: quartz weights, "Balances," 124
Lavoisier: (Haüy and) standard kilogramme, "Volume, Measurements of," 776
Layton: arithmometer, "Calculating Machines," 198
Lea: water flow recorder, "Meters," 569
Le Cadet, G.: balloon observations, "Atmospheric Electricity," 90
Le Chatelier: (Wologdine and) density and purity of graphite, "Balances," 135
Leduc: determination of density of gases, "Balances," 136; high-precision work, "Barometers and Manometers," 190
Lee: (Lamb and) float method, "Balances," 129
Lee, A. W.: entropy-temperature tables, "Atmosphere, Thermodynamics of the," 81

Leeds: (Northrupp and) tachometer calibration,
“Meters,” 544, 545
Lefèvre-Gineau: (Fabbroni and) standard kilogramme,
“Volume, Measurements of,” 776, 777
Leffmann: -Beam milk-test bottle, “Volume,
Measurements of,” 796
Lehmann: (Miethe and) purple-light, “Meteoro-
logical Optics,” 522
Leibnitz: stepped reckoner, “Calculating Machines,”
194
Lenard, P.: condensation in thunderstorms, “Atmo-
sphere, Physics of the,” 43; theory of fine-
weather field, “Atmospheric Electricity,” 96, 9[illegible]
Lepinay: (Buisson, Benoit, and) determination of
litre, “Volume, Measurements of,” 778
Lépine: “Calculating Machines,” 195
Leupold: “Calculating Machines,” 195
Liesche: manostat, “Barometers and Manometers,”
192
Ling: (Eynon, Lane, and) solution-factors, “Sac-
charometry,” 733
Linss: conduction through the air, “Atmospheric
Electricity,” 92; electricity of rain, *ibid.* 92
Lister, S.: nomogram, “Nomography,” 643
Loewe, L.: tolerance system, “Metrology,” 616
Loewy: (Fayet and) scale errors, “Micrometers,” 634
Lorgna: “Map Projections,” 486
Lo Surdo: radiation from the earth, “Radiation,” 712
Love, A. E. H.: on isostasy, “Gravity Survey,” 412,
413
Lovi, Mrs. I.: (Irving and) Lovi's beads, “Alcoholo-
metry,” 16
Lowenherz: thread, “Gauges,” 348, 351
Lowitz: alcohol densities obtained by, “Alcoholo-
metry,” 11; arcs of, “Meteorological Optics,”
529, 530, 532
Lowry: (Bousfield and) density of caustic soda,
“Balances,” 130
Lubbock, Sir J.: method of prediction, “Tides and
Tide-prediction,” 759, 764
Lully, R.: test of strength of spirits, “Alcoholo-
metry,” 9
Lunge: (Wienick and) density of ammonia, “Bal-
ances,” 130

McCaw, G. T.: “Map Projections,” 485
M'Dowall: pointer method, “Balances,” 112
MacInnes: (Kreiling and) vapour densities, “Bal-
ances,” 137
M'Kelvy: (Osborne, Bearce, and) specific gravity
and temperature, “Alcoholometry,” 14; density
investigations, “Balances,” 130
Mainka: seismograph, “Seismometry,” 736, 740
Majorana, Q.: quenching factor, “Gravity Survey,”
413
Mandet: (Pérard and) measurement of wrung block
gauges, “Gauges,” 300
Manley: investigation of thermal effects, “Bal-
ances,” 116
Mannheim: logarithmic scale, “Draughting De-
vices,” 276
Marchant: “Calculating Machines,” 198
Marek: density of mercury, “Balances,” 131
Margules: effect of rotation and friction of the air
on oscillation, “Atmosphere, Physics of the,” 38
Marinus of Tyre: projection, “Map Projections,” 483
Marten, W.: comparison of sunshine recorders,
“Radiant Heat and its Spectrum Distribution,”
700
Martin: hydrometer, “Alcoholometry,” 15
Marvin: compensated barograph, “Barometers and
Manometers,” 148; sunshine recorder, “Radiant
Heat and its Spectrum Distribution,” 700, 701;
pyrheliometer, “Radiation, The Measurement
of,” 716
(Wiechert and) inverted pendulum, “Seismometry,”
736
Mascart, E.: sealed mercury barometer, “Gravity
Survey,” 404; *Traité d'optique*, “Meteorological
Instruments,” 518, 524
Maskelyne: (Hutton and) mountain observations o[illegible]
“G,” “Earth, The Mean Density of the,” 279,
285
Masson, O.: hygroscopic properties of cotton,
“Humidity,” 428
Mategczeck: (Scheibler and) sugar table, “Saccharo-
metry,” 727
Mather: (Platt and) vortex humidifier, “Humidity,”
429, 430
Maurer: radiation from the earth, “Radiation,”
712
Maxwell, J. C.: theory of wet bulb thermometer,
“Humidity,” 421, 422, 423; geometric design,
“Instruments, The Design of Scientific,” 449
Mayo: (Tyndall and) volumetric apparatus for water
vapour, “Humidity,” 427, 428
Meissner, J. W.: areometer, “Alcoholometry,” 19
Menard: manometer, “Barometers and Mano-
meters,” 186
Mendeléeff: research on alcohol, “Alcoholometry,” 12
(Gutkowski and) capillary depression, “Barometers
and Manometers,” 159 *n.* 2
Mendenhall, C. E.: experiments on the determination
of “G,” “Earth, The Mean Density of the,”
280, 285
Mercator, G.: projection, “Map Projections,” 479,
480, 482, 483, 484, 486
Merriman: (Wade and) manostat, “Barometers and
Manometers,” 192
Meyer, V.: method of measuring densities of vapours,
“Balances,” 137
(Biltz and) “Balances,” 137
Michell, Rev. J.: torsion balance, “Earth, The
Mean Density of the,” 280
Michelson, A. A.: metre measured in terms of wave-
lengths, “Line Standards of Length,” 475, 476,
477, “Metrology,” 574; pyrheliometer,
“Meteorological Instruments,” 514, “Radiant
Heat and its Spectrum Distribution,” 703;
interferometer, “Metrology,” 631, “Volume,
Measurement of,” 778; micrometer screw,
“Micrometers,” 633
Mierdel, F.: glories, “Meteorological Optics,” 527
Miethe: (Lehmann and) purple-light, “Meteoro-
logical Optics,” 522
Millar: spirit balance, “Alcoholometry,” 10
(Brown, Morris, and) solution-factors, “Saccharo-
metry,” 733
Miller: -Casella maximum-minimum thermometer,
“Oceanography, Physical,” 673
Milne, J.: seismograph, “Earthquakes and Earth-
quake Waves,” 28[illegible], “Seismometry,” 736, 737,
739, 740; records, “Earthquakes and Earth-
quake Waves,” 286; surface waves, *ibid.* 287;
locating an earthquake, *ibid.* 288; Milne-Shaw
seismograph, “Seismometry,” 736, 741
Mohn, H.: relation between velocities of wind and
current, “Oceanography, Physical,” 680; density
temperature, and current, *ibid.* 689
Möhr: hydrostatic balance, “Alcoholometry,” 10;
chemical determination of chlorine in sea-water,
“Oceanography, Physical,” 677, 678; cubic
centimetre, “Saccharometry,” 725; system of
units, “Volume, Measurements of,” 779
Mollweide: projection, “Map Projections,” 484
Moncony: areometer, “Alcoholometry,” 15
Montgomerie: coefficient of refraction, “Trigono-
metrical Heights,” 7[illegible]9
Moreland, Sir C.: apparatus, “Calculating
Machines,” 194
Morley: determination of density of gases, “Bal-
ances,” 136
Morris: (Brown, Millar, and) solution-factors,
“Saccharometry,” 733
Moulin, M.: on collectors, “Atmospheric Electricity,”
89
Mouton: wave-lengths, “Radiant Heat and its
Spectrum Distribution,” 708
Müller: stepped reckoner, “Calculating Machines,”
195; magnetic gauge, “Meters,” 546
Munro: -Rooker wind direction recorder, “Meteoro-
logical Instruments,” 509, 510
Murray: spirit balance, “Alcoholometry,” 10

Nagaoka, H.: invariable pendulums, “Gravity
Survey,” 401
Nansen, F.: “Hydrometers,” 438 *n.*; buoy,
“Oceanography, Physical,” 67[illegible]; current meter,
ibid. 676; relation between velocities of wind
and current, *ibid.* 680; salinity of bank water,
ibid. 682; bottom water, *ibid.* 682; wind-drift
current, *ibid.* 684; Nansen-Pettersson water
bottle, *ibid.* 67[illegible]
Napier: numbering rods, “Calculating Machines,”
193, 194
Nemetz, J.: weights, “Balances,” 124
Nernst, W.: micro-balances, “Balances,” 120
Neuhoff, O.: treatment of isentropic equations for air,

" Atmosphere, Thermodynamics of the," 78, 79, 80
Newall : end gauges, " Gauges," 299 ; measuring machine, *ibid.* 364, 365, 366, 367, " Metrology," 630 ; micrometer, " Gauges," 392 ; limit system, " Metrology," 613, 616
Newcomb, S. : *Theory* and *Tables of the Sun*, " Clocks and Time-keeping," 203
Newman : barometer, " Barometers and Manometers," 145, 146
Newnham, E. V. : entropy temperature diagram, " Atmosphere, Thermodynamics of the," 82
Newton, Sir I. : constant of gravitation, " Earth, The Mean Density of the," 279, " Gravitation, Constant of," 397 ; mean density, " Earth, The Mean Density of the," 2[illegible] ; on rainbows " Meteorological Optics," 524 ; laws of motion, " Metrology," 572, 573
Nichol : radiometer, " Radiant Heat and its Spectrum Distribution," 707
Nichols : density of ice, " Balances," 135
Nicholson : hydrometer, " Balances," 133, " Hydrometers," 436
Nicholson, P. : centrolinead, " Draughting Devices," 260
Nicolosi : projection, " Map Projections," 483
Nipher : snow shield, " Meteorological Instruments," 499
Nordenskjöld : snow crystals, " Meteorological Optics," 529
Normand, C. W. B. : " Atmosphere, Thermodynamics of the," 81
Northrup : (Leeds and) tachometers, " Meters," 544, 545
Nutting : " Radiant Heat and its Spectrum Distribution," 706

Oberbeck : cyclones, " Atmosphere, Physics of the," 36 ; volumenometer, " Balances," 135
Ocagne, D' : nomogram, " Nomography," 641
Oertling, L., Ltd. : assay balance, " Balances," 120
Oldham, R. D. : preliminary tremors, " Earthquakes and Earthquake Waves," 286, 287
Omori : seismograph, " Seismometry," 736, 737, 740
Osborne, N. S. : (M'Kelvy, Bearce, and) specific gravity and temperature, " Alcoholometry," 11 ; density investigations, " Balances," 130
(Veazey and) testing of flasks, " Volume, Measurements of," 793 *n.* 2, 795, 796
Osler : pressure-plate anemometer, " Meteorological Instruments," 503
Ostwald : calibration of burettes, " Volume, Measurements of," 803, 809
O'Sullivan : solution-factors, " Saccharometry," 733
Oti : (Fujiwhara and) arcs of Lowitz, " Meteorological Optics," 532 *n.* 2

Parry : upper arc, " Meteorological Optics," 582
Pascal, B. : machine, " Calculating Machines," 194 ; integraphs, " Integration, Mechanical Methods of," 456, 457
Paschwitz, von R. : horizontal pendulum record, " Earthquakes and Earthquake Waves," 286
Patterson, J. : upper air investigations, " Atmosphere, Thermodynamics of the," 56, 57
Payen : arithmometer, " Calculating Machines," 195
Pearson, K. : " Observations, The Combination of," 644 *n.*
Pechmann : determination, " Earth, The Mean Density of the," 285
Peltier, A. : measurement of electric field, " Atmospheric Electricity," 87
Peppler, A. : temperature and pressure, " Atmosphere, Thermodynamics of the," 56
Pérard : (Mandet and) measurement of wrung block gauges, " Gauges," 300
Pereire : " Calculating Machines," 195
Pernter, J. M. : formulae for vapour pressure, " Humidity," 420, 422, 424 ; *Meteorologische Optik*, " Meteorological Optics," 518, 523, 524 ; on rainbows, *ibid.* 524, 525, 526 ; on arcs of Lowitz, *ibid.* 532 *n.* 2, 3 ; radiation from the earth, " Radiation," 712
Perot : (Benoit, Fabry, and) metre determined in terms of wave-lengths, " Line Standards of Length," 177
Perry : log-log rule, " Draughting Devices," 273
Peters : logarithm tables, " Latitude, Longitude and Azimuth," 459
Pettersson, O. : micro-balance, " Balances," 122, " Earth, The Mean Density of the," 285 ; paper by, " Oceanography, Physical," 673 ; ocean currents, *ibid.* 679 ; currents and melting ice, *ibid.* 681, 682 ; Nansen-Pettersson water-bottle, *ibid.* 674
(Ångström and) float method of determining density, " Balances," 129
Pitot : tube meter, " Meters," 557
Plato : density of sugar solutions, " Balances," 128, 130, " Saccharometry," 726 ; sugar tables, " Saccharometry," 725, 726, 727, 728, 729
Platt : (Matter and) vortex humidifier, " Humidity," 429, 430
Poleni : wheel with movable teeth, " Calculating Machines," 198
Pollock, J. A. : (Threlfall and) gravity balance, " Gravity Survey," 403
Potter : rainbow, " Meteorological Optics," 524
Pouillet : pyrheliometer, " Radiant Heat and its Spectrum Distribution," 701
Powers : " Calculating Machines," 201
Poynting : " common balance " apparatus, " Balances," 113, " Earth, The Mean Density of the," 284 ; monograph by, " Earth, The Mean Density of the," 280 ; " common balance " determinations, *ibid.* 284, 285 ; radiation from the solar system, " Radiation, The Measurement of," 717
(Grey and) torsion balance used by, " Earth, The Mean Density of the," 283
Pratt, Archdeacon : isostasy hypothesis, " Gravity Survey," 412, 413
Pratt : (Whitney and) measuring machine, " Gauges," 367, 368, 369, 370
Preston : determination of " G," " Earth, The Mean Density of the," 285
Prestwich, J. A. : fluid gauge, " Gauges," 383, 384, " Metrology," 630
Prenner : quartz manometer, " Barometers and Manometers," 189
Price : water flow meter, " Oceanography, Physical," 676
Ptolemy : projection, " Map Projections," 481, 482

Quevennes : lactometer, " Hydrometers," 144

Rae : (Reilly and) conditions and methods of use of balance, " Balances," 117, 118
Ramsay : determination of density of gases, " Balances," 136
(Gray and) quartz micro-balance, " Balances," 121 ; density of radium emanation, *ibid.* 121, 137
(Steele and) determination of vapour densities, " Balances," 137
(Travers and) density of gases, " Balances," 136
Ramsden : spirit balance, " Alcoholometry," 10
Rankin : halo, " Meteorological Optics," 533
Rayleigh, Lord : moving cyclones, " Atmosphere, Physics of the," 35, 36 ; free periods of atmosphere, *ibid.* 37, 38 ; experiments on jets of water, " Atmospheric Electricity," 105 ; method of determining density of gases, *ibid.* 136 ; manometers, " Barometers and Manometers," 187, 190, " Manometer," 477 ; waves on an elastic solid, " Earthquakes and Earthquake Waves," 289 ; astronomical refraction formula, " Meteorological Optics," 519 ; diffraction theory, *ibid.* 521 ; interferometer, " Oceanography, Physical," 679 ; silvered trumpet, " Quartz Fibres," 698 ; tono-micrometer, " Radio-micrometer and some other Instruments," 723
Regnard : transparency of sea-water, " Oceanography, Physical," 672
Regnault : water-vapour pressure table, " Atmosphere, Thermodynamics of the," 74, " Humidity," 416 ; hygrometer, " Humidity," 417, 418, 424 ; formulae, *ibid.* 419 ; wet-bulb thermometer theory, *ibid.* 423
(Broch and) table, " Humidity," 416
Reich : Cavendish experiment repeated by, " Earth, The Mean Density of the," 281, 283, 285
Reid : measuring machine, " Gauges," 381
Reilly : (Rae and) conditions and methods of use of balance, " Balances," 117, 118
Reimerdes, E. : (Domke and) work by, " Hydrometers," 444

Reiser: *Hardening and Tempering of Steel*, "Gauges," 346
Rietz: rule, "Draughting Devices," 278
Remy: (Josse and) sugar table, "Saccharometry," 726
Richard: reversing water-bottle, "Oceanography, Physical," 675
Richardson, L. F.: opacity of cloud, "Meteorological Optics," 528; radiation from the air, "Radiation," 711
Richarz: (Kriger-Menzel and) horizontal balance beam determinations, "Earth, The Mean Density of the," 281, 285
Richmond, H. D.: hydrometer scales, "Hydrometry," 444
Richter, V.: account of Mendeléeff's researches, "Alcoholometry," 12
Riefler: escapement, "Clocks and Time-keeping," 204, 215, 216; pendulum, *ibid.* 205, 206, 208, 211, 217, 218; microscope for reading arc of oscillation, *ibid.* 223; system of control, *ibid.* 225; clock, change of rate and temperature, *ibid.* 229, 230; analysis of performance, *ibid.* 231
Ritchie, J., and Son: electric control, "Clocks and Time-keeping," 226
Roberval, Prof.: balance, "Weighing Machines," 820, 821
Robinson: attachment, "Clocks and Time-keeping," 209; anemometer, "Meteorological Instruments," 500, 501; Robinson-Beckley anemometers, *ibid.* 501, 502, 507
Roget, Dr.: log-log rule principle, "Draughting Devices," 273
Rooker: Munro-Rooker wind-direction recorder, "Meteorological Instruments," 509, 510
Rotch, L.: investigation of the upper air, "Air, The Investigation of the Upper," 1, 2
Rouch, J.: annual variation of electric field, Antarctic, "Atmospheric Electricity," 96
Rowell, H. S.: effective diameter gauge, "Gauges," 339, 340
Rowland: ruled gratings, "Micrometers," 633, "Radiant Heat and its Spectrum Distribution," 708
Rozhdestvenski: (Doroshevski and) density determination, "Balances," 130
Rubens: multiple junction thermoelement, "Radiant Heat and its Spectrum Distribution," 706
Rudberg: spirit tables, "Alcoholometry," 12
Rudzki, M. P.: earth waves, "Earthquakes and Earthquake Waves," 289
Ruppin: composition of sea-water, "Oceanography, Physical," 666
(Krümmel and) viscosity of sea-water, "Oceanography, Physical," 671; electrical conductivity, *ibid.* 671, 672
Rutherfurd, L. M.: micrometer screw, "Micrometers," 633
Ryder: (Shrader and) manometer, "Barometers and Manometers," 191
Rykatchew, M. (jun.): air temperatures, "Atmosphere, Thermodynamics of the," 55, 58

St. Claire-Deville, H.: platinum-iridium, "Metrology," 587
Sandstrom, J. W.: melting ice and currents, "Oceanography, Physical," 681
(Helland-Hansen and) calculation of current velocity, "Oceanography, Physical," 690, 691
Sanson: (or Sanson-Flamsteed) projection, "Map Projections," 485, 486
Sartorius: attachment of terminal knife-edges, "Balances," 110, 112
Schaffer: (Budenberg and) tachometers, "Meters," 534, 535
Scheel: tables of vapour pressure, "Humidity," 416
(Heuse and) manometer, "Barometers and Manometers," 187
(Thiesen and) density of mercury, "Balances," 131
Scheibler: equation for specific gravity and sucrose content, "Saccharometry," 726; specific gravity table, *ibid.* 727
(Mategezeek and) specific gravity table, "Saccharometry," 727
Scheiner: halo, "Meteorological Optics" 533
Schilt, V.: adding machine, "Calculating Machines," 199
Schindelhauer, F.: electricity of rain, "Atmospheric Electricity," 101
Schloesing: determination of density of gases, "Balances," 137; hygroscopic curves, "Humidity," 428
Schloesser, W.: Möhr system of units, "Volume, Measurements of," 779 *n.*; method of using pipettes, *ibid.* 797, 798, 801, 802
Schlüter: "Earthquakes and Earthquake Waves," 290
Schmidt, W.: waves in surfaces of discontinuity, "Atmosphere, Physics of the," 38; audibility of thunder, *ibid.* 43; interchange of air masses, "Radiation," 713
Schneider, E.: pendulum apparatus, "Gravity Survey," 399
Schönrock: sugar formula, "Saccharometry," 726
Schott, G.: temperature of Western Pacific, "Oceanography, Physical," 684
Schulz, B.: paper by, "Oceanography, Physical," 673; vertical equilibrium of sea-water, *ibid.* 683
Schulze: density of vapours, "Balances," 137
Schultz: extended parhelia, "Meteorological Optics," 532
Schumann: rigidity correction, "Gravity Survey," 401
Schuster, Sir A.: oscillation effects on the barometer at sea, "Gravity Survey," 404
Schwackhöfer: vapour pressure apparatus, "Humidity," 427
Schwartzschild: radiative equilibrium, "Atmosphere, Physics of the," 26 *n.* 4
Schweidler, E. von: fine-weather field theories criticised by, "Atmospheric Electricity," 97
Sears, J. E. (jun.): comparator, "Gauges," 385, 386, 387; "envelope" in gauging, "Metrology," 621; chronograph, "Volume, Measurements of," 814, 815
(Glazebrook, Brookes, and) method of making block gauges, "Gauges," 301 *n.*, "Metrology," 621
Sellers: thread, "Gauges," 347, 348, "Metrology," 606, 607
Shaw, J. J.: Milne-Shaw seismograph, "Seismometry," 736, 741
Shaw, P. E.: temperature variation of "G," "Earth, The Mean Density of the," 283, "Gravity Survey," 413; screw-measuring machine (1919), "Gauges," 332, 333, 334; measuring machine (1912), *ibid.* 362, 375, 376, 377, "Metrology," 628
Shaw, Sir W. N.: persistence of moving cyclones, "Atmosphere, Physics of the," 35, 36; *Forecasting Weather* by, *ibid.* 39; gravimetric method of finding vapour pressure, "Humidity," 426, 427; volumetric method, *ibid.* 427
(Cave and) formula for variation of wind with height, "Atmosphere, Physics of the," 32
(Dines and) microbarograph, "Barometers and Manometers," 148, "Microbarograph," 633
Sheepshanks: comparison of length standards, "Metrology," 591
Shortt, W. H.: synchronome escapement, "Clocks and Time-keeping," 214, 215
Shrader: (Ryder and) manometer, "Barometers and Manometers," 191
Sicklen, Van: chronometric instrument, "Meters," 537, 536
Sidersky: sugar tables, "Saccharometry," 725, 726, 729
Sikes, B.: spirit tables, "Alcoholometry," 12, 17, 18; hydrometer, *ibid.* 16, 17, 18, 19, "Hydrometers," 432, 438, 443
Simpson, G. C.: Antarctic observations, "Air, The Investigation of the Upper," 7; raindrops and thunderstorm electricity, "Atmosphere, Physics of the," 43, "Atmospheric Electricity," 97, 101, 103; variations, "Atmospheric Electricity," 92, 99; theories of fine-weather field criticised, *ibid.* 97; theory of field of thunderstorms, *ibid.* 105; electricity of snow, *ibid.* 105 *n.*; rule, "Integration, Mechanical Methods of," 451; iridescent clouds, "Meteorological Optics," 527
Smethurst: humidifier, "Humidity," 430
Smith: calculating typewriter, "Calculating Machines," 200
Smith, F. E.: arrangement for maintaining vibration, "Clocks and Time-keeping," 222
Smits: manostat, "Barometers and Manometers," 192
Sorean: formula connecting pressure and height, "Barometers and Manometers," 182

Soret: hypothesis of secondary diffraction, "Meteorological Optics," 523; paranthelia, *ibid.* 533
Southern, L.: temperature and gravitation, "Gravity Survey," 413
Soxhlet: lactometer, "Hydrometers," 444
Speer: hydrometer, "Alcoholometry," 16
Speranski: manostat, "Barometers and Manometers," 192
Sprengel: pyknometer, "Balances," 128
Squibb: alcoholic densities, "Alcoholometry," 12
Stanley, W. F.: bow curve, "Draughting Devices," 265; drawing table, *ibid.* 269
Stanton, T. E.: flow round a cylinder, "Humidity," 419; steam meter calibration, "Meters," 554
Stas: pipette, "Volume, Measurements of," 803, 804
Stebbins: "Radiation, The Measurement of," 718
Steele: (Grant and) micro-balances, "Balances," 120, 121
(Ramsay and) determination of vapour densities, *ibid.* 137
Stefan: conductivity of air, "Humidity," 421; Stefan-Boltzmann law, "Radiation," 711, 714
Steiger, O.: Millionaire Machine, "Calculating Machines," 200
Stenius, S.: osmotic pressure of sea-water, "Oceanography, Physical," 670, 671
Sterneck, von: colliery experiments on "G," "Earth, The Mean Density of the," 280, 285; apparatus, "Gravity Survey," 399
Stevenson, T.: screen, "Meteorological Instruments," 490, 491, 493
Stewart: tachometer, "Meters," 537, 538
Stöckle: surface tension of mercury, "Barometers and Manometers," 159 *n.*
Stokes, Sir G. G.: friction researches, "Clocks and Time-keeping," 204; formula, "Gravity Survey," 411; polarisation, "Meteorological Optics," 523; sunshine recorder "Radiant Heat and its Spectrum Distribution," 699, 700; Campbell-Stokes sunshine recorder, "Meteorological Instruments," 188, 510, 511, 512, 513
Stroudley: tachometer, "Meters," 537
Sverdrup, H. U.: (Hesselberg and) eddy diffusivity, "Atmosphere, Physics of the," 34, 35; stability of sea-water, "Oceanography, Physical," 683; tables, *ibid.* 691
Swann, W. F. G.: measurement of conductivity, "Atmospheric Electricity," 94; corpuscular radiation, *ibid.* 97; Ebert theory criticised, *ibid.* 97
(Bauer and) land and ocean values, "Atmospheric Electricity," 95, 96

Tagliabue: hydrometer, "Alcoholometry," 16
Talcott: determination of latitude, "Gravity Survey," 407, 408, "Latitude, Longitude, and Azimuth," 462; latitude observations, "Trigonometrical Heights," 773, 774
Tate: arithmometer, "Calculating Machines," 198
Taylor, C. H.: (Fowler, Alpe, and) gauge for plug screws, "Gauges," 343, 344
Taylor, G. I.: variation of wind with height, "Atmosphere, Physics of the," 32; eddy motion, *ibid.* 32, 33, "Eddy," 294, "Oceanography, Physical," 686; eddy diffusivity, "Atmosphere, Physics of the," 32, 33, 35; theory of wet bulb thermometer in moving air, "Humidity," 423; friction of wind over grass, "Oceanography, Physical," 680
Taylor, T. S.: quartz balance, "Balances," 121, 122; density of helium, *ibid.* 121, 137
Taylor, W.: screw gauges, "Gauges," 298; tests by plug checks, *ibid.* 327, "Metrology," 600; gap gauge for screws, "Gauges," 339; expanding effective diameter plug screw gauge, *ibid.* 340; "scissor" gauge, *ibid.* 340, 341
(Taylor, Hobson, and) floating micrometer measuring machine, "Gauges," 319
Thachers: rule, "Draughting Devices," 279
Thales: gnomonic projection, "Map Projections," 483
Theodorich: rainbow, "Meteorological Optics," 521
Thiesen, M.: "Balances," 113; density tables, *ibid.* 131
(Scheel and) density of mercury, "Balances," 131
Thomas, C. C.: electrical gas meter, "Meters," 561, 562, 563
Thomas, C. X.: arithmometer, "Calculating Machines," 195, 196, 197, 198
Thomson, Hedley: limit gauges, "Metrology," 615, 616
Thomson, James: water-vapour pressure and change of state, "Humidity," 416; notch meter formula, "Meters," 569
Thomson, Sir J. J.: thunderstorms and condensation effects, "Atmospheric Electricity," 104
Thorade, H.: relation between the velocities of wind and current, "Oceanography, Physical," 680, 681
Thorpe, Sir E.: revised spirit tables, "Alcoholometry," 12, 13, 17; vapour density determinations, "Balances," 137
Thoulet: (Chevalier and) "Oceanography, Physical," 670
Threlfall, R.: determination of density of gases, "Balances," 137
(Pollock and) gravity balance, "Gravity Survey," 403
Thuras, A. L.: float method, "Balances," 129, 130
Tiede: clock, "Clocks and Time-keeping," 204, 213
Tippler: water flow meter, "Meters," 570
Tissot: projection, "Map Projections," 485
Tomlinson, G. A.: testing of ring gauges, "Gauges," 304, 334
Tourpaian: (Jaquerod and) determination of density of gases, "Balances," 137
Toussaint: exponential altimeter formula, "Barometers and Manometers," 182, "Temperature, Atmospheric," 756
Trabert: account of Margules' work, "Atmosphere, Physics of the," 38 *n.* 1
Tralles: spirit tables compiled by, "Alcoholometry," 11; hydrometer, *ibid.* 19; report by, "Volume, Measurements of," 777
Travers: (Ramsay and) density of gases, "Balances," 136
Tresca, G.: section, "Line Standards of Length," 467, "Metrology," 587
Trouton, F. T.: hygroscopic properties of wool, "Humidity," 428
Tutton, A. E. H.: wave-length comparator, "Comparators," 241, 242, 243, "Line Standards of Length," 468, "Metrology," 631; interferometer, "Comparators," 241, 243; line standard, "Metrology," 575
Twaddell: hydrometer, "Saccharometry," 730
Tyndall: (Mayo and) volumetric humidity apparatus, "Humidity," 427, 428

Ulloa ring, "Meteorological Optics," 526, 528
Underwood: calculating typewriter, "Calculating Machines," 200

Valentine, B.: deflagration method, "Alcoholometry," 9
Vauraborg, C.: refractive index of sea-water, "Oceanography, Physical," 672
Veazey, B. H.: (Osborne and) testing of flasks, "Volume, Measurements of," 793 *n.* 2, 795, 796
Venturi: tube meter, "Meters," 557, 558, 559; water meter, *ibid.* 567, 568, 569
Vicentini, Dr.: seismograph, "Seismometry," 735
Vick, De (or Wieck): clock, "Clocks and Time-keeping," 203
Vieths: lactometer, "Hydrometers," 444
Vincent: density of ice, "Balances," 135
Violle: actinometer, "Radiant Heat and its Spectrum Distribution," 701, 702
Vivien: sugar tables, "Saccharometry" 727
Vollhard: chlorine determination, "Oceanography, Physical," 677
Volquartz: hydrometer, "Saccharometry," 731
Volta: measurement by collector, "Atmospheric Electricity," 89
Vosatka: hydrometer, "Saccharometry," 731

Wade: (Merriman and) manostat, "Barometers and Manometers," 192
Wagner, A.: temperature in upper air, "Atmosphere, Thermodynamics of the," 55, 58
Wahl: calculating typewriter, "Calculating Machines," 200
Walker, G. T.: correlation coefficients, "Radiation," 708
Walker, G. W.: earth surface reflected waves, "Earthquakes and Earthquake Waves," 293
Walker, J. T.: coefficient of refraction, "Trigonometrical Heights," 769
Warner: tachometer, "Meters," 537, 538

Watson: upper air experiments, "Air, The Investigation of the Upper," 1
Watson, W.: (Boys and) stellar observations with radio-micrometer, "Radio-micrometer and some other Instruments," 722
Weiser: vapour density determination, "Balances," 137
Werner: projection, "Map Projections," 486
Westphal: hydrostatic balance, "Alcoholometry," 10, "Balances," 129
Wharton: (Field and) current meters, "Oceanography, Physical," 676
Wheatstone: electric control, "Clocks and Time-keeping," 226
Whiddington: (Dowling and) ultra-micrometer, "Metrology," 631
Whipple, F. J. W.: height and pressure tables, "Barometers and Manometers," 182; spirit thermometers, "Meteorological Instruments," 402; on Robinson anemometer, *ibid.* 500, 501
Whipple, G. M.: black-bulb thermometer and sunshine recorder, "Radiant Heat and its Spectrum Distribution," 702
Whipple, R. S.: paper by, "Meteorological Instruments," 514, "Radiant Heat and its Spectrum Distribution," 699, 708, "Radiation, The Measurement of," 716; radio integrator improvement, "Radiant Heat and its Spectrum Distribution," 703
White: (Bingham and) viscosity of pure water, "Oceanography, Physical," 671
Whitney: (Pratt and) measuring machine, "Gauges," 367, 368, 369, 370
Whittaker, E. T.: nomogram, "Nomography," 641
Whitworth, Sir J.: gauges, "Gauges," 298, 299, "Metrology," 596, 600; screws, standard thread, "Gauges," 347, 350, "Metrology," 596, 606, 607
Wiebe: vapour pressure table, "Humidity," 416
Wiechert, E.: seismograph, "Seismometry," 740
(Marvin and) inverted pendulum, "Seismometry," 736
(Zöppritz and) time graphs, "Earthquakes and Earthquake Waves," 287, 288; waves in the interior of the earth, *ibid.* 290, 293
Wien: theory of waves in surfaces of discontinuity, "Atmosphere, Physics of the," 38
Wiernick: (Lunge and) ammonia density, "Balances," 130
Wilsing: metronome balance determinations, "Earth, The Mean Density of the," 284, 285
Wilson, Dr.: Lovi's beads devised by, "Alcoholometry," 16
Wilson, R. P.: projection comparator, "Gauges," 360, 361, 362
Wilson, W. E.: radio integrator, "Radiant Heat and its Spectrum Distribution," 702; temperature of the sun, "Radiation, The Measurement of," 717; "Radio-micrometer and some other Instruments," 722
Winstanley: radiograph, "Radiant Heat and its Spectrum Distribution," 702
Witting, R.: relation between the velocities of wind and current, "Oceanography, Physical," 580
Wolff, A. J.: work by, "Map Projections," 481
Wologdine: (Le Chatelier and) density and purity of graphite, "Balances," 135
Wood, R. W.: diffraction experiments, "Meteorological Optics," 522
Worthington: displacement meter, "Meters," 564; turbine meter, *ibid.* 566
Wright, E.: theory of Mercator's projection, "Map Projections," 484
Wust: evaporation of sea-water, "Oceanography, Physical," 671

Yorke: notch meter, "Meters," 570
Young, A. E.: projection, "Map Projections," 481, 486
Young, T.: on rainbow, "Meteorological Optics," 524

Zeetzmann: "Calculating Machines," 198
Zehnder: volumenometer, "Balances," 135
Zeigler: contraction of sugar solutions, "Saccharometry," 726
Zöllner: seismograph, "Seismometry," 736
Zöppritz, K.: (Geiger, Gutenberg, and) earth surface reflected waves, "Earthquakes and Earthquake Waves," 292, 293
(Wiechert and) time graphs, *ibid.* 287, 288, 290

Printed in Great Britain by R. & R. CLARK, LIMITED, *Edinburgh.*